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# Electroweak corrections to double Higgs production at the LHC

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January 22nd, 2024

The 27th LHC Mini-Workshop



中山大學  
SUN YAT-SEN UNIVERSITY

Based on: arxiv: 2311.16963

In cooperation with:

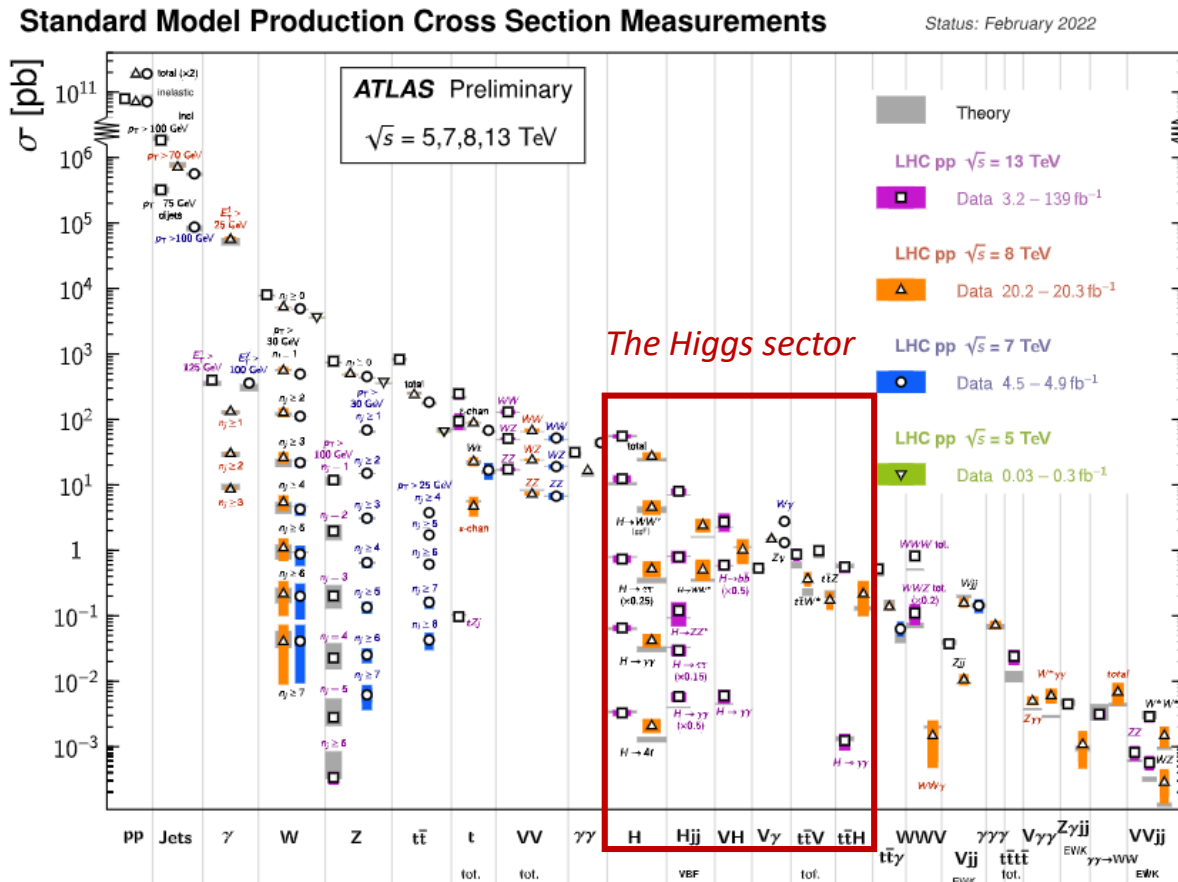
Huan-Yu Bi, Li-Hong Huang, Rui-Jun Huang, Yan-Qing Ma

# Introduction to Higgs



- Discovery of Higgs boson(2012,LHC): the last found elementary particle in SM
- Experiments at the ATLAS and CMS: consistent with the results predicted by SM

Azzurri: *Int.J.Mod.Phys.A* 38 (2023) 09n10, 23300077

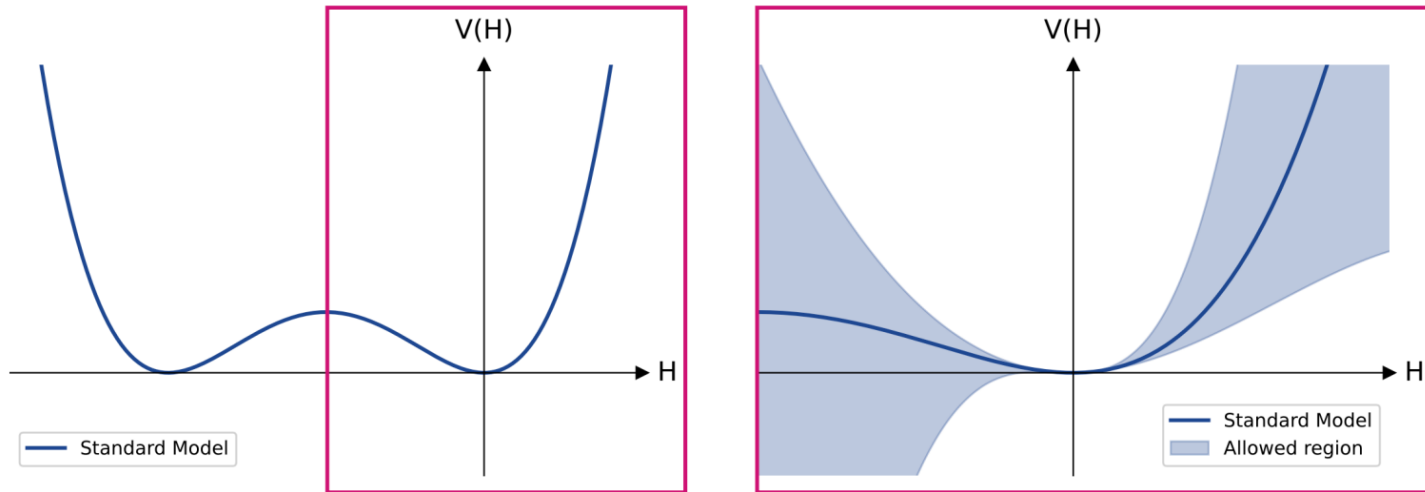


# Higgs Potential



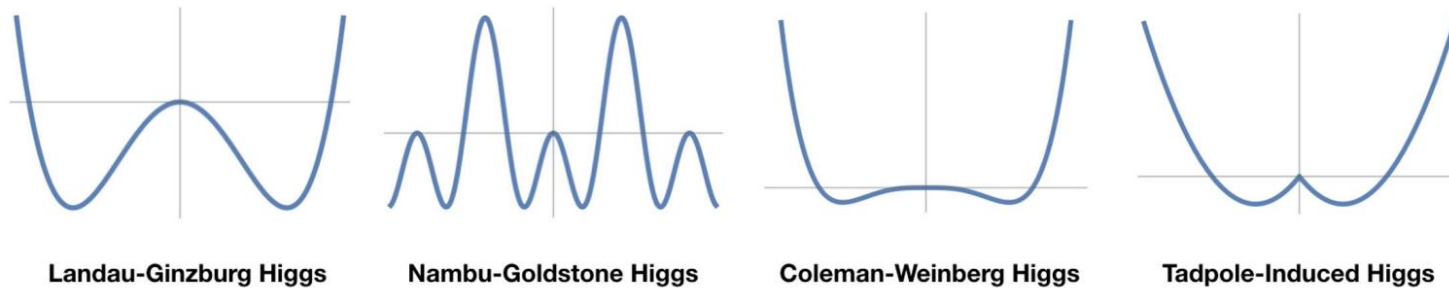
- **Problems not clear:** shape of Higgs potential, new physics beyond SM...

Plot taken from Moser: [Higgs 2023](#)



[using current ATLAS limits @ 95% CL]

[Agrawal et al: 1907.02078](#)



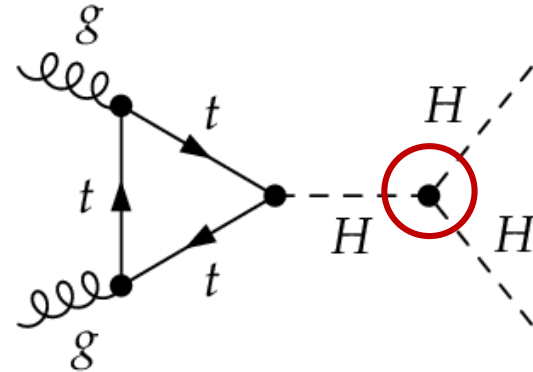
Higgs potential predicted by other BSM theories.

# Higgs trilinear coupling



- Higgs potential is probed through determining the strength of Higgs boson self-interactions in searches for HH production. ( $\lambda^{SM} \approx 1/8$ )

$$V(h) = \frac{m_h^2}{2} h^2 + \lambda^{SM} v h^3 + \frac{1}{4} \lambda^{SM} h^4$$



- Experiment constraints on Higgs boson self-interactions:

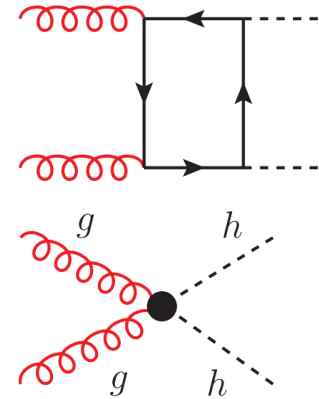
ATLAS: 2007.02873

CMS: 2202.09617

Jones: LHEP 2023 (2023) 442

- Current:  $-1.5 < \lambda_{hhh}^{EX}/\lambda^{SM} < 6.7$  for ATLAS,  $-2.3 < \lambda_{hhh}^{EX}/\lambda^{SM} < 9.4$  for CMS.
- Future: a limit of  $-0.5 < \lambda_{hhh}^{EX}/\lambda^{SM} < 1.5$  will be achieved.

# QCD corrections status



Ajjath et al:2209.03914

- QCD corrections
  - NLO QCD with full top-quark mass dependence, [Borowka et al:1604.06447](#)
  - NLO QCD matched to parton shower, [Heinrich et al:1703.09252](#)
  - NLO QCD with soft-gluon resummation, [Ferrera et al: 1609.01691](#)
  - NNLO QCD in heavy-top limit (**HTL**) approximation, [Florian et al:1305.5206](#)
  - NNLO in **HTL**+ NLO with full top-quark mass dependence, [Florian et al:2106.14050](#)
  - NNLO QCD in **HTL** matched to parton shower, [Alioli et al: 2212.10489](#)
  - NNNLO QCD in **HTL**, [Chen et al:1909.06808](#)
  - NNNLO in **HTL** include the top-quark mass effects, [Chen et al:1912.13001](#)
  - NNNLO in **HTL**+ NLO with full top-quark mass dependence + soft-gluon resummation,

- Current theoretical uncertainties:  $\mathcal{O}(1\%)$  [Jones: LHEP 2023 \(2023\) 442](#)

Process	QCD	$\sigma_{th}[pb]$	$\delta_{th}[\%]$
HH production via gg fusion	$N^3LO_{HTL}$ $NLO_{QCD}$	0.03105	+2.2 -5.0

# Why EW corrections

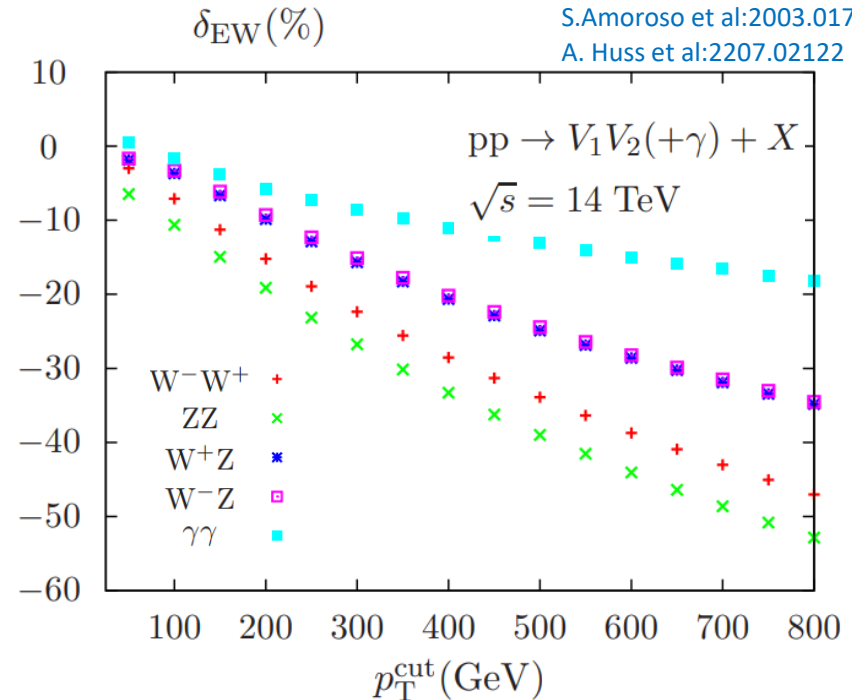
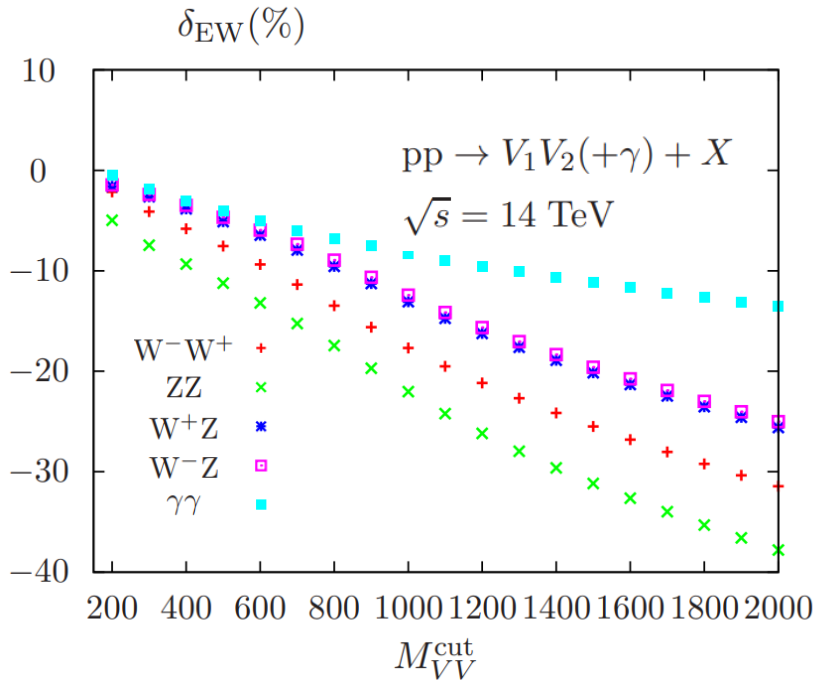


- EW corrections

- $\alpha \sim \mathcal{O}(1\%)$ , the biggest uncertainty from theoretical side!
- Sudakov enhancement,  $\mathcal{O}(10\% \sim 30\%)$  corrections at high energy region. [A Bierweiler et al:1305.5402](#)
- NLO EW corrections are crucial**, a focal point in 2015, 2017, 2019 and 2021 Les Houches

precision wish lists

[J.R. Anderson et al:1605.04692](#)  
[Les Houches 2017:1803.07977](#)  
[S.Amoroso et al:2003.01700](#)  
[A. Huss et al:2207.02122](#)



# EW corrections status



- Partial results
  - two-loop box diagrams, [Davies et al:2207.02587](#)
  - top-quark Yukawa corrections, [Muhlleitner et al:2207.02524](#)
  - Higgs self-coupling corrections, [Borowka et al: 1811.12366](#)
- Most recent work: [Davies et al: 2308.01355](#)
  - HTL and the convergent behavior is not good.
  - Neglecting diagrams with massless fermion loops.
  - Only prediction at the matrix element squared level.
- Groups working on this topic: <https://indico.ihep.ac.cn/event/18025/>
  - See talk by KIT group at Higgs 2023: [HTL + partial results](#)
  - See talk by ShanDong University group at Higgs 2023: [partial results](#)
  - See talk by Durham University group at Higgs 2023: [partial results](#)

# Production Rate

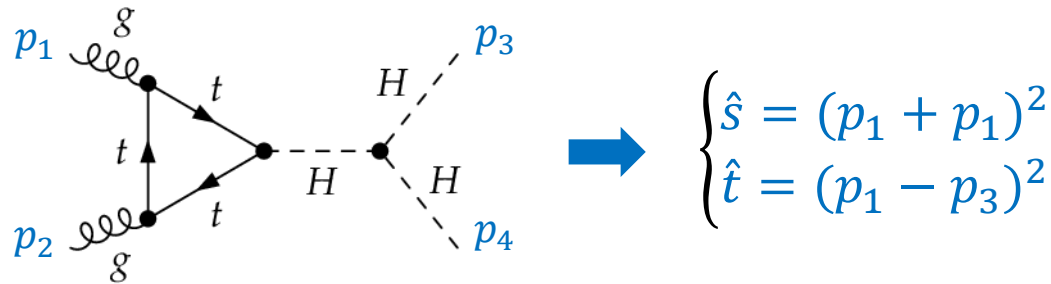


- Di-Higgs production cross section:

$$\sigma(pp \rightarrow HH) = \int dx_1 dx_2 f_g(x_1) f_g(x_2) \hat{\sigma}_{gg \rightarrow HH}(\hat{s}, m^2)$$

gluon PDF

- Kinematics invariants and mass scales include  $m_h, m_t, m_W, m_Z, \hat{s}, \hat{t}$ ,



- Multiple mass scales, analytic result for  $\hat{\sigma}$  is challenging. ❌
- Monte Carlo integration method can be adopted. ✅

$$\int dx_1 dx_2 f_g(x_1) f_g(x_2) d\hat{t} \frac{d\hat{\sigma}}{d\hat{t}}(\hat{s}, \hat{t}) = \sum_{i,j} \Delta_{i,j} \times \frac{d\hat{\sigma}}{d\hat{t}}(\hat{s}_i, \hat{t}_j)$$

- Lots of numerical results for  $d\hat{\sigma}/d\hat{t}$  at different phase space points are required.

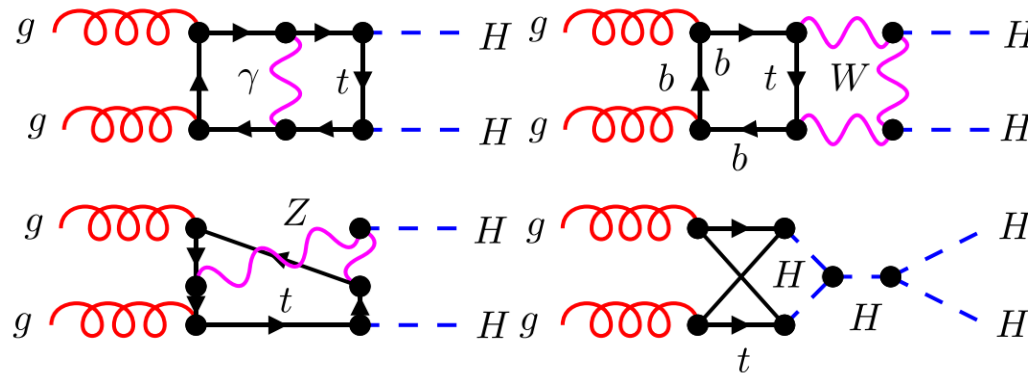


# Calculation procedure



- Numerical results for  $d\hat{\sigma}/d\hat{t}$  at a phase space point  $(\hat{s}_i, \hat{t}_j)$  can be evaluated using following steps:

- Generate Feynman amplitudes: **8 LO diagrams, 2020 NLO diagrams** T. Hahn:0012260



- Manipulate amplitudes to obtain scalar integrals.
- Reduce scalar integrals to master integrals.
- Calculate master integrals.
- Remove divergence via renormalization

CalcLoop  
Blade  
AMFlow

ABC trilogy for  
multi-loop calculation



<https://gitlab.com/multiloop-pku>

# Manipulate amplitudes



- Amplitudes for  $g(p_1)g(p_2) \rightarrow H(p_3)H(p_4)$ :

$$M_{ab} = \delta_{ab} \epsilon_1^\mu \epsilon_2^\nu M_{\mu\nu}$$

- Decomposition to form factor

$$M_{\mu\nu} = F_1(\hat{s}, \hat{t}, m^2) T_1^{\mu\nu} + F_2(\hat{s}, \hat{t}, m^2) T_2^{\mu\nu}$$

- Decomposition to scalar integrals

$$F_i(\hat{s}, \hat{t}, m^2) = \sum_j C_{i,j}(\hat{s}, \hat{t}, m^2) \times \boxed{I_{i,j}(\hat{s}, \hat{t}, m^2)}$$

To be reduced

To be calculated, results for  $\mathcal{O}(10^4)$

$(\hat{s}_i, \hat{t}_i)$  are required

- Reduction to master integrals

$$I_{i,j,k}(\hat{s}, \hat{t}, m^2) = \sum_k P_{i,j,k}(\hat{s}, \hat{t}, m^2) \times \boxed{M_{i,j,k}(\hat{s}, \hat{t}, m^2)}$$

# Calculate integrals



- Calculate integrals for a phase space point
  - Mass parameters input:  $\frac{m_H^2}{m_t^2} = \frac{12}{23}, \frac{m_Z^2}{m_t^2} = \frac{23}{83}, \frac{m_W^2}{m_t^2} = \frac{14}{65}, m_t^2 = 1$
  - Only two dimensional regulator  $\epsilon$  points ( $\epsilon = \pm \frac{1}{1000}$ ) are required, efficiency enhanced
  - 3000 cpu.h run time
- Calculate integrals for  $\mathcal{O}(10^4)$  phase space points
  - $3 \times 10^4$  phase space points
  - $\mathcal{O}(10^8)$  cpu.h run time with all points calculated by AMFlow
  - Differential equation running method,  $\mathcal{O}(10^5)$  cpu.h run time

# Total cross sections



- Input parameters

- $\alpha = \frac{\sqrt{2}}{\pi} G_\mu m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right) \quad m_t = 172.69 \text{ GeV} \quad \alpha = 7.512 \times 10^{-3}$

- $G_\mu = 1.166378 \times 10^{-5} \text{ GeV}^{-2}$

- NNPDF3.1 PDF set

- NNPDF31\_nlo\_αs\_0118 for both LO and NLO calculations

- Renormalization

- On-shell renormalization for masses and fields

- $G_\mu$ -scheme renormalization for electromagnetic coupling [A. Denner: 1912.06823](#)

- Results:  $1.8 \times 10^4$  events

- observed differences with varying  $\mu$  are around 20%

- Correction factor  $[-4.6\%, -4.3\%]$  for different  $\mu$ ,  $\mathcal{O}(0.1\%)$  EW corrections beyond NLO

$\mu$	$M_{HH}/2$	$\sqrt{p_T^2 + m_H^2}$	$m_H$
LO	19.96(6)	21.11(7)	25.09(8)
NLO	19.12(6)	20.21(6)	23.94(8)
$\mathcal{K}$ -factor	0.958(1)	0.957(1)	0.954(1)

- $-4\%$  NLO EW corrections

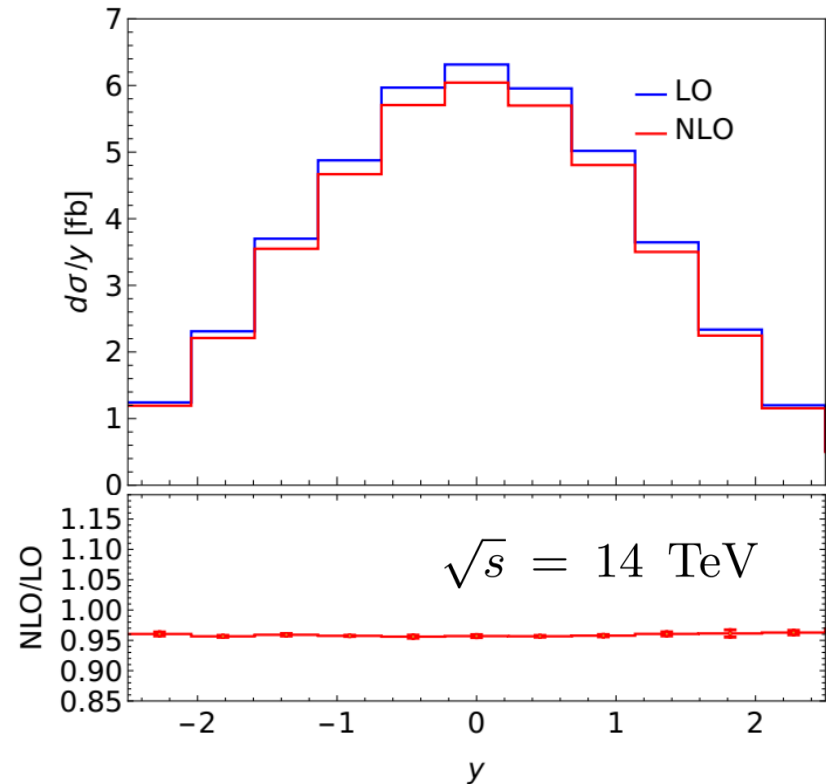
# $y$ differential distribution



- The differential K factor can get a controllable error with far fewer events

$$\sigma^{\text{NLO}} = K \times \sigma^{\text{LO}}$$

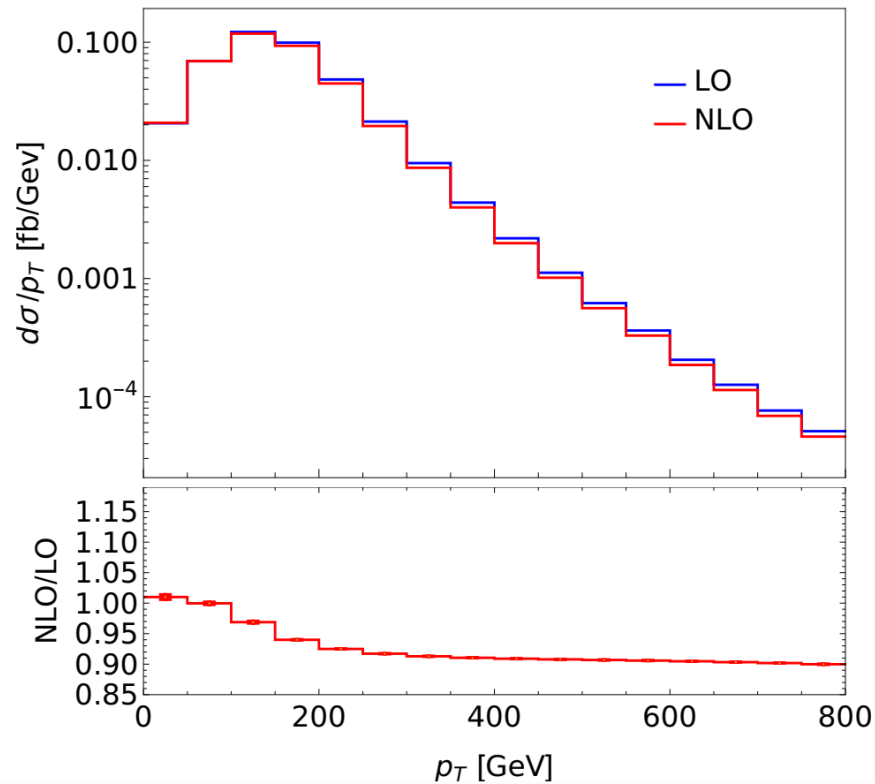
- K uses  $1.8 \times 10^4$  events for  $\sigma$  and additional 400 events for each bin
- $\sigma^{\text{LO}}$  uses  $3 \times 10^5$  events
- Up to NLO,  $K \approx 0.96$



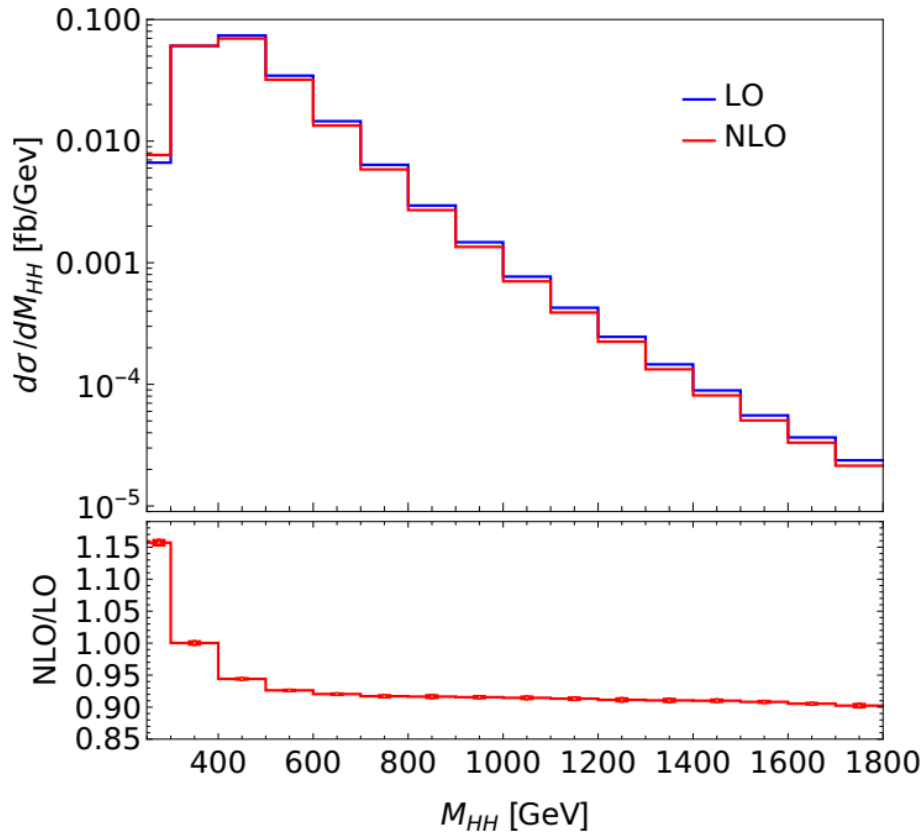
# $p_T$ differential distribution



- $-10\%$  NLO corrections in the tail



# $M_{HH}$ differential distribution



- +15% NLO corrections at the beginning of spectrum
- -10% corrections in the tail, similar to  $p_T$  differential distribution
- Sudakov enhancement

# Summary



- Higgs trilinear coupling is important
- Differential equation running method is efficient
- NLO EW corrections to total cross sections is about  $-4\%$
- $-4\%$  NLO EW corrections to rapidity distribution
- $+15\%$  NLO corrections at the beginning of spectrum for the  $M_{HH}$ , Sudakov effect was observed for both  $p_T$  and  $M_{HH}$  distribution
- The uncertainties from  $N^2$ LO EW corrections are overall about  $\mathcal{O}(0.1\%)$ , sufficient precision from current QCD corrections and NLO EW corrections for measurements at HL-LHC

Thanks for your attention!