27th LHC Mini-Workshop

PhaseTracer 2

From effective potential to transition properties

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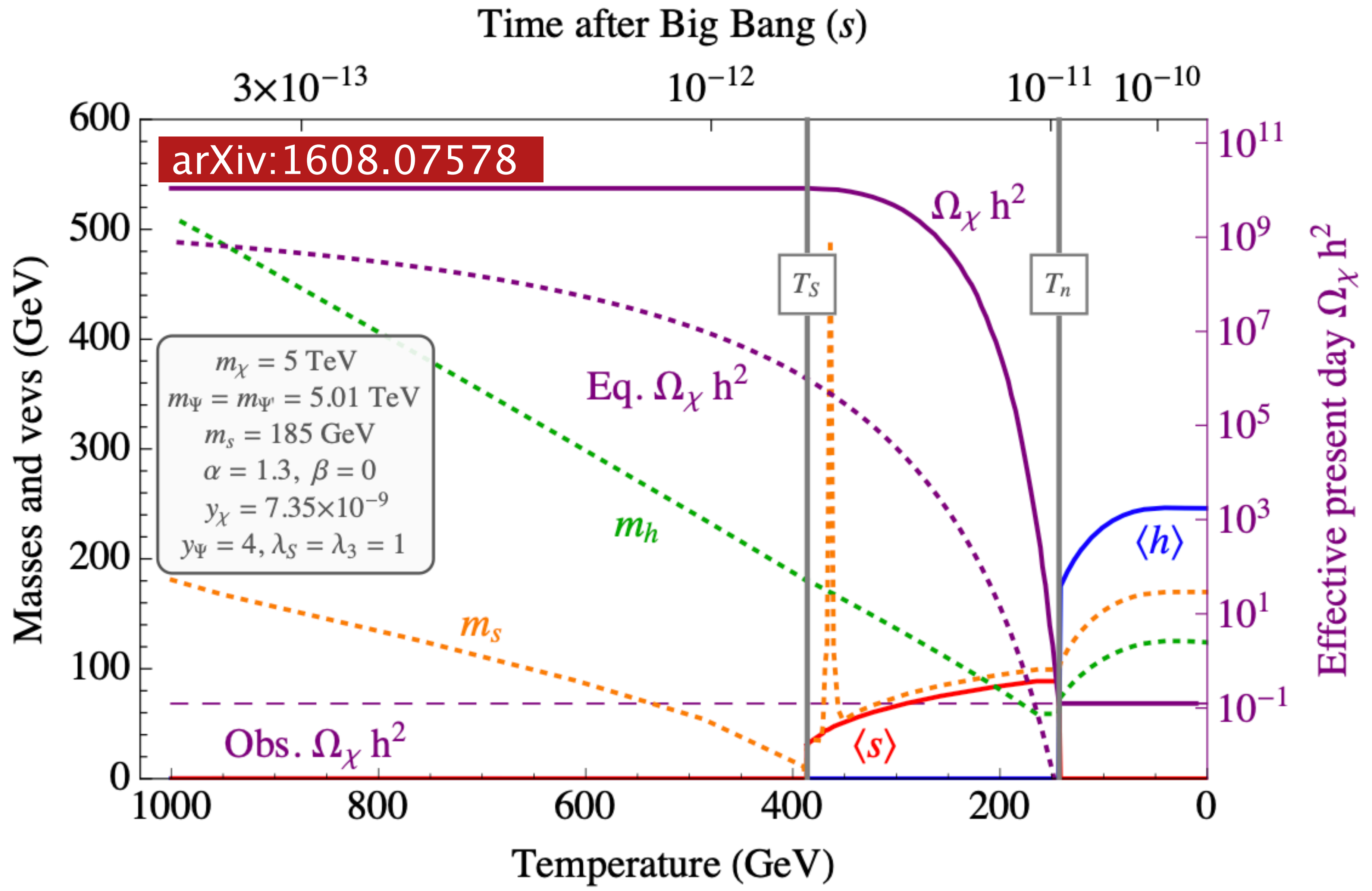
Motivation

➤ Evolution of the Higgs effective potential in the early universe

- Electroweak phase transition
- Stochastic gravitational wave background
- Baryon asymmetry of the Universe
- Higgs vacuum stability
- Dark matter relic density
- ……



➤ Find minimums of potential as function of temperature — PhaseTracer

PhaseTracer 1

build unknown | license GPL-3.0 | arXiv 2003.02859

PhaseTracer

arXiv:2003.02859

`PhaseTracer` is a C++ software package for mapping out cosmological phases, and potential transitions between them, for Standard Model extensions with any number of scalar fields.

Building

To build the shared library and the examples:

```
mkdir build
cd build
cmake ..
make
```

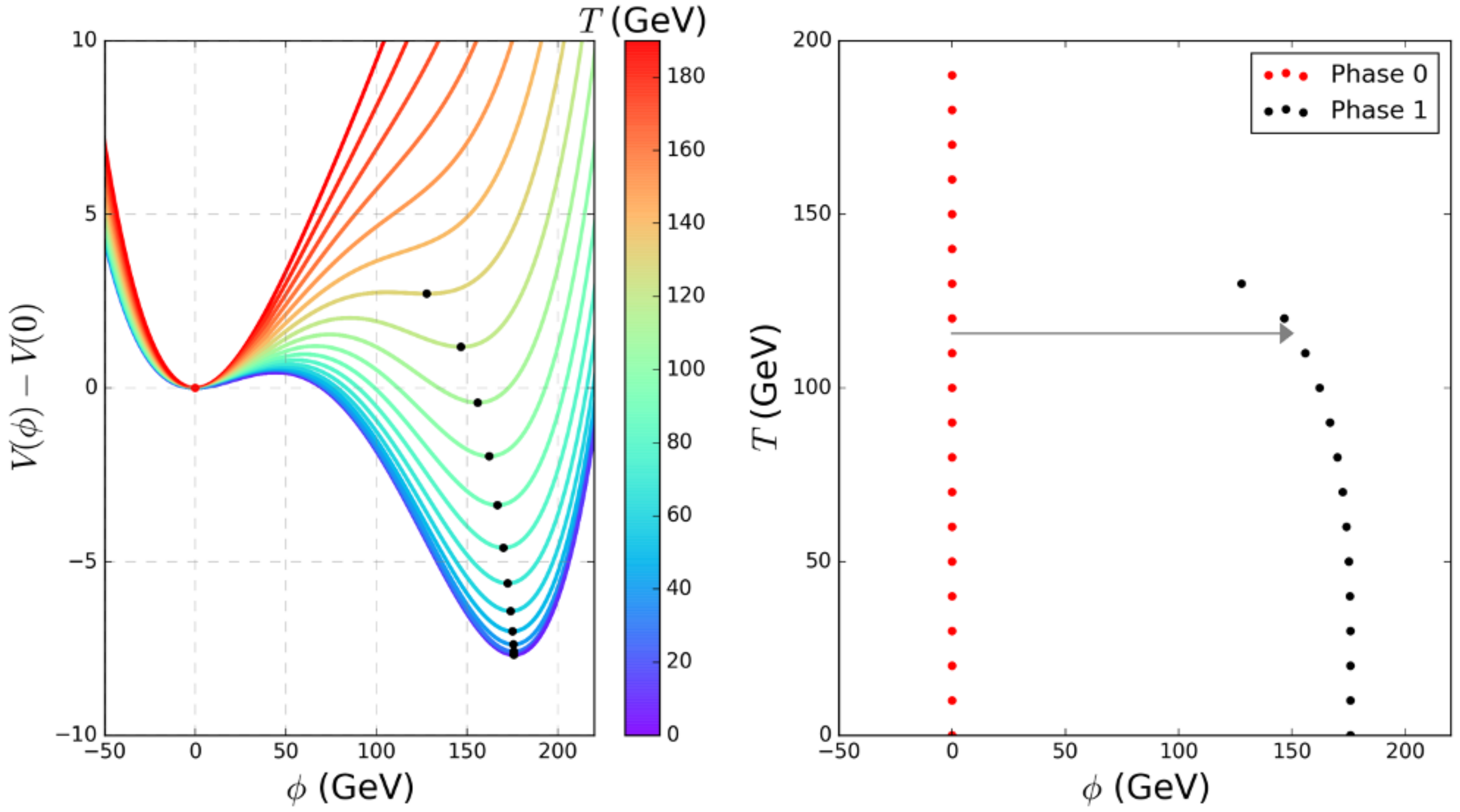
Running

If the build was succesful, run the examples with:

```
cd ..
./bin/run_1D_test_model
./bin/run_2D_test_model
./bin/scan_Z2_scalar_singlet_model
```

Input: effective potential

- $\overline{\text{MS}}$ scheme of Landau gauge
- High temperature approximation (HT)



Output: phases

- Critical temperature T_C
- Transition strength $\gamma = \phi / T_C$

PhaseTracer

- Method 1
  Analytic calculation
- Method 2
- Method 3

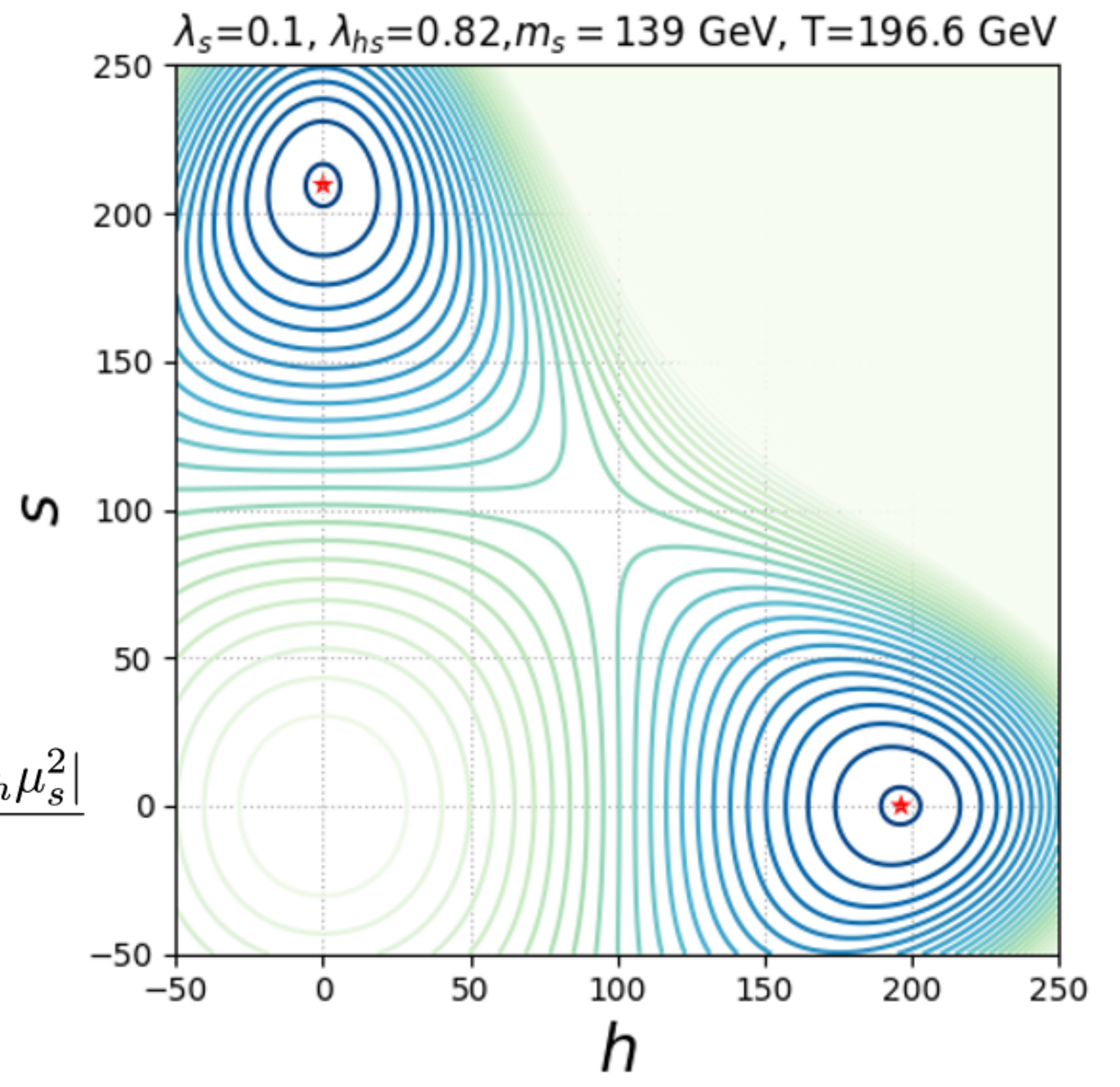
➤ For simplified potential,

$$V(H,s,T)=(\mu_h^2+c_hT^2)H^\dagger H+\lambda_h(H^\dagger H)^2+\frac{\lambda_{hs}}{2}(H^\dagger H)s^2+\frac{(\mu_s^2+c_sT^2)}{2}s^2+\frac{\lambda_s}{4}s^4$$

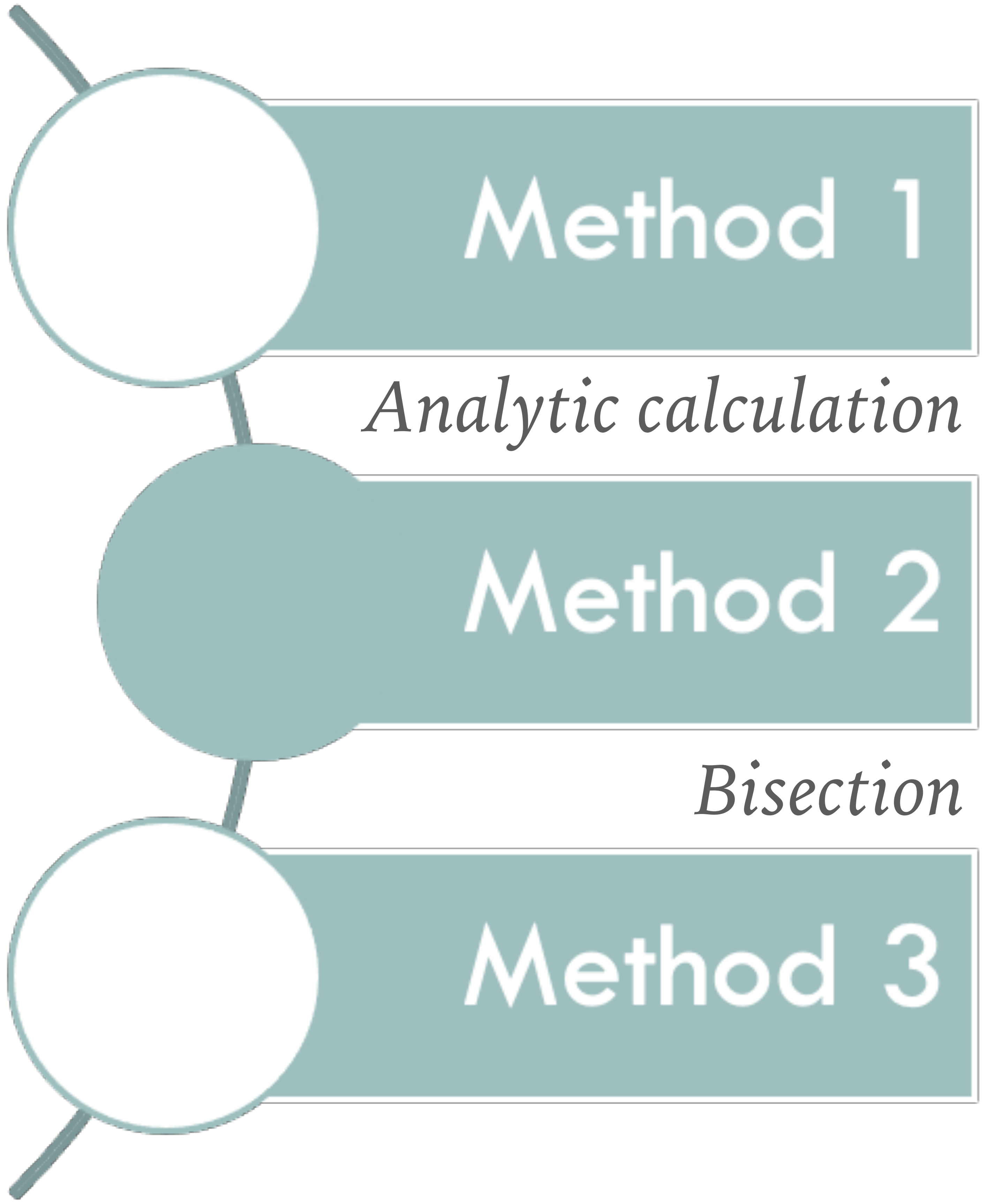
$$\left.\frac{\partial V}{\partial v}\right|_{v_h,v_s=0}=\left.\frac{\partial V}{\partial v}\right|_{v_s,v_h=0}=0$$

$$V(v_h,0,T_c)=V(0,v_s,T_c)$$

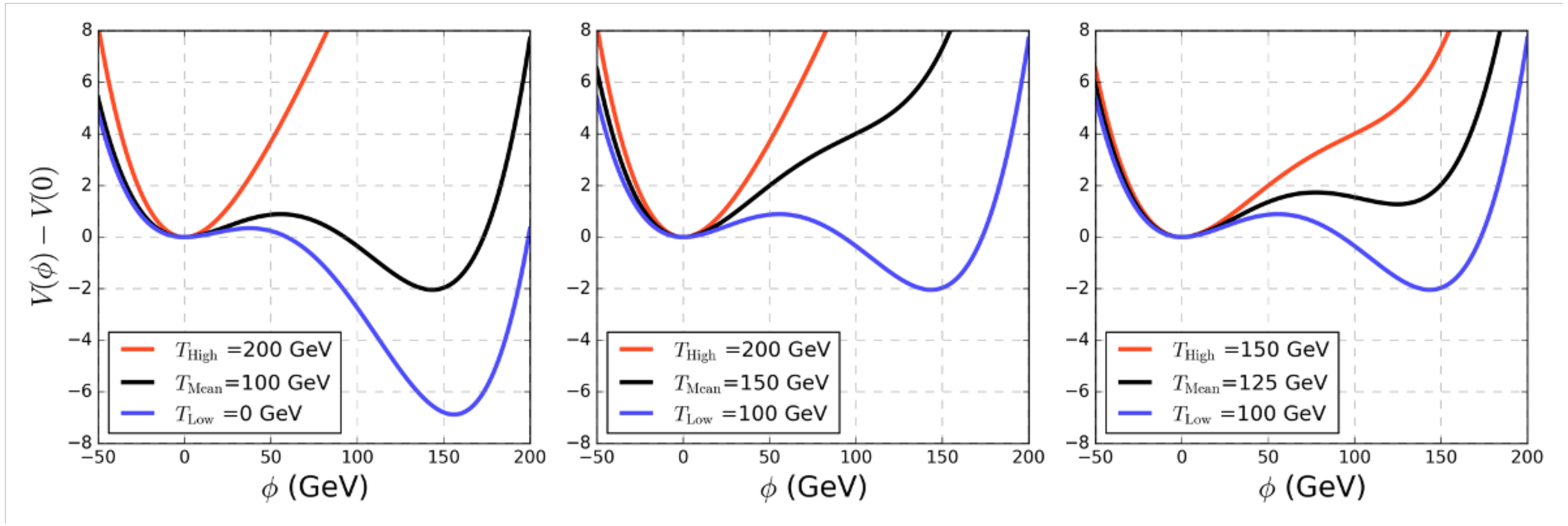
$$T_c^2=\frac{\lambda_s c_h\mu_h^2-\lambda_h c_s\mu_s^2-\sqrt{\lambda_h\lambda_s}|c_s\mu_h^2-c_h\mu_s^2|}{\lambda_s c_h^2-\lambda_h c_s^2}$$



PhaseTracer



- Starting with an un-broken phase at high temperature and a broken phase at low temperature, find a temperature that these two phase degenerate using bisection method.



- Used in BSMPT (arXiv: 1803.02846, 2007.01725)

PhaseTracer

Method 1

Analytic calculation

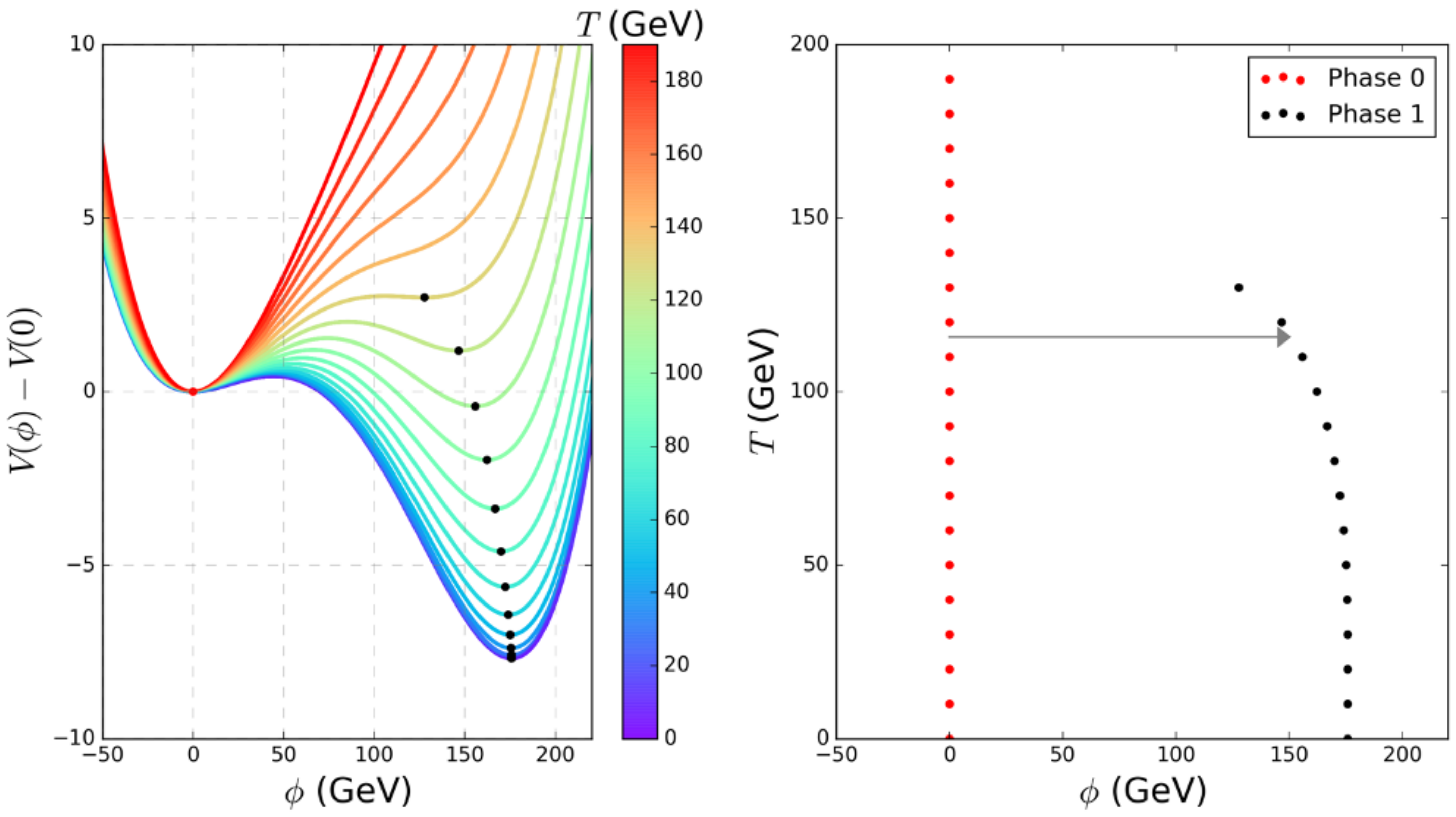
Method 2

Bisection

Method 3

Trace minimums

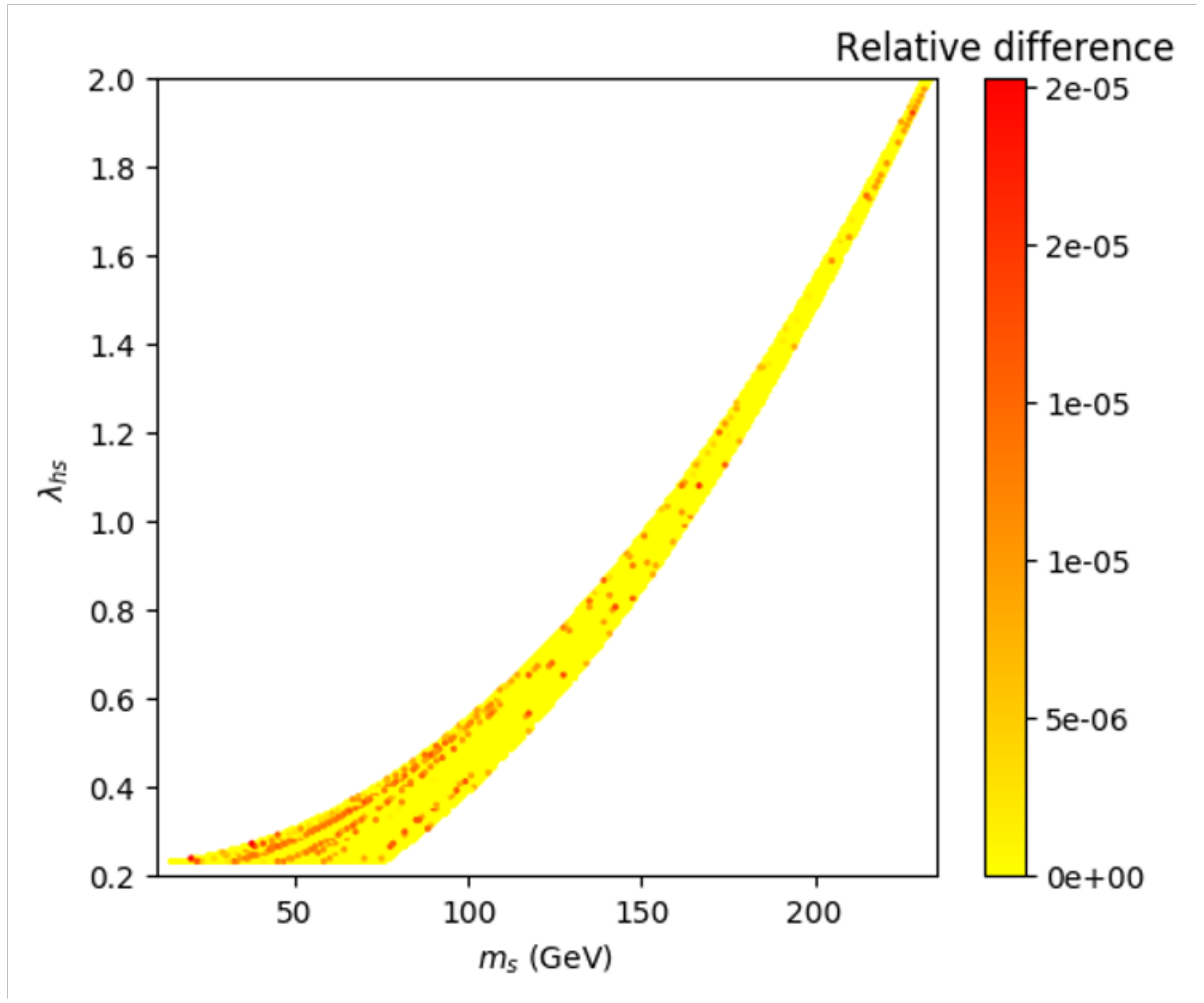
- Starting with a minimum at low/high temperature, trace the location of minimum at different temperatures.

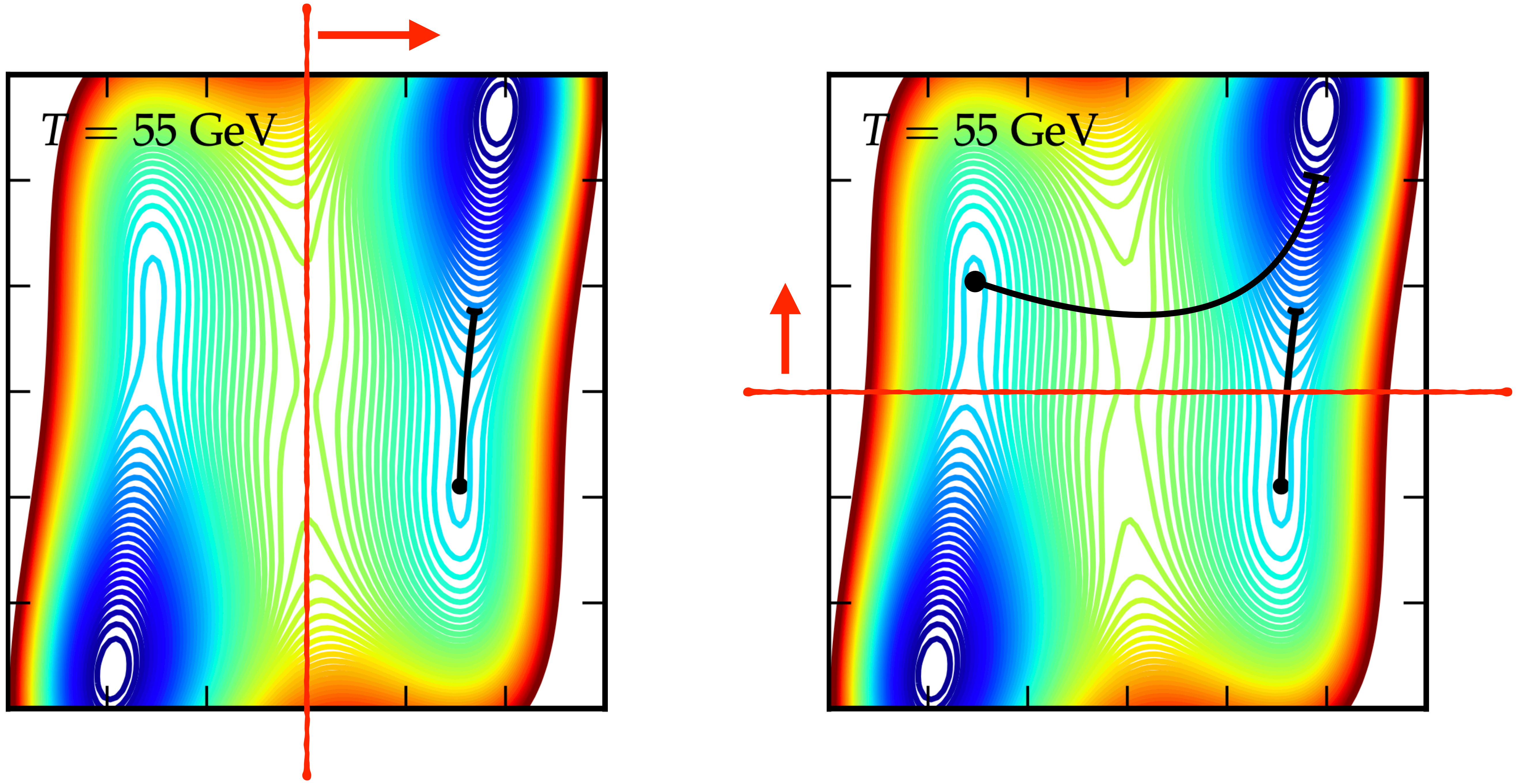


- Used in CosmoTransitions (arXiv:1109.4189) and PhaseTracer (arXiv:2003.02859).

PhaseTracer vs. CosmoTransitions

	2D simplified potential	NMSSM
CosmoTransitions	4.97 seconds	4 minutes
PhaseTracer	0.063 second	8.9 seconds





PhaseTracer 1 vs. PhaseTracer 2

- PhaseTracer 2 : More flexible input and more outputs
- Effective potential in PhaseTracer 1
  - $\overline{\text{MS}}$ scheme
  - R_ξ gauge
  - Arnold-Espinosa approach
  - High temperature approximation (HT)

$$V_{\text{eff}} = V^{\text{tree}} + \Delta V^{\text{1-loop}} + \Delta V_T^{\text{1-loop}} + V_{\text{daisy}}.$$

$$\begin{aligned}\Delta V^{\text{1-loop}} = \frac{1}{64\pi^2}\Bigg(&\sum_\phi n_\phi m_\phi^4(\xi)\left[\ln\left(\frac{m_\phi^2(\xi)}{Q^2}\right) - 3/2\right] \\ &+ \sum_V n_V m_V^4 \left[\ln\left(\frac{m_V^2}{Q^2}\right) - 5/6\right] \\ &- \sum_V \tfrac{1}{3} n_V (\xi m_V^2)^2 \left[\ln\left(\frac{\xi m_V^2}{Q^2}\right) - 3/2\right] \\ &- \sum_f n_f m_f^4 \left[\ln\left(\frac{m_f^2}{Q^2}\right) - 3/2\right]\Bigg).\end{aligned}$$

$$V_{\text{daisy}} = -\frac{T}{12\pi}\left(\sum_\phi n_\phi \left[\left(\overline{m}_\phi^2\right)^{\frac{3}{2}} - \left(m_\phi^2\right)^{\frac{3}{2}}\right] + \sum_V \tfrac{1}{3} n_V \left[\left(\overline{m}_V^2\right)^{\frac{3}{2}} - \left(m_V^2\right)^{\frac{3}{2}}\right]\right)$$

PhaseTracer 1 vs. PhaseTracer 2

- PhaseTracer 2 : More flexible input and more outputs
- Effective potential in PhaseTracer 2
  - $\overline{\text{MS}}$ scheme, on-shell like scheme
  - R_ξ gauge, covariant gauge
  - Arnold-Espinosa approach, Parwani approach
- High temperature approximation (HT), PRM
- 3D EFT potential

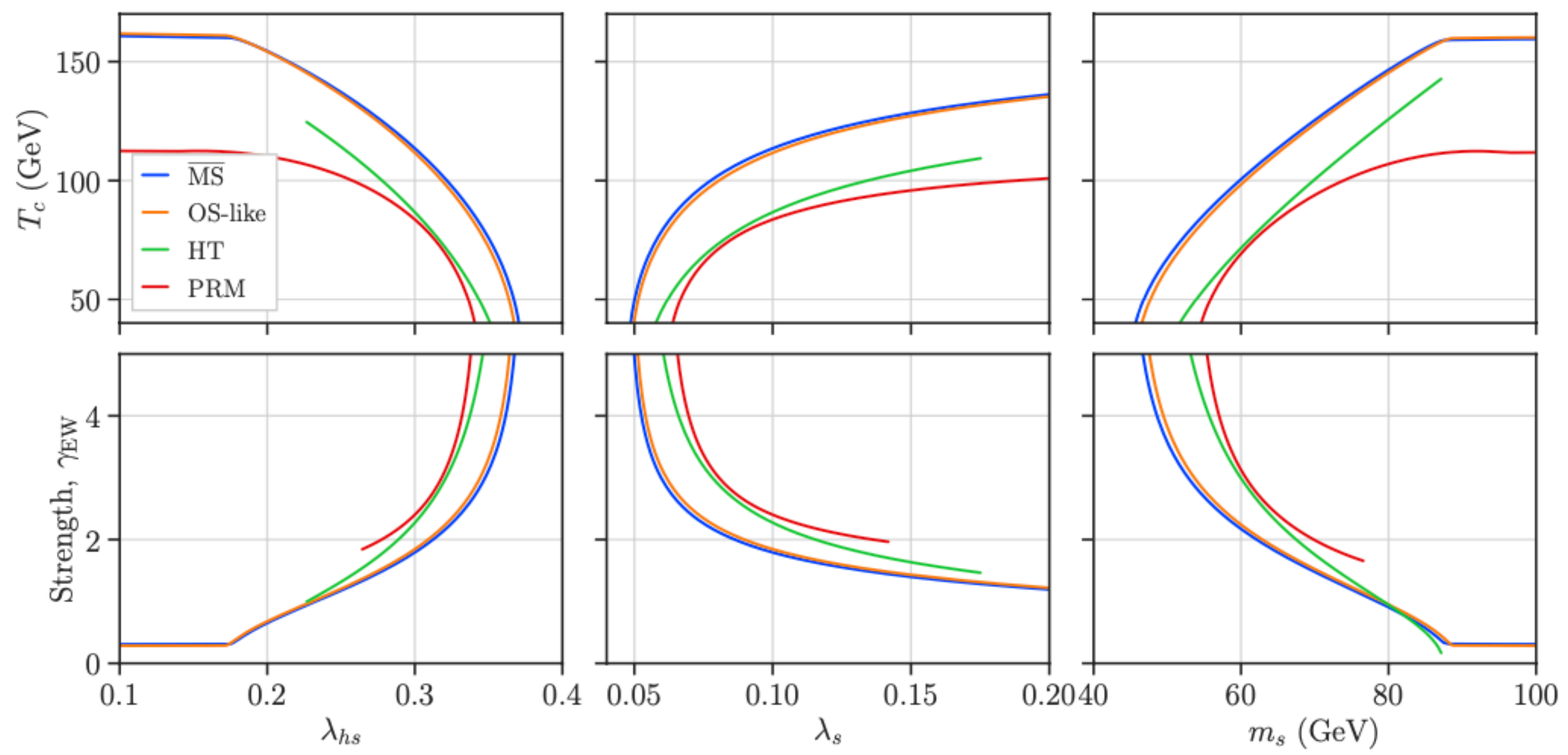
$$V_{\rm CW}(\boldsymbol{\phi}) = \frac{1}{64\pi^2}\sum_i \tilde{n}_i \left[m_i^4(\boldsymbol{\phi})\left(\log\left(\frac{m_i^2(\boldsymbol{\phi})}{m_i^2(\boldsymbol{v})}\right) - \frac{3}{2}\right) + 2m_i^2(\boldsymbol{v})m_i^2(\boldsymbol{\phi}) - \frac{1}{2}m_i^4(\boldsymbol{v})\right]$$

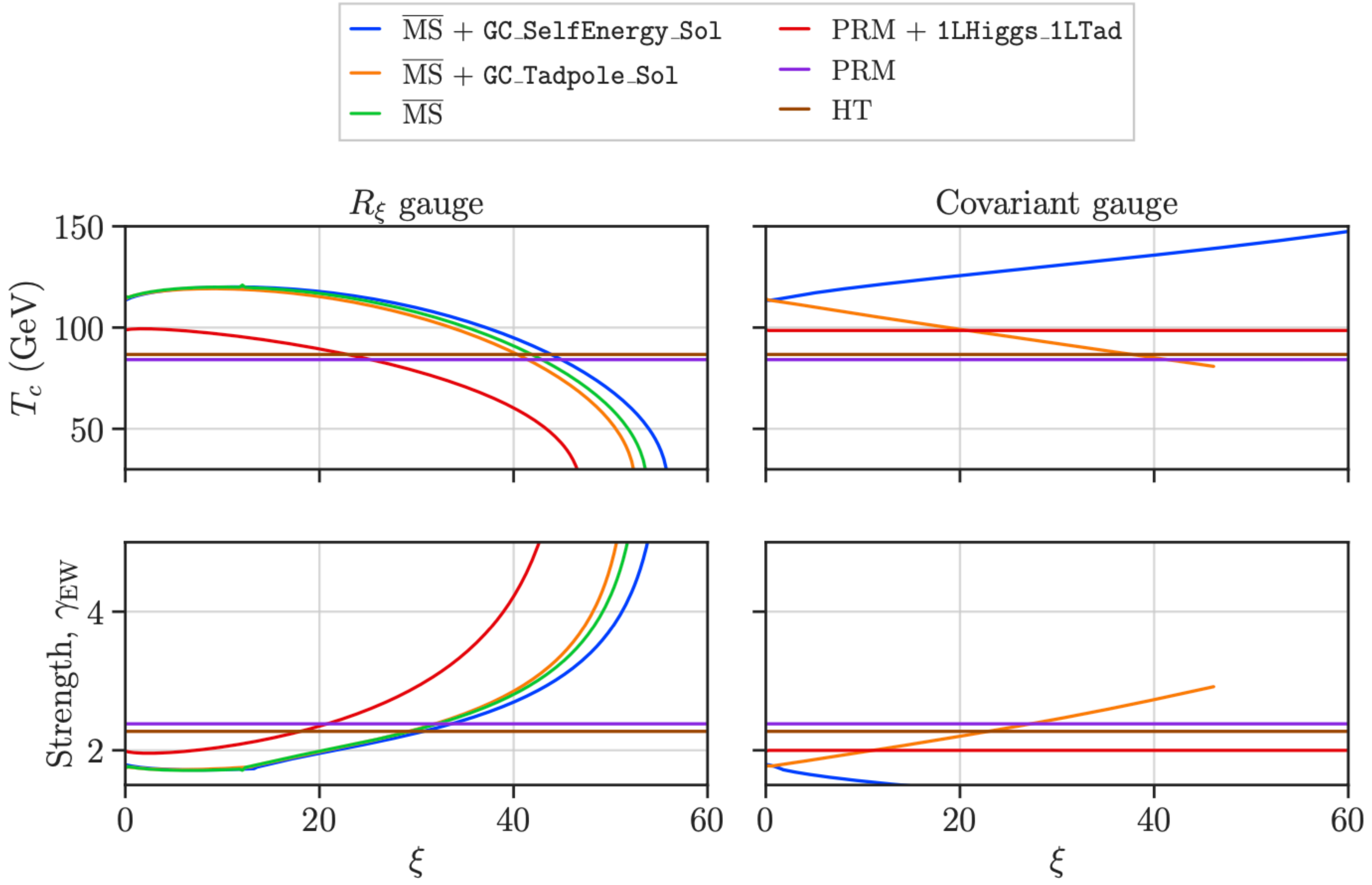
$$m_{1,\pm}^2 = \frac{1}{2}\left(\chi \pm \mathrm{Re}\sqrt{\chi^2 - \Upsilon_W}\right)$$

$$m_{2,\pm}^2 = \frac{1}{2}\left(\chi \pm \mathrm{Re}\sqrt{\chi^2 - \Upsilon_Z}\right)$$

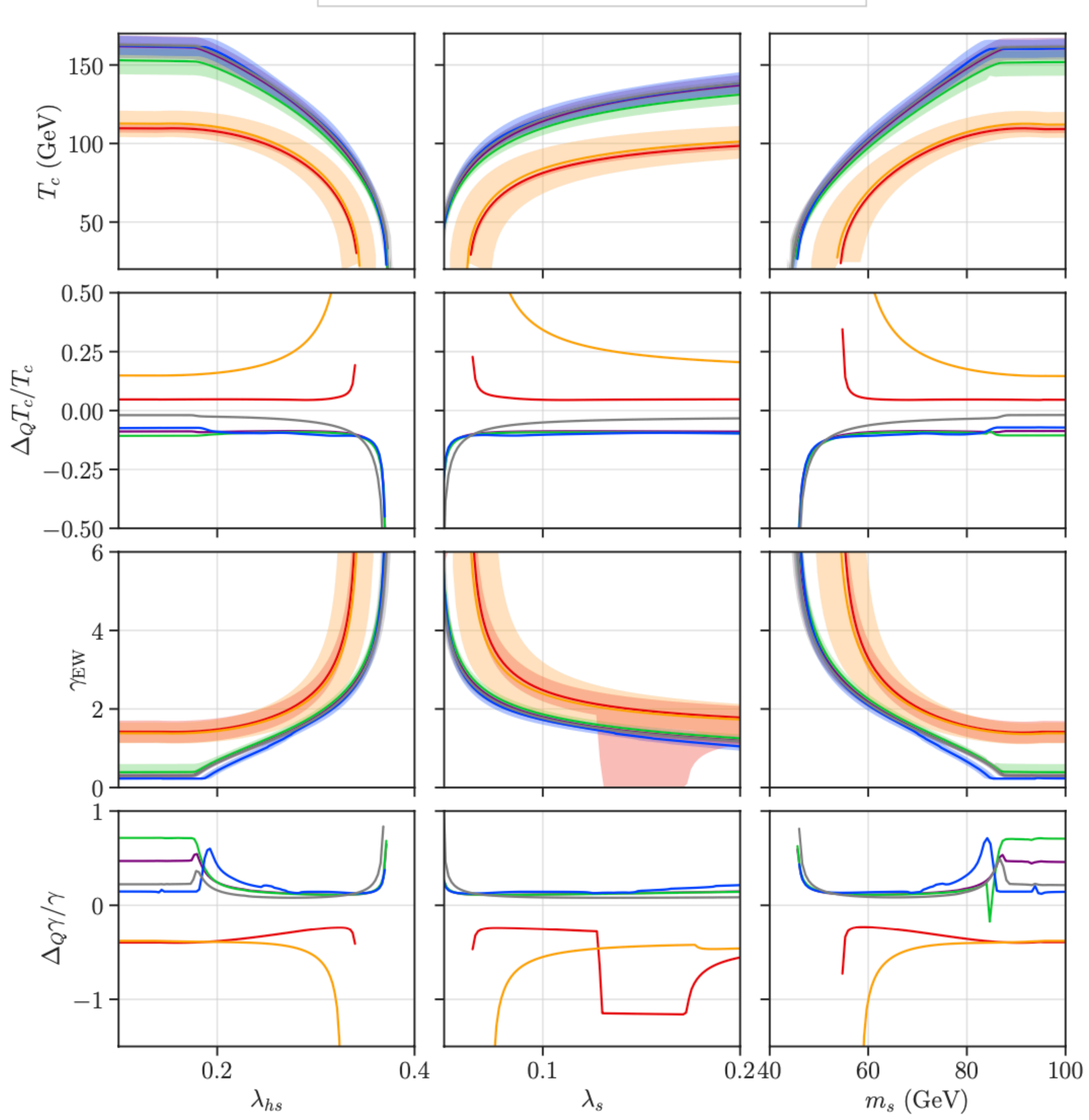
$$V_{\rm HT}(\boldsymbol{\phi}, T) = V_0(\boldsymbol{\phi}) + \frac{1}{2}c_h T^2 \phi_h^2 + \frac{1}{2}c_s T^2 \phi_s^2$$

PhaseTracer 1 vs. PhaseTracer 2





arXiv:2208.01319

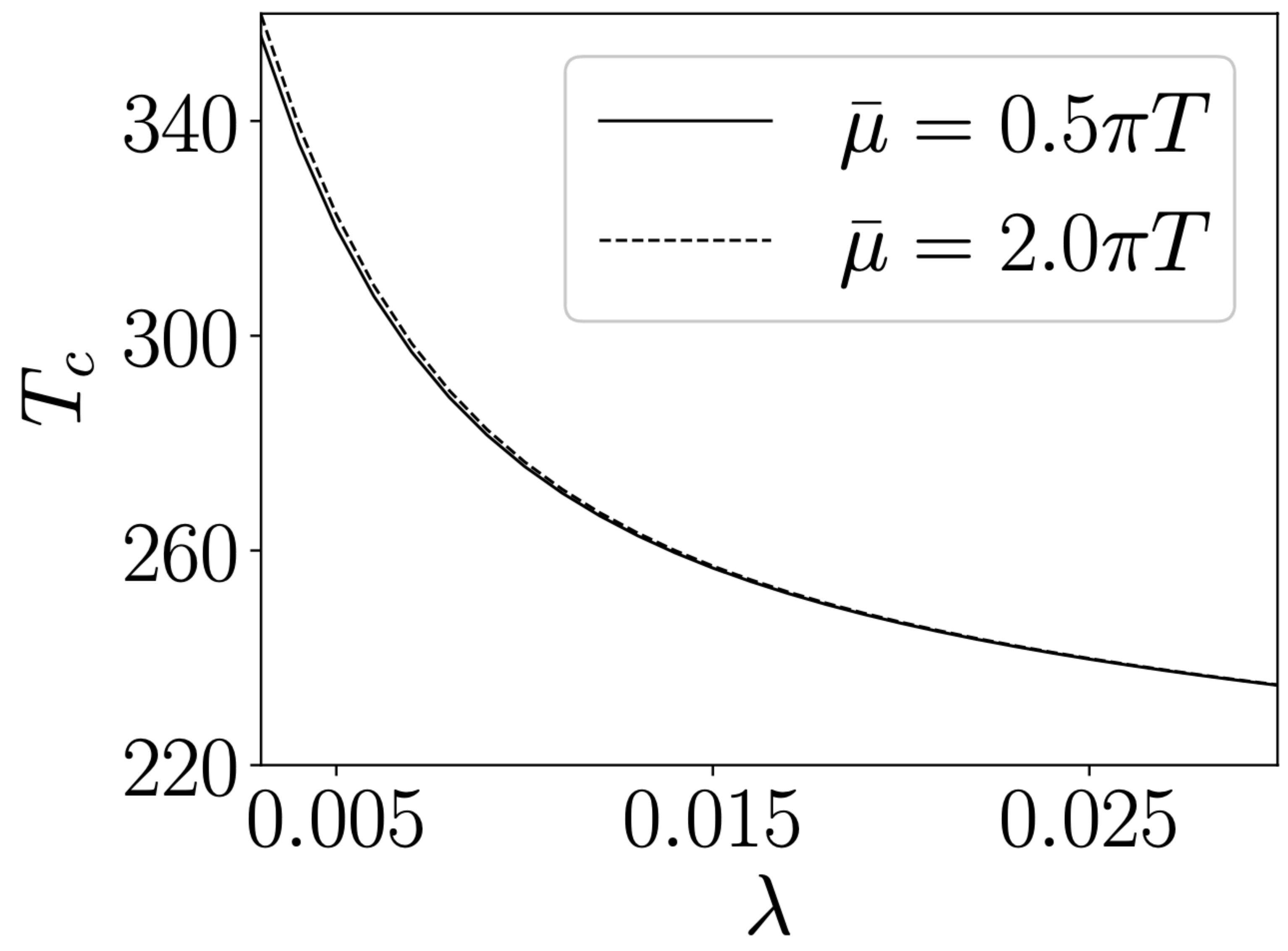


arXiv:2205.08815

PhaseTracer 1 vs. PhaseTracer 2

➤ 3D EFT potential (DRalgo)

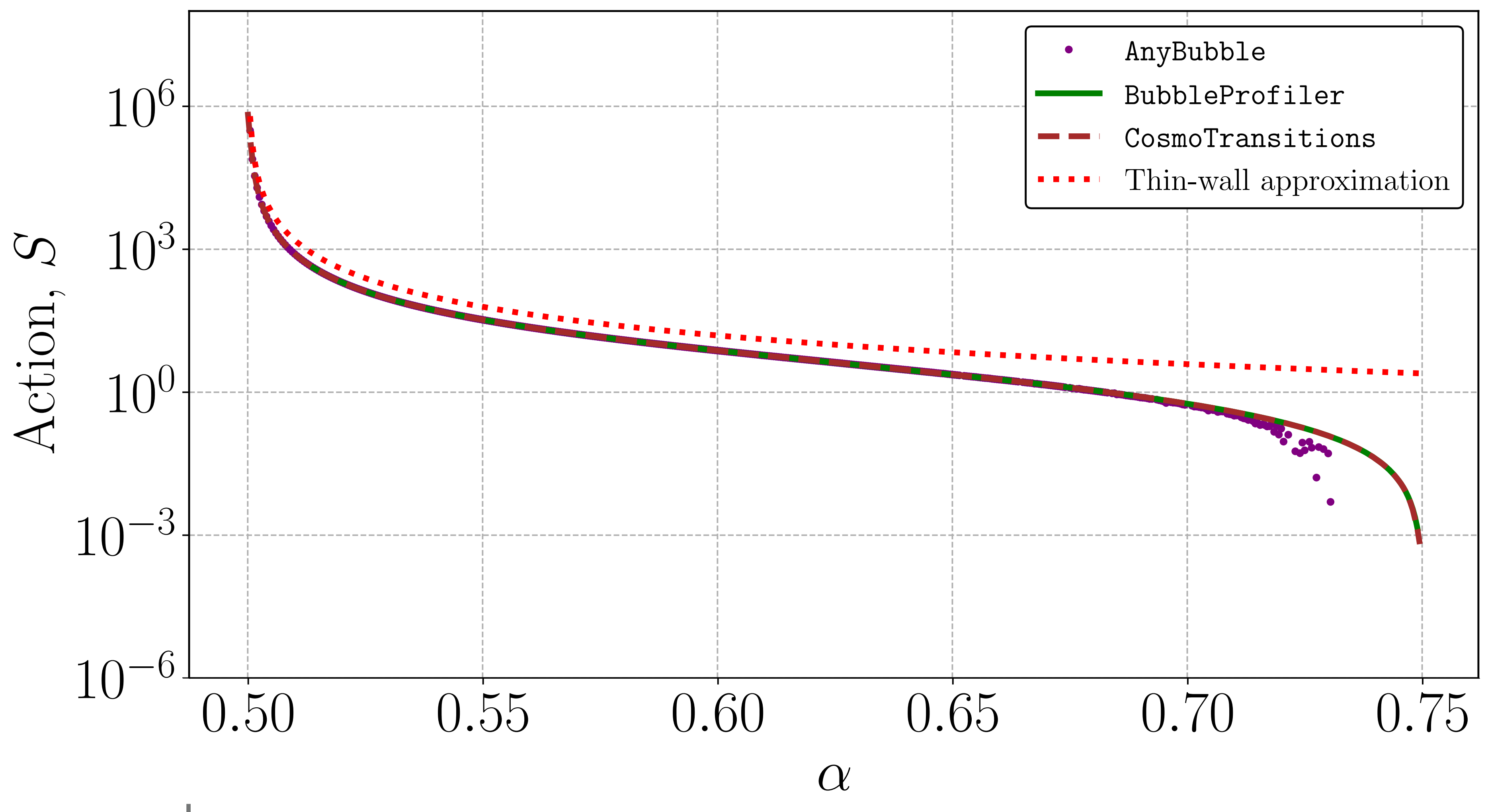
- ➤ Integrate out ultraviolet modes (non-zero Matsubara modes in the imaginary time formalism).
- ➤ Two-loop thermal masses, effective couplings, and beta functions.
- ➤ Reduce theoretical uncertainties.



Input: Four-dimensional Lagrangian $\mathcal{L}_{4d}$ with parameters $\{c_1, \ldots, c_n\}$, temperature T, physical parameters, heavy masses $\boldsymbol{M}$
Start: Initialize model
Call `PerformDRhard[]` {
  for all $c_i \in \{c_1, \ldots, c_n\}$ **do**
    Compute 4-dimensional β-functions $\beta(c_i)$
    Compute $c_{i,3d}(T, \boldsymbol{M})$ by integrating out non-zero Matsubara modes
  end for
  Compute thermal (Debye) masses $m_{D,i}(T, \boldsymbol{M})$
  Compute couplings that involve temporal vector fields
}
Output/Input: Three-dimensional soft Lagrangian $\mathcal{L}_{3d}$ with parameters $\{c_{1,3d}, \ldots, c_{n,3d}\}$
Call `PerformDRsoft[]` {
  for all $c_{i,3d} \in \{c_{1,3d}, \ldots, c_{n,3d}\}$ **do**
    Compute 3-dimensional β-functions $\beta_{3d}(c_{i,3d})$
    Compute $\bar{c}_{i,3d}(T, c_{1,3d}, \ldots, c_{n,3d})$ by integrating out massive temporal scalars
  end for
}
Output: Three-dimensional ultrasoft Lagrangian $\bar{\mathcal{L}}_{3d}$ with parameters $\{\bar{c}_{1,3d}, \ldots, \bar{c}_{n,3d}\}$
if Lattice resources **then**
  Compute lattice continuum relations to construct $\mathcal{L}_{3d}^{\text{lattice}}$
  Monte Carlo simulation
else
  Call `CalculatePotentialUS[]` {
    Compute effective potential $V_{\text{eff}}^{3d}(m_{i,3d}^2)$ up to two-loops
  }
end if
Compute thermodynamic parameters T_c, L/T_c^4

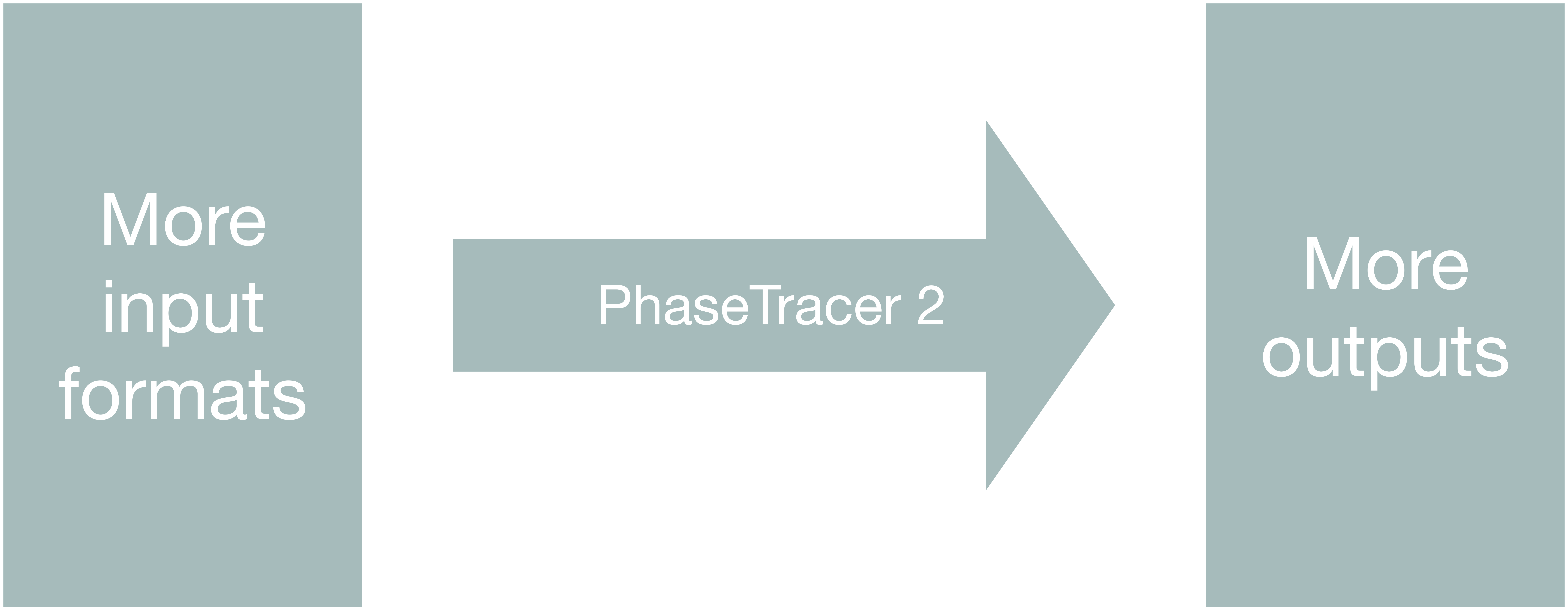
PhaseTracer 1 vs. PhaseTracer 2

- More outputs:
  - Critical temperature
  - Transition strength
  - Action ⟵ BubbleProfiler and others
  - Latent heat
  - Transition's duration
  - Approximated GW
- +TransitionSolver




```
$ ./../bin/run_1D_test_model
=== transition from phase 0 to phase 1 ===
changed = [true]
TC = 59.1608
false vacuum (TC) = [-5.04017e-06]
true vacuum (TC) = [50.0003]
gamma (TC) = 0.845159
delta potential (TC) = 0.00117818
TN = 56.9923
false vacuum (TN) = [-5.80726e-06]
true vacuum (TN) = [54.2993]
action (TN) = 7751.9
```

Summary



Thank you!

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