Possible evidences from $H(z)$ parameter data for physics beyond ΛCDM

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Outline

Motivation

- The Universe is experiencing an accelerated expansion.
- Hubble tension may also provide evidences for physics beyond ΛCDM.
- Statistical methods, such as the maximum likelihood, are generally used to analyze the observational data to fit the parameters.
- These statistical method yields the best statistical results, but it is easy to eliminate some interesting (possibly important) data.
- Here we propose a model-independent method by using the Lagrange mean value theorem to analyze $H(z)$ parameter data.

Outline

consider a spatially flat FRWL spacetime here, the Friedmann equations take the form

$$H^2 = \frac{8\pi G}{3}\rho, \tag{1}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \tag{2}$$

or equivalently

$$\dot{H} = -4\pi G(\rho + p), \tag{3}$$

where the $H \equiv \dot{a}/a$ is the Hubble parameter with the dot denoting the derivative with respect to the cosmic time t. The total energy density ρ and pressure p contain contributions coming from the radiation, nonrelativistic matter, and other components.

Because $dz = -(1+z)Hdt$, we have

$$\dot{H} = -(1+z)H\frac{dH}{dz}. \tag{4}$$

Combining Eqs. (3) and (4), yields

$$\frac{dH}{dz} = \frac{4\pi G}{(1+z)H}(\rho + p) = \frac{4\pi G\rho(1+w_{\rm t})}{(1+z)H}, \tag{5}$$

where $w_{\rm t}$ is the total EoS. From this equation, we can judge whether the total EoS is greater than, equal to, or less than -1: see for example, if $dH/dz < 0$, we have $w_{\rm x} \leq w_{\rm t} \leq -1$ because of the positive of H and ρ. In an era dominated by dark energy, we can also determine with Eq. (5) wether the EoS of dark energy is equal to -1: if $dH/dz = 0$, then one have $w_{\rm x} \simeq w_{\rm t} = -1$. If $dH/dz \leq 0$, we know the Universe is experiencing an accelerated expansion.

But if $dH/dz > 0$, we can't judge whether the Universe speeds up. At this point, we need another important physical quantity, the deceleration parameter, which is defined as

$$q = -1 + (1+z)\frac{1}{H}\frac{dH}{dz}. \tag{6}$$

Now a question naturally rise: if we have some $H(z)$ parameter data, how can we use them to directly determine dH/dz or q?

Think of Lagrange mean value theorem in Calculus, which states: for a continuous and differentiable function $f(x)$, there exists $x_1 < x_{12} < x_2$ satisfying

$$\frac{df}{dx}\Big|_{x=x_{12}} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}. \tag{7}$$

Applying this theorem to Hubble parameter which we assume is a continuous and differentiable function of z, and taking function $H(z)$ as $f(x)$ in (7), we have

$$H'(z_{ij}) \equiv \frac{dH}{dz}\Big|_{z=z_{ij}} = \frac{H(z_i) - H(z_j)}{z_i - z_j}, \tag{8}$$

where $z_j < z_{ij} < z_i$. Combining this equation and Hubble parameter data, namely, taking the $H(z)$ data from the table 14 for $H(z_i)$ and $H(z_j)$ and the corresponding z data for z_i and z_j, we can obtain a lot of data for $H'(z_{ij})$.

Since the difference between different data of Hubble parameters $H(z)$ in table 14 in general is large, $H'(z_{ij})$ will be larger if $z_i - z_j \ll 1$, which will make relevant results less credible. In order to make the results credible, we will impose restrictions on $H(z_i) - H(z_j)$ and $z_i - z_j$ during the process of data analysis. When applying equation (6) to analyze the data in table 14, z_{ij} and $H(z_{ij})$ are unknown, we take approximatively: $z_{ij} = (z_i + z_j)/2$ and $H(z_{ij}) \simeq [H(z_i) + H(z_j)]/2$, which can be called as mid-value approximate method.

if the difference between $H(z_i)$ and $H(z_j)$ and the difference between z_i and z_j are reasonable, this approximate method in general is credible. Then we have

$$q(z_{ij}) \simeq -1 + \frac{(2 + z_i + z_j)\, H'(z_{ij})}{H(z_i) + H(z_j)}. \tag{9}$$

The uncertainties associated to the $H'(z)$ and $q(z)$ are given by, respectively

$$\sigma_{H'} = \frac{\sqrt{\sigma_{H_i}^2 + \sigma_{H_j}^2}}{z_i - z_j}, \tag{10}$$

and

$$\sigma_q = \frac{2(2 + z_i + z_j)H_i}{(H_i + H_j)^2} \frac{\sqrt{\sigma_{H_i}^2 + \sigma_{H_j}^2}}{z_i - z_j}. \tag{11}$$

With $\sigma_{H'}$ and σ_q, we can determine whether the results are credible at 1 σ confidence level.

Outline

1. Motivation
2. Theoretical method and $H(z)$ parameter data
   - Theoretical method
   - $H(z)$ parameter data
3. Applications
4. Summary

index	z	$H(z)$[km s^{-1}Mpc^{-1}]	σ_H	Reference	Method	index	z	$H(z)$	σ_H	Reference	Method
z_1	0	74.03	1.42	[8]	SN Ia/Cepheid	z_{33}	0.51	90.4	1.9	[21]	Clustering
z_2	0.07	69	19.6	[22]	DA	z_{34}	0.52	94.35	2.65	[23]	Clustering
z_3	0.1	69	12	[24]	DA	z_{35}	0.56	93.33	2.32	[23]	Clustering
z_4	0.12	68.6	26.2	[22]	DA	z_{36}	0.57	92.9	7.8	[25]	Clustering
z_5	0.17	83	8	[24]	DA	z_{37}	0.59	98.48	3.19	[23]	Clustering
z_6	0.1797	75	4	[26]	DA	z_{38}	0.5929	104	13	[26]	DA
z_7	0.1993	75	5	[26]	DA	z_{39}	0.6	87.9	6.1	[27]	Clustering
z_8	0.2	72.9	29.6	[22]	DA	z_{40}	0.61	97.3	2.1	[21]	Clustering
z_9	0.24	79.69	2.65	[28]	Clustering	z_{41}	0.64	98.82	2.99	[23]	Clustering
z_{10}	0.27	77	14	[24]	DA	z_{42}	0.6797	92	8	[26]	DA
z_{11}	0.28	88.8	36.6	[22]	DA	z_{43}	0.73	97.3	7	[27]	Clustering
z_{12}	0.3	81.7	6.22	[29]	Clustering	z_{44}	0.75	98.8	33.6	[30]	Clustering
z_{13}	0.31	78.17	4.74	[23]	Clustering	z_{45}	0.7812	105	12	[26]	DA
z_{14}	0.34	83.8	3.66	[28]	Clustering	z_{46}	0.8754	125	17	[26]	DA
z_{15}	0.35	82.7	8.4	[31]	Clustering	z_{47}	0.88	90	40	[24]	DA
z_{16}	0.3519	83	14	[26]	DA	z_{48}	0.9	117	23	[24]	DA
z_{17}	0.36	79.93	3.39	[23]	Clustering	z_{49}	0.978	113.72	14.63	[32]	Clustering
z_{18}	0.38	81.5	1.9	[21]	Clustering	z_{50}	1.037	154	20	[26]	DA
z_{19}	0.3802	83	13.5	[33]	DA	z_{51}	1.23	131.44	12.42	[32]	Clustering
z_{20}	0.40	82.04	2.03	[23]	DA	z_{52}	1.3	168	17	[24]	DA
z_{21}	0.4	95	17	[24]	DA	z_{53}	1.363	160	33.6	[34]	DA
z_{22}	0.4004	77	10.2	[33]	DA	z_{54}	1.43	177	18	[24]	DA
z_{23}	0.4247	87.1	11.2	[33]	DA	z_{55}	1.526	148.11	12.71	[32]	Clustering
z_{24}	0.4293	91.8	5.3	[33]	DA	z_{56}	1.53	140	14	[24]	DA
z_{25}	0.43	86.45	3.68	[28]	Clustering	z_{57}	1.75	202	40	[24]	DA
z_{26}	0.44	82.6	7.8	[27]	Clustering	z_{58}	1.944	172.63	14.79	[32]	Clustering
z_{27}	0.44	84.81	1.83	[23]	Clustering	z_{59}	1.965	186.5	50.4	[34]	DA
z_{28}	0.4497	92.8	12.9	[33]	DA	z_{60}	2.3	224	8	[35]	Clustering
z_{29}	0.47	89	34	[36]	DA	z_{61}	2.33	224	8	[37]	Clustering
z_{30}	0.4783	80.9	9	[33]	DA	z_{62}	2.34	222	7	[38]	Clustering
z_{31}	0.48	87.79	2.03	[23]	DA	z_{63}	2.36	226	8	[39]	Clustering
z_{32}	0.48	97	62	[24]	DA						

TABLE I: Hubble parameter compilation from cosmic chronometers (DA) or from the radial BAO surveys (clustering).

index	$H'(z)$	$\sigma_{H'}$	$q(z)$	σ_q	index	$H'(z)$	$\sigma_{H'}$	$q(z)$	σ_q
$z_{61} \in (0, 0.1797)$	5.398	23.62	-0.921	0.348	$z_{2714} \in (0.34, 0.44)$	10.1	40.92	-0.833	0.679
$z_{71} \in (0, 0.1993)$	4.867	26.08	-0.928	0.387	$z_{359} \in (0.24, 0.56)$	42.625	11.006	-0.31	0.192
$z_{91} \in (0, 0.24)$	23.583	12.527	-0.656	0.189	$z_{409} \in (0.24, 0.61)$	47.595	9.138	-0.233	0.162
$z_{131} \in (0, 0.31)$	13.355	15.962	-0.797	0.249	$z_{419} \in (0.24, 0.64)$	47.825	9.988	-0.228	0.178
$z_{141} \in (0, 0.34)$	28.745	11.547	-0.574	0.182	$z_{3418} \in (0.38, 0.52)$	91.786	23.291	0.514	0.412
$z_{171} \in (0, 0.36)$	16.389	10.209	-0.749	0.162	$z_{3420} \in (0.40, 0.52)$	102.583	27.818	0.698	0.493
$z_{181} \in (0, 0.38)$	19.658	6.242	-0.699	0.1	$z_{3718} \in (0.38, 0.59)$	80.857	17.68	0.334	0.319
$z_{201} \in (0, 0.4)$	20.025	6.193	-0.692	0.1	$z_{3431} \in (0.48, 0.52)$	164.00	83.454	1.701	1.424
$z_{251} \in (0.0, 0.43)$	28.884	9.173	-0.562	0.15	$z_{3727} \in (0.44, 0.59)$	91.133	24.518	0.507	0.436
$z_{271} \in (0.0, 0.44)$	24.5	5.264	-0.624	0.086	$z_{3731} \in (0.48, 0.59)$	97.182	34.374	0.602	0.599
$z_{311} \in (0.0, 0.48)$	28.667	5.161	-0.561	0.086	$z_{4324} \in (0.4293, 0.73)$	18.291	18.959	-0.694	0.326
$z_{139} \in (0.24, 0.31)$	-21.714	77.578	-1.351	1.241	$z_{5150} \in (1.037, 1.23)$	-116.891	121.983	-2.747	1.679
$z_{179} \in (0.24, 0.36)$	2.00	35.857	-0.967	0.585	$z_{5550} \in (1.037, 1.526)$	-12.045	48.46	-1.182	0.718
$z_{189} \in (0.24, 0.38)$	12.929	23.291	-0.79	0.383	$z_{5650} \in (1.037, 1.53)$	-28.398	49.519	-1.441	0.733
$z_{209} \in (0.24, 0.40)$	14.688	20.864	-0.76	0.346	$z_{5552} \in (1.3, 1.526)$	-88.009	93.92	-2.344	1.344
$z_{259} \in (0.24, 0.43)$	35.579	23.868	-0.428	0.399	$z_{5652} \in (1.3, 1.53)$	-121.739	95.75	-2.909	1.365
$z_{279} \in (0.24, 0.44)$	25.60	16.102	-0.583	0.271	$z_{5554} \in (1.43, 1.526)$	-300.938	229.532	-5.588	3.188
$z_{319} \in (0.24, 0.48)$	33.75	13.9091	-0.452	0.237	$z_{5654} \in (1.43, 1.53)$	-370	228.035	-6.789	3.152
$z_{339} \in (0.24, 0.51)$	39.667	12.077	-0.359	0.208	$z_{5854} \in (1.43, 1.944)$	-8.502	45.324	-1.131	0.688
$z_{2014} \in (0.34, 0.40)$	-29.333	69.755	-1.485	1.14					

TABLE II: $H'(z)$ and $q(z)$ data obtained from $H(z)$ parameter data.

- At redshifts z_{61}, z_{71}, z_{91}, z_{131}, z_{141}, z_{171}, z_{181}, z_{201}, z_{251}, z_{271}, z_{311}, z_{139}, z_{179}, z_{189}, z_{209}, z_{259}, z_{279}, z_{319}, z_{339}, z_{2014}, z_{2714}, z_{359}, z_{409}, z_{419}, z_{4324}, z_{5150}, z_{5550}, z_{5650}, z_{5552}, z_{5652} z_{5554}, z_{5654}, and z_{5854}, the Universe experiences an accelerated expansion at 1 σ confidence level.
- At redshifts z_{3418}, z_{3420}, z_{3718}, z_{3431}, z_{3727}, and z_{3731}, the Universe experiences an decelerated expansion at 1 σ confidence level.
- At redshifts z_{139}, z_{2014}, z_{5150}, z_{5550}, z_{5650}, z_{5552}, and z_{5854}, since $H'(z) < 0$, implying $w_{\rm x} \leq w_{\rm t} < -1$, but not at at 1 σ confidence level. However, we can infer that $w_{\rm x} \leq w_{\rm t} < -1$ at redshifts z_{5652}, z_{5554}, and z_{5654} at at 1 σ confidence level.

- There exists decelerated phase at 1 σ confidence level in the redshift range $(0.38, 0.59)$;
- The EoS of dark energy may be less than -1 at 1 σ confidence level at some redshifts in the redshift range $(1.3, 1.53)$;
- There exist accelerated phase at 1 σ confidence level in the redshift range $(1.037, 1.944)$.
- These results suggest that the dark energy maybe dynamic with EoS crossing -1 and the Universe may be accelerated first, then decelerated, and accelerated again recently.

Thanks!