Axion, ALP and Dark Photon Effective Field Theory

Bosonic Extention of the SM

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Motivation

Dark matter candidates:

- WIMP ~ 10GeV-TeV
- Axion, ALP $\leq$ GeV
- Dark photon $\leq$ GeV
- Primordial black hole
- ......

Light bosons such as axion, ALP and dark photon can be the candidates of dark matter.
Because the Goldstone is needed when the dark photon is described under the **Stuekelberg mechanism**, thus the EFT treatment of axion, ALP and dark photon can be considered simultaneously.

Origin

Axion is a (pseudoscalar) Goldstone originated from Peccei-Quinn symmetry $U(1)_{PQ}$ breaking to solve the "strong CP problem".[1]

$$U(1)_{PQ} \xrightarrow{\text{break}} a(t,\vec{x}) \xrightarrow{\text{parity transform}} -a(t,-\vec{x})\,. \tag{1}$$

However, such a pseudoscalar field satisfying shift symmetry has been extended to other sectors without the requirement that it is from $U(1)_{PQ}$ break, and is often named as axion-like particle (ALP).

Regardless its UV completion, ALP can be treated as a SM singlet extending the SMEFT.

[1] Phys.Rev.Lett. 38 (1977) 1440-1443, Phys.Rev. D16 (1977) 1791–1797

Effective Lagrangian

The GEFT is the SMEFT with an ALP $a(x)$ extension, its effective Lagrangian takes form that

$$\mathcal{L}_{GEFT} = \mathcal{L}_{SMEFT} + \mathcal{L}_{ALP} + \mathcal{L}_{int} \,, \tag{2}$$

where $\mathcal{L}_{SMEFT}$ is the SMEFT Lagrangian, $\mathcal{L}_{ALP}$ is the effective Lagrangian containing ALP solely,

$$\mathcal{L}_{ALP} = \frac{1}{2}\partial_\mu a \partial^\mu a - V(a) \,, \tag{3}$$

and $\mathcal{L}_{int}$ contains the operators of ALPs and the standard model particles.

Leading Order Lagrangian

The leading terms of $\mathcal{L}_{int}$ are [2]

$$\mathcal{L}_{int} = c_{\widetilde{B}} \frac{a}{f_a} B_{\mu\nu} \widetilde{B}^{\mu\nu} + c_{\widetilde{W}} \frac{a}{f_a} W^a_{\mu\nu} \widetilde{W}^{a\mu\nu} + c_{\widetilde{G}} \frac{a}{f_a} G^\lambda_{\mu\nu} \widetilde{G}^{\lambda\mu\nu}$$
$$+ c_u \frac{\partial_\mu a}{f_a} \left(\overline{u}_R \gamma^\mu u_R\right) + c_d \frac{\partial_\mu a}{f_a} \left(\overline{d}_R \gamma^\mu d_R\right) + c_e \frac{\partial_\mu a}{f_a} \left(\overline{e}_R \gamma^\mu e_R\right)$$
$$+ c_Q \frac{\partial_\mu a}{f_a} \left(\overline{Q}_L \gamma^\mu Q_L\right)_{i,j\neq 1,1} + c_L \frac{\partial_\mu a}{f_a} \left(\overline{L}_L \gamma^\mu L_L\right)_{i\neq j} \tag{4}$$

In terms of the canonical dimension, the leading operators are of dim-5.

[2] Phys. Lett. B 169 (1986) 73–78

Shift Symmetry

The shift symmetry of the Goldstone implies that ALPs interact with other particles via derivatives, but at the renomalizable level, the shift symmetry of the operators involve gauge bosons is implicit.

$$
\begin{aligned}
\frac{a}{f_a} F_{\mu\nu}\tilde{F}^{\mu\nu} &= 2\frac{a}{f_a}\partial_\mu A_\nu \tilde{F}^{\mu\nu} \\
\xrightarrow{\text{Integrat by part}} & -2\frac{\partial_\mu a}{f_a} A_\nu \tilde{F}^{\mu\nu} - \frac{a}{f_a} A_\nu \partial_\mu \tilde{F}^{\mu\nu} \\
\xrightarrow{\text{Bianchi identity}} & -2\frac{\partial_\mu a}{f_a} A_\nu \tilde{F}^{\mu\nu} .
\end{aligned} \tag{5}
$$

Majoron(1)

Considering a special case where the breaking $U(1)$ is $U(1)_{B-L}$ symmetry, the consequent ALP is called majoron. [3]

Type-I seesaw	
UV particles	$(B-L)$
N_R	-1
ϕ	2

There are UV renormalizable operators such as

$$\mathcal{L} \supset -y_R \overline{L}\tilde{H}N_R + \frac{1}{2}\lambda\phi\overline{N_R^c}N_R + h.c. \qquad (6)$$

The VEV of ϕ, $\phi(x) = \frac{1}{\sqrt{2}}(f + \sigma(x) + iJ(x))$, breaks the $U(1)_{B-L}$ spontaneously.

[3] Phys. Lett. B 99 (1981) 411–415

Majoron(2)

If the heavy particles $N_R(x)$ and $\sigma(x)$ are integrated out, we can get effective operators coupled to the light Goldstone $J(x)$ such as

$$c_J J(\overline{L^c}\tilde{H}^*)(\tilde{H}^\dagger L) + h.c. \tag{7}$$

This operator can be transformed to be explicitly shift-symmetric,

$$J(\overline{L^c}\tilde{H}^*)(\tilde{H}^\dagger L) \to \frac{1}{\mu_H^2}\epsilon^{ik}\epsilon^{jl}(D_\mu J)H_k(D_\mu H_l)(L_i L_j)\,. \tag{8}$$

Thus all the majoron effective interactions are a subset of the general ALP Lagrangian, and are dominated by high-dimension (dim-8) operators.

Dark Gauge Symmetry

Dark photon is a massive vector field X_μ, it can be assumed to be gauged under a "dark" $U(1)_X$ group.

SM sector			dynamic mixing	dark sector
$SU(3)_c \times$	$SU(2)_{ew} \times$	$U(1)_Y$		$U(1)_X$
G	W	B	$\epsilon B_{\mu\nu} X^{\mu\nu}$	X

(9)

Including a "dark" higgs S, the unbroken Lagrangian takes the form that

$$\mathcal{L} = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + D_\mu S^\dagger D^\mu S + \mu^2 S^\dagger S - \lambda (S^\dagger S)^2 \,, \tag{10}$$

Higgs Mechanism

According to the Higgs Mechanism, v_S, the VEV of S triggers the spontaneous symmetry breaking. Suppose

$$S = \frac{1}{\sqrt{2}}(v_S + \sigma)\exp(-i\frac{\pi}{v_S}), \tag{11}$$

the broken Lagrangian is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}g^2 v_S^2 \left(X_\mu - \frac{1}{g v_S}\partial_\mu \pi\right)\left(X^\mu - \frac{1}{g v_S}\partial^\mu \pi\right) \\ &+ \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2}2\lambda v_S^2 \sigma^2 - \lambda v \sigma^3 - \frac{1}{4}\lambda \sigma^4 \\ &+ \left(\frac{\sigma}{v} + \frac{\sigma^2}{2v^2}\right)\left(g v X_\mu - \partial_\mu \pi\right)\left(g v X^\mu - \partial^\mu \pi\right)\end{aligned} \tag{12}$$

Stuekelberg Mechanism(1)

Without the assumption that the Dark photon is a gauge boson, it can be described by the Stueckelberg mechanism.
Regardless the UV completion the Lagrangian of an IR massive vector boson should be of Proca form

$$\mathcal{L}_{Proca} = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2} m_X^2 X_\mu X^\mu \,. \tag{13}$$

Under the gauge transformation:

$$X_\mu \to X'_\mu = X_\mu + \partial_\mu \sigma \tag{14}$$

The Lagrangian is not invariant,

$$\mathcal{L} \to \mathcal{L}' = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + m_X^2 (X_\mu - \frac{1}{m_X} \partial_\mu \sigma)(X^\mu - \frac{1}{m_X} \partial^\mu \sigma) \tag{15}$$

Stueckelberg Mechanism(2)

The sollution is to introduce another scalar field to fix the gauge. Considering a scalar field π, satisfying equation of motion that

$$(\partial^2 + m_X^2)\pi = 0$$

the Stueckelberg Lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{Stueck}} = &-\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_X^2(X_\mu - \frac{1}{m_X}\partial_\mu\pi)(X^\mu - \frac{1}{m_X}\partial^\mu\pi) \\ &- \frac{1}{2}(\partial^\mu X_\mu + m_X\pi)(\partial^\nu X_\nu + m_X\pi)\,. \end{aligned} \tag{16}$$

Under the gauge transformation

$$X_\mu \to X_\mu + \partial_\mu\sigma\,, \quad \pi \to \pi + m_X\sigma\,, \tag{17}$$

the first line is invariant, and the second line is not , which is the gauge-fix terms.

Stueckelberg Mechanism(3)

The gauge-fix terms can be generalized to the 't Hooft-like gauge function, and the general Lagrangian takes the form [4]

$$\mathcal{L}'_{\text{Stueck}} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_X^2(X_\mu - \frac{1}{m_X}\partial_\mu\pi)(X^\mu - \frac{1}{m_X}\partial^\mu\pi) - \frac{1}{2\xi}(\partial^\mu X_\mu + \xi m_X\pi)(\partial^\nu X_\nu + \xi m_X\pi)\,. \tag{18}$$

Gauge choices:

- Unitary gauge ($\xi = \infty$):

$$\mathcal{L}'_{\text{Stueck}} \xrightarrow{X_\mu \to X_\mu + \partial_\mu\pi/m_X} \mathcal{L}_{\text{Proca}} \tag{19}$$

- Feynman gauge ($\xi = 1$):

$$\mathcal{L}'_{\text{Stueck}} \to \mathcal{L}_{\text{Stueck}}\,. \tag{20}$$

[4] Int.J.Mod.Phys.A 19 (2004) 3265-3348

Comparison

$$\mathcal{L} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}g^2 v_S^2 \left(X_\mu - \frac{1}{g v_S}\partial_\mu \pi\right)\left(X^\mu - \frac{1}{g v_S}\partial^\mu \pi\right) + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}2\lambda v_S^2\sigma^2 - \lambda v \sigma^3 - \frac{1}{4}\lambda\sigma^4 + \left(\frac{\sigma}{v} + \frac{\sigma^2}{2v^2}\right)(g v X_\mu - \partial_\mu\pi)(g v X^\mu - \partial^\mu \pi) \tag{21}$$

The Stueckelberg Lagrangian (apart the gauge fix term) can be related to the one from Higgs mechanism once we identify

$$m_X^2 = g^2 v_S^2 \,. \tag{22}$$

provided $m_\sigma^2 = \lambda^2 v_S^2$ is so large that σ decouples, which means the self coupling λ of the "dark higgs" S is strong.

Building Blocks

The SMEFT extended by dark photon is XEFT.
Generally, defining $U = \exp(i\pi/m_X)$, and covarint derivative

$$D_\mu U = (\partial_\mu - iX_\mu)U = iU(\frac{1}{m_X}\partial_\mu \pi - X_\mu)\,, \tag{23}$$

the Stueckelberg Lagrangian can be rewritten as

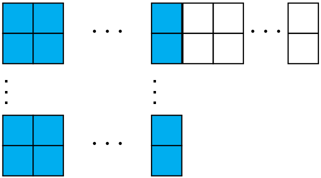
$$\mathcal{L}_{\text{Stueck}} = -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{m_X^2}{2}(D_\mu U)^\dagger(D^\mu U) \tag{24}$$

which is identical with the nonlinear Lagrangian. [5]
Thus within the Stueckelberg mechanism there are two building blocks, a massless vector particle X_μ and a Goldstone $u_\mu = iU^\dagger D_\mu U$ of chiral dimension-1.

[5] JHEP 03 (2008) 047

Independent Lorentz Structures

The independent Lorentz structures are isomorphic to the SSYTs of the primary Young diagram.



$$\rightarrow \prod_{n} \langle ij \rangle \prod_{\tilde{n}} [kl] \,, \tag{25}$$

The shift symmetry implies the corresponding amplitudes satisfy Adler's zero condition,

$$\lim_{p\to 0} \mathcal{M}(p) = 0 \,, \tag{26}$$

Overview

We construct the efective operators of ALP and dark photon upto dim-8. In detail,

- We consider the Majoron as a special case of ALP and find its leading operators are of dimension-8;
- For dark photon, both unitary and Feynman gauge are considered.
- The ALP result is consistent with the Hilbert series method[6];

[6] JHEP 11 (2023) 196

ALP Operators(1)

dim-5 operators					
Class	**Type**	**Real**	**F**	**Axion**	**Majoron**
$F_L^2\phi$	$B_L^2 s$	$sB_{L\mu\nu}B_L{}^{\mu\nu}$		✓	
	$W_L^2 s$	$sW_L{}^I{}_{\mu\nu}W_L{}^{I\mu\nu}$		✓	
	$G_L^2 s$	$sG_L{}^A{}_{\mu\nu}G_L{}^{A\mu\nu}$		✓	
$\psi^2\phi^2$	$e_c H^\dagger L s$	$sH^{\dagger i}(e_{cp}L_{ri})$		✓	
	$d_c H^\dagger Q s$	$sH^{\dagger i}(d_{cp}{}^a Q_{rai})$		✓	
	$HQu_c s$	$\epsilon^{ij} sH_j(Q_{pai}u_{cr}{}^a)$		✓	
ϕ^5	s^5	s^5			
	$HH^\dagger s^3$	$s^3 H_i H^{\dagger i}$			
	$H^2 H^{\dagger 2} s$	$sH_iH_jH^{\dagger i}H^{\dagger j}$			

ALP Operators(2)

Class	Type	Real	F	Axion	Majoron
		***dim*-6 operators**			
$F_L^2\phi^2$	$B_L^2 s^2$	$s^2 B_{L\mu\nu} B_L{}^{\mu\nu}$			
	$W_L^2 s^2$	$s^2 W_L{}^I{}_{\mu\nu} W_L{}^{I\mu\nu}$			
	$G_L^2 s^2$	$s^2 G_L{}^A{}_{\mu\nu} G_L{}^{A\mu\nu}$			
$\phi^3\psi^2$	$H^2 L^2 s$	$\epsilon^{ik}\epsilon^{jl} s H_k H_l (L_{pi} L_{rj})$			
	$e_c H^\dagger L s^2$	$s^2 H^{\dagger i}(e_{cp} L_{ri})$			
	$d_c H^\dagger L s^2$	$s^2 H^{\dagger i}(d_{cp}{}^a Q_{rai})$			
	$H Q u_c s^2$	$\epsilon^{ij} s^2 H_j (Q_{pai} u_{cr}{}^a)$			
$D^2\phi^4$	$D^2 H H^\dagger s^2$	$(D_\mu s)(D^\mu s) H_i H^{\dagger i}$		✓	
ϕ^6	$H^2 H^{\dagger 2} s^2$	$s^2 H_i H_j H^{\dagger i} H^{\dagger j}$			
	$H H^\dagger s^4$	$s^4 H_i H^{\dagger i}$			
	s^6	s^6			

ALP Operators(3)

dim-7 operators					
Class	**Type**	**Real**	**F**	**Axion**	**Majoron**
$F_L^3\phi$	$W_L^3 s$	$\epsilon^{IJK} s {W_L}^I{}_{\mu\nu} {W_L}^{K\lambda\mu} {W_L}^J{}_\lambda{}^\nu$			
	$G_L^3 s$	$f^{ABC} s {G_L}^A{}_{\mu\nu} {G_L}^{C\lambda\mu} {G_L}^B{}_\lambda{}^\nu$			
$DF_L\phi\psi\bar{\psi}$	$DB_L e_c e_c^\dagger s$	${B_L}^{\mu\nu}(D_\nu s)(e_{cp}\sigma_\mu e^\dagger_{cr})$		✓	
	$DB_L L L^\dagger s$	$(D_\nu s){B_L}^{\mu\nu}(L_{pi}\sigma_\mu {L^\dagger}_r{}^i)$		✓	
	$DB_L d_c d_c^\dagger s$	$(D_\nu s){B_L}^{\mu\nu}({d_{cp}}^a\sigma_\mu d^\dagger_{cra})$		✓	
	$DB_L u_c u_c^\dagger s$	$(D_\nu s){B_L}^{\mu\nu}({u_{cp}}^a\sigma_\mu u^\dagger_{cra})$		✓	
	$DB_L Q Q^\dagger s$	$(D_\nu s){B_L}^{\mu\nu}(Q_{pai}\sigma_\mu {Q^\dagger}_r{}^{ai})$		✓	
	$DW_L L L^\dagger s$	${\tau^I}^i_j(D_\nu s){W_L}^{I\mu\nu}(L_{pi}\sigma_\mu {L^\dagger}_r{}^j)$		✓	
	$DW_L Q Q^\dagger s$	${\tau^I}^i_j(D_\nu s){W_L}^{I\mu\nu}(Q_{pai}\sigma_\mu {Q^\dagger}_r{}^{aj})$		✓	
	$DG_L d_c d_c^\dagger s$	${\lambda^A}^b_a(D_\nu s){G_L}^{A\mu\nu}({d_{cp}}^a\sigma_\mu d^\dagger_{crb})$		✓	
	$DG_L u_c u_c^\dagger s$	${\lambda^A}^b_a(D_\nu s){G_L}^{A\mu\nu}({u_{cp}}^a\sigma_\mu u^\dagger_{crb})$		✓	
	$DG_L Q Q^\dagger s$	${\lambda^A}^a_b(D_\nu s){G_L}^{A\mu\nu}(Q_{pai}\sigma_\mu {Q^\dagger}_r{}^{bi})$		✓	

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ALP Operators(4)

dim-8 operators					
Class	**Type**	**Real**	**F**	**Axion**	**Majoron**
$D\phi\psi^3\bar{\psi}$	$Dd_cL^2u_c^\dagger s$	$\epsilon^{ij}s({d_{cp}}^a\sigma_\mu u_{cta}^\dagger)(L_{ri}(D^\mu L_{sj}))$			
		$\epsilon^{ij}(D^\mu s)({d_{cp}}^a L_{ri})(L_{sj}\sigma_\mu u_{cta}^\dagger)$	$\mathcal{Y}[\boxed{r}\boxed{s}]$ / $\mathcal{Y}[\begin{smallmatrix}\boxed{r}\\\boxed{s}\end{smallmatrix}]$	✓	✓
	$Dd_c^2LQ^\dagger s$	$\epsilon_{abc}s({d_{cp}}^a\sigma_\mu {Q^\dagger}_t{}^{ci})({d_{cr}}^b(D^\mu L_{si}))$			
		$\epsilon_{abc}(D^\mu s)({d_{cp}}^a L_{si})({d_{cr}}^b\sigma_\mu {Q^\dagger}_t{}^{ci})$	$\mathcal{Y}[\boxed{p}\boxed{r}]$ / $\mathcal{Y}[\begin{smallmatrix}\boxed{p}\\\boxed{r}\end{smallmatrix}]$	✓	✓
	$Dd_c^3e_c^\dagger s$	$\epsilon_{abc}s({d_{cp}}^a\sigma_\mu e_{ct}^\dagger)({d_{cr}}^b(D^\mu {d_{cs}}^c))$			
		$\epsilon_{abc}(D^\mu s)({d_{cp}}^a {d_{cr}}^b)({d_{cs}}^c\sigma_\mu e_{ct}^\dagger)$		✓	✓
$D^2F_L^2\phi^2$	$D^2B_L^2s^2$	$(D_\mu s)(D^\mu s)B_{L\nu\lambda}{B_L}^{\lambda\nu}$		✓	
	$D^2W_L^2s^2$	$(D_\mu s)(D^\mu s){W_L}^I{}_{\nu\lambda}{W_L}^{I\lambda\nu}$		✓	
	$D^2G_L^2s^2$	$(D_\mu s)(D^\mu s){G_L}^A{}_{\nu\lambda}{G_L}^{A\lambda\nu}$		✓	

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Dark Photon Operarors(1)

dim-4 operators				
Class	**Type**	**Stueckelberg**	**Unitary**	**F**
F_L^2	$X_L B_L$	$B_{L\mu\nu} X_L{}^{\mu\nu}$	✓	
$u\psi\bar{\psi}$	$u e_c e_c^\dagger$	$(e_{cp}\sigma^\mu e^\dagger_{cr})u_\mu$	$(e_{cp}\sigma^\mu e^\dagger_{cr})X_\mu$	
	$uLL^\dagger$	$(L_{p_i}\sigma^\mu L^\dagger{}_r{}^i)u_\mu$	$(L_{p_i}\sigma^\mu L^\dagger{}_r{}^i)X_\mu$	
	$uQQ^\dagger$	$(Q_{p_{ai}}\sigma^\mu Q^\dagger{}_r{}^{ai})u_\mu$	$(Q_{p_{ai}}\sigma^\mu Q^\dagger{}_r{}^{ai})X_\mu$	
	$u d_c d_c^\dagger$	$(d_{cp}{}^a\sigma^\mu d^\dagger_{cra})u_\mu$	$(d_{cp}{}^a\sigma^\mu d^\dagger_{cra})X_\mu$	
	$u u_c u_c^\dagger$	$(u_{cp}{}^a\sigma^\mu u^\dagger_{cra})u_\mu$	$(u_{cp}{}^a\sigma^\mu u^\dagger_{cra})X_\mu$	
$u^2\phi^2$	$u^2 H H^\dagger$	$H_i H^{\dagger i} u_\mu u^\mu$	$H_i H^{\dagger i} X_\mu X^\mu$	
u^4	u^4	$u_\mu u^\mu u_\nu u^\nu$	$X_\mu X^\mu X_\nu X^\nu$	

Dark Photon Operarors(2)

dim-6 operators				
Class	**Type**	**Stueckelberg**	**Unitary**	**F**
$F_L^2\phi^2$	$X_L B_L H H^\dagger$	$H_i {H^\dagger}^i B_{L\mu\nu} {X_L}^{\mu\nu}$	✓	
	$X_L W_L H H^\dagger$	${\tau^I}^i_j H_i {H^\dagger}^j {W_L}^I{}_{\mu\nu} {X_L}^{\mu\nu}$	✓	
	$X_L^2 H H^\dagger$	$H_i {H^\dagger}^i X_{L\mu\nu} {X_L}^{\mu\nu}$	✓	
$F_L\phi\psi^2$	$X_L e_c H^\dagger L$	${H^\dagger}^i X_{L\mu\nu}(e_{cp}\sigma^{\mu\nu} L_{ri})$	✓	
	$X_L d_c H^\dagger Q$	${H^\dagger}^i X_{L\mu\nu}({d_{cp}}^a \sigma^{\mu\nu} Q_{rai})$	✓	
	$X_L H Q u_c$	$\epsilon^{ij} H_j X_{L\mu\nu}(Q_{pai}\sigma^{\mu\nu} {u_{cr}}^a)$	✓	
$u\phi^2\psi\bar{\psi}$	$uHH^\dagger e_c e_c^\dagger$	$H_i {H^\dagger}^i (e_{cp}\sigma^\mu e^\dagger_{cr}) u_\mu$	$H_i {H^\dagger}^i (e_{cp}\sigma^\mu e^\dagger_{cr}) X_\mu$	
	$uHH^\dagger LL^\dagger$	$H_j {H^\dagger}^i (L_{p_i}\sigma^\mu {L^\dagger}_r{}^j) u_\mu$	$H_j {H^\dagger}^i (L_{p_i}\sigma^\mu {L^\dagger}_r{}^j) X_\mu$	
		$H_j {H^\dagger}^j (L_{p_i}\sigma^\mu {L^\dagger}_r{}^i) u_\mu$	$H_j {H^\dagger}^j (L_{p_i}\sigma^\mu {L^\dagger}_r{}^i) X_\mu$	
	$uHH^\dagger QQ^\dagger$	$H_j {H^\dagger}^i (Q_{p_{ai}}\sigma^\mu {Q^\dagger}_r{}^{aj}) u_\mu$	$H_j {H^\dagger}^i (Q_{p_{ai}}\sigma^\mu {Q^\dagger}_r{}^{aj}) X_\mu$	
		$H_j {H^\dagger}^j (Q_{p_{ai}}\sigma^\mu {Q^\dagger}_r{}^{ai}) u_\mu$	$H_j {H^\dagger}^j (Q_{p_{ai}}\sigma^\mu {Q^\dagger}_r{}^{ai}) X_\mu$	
	$uHH^\dagger d_c d_c^\dagger$	$H_i {H^\dagger}^i ({d_{cp}}^a \sigma^\mu d^\dagger_{cra}) u_\mu$	$H_i {H^\dagger}^i ({d_{cp}}^a \sigma^\mu d^\dagger_{cra}) X_\mu$	
	$uHH^\dagger u_c u_c^\dagger$	$H_i {H^\dagger}^i ({u_{cp}}^a \sigma^\mu u^\dagger_{cra}) u_\mu$	$H_i {H^\dagger}^i ({u_{cp}}^a \sigma^\mu u^\dagger_{cra}) X_\mu$	

Dark Photon Operarors(3)

dim-7 operators

Class	Type	Stueckelberg	Unitary	F
$F_L\phi^2\psi^2$	$X_LH^2L^2$	$\epsilon^{ik}\epsilon^{jl}H_kH_lX_{L\mu\nu}(L_{p_i}\sigma^{\mu\nu}L_{rj})$	✓	
$u\psi^3\bar{\psi}$	$ud_cL^2u_c^\dagger$	$\epsilon^{ij}(u^\dagger_{cta}\overline{\sigma}_\nu d_{cp}{}^a)(L_{ri}\sigma^{\nu\mu}L_{sj})u_\mu$	$\epsilon^{ij}(u^\dagger_{cta}\overline{\sigma}_\nu d_{cp}{}^a)(L_{ri}\sigma^{\nu\mu}L_{sj})X_\mu$	
		$\epsilon^{ij}(u^\dagger_{cta}\overline{\sigma}^\mu d_{cp}{}^a)(L_{ri}L_{sj})u_\mu$	$\epsilon^{ij}(u^\dagger_{cta}\overline{\sigma}^\mu d_{cp}{}^a)(L_{ri}L_{sj})X_\mu$	
	$ud_c^2LQ^\dagger$	$\epsilon_{abc}(Q^\dagger{}_t{}^{ci}\overline{\sigma}^\mu d_{cp}{}^a)(d_{cr}{}^bL_{si})u_\mu$	$\epsilon_{abc}(Q^\dagger{}_t{}^{ci}\overline{\sigma}^\mu d_{cp}{}^a)(d_{cr}{}^bL_{si})X_\mu$	$\mathcal{Y}[\boxed{p}\boxed{r}]$ $\mathcal{Y}\left[\begin{smallmatrix}\boxed{p}\\ \boxed{r}\end{smallmatrix}\right]$
	$ud_c^3e_c^\dagger$	$\epsilon_{abc}(e^\dagger_{ct}\overline{\sigma}^\mu d_{cp}{}^a)(d_{cr}{}^bd_{cs}{}^c)u_\mu$	$\epsilon_{abc}(e^\dagger_{ct}\overline{\sigma}^\mu d_{cp}{}^a)(d_{cr}{}^bd_{cs}{}^c)X_\mu$	
$Du\phi^2\psi^2$	DuH^2L^2	$\epsilon^{ik}\epsilon^{jl}H_kH_l((D^\mu L_{p_i})L_{rj})u_\mu$	$\epsilon^{ik}\epsilon^{jl}H_kH_l((D^\mu L_{p_i})L_{rj})X_\mu$	$\mathcal{Y}[\boxed{p}\boxed{r}]$ $\mathcal{Y}\left[\begin{smallmatrix}\boxed{p}\\ \boxed{r}\end{smallmatrix}\right]$
		$\epsilon^{ik}\epsilon^{jl}H_l(D^\mu H_k)(L_{p_i}L_{rj})u_\mu$	$\epsilon^{ik}\epsilon^{jl}H_l(D^\mu H_k)(L_{p_i}L_{rj})X_\mu$	
		$\epsilon^{ik}\epsilon^{jl}H_l(D_\nu H_k)(L_{p_i}\sigma^{\nu\mu}L_{rj})u_\mu$	$\epsilon^{ik}\epsilon^{jl}H_l(D_\nu H_k)(L_{p_i}\sigma^{\nu\mu}L_{rj})X_\mu$	
$u^2\phi^2\psi^2$	$u^2H^2L^2$	$\epsilon^{ik}\epsilon^{jl}H_kH_l(L_{p_i}L_{rj})u_\mu u^\mu$	$\epsilon^{ik}\epsilon^{jl}H_kH_l(L_{p_i}L_{rj})X_\mu X^\mu$	

Dark Photon Operarors(4)

dim-8 operators				
Class	**Type**	**Stueckelberg**	**Unitary**	**F**
$F_L\phi^3\psi^2$	$X_LHH^{\dagger 2}Le_c$	$H_jH^{\dagger i}H^{\dagger j}X_{L\mu\nu}(e_{cp}\sigma^{\mu\nu}L_{ri})$	✓	
	$X_LHH^{\dagger 2}Qd_c$	$H_jH^{\dagger i}H^{\dagger j}X_{L\mu\nu}(d_{cp}{}^a\sigma^{\mu\nu}Q_{rai})$	✓	
	$X_LH^2H^{\dagger}Qu_c$	$\epsilon^{ij}H_jH_kH^{\dagger k}X_{L\mu\nu}(Q_{pai}\sigma^{\mu\nu}u_{cr}{}^a)$	✓	
$F_L\psi^4$	$X_Le_cu_c^2$	$\epsilon_{abc}X_{L\mu\nu}(d_{cp}{}^a e_{cr})(u_{cs}{}^b\sigma^{\mu\nu}u_{ct}{}^c)$	✓	$\mathcal{Y}[\boxed{s}\boxed{t}]$
		$\epsilon_{abc}X_{L\mu\nu}(d_{cp}{}^a\sigma_\lambda{}^\nu e_{cr})(u_{cs}{}^b\sigma^{\mu\lambda}u_{ct}{}^c)$	✓	$\mathcal{Y}[\boxed{s}\boxed{t}]$
		$\epsilon_{abc}X_{L\mu\nu}(d_{cp}{}^a\sigma^{\mu\nu}e_{cr})(u_{cs}{}^b u_{ct}{}^c)$	✓	$\mathcal{Y}\left[\begin{smallmatrix}\boxed{s}\\ \boxed{t}\end{smallmatrix}\right]$
	$X_Le_cLQu_c$	$\epsilon^{ij}X_{L\mu\nu}(e_{cp}\sigma^{\mu\nu}L_{ri})(Q_{saj}u_{ct}{}^a)$	✓	
		$\epsilon^{ij}X_{L\mu\nu}(e_{cp}L_{ri})(Q_{saj}\sigma^{\mu\nu}u_{ct}{}^a)$	✓	
		$\epsilon^{ij}X_{L\mu\nu}(e_{cp}\sigma_\lambda{}^\nu L_{ri})(Q_{saj}\sigma^{\mu\lambda}u_{ct}{}^a)$	✓	
	$X_Ld_cQ^2u_c$	$\epsilon^{ij}X_{L\mu\nu}(d_{cp}{}^a\sigma^{\mu\nu}Q_{rbi})(Q_{saj}u_{ct}{}^b)$	✓	$\mathcal{Y}[\boxed{r}\boxed{s}]$
				$\mathcal{Y}\left[\begin{smallmatrix}\boxed{r}\\ \boxed{s}\end{smallmatrix}\right]$
		$\epsilon^{ij}X_{L\mu\nu}(d_{cp}{}^a Q_{rbi})(Q_{saj}\sigma^{\mu\nu}u_{ct}{}^b)$	✓	$\mathcal{Y}[\boxed{r}\boxed{s}]$
				$\mathcal{Y}\left[\begin{smallmatrix}\boxed{r}\\ \boxed{s}\end{smallmatrix}\right]$

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As extension of the standard model, the ALPs and the dark photon can regarded as candidates of the dark matter, and mediaters between the standard model particles and the dark matter.
Treating them as low-energy degrees of freedom, we develop the EFT descriptions of them, including their building blocks, symmetries and power-counting scheme.
In particular, we construct the effective operators involve ALP and dark photon upto dimension-8, which will play an important role in the high-loop calculations and high luminosity experiments.

Thank you!