



# HIAF能区重离子碰撞中超子产生和超核物理研究

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**School of Physics and Optoelectronics, South China University  
of Technology, Guangzhou**

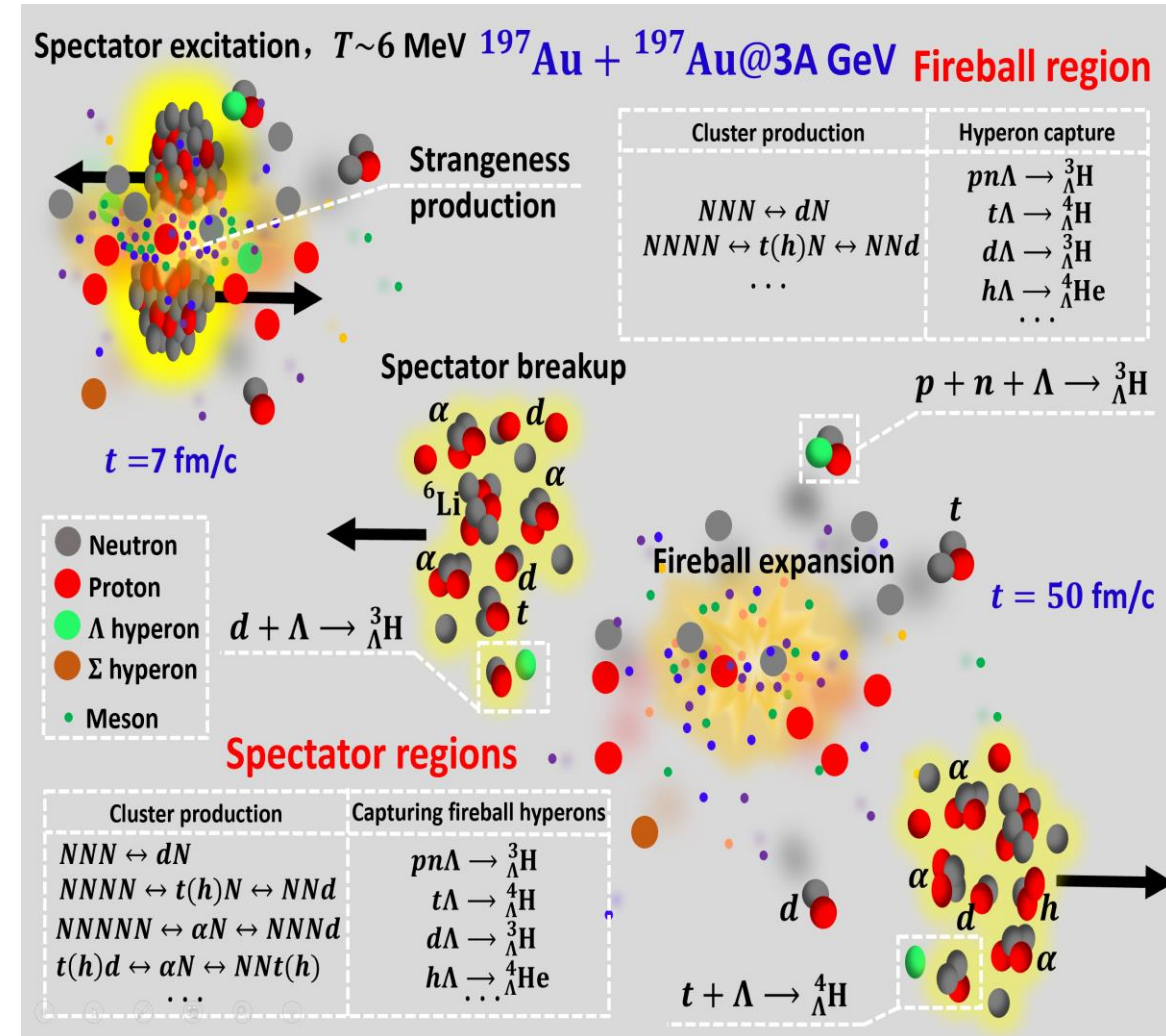
**\*Email: [fengzhq@scut.edu.cn](mailto:fengzhq@scut.edu.cn)**

**Collaborators: Hui-Gan Cheng and Si-Na Wei**



# Outline

- Hyperon, cluster and hypernuclear cluster production via heavy-ion collision and hadron induced reactions-  
**overview**
- LQMD transport model and recognizing cluster
- Fragmentation reaction and hyperfragment production
- Summary and perspective



# I. Progress on hyperon, nuclear cluster and hypernuclear cluster production via heavy-ion collisions

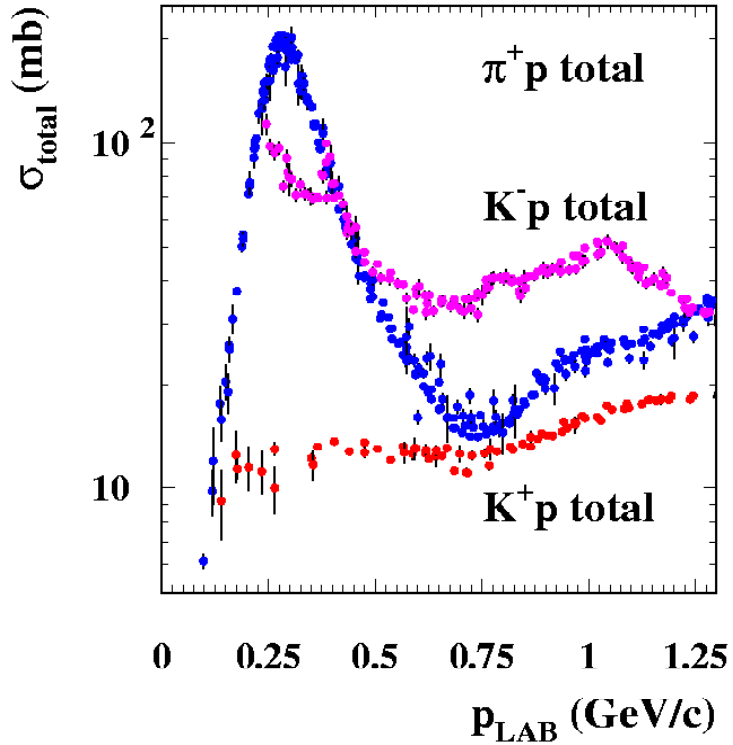


## Strange particles production in hadron-hadron collisions

科学通报 2018年 第63卷 第8期: 735-744

中高能重离子碰撞中奇异粒子产生和超核形成机制

冯北庆

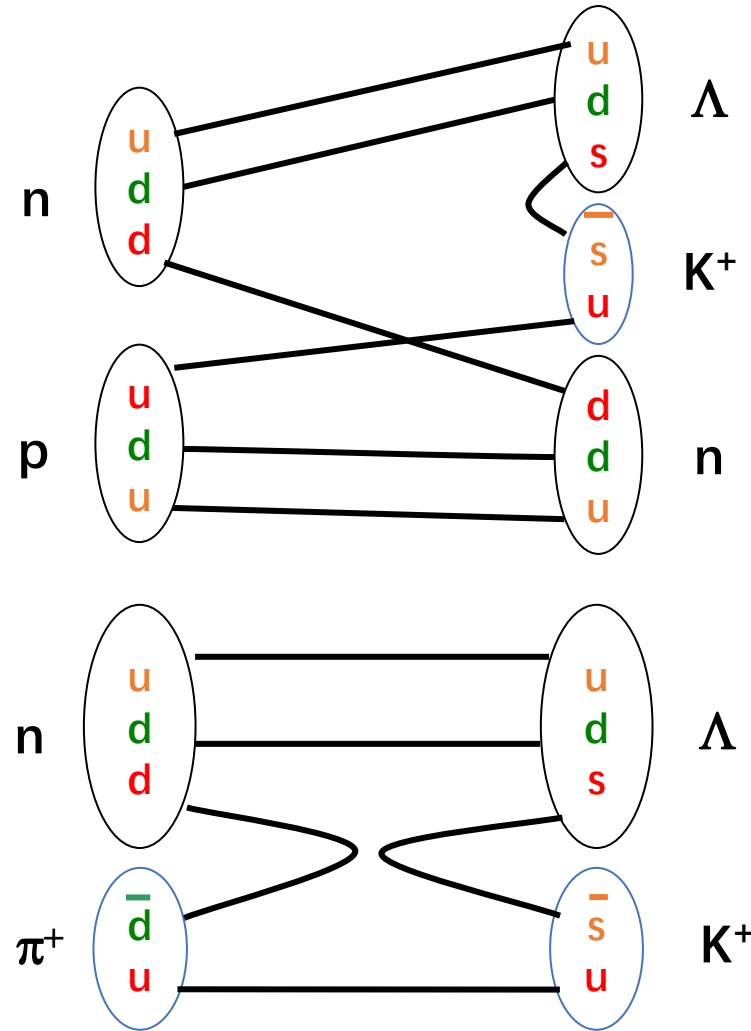


mean free path at  $\rho_0$ :

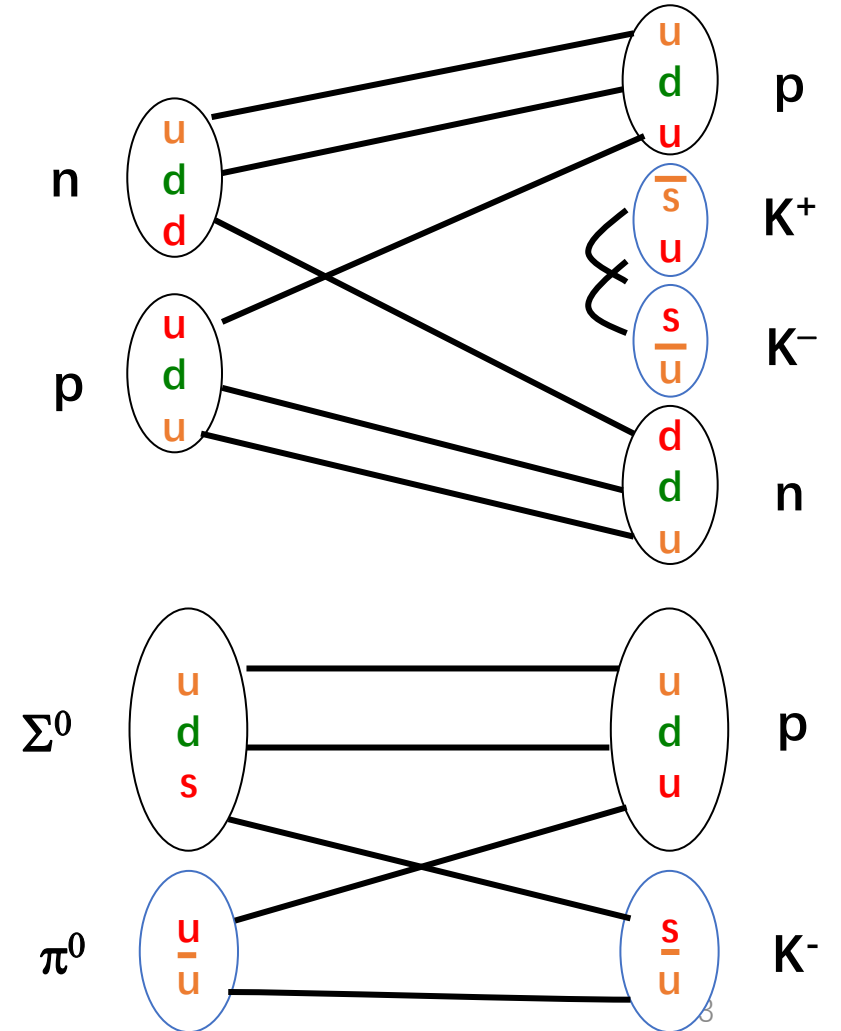
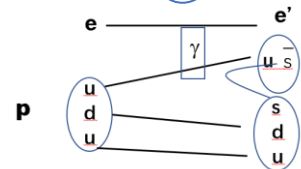
$$\lambda(\pi) = 0.3 \text{ fm}$$

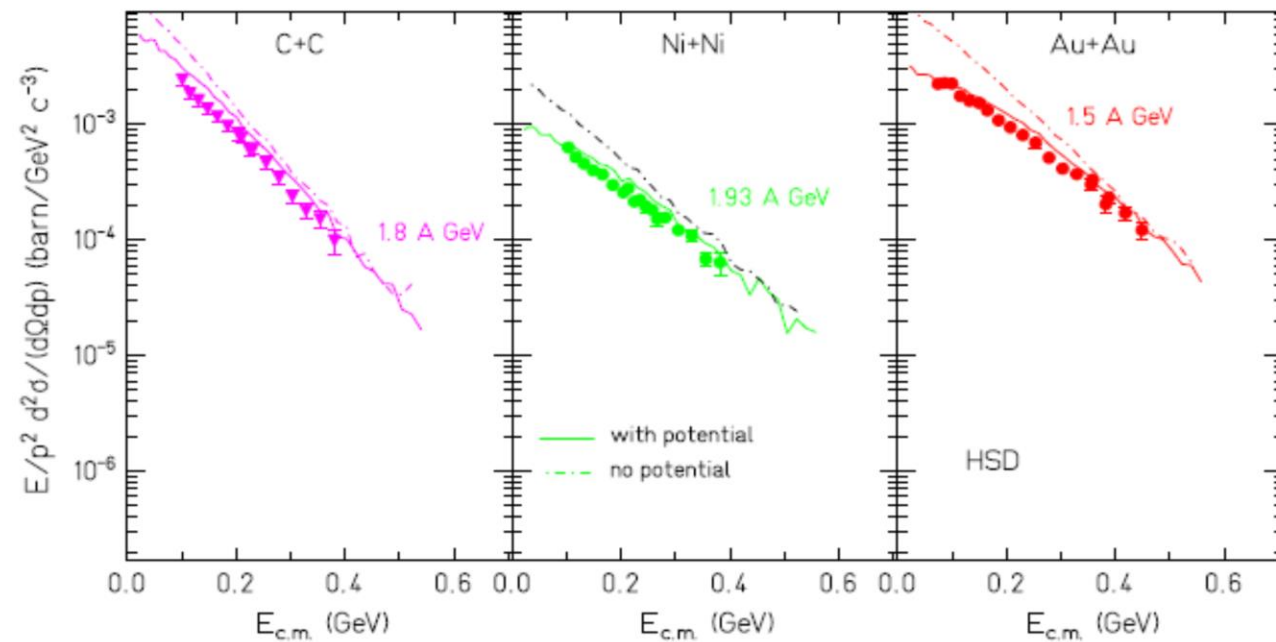
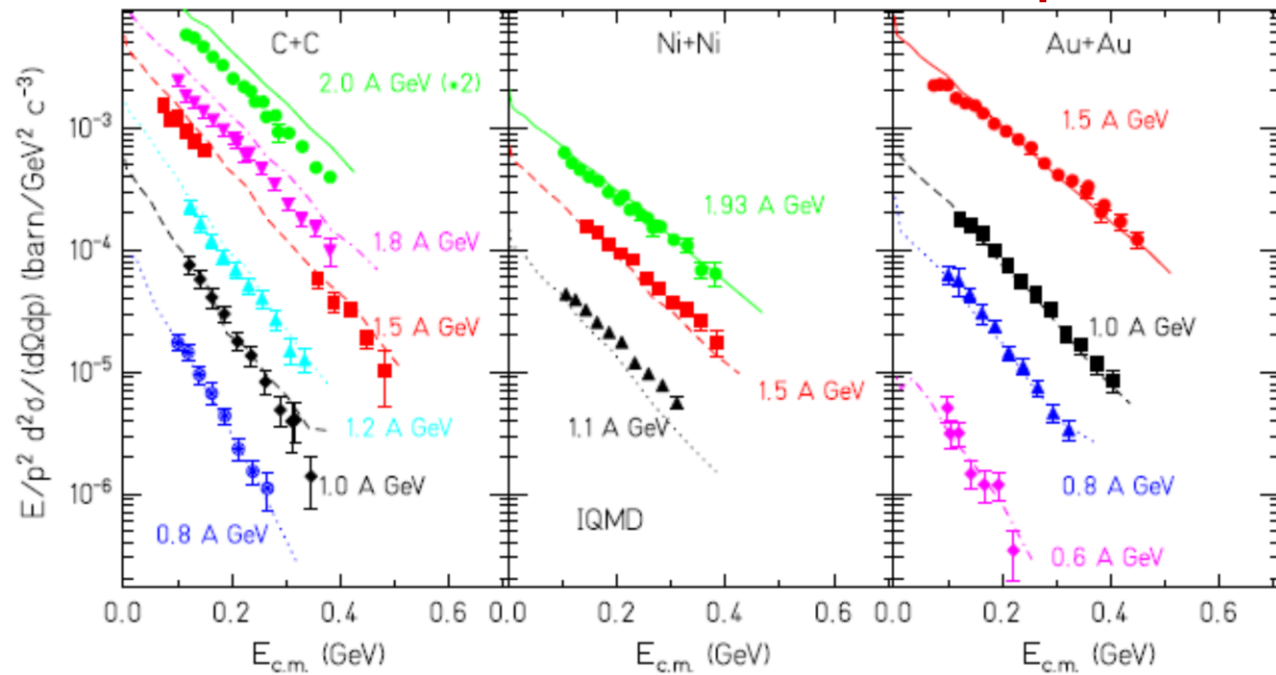
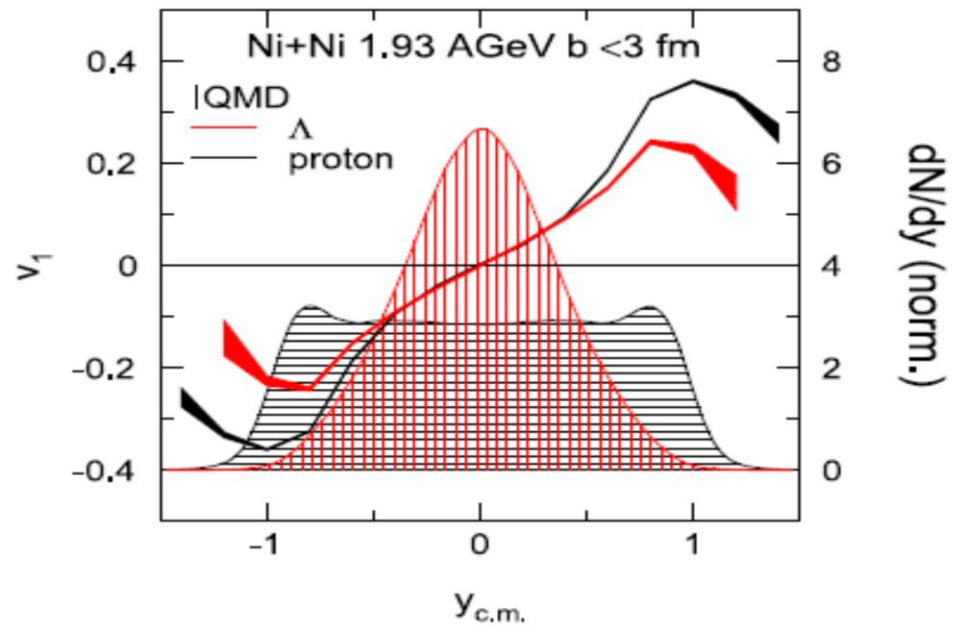
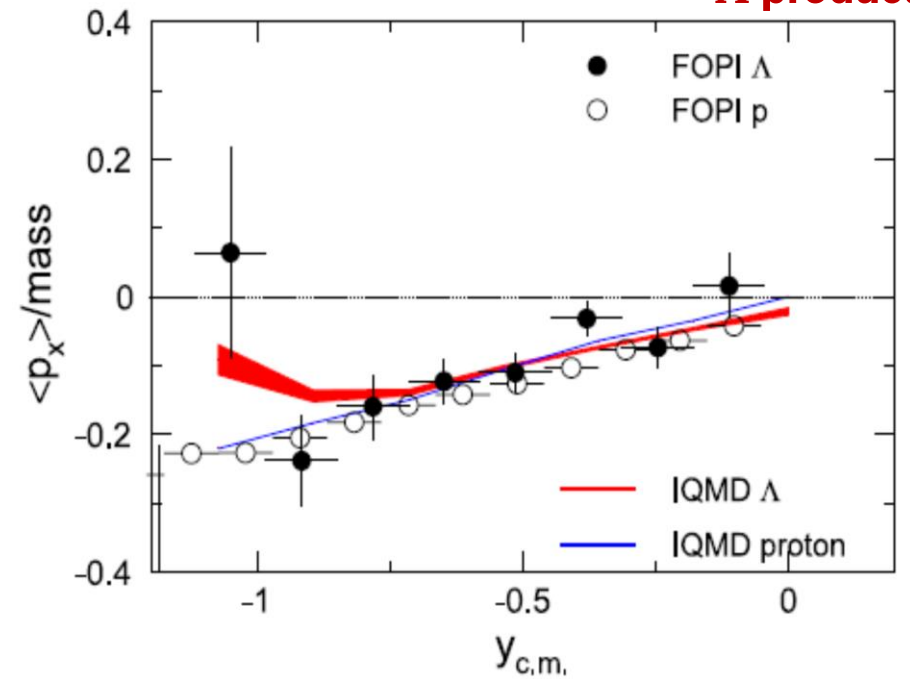
$$\lambda(K^+) = 5 \text{ fm}$$

$$\lambda(K^-) = 0.8 \text{ fm}$$



$$e+p \rightarrow e' + K^+ + \Lambda$$



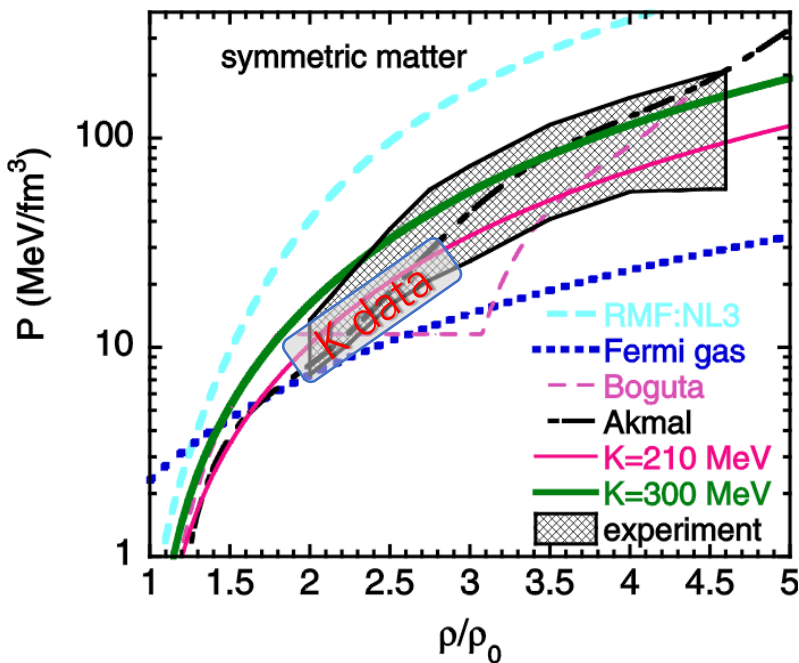
**K<sup>+</sup> production** **$\Lambda$  production**

# 同位旋对称核物质状态方程

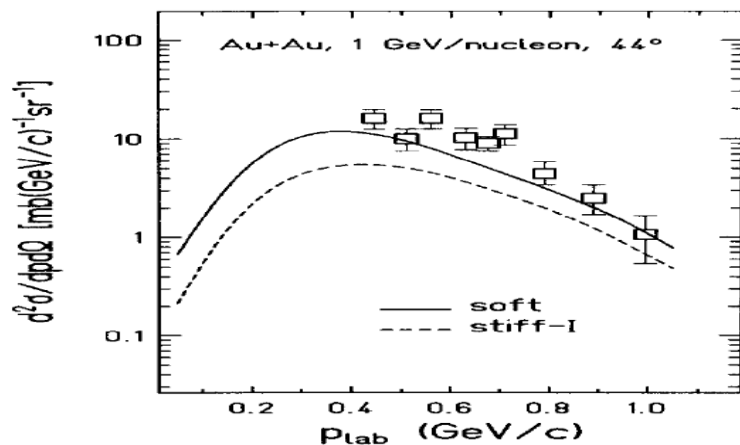
$K^+$  介子探针:

- KN相互作用势
- $\Delta$ 衰变宽度
- $N\Delta$ 吸收截面

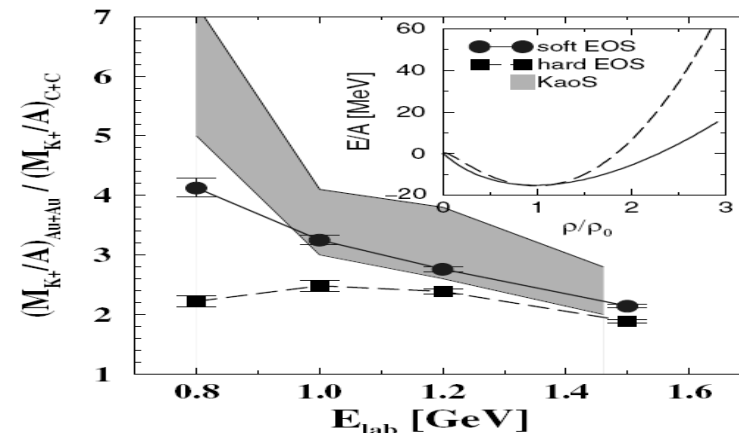
P. Danielewicz, R. Lacey and W.G. Lynch, Science 298, 1592 (2002)



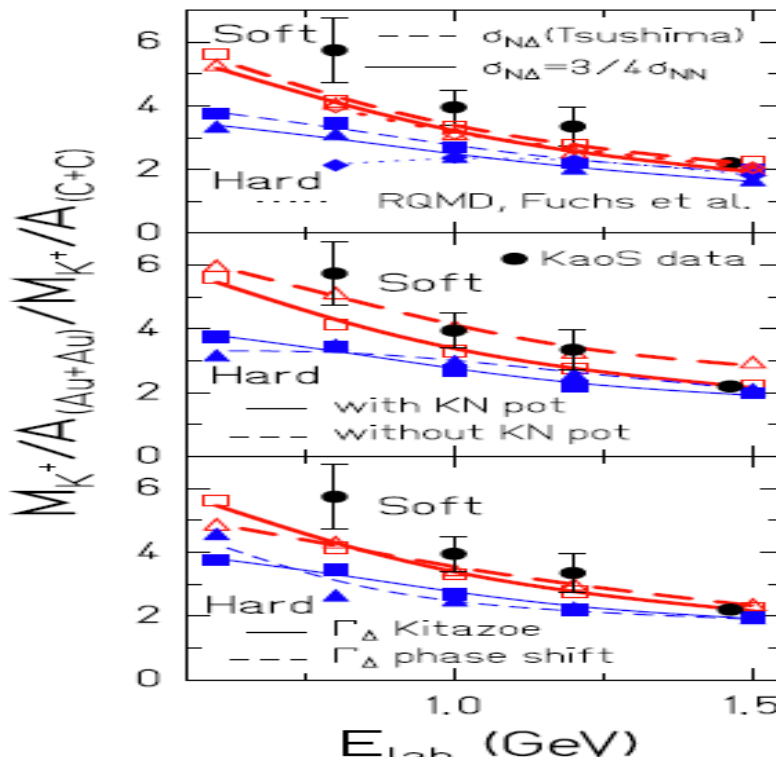
G.Q. Li, C. M. Ko, PLB349(1995)405. BUU



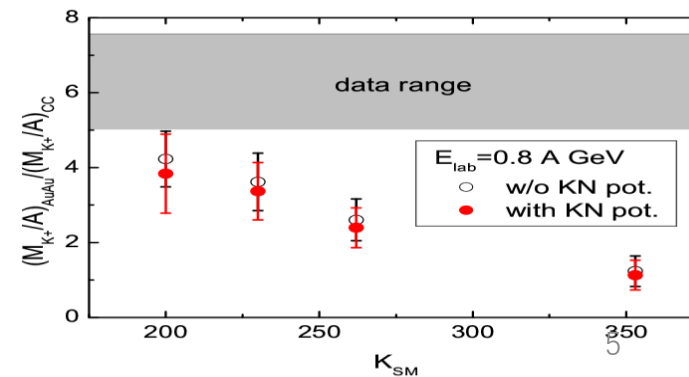
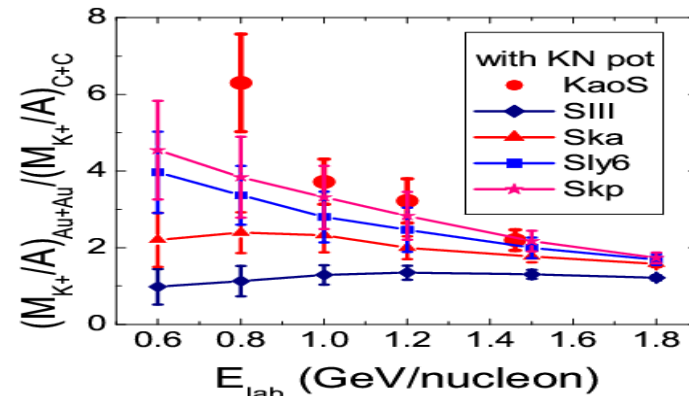
C. Fuchs et al., PRL86(2001)1974. QMD



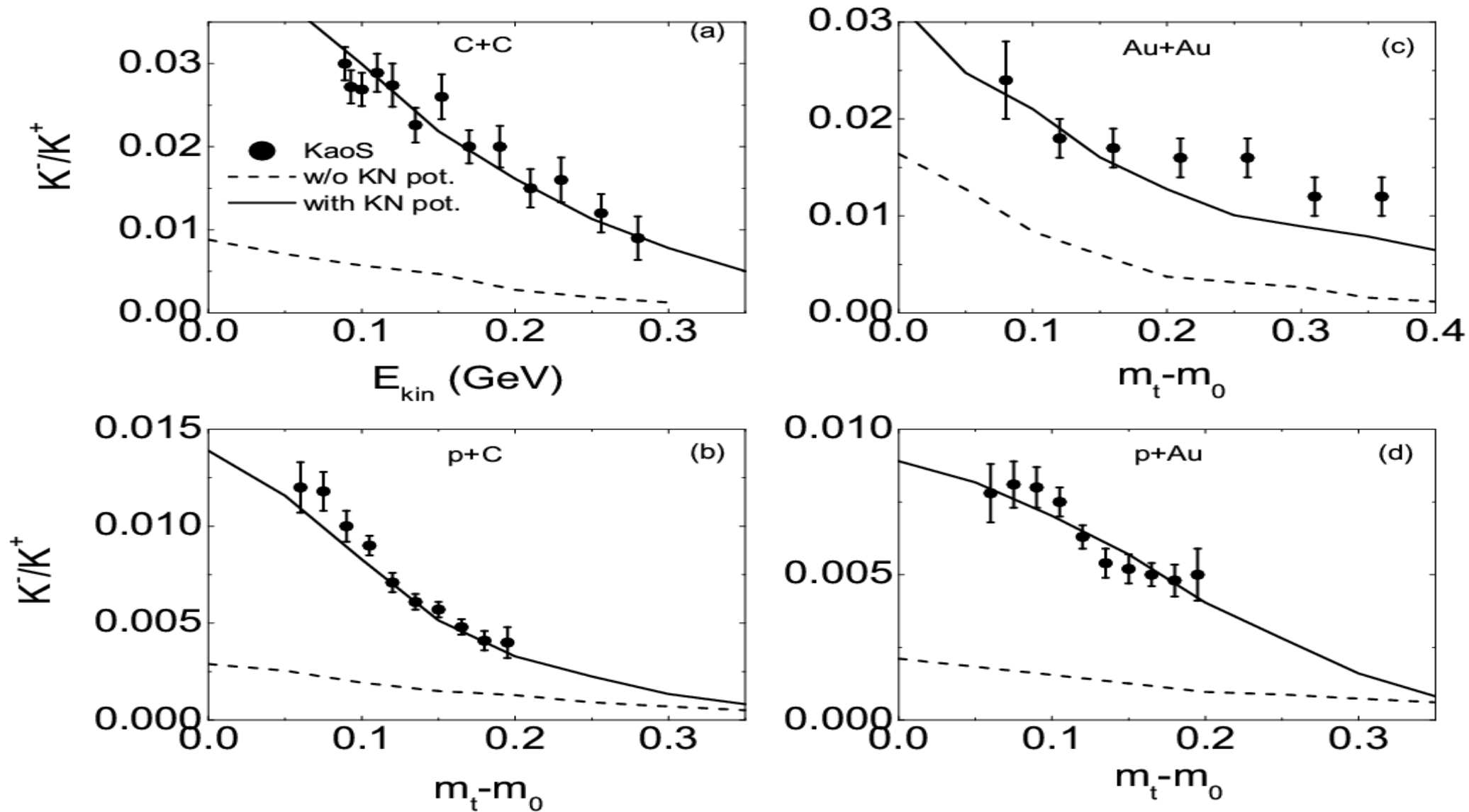
Ch. Hartnack et al., PRL96 (2006) 012302. IQMD



Z. Q. Feng, PRC 83 (2011) 067604. LQMD



The ratio of  $K^-/K^+$  as a function of transverse mass (kinetic energy) in collisions of  $^{12}\text{C} + ^{12}\text{C}$  and protons on  $^{12}\text{C}$  and  $^{197}\text{Au}$  at the beam energies of 1.8A GeV and 2.5 GeV (Z. Q. Feng et al., Phys. Rev. C 90, 064604 (2014) )



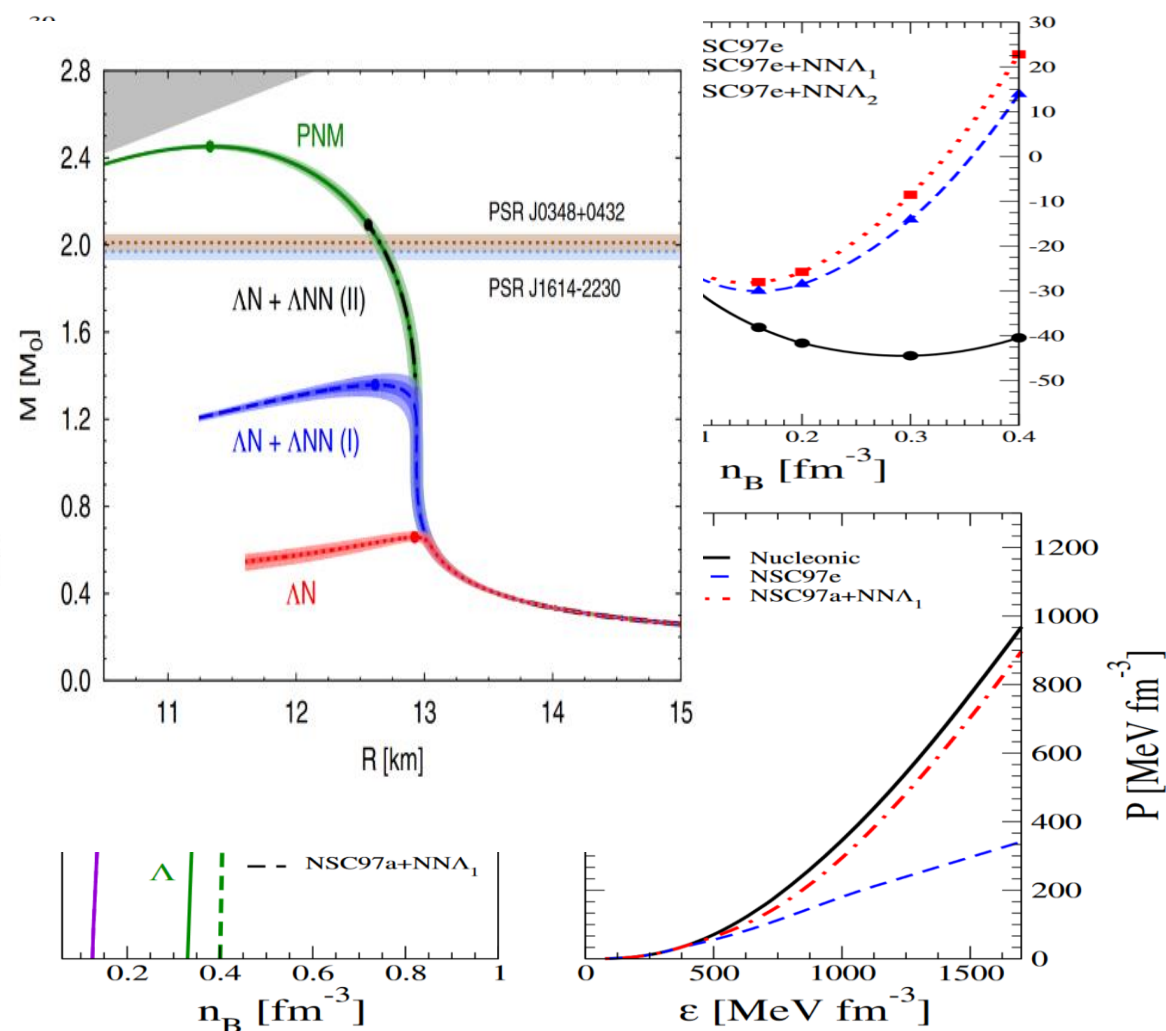
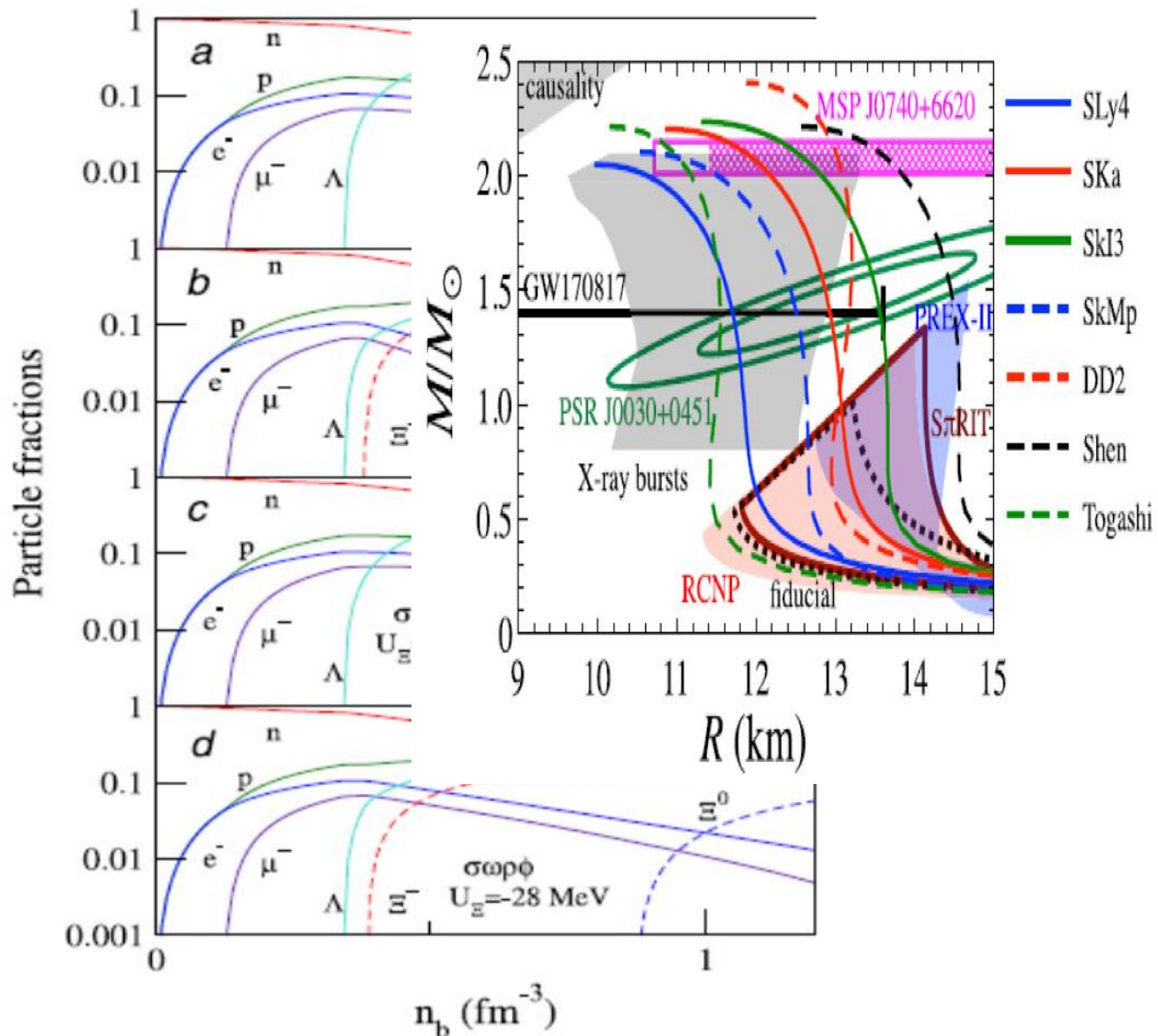
$$V_{K^+}(\rho_0) = 28 \text{ MeV}, \quad V_{K^-}(\rho_0) = -100 \text{ MeV}$$

# Hyperons in neutron stars (NS)

S. Weissenborn, D. Chatterjee, J. Schaffner-Bielich, Nucl. Phys. A 881, 62 (2012)

W. Z. Jiang, R. Y. Yang, and D. R. Zhang, Phys. Rev. C 87, 064314 (2013)

Diego Lonardonì, Alessandro Lovato, Stefano Gandolfi, and Francesco Pederiva, Phys. Rev. Lett. 114, 092301 (2015)

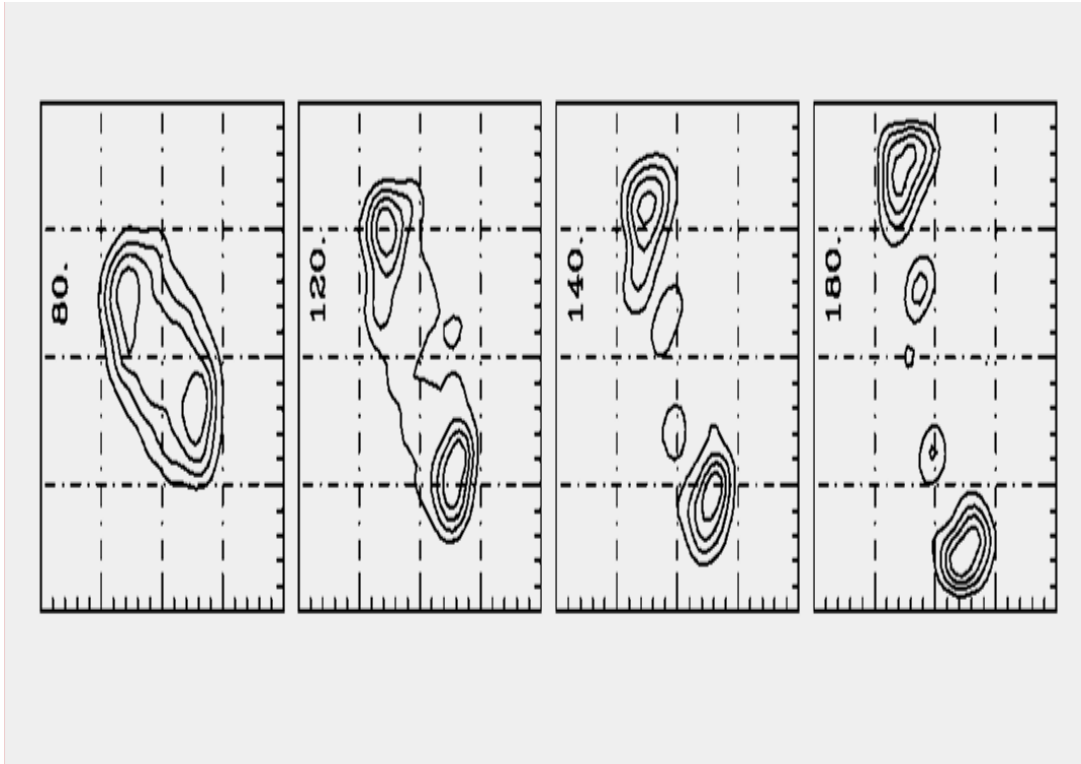


# Cluster production in heavy-ion collisions

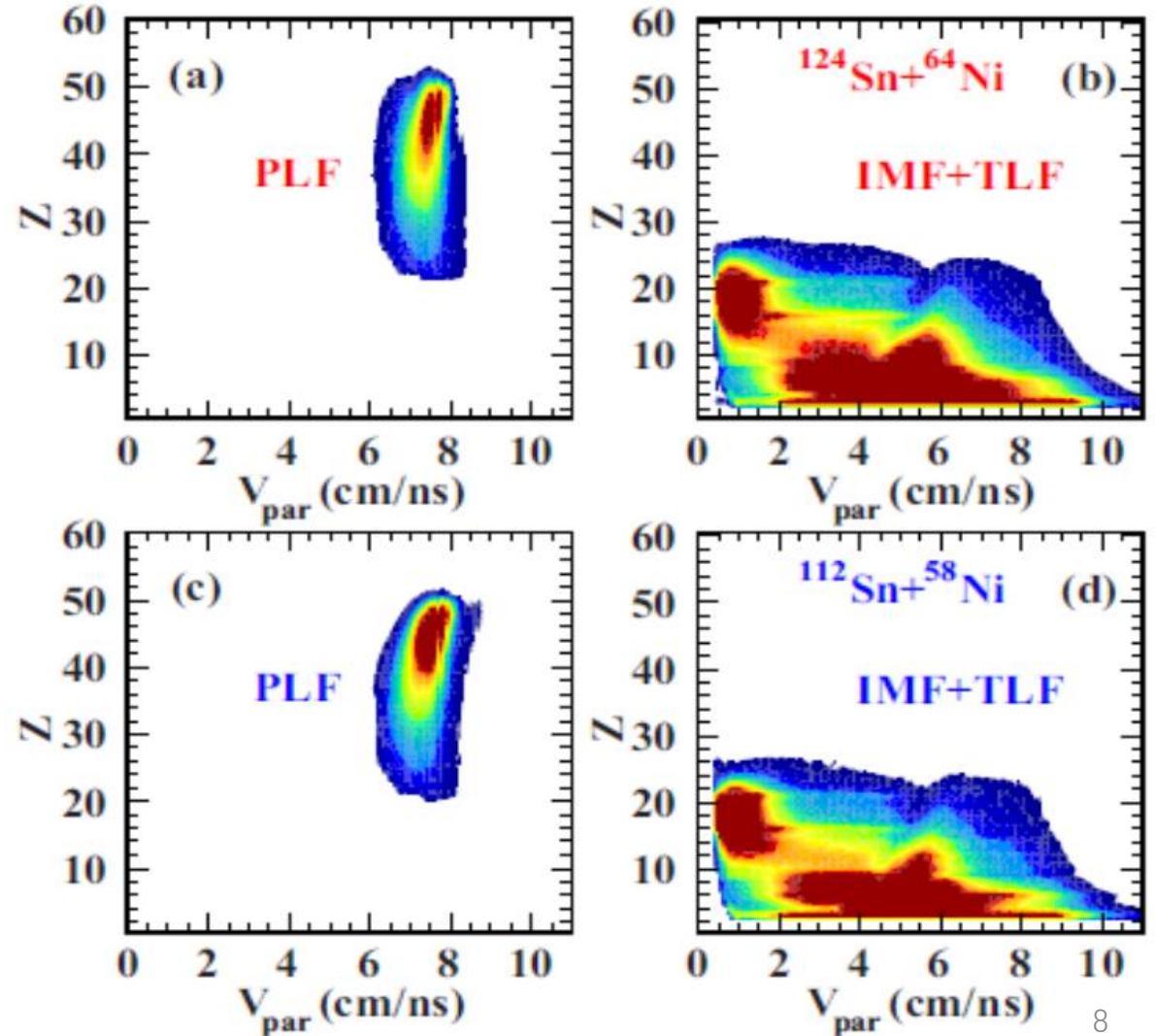
## Experiments:

**SSC and CSR(HIRFL), INDRA (GANIL), CHIMERA (LNS), NSCL (MSU), FOPI and HADES (GSI) ...**

Temporal evolution of density profile



P. Russotto *et al.*, PRC 91, 014610 (2015)





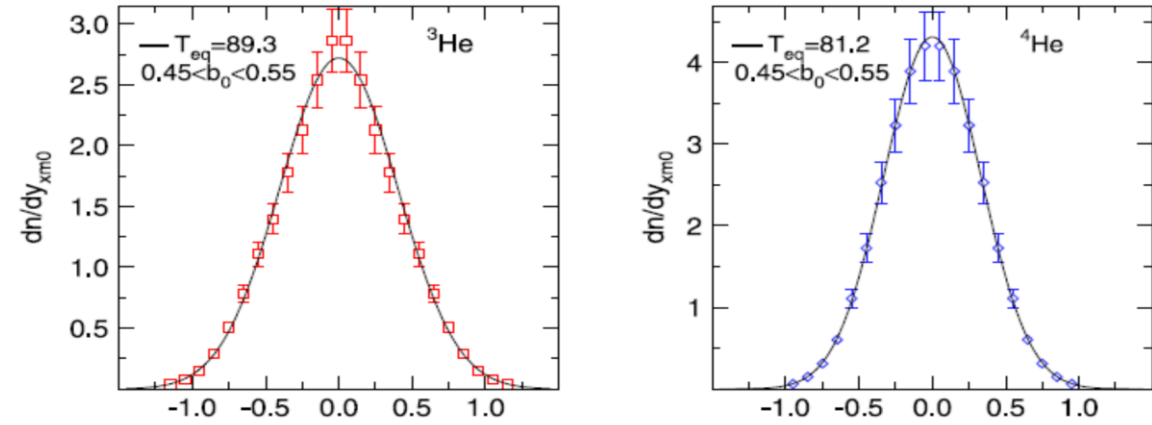
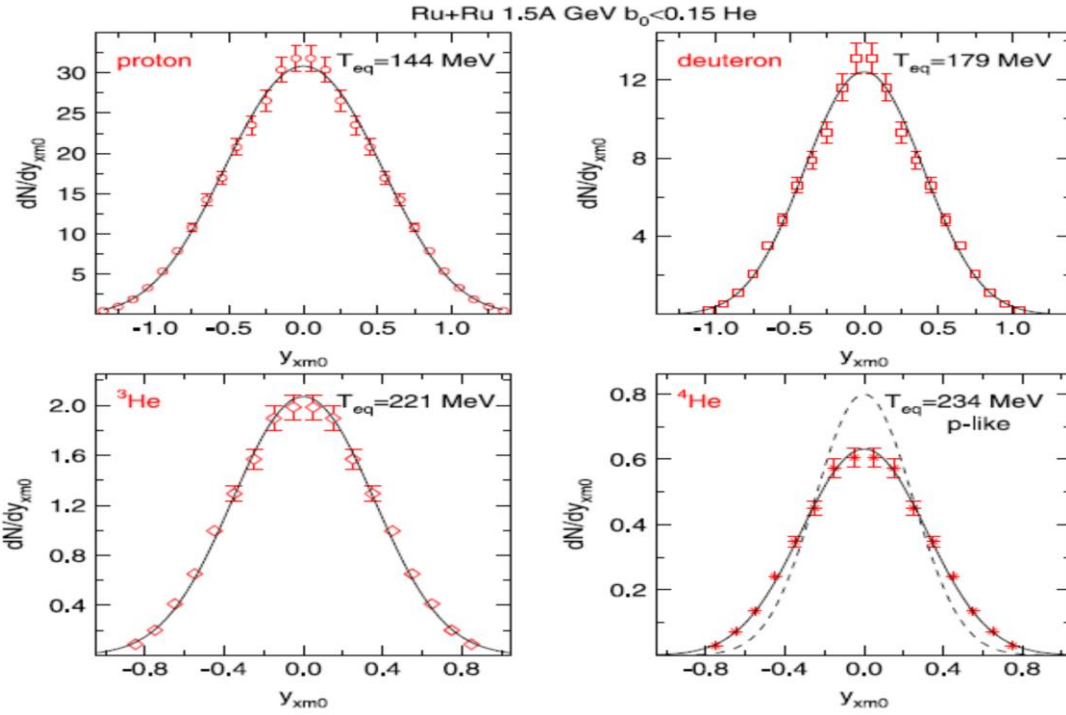
# Cluster production measurement at FOPI



## Systematics of central heavy ion collisions in the 1 A GeV regime

FOPI Collaboration

Au+Au 0.4A GeV

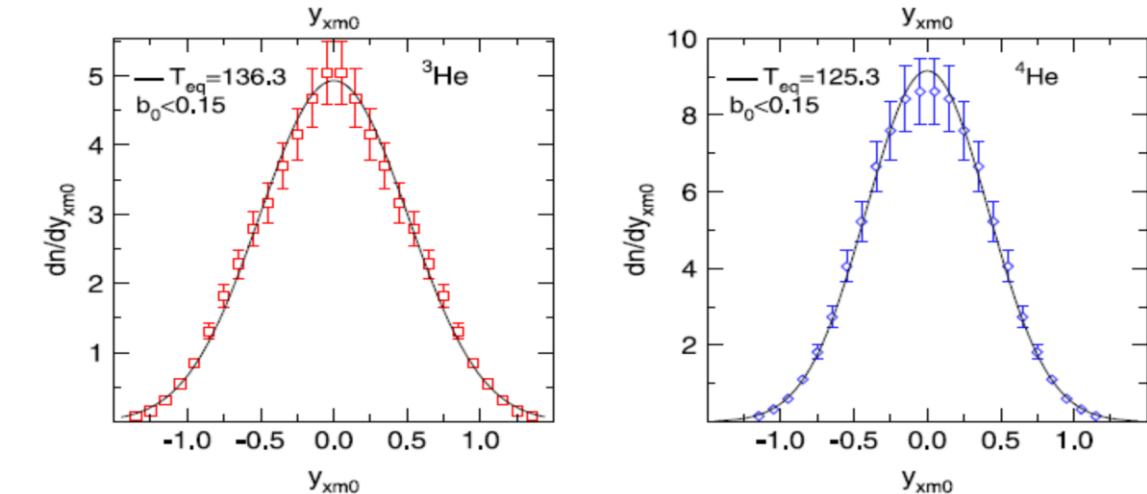


Multiplicities and charge balance for Au + Au at  $E/A = 0.15$  GeV and  $b_0 < 0.15$ .

Z = 1	$59.9 \pm 3.0$	p	$24.1 \pm 1.2$	Z = 2	$25.0 \pm 1.8$	${}^3\text{He}$	$8.4 \pm 0.9$
		d	$20.4 \pm 1.4$			${}^4\text{He}$	$16.6 \pm 1.7$
		t	$15.1 \pm 1.5$				
		Li	$5.0 \pm 0.5$	Be	$1.69 \pm 0.17$		
		B	$1.44 \pm 0.15$	C	$0.90 \pm 0.09$		
		N	$0.50 \pm 0.05$	O	$0.26 \pm 0.03$		

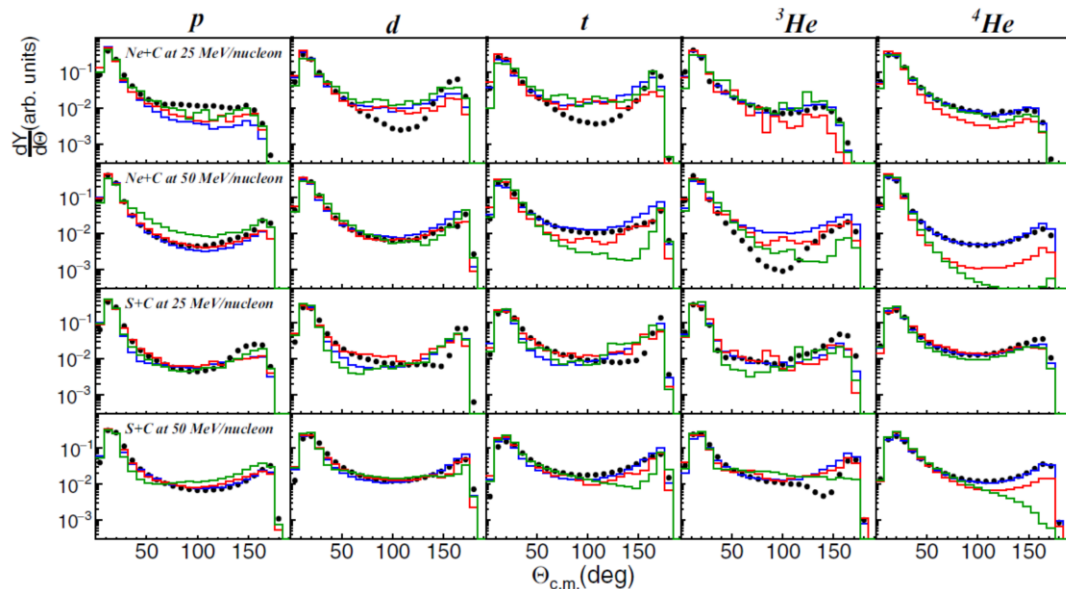
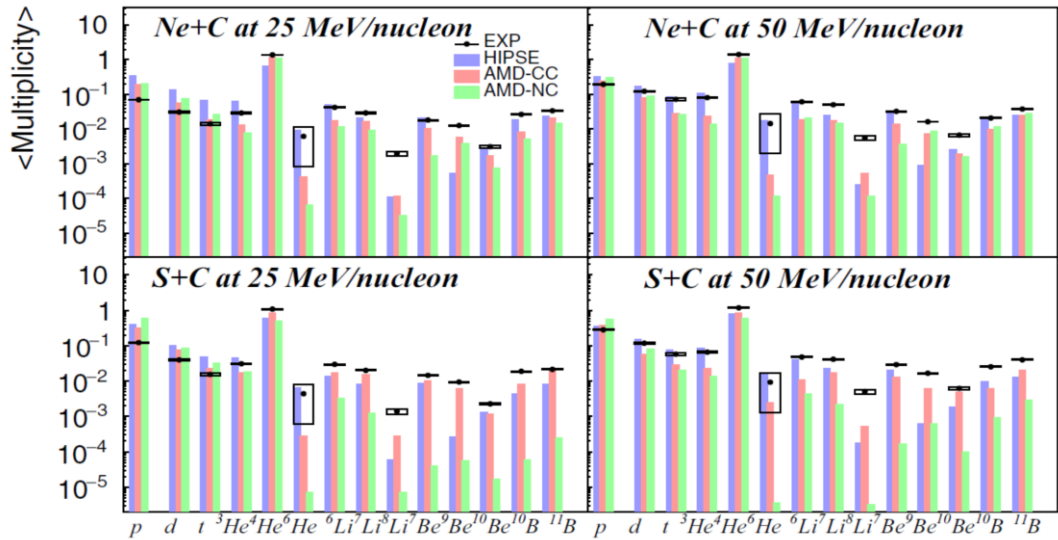
Multiplicities and charge balance for Ni + Ni at  $E/A = 0.25$  GeV and  $b_0 < 0.15$ .

Z = 1	$34.9 \pm 1.8$	p	$19.2 \pm 1.0$	Z = 2	$9.0 \pm 0.7$	${}^3\text{He}$	$3.24 \pm 0.33$
		d	$10.5 \pm 0.8$			${}^4\text{He}$	$5.79 \pm 0.58$
		t	$5.1 \pm 0.5$				
		Li	$0.91 \pm 0.09$	Be	$0.26 \pm 0.03$		
		B	$0.10 \pm 0.01$				



## Cluster production measurement

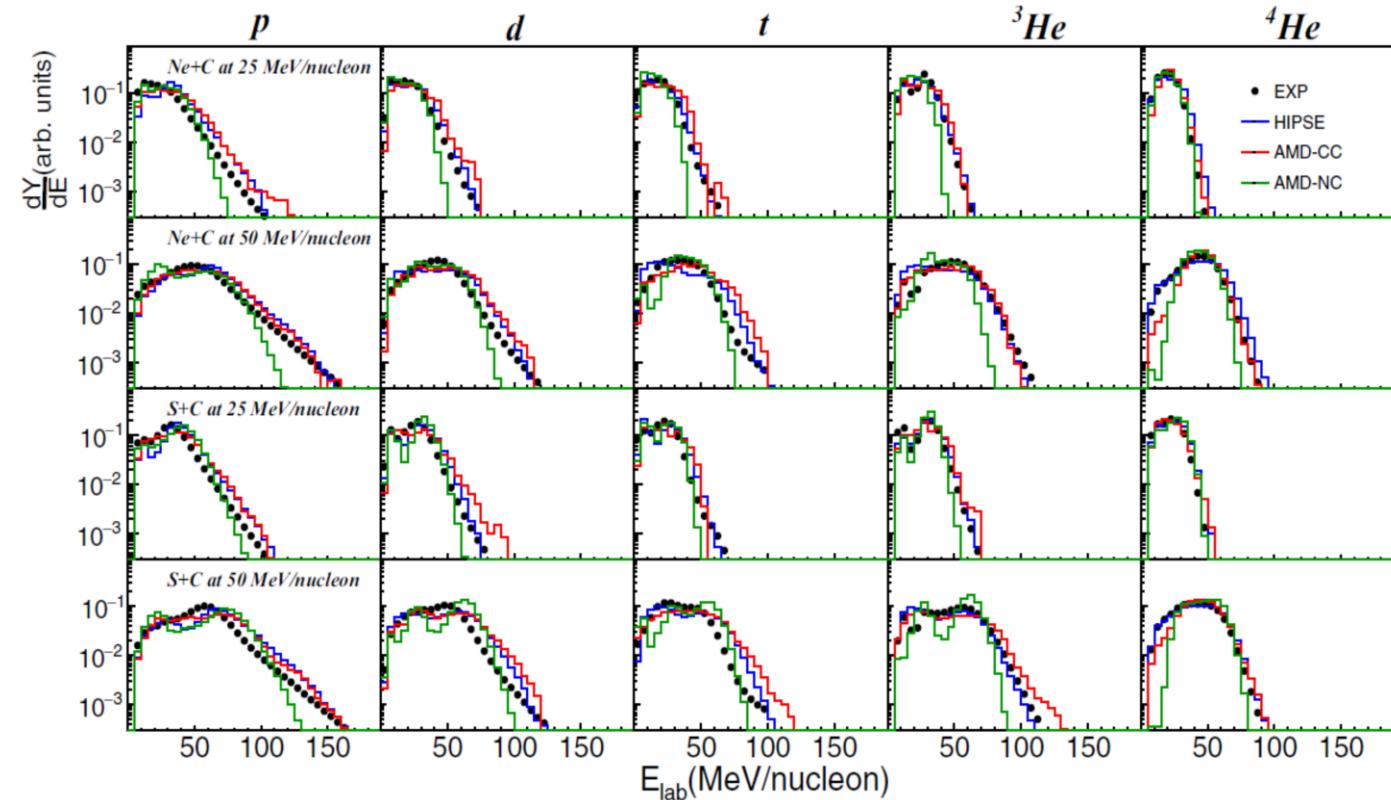
## at FOPI INDRA-FAZIA



## Examination of cluster production in excited light systems at Fermi energies from new experimental data and comparison with transport model calculations

C. Frosin<sup>1,2,\*</sup>, S. Piantelli,<sup>2</sup> G. Casini,<sup>2</sup> A. Ono,<sup>3</sup> A. Camaiani,<sup>4</sup> L. Baldesi,<sup>2</sup> S. Barlini,<sup>1,2</sup> B. Borderie,<sup>5</sup> R. Bougault,<sup>6</sup> C. Ciampi,<sup>1,2</sup> M. Cicerchia,<sup>7</sup> A. Chbihi,<sup>8</sup> D. Dell'Aquila,<sup>9,10</sup> J. A. Dueñas,<sup>11</sup> D. Fabris,<sup>12</sup> Q. Fable,<sup>13</sup> J. D. Frankland,<sup>8</sup> T. Génard,<sup>8</sup> F. Gramegna,<sup>7</sup> D. Gruyer,<sup>6</sup> M. Henri,<sup>8</sup> B. Hong,<sup>14,15</sup> M. J. Kweon,<sup>16</sup> S. Kim,<sup>17</sup> A. Kordyasz,<sup>18</sup> T. Kozik,<sup>19</sup> I. Lombardo,<sup>20,21</sup> O. Lopez,<sup>6</sup> T. Marchi,<sup>7</sup> K. Mazurek,<sup>22</sup> S. H. Nam,<sup>14,15</sup> J. Lemarié,<sup>8</sup> N. LeNeindre,<sup>6</sup> P. Ottanelli,<sup>2</sup> M. Parlog,<sup>6,23</sup> J. Park,<sup>14,15</sup> G. Pasquali,<sup>1,2</sup> G. Poggi,<sup>1,2</sup> A. Rebillard-Soulié,<sup>6</sup> B. H. Sun,<sup>24</sup> A. A. Stefanini,<sup>1,2</sup> S. Terashima,<sup>24</sup> S. Upadhyaya,<sup>19</sup> S. Valdré,<sup>2</sup> G. Verde,<sup>20,13</sup> E. Vient,<sup>6</sup> and M. Vigilante<sup>25,26</sup>

(INDRA-FAZIA Collaboration)

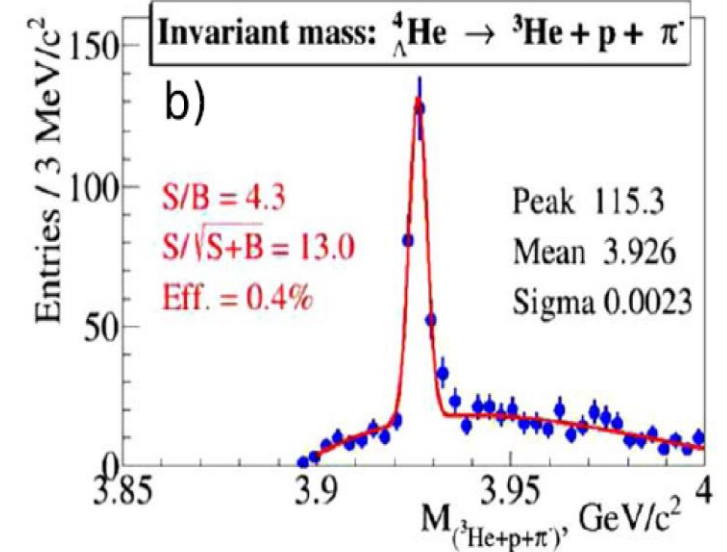
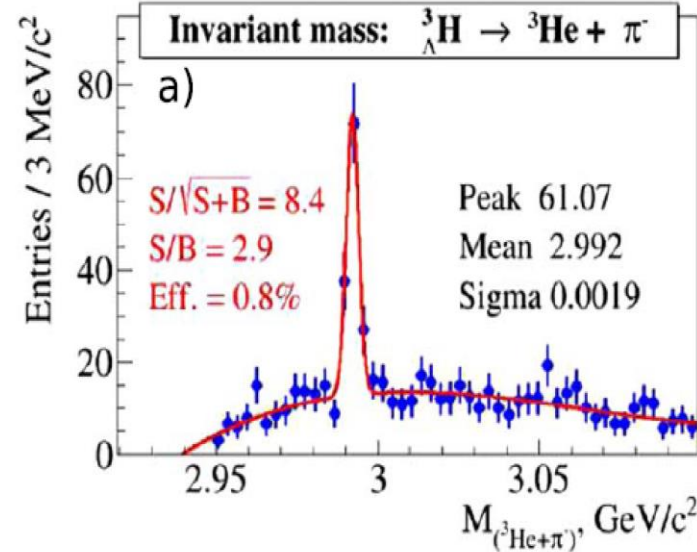
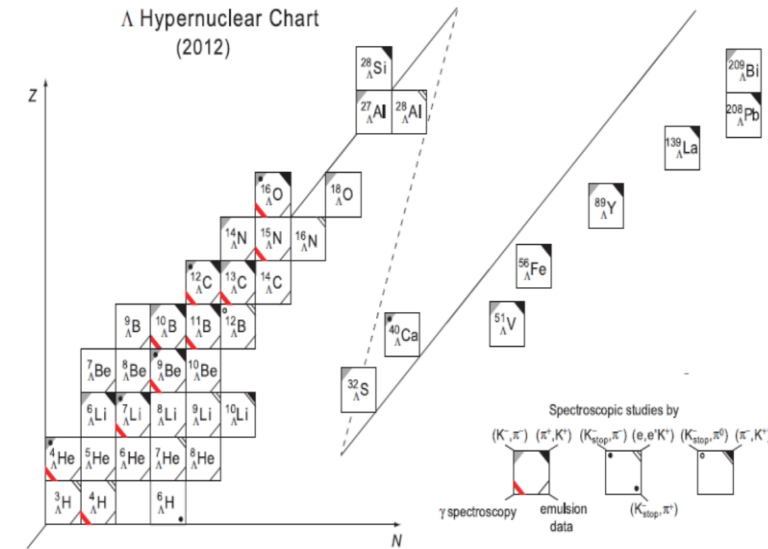
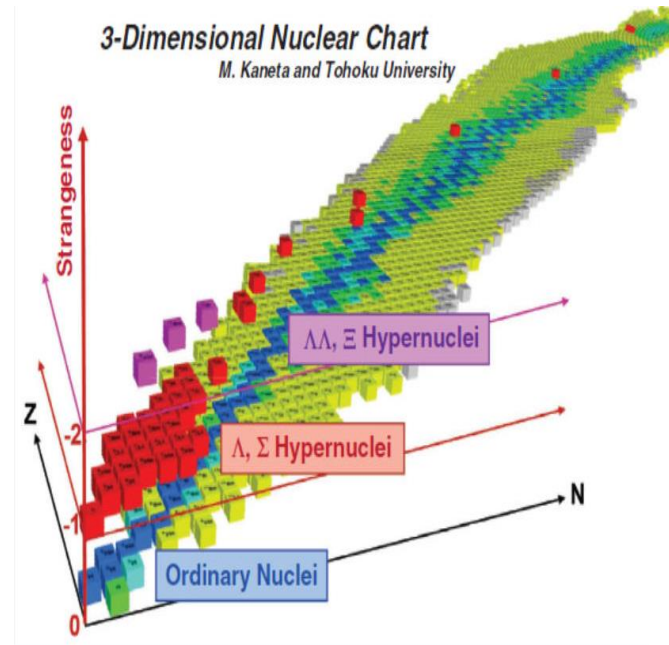
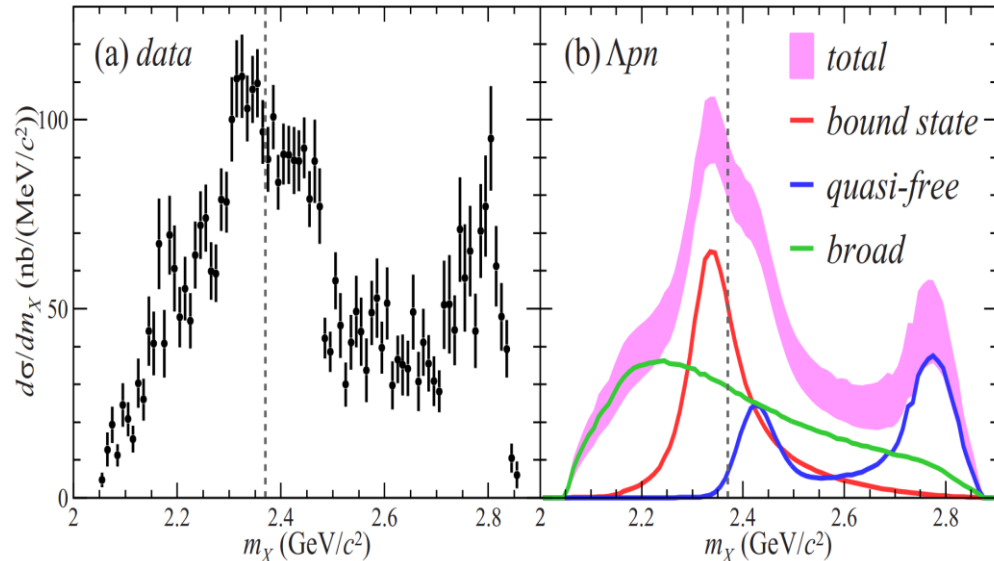


# Advantage of HICs for HN production

1. Neutron-rich/proton-rich HN nuclei and spectroscopies
2. Multistrangeness HN ( $S=-2$ )  $\Lambda\Lambda$ X和 $\Xi$ X
3. Interaction potentials of  $N\Lambda$ ,  $N\Xi$ ,  $NN\Lambda$ , etc

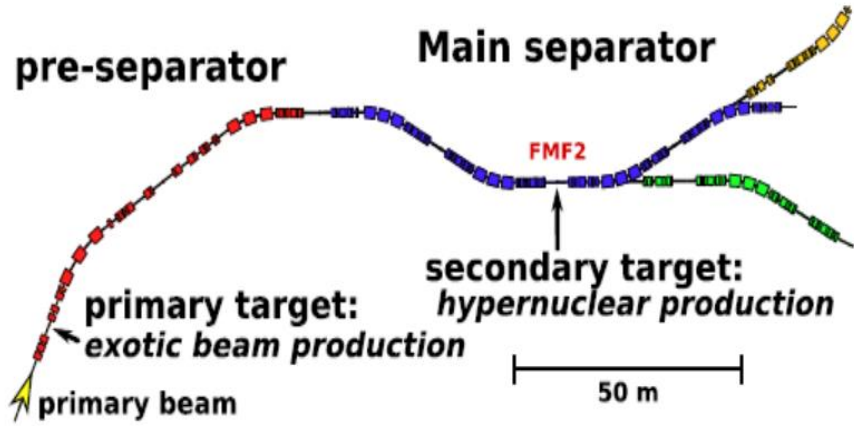
PHYSICAL REVIEW C **102**, 044002 (2020)

Observation of a  $\bar{K}NN$  bound state in the  ${}^3\text{He}(K^-, \Lambda p)n$  reaction

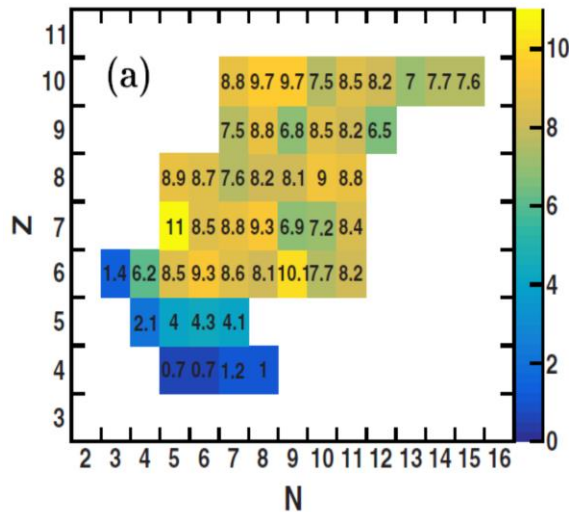


# Estimation of proton-rich and neutron rich HN with GSI exotic beam

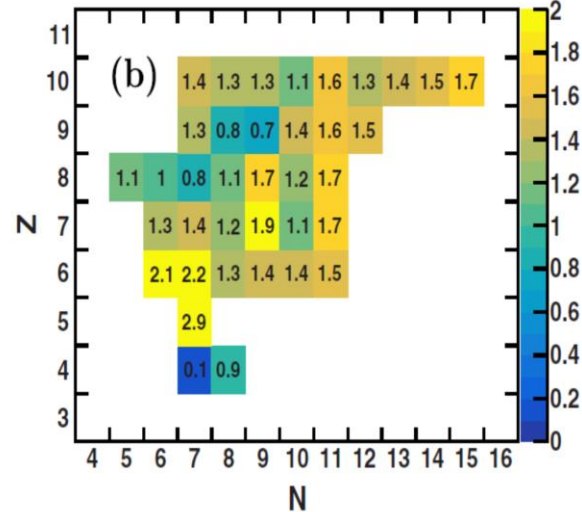
C. Rappold, J. López-Fidalgo, PRC 94, 044616 (2016)



$^8_{\Lambda}\text{Be}$

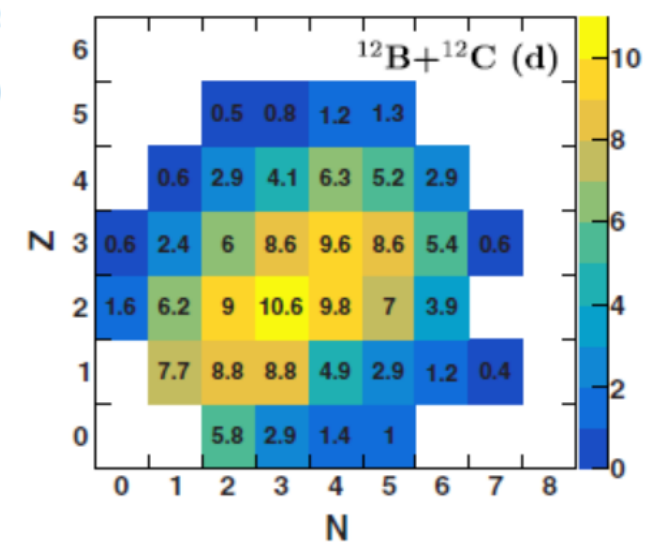
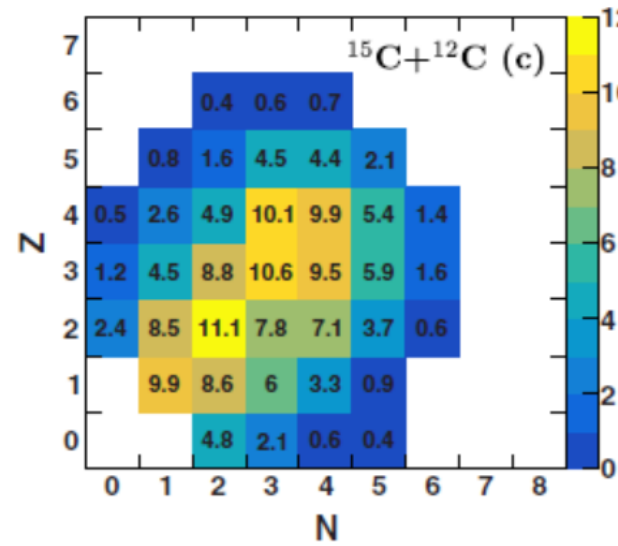
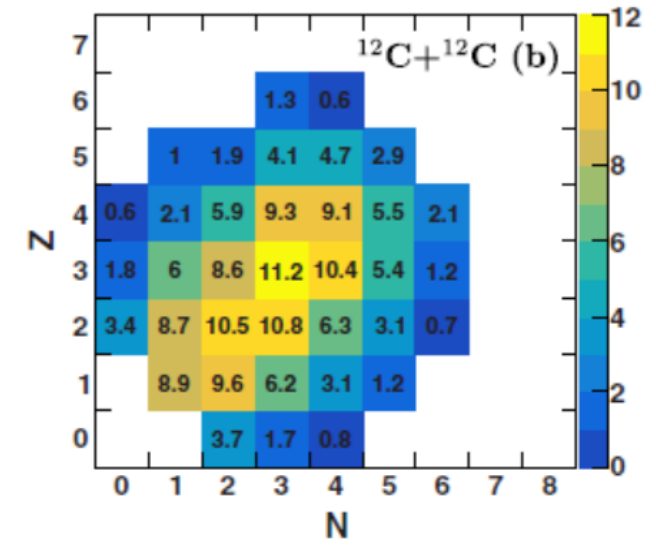
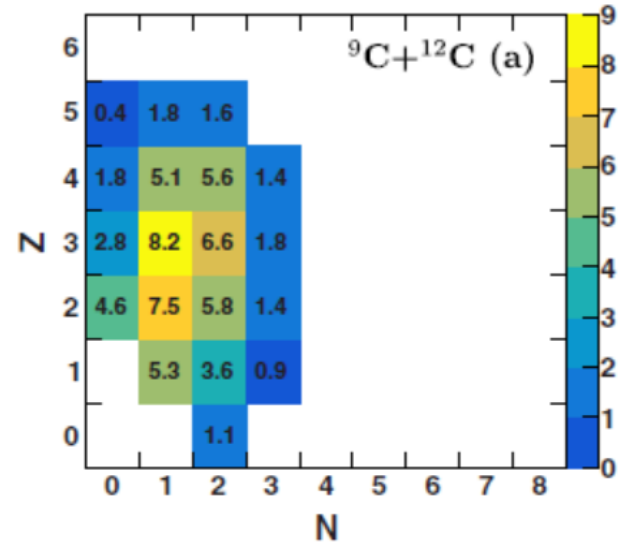


$^{11}_{\Lambda}\text{Be}$

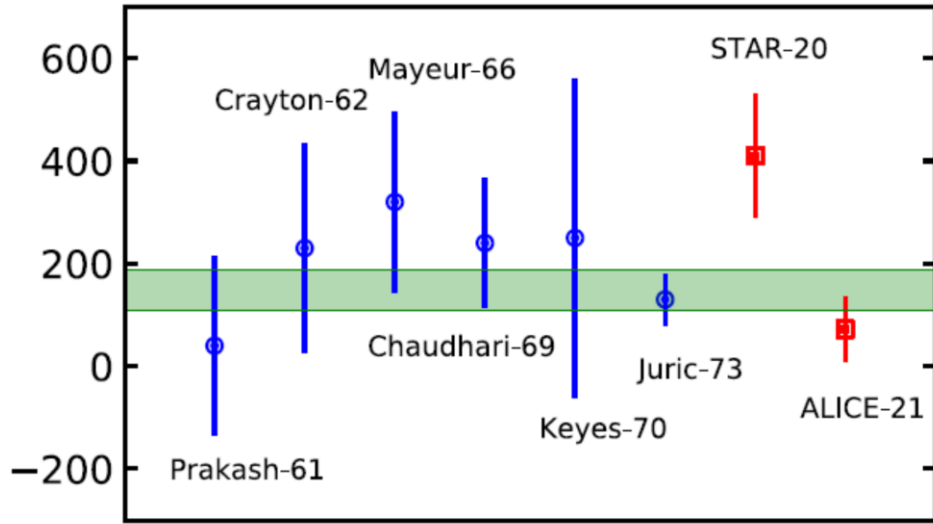


## $\Lambda$ -hypernuclei

$\mu\text{b}$



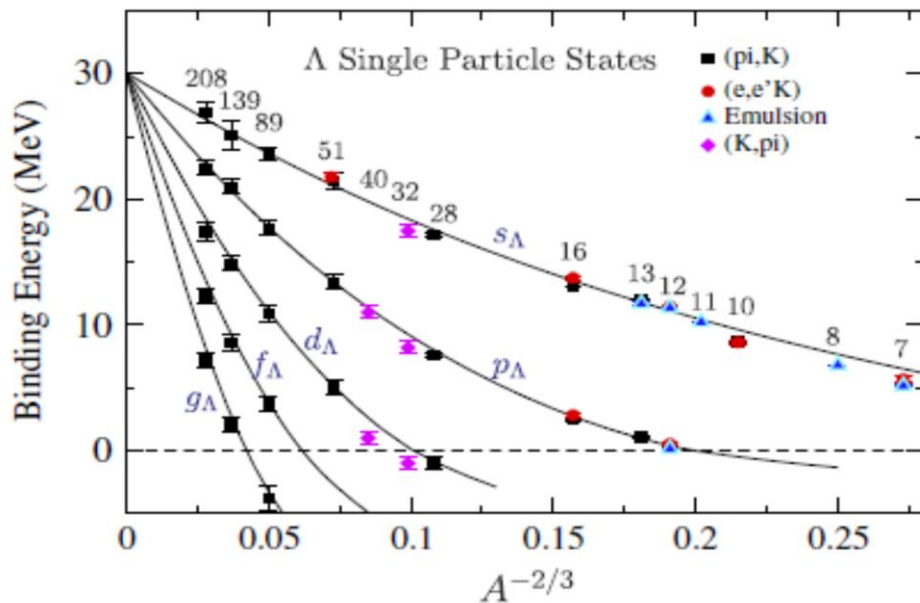
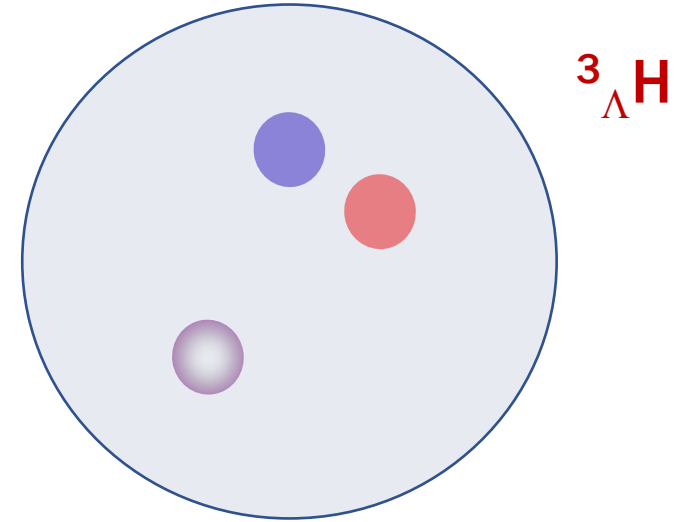
## Radius and binding energy HN



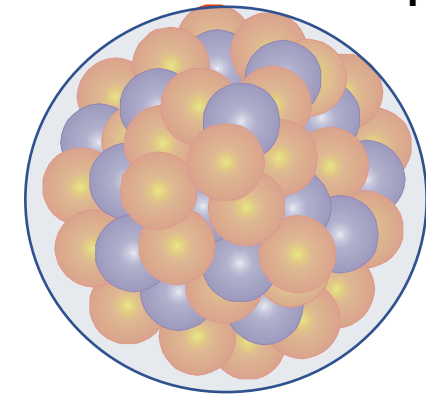
$$B_{\Lambda} = 72^{+63}_{-36} \text{ keV}$$

$$R \sim 10 \text{ fm}$$

$$B_{\Lambda} = 148 \pm 40 \text{ keV}$$



$^{208}\text{Pb}$ ,  $R \sim 7 \text{ fm}$

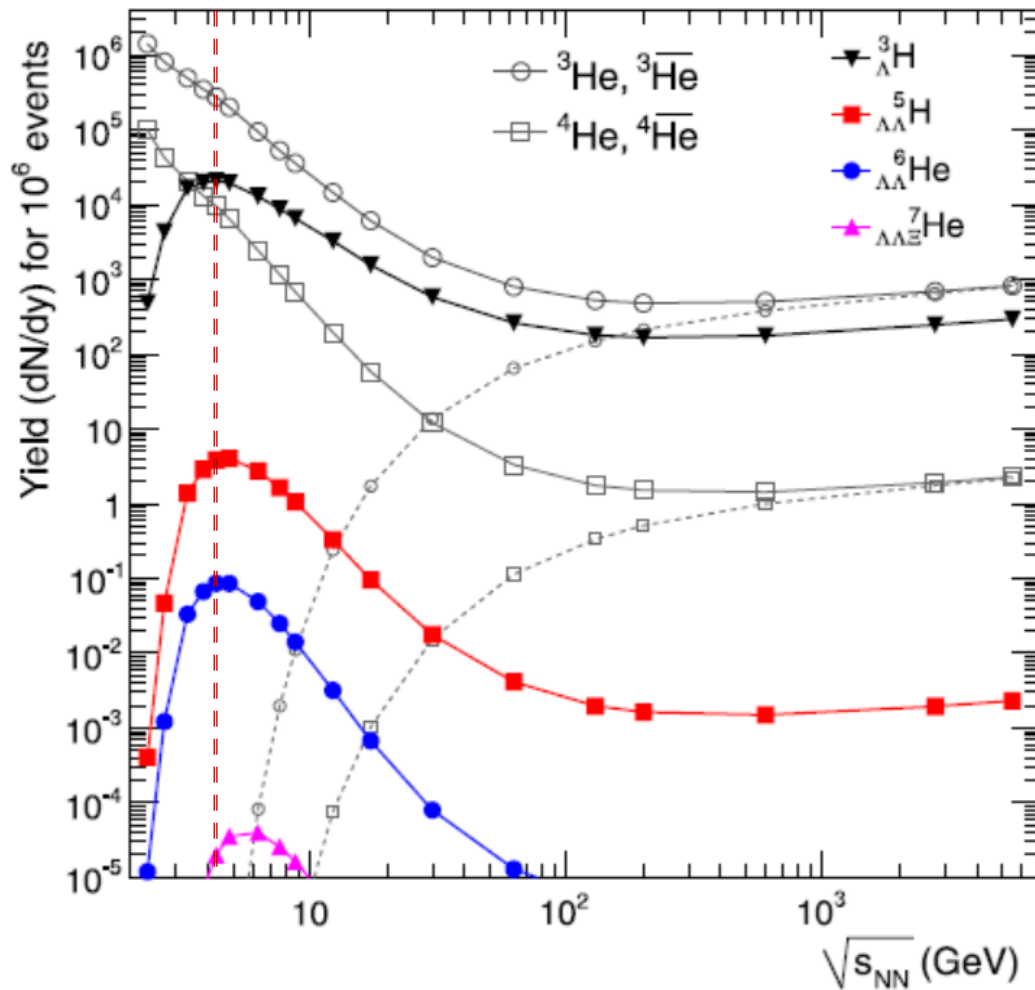


Millener, D. J., C. B. Dover, and A. Gal, 1988, Phys. Rev. C 38, 2700

# (Hyper-)cluster production in HICs-statistical approach

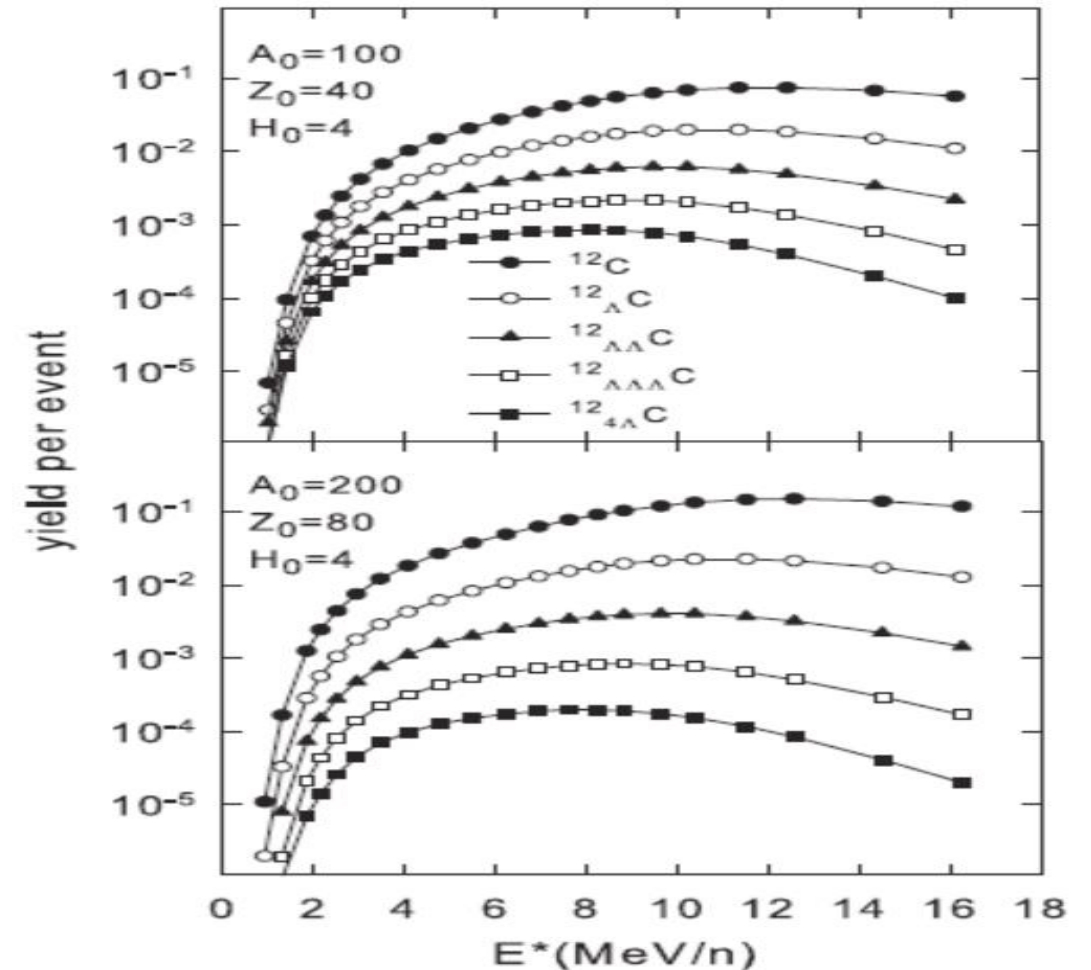
A. Andronic, P. Braun-Munzinger, J. Stachel, H. Stöcker,  
Physics Letters B 697 (2011) 203–207

**Pb+Pb**



N. Buyukcizmeci, R. Ogul, A. S. Botvina, M.  
Bleicher, Phys. Scr. 95 075311 (2020)

**Statistical multifragmentation model (SMM)**

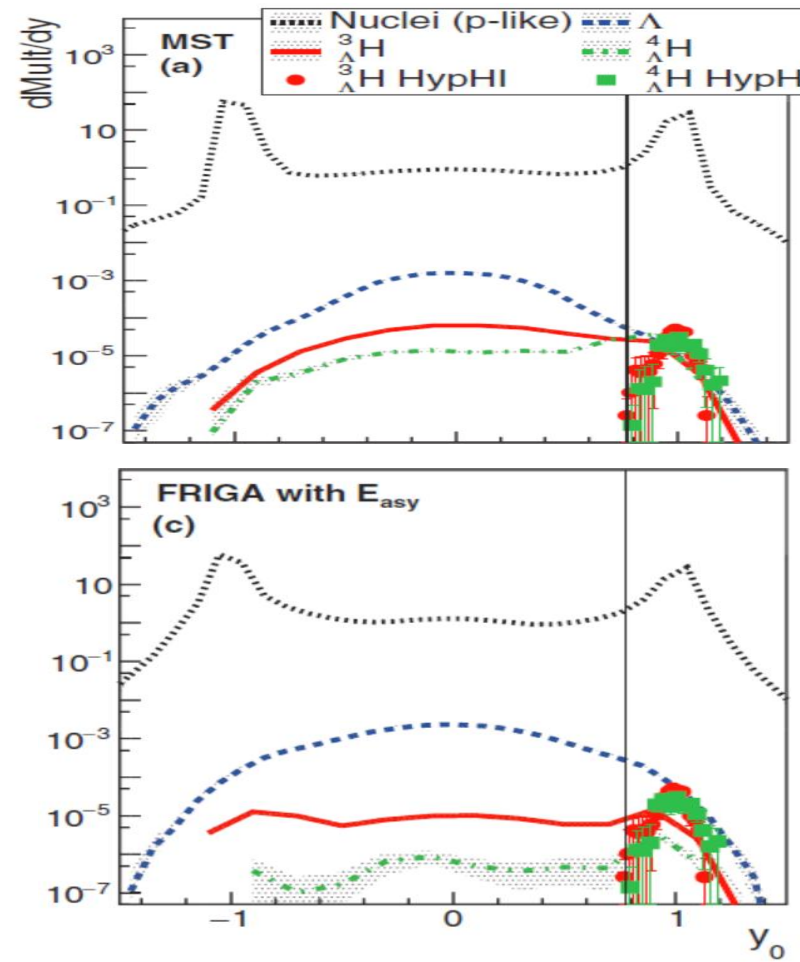
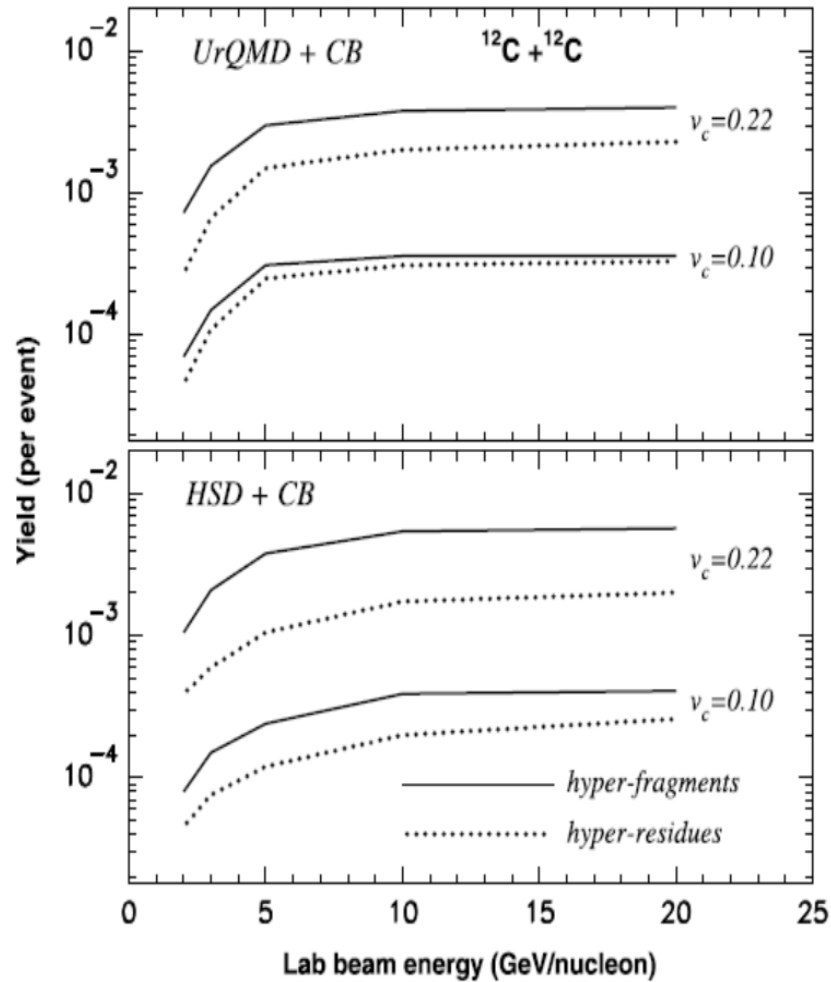


# Transport model + coalescence approach

A.S. Botvina, J. Steinheimer, E. Bratkovskaya et al., Physics Letters B 742 (2015) 7-14

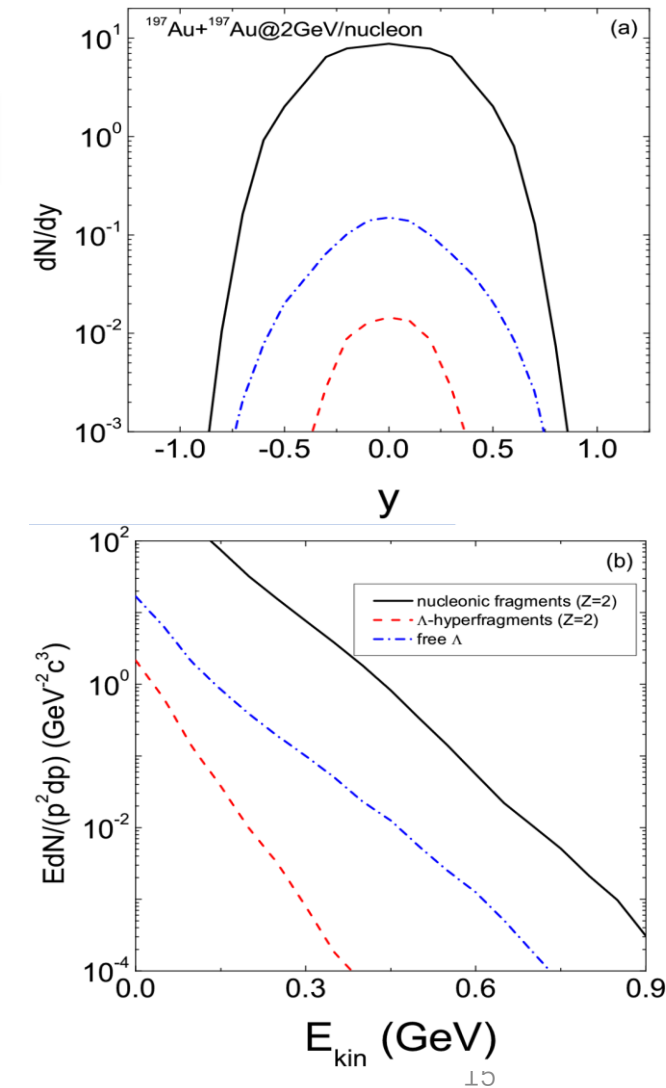
J. Aichelin, E. Bratkovskaya, A. Le Fèvre et al., Physical Review C 101, 044905 (2020)  
 A. Le Fèvre, J. Aichelin, C. Hartnack and Y. Leifels 100, Physical Review C 034904 (2019)

**${}^6\text{Li}+{}^{12}\text{C}@2\text{A GeV}$**

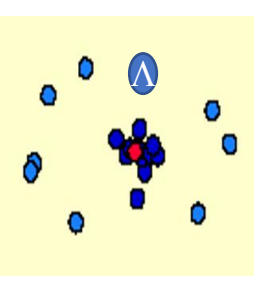


中高能重离子碰撞中奇异粒子产生和超核形成机制

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# Recognizing a nuclear cluster or hypernuclear cluster



**1) Classical coalescence approach in phase space for nuclides of  $Z > 2$  combined with the GEMINI decay code (minimum spanning tree (MST) procedure)**

$$|r_i - r_j| \leq 3 \text{ fm}, |r_i - r_Y| \leq 4.5 \text{ fm}, |p_i - p_j| \leq 0.3 \text{ GeV}/c$$

C. Samanta et al, J. Phys. G: Nucl. Part. Phys. 32 (2006) 363

Z. Q. Feng, Phys. Rev. C 102, 044604 (2020)

**Binding energy:**  $E_B(Z_i, N_i) = \sum_j \sqrt{p_j^2 + m_j^2} - m_j$

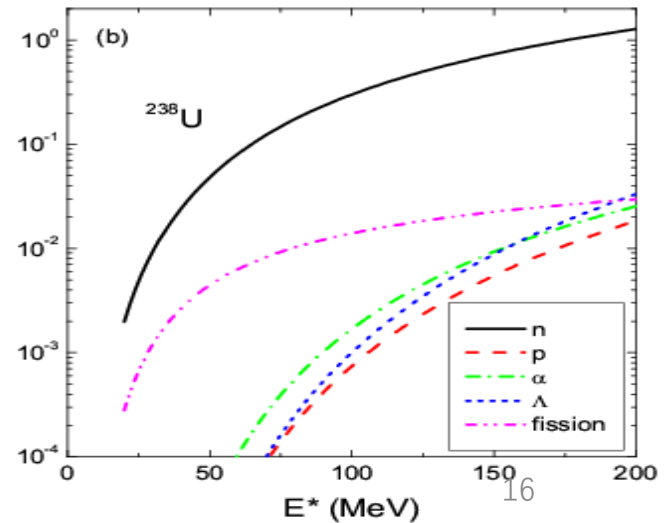
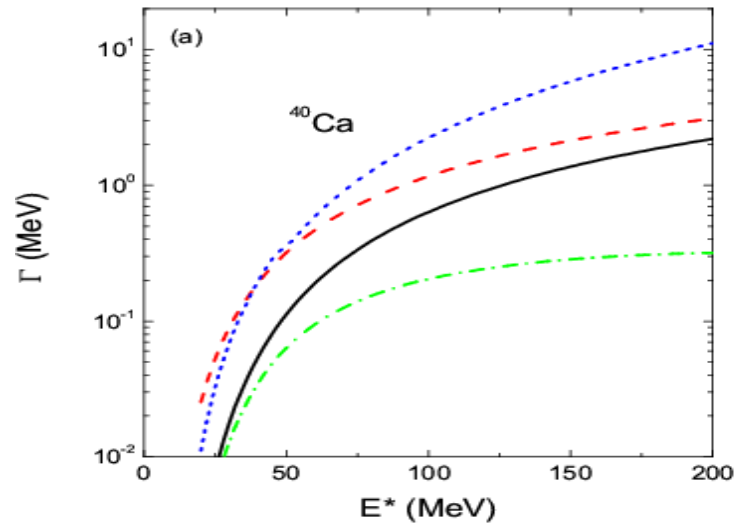
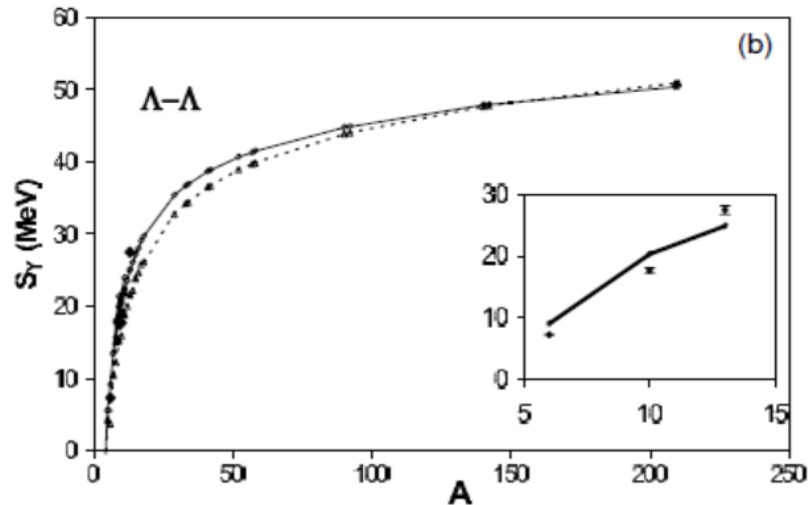
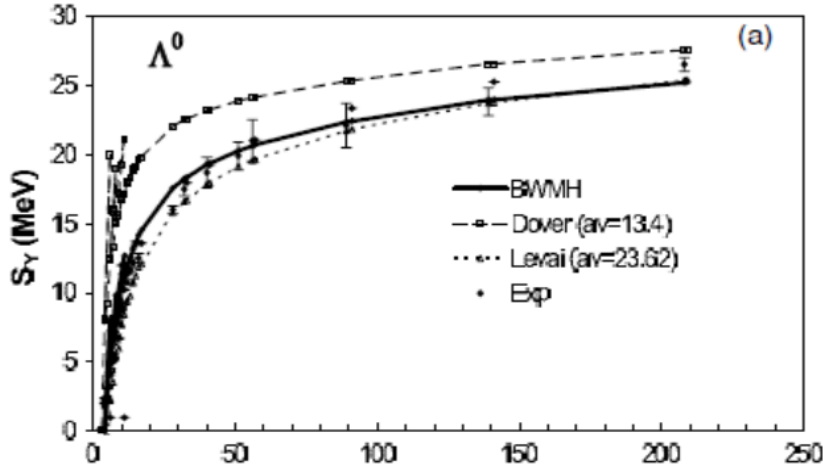
**Excitation energy  $E^*(Z_\nu, N_\nu, nY)$**

$$= E_B(Z_\nu, N_\nu, nY) - E_{LD}(Z_\nu, N_\nu, nY)$$

**The decay of excited hypernucleus is described by the GEMINI code!**

$$+ \frac{1}{2} \sum_{j,k,k \neq j} \int f_j(\mathbf{r}, \mathbf{p}, t) f_k(\mathbf{r}', \mathbf{p}', t) \times v(\mathbf{r}, \mathbf{r}', \mathbf{p}, \mathbf{p}') d\mathbf{r} d\mathbf{r}' d\mathbf{p} d\mathbf{p}'$$

$$+ \frac{1}{6} \sum_{j,k,l,k \neq j, k \neq l, j \neq l} \int f_j(\mathbf{r}, \mathbf{p}, t) f_k(\mathbf{r}', \mathbf{p}', t) \times f_l(\mathbf{r}'', \mathbf{p}'', t) v(\mathbf{r}, \mathbf{r}', \mathbf{r}'', \mathbf{p}, \mathbf{p}', \mathbf{p}'') \times d\mathbf{r} d\mathbf{r}' d\mathbf{r}'' d\mathbf{p} d\mathbf{p}' d\mathbf{p}''$$



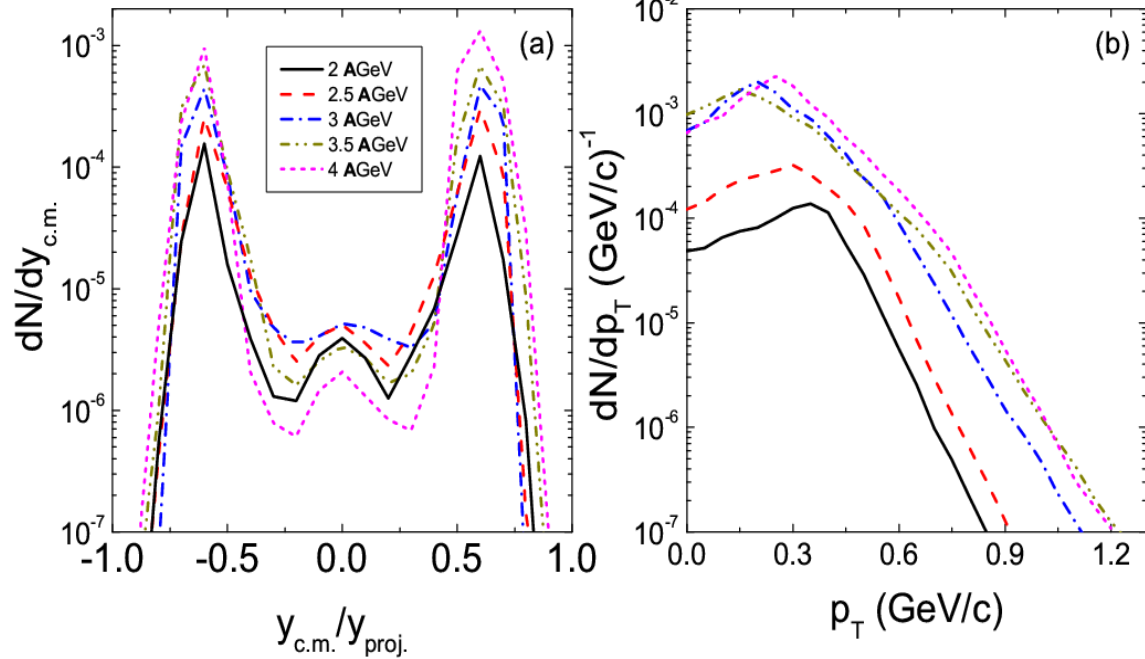


## 2) Wigner density approach for $Z \leq 2$

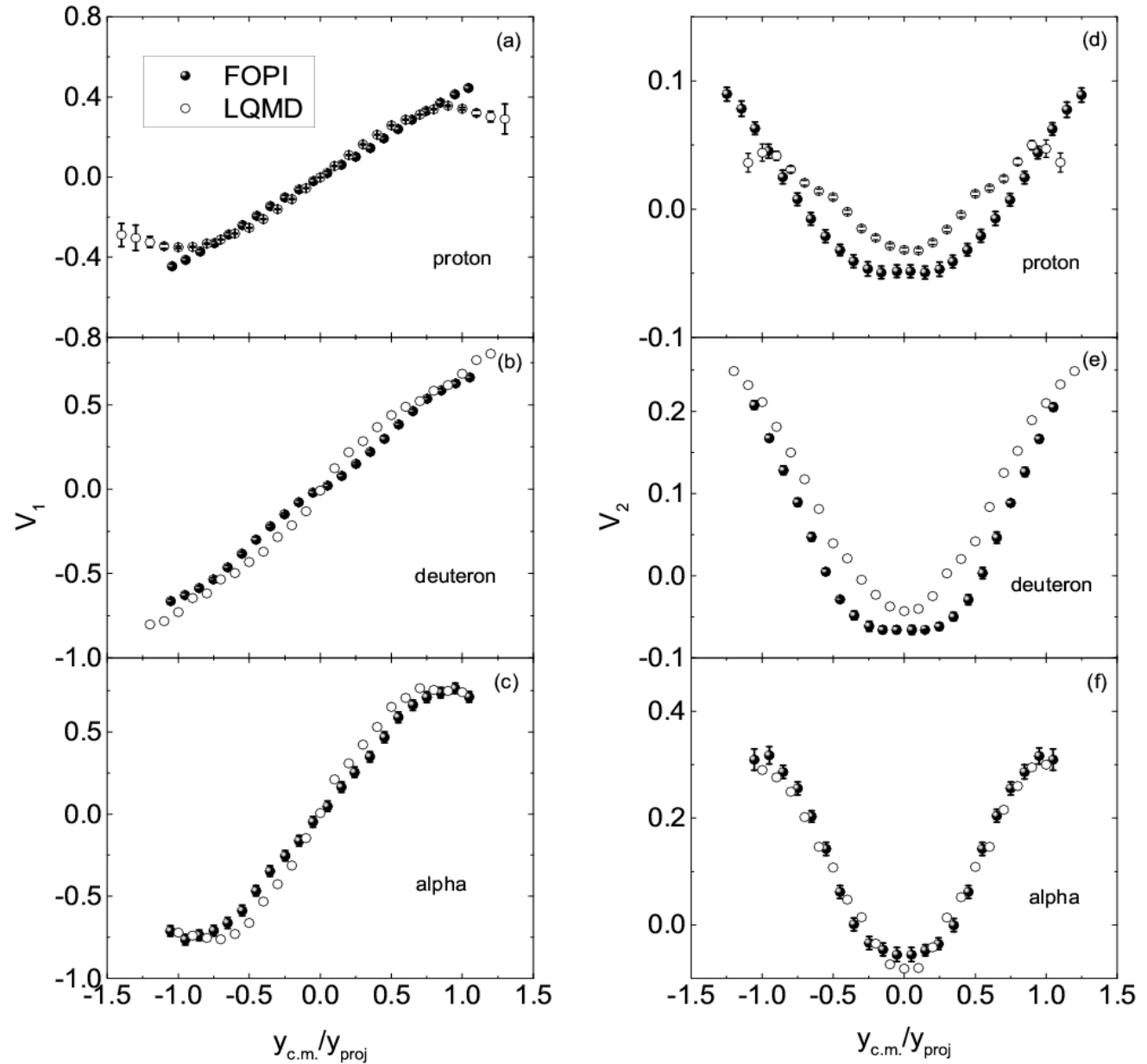
R. Mattiello et al., Phys. Rev. C 55, 1443 (1997)

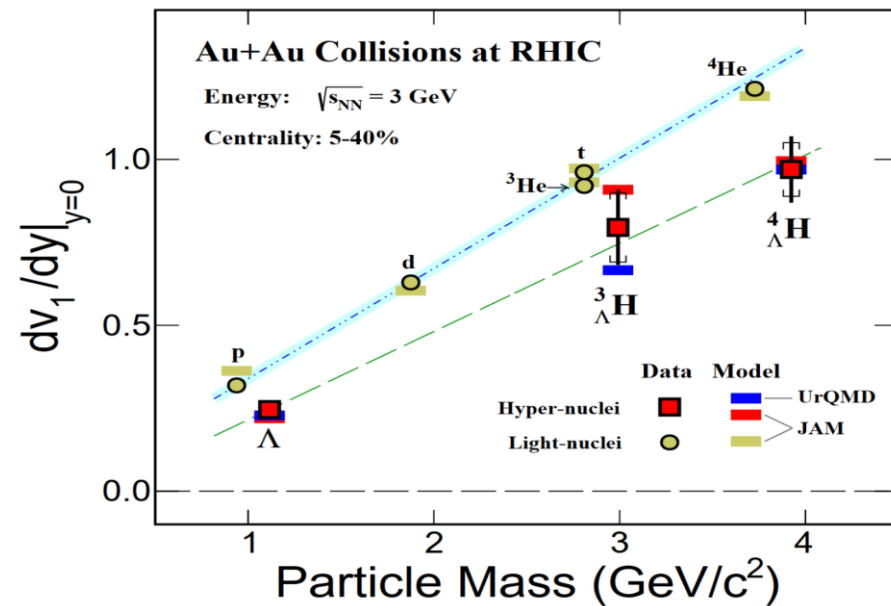
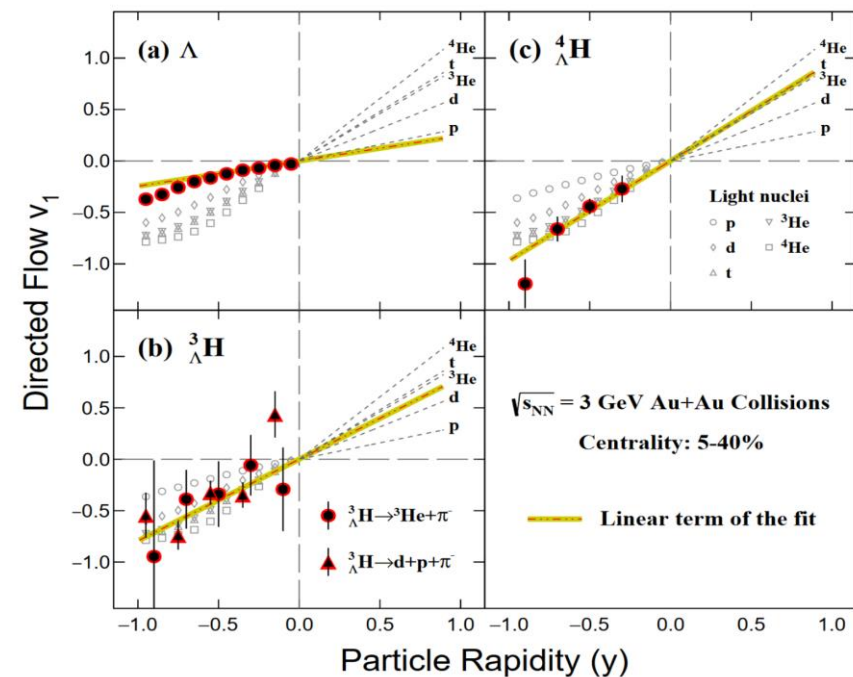
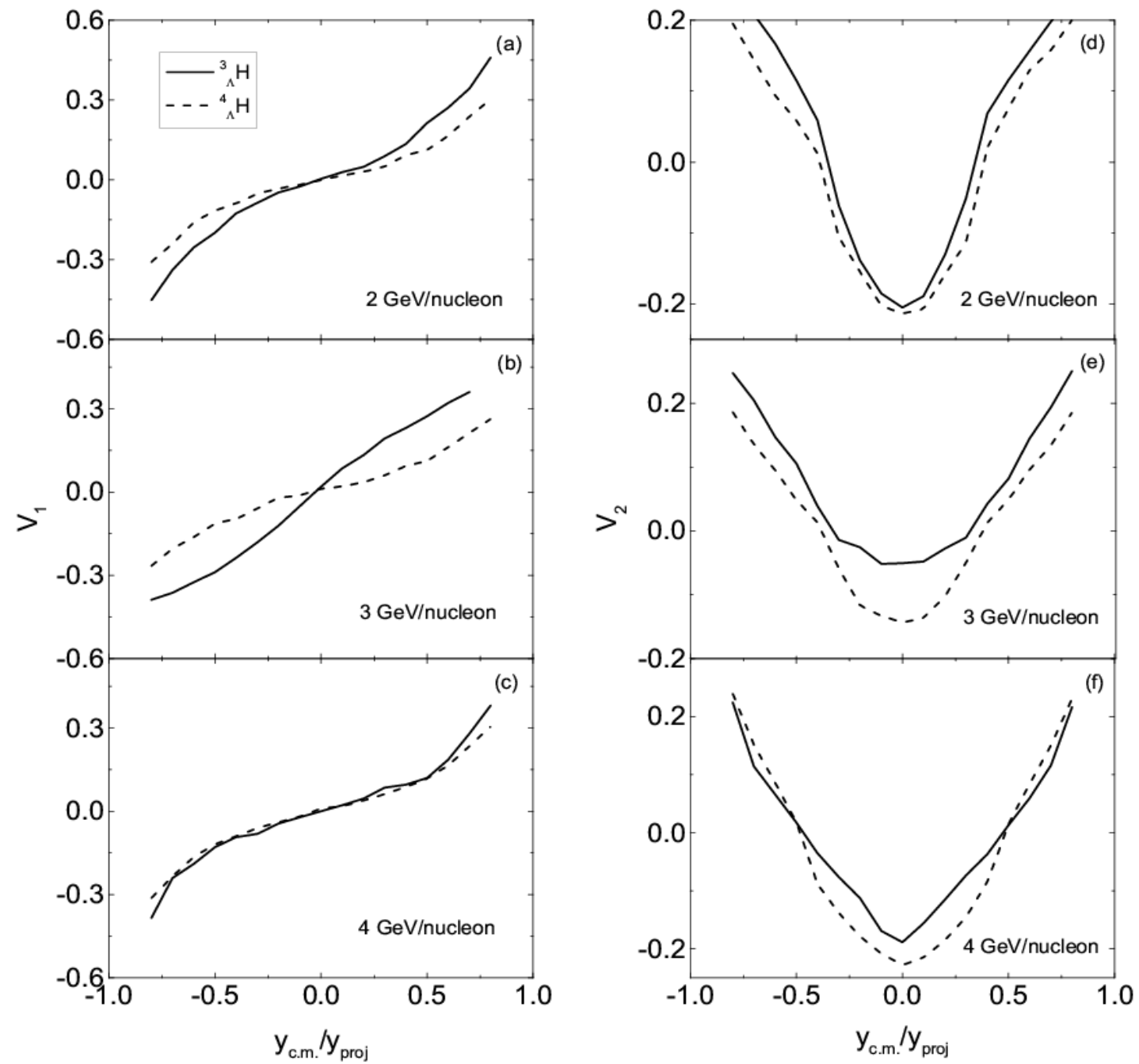
$$\frac{dN_M}{d^3P} = G_M \binom{A}{M} \binom{M}{Z} \frac{1}{A^M} \int \prod_{i=1}^Z f_p(\mathbf{r}_i, \mathbf{p}_i) \prod_{i=Z+1}^M f_n(\mathbf{r}_i, \mathbf{p}_i) \times \rho^W(\mathbf{r}_{k_1}, \mathbf{p}_{k_1}, \dots, \mathbf{r}_{k_{M-1}}, \mathbf{p}_{k_{M-1}}) \delta(\mathbf{P} - (\mathbf{p}_1 + \dots + \mathbf{p}_M)) d\mathbf{r}_1 d\mathbf{p}_1 \dots d\mathbf{r}_M d\mathbf{p}_M$$

### $^3_1\text{H}$ via $^{197}\text{Au} + ^{197}\text{Au}$



LQMD calculation: Eur. Phys. J. A, 57 (2021) 18; FOPI data from Nucl. Phys. A 876, 1 (2012)





### 3) Surface coalescence for cluster construction

A. Boudard, J. Cugnon, S. Leray et al., Nucl. Phys. A 740, 195-210 (2004)

Hui-Gan Cheng, Zhao-Qing Feng, Chinese Physics C 45, 084107 (2021)

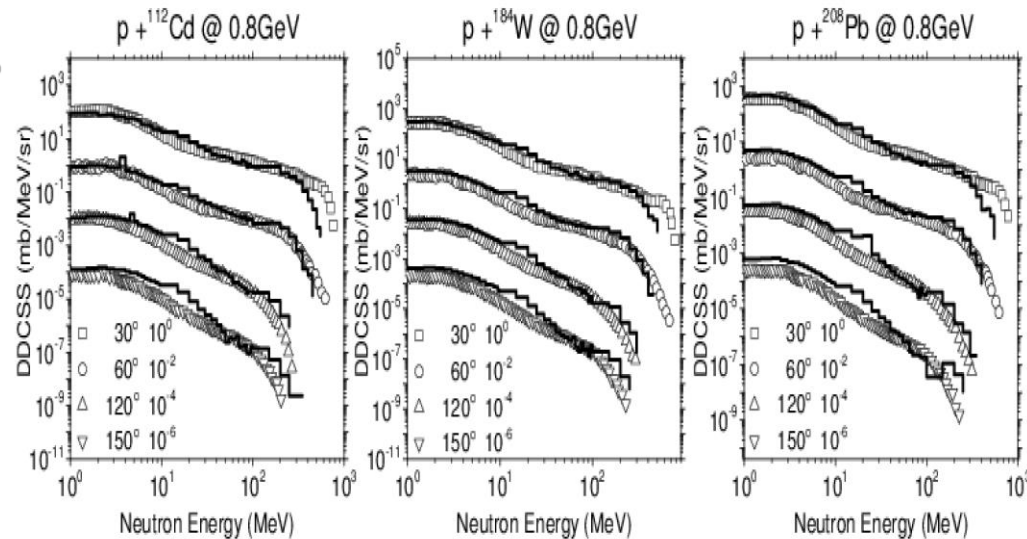
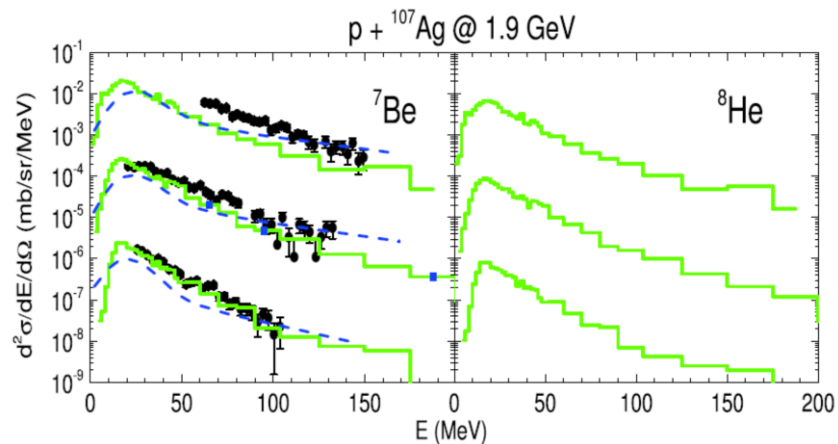
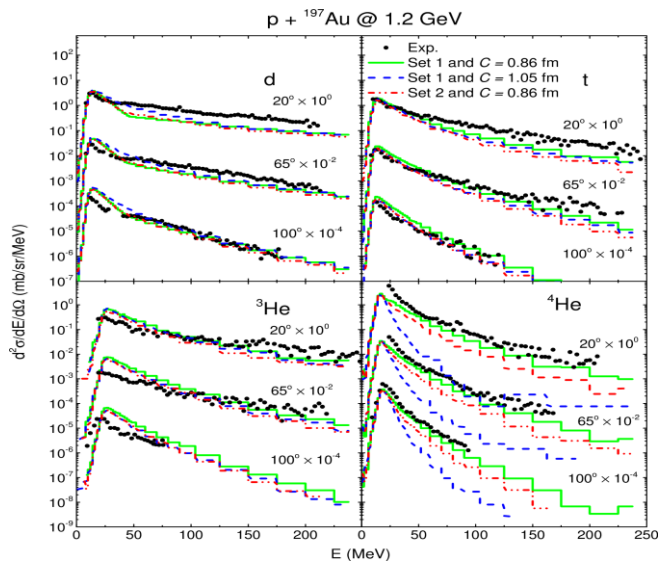
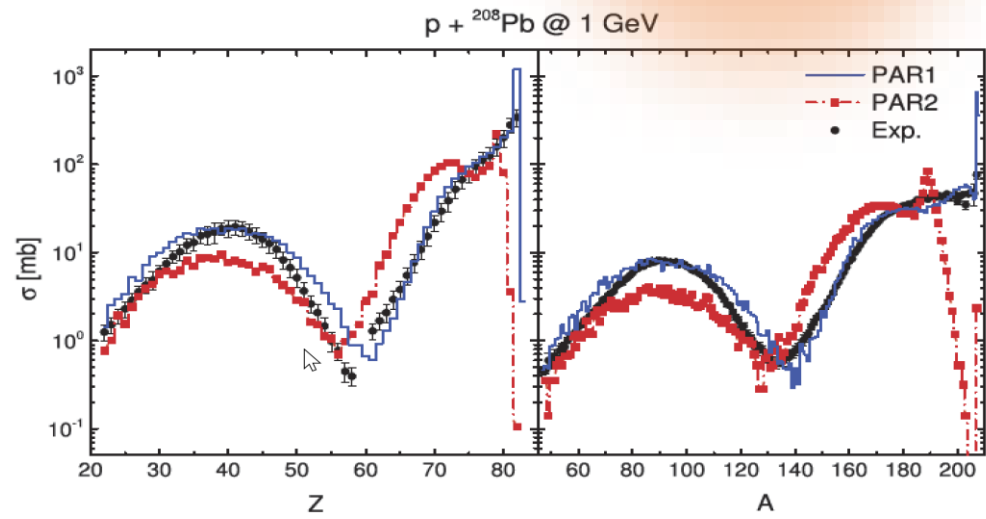
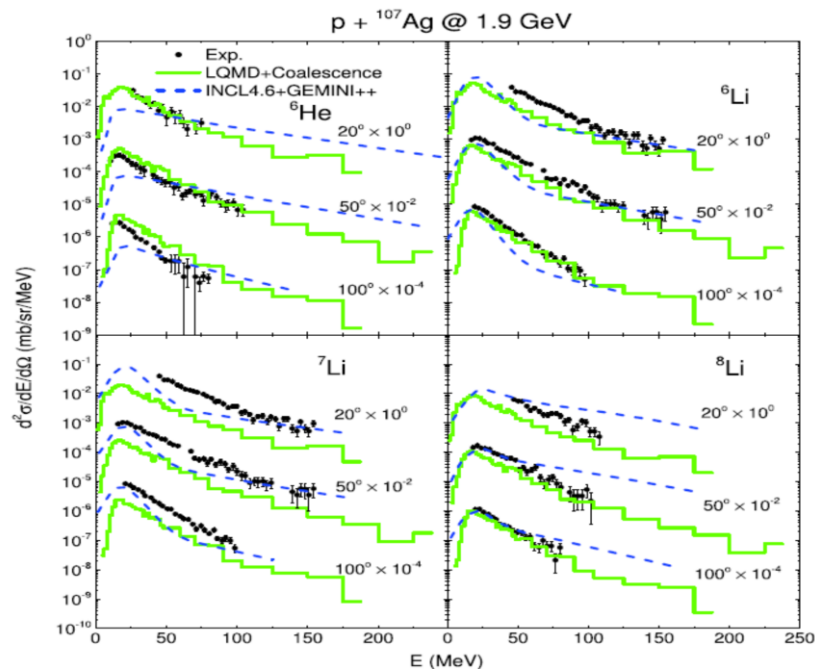
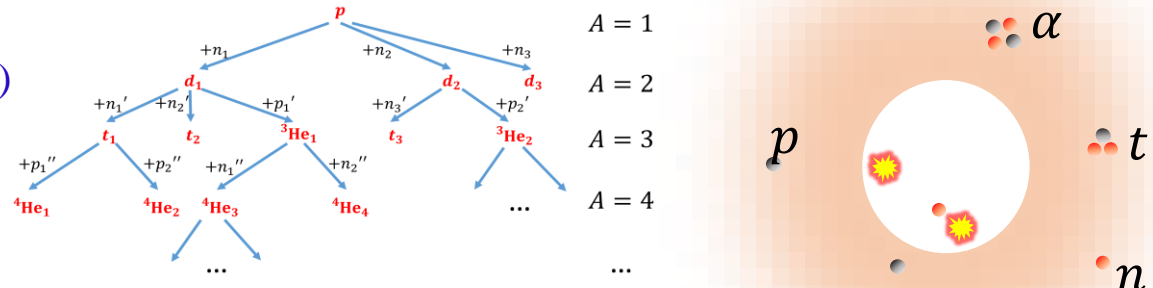
$$R_{Nj} P_{Nj} \leq h_0, \quad R_{Nj} \geq 1 \text{ fm},$$

$$R_{Nj} = |\mathbf{R}_N - \mathbf{r}_j|,$$

$$P_{Nj} = \left| \frac{m_j}{M_N + m_j} \mathbf{p}_N - \frac{M_N}{M_N + m_j} \mathbf{p}_j \right|.$$

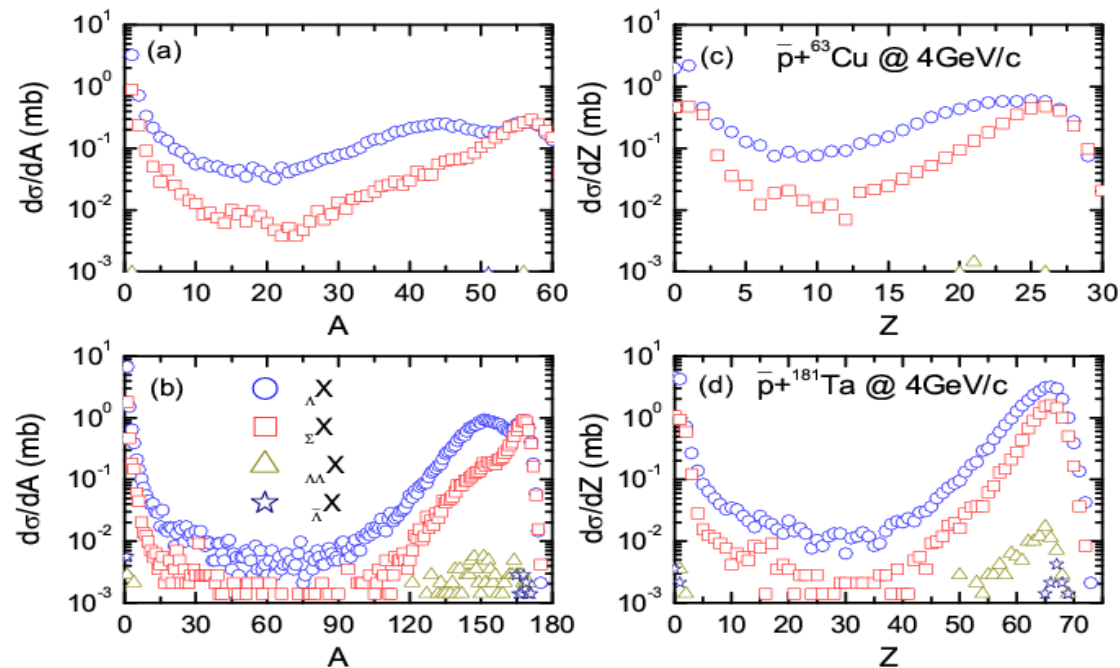
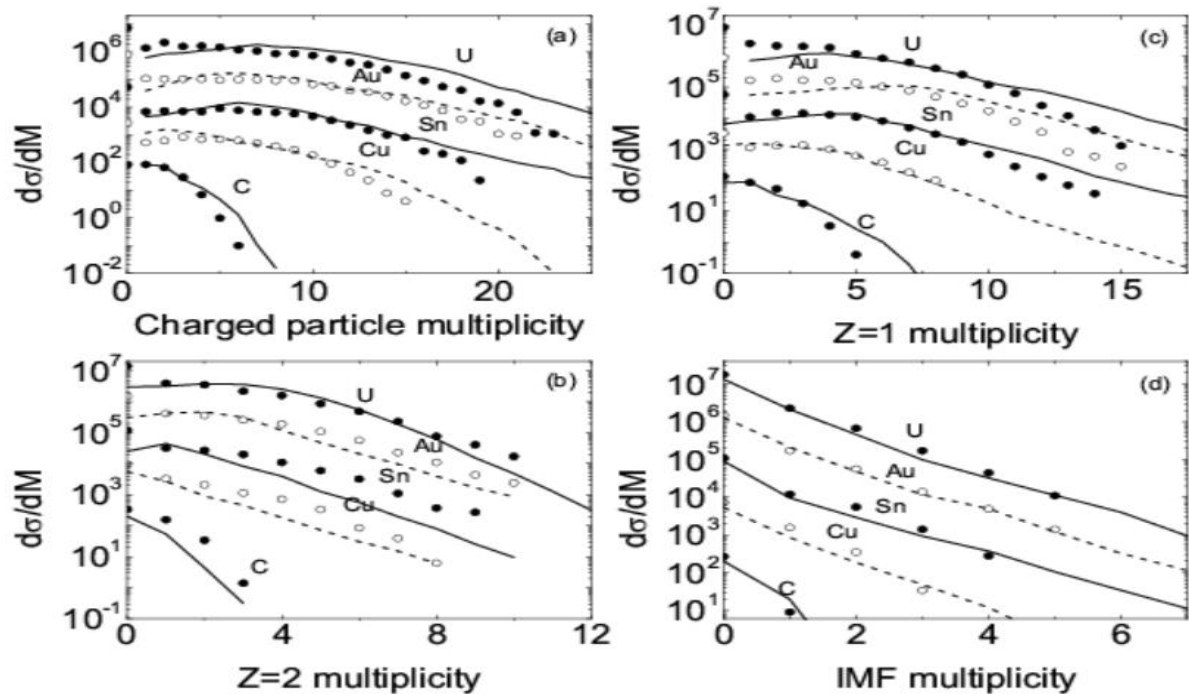
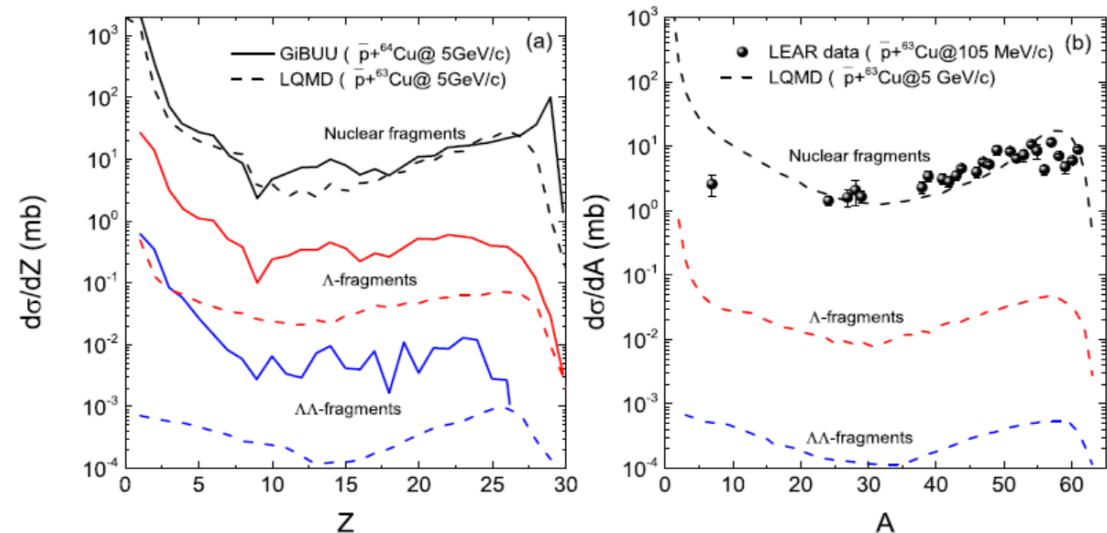
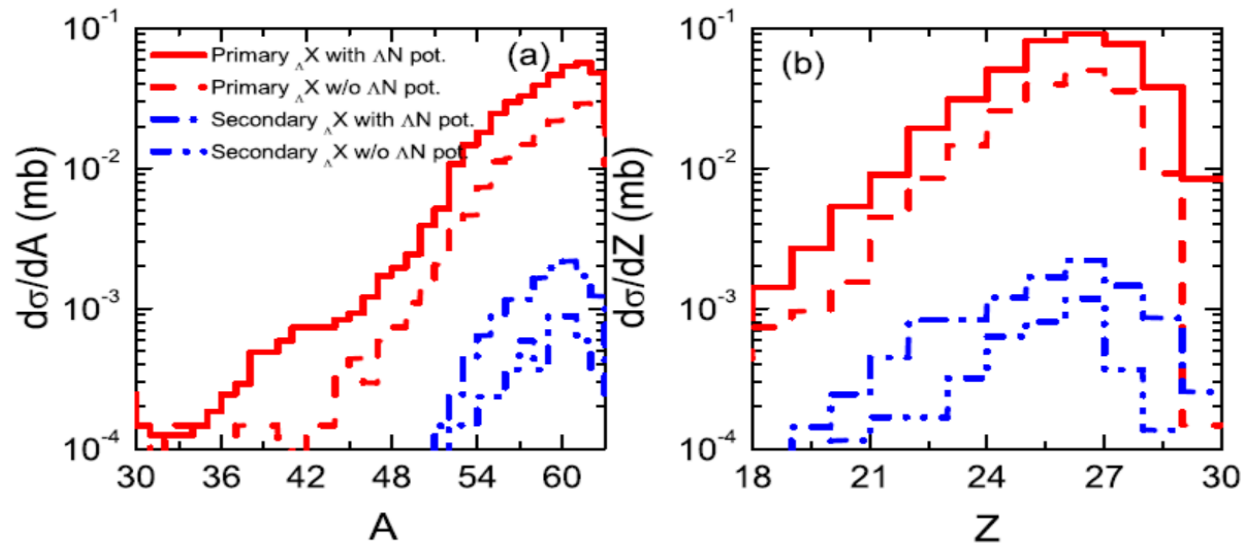
**Table 2.** Surface coalescence parameters of Set 1 and Set 2.

Construction	$h_0 / (\text{fm MeV}/c)$	
	Set 1	Set 2
$p+n \rightarrow d$	387	336
$d+n \rightarrow t$	387	315
$d+p \rightarrow {}^3\text{He}$	387	315
$t+p \rightarrow {}^4\text{He}$	387	300
${}^3\text{He}+n \rightarrow {}^4\text{He}$	387	300



# (Hyper) nuclear fragments with antiproton induced reactions

Zhao-Qing Feng, Physical Review C 101, 064601 (2020); 93, 041601(R) (2016)



## 4) Kinetic approach for cluster production

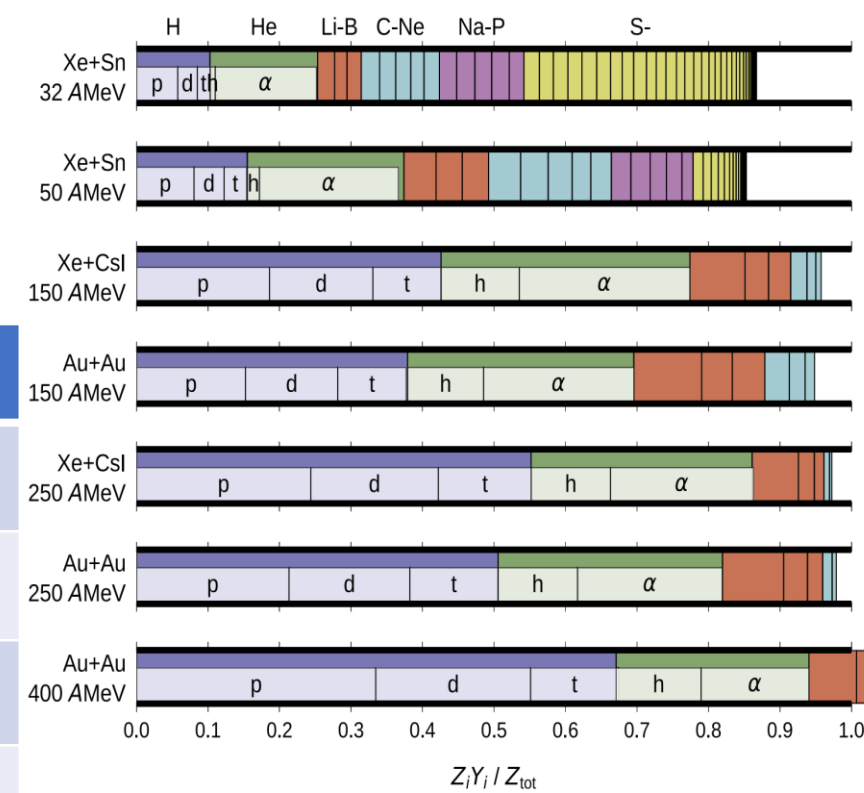
P. Danielewicz, G. F. Bertsch, Nuclear Physics A 533 (1991) 712-748

Akira Ono, Prog. Part. Nucl. Phys. 105, 139-179 (2019)

R. Wang, Y. G. Ma, L. W. Chen et al., Phys. Rev. C 108, L031601 (2023)

Hui-Gan Cheng, Zhao-Qing Feng, Phys. Rev. C 109, L021602 (2024)

Year	models	Author(s)	Cluster(s)	Energy	Treatment(s)
1991	pBUU	P. Danielewicz et al.	$d, t, h$	fermi /intermediate energies	kinetic, Mott cut
2013	AMD-cluster	A. Ono	$2N, 3N, \alpha$	fermi /intermediate energies	kinetic, fermionic mean field
2021	SMASH	J. Staudenmaier et al.	$d$	GeV and higher	kinetic
2022	PHQMD	G. Coci et al.	$d$	GeV and higher	kinetic
2023	IBUU	R. Wang et al.	$d, t, h, \alpha$	intermediate energies	kinetic, Mott cut
2023	LQMD	H. G. Cheng and Z. Q. Feng	$d, t, h, \alpha$	fermi /intermediate energies	Kinetic, binding energy, Pauli effects



**Clusters are produced by multinucleon or nucleon-cluster collisions**

$N_1 + N_2 \leftrightarrow \text{deuteron}, N_1 + N_2 + D_1 \rightarrow \text{deuteron} + N'_1, N_1 + N_2 + N_3 \leftrightarrow \text{triton (helium-3)},$

$N_1 + N_2 + N_3 + D_1 \rightarrow \text{triton (helium-3)} + N'_1, N_1 + N_2 + N_3 + N_4 \leftrightarrow \text{alpha}$

$p + \Lambda \leftrightarrow \Lambda^2\text{H}, p + \Lambda + n \leftrightarrow \Lambda^3\text{H}, p + \Lambda + n + n \leftrightarrow \Lambda^4\text{H}, p + \Lambda + \Lambda \leftrightarrow \Lambda\Lambda^3\text{H}, p + n + \Lambda + \Lambda \leftrightarrow \Lambda\Lambda^4\text{H}$

...

$$\frac{d\sigma}{d\Omega} = P(C_1 + C_2 \rightarrow C_3 + C_4) \times \frac{v_{\tilde{p}_{\text{rel}}}}{v} \frac{|\partial e(k)/\partial k|_{k=\tilde{p}_{\text{rel}}}}{|\partial H(p_f)/\partial p_f|_{p_f=p_{\text{rel}}}} \frac{p_{\text{rel}}^2}{\tilde{p}_{\text{rel}}^2} \left[ \frac{d\sigma_{\text{NN}}}{d\Omega} \right]_{\tilde{p}_{\text{rel}}}$$

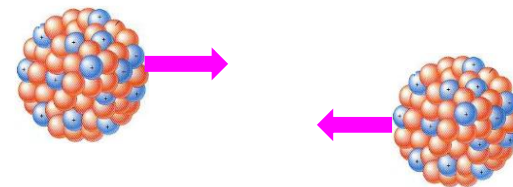
# III. LQMD transport model



## 兰州量子分子动力学模型 (LQMD)

Heavy-ion collisions (5 MeV – 5 GeV/nucleon) and hadron induced reaction ( $p$ ,  $\bar{p}$ ,  $\pi$ ,  $K$ ,  $e$ , etc)

- **LQMD transport model** (Skyrme interaction, Walecka model with  $\sigma$ ,  $\omega$ ,  $\rho$ ,  $\delta$ )
- **Neutron star equation of state** (nuclear **symmetry energy** at sub- and supra- saturation densities in HICs, isospin splitting of nucleon effective mass from HICs, particle production, 2-body and 3-body potential, multi-body correlation)
- **In-medium effects of hadrons** (optical potentials, energy conservation and in-medium effects, i.e.,  $\Delta(1232)$ ,  $N^*(1440)$ ,  $N^*(1535)$ ), hyperons ( $\Lambda, \Sigma, \Xi$ ) and mesons ( $\pi, K, \eta, \rho, \omega, \phi \dots$ )
- **Kinetic production of (hyper)clusters and nuclear fragmentation reactions** (production cross section, phase-space distribution, collective flows, cluster transportation, Mott effect, e.g., deuteron, triton,  $^3\text{He}$ ,  $\alpha$ ,  $_{\Lambda(\Sigma)}X$ ,  $_{\Lambda\Lambda}X$ ,  $_{\Xi}X$ ,  $_{\bar{\Lambda}}X$ )
- **Nuclear fusion near Coulomb barrier energies** (barrier distribution, neck dynamics, fusion cross section etc)
- **Hadron induced nuclear reactions** (spallation reaction, physics at PANDA such as hypernuclear, neutron skin thickness etc)



# Transport models for heavy-ion collisions

Progress in Particle and Nuclear Physics 125 (2022) 103962

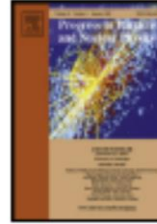


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Review

## Transport model comparison studies of intermediate-energy heavy-ion collisions

Hermann Wolter<sup>1,\*</sup>, Maria Colonna<sup>2</sup>, Dan Cozma<sup>3</sup>, Pawel Danielewicz<sup>4,5</sup>, Che Ming Ko<sup>6</sup>, Rohit Kumar<sup>4</sup>, Akira Ono<sup>7</sup>, ManYee Betty Tsang<sup>4,5</sup>, Jun Xu<sup>8,9</sup>, Ying-Xun Zhang<sup>10,11</sup>, Elena Bratkovskaya<sup>12,13</sup>, Zhao-Qing Feng<sup>14</sup>, Theodoros Gaitanos<sup>15</sup>, Arnaud Le Fèvre<sup>12</sup>, Natsumi Ikeno<sup>16</sup>, Youngman Kim<sup>17</sup>, Swagata Mallik<sup>18</sup>, Paolo Napolitani<sup>19</sup>, Dmytro Oliinychenko<sup>20</sup>, Tatsuhiko Ogawa<sup>21</sup>, Massimo Papa<sup>2</sup>, Jun Su<sup>22</sup>, Rui Wang<sup>9,23</sup>, Yong-Jia Wang<sup>24</sup>, Janus Weil<sup>25</sup>, Feng-Shou Zhang<sup>26,27</sup>, Guo-Qiang Zhang<sup>9</sup>, Zhen Zhang<sup>22</sup>, Joerg Aichelin<sup>28</sup>, Wolfgang Cassing<sup>25</sup>, Lie-Wen Chen<sup>29</sup>, Hui-Gan Cheng<sup>14</sup>, Hannah Elfner<sup>12,13,20</sup>, K. Gallmeister<sup>25</sup>, Christoph Hartnack<sup>28</sup>, Shintaro Hashimoto<sup>21</sup>, Sangyong Jeon<sup>30</sup>, Kyungil Kim<sup>17</sup>, Myungkuk Kim<sup>31</sup>, Bao-An Li<sup>32</sup>, Chang-Hwan Lee<sup>33</sup>, Qing-Feng Li<sup>24,34</sup>, Zhu-Xia Li<sup>10</sup>, Ulrich Mosel<sup>25</sup>, Yasushi Nara<sup>35</sup>, Koji Niita<sup>36</sup>, Akira Ohnishi<sup>37</sup>, Tatsuhiko Sato<sup>21</sup>, Taesoo Song<sup>12</sup>, Agnieszka Sorensen<sup>38,39</sup>, Ning Wang<sup>11,40</sup>, Wen-Jie Xie<sup>41</sup>, (TMEP collaboration)

**Table 1**

List of transport models that participated in the TMEP code comparisons discussed in this paper. The columns give the information on the name of the code, the main correspondents of the code, the energy range intended for the code, the treatment of effects of relativity (see Section 2.1), and the comparisons in which the code participated. The different comparisons are listed in the last column in the table by a numbers  $n$ , which refer to the subsections 3.n, where they are described in detail:  $n = 1$  for Au+Au collisions around 1 AGeV,  $n = 2$  for Au+Au collision at 100 and 400 AMeV,  $n = 3$  for box-Vlasov,  $n = 4$  for box-cascade with only nucleons,  $n = 5$  for box-cascade with pion and  $\Delta$  resonance production, and  $n = 6$  for the prediction of pion ratios for Sn+Sn collisions.

BUU Type	Code Correspondents	Energy Range [A GeV]	Relativity	Comparisons
BLOB	P. Napolitani, M. Colonna	0.01–0.5	non-rel	2
BUU-VM	S. Mallik	0.02–1	rel	3,4,5
DJBUU	Y. Kim, S. Jeon, M. Kim, C.-H. Lee, K. Kim	0.05–2	cov	3
GiBUU	J. Weil, T. Gaitanos, K. Gallmeister, U. Mosel	0.05–40	rel/cov	1,2,3,4
IBL	W.J. Xie, F.S. Zhang	0.05–2	rel	2
IBUU	J. Xu, L.W. Chen, B.A. Li	0.05–2	rel	2,3,4,5
LBUU(LHV)	R. Wang, Z. Zhang, L.-W. Chen	0.01–1.5	rel	3
pBUU	P. Danielewicz	0.01–12	rel	1,2,3,4,5,6
PHSD	E. Bratkovskaya, W. Cassing	0.1–200	rel/cov	1,6
RBUU	T. Gaitanos	0.05–2	cov	1,2
RVUU	Z. Zhang, C.M. Ko, T. Song	0.05–2	cov	1,2,3,4,5
SMASH	D. Oliinychenko, H. Elfner, A. Sorensen	0.5–200	cov	3,4,5,6
SMF	M. Colonna, P. Napolitani	0.01–0.5	non-rel	2,3,4
$\chi$ BUU	Z. Zhang, C.M. Ko	0.01–0.5	non-rel	6

QMD Type	Code Correspondents	Energy Range [AGeV]	Relativity	Comparisons
AMD	A. Ono	0.01–0.3	non-rel	2
AMD+JAM	N. Ikeno, A. Ono	0.01–0.3	non-rel+rel	6
BQMD/IQMD	A. Le Fèvre, J. Aichelin, C. Hartnack, R. Kumar	0.05–2	rel	1,2,6
CoMD	M. Papa	0.01–0.3	non-rel	2,4
ImQMD	Y.X. Zhang, N. Wang, Z.X. Li	0.02–0.4	rel	2,3,4
IQMD-BNU	J. Su, F.S. Zhang	0.05–2	rel	2,3,4,5,6
IQMD-SINAP	G.Q. Zhang	0.05–2	rel	2
JAM	A. Ono, N. Ikeno, Y. Nara, A. Ohnishi	1–158	rel	4,5
JQMD 2.0	T. Ogawa, K. Niita, S. Hashimoto, T. Sato	0.01–3	rel	4,5
LQMD(IQMD-IMP)	Z.Q. Feng, H.G. Cheng	0.01–10	rel	2,3,4,5
TuQMD/dcQMD	D. Cozma	0.1–2	rel	1,2,3,4,5,6
UrQMD	Y. J. Wang, Q. F. Li, Y. X. Zhang	0.05–200	rel	1,2,3,4,6

# 1. Lanzhou quantum molecular dynamics transport model (LQMD-Skyrme)

PHYSICAL REVIEW C 84, 024610 (2011)

Momentum dependence of the symmetry potential and its influence on nuclear reactions

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(Received 11 July 2011; published 19 August 2011)

$$H_B = \sum_i \sqrt{\mathbf{p}_i^2 + \mathbf{m}_i^2} + U_{\text{int}} + U_{\text{mom}}$$

$$U_{\text{loc}} = \int V_{\text{loc}}(\rho(\mathbf{r})) d\mathbf{r}$$

$$V_{\text{loc}}(\rho) = \frac{\alpha}{2} \frac{\rho^2}{\rho_0} + \frac{\beta}{1+\gamma} \frac{\rho^{1+\gamma}}{\rho_0^\gamma} + E_{\text{sym}}^{\text{loc}}(\rho) \rho \delta^2 + \frac{g_{\text{sur}}}{2\rho_0} (\nabla \rho)^2 + \frac{g_{\text{sur}}^{\text{iso}}}{2\rho_0} [\nabla(\rho_n - \rho_p)]^2,$$

$$U_{\text{mom}} = \frac{1}{2\rho_0} \sum_{i,j,j \neq i} \sum_{\tau,\tau'} C_{\tau,\tau'} \delta_{\tau,\tau_i} \delta_{\tau',\tau_j} \int \int \int d\mathbf{p} d\mathbf{p}' d\mathbf{r} f_i(\mathbf{r}, \mathbf{p}, t) \times [\ln(\epsilon(\mathbf{p} - \mathbf{p}')^2 + 1)]^2 f_j(\mathbf{r}, \mathbf{p}', t).$$

$$E_{\text{sym}}(\rho) = \frac{1}{3} \frac{\hbar^2}{2m} \left( \frac{3}{2} \pi^2 \rho \right)^{2/3} + E_{\text{sym}}^{\text{loc}}(\rho) + E_{\text{sym}}^{\text{mom}}(\rho).$$

$$E_{\text{sym}}^{\text{loc}}(\rho) = \frac{1}{2} C_{\text{sym}} (\rho/\rho_0)^{\gamma_s}$$

$$E_{\text{sym}}^{\text{loc}}(\rho) = a_{\text{sym}} (\rho/\rho_0) + b_{\text{sym}} (\rho/\rho_0)^2.$$

Table 1: The parameters and properties of isospin symmetric EoS used in the LQMD model at the density of 0.16 fm<sup>-3</sup>.

Parameters	$\alpha$ (MeV)	$\beta$ (MeV)	$\gamma$	$C_{\text{mom}}$ (MeV)	$\epsilon$ (c <sup>2</sup> /MeV <sup>2</sup> )	$m_{\infty}^*/m$	$K_{\infty}$ (MeV)
PAR1	-215.7	142.4	1.322	1.76	$5 \times 10^{-4}$	0.75	230
PAR2	-226.5	173.7	1.309	0.	0.	1.	230

$$\begin{aligned} C_{\text{sym}} &= 38 \text{ MeV} \\ a_{\text{sym}} &= 37.7 \text{ MeV} \\ b_{\text{sym}} &= -18.7 \text{ MeV} \end{aligned}$$



## 2. Covariant energy-density functional (LQMD.RMF)

Si-Na Wei, Zhao-Qing Feng,  
Nuclear Science and Techniques 35, 15 (2024)  
arXiv:2302.09984

$$L = \bar{\psi}[i\gamma_\mu \partial^\mu - (M_N - g_\sigma \varphi - g_\delta \vec{\tau} \cdot \vec{\delta}) - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \vec{\tau} \cdot \vec{b}^\mu] \psi \\ + \frac{1}{2} (\partial_\mu \varphi \partial^\mu \varphi - m_\sigma^2 \varphi^2) - U(\varphi) + \frac{1}{2} (\partial_\mu \vec{\delta} \partial^\mu \vec{\delta} - m_\delta^2 \vec{\delta}^2) \\ + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{b}_\mu \vec{b}^\mu - \frac{1}{4} \vec{G}_{\mu\nu} \vec{G}^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \\ G_{\mu\nu} = \partial_\mu \vec{b}_\nu - \partial_\nu \vec{b}_\mu, \\ U(\varphi) = \frac{g_2}{3} \varphi^3 + \frac{g_3}{4} \varphi^4$$

### Energy density functional

$$\varepsilon = \sum_{i=n,p} 2 \int \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + M_i^{*2}} + \frac{1}{2} m_\sigma^2 \varphi^2 + U(\varphi) + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 b_0^2 + \frac{1}{2} m_\delta^2 \delta_3^2$$

### Temporal evolution in phase space

$$\dot{\mathbf{x}} = \frac{\mathbf{p}_i^*}{p_0^*} + \sum_{i \neq j}^N \left\{ \frac{g_v^2}{2m_v^2} z_j^{*\mu} u_{i,\mu} B_i B_j \frac{\partial \rho_{ij}}{\partial \mathbf{p}_i} + \frac{g_v^2}{2m_v^2} z_i^{*\mu} u_{j,\mu} B_i B_j \frac{\partial \rho_{ji}}{\partial \mathbf{p}_i} + \frac{g_v^2}{2m_v^2} z_j^{*\mu} \rho_{ji} B_i B_j \frac{\partial u_{i,\mu}}{\partial \mathbf{p}_i} \right. \\ \left. + z_j^{*\mu} \frac{B_i B_j \bar{g}_v^2}{2m_v^2} \left[ \frac{\rho_{ij}}{1 - p_{T,ij}^2/\Lambda_v^2} \frac{\partial u_{i,\mu}}{\partial \mathbf{p}_i} + \frac{u_{i,\mu}}{1 - p_{T,ij}^2/\Lambda_v^2} \frac{\partial \rho_{ij}}{\partial \mathbf{p}_i} + u_{i,\mu} \rho_{ij} \frac{\partial [1/(1 - p_{T,ij}^2/\Lambda_v^2)]}{\partial \mathbf{p}_i} \right] \right. \\ \left. + z_i^{*\mu} \frac{B_i B_j \bar{g}_v^2}{2m_v^2} \left[ \frac{u_{j,\mu}}{1 - p_{T,ji}^2/\Lambda_v^2} \frac{\partial \rho_{ji}}{\partial \mathbf{p}_i} + u_{j,\mu} \rho_{ji} \frac{\partial [1/(1 - p_{T,ji}^2/\Lambda_v^2)]}{\partial \mathbf{p}_i} \right] \right. \\ \left. - \frac{m_j^*}{p_j^{*0}} \frac{\partial S_j}{\partial \mathbf{p}_i} - \frac{m_i^*}{p_i^{*0}} \frac{\partial S_i}{\partial \mathbf{p}_i} \right\},$$

$$\dot{\mathbf{p}} = - \sum_{i \neq j}^N \left\{ \frac{g_v^2}{2m_v^2} z_j^{*\mu} u_{i,\mu} B_i B_j \frac{\partial \rho_{ij}}{\partial \mathbf{r}_i} + \frac{g_v^2}{2m_v^2} z_i^{*\mu} u_{j,\mu} B_i B_j \frac{\partial \rho_{ji}}{\partial \mathbf{r}_i} \right. \\ \left. + z_j^{*\mu} \frac{B_i B_j \bar{g}_v^2}{2m_v^2} \frac{u_{i,\mu}}{1 - p_{T,ij}^2/\Lambda_v^2} \frac{\partial \rho_{ij}}{\partial \mathbf{r}_i} \right. \\ \left. + z_i^{*\mu} \frac{B_i B_j \bar{g}_v^2}{2m_v^2} \frac{u_{j,\mu}}{1 - p_{T,ji}^2/\Lambda_v^2} \frac{\partial \rho_{ji}}{\partial \mathbf{r}_i} \right. \\ \left. - \frac{m_j^*}{p_j^{*0}} \frac{\partial S_j}{\partial \mathbf{r}_i} - \frac{m_i^*}{p_i^{*0}} \frac{\partial S_i}{\partial \mathbf{r}_i} \right\},$$

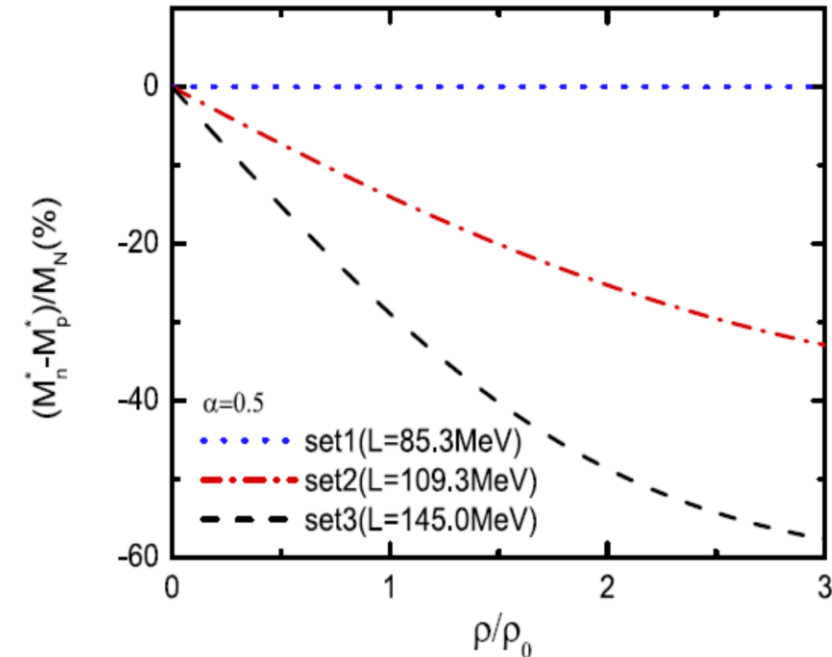
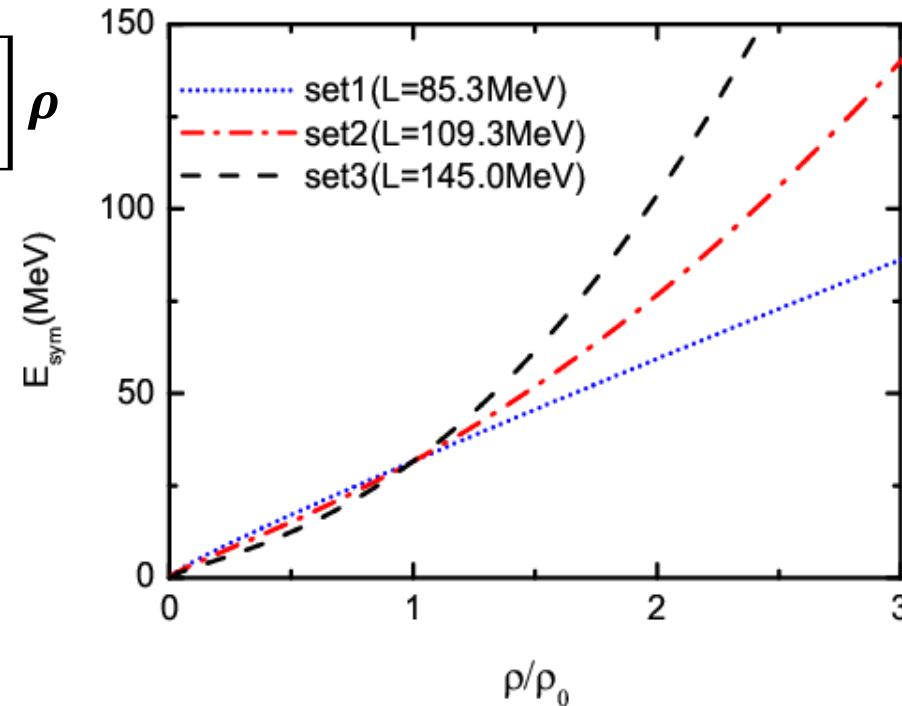
TABLE I: Parameter sets for RMF. The saturation density  $\rho_0$  is set to be  $0.16 \text{ fm}^{-3}$ . The binding energy of saturation density is  $E/A - M_N = -16 \text{ MeV}$ . The isoscalar-vector  $\omega$  and isovector-vector  $\rho$  masses are fixed to their physical values,  $m_\omega = 783 \text{ MeV}$  and  $m_\rho = 763 \text{ MeV}$ . The remaining meson mass  $m_\sigma$  is set to be  $550 \text{ MeV}$ .

model	$g_\sigma$	$g_\omega$	$g_2 \text{ (fm}^{-1}\text{)}$	$g_3$	$g_\rho$	$g_\delta$	$K \text{ (MeV)}$	$E_{sym}(\rho_0) \text{ (MeV)}$	$L(\rho_0) \text{ (MeV)}$
set1	8.145	7.570	31.820	28.100	4.049	-	230	31.6	85.3
set2	8.145	7.570	31.820	28.100	8.673	5.347	230	31.6	109.3
set3	8.145	7.570	31.820	28.100	11.768	7.752	230	31.6	145.0

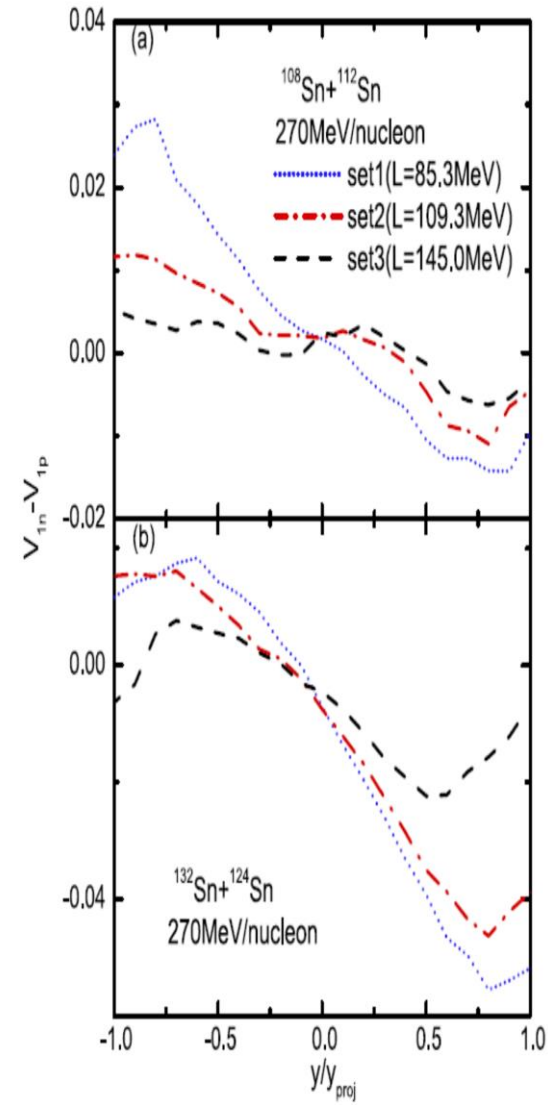
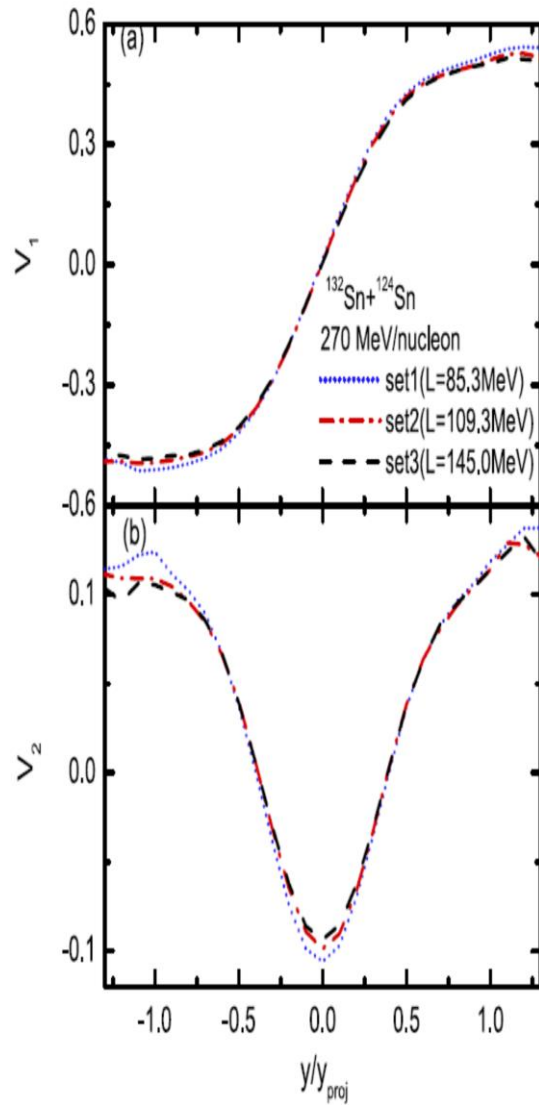
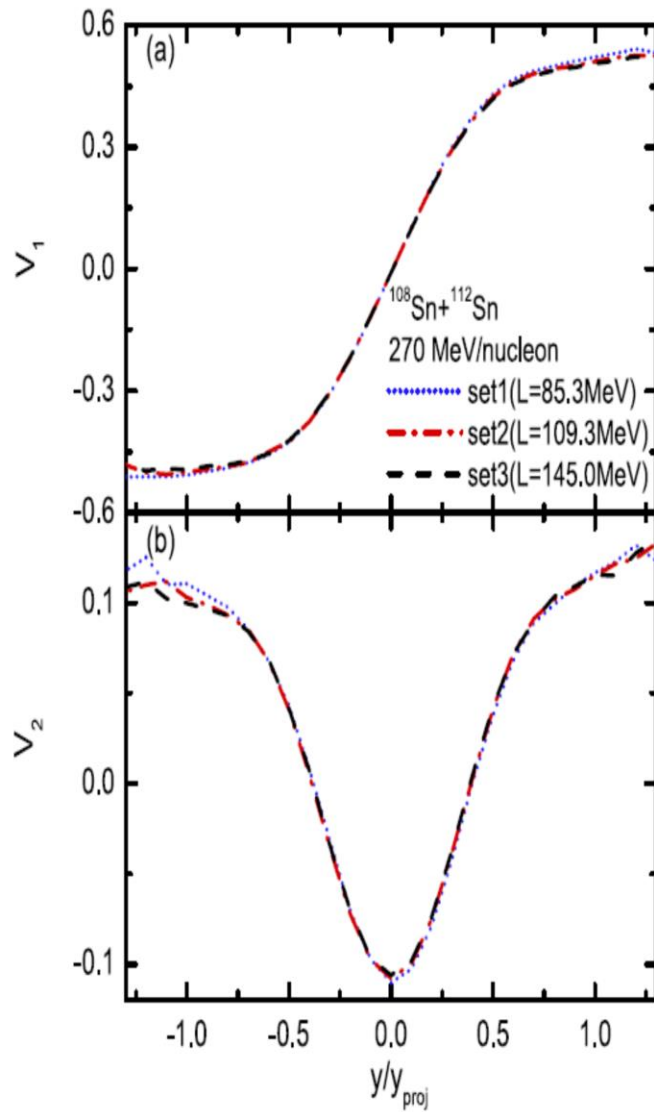
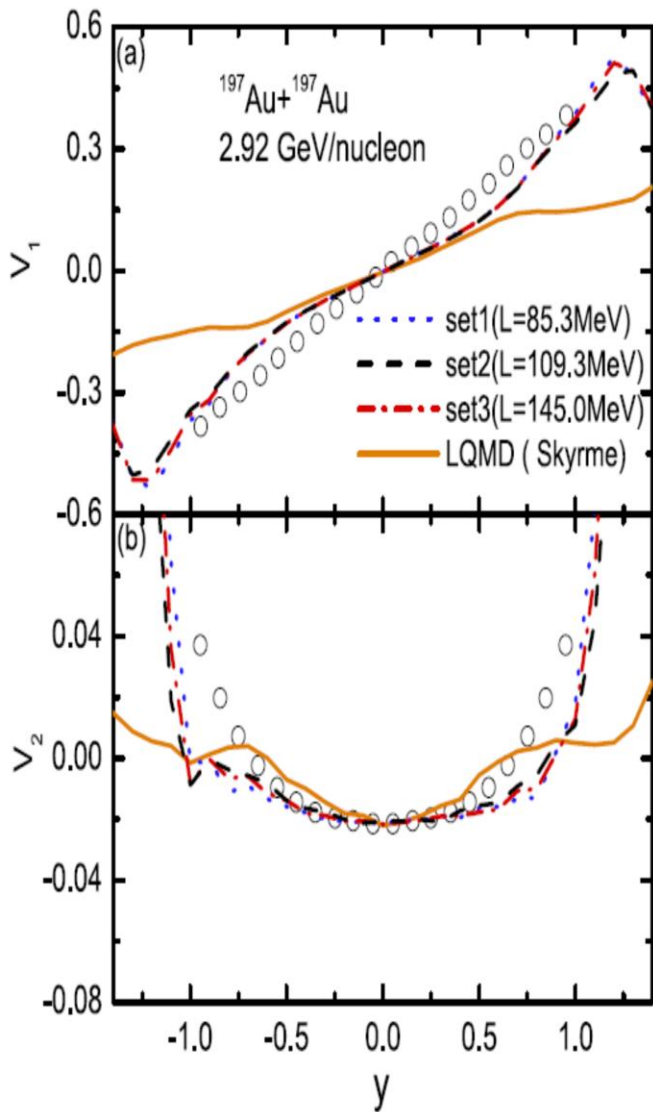
## Symmetry energy in LQMD.RMF

$$E_{sym} = \frac{1}{6} \frac{k_F^2}{E_F^*} + \frac{1}{2} \left[ f_\rho - f_\delta \left( \frac{M^*}{E_F^*} \right) \right] \rho$$

$$f_{\rho,\delta} = g_{\rho,\delta} / m_{\rho,\delta}$$



# Results from LQMD.RMF calculation: collective flows for free protons



### 3. Particle production

$\pi$  and resonances ( $\Delta(1232)$ ,  $N^*(1440)$ ,  $N^*(1535)$ , ...) production:

$$\begin{aligned}
 NN &\leftrightarrow N\Delta, & NN &\leftrightarrow NN^*, & NN &\leftrightarrow \Delta\Delta, & \Delta &\leftrightarrow N\pi, \\
 N^* &\leftrightarrow N\pi, & NN &\leftrightarrow NN\pi(s\text{-state}), & N^*(1535) &\leftrightarrow N\eta
 \end{aligned}$$

Collisions between resonances,  $NN^* \leftrightarrow N\Delta$ ,  $NN^* \leftrightarrow NN^*$

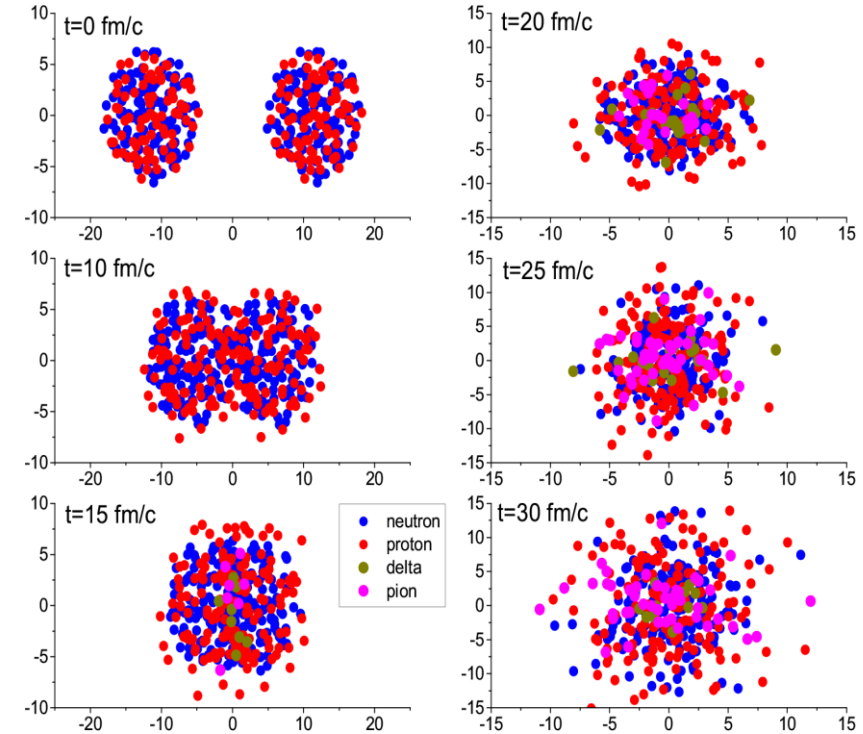
Strangeness channels:

$$\begin{aligned}
 BB &\rightarrow BYK, & BB &\rightarrow BBK\bar{K}, & B\pi(\eta) &\rightarrow YK, & YK &\rightarrow B\pi, \\
 B\pi &\rightarrow NK\bar{K}, & Y\pi &\rightarrow B\bar{K}, & B\bar{K} &\rightarrow Y\pi, & YN &\rightarrow \bar{K}NN, \\
 BB &\rightarrow B\Xi KK, & \bar{K}B &\leftrightarrow K\Xi, & YY &\leftrightarrow N\Xi, & \bar{K}Y &\leftrightarrow \pi\Xi.
 \end{aligned}$$

Reaction channels with antiproton:

$$\bar{p}N \rightarrow \bar{N}N, \quad \bar{N}N \rightarrow \bar{N}N, \quad \bar{N}N \rightarrow \bar{B}B, \quad \bar{N}N \rightarrow \bar{Y}Y$$

$$\bar{N}N \rightarrow \text{annihilation}(\pi, \eta, \rho, \omega, K, \bar{K}, K^*, \bar{K}^*, \phi)$$



Statistical model with SU(3)

symmetry for annihilation

(E.S. Golubeva et al., Nucl. Phys. A 537, 393 (1992))

The **PYTHIA** and **FRITIOF** code are used for baryon(meson)-baryon and antibaryon-baryon collisions at high invariant energies

# 4. hyperon-nucleon interaction in dense nuclear matter

$$H_Y = \sum_{i=1}^{N_Y} V_i^{Coul} + V_{opt}^Y(\mathbf{p}_i, \rho_i) + \sqrt{\mathbf{p}_i^2 + m_Y^2}$$

$$V_{opt}^Y(\mathbf{p}_i, \rho_i) = \omega_Y(\mathbf{p}_i, \rho_i) - \sqrt{\mathbf{p}_i^2 + m_Y^2}$$

$$\omega_Y(\mathbf{p}_i, \rho_i) = \sqrt{(m_Y + \Sigma_S^Y)^2 + \mathbf{p}_i^2} + \Sigma_V^Y,$$

Phenomenological potential by fitting the results of chiral effective field theory

$$V_{opt}^\Lambda(\mathbf{p}_i, \rho_i) = V_a(\rho_i/\rho_0) + V_b(\rho_i/\rho_0)^2 + C_{mom}(\rho_i/\rho_0) \ln(\epsilon \mathbf{p}_i^2 + 1)$$

$$V_{opt}^\Sigma(\mathbf{p}_i, \rho_i) = V_0(\rho_i/\rho_0)^{\gamma_s} + V_1(\rho_n - \rho_p) t_\Sigma \rho_i^{\gamma_s} + C_{mom}(\rho_i/\rho_0) \ln(\epsilon \mathbf{p}_i^2 + 1).$$

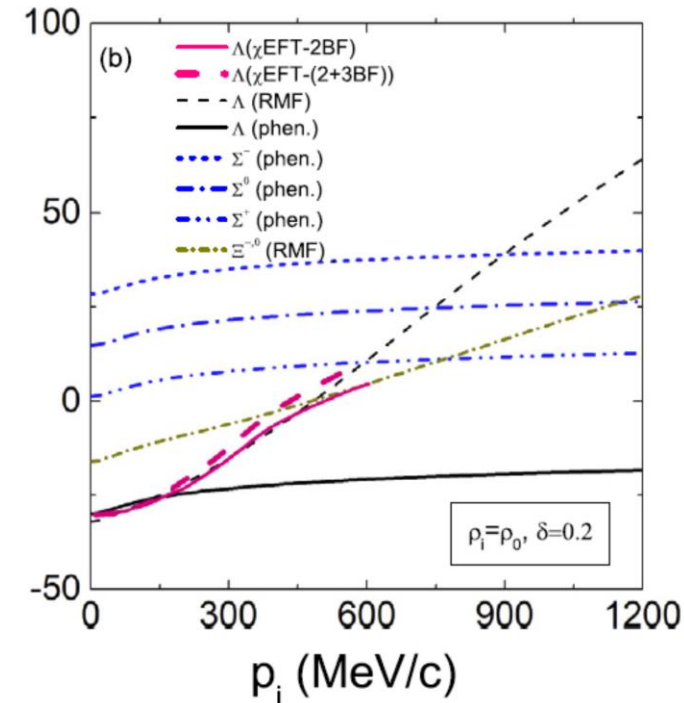
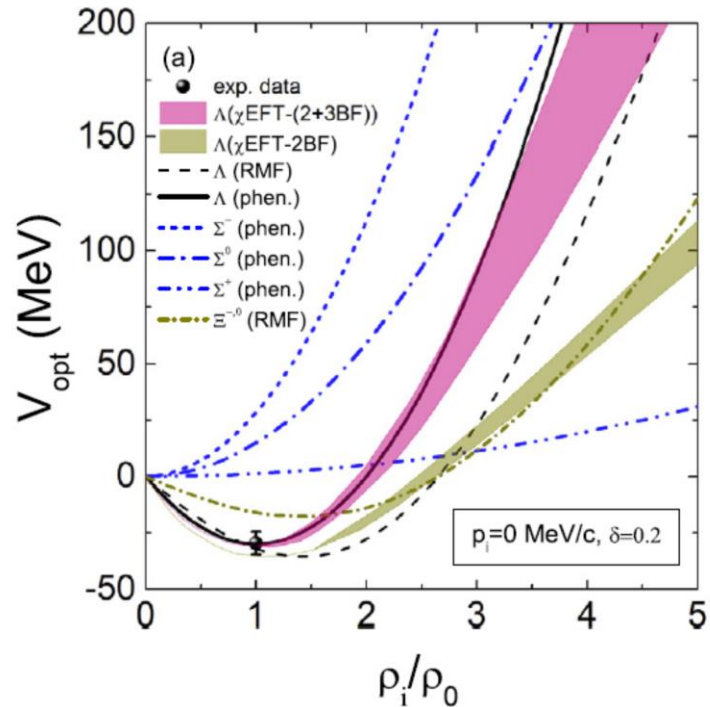


Letter

Extracting the hyperon-nucleon interaction via collective flows in heavy-ion collisions

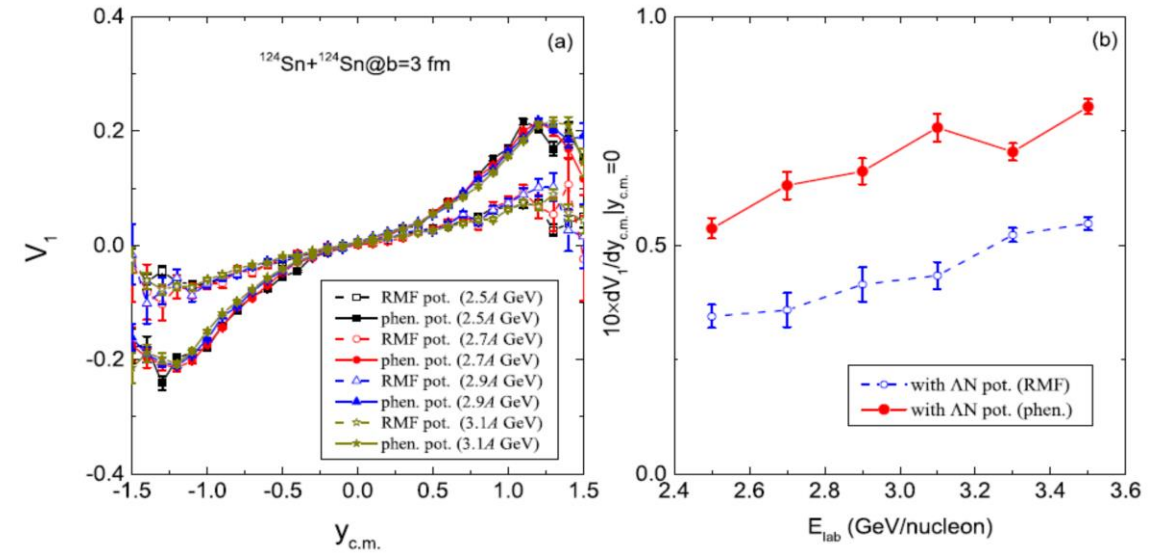
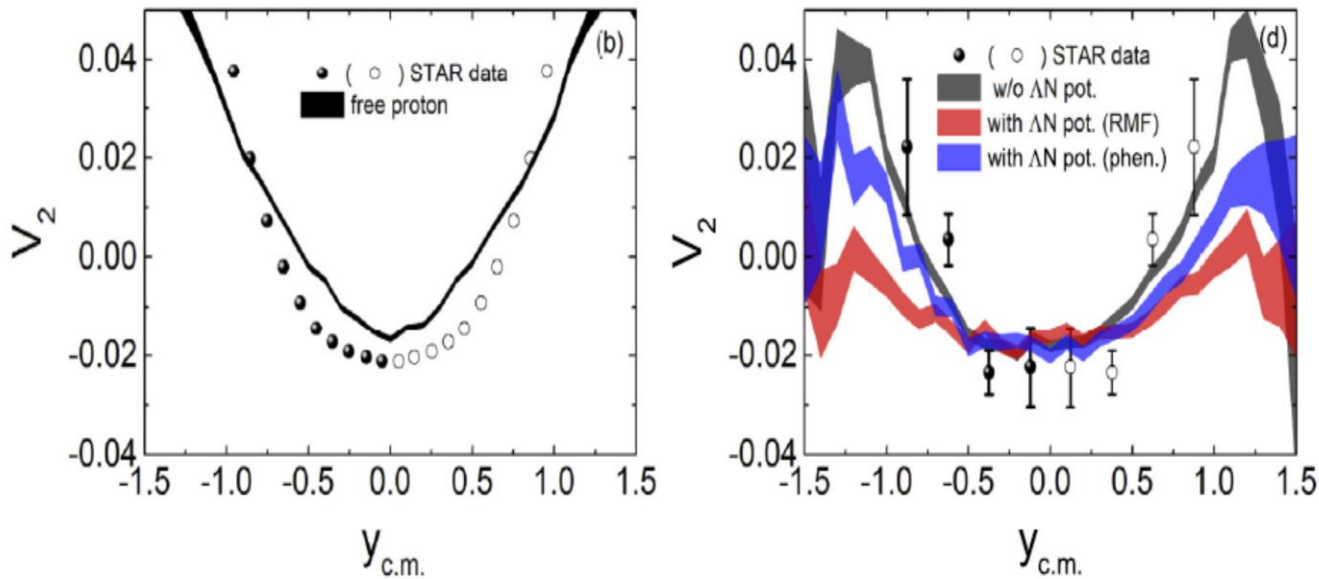
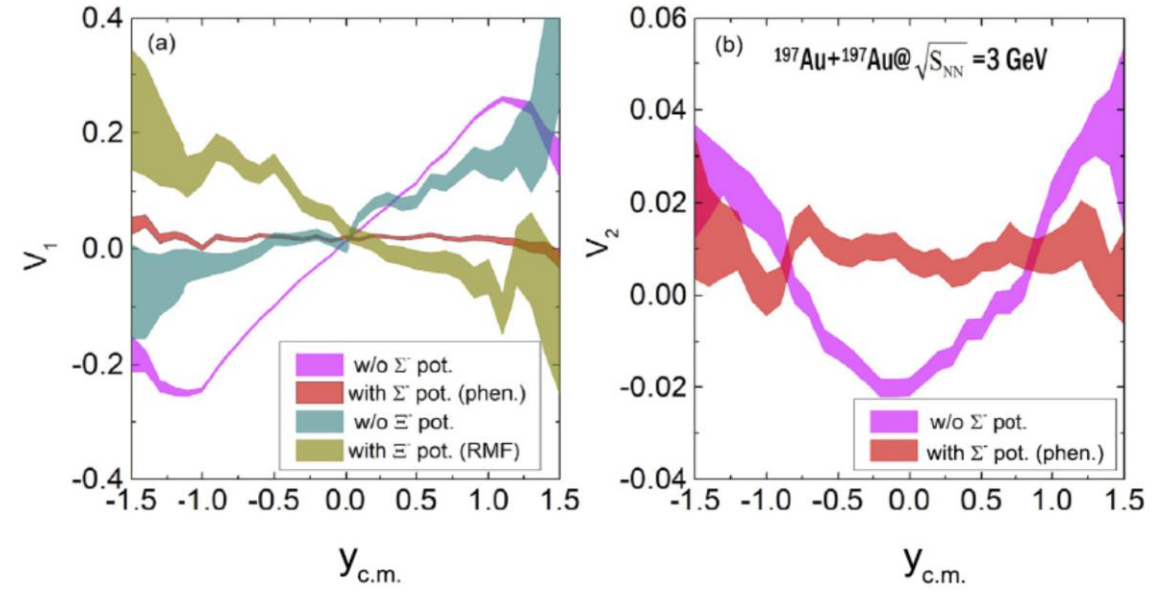
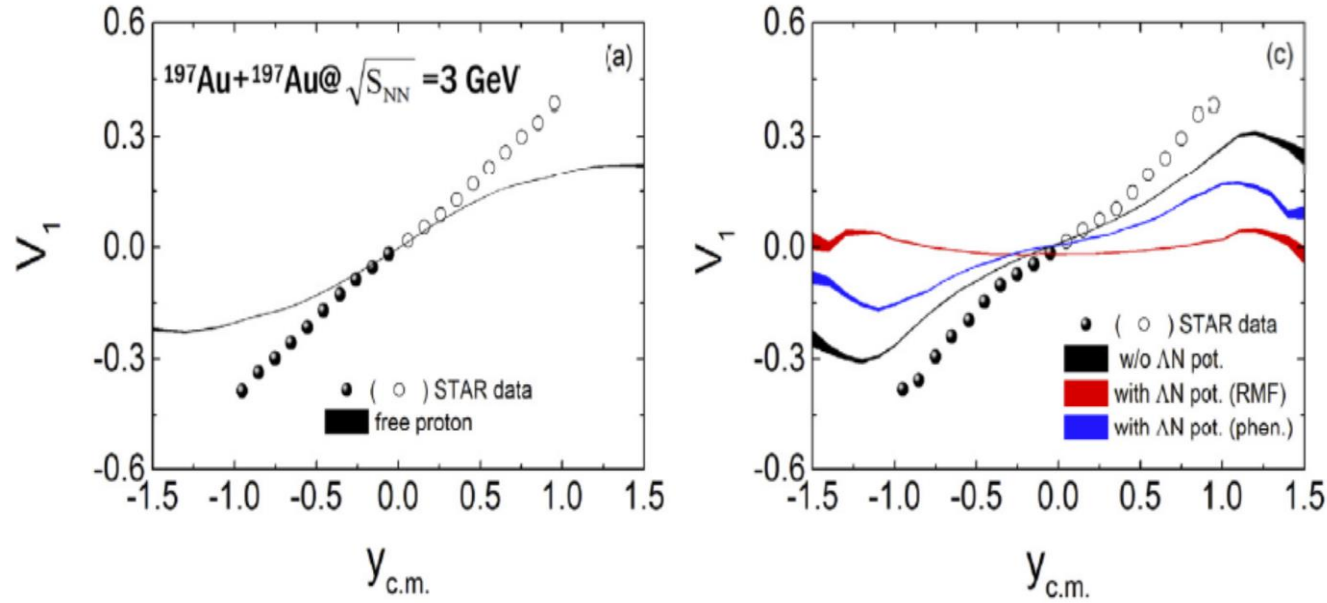
Zhao-Qing Feng <sup>ID</sup>

School of Physics and Optoelectronics, South China University of Technology, Guangzhou 510640, China



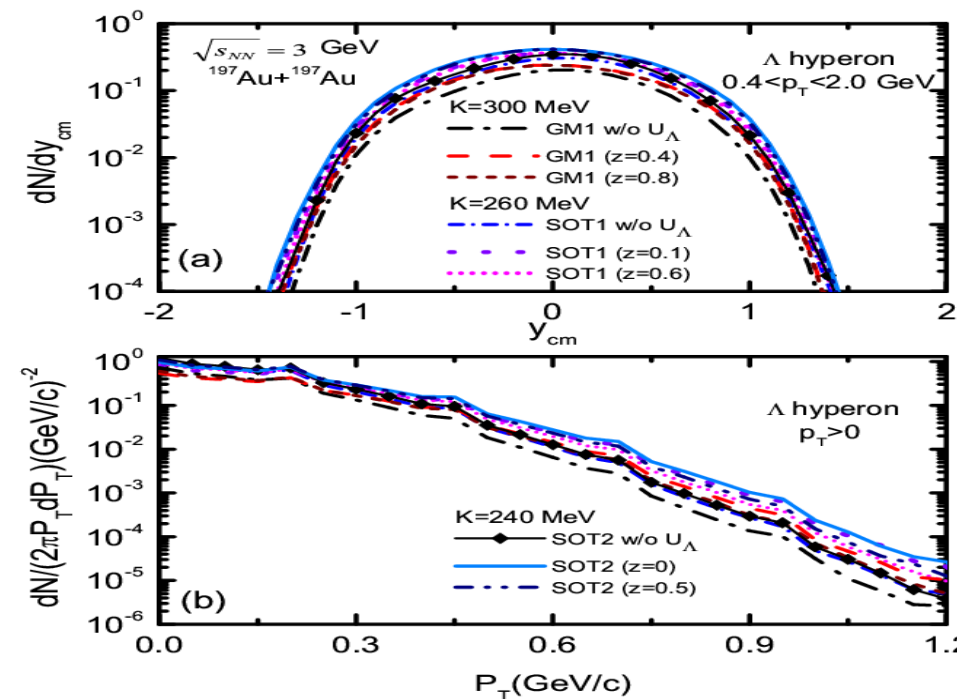
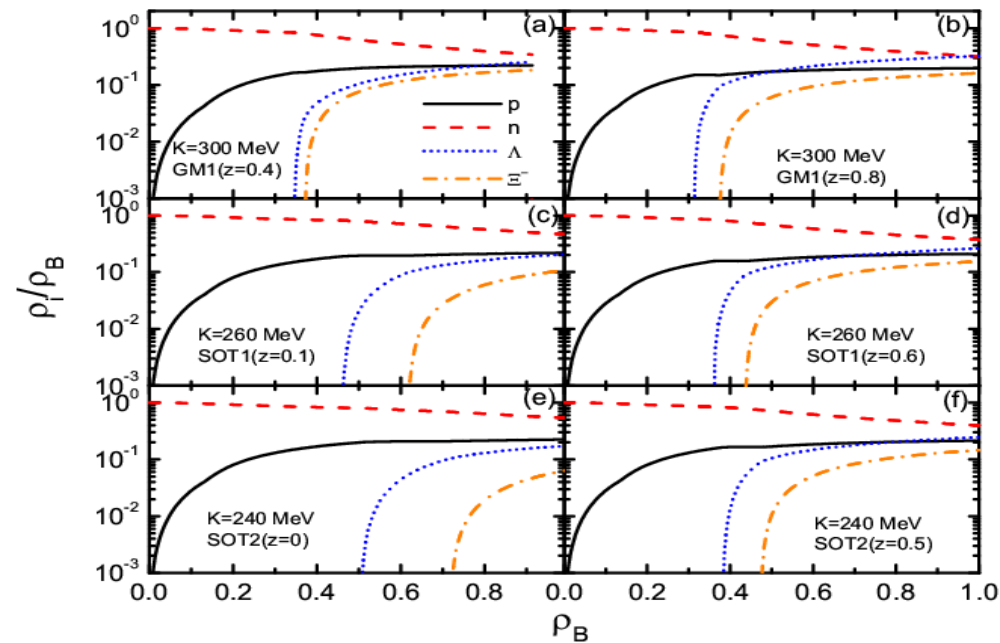
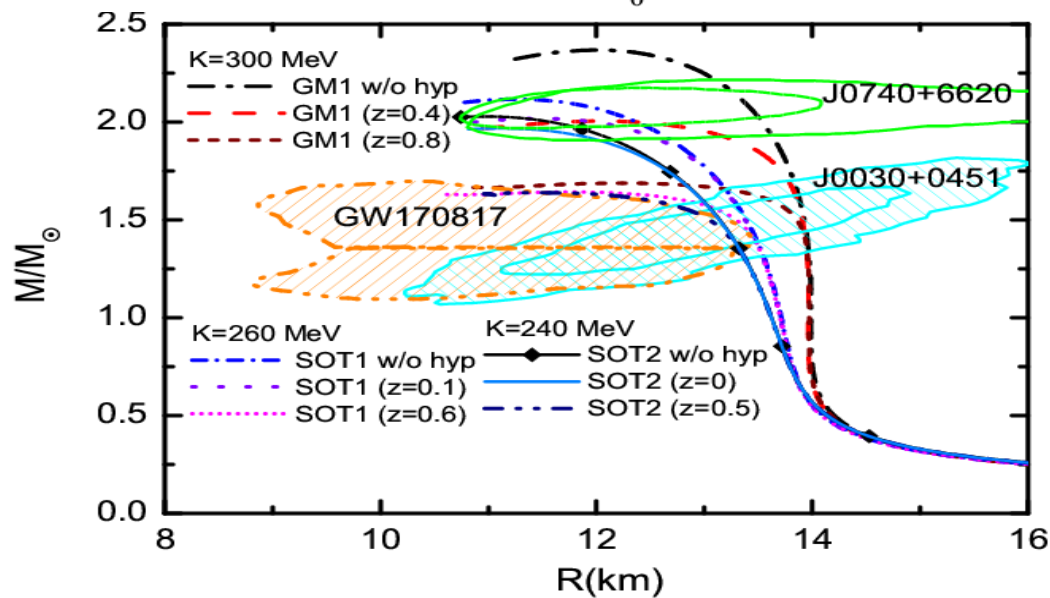
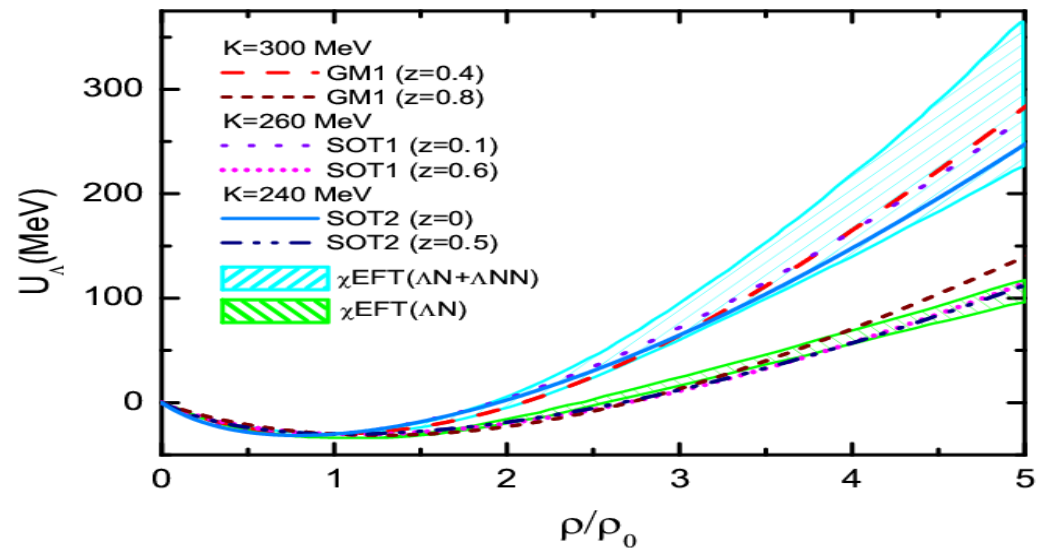
# Extracting the hyperon-nucleon interaction via collective flows in heavy-ion collisions

Phys. Lett. B 851 (2024) 138580



# Correlation of the hyperon potential stiffness with hyperon constituents in neutron stars and heavy-ion collisions

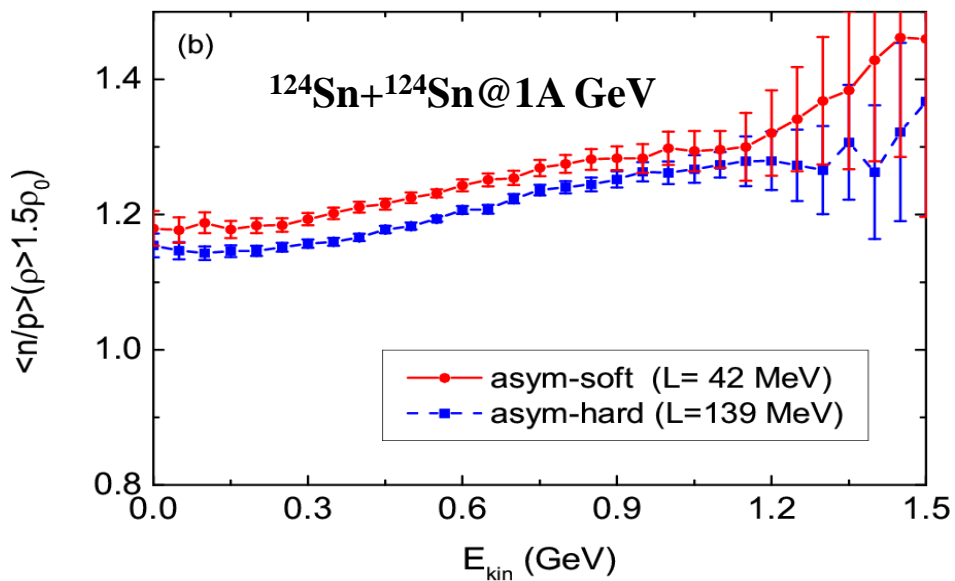
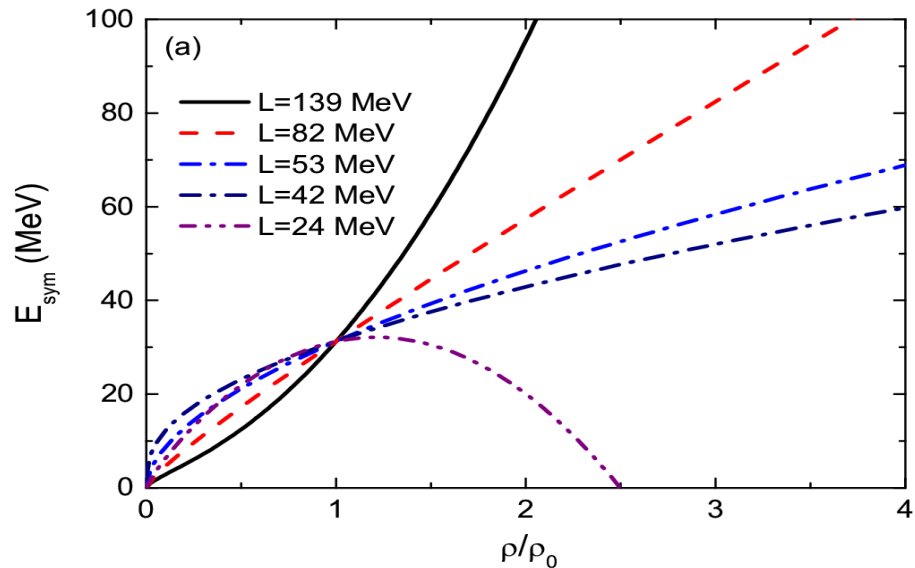
Si-Na Wei, Zhao-Qing Feng, Wei-Zhou Jiang, PLB (accepted)



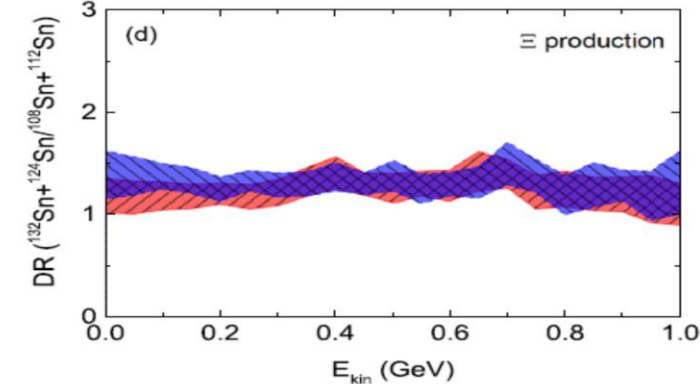
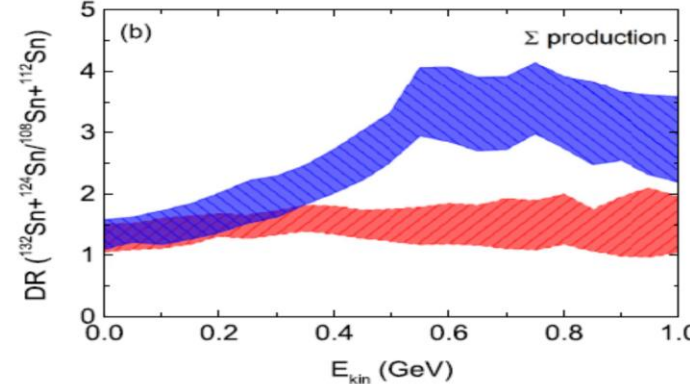
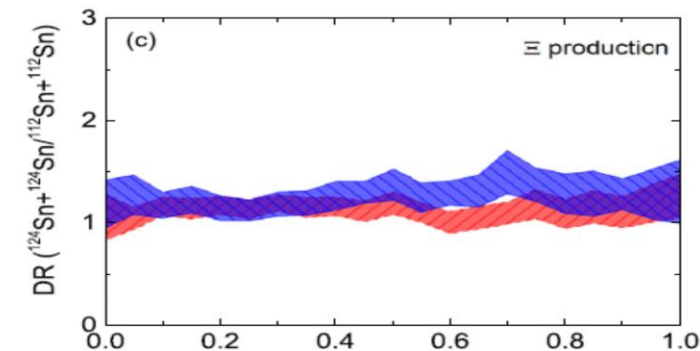
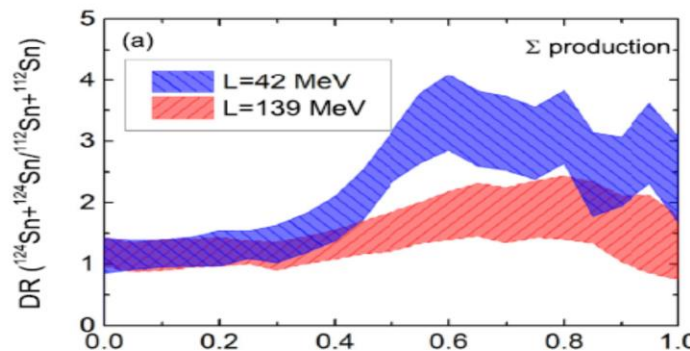
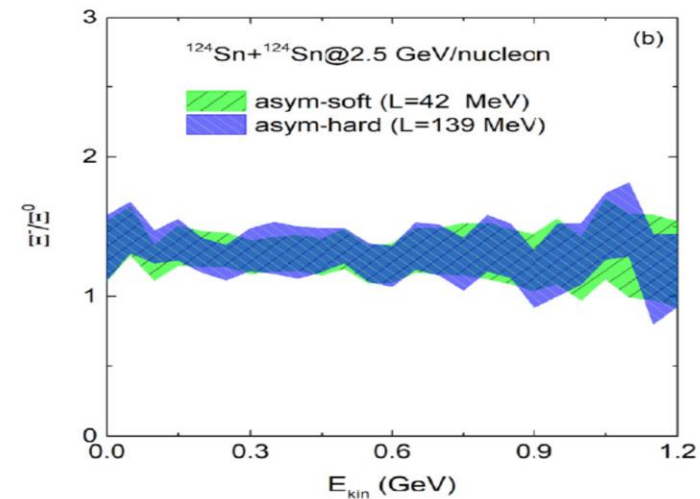
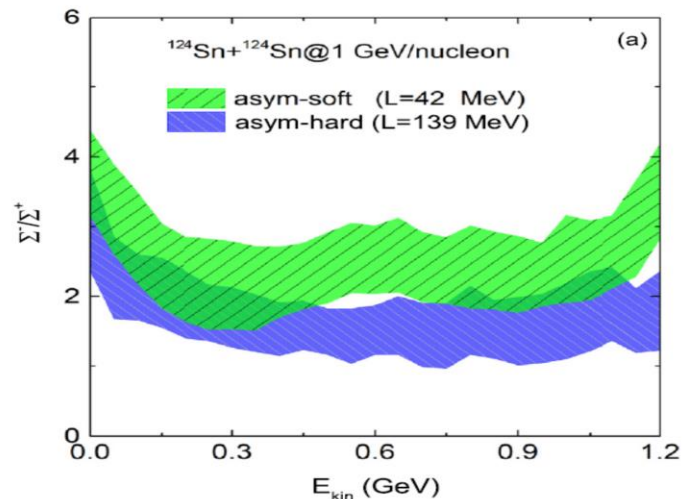
Probing the high-density symmetry energy from subthreshold hyperon production in heavy-ion collisions

Zhao-Qing Feng

School of Physics and Optoelectronics, South China University of Technology, Guangzhou 510640, China



# High-density symmetry energy from hyperon production in heavy-ion collisions, **Physics Letters B 846 (2023) 138180**

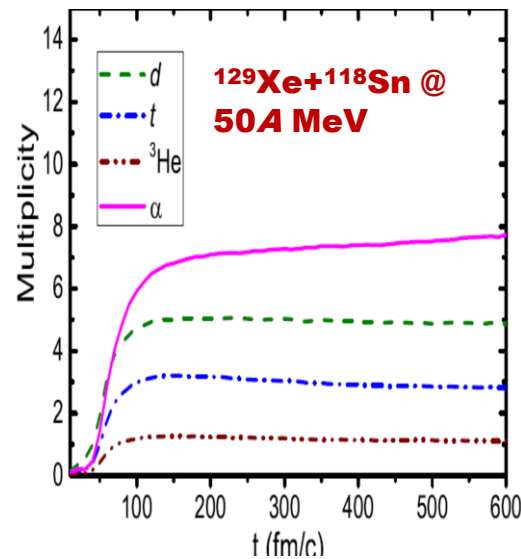
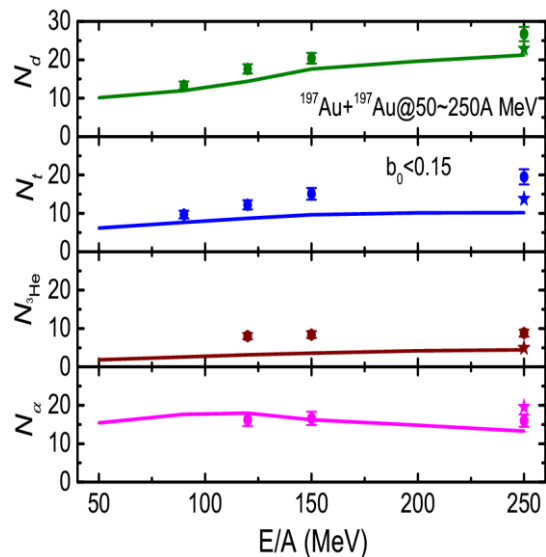




# IV. Nuclear cluster and hypercluster production in HICs

## Kinetic approach for cluster production

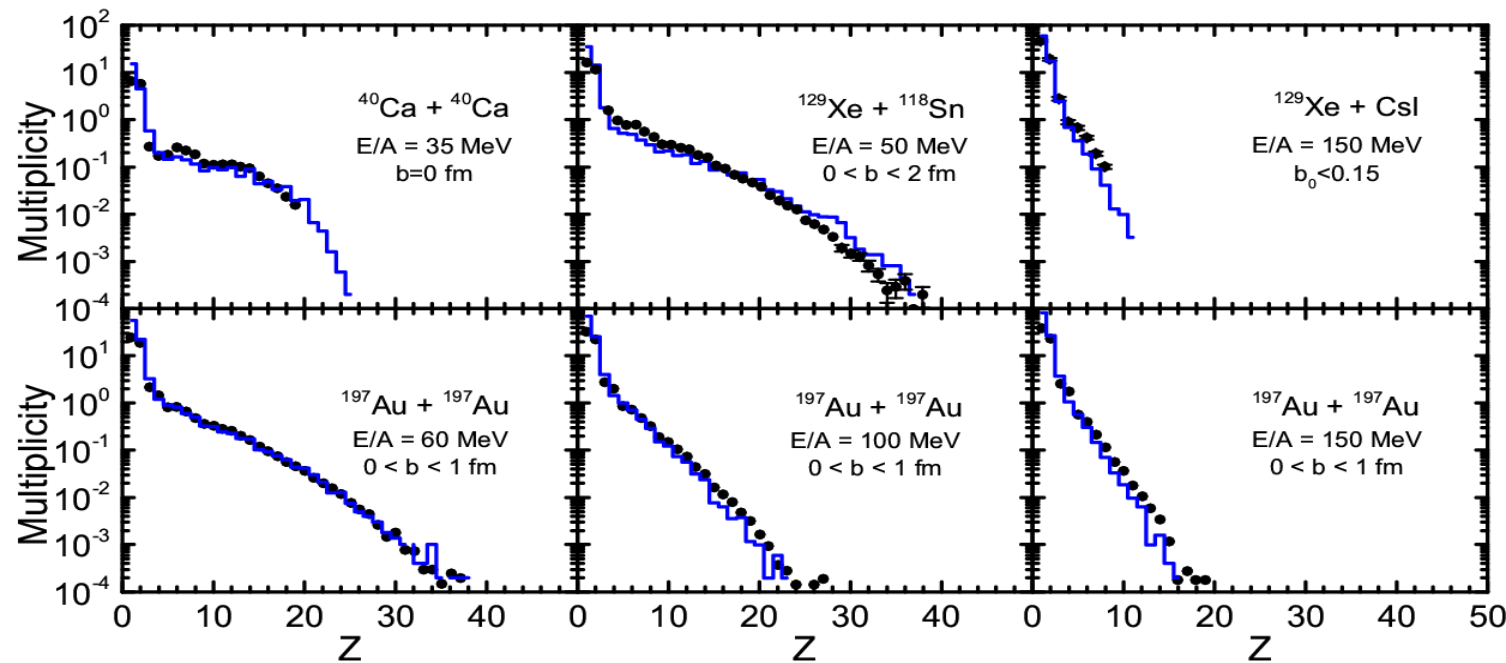
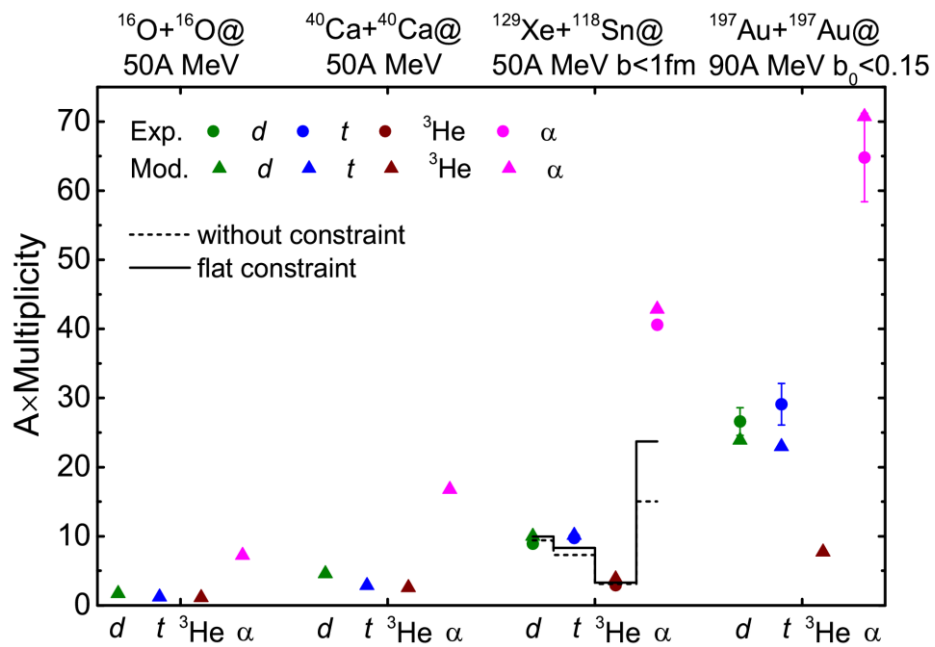
$$\begin{aligned}
 H = & \sum_i \frac{\mathbf{p}_i^2}{2m} + \frac{\alpha}{2} \sum_{i,j} \frac{\rho_{ij}}{\rho_0} + \frac{\beta}{1+\gamma} \sum_i \left( \sum_{j,j \neq i} \frac{\rho_{ij}}{\rho_0} \right)^\gamma \\
 & + \frac{C_{sym}}{2} \sum_{i,j} t_{z_i} t_{z_j} \frac{\rho_{ij}}{\rho_0} + \frac{g_{sur}}{2} \sum_{i,j} \left[ \frac{3}{2L} - \left( \frac{\mathbf{r}_i - \mathbf{r}_j}{2L} \right)^2 \right] \frac{\rho_{ij}}{\rho_0} \\
 & + \sum_i^{N_C} E_{z.p.}^i + \sum_i^{N_d} V_{corr} e^{-r_i^2/4L}
 \end{aligned}$$



博士学位论文

中高能区重离子碰撞中结团产生机制研究

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 指导教师 冯兆庆教授  
 所在学院 物理与光电学院  
 论文提交日期 2023年10月16日



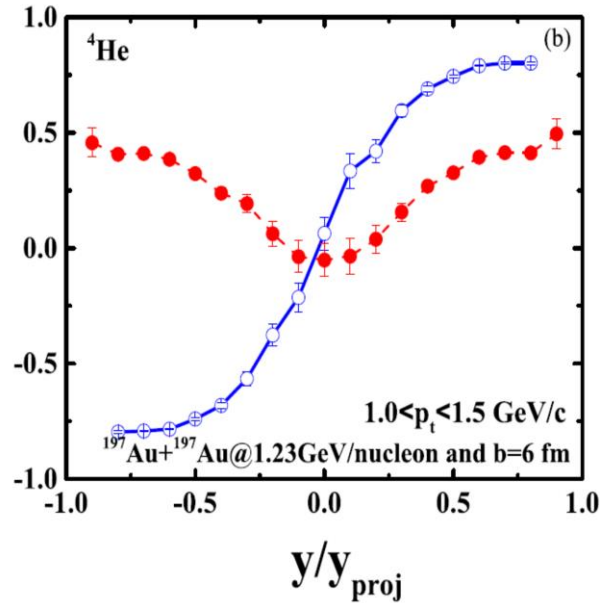
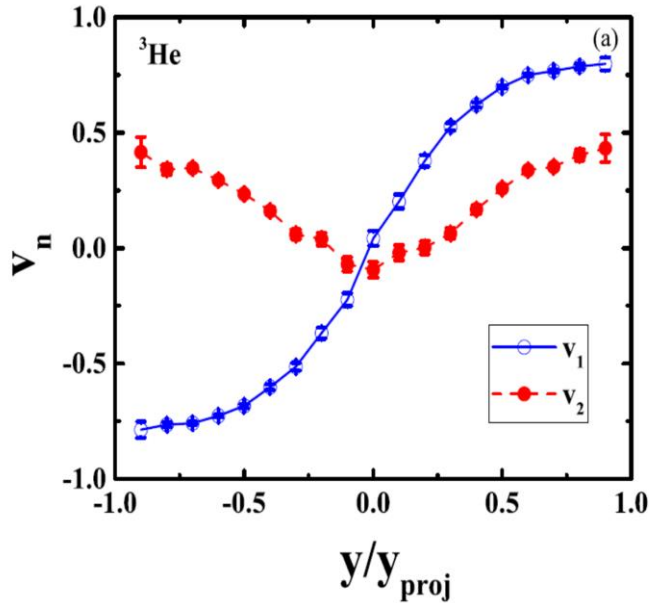
# Collective flows (d, t, $^3\text{He}$ , $\alpha$ ) (Wigner density approach)

Heng-Jin Liu, Hui-Gan Cheng, ZQF, Physical Review C 108, 024614 (2023)

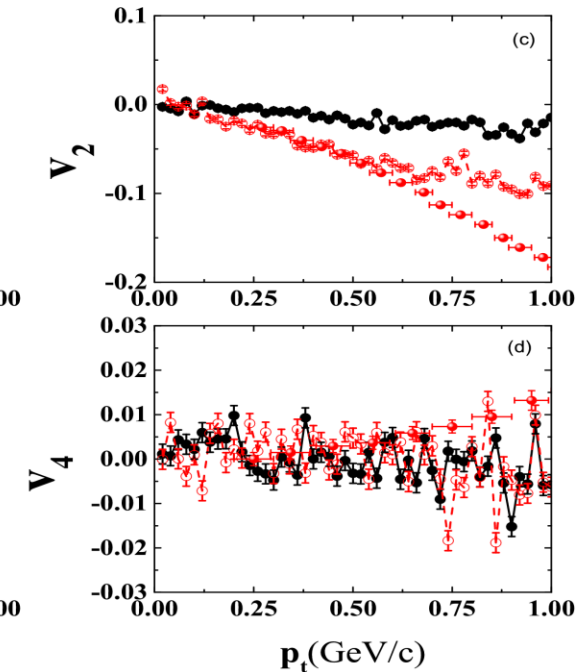
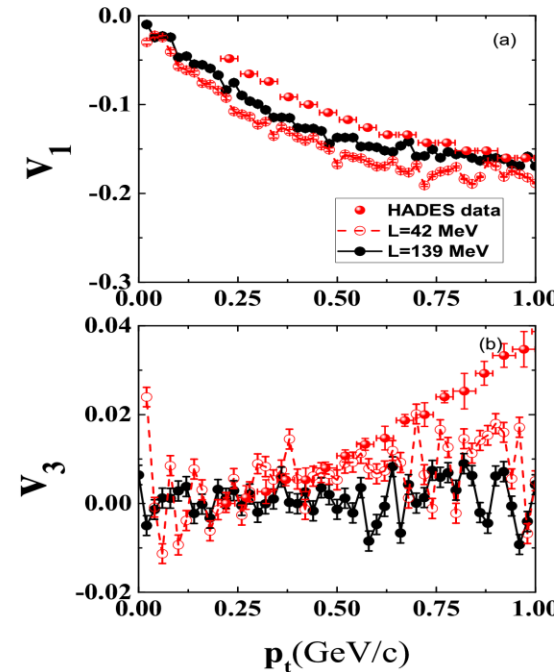
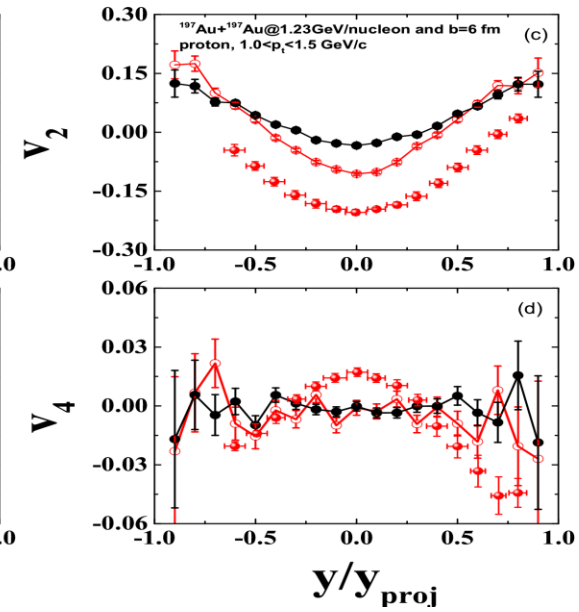
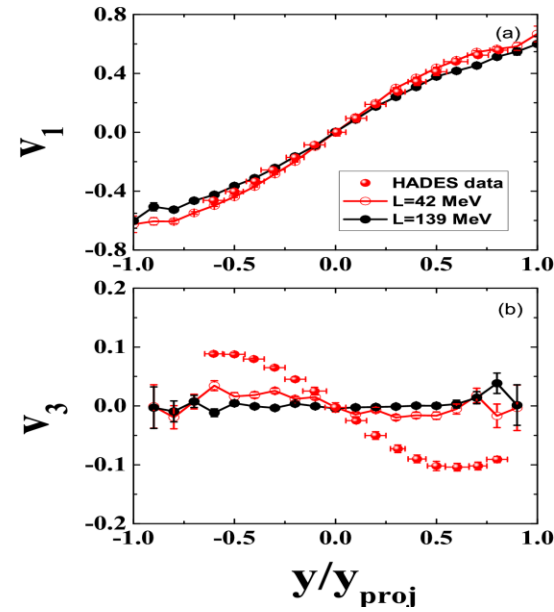
$$\frac{dN}{N_0 d\phi}(y, p_t) = 1 + 2v_1(y, p_t) \cos(\phi) + 2v_2(y, p_t) \cos(2\phi) + 2v_3(y, p_t) \cos(3\phi) + 2v_4(y, p_t) \cos(4\phi)$$

$$p_t = \sqrt{p_x^2 + p_y^2}$$

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$



HADES Collaboration, Phys. Rev. Lett. 125, 262301 (2020)



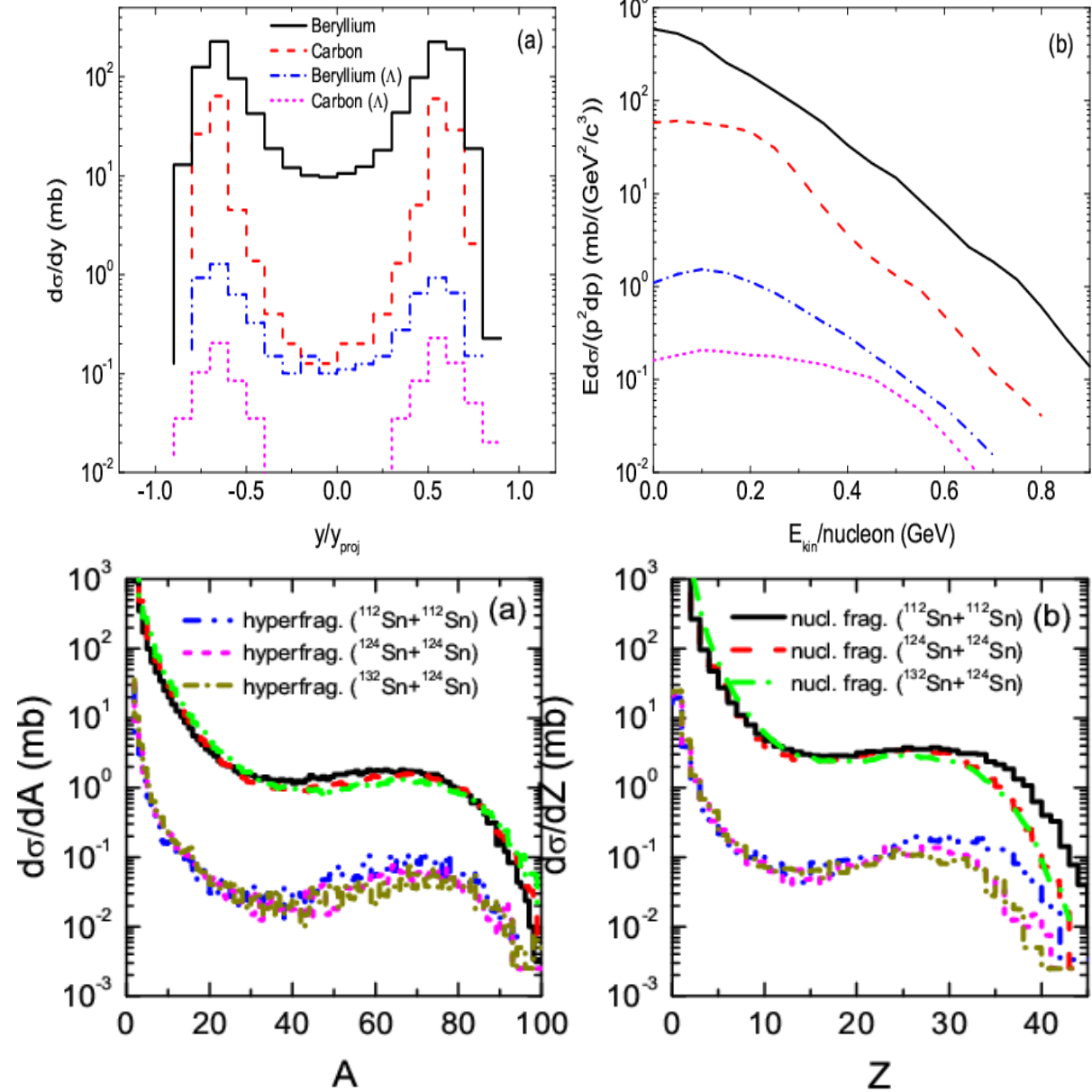
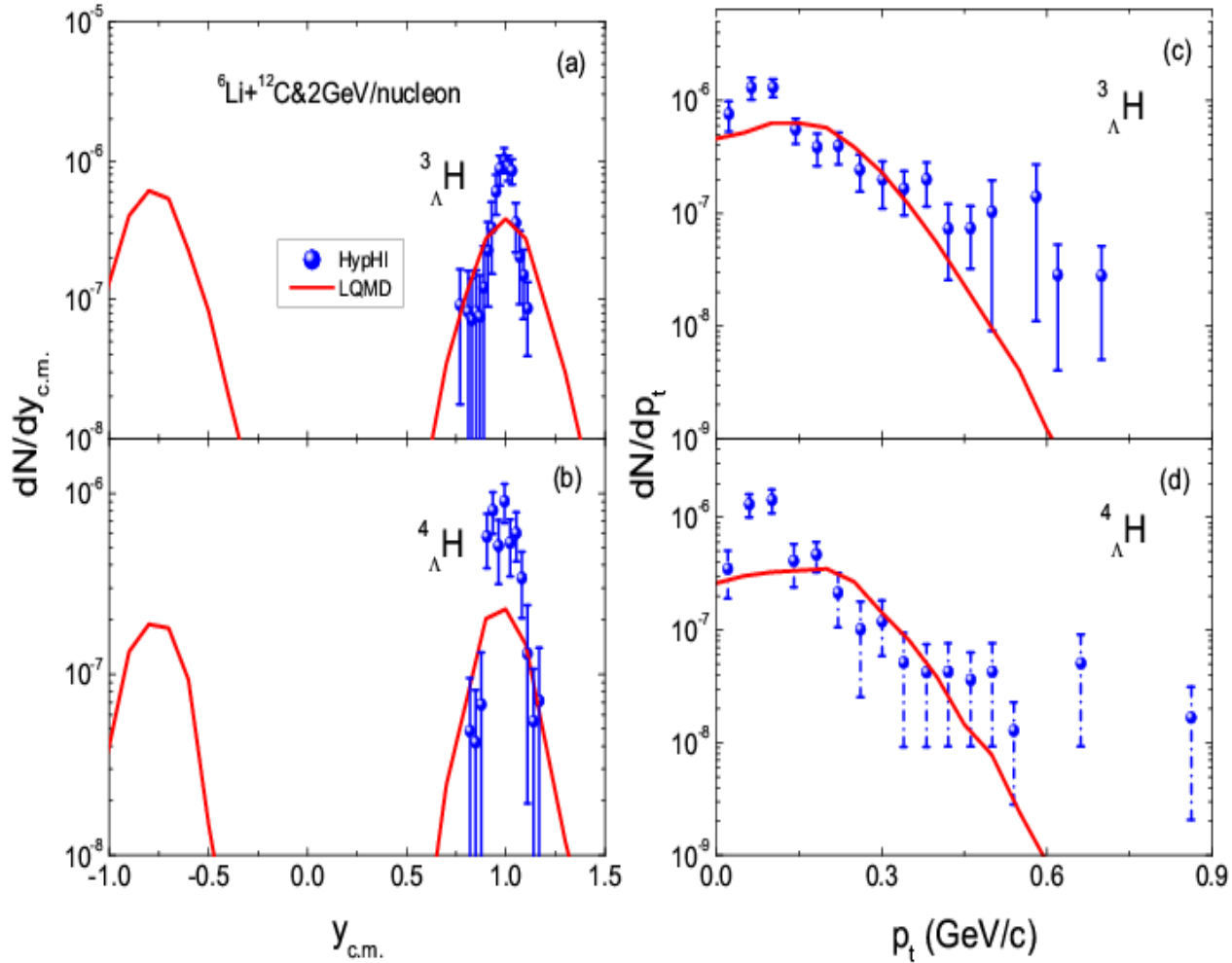
# Hypernuclide production via HICs (Wigner density approach)

Z. Q. Feng, Phys. Rev. C 102, 044604 (2020)

Data: C. Rappold et al., (HypHI collaboration)

Phys. Lett. B 747, 129 (2015)

$^{124}\text{Sn}+^{124}\text{Sn}@2\text{A GeV}$



# Multi-strangeness hypernucleide production

H.G. Cheng, Z. Q. Feng, Phys. Lett. B 824 (2022) 136849

$^{197}\text{Au} + ^{197}\text{Au} @ 3A \text{ GeV}$

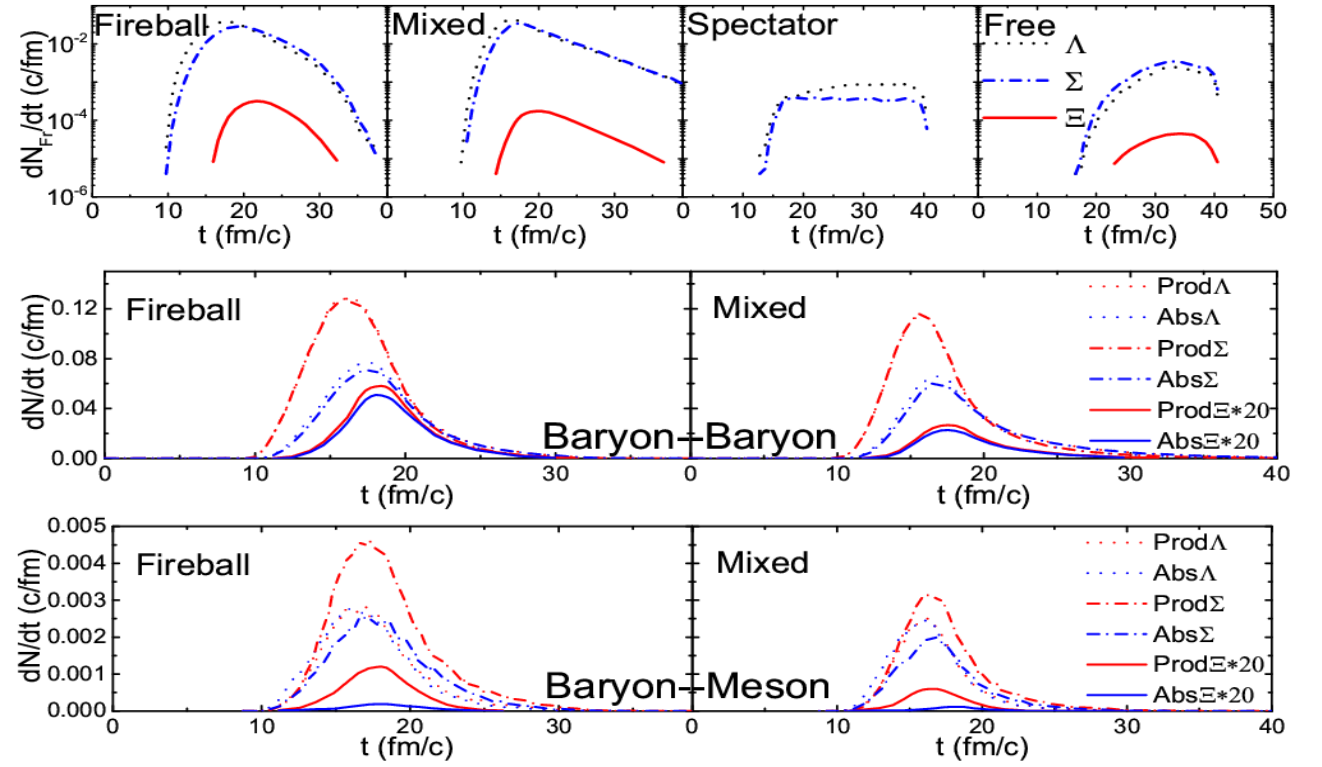
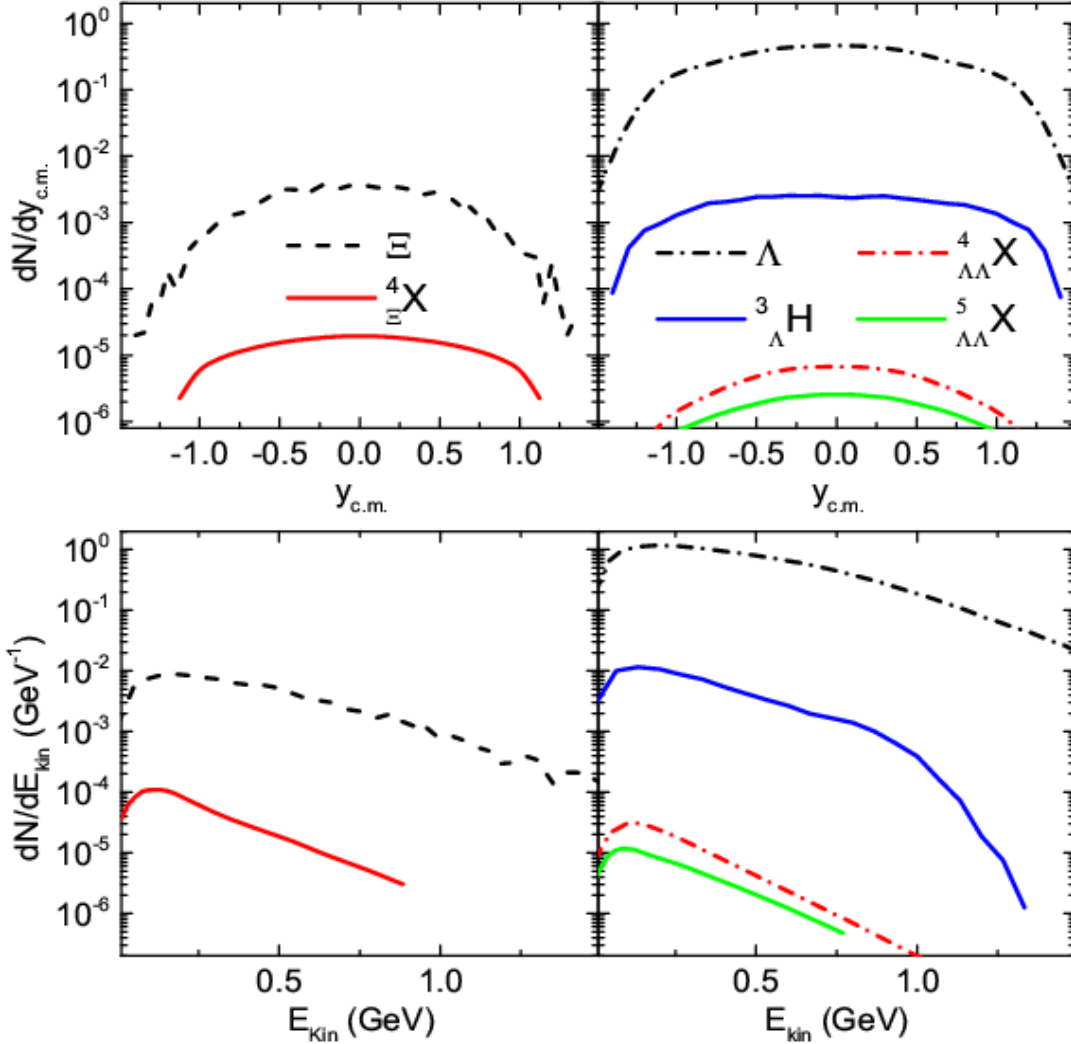
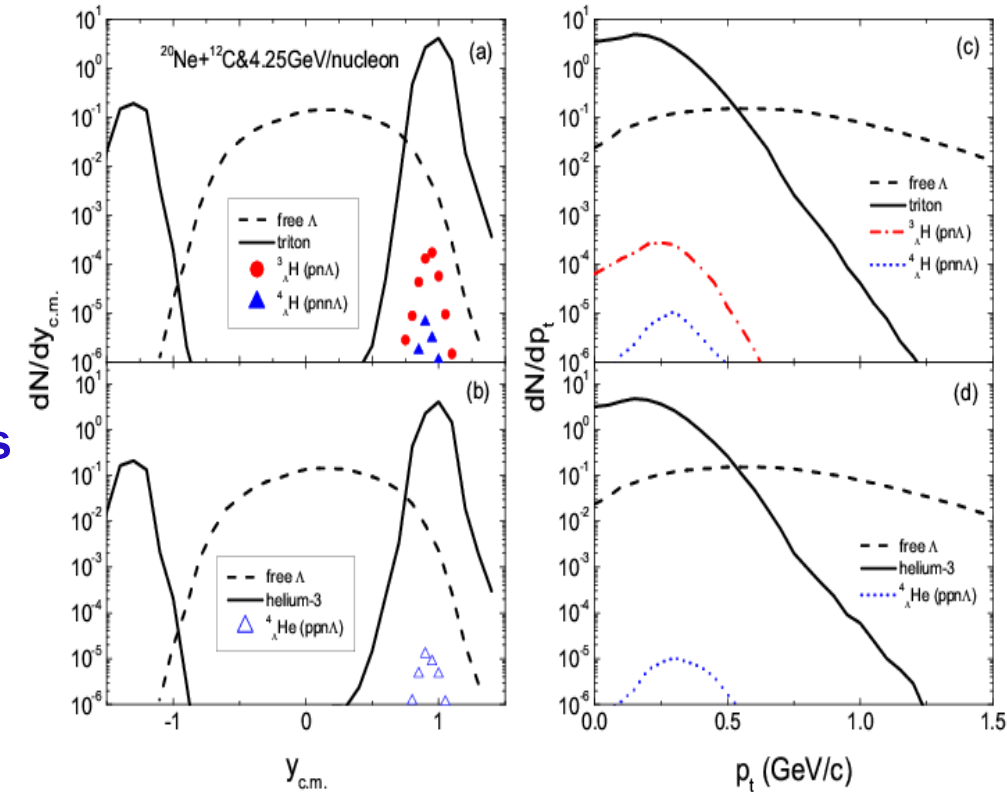


TABLE I. Comparison between cross sections of double lamda hypernuclei calculated with  $r_0 = 3.5 \text{ fm}$  for  $\Lambda$  in  $^{197}\text{Au} + ^{197}\text{Au}$  and  $^{40}\text{Ca} + ^{40}\text{Ca}$  collisions at  $3A \text{ GeV}$

Hypernuclei	Cross sections (mb)	
	$^{197}\text{Au} + ^{197}\text{Au}$	$^{40}\text{Ca} + ^{40}\text{Ca}$
${}^4_{\Lambda\Lambda}H$	$2.6 \times 10^{-2}$	$1.0 \times 10^{-4}$
${}^4_{\Lambda\Lambda}He$	$1.0 \times 10^{-2}$	$\sim 10^{-5}$
${}^5_{\Lambda\Lambda}H$	$5.9 \times 10^{-3}$	$\sim 10^{-5}$
${}^5_{\Lambda\Lambda}He$	$5.1 \times 10^{-3}$	$\sim 10^{-5}$
${}^5_{\Lambda\Lambda}Li$	$1.4 \times 10^{-3}$	$\sim 10^{-6}$
${}^6_{\Lambda\Lambda}He$	$2.2 \times 10^{-3}$	$\sim 10^{-6}$
${}^7_{\Lambda\Lambda}He$	$6.8 \times 10^{-4}$	$\sim 10^{-6}$

## V. Summary

- The Extremely proton-rich/neutron-rich hypernucleides might be created via heavy-ion collisions at HIAF energies, e.g.,  ${}^3_{\Lambda}\text{H}$  and  ${}^4_{\Lambda}\text{H}$  production in the reaction of  ${}^{20}\text{Ne}+{}^{12}\text{C}$  at HIAF.
- The high-density symmetry probes single and double ratios of  $\Sigma^-/\Sigma^+$  (double ratio) via the isotopic reactions  ${}^{112}\text{Sn}+{}^{112}\text{Sn}$  and  ${}^{124}\text{Sn}+{}^{124}\text{Sn}$ , in particular above 0.4 GeV.
- The 3-body interaction potentials, e.g.,  $\Lambda\text{NN}, \Sigma\text{NN}, \Xi\text{NN}$  etc, might be constrained via heavy-ion collisions at HIAF.
- Kinetic approach is implemented in the LQMD transport model for the nuclear cluster production in Fermi energy heavy-ion collisions, hypercluster in the near future, in which the binding energy, multinucleon (cluster) collisions, Pauli principle, Mott effect etc are taken into account.



Thanks for your attention!