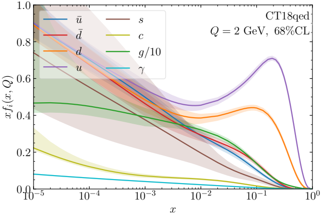
Parton distributions for the LHC precision era

Photon PDF for NLO EW corrections

Keping Xie

Michigan State University



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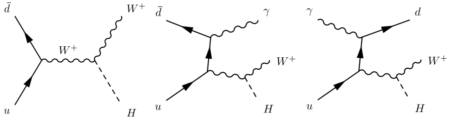
In collaboration with T. J. Hobbs (ANL), T.-J. Hou (South China U.), C. Schmidt (MSU), M. Yan (PKU), C.-P. Yuan (MSU), and B. Zhou (Fermilab)

2106.10299, 2107.13580, 2305.10497

The precision requirements



The precision requirements

- The LHC becomes a precision machine.
- Theoretical cross sections have been achieved at NNLO in QCD, $\mathcal{O}(\alpha_s^2)$, for many processes.
- Due to $\alpha_e \sim \alpha_s^2$, we expect the QED/EW corrections are the same level.
- The photon-initiated processes $(\gamma + \gamma, q, g \to X)$ will have observable effects.

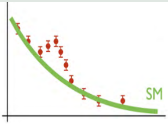
Many applications

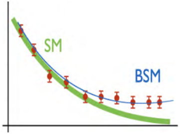
The SM processes

- Drell-Yan: $\ell^+\ell^-$
- $W^\pm H$
- W^+W^-

BSM scenarios

- Heavy leptons: L^+L^-
- Charged Higgs: $H^\pm, H^{\pm\pm}$ [2107.13580]





The existing photon PDFs

The first generation

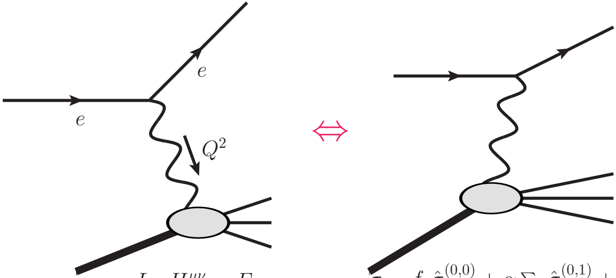
- MRST2004QED [0411040] models the photon PDF with an effective mass scale.
- NNPDF23QED [1308.0598] and NNPDF3.0QED [1410.8849] constrains photon PDF with the LHC Drell-Yan data, $q\bar{q}, \gamma\gamma \to \ell^+\ell^-$
- CT14qed_inc fits the inelastic ZEUS $ep \to e\gamma + X$ data [1509.02905], and include elastic component as well.

The second generation

- LUXqed directly takes the structure functions $F_{2,L}(x, Q^2)$ to constrain photon PDF uncertainty down to a percent level [1607.04266,1708.01256]
- NNPDF3.1luxqed [1712.07053] initializes photon PDF with LUX formula at $\mu_0 = 100\ \mathrm{GeV}$ (a high scale) and evolves DGLAP equation both upwardly and downwardly.
- MMHT2015qed [1907.02750] initializes photon at $\mu_0 = 1$ GeV (a low scale) and evolve DGLAP upwardly. It's updated as MSHT20qed by the recent fit [2111.05357].
- Our work incorporates the LUX formalism with the CT18 [1912.10053] global analysis.

The LUX formalism [1607.04266,1708.01256]

- The DIS process: $ep \to e + X$



$\sigma \sim L_{\mu\nu} H^{\mu\nu} \sim F_{2,L}$ $\quad\quad$ $\sigma \sim f_\gamma \hat{\sigma}^{(0,0)}_{\ell\gamma} + \alpha \sum_j \hat{\sigma}^{(0,1)}_{\ell j} + \cdots$

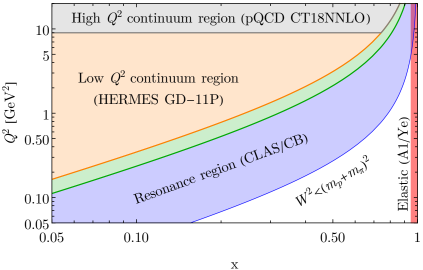
- Matching these two approaches leads to the LUX master formula:

$$x\gamma(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{\mathrm{d}z}{z} \left\{ \int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{\mathrm{d}Q^2}{Q^2} \alpha_{\mathrm{ph}}^2(-Q^2) \left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_q^2}{Q^2} \right) \times F_2(x/z, Q^2) - z^2 F_L(x/z, Q^2) \right] - \alpha^2(\mu^2) z^2 F_2(x/z, \mu^2) \right\}.$$

The square bracket term corresponds to the "physical factorization" scheme, while the second term is referred as the "$\overline{\text{MS}}$-conversion" term.

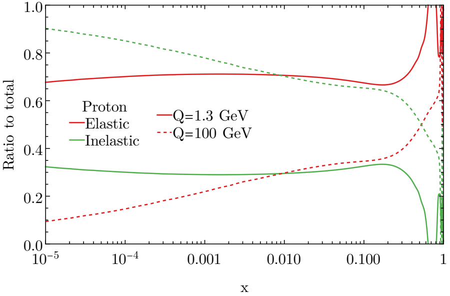
- The structure functions $F_{2,L}$ can be directly measured, or calculated through pQCD in the high-energy regime.

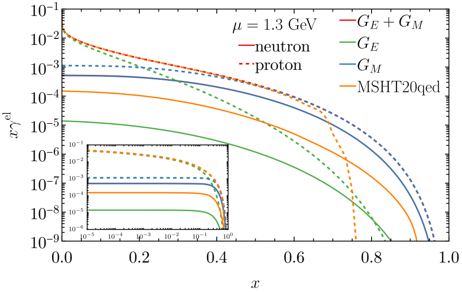
The breakup of (x, Q^2) plane: nonperturbative resources



- In the resonance region $W^2 = m_p^2 + Q^2(1/x - 1) < W_{\rm lo}^2 = 3\ \ {\rm GeV}^2$, the structure functions are taken from CLAS [0301204] or Christy-Bosted [0712.3731] fits.
- In the low-Q^2 continuum region $W^2 > W_{\rm hi}^2 = 4\ \ {\rm GeV}^2$, the HERMES GD11-P [1103.5704] fits with ALLM [PLB1991] functional form.
- In the high-Q^2 region ($Q^2 > Q^2_{\rm PDF}$), $F_{2,L}$ are determined through pQCD.
- The elastic form factors are taken from A1 [1307.6227] or Ye [1707.09063] fits of world data.

Elastic vs inelastic photons

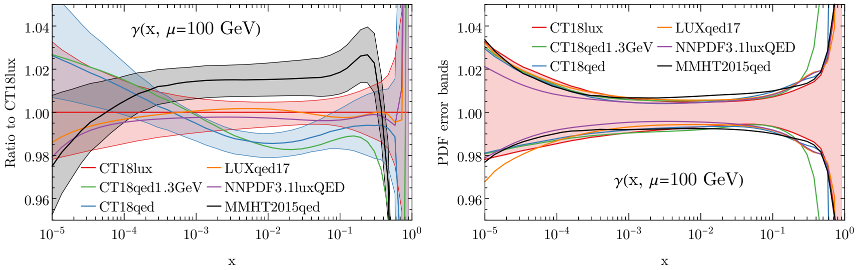




- At a low Q, the elastic photon dominates, which inelastic one dominates at high Q, due to its rapid evolution ($q \to q\gamma$).
- In the elastic photon, the electric form factor dominates at low x, while the magnetic one dominates at large x.
- The neutron's elastic photon is small, due to its zero electric charge. It is resulted from the magnetic form factor.

Two approaches: LUX vs DGLAP

- CT18lux: directly calculate the photon PDF with the LUX formalism
- CT18qed: initialize the inelastic photon PDF with the LUX formalism at low scales, and evolve the $\mathrm{QED_{NLO}} \otimes \mathrm{QCD_{NNLO}}$ DGLAP equations up to high scales, similar to MMHT2015qed.



The take-home message:

- In the intermediate-x region, all photon PDFs give similar error bands.
- CT18lux photon PDF is in between LUXqed (also, NNPDF3.1luxQED) and MMHT2015qed, while CT18qed gives a smaller photon PDF.
- In the large-x region, the DGLAP approach (for both MMHT2015qed and CT18qed) gives a smaller photon than the LUX approach.

The difference between LUX and DGLAP

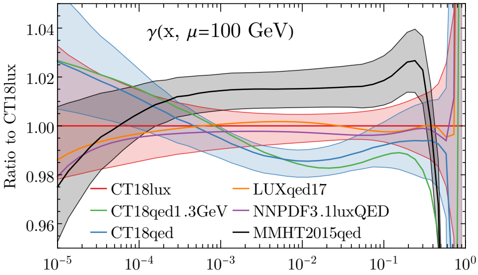
- The DGLAP only evolves the inelastic photon

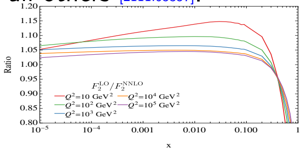
$$\frac{\mathrm{d}x\gamma^{\rm inel}}{\mathrm{d}\log\mu^2}=\frac{\alpha}{2\pi}\left(xP_{\gamma\gamma}\otimes x\gamma^{\rm inel}+\sum_i e_i^2 xP_{\gamma q}\otimes xq_i\right)$$

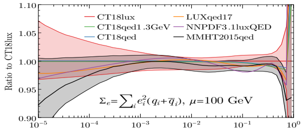
- The first-order solution corresponds to the LO F_2 in LUX formalism

$$x\gamma^{\rm inel}(x,\mu^2)\sim\int^{\mu^2}\mathrm{d}\log Q^2\,\frac{\alpha}{2\pi}\sum_i e_i^2 xP_{\gamma q}\otimes {\color{red}xf_{q_i}}\to F_2^{\rm LO}\text{ in LUX formula}$$

- It explains CT18qed gives larger photon at small x than CT18lux.
- MMHT2015qed gives smaller photon at small x, because the smaller charge-weighted singlet quark distributions.
- The updated MMHT20qed gets very closed to all others [2111.05357].

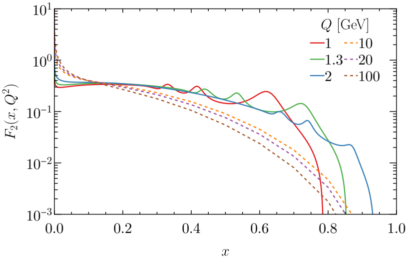


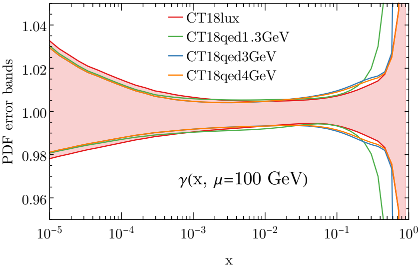




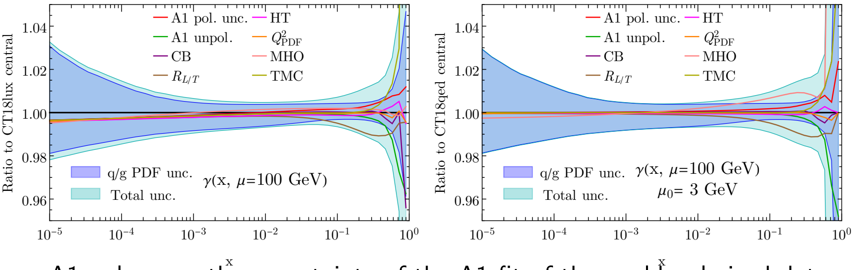
The large x behavior: nonperturbative contribution

- At large x, the LUX approach gives significantly larger PDF than the DGLAP one.
- It is resulted from the non-perturbative F_2 at low energy (resonance and low-Q^2 continuum regions).
- It induces a big uncertainty with the DGLAP low initialization scale approach, because scaling violation is not well behaved in the non-perturbative F_2.
- It can be rescued with a slightly higher initialization scale above the pQCD matching scale $Q_{\rm PDF} \sim 3$ GeV, as compared to CT18's 1.3 GeV.



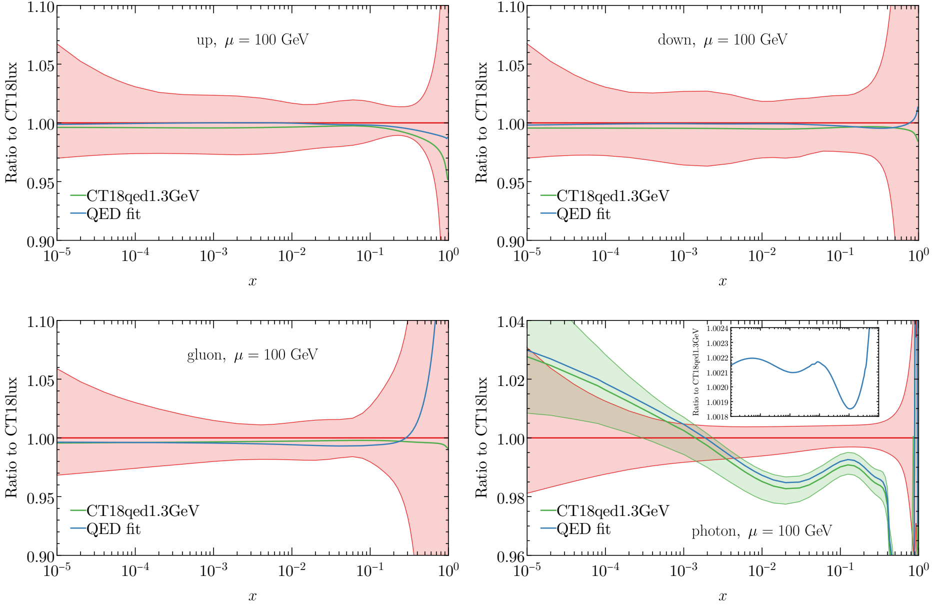


Photon PDF uncertainties



- A1 pol. unc.: the uncertainty of the A1 fit of the world polarized data
- A1 unpol.: Switching to A1 fit of the world unpolarized data
- CB: Changing resonance SF from CLAS to Christy-Bosted fit
- Variations of $R_{L/T} = \sigma_L/\sigma_T$ by 50% [1708.01256]
- HT: Adding higher-twist contribution to F_L [1708.01256] and F_2 [1602.03154].
- $Q^2_{\rm PDF}$: changing the matching scale $9 \to 5\ \ {\rm GeV}^2$
- MHO: varying the scale to estimate the missing high-order uncertainty
- TMC: adding the target mass correction to the SFs.

Global fit with QCD+QED evolution ("QEDfit")



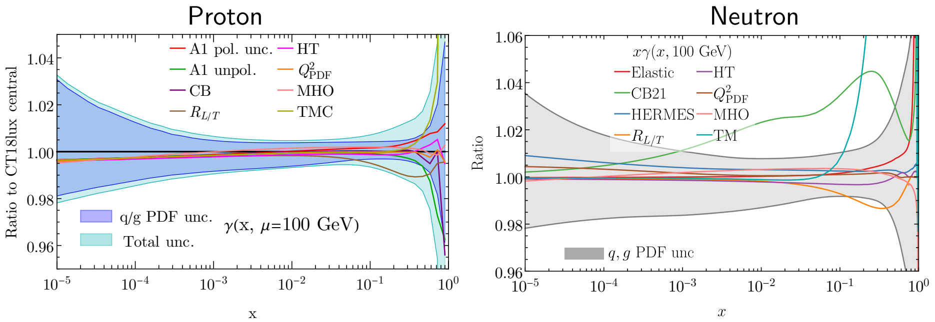
Fitting quality: χ^2

ID	Experimental dataset	References	N_{pt}	CT18lux	CT18qed	QED fit
160	HERAI + II 1 fb^{-1}, H1 and ZEUS combined	[61]	1120	1406	1405	1405
101	BCDMS F_2^p	[60]	337	375	381	377
102	BCDMS F_2^d	[62]	250	281	283	281
104	NMC F_2^d/F_2^p	[63]	123	126	126	126
108	CDHSW F_2^p	[64]	85	85.6	86.6	86.6
109	CDHSW $x_B F_3^p$	[64]	96	86.4	87.1	86.0
110	CCFR F_2^p	[65]	69	78.4	77.6	77.7
111	CCFR $x_B F_3^p$	[66]	86	33.4	32.3	33.9
124	NuTeV $\nu\mu\mu$ SIDIS	[67]	38	18.6	18.8	18.4
125	NuTeV $\bar{\nu}\mu\mu$ SIDIS	[67]	33	38.4	38.5	37.8
126	CCFR $\nu\mu\mu$ SIDIS	[68]	40	29.8	29.7	29.8
127	CCFR $\bar{\nu}\mu\mu$ SIDIS	[68]	38	19.8	19.7	19.8
145	H1 σ_r^b	[69]	10	6.81	6.81	6.91
147	Combined HERA charm production	[70]	47	58.7	58.7	57.7
169	H1 F_L	[71]	9	17.0	17.0	16.9
201	E605 Drell-Yan $sd^2\sigma/(d\sqrt{\tau}dy)$	[72]	119	103	104	103
203	E866 Drell-Yan $\sigma_{pd}/(2\sigma_{pp})$	[73]	15	16.2	16.4	16.6
204	E866 Drell-Yan $Q^3 d^2\sigma_{pp}/(dQ dx_F)$	[74]	184	244	245	246
225	CDF Run-1 lepton A_{ch}, $p_{T\ell} > 25$ GeV	[75]	11	9.04	9.30	9.17
227	CDF Run-2 electron A_{ch}, $p_{T\ell} > 25$ GeV	[76]	11	13.5	12.8	13.4
234	DØ Run-2 muon A_{ch}, $p_{T\ell} > 20$ GeV	[77]	9	8.91	10.2	9.36
260	DØ Run-2 Z rapidity	[78]	28	16.8	16.8	16.8
261	CDF Run-2 Z rapidity	[79]	29	49.1	50.5	49.1
266	CMS 7 TeV 4.7 fb^{-1}, muon A_{ch}, $p_{T\ell} > 35$ GeV	[80]	11	7.72	8.23	7.92
267	CMS 7 TeV 840 pb^{-1}, electron A_{ch}, $p_{T\ell} > 35$ GeV	[81]	11	11.0	12.4	12.0
268	ATLAS 7 TeV 35 pb^{-1}, W/Z cross sec., A_{ch}	[82]	41	44.8	44.1	44.0
281	DØ Run-2 9.7 fb^{-1}, electron A_{ch}, $p_{T\ell} > 25$ GeV	[83]	13	22.9	23.6	22.4
504	CDF Run-2 inclusive jet production	[84]	72	125	126	124
514	DØ Run-2 inclusive jet production	[85]	110	114	113	114

ID	Experimental dataset	Ref.	N_{pt}	CT18lux	CT18qed	QED fit
245	LHCb 7 TeV 1.0 fb^{-1}, forward W/Z	[59]	33	53.4	49.9	53.9
246	LHCb 8 TeV 2.0 fb^{-1}, forward $Z \to e^- e^+$	[86]	17	25.5	23.7	25.5
249	CMS 8 TeV 18.8 fb^{-1}, muon A_{ch}	[58]	11	12.4	15.5	11.7
250	LHCb 8 TeV 2.0 fb^{-1}, forward W/Z	[87]	34	73.2	69.2	72.6
253	ATLAS 8 TeV 20.3 fb^{-1}, Z p_T	[88]	27	30.0	29.4	31.1
542	CMS 7 TeV 5 fb^{-1}, single incl. jet $R = 0.7$	[89]	158	195	193	195
544	ATLAS 7 TeV 4.5 fb^{-1}, single incl. jet $R = 0.6$	[90]	140	202	200	204
545	CMS 8 TeV 19.7 fb^{-1}, single incl. jet $R = 0.7$	[91]	185	213	220	210
573	CMS 8 TeV 19.7 fb^{-1}, $t\bar{t}$ $(1/\sigma)d^2\sigma/(dp_T^t dy^t)$	[92]	16	18.9	18.8	18.9
580	ATLAS 8 TeV 20.3 fb^{-1}, $t\bar{t}$ $d\sigma/dp_T^t$ and $d\sigma/dm_{t\bar{t}}$	[93]	15	9.51	9.49	9.70
	Total χ^2 for all 39 datasets		3681	4293	4302	4296

- The CT18lux share the same χ^2 as CT18, as quark and gluon PDFs remain the same.
- CT18QED gives a small corrections to up and down quark PDFs, which increases χ^2 a little.
- Global fit with QCD+QED evolution ("QEDfit") pull the PDFs and χ^2 back, very closed to CT18lux.

Photon content of the neutron [2305.10497]



- The proton's photon PDF uncertainty is about 1% level.
- The neutron's photon is $(2 \sim 4)\%$ in the moderate-x region.
- A significant improvement in comparison with the 1st generation of photon PDFs.

Isospin symmetry violation

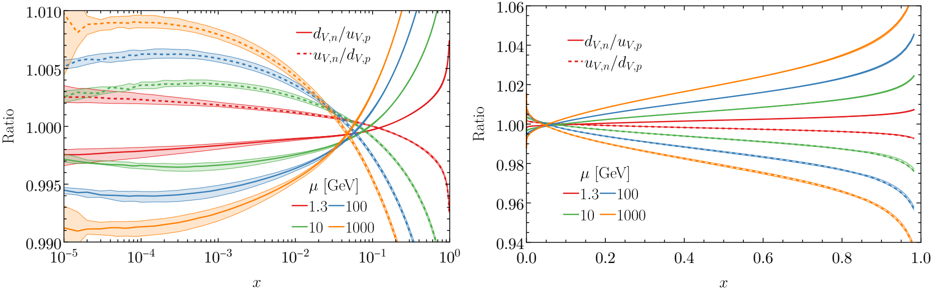
$$\varepsilon = \frac{\int \mathrm{d}x x (\gamma_p^{\mathrm{inel}}(x,\mu_0^2) - \gamma_n^{\mathrm{inel}}(x,\mu_0^2))}{\int \mathrm{d}x x \left(\frac{3}{4} u_{V,p}^{(\mathrm{QED})}(x,\mu_0^2) - 3 d_{V,p}^{(\mathrm{QED})}(x,\mu_0^2)\right)}$$

- Model the initial isospin violation with QED interaction

$$\Delta d_{V,n}(x,\mu_0^2) = d_{V,n}(x,\mu_0^2) - u_{V,p}(x,\mu_0^2) = \varepsilon \left(1 - \frac{e_d^2}{e_u^2}\right) u_{V,p}^{(\mathrm{QED})}(x,\mu_0^2),$$

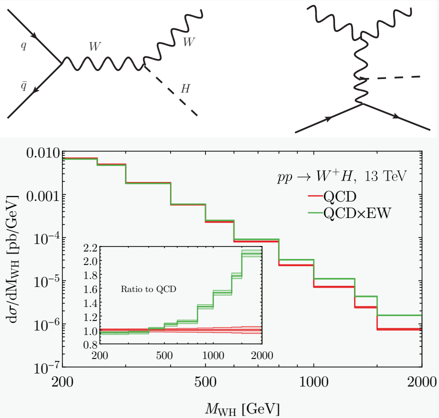
$$\Delta u_{V,n}(x,\mu_0^2) = u_{V,n}(x,\mu_0^2) - d_{V,p}(x,\mu_0^2) = \varepsilon \left(1 - \frac{e_u^2}{e_d^2}\right) d_{V,p}^{(\mathrm{QED})}(x,\mu_0^2).$$

- The ε parameter can be self-consistently determined through sum rules.
- Our ISV is Smaller than MSHT20qed due to only γ^{inel} in ε [2305.10497]

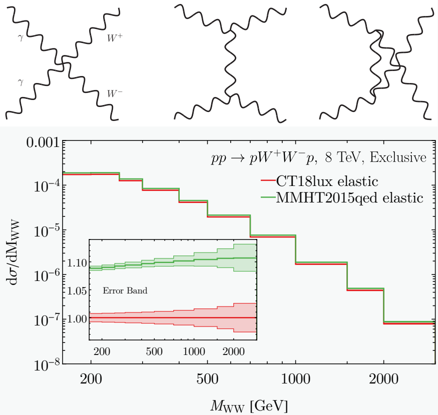


The applications

W^+H production

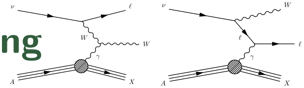


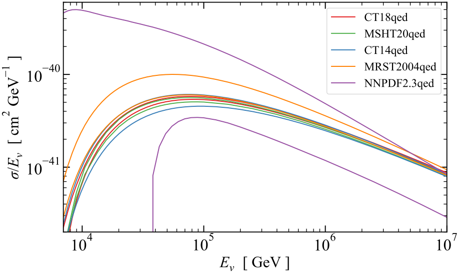
Exclusive $\gamma\gamma \to W^+W^-$

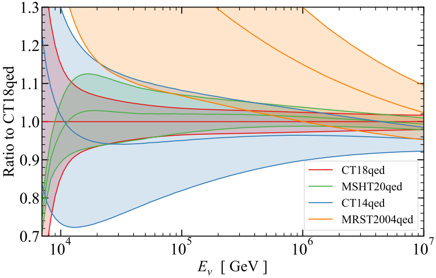


- At a large invariant mass, the photon initiated processes make a significant contribution
- CT18lux elastic photon (α_e running includes both quarks and leptons) is smaller than MMHT2015qed one (where only quarks are included).

W boson production in ν-A scattering

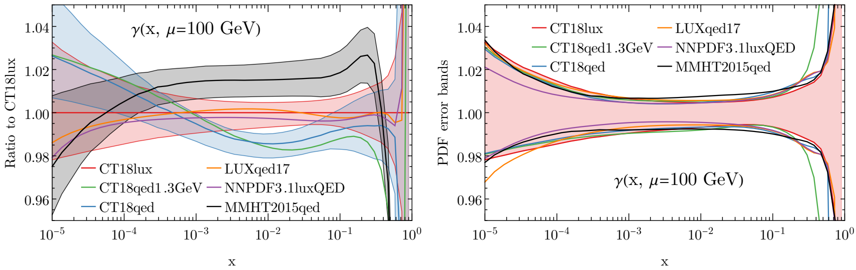






- W-boson production can be measured at in high-energy neutrino telescopes, e.g., IceCube, KM3NET, as well as collider, *i.e.*, FASER and future FPFs
- Our photon PDF directly contributes to the photon-initiated sub-process [2305.10497]
- The photon PDF uncertainty is reduced to a percent level.

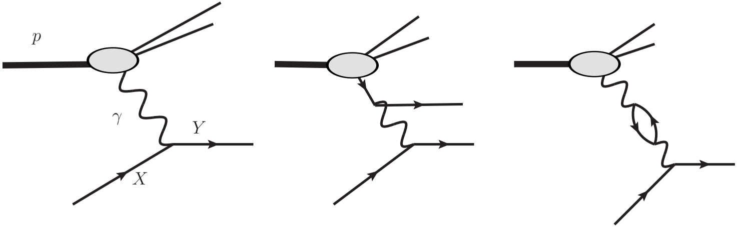
Summary and conclusions



- Photon PDF is essential for the precision NLO EW calculations.
- Photon PDF can be precisely determined by mapping the structure functions to the PDF, the LUXqed formalism.
- We published two photon PDF sets, CT18lux and CT18qed [http://cteq-tea.gitlab.io/project/00pdfs/], based on the LUX and DGLAP approach, respectively.
- The photon PDF precision is significantly improved, up to a percent level.
- The overall uncertainties agree with the LUXqed (also NNPDF3.1luxQED) and MMHT2015qed.
- The isospin symmetry violation due to the QED effect is within (a few) 1% at a (large) small x [2305.10497].
- Many phenomenological implications have been explored [2106.10299, 2107.13580, 2305.10497].

The cancellation in a higher order calculation

- Suppose we want to calculate a process $\gamma + X \to Y$.



- At one order higher, both photon and quark parton will participate.
- The PDFs are related with the DGLAP evolution, with divergence properly canceled.
- This can be also achieved in the LUX approach, with proper $\overline{\mathrm{MS}}$ conversion terms order by order.

The scale variation of the $\overline{\mathrm{MS}}$ conversion term

- In the default scale choice $\mu^2/(1-z)$, the $\overline{\mathrm{MS}}$-conversion term is
$$x\gamma^{\mathrm{con}} \sim (-z^2)F_2(x/z,\mu^2),$$
which is negative
- When varying the scale as μ^2, the conversion term should be change as well,
$$x\gamma^{\mathrm{con}}([M]) = x\gamma^{\mathrm{con}} + \frac{1}{2\pi\alpha}\int_x^1 \frac{\mathrm{d}z}{z}\int_{M^2[z]}^{\frac{\mu^2}{1-z}} \frac{\mathrm{d}Q^2}{Q^2}\alpha^2 z p_{\gamma q}(z) F_2(x/z,Q^2).$$
With $M^2[z]=\mu^2$, we have $\int_{\mu^2}^{\frac{\mu^2}{1-z}} \frac{\mathrm{d}Q^2}{Q^2} = \log\frac{1}{1-z}$.
- The central MMHT2015qed corresponds to $M^2[z]=\mu^2$ choice at low scale $\mu_0 = 1 \ \ \mathrm{GeV}$.
- The DGLAP approach at low scale DOES give larger uncertainty due to the large non-perturbative contributions to structure functions.
- One method to avoid it is to start γ PDF at a higher scale in the pQCD region, i.e., $\mu_0^2 \geq Q^2_{\mathrm{PDF}}$.

The DGLAP approach gives smaller PDFs at large x

- MMHT2015qed divides the integration into two regions:

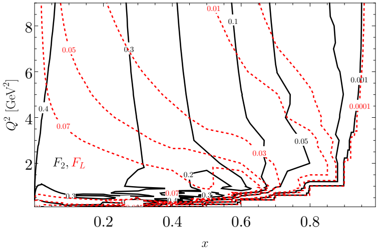
$$\left(\int_{\frac{x^2 m_p^2}{1-z}}^{\mu_0^2} + \int_{\mu_0^2}^{\frac{\mu_0^2}{1-z}}\right)[\cdots]$$

The second part is integrated semi-analytically:

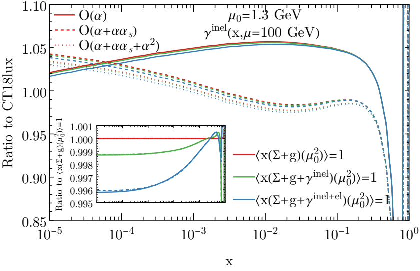
$$\int_{\mu_0^2}^{\frac{\mu_0^2}{1-z}} \frac{\mathrm{d}Q^2}{Q^2}\alpha^2\left(zp_{\gamma q}+\frac{2x^2m_p^2}{Q^2}\right)F_2(x/z,\mu_0^2)=\alpha^2(\mu_0^2)\left(zp_{\gamma q}\log\frac{1}{1-z}+\frac{2x^2m_p^2 z}{\mu_0^2}\right)F_2\left(\frac{x}{z},\mu_0^2\right)$$

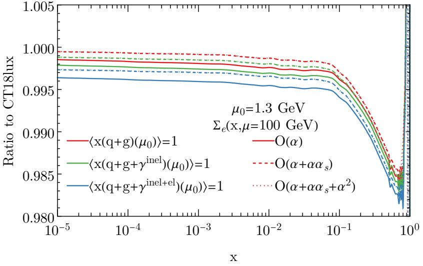
The F_L is dropped because $F_L \sim \mathscr{O}(\alpha_s) \ll F_2$.

- In contrast, we integrate over $F_2(x/z, Q^2)$ rather than $F_2(x/z, \mu_0^2)$.
- It explains the MMHT2015qed gives smaller photon at large x than CT18qed.
- MMHT15 does not include the uncertainty induced by μ_0 variation.



The NLO QED evolution and momentum sum rules





- The NLO QED corrections to splitting functions

$$P_{ij} = \frac{\alpha}{2\pi} P_{ij}^{(0,1)} + \frac{\alpha}{2\pi}\frac{\alpha_S}{2\pi} P_{i,j}^{(1,1)} + \left(\frac{\alpha}{2\pi}\right)^2 P_{ij}^{(0,2)} + \cdots$$

- The NLO QED correction is negative.
- The momentum sum rules: the impact is $\mathscr{O}(0.1\%)$, negligible compared with higher order QED evolution.

$$\langle x(\Sigma + g + \gamma^{\mathrm{inel+el}} \rangle = 1$$