Exploring *HVV* amplitudes with *CP* violation by decomposition and on-shell scattering amplitude methods

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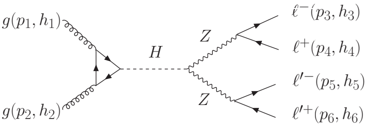
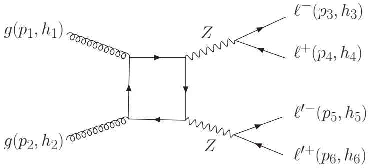
Importance of studying *CP* violation

- Matter-antimatter asymmetry in our Universe.
  CP violation in electroweak Baryogenesis.
- Two sources in SM: CKM and strong *CP* problem.
  Not enough
- SM Higgs boson is a *CP*-even scalar
  Mixing between scalar and pseudoscalar cause *CP* violated Higgs couplings in BSM. Eg. THDM, MSSM, Composite Higgs Model.
- Effective Lagrangian by adding *CP*-violated terms

- $H \to \gamma\gamma$, $H \to \gamma\ell\ell$ and $H \to ZZ \to 4\ell$ processes
  Golden channels for precise measurement of Higgs properties.
  CP violation phase could not be probed solely in $H \to \gamma\gamma$ or $H \to \gamma\ell\ell$ processes without interference from background.
  By contrast, in $H \to ZZ \to 4\ell$ processes CP violation could be probed lonely through its kinematic angles.
- Two independent ways.
  (1)A decomposition relation between these amplitudes is illustrated in an interesting diagrammatic way.
  (2)Calculate the same amplitude from the on-shell method (BCFW recursion relation)
  A parallel proof of the decomposition relation.





$$\mathcal{M}^{gg\to H}(1_g^+, 2_g^+) = \frac{2c_g}{v}[12]^2 ,$$

$$\mathcal{M}^{gg\to H}(1_g^-, 2_g^-) = \frac{2c_g}{v}\langle 12\rangle^2 .$$

$$\frac{c_g}{v} = \frac{1}{2}\sum_f \frac{\delta^{ab}}{2}\frac{i}{16\pi^2} g_s^2 4e \frac{m_f^2}{2M_W s_W}\frac{1}{M_H^2}(2 + s_{12}(1-\tau_H)C_0^{\gamma\gamma}(m_f^2)) ,$$

$$\langle ij\rangle \equiv \langle i^-|j^+\rangle = \overline{u_-(p_i)}u_+(p_j), \quad [ij] \equiv \langle i^+|j^-\rangle = \overline{u_+(p_i)}u_-(p_j),$$

$$\langle ij\rangle[ji] = 2p_i\cdot p_j, \quad s_{ij} = (p_i+p_j)^2, \quad \epsilon_\mu^\pm(p_i, q) = \pm\frac{\langle q^\mp|\gamma_\mu|p_i^\mp\rangle}{\sqrt{2}\langle q^\mp|p_i^\pm\rangle},$$

$$\mathcal{M}^{H\to ZZ\to 4\ell}(3^-_{\ell^-},4^+_{\ell^+},5^-_{\ell^-},6^+_{\ell^+}) = f\times l_e^2\frac{M_W^2}{\cos^2\theta_W}\langle 35\rangle[46],$$

$$\mathcal{M}^{H\to ZZ\to 4\ell}(3^-_{\ell^-},4^+_{\ell^+},5^+_{\ell^-},6^-_{\ell^+}) = f\times l_e r_e\frac{M_W^2}{\cos^2\theta_W}\langle 36\rangle[45],$$

$$\mathcal{M}^{H\to ZZ\to 4\ell}(3^+_{\ell^-},4^-_{\ell^+},5^-_{\ell^-},6^+_{\ell^+}) = f\times l_e r_e\frac{M_W^2}{\cos^2\theta_W}\langle 45\rangle[36],$$

$$\mathcal{M}^{H\to ZZ\to 4\ell}(3^+_{\ell^-},4^-_{\ell^+},5^+_{\ell^-},6^-_{\ell^+}) = f\times r_e^2\frac{M_W^2}{\cos^2\theta_W}\langle 46\rangle[35],$$

$$f = -2ie^3\frac{1}{M_W\sin\theta_W}P_Z(s_{34})P_Z(s_{56}),$$

$$P_X(s) = \frac{1}{s-M_X^2+iM_X\Gamma_X}$$

$$\mathcal{M}^{gg\to H\to ZZ\to 4\ell} = \mathcal{M}^{gg\to H}\times P_H(s_{12})\times\mathcal{M}^{H\to ZZ\to 4\ell}.$$

BSM HVV helicity amplitudes

In SMEFT

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\sum_k C_k^5 \mathcal{O}_k^5 + \frac{1}{\Lambda^2}\sum_k C_k^6 \mathcal{O}_k^6 + \mathcal{O}(\frac{1}{\Lambda^3}) \ ,$$

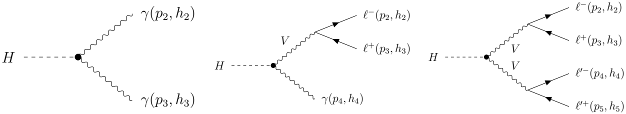
BSM HVV in Warsaw basis

$$\mathcal{O}^6_{\Phi D} = (\Phi^\dagger D^\mu \Phi)^* (\Phi^\dagger D^\mu \Phi),$$
$$\mathcal{O}^6_{\Phi W} = \Phi^\dagger \Phi W^I_{\mu\nu} W^{I\mu\nu}, \quad \mathcal{O}^6_{\Phi B} = \Phi^\dagger \Phi B_{\mu\nu} B^{\mu\nu}, \quad \mathcal{O}^6_{\Phi WB} = \Phi^\dagger \tau^I \Phi W^I_{\mu\nu} B^{\mu\nu},$$
$$\mathcal{O}^6_{\Phi \tilde{W}} = \Phi^\dagger \Phi \tilde{W}^I_{\mu\nu} W^{I\mu\nu}, \quad \mathcal{O}^6_{\Phi \tilde{B}} = \Phi^\dagger \Phi \tilde{B}_{\mu\nu} B^{\mu\nu}, \quad \mathcal{O}^6_{\Phi \tilde{W} B} = \Phi^\dagger \tau^I \Phi \tilde{W}^I_{\mu\nu} B^{\mu\nu},$$

After SSB, HVV effective interactions,

$$\mathcal{L}^{int} = -\frac{c_{VV}}{v} H V^{\mu\nu} V_{\mu\nu} - \frac{\tilde{c}_{VV}}{v} H V^{\mu\nu} \tilde{V}_{\mu\nu} \ ,$$

$$\xi \ \equiv \ \tan^{-1}(\tilde{c}_{VV}/c_{VV}) \ , \qquad \text{when} \quad \mathrm{Arg}(\tilde{c}_{VV}/c_{VV}) = 0 \quad \text{or} \quad \pi \ ,$$



- For process $H \to \gamma\gamma$,

$$
\begin{aligned}
\mathcal{M}(2_\gamma^+, 3_\gamma^+) &= \frac{2c^S_{\gamma\gamma}}{v} e^{i\xi} [23]^2 \,, \\
\mathcal{M}(2_\gamma^-, 3_\gamma^-) &= \frac{2c^S_{\gamma\gamma}}{v} e^{-i\xi} \langle 23\rangle^2 \,, \\
\mathcal{M}(2_\gamma^+, 3_\gamma^-) &= 0 \,, \\
\mathcal{M}(2_\gamma^-, 3_\gamma^+) &= 0 \,,
\end{aligned}
$$

- For process $H \to V\gamma \to \ell\ell\gamma$,

$$
\begin{aligned}
\mathcal{M}(2_{\ell^-}^-, 3_{\ell^+}^+, 4_\gamma^-) &= f_V^-(s_{23}) \times \frac{2c^S_{\gamma V}}{v} e^{-i\xi} [23]\langle 24\rangle^2, \\
\mathcal{M}(2_{\ell^-}^-, 3_{\ell^+}^+, 4_\gamma^+) &= f_V^-(s_{23}) \times \frac{2c^S_{\gamma V}}{v} e^{i\xi} \langle 23\rangle [34]^2, \\
\mathcal{M}(2_{\ell^-}^+, 3_{\ell^+}^-, 4_\gamma^+) &= f_V^+(s_{23}) \times \frac{2c^S_{\gamma V}}{v} e^{i\xi} \langle 23\rangle [24]^2, \\
\mathcal{M}(2_{\ell^-}^+, 3_{\ell^+}^-, 4_\gamma^-) &= f_V^+(s_{23}) \times \frac{2c^S_{\gamma V}}{v} e^{-i\xi} [23]\langle 34\rangle^2,
\end{aligned}
$$

- For process $H \to VV \to 2\ell 2\ell'$,

$$\mathcal{M}(2^-_{\ell-}, 3^+_{\ell+}, 4^-_{\ell'-}, 5^+_{\ell'+}) = f_V^-(s_{23}) f_V^-(s_{45}) \frac{2c^S_{VV}}{v} \left(e^{i\xi}\langle 23\rangle\langle 45\rangle[35]^2 + e^{-i\xi}[23][45]\langle 24\rangle^2\right),$$

$$\mathcal{M}(2^-_{\ell-}, 3^+_{\ell+}, 4^+_{\ell'-}, 5^-_{\ell'+}) = f_V^-(s_{23}) f_V^+(s_{45}) \frac{2c^S_{VV}}{v} \left(e^{i\xi}\langle 23\rangle\langle 45\rangle[34]^2 + e^{-i\xi}[23][45]\langle 25\rangle^2\right),$$

$$\mathcal{M}(2^+_{\ell-}, 3^-_{\ell+}, 4^-_{\ell'-}, 5^+_{\ell'+}) = f_V^+(s_{23}) f_V^-(s_{45}) \frac{2c^S_{VV}}{v} \left(e^{i\xi}\langle 23\rangle\langle 45\rangle[25]^2 + e^{-i\xi}[23][45]\langle 34\rangle^2\right),$$

$$\mathcal{M}(2^+_{\ell-}, 3^-_{\ell+}, 4^+_{\ell'-}, 5^-_{\ell'+}) = f_V^+(s_{23}) f_V^+(s_{45}) \frac{2c^S_{VV}}{v} \left(e^{i\xi}\langle 23\rangle\langle 45\rangle[24]^2 + e^{-i\xi}[23][45]\langle 35\rangle^2\right),$$

Multilinear vertices

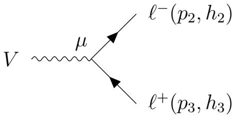
In dimension-6 operators, only three kinds of effective operators $\psi^2 X\varphi$, $X^2\varphi^2$ and X^3 are multilinear vertices

$$\begin{aligned}(\bar{\psi}\gamma^{\mu\nu}\psi)\varphi X_{\mu\nu}: \quad & \Gamma^{\mu}(k_1) = [l_1|k_1\gamma^{\mu}|l_2] + [l_1|\gamma^{\mu}k_1|l_2], \\ \varphi^2 X^{\mu\nu}X_{\mu\nu}: \quad & \Gamma^{\mu\nu}(k_1,k_2) = k_1^{\mu}k_2^{\nu} - g^{\mu\nu}k_1\cdot k_2, \\ \mathrm{tr}(X^{\mu}_{\nu}X^{\nu}_{\rho}X^{\rho}_{\mu}): \quad & \Gamma^{\mu\nu\rho}(k_1,k_2,k_3) = k_1^{\nu}k_2^{\rho}k_3^{\mu} + \cdots .\end{aligned}$$

$$\Gamma^{\mu\nu}(k,k') = -i\frac{4}{v}[c_{VV}\ (k^{\nu}k'^{\mu} - k\cdot k' g^{\mu\nu}) + \tilde{c}_{VV}\ \epsilon^{\mu\nu\rho\sigma}k_{\rho}k'_{\sigma}]\ ,$$

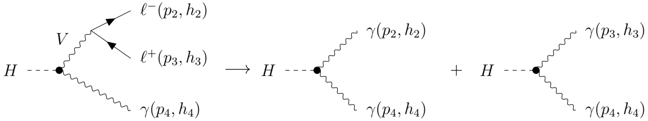
$$\begin{aligned} & \Gamma^{\mu\nu}(k,k') \\ = \ & \Gamma^{\mu\nu}(p_2+p_3,k') = \Gamma^{\mu\nu}(p_2,k') + \Gamma^{\mu\nu}(p_3,k') \\ = \ & \Gamma^{\mu\nu}(p_2+p_3,p_4+p_5) \\ = \ & \Gamma^{\mu\nu}(p_2,p_4) + \Gamma^{\mu\nu}(p_2,p_5) + \Gamma^{\mu\nu}(p_3,p_4) + \Gamma^{\mu\nu}(p_3,p_5).\end{aligned}$$

Current J_μ of $V \to \ell^+\ell^-$ proportional to a photon's polarization vector



$$
\begin{aligned}
J_\mu^{(2)}(p_2^{\mp\frac{1}{2}}, p_3^{\pm\frac{1}{2}}) &= \frac{f_V^{\mp}(s_{23})}{\sqrt{2}}\langle 2^{\mp}|\gamma_\mu|3^{\mp}\rangle \\
&= \pm f_V^{\mp}(s_{23})\langle 2^{\mp}|3^{\pm}\rangle \epsilon_\mu^{\pm}(3,2) \\
&= \pm f_V^{\mp}(s_{23})\langle 2^{\pm}|3^{\mp}\rangle \epsilon_\mu^{\mp}(2,3),
\end{aligned}
$$

Decomposition of amplitudes of $H \to V\gamma \to \ell\ell\gamma$

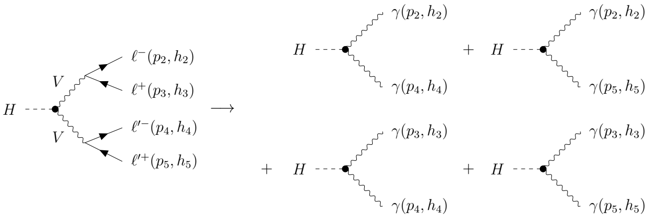


$$F^{(2)}_{q_{i1}}(q_{i1}^{-\frac{1}{2}}, q_{i2}^{+\frac{1}{2}}) = f_V^-(k_i^2)[q_{i1}q_{i2}], \qquad F^{(2)}_{q_{i2}}(q_{i1}^{-\frac{1}{2}}, q_{i2}^{+\frac{1}{2}}) = f_V^-(k_i^2)\langle q_{i1}q_{i2}\rangle,$$

$$F^{(2)}_{q_{i1}}(q_{i1}^{+\frac{1}{2}}, q_{i2}^{-\frac{1}{2}}) = -f_V^+(k_i^2)\langle q_{i1}q_{i2}\rangle, \qquad F^{(2)}_{q_{i2}}(q_{i1}^{+\frac{1}{2}}, q_{i2}^{-\frac{1}{2}}) = -f_V^+(k_i^2)[q_{i1}q_{i2}].$$

$$\begin{aligned}
&\mathcal{M}(2^-_{\ell^-}, 3^+_{\ell^+}, 4^-_\gamma)\\
=&F^{(2)}_{p_2}(p_2^{-\frac{1}{2}}, p_3^{+\frac{1}{2}})\mathcal{M}(2^-_\gamma, 4^-_\gamma) + F^{(2)}_{p_3}(p_2^{-\frac{1}{2}}, p_3^{+\frac{1}{2}})\mathcal{M}(3^+_\gamma, 4^-_\gamma)\\
=&f_V^l(s_{23}) \times ([23]\mathcal{M}(2^-_\gamma, 4^-_\gamma) + \langle 23\rangle\mathcal{M}(3^+_\gamma, 4^-_\gamma)),
\end{aligned}$$

Decomposition of amplitudes of $H \to VV \to 4\ell$



$$
\begin{aligned}
& \mathcal{M}(2^-_{\ell^-}, 3^+_{\ell^+}, 4^-_{\ell'^-}, 5^+_{\ell'^+}) \\
= \quad & f_V^I(s_{23}) f_V^I(s_{45}) \times (\\
& \quad [23][45]\mathcal{M}(2^-_\gamma, 4^-_\gamma) + [23]\langle 45\rangle \mathcal{M}(2^-_\gamma, 5^+_\gamma) \\
& + \langle 23\rangle [45]\mathcal{M}(3^+_\gamma, 4^-_\gamma) + \langle 23\rangle\langle 45\rangle \mathcal{M}(3^+_\gamma, 5^+_\gamma) \ ),
\end{aligned}
$$

The form of the momentum dependence between the $H \to 4\ell$ and the $H \to \gamma\gamma$ amplitudes.

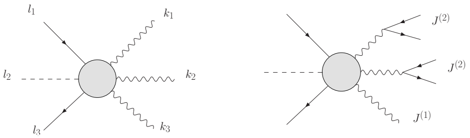
CP violation phase in helicity amplitudes

Amplitudes of $H \to \gamma\gamma$ are basis for other amplitudes. $\mathcal{M}(+,-) = \mathcal{M}(-,+) = 0$, left basis $\mathcal{M}(+,+)$ and $\mathcal{M}(-,-)$. *CP* violation phases are reverse in the two bases. In $H \to \gamma\gamma$ and $H \to V\gamma \to \ell\ell\gamma$ processes, *CP* violation phase is a global phase in each amplitude. It is an unobservable phase if one doesn't consider interference between this amplitude and the background amplitudes.

In $H \to 4\ell$ process, two bases coexist in each amplitude, thus the *CP* violation phase appear as a physical observable.

Meanwhile, it means that the interference between *CP*-even term and *CP*-odd term exists at differential cross section level after squaring the amplitude. So the interference could be probed through kinematic angles. An obvious effect is a shift of azimuthal angle caused by the interference between *CP*-even and *CP*-odd term.

Proof



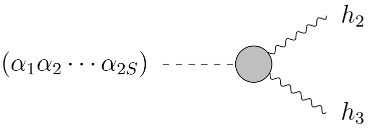
$$M_{\rm lower}(l_1,\ldots,l_{m-n};k_1^{h_1},\ldots,k_n^{h_n})=\Gamma^{\mu_1\cdots\mu_n}(k_1,\cdots,k_n)\prod_i\epsilon_{\mu_i}^{h_i}(k_i,r_i)\ ,$$

$$\Gamma^{\mu_1\cdots\mu_n}(k_1,\ldots,k_n)=\sum_{j_1,\cdots,j_n}\Gamma^{\mu_1\cdots\mu_n}(q_{1j_1},\ldots,q_{nj_n})\ ,$$

$$J_\mu^{(n_i)}(q_{i1}^{h_{i1}},\ldots,q_{in_i}^{h_{in_i}})=F_{q_{ij_i}}^{(n_i)}(q_{i1}^{h_{i1}},\ldots,q_{in_i}^{h_{in_i}})\epsilon_\mu^{H_{ij_i}}(q_{ij_i},r_{ij_i})\ ,$$

$$\begin{aligned}
M_{\rm higher}&=\Gamma^{\mu_1\cdots\mu_n}(k_1,\ldots,k_n)\prod_i J_\mu^{(n_i)}(q_{i1}^{h_{i1}},\ldots,q_{in_i}^{h_{in_i}})\\
&=\sum_{j_1,\cdots,j_n}\Gamma^{\mu_1\cdots\mu_n}(q_{1j_1},\ldots,q_{nj_n})\prod_i F_{q_{ij_i}}^{(n_i)}(q_{i1}^{h_{i1}},\ldots,q_{in_i}^{h_{in_i}})\epsilon_\mu^{H_{ij_i}}(q_{ij_i},r_{ij_i})\\
&=\sum_{j_1,\cdots,j_n}F_{q_{ij_i}}^{(n_i)}(q_{i1}^{h_{i1}},\ldots,q_{in_i}^{h_{in_i}})\Gamma^{\mu_1\cdots\mu_n}(q_{1j_1},\ldots,q_{nj_n})\prod_i\epsilon_\mu^{H_{ij_i}}(q_{ij_i},r_{ij_i})\\
&=\sum_{j_1,\cdots,j_n}F_{q_{ij_i}}^{(n_i)}(q_{i1}^{h_{i1}},\ldots,q_{in_i}^{h_{in_i}})M_{\rm lower}(q_{i1}^{H_{ij_1}},\ldots,q_{in_i}^{H_{ij_n}})
\end{aligned}$$

Amplitudes of $H \to \gamma\ell\ell$



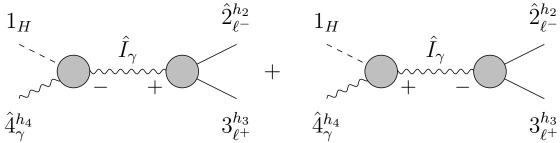
$$\mathcal{M}^{h_2 h_3}\{\alpha_1\alpha_2\cdots\alpha_{2S}\}$$

$$\mathcal{M}_3(1_H, 2_\gamma^{h_2}, 3_\gamma^{h_3}) = e^{i\xi^{h_2,h_3}} \frac{g}{m^{h_2+h_3-1}} [23]^{h_2+h_3} \ ,$$

$$\begin{aligned} \mathcal{M}_3(1_H, 2_\gamma^{+}, 3_\gamma^{+}) &= e^{i\xi'} \frac{g}{m} [23]^2 \ , \\ \mathcal{M}_3(1_H, 2_\gamma^{-}, 3_\gamma^{-}) &= e^{-i\xi'} \frac{g}{m} \langle 23 \rangle^2 \ , \end{aligned}$$

Amplitudes of $H \to \gamma \ell\ell$

$\mathcal{M}_4(1_H, 2^{h_2}_{\ell-}, 3^{h_3}_{\ell+}, 4^{h_4}_{\gamma}) =$

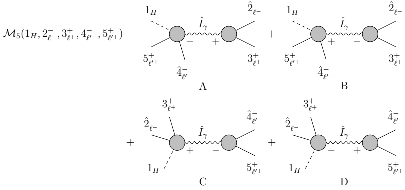


BCFW recursion relation

$$\begin{aligned}\mathcal{M}(1_H, 2^{h_2}_{\ell-}, 3^{h_3}_{\ell+}, 4^{h_4}_{\gamma}) = &\quad P_\gamma(s_{23})\mathcal{M}(1_H, \hat{4}^{h_4}_\gamma, -\hat{P}^-_{I\gamma})\mathcal{M}(\hat{P}^+_{I\gamma}, \hat{2}^{h_2}_{\ell-}, 3^{h_3}_{\ell+}) \\ + &\quad P_\gamma(s_{23})\mathcal{M}(1_H, \hat{4}^{h_4}_\gamma, -\hat{P}^+_{I\gamma})\mathcal{M}(\hat{P}^-_{I\gamma}, \hat{2}^{h_2}_{\ell-}, 3^{h_3}_{\ell+}) ,\end{aligned}$$

$$\begin{aligned}\mathcal{M}(1_H, 2^-_{\ell-}, 3^+_{\ell+}, 4^-_\gamma) &= \tilde{e}P_\gamma(s_{23}) \times \frac{2c^S_{\gamma V}}{v} e^{-i\xi} \frac{\langle \hat{I}\hat{4}\rangle^2 [\hat{I}3]^2}{[\hat{2}3]}, \\ &= \tilde{e}P_\gamma(s_{23}) \times \frac{2c^S_{\gamma V}}{v} e^{-i\xi} [23]\langle 24\rangle^2,\end{aligned}$$

Amplitudes of $H \to 4\ell$



$$
\begin{aligned}
\mathcal{M}_5(1_H, 2^-_{\ell-}, 3^+_{\ell+}, 4^-_{\ell'-}, 5^+_{\ell'+}) = \quad & P_\gamma(s_{23}) \mathcal{M}(1_H, \hat{4}^-_{\ell'-}, 5^+_{\ell'+}, -\hat{P}^-_{l\gamma}) \mathcal{M}(\hat{P}^+_{l\gamma}, \hat{2}^-_{\ell-}, 3^+_{\ell+}) \\
+ \quad & P_\gamma(s_{23}) \mathcal{M}(1_H, \hat{4}^-_{\ell'-}, 5^+_{\ell'+}, -\hat{P}^+_{l\gamma}) \mathcal{M}(\hat{P}^-_{l\gamma}, \hat{2}^-_{\ell-}, 3^+_{\ell+}) \\
+ \quad & P_\gamma(s_{45}) \mathcal{M}(1_H, \hat{2}^-_{\ell-}, 3^+_{\ell+}, -\hat{P}^+_{l\gamma}) \mathcal{M}(\hat{P}^-_{l\gamma}, \hat{4}^-_{\ell'-}, 5^+_{\ell'+}) \\
+ \quad & P_\gamma(s_{45}) \mathcal{M}(1_H, \hat{2}^-_{\ell-}, 3^+_{\ell+}, -\hat{P}^-_{l\gamma}) \mathcal{M}(\hat{P}^+_{l\gamma}, \hat{4}^-_{\ell'-}, 5^+_{\ell'+}),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}(1_H, 2^-_{\ell-}, 3^+_{\ell+}, 4^-_{\ell'-}, 5^+_{\ell'+}) \quad = \quad & \frac{2c^S_{\gamma\gamma}}{v} e^{-i\xi} P_\gamma(s_{23}) P_\gamma(s_{45}) [45][23]\langle 24\rangle^2 \\
& + \frac{2c^S_{\gamma\gamma}}{v} e^{i\xi} P_\gamma(s_{45}) P_\gamma(s_{23}) \langle 23\rangle\langle 45\rangle[35]^2 \ ,
\end{aligned}
$$

Summary and Prospect

- The *HVV* amplitudes with *CP* violation in BSM are analyzed in two ways.
- Off-shell way under field theory framework. we decompose helicity amplitudes of $H \to \gamma V \to \gamma \ell\ell$ and $H \to VV \to 4\ell$ into helicity amplitudes of $H \to \gamma\gamma$.
  Two preconditions: 1.The multilinear momentum dependence of the *HVV* vertexes, decompose the vertexes of the overall momentum into a summation of momentum of sub-processe.
  2. The current of J_μ in $V \to \ell^+\ell^-$ is formally proportional to a photon's polarization vector, to replace such a sub-process by an equivalent photon.
- On-shell scattering amplitude approach. Massless propagator, BCFW recursion relations. Massive propagator case, little-group covariant massive-spinor formalism.
- Consistent results through off-shell and on-shell ways.

Convenient for the future CP violation searches in HVV couplings.

Thanks!