手征有效场论对重子分立电磁形状因子的一些研究

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第一届超子物理会议
惠州，2024.04

湖南大學
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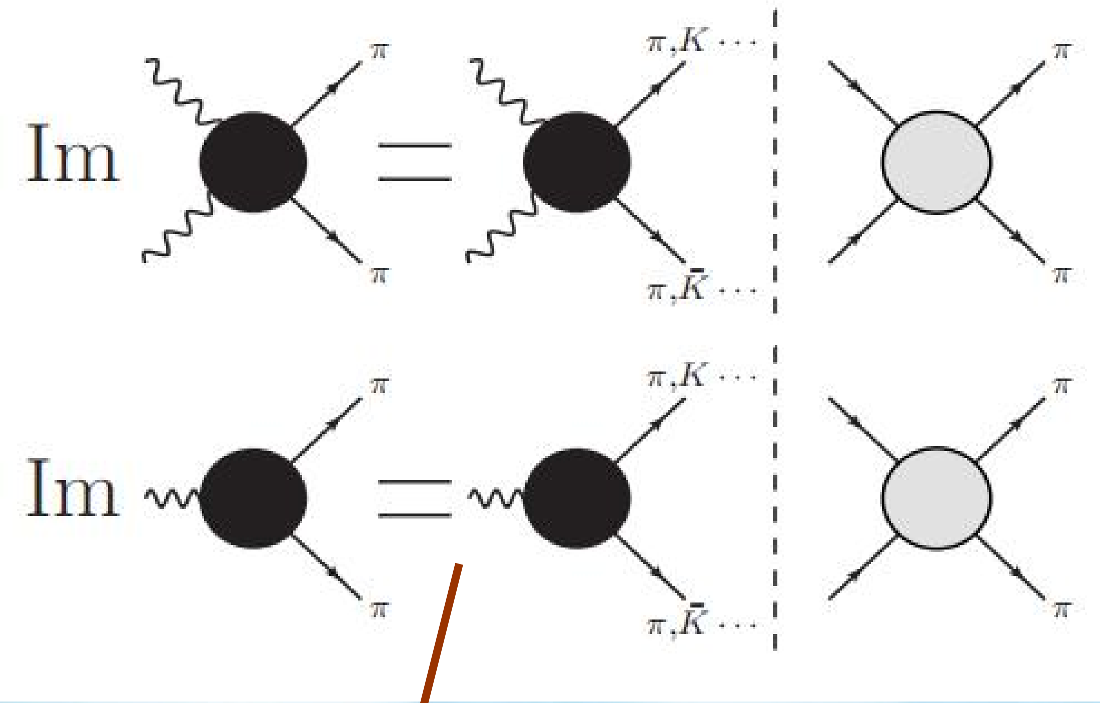
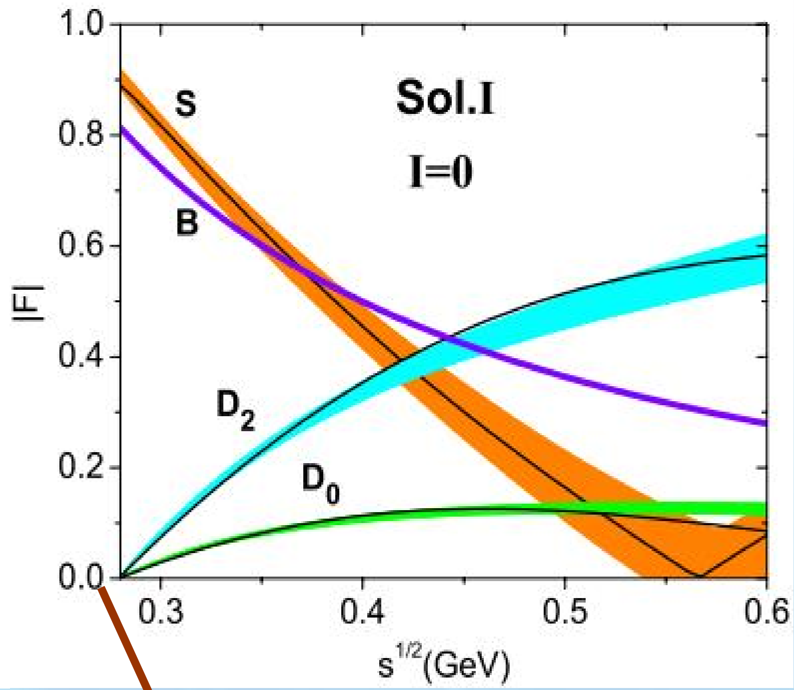
Outline

1. Introduction

- FSI plays important role to describe the re-scattering of hadrons
- The Born term could be 'enhanced' by FSI



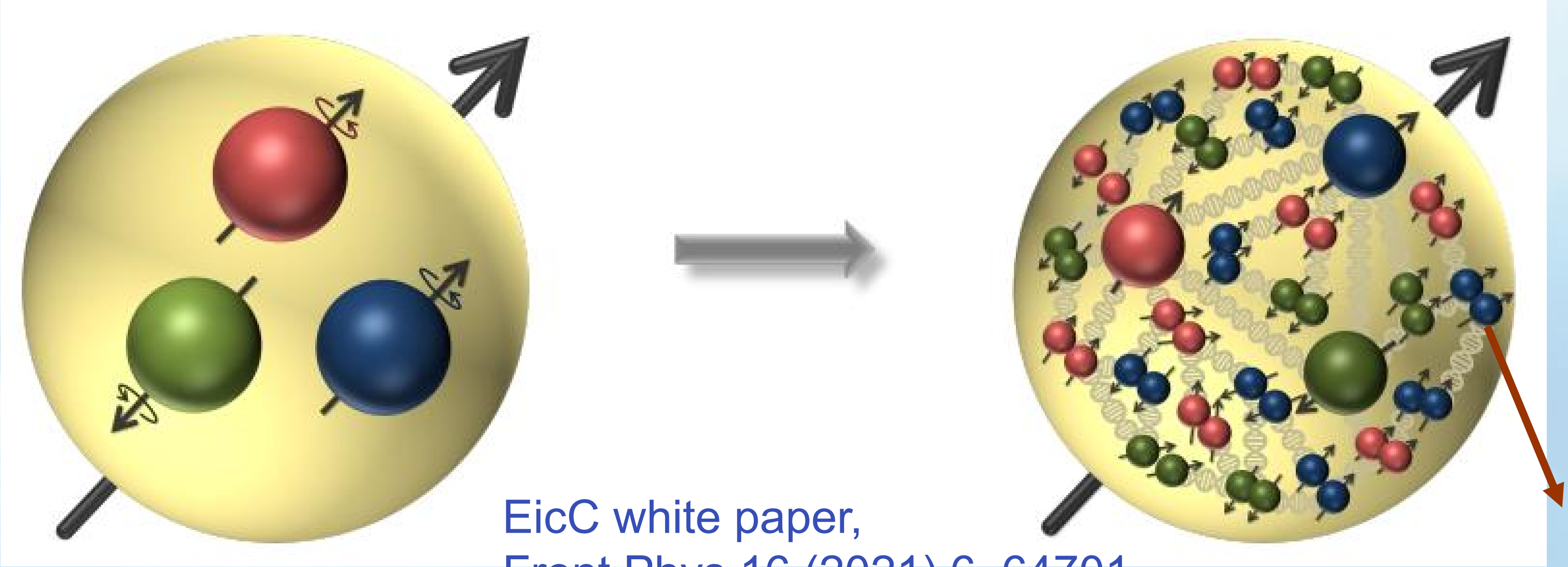


Yao, Dai#, Zheng, Zhou, RPP84(2021)076201

Dai&Pennington, PRD90 (2014) 036004

Baryon

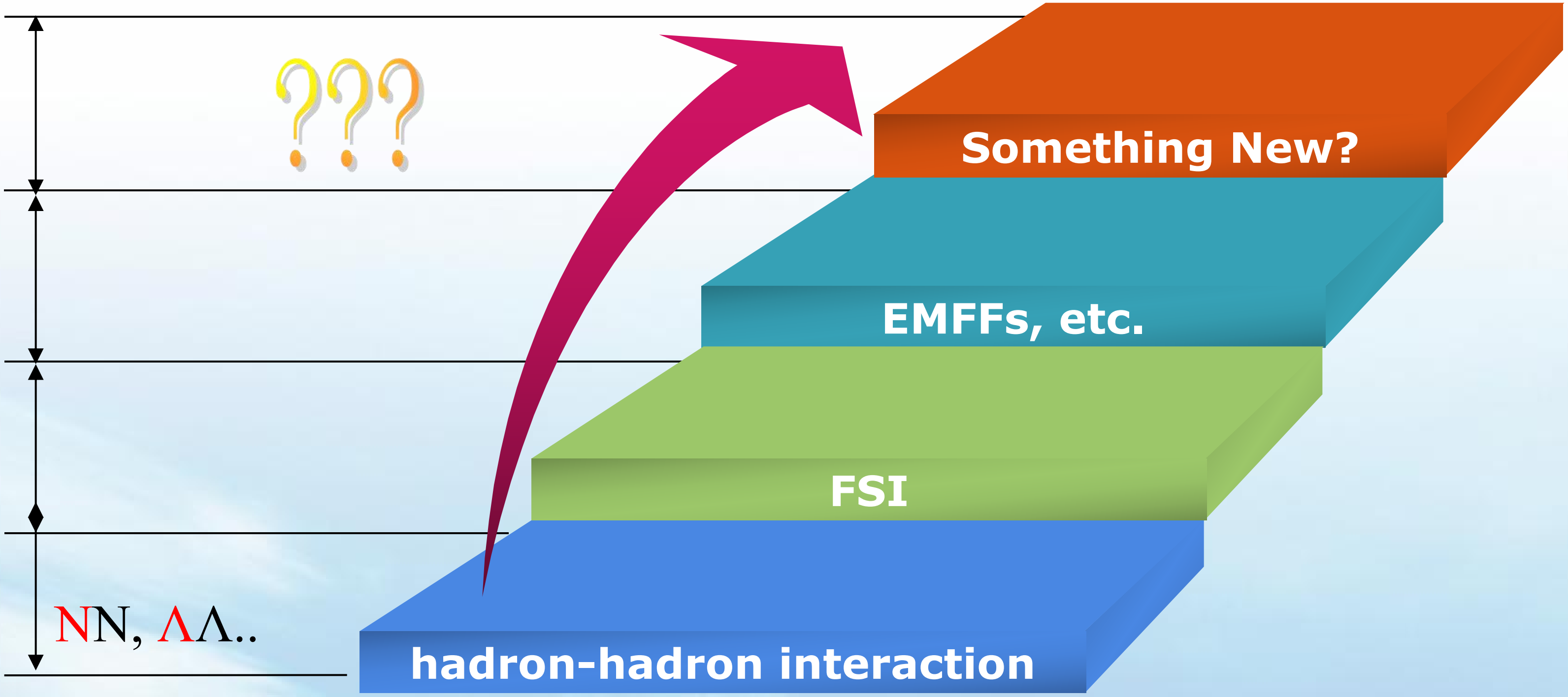
- Baryon inner structure?
- EIC, EicC: 3D structure of proton?
- Mass, spin,radius?
- EMFF: the inner structure of baryons?
- Threshold enhancement?



EicC white paper,
Front.Phys.16 (2021) 6, 64701

Strategy

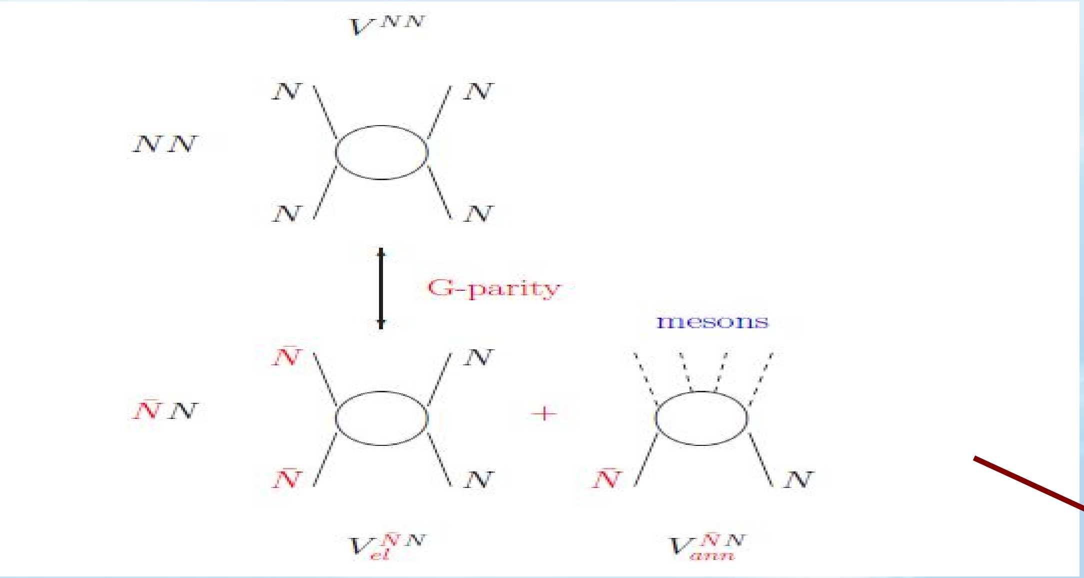
- New insights in strong interactions?



2. NN scattering amplitudes

- **SU(2) NN scattering amplitude**
  - elastic NN scattering:

    E.Epelbaum *et.al.*, EPJA51 (2015) , 53
    - pion(s) exchange: NN Chiral EFT+G-parity
    - LECs of contact term: to be fixed by data
  - annihilation: unitarity, fit to the data

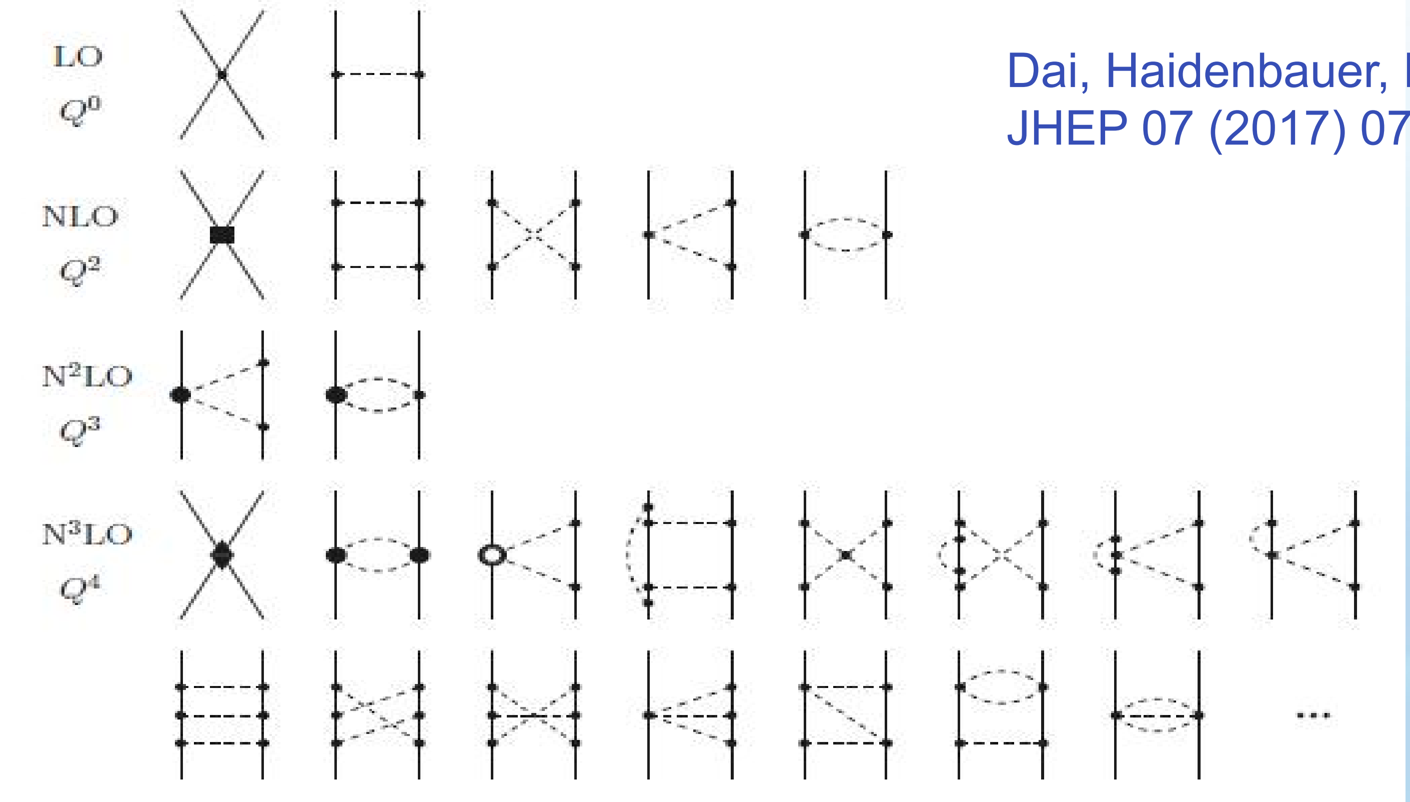


$$V^{NN} = V_{1\pi} + V_{2\pi} + V_{3\pi} + \ldots + V_{cont}$$
$$V_{el}^{\bar{N}N} = -V_{1\pi} + V_{2\pi} - V_{3\pi} + \ldots + V_{cont}$$
$$V_{ann}^{\bar{N}N} = \sum_X V^{\bar{N}N \to X}$$

J.Haidenbauer, talk at Bochum

ChEFT

- Up to N³LO, in time ordered ChEFT:
  - only irreducible diagrams contributes
  - Lippmann-Shwinger equation



Dai, Haidenbauer, Meissner, JHEP 07 (2017) 078

ChEFT: potentials

- pion(s) exchange potentials:

$$V_{1\pi}(q) = \left(\frac{g_A}{2F_\pi}\right)^2 \left(1 - \frac{p^2 + p'^2}{2m^2}\right) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q}\, \boldsymbol{\sigma}_2 \cdot \mathbf{q}}{\mathbf{q}^2 + M_\pi^2}$$

$$V_{2\pi} = V_C + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_C + [V_S + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_S]\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + [V_T + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_T]\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q} + [V_{LS} + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_{LS}]\, i(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{q} \times \boldsymbol{k}),$$

- Fourier transformation: change it into corordinat space to do regularization

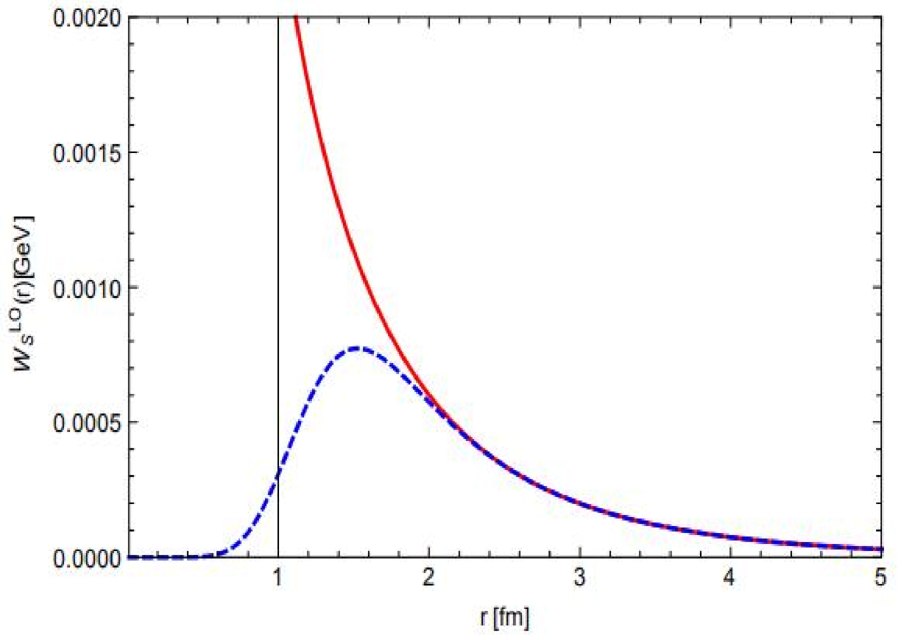
$$V_C(q) = 4\pi \int_0^\infty f(r)\, V_C(r) j_0(qr) r^2 dr,$$

$$V_S(q) = 4\pi \int_0^\infty f(r) \left(V_S(r) j_0(qr) + \tilde{V}_T(r) j_2(qr)\right) r^2 dr,$$

$$V_T(q) = -\frac{12\pi}{q^2} \int_0^\infty f(r)\, \tilde{V}_T(r) j_2(qr) r^2 dr,$$

$$V_{SL}(q) = \frac{4\pi}{q} \int_0^\infty f(r)\, V_{LS}(r) j_1(qr) r^3 dr.$$

$$f(r) = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^n.$$



ChEFT: potentials

- Contact terms: short distance

$$
\begin{aligned}
V(^1S_0) &= \tilde{C}_{^1S_0} + C_{^1S_0}(p^2 + p'^2) + D^1{}_{^1S_0} p^2 p'^2 + D^2{}_{^1S_0}(p^4 + p'^4)\,,\\
V(^3S_1) &= \tilde{C}_{^3S_1} + C_{^3S_1}(p^2 + p'^2) + D^1{}_{^3S_1} p^2 p'^2 + D^2{}_{^3S_1}(p^4 + p'^4)\,,\\
V(^1P_1) &= C_{^1P_1}\, p p' + D_{^1P_1}\, p p'(p^2 + p'^2)\,,\\
V(^3P_1) &= C_{^3P_1}\, p p' + D_{^3P_1}\, p p'(p^2 + p'^2)\,,\\
V(^3P_0) &= C_{^3P_0}\, p p' + D_{^3P_0}\, p p'(p^2 + p'^2)\,,\\
V(^3P_2) &= C_{^3P_2}\, p p' + D_{^3P_2}\, p p'(p^2 + p'^2)\,,\\
V(^3D_1 - {}^3S_1) &= C_{\epsilon_1}\, p'^2 + D^1{}_{\epsilon_1} p^2 p'^2 + D^2{}_{\epsilon_1} p'^4\,,\\
V(^3S_1 - {}^3D_1) &= C_{\epsilon_1}\, p^2 + D^1{}_{\epsilon_1} p^2 p'^2 + D^2{}_{\epsilon_1} p^4\,,
\end{aligned}
$$

- Non-local regularization

$$f(p', p) = \exp\left(-\frac{p'^m + p^m}{\Lambda^m}\right)$$

- Annihilation terms: short distance physics, around 1 fm or less

$$V_{\rm ann} = V_{\bar{N}N\to X}\, G_X\, V_{X\to\bar{N}N}$$

the same form as that of contact terms

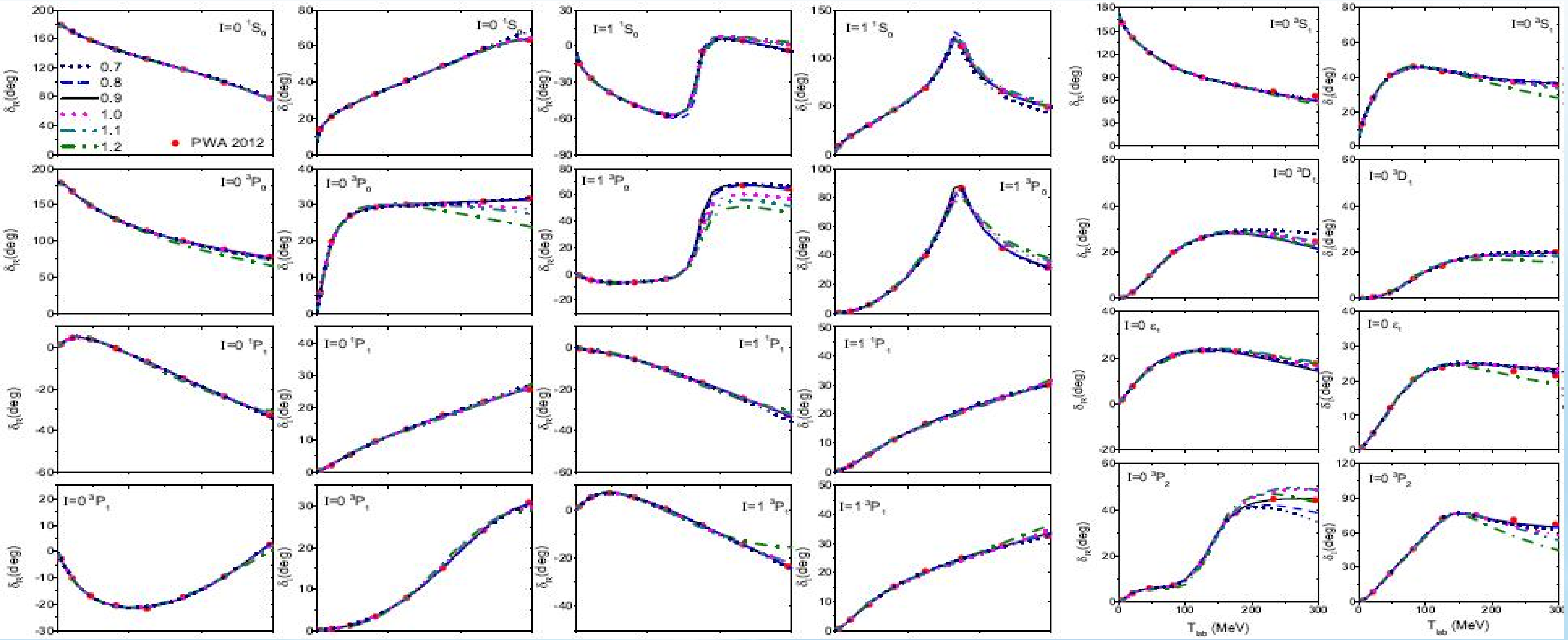
Ignore the transition between annihilation channels

Phase shifts of different cutoff

- LS equation to solve amplitrudes

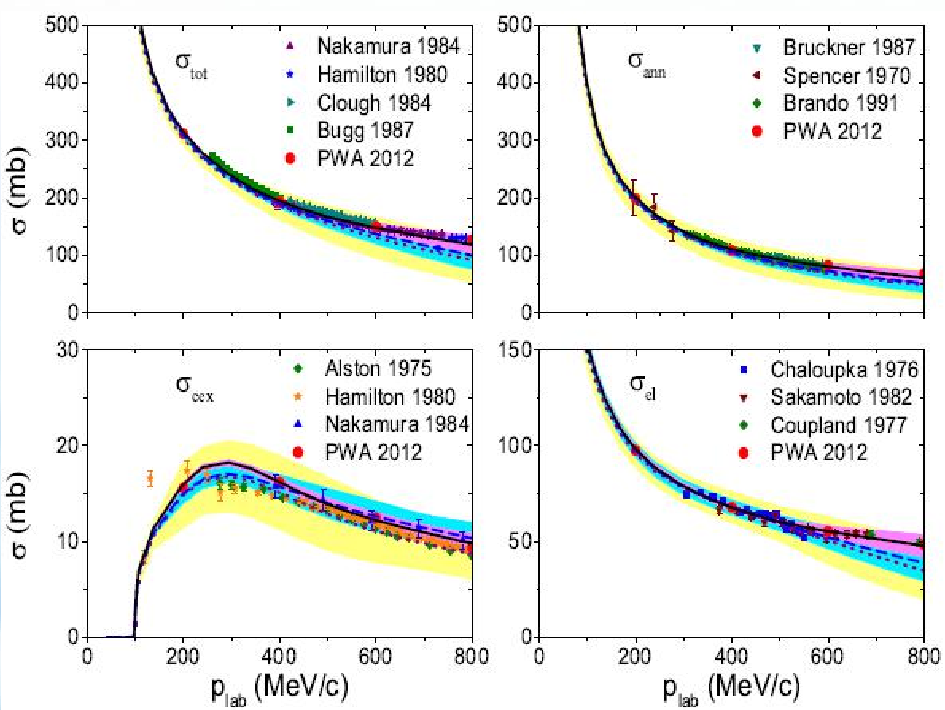
$$T_{L''L'}(p'',p';E_k) = V_{L''L'}(p'',p') + \sum_L \int_0^\infty \frac{dpp^2}{(2\pi)^3} V_{L''L}(p'',p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p,p';E_k)$$

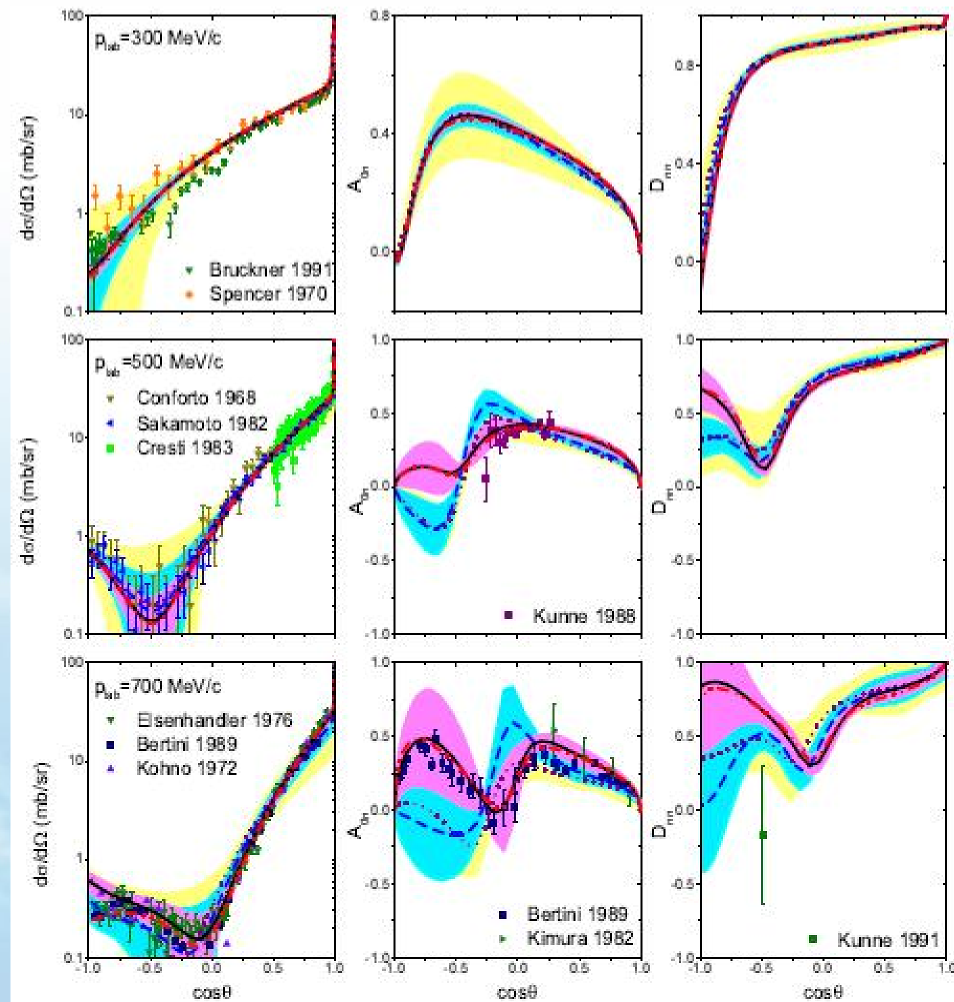
- Lowest partial waves are perfect up to 300 MeV



Observables

- Cross sections
- Angular distributions

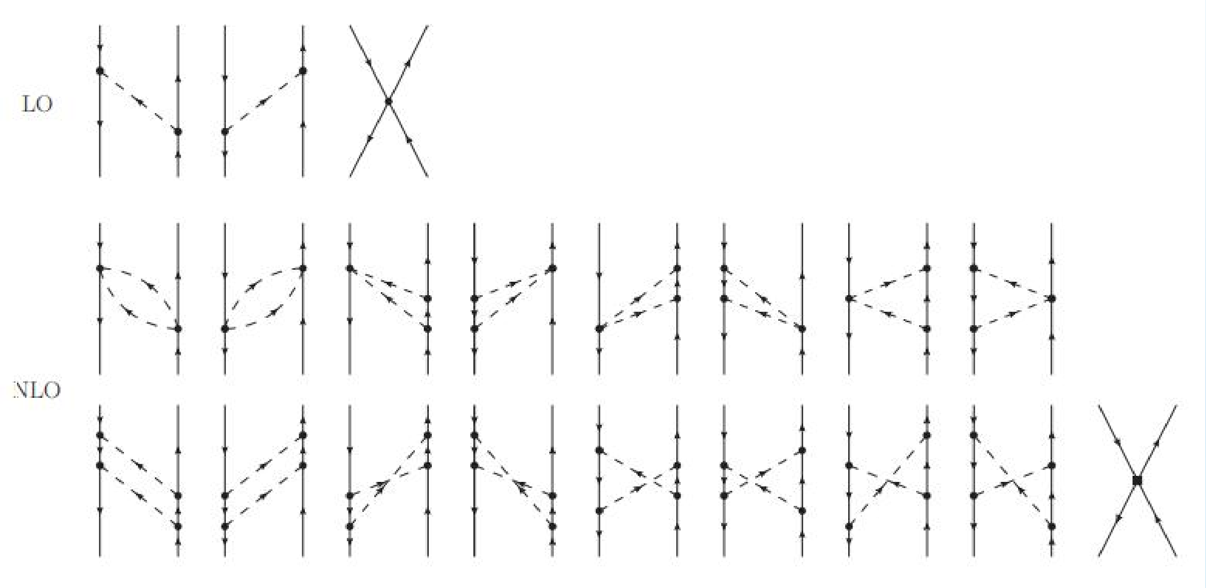




Why SU(3) ChEFT

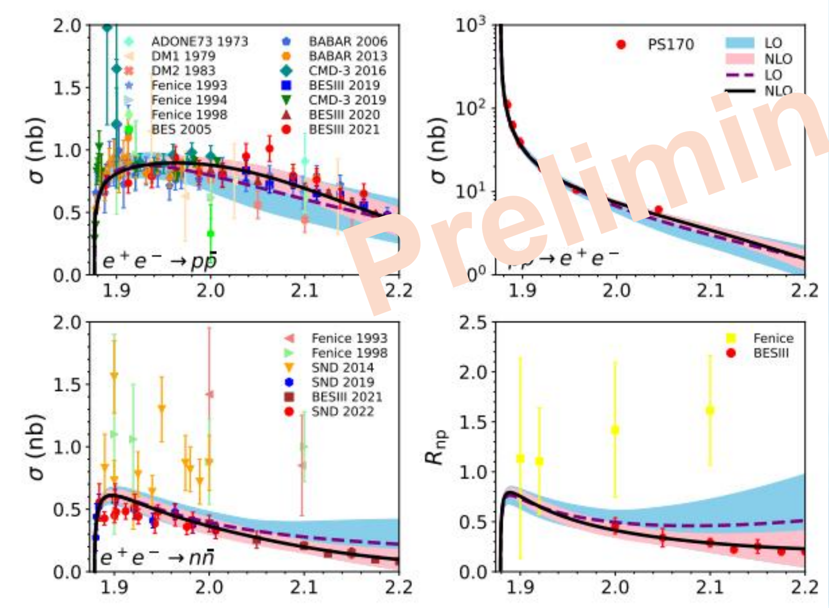
- SU(2): so far, so good, but
  - only pion exchanges
  - only works for nucleons
- SU(3) G-parity transformation is not OK as kaon does not have definitive G-parity
  - Direct calculation of BB scattering

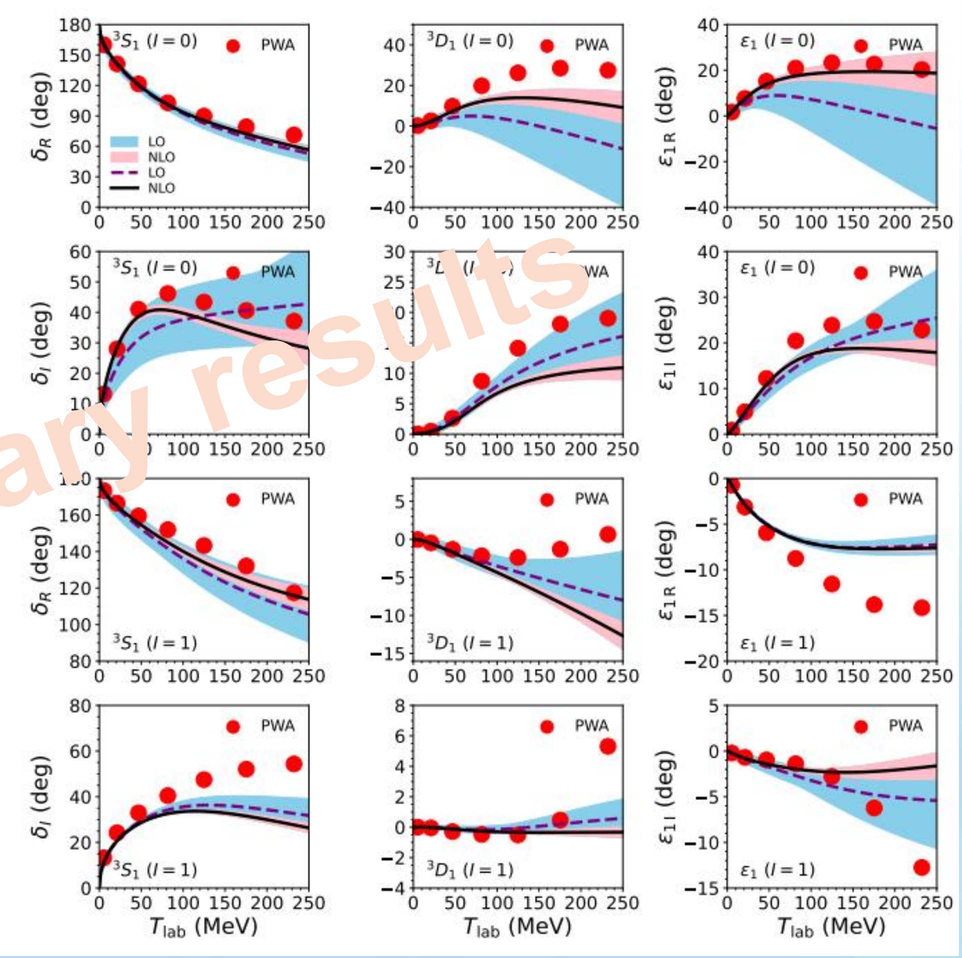
$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$



SU(3) ChEFT

- Fit results
- Phase shifts
- Cross sections
- differential cross sections
- ratios, etc.



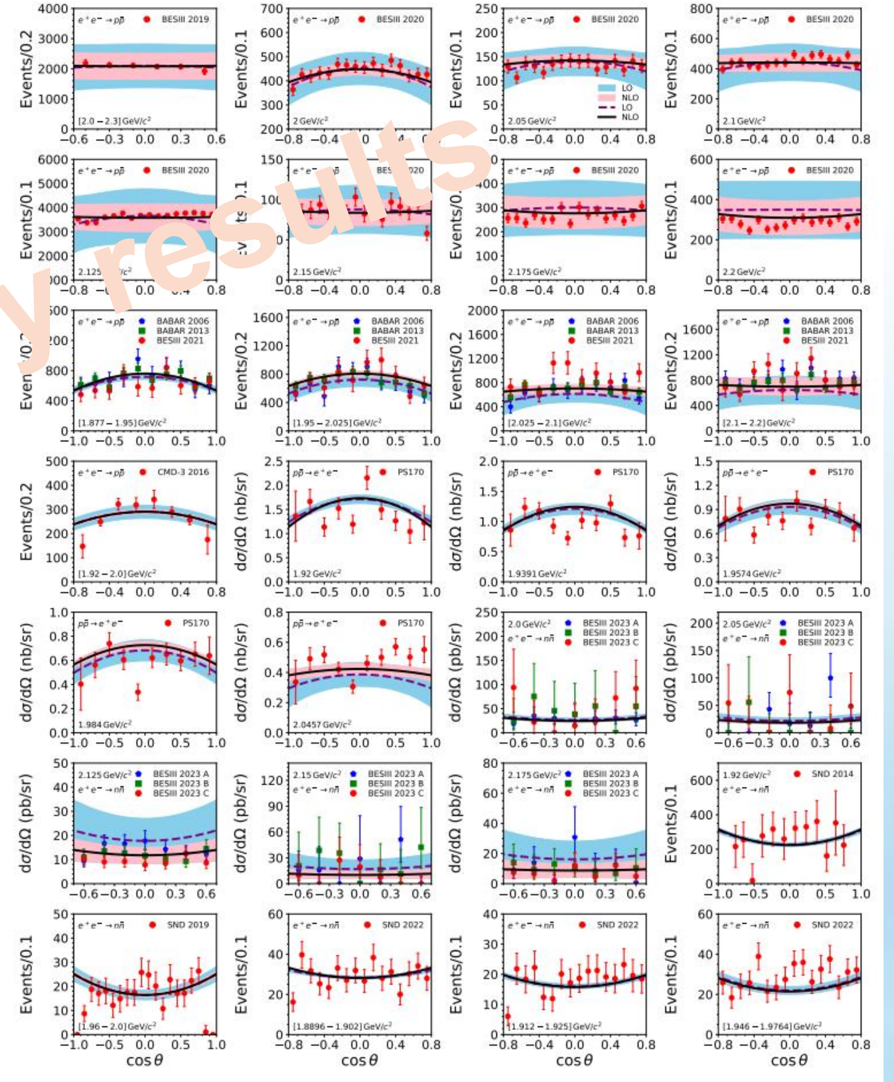


Preliminary results

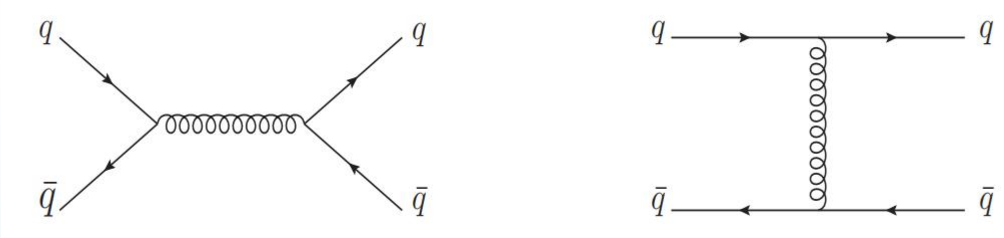
SU(3) ChEFT

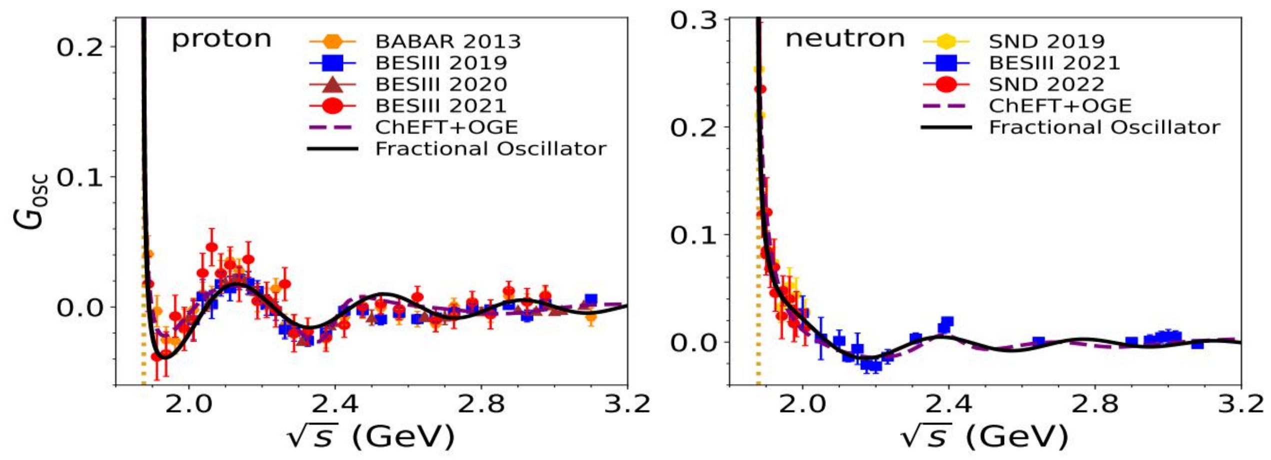
- Angular distributions also help to fix partial wave amplitudes

	LO		NLO					
	N	χ^2/N	N			χ^2/N		
Λ (MeV)		850		750	800	[illegible]	[illegible]	[illegible]
Cross Section	105	1.77	154	[illegible]	[illegible]	[illegible]	[illegible]	1.60
Differential cross section	221	1.33	477	[illegible]	[illegible]	[illegible]	1.50	1.47
R_{np}	1	[illegible]	[illegible]	[illegible]	[illegible]	1.08	1.52	1.91
Effective EMFFs	88	[illegible]	[illegible]	[illegible]	1.61	1.60	1.62	1.68
$\lvert G_E/G_M \rvert$, $\lvert G_E \rvert$ and $\lvert G_M \rvert$	13	[illegible]	44	1.73	1.70	1.60	1.41	1.23
G_{osc}	33	2.65	71	1.66	1.67	1.68	1.73	1.88
Phase shift	24	0.008	36	0.003	0.004	0.004	0.005	0.006
Scattering length	4	1.40	4	0.88	0.94	0.94	0.88	0.84
$\chi^2_{d.o.f.}$	–	1.59	–	1.63	1.61	1.57	1.54	1.53



ChEFT+OGE?



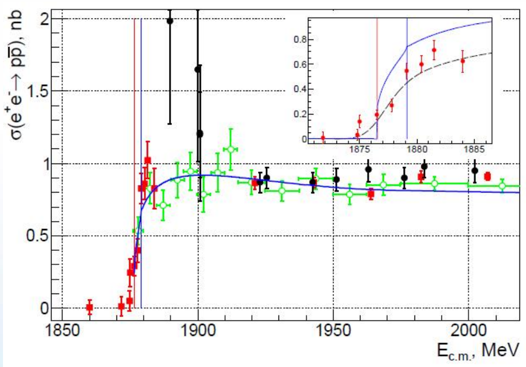


- Consider one-gluon exchange potential in the high energy region
- It can reproduce the fractional oscillations.
- An efficient way to describe the strong interaction in both low energy region and high energy region?

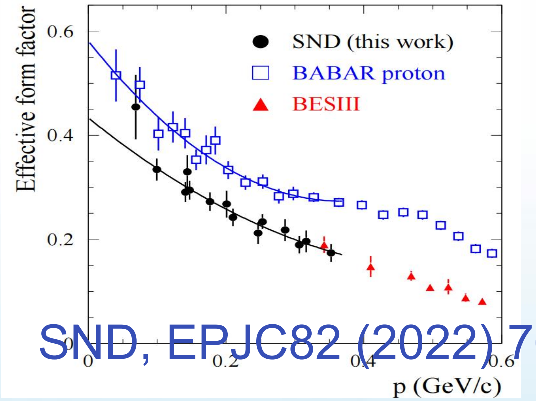
Yang, Guo, Dai, et. al., Sci.Bull. 68 (2023) 2729;

3 EMFFs of nucleons

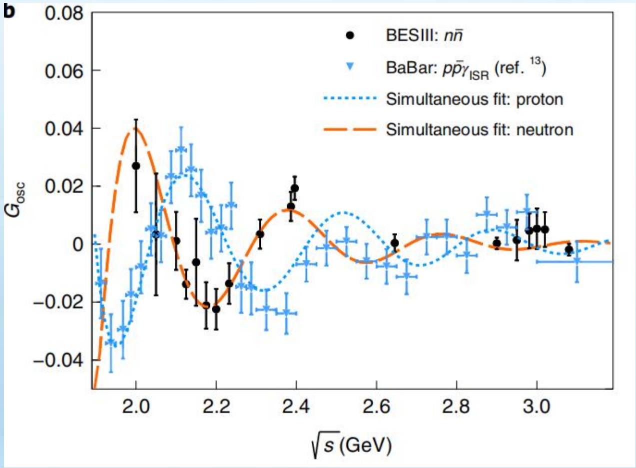
- CMD-3 has excellent measurement in low energy region
- BESIII's high statistics' measurements

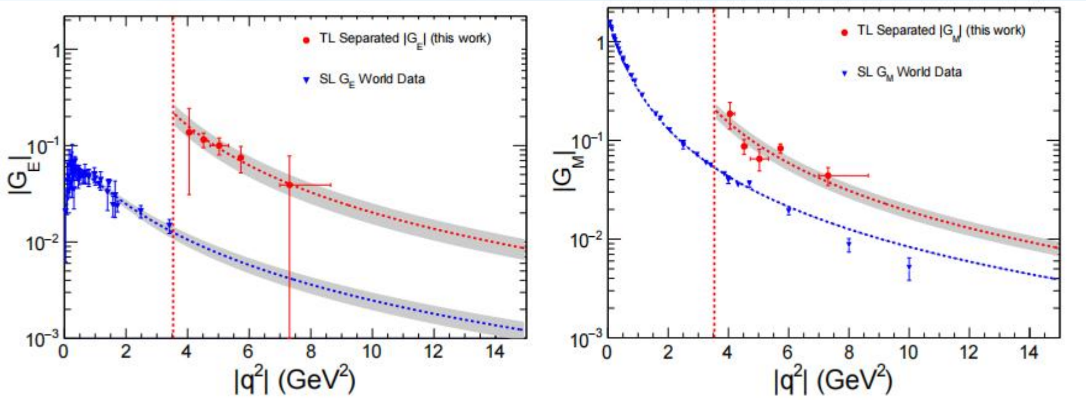


CMD-3, PLB05 (2019) 032



SND, EPJC82 (2022) 761





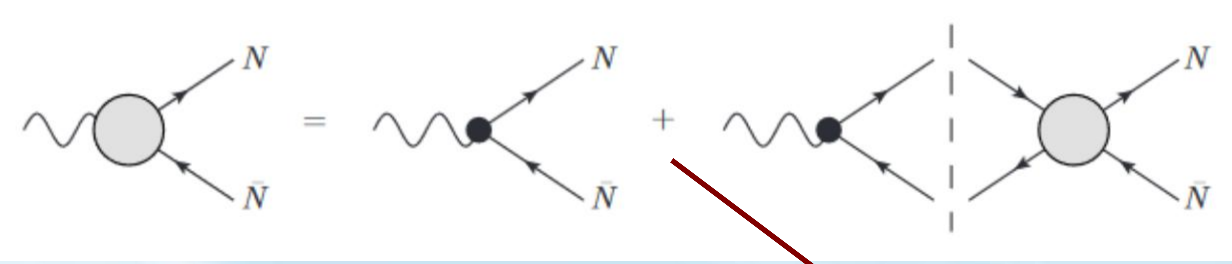
BESIII: PRL 130 (2023) 15, 151905

BESIII: PRD 99 (2019) 092002; PRL 124 (2020) 4, 042001, Nature Phys.17 (2021) 1200

FSI

- To analyze ee→NN, we need to consider FSI
- Distorted-wave Born approximation (DWBA):

$$f_{L'}(k;E_k) = f^0_{L'}(k) + \sum_L \int_0^\infty \frac{dpp^2}{(2\pi)^3} f^0_L(p) \frac{1}{2E_k - 2E_p + i0^+} T_{LL'}(p,k;E_k)$$



SU(3) ChEFT: Yang, Dai, et. al., Sci.Bull. 68 (2023) 2729;

- Vector meson dominance: $^3S_1$-$^3D_1$

SU(2)ChEFT: J.Haidenbauer, X.-W. Kang, U.-G. Meißner, NPA 929 (2014) , PRD91 (2015) 074003.

EMFFs of nucleons

- NLO result of SU(3) ChEFT.
- Combining proton's and neutron's.

- Relation between amplitudes and EMFFs

$$f_0^{\bar{N}N} = G_M + \frac{M_N}{\sqrt{s}} G_E, \quad f_2^{\bar{N}N} = \frac{1}{2}\left(G_M - \frac{2M_N}{\sqrt{s}} G_E\right)$$

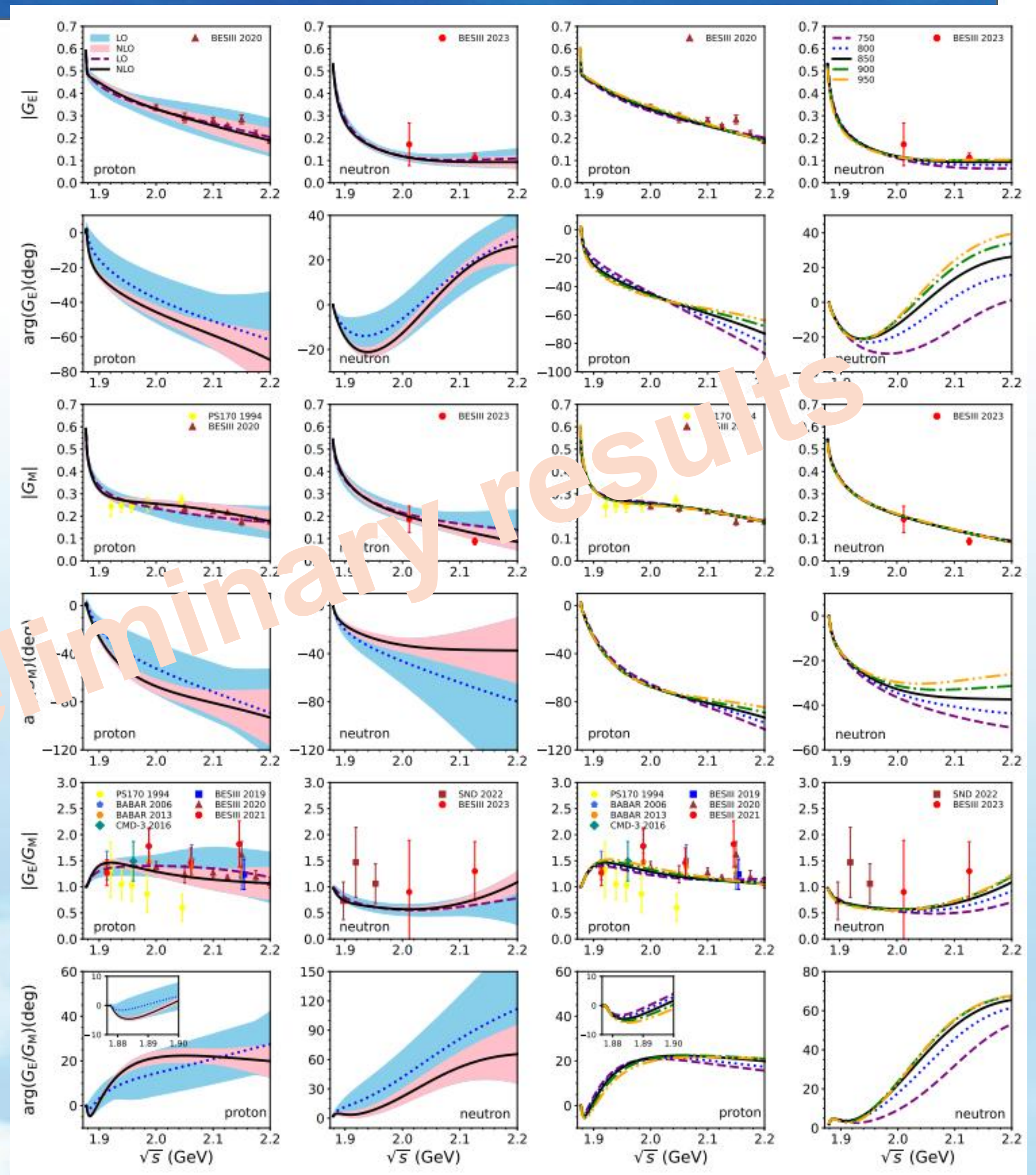
- Relation between EMFFs and physical observables

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2 \beta}{4s} C(s) \left[|G_{\mathrm{M}}^N(s)|^2 (1 + \cos^2\theta) + \frac{4M_N^2}{s} |G_{\mathrm{E}}^N(s)|^2 \sin^2\theta \right]$$

Individual EMFFs of nucleons

- Modulus: $|G_E|=|G_M|$at threshold, and will restore in high energy region

- Phases:
  - An overall phase is unobservable
  - relative phase chenges rapidly near threshold



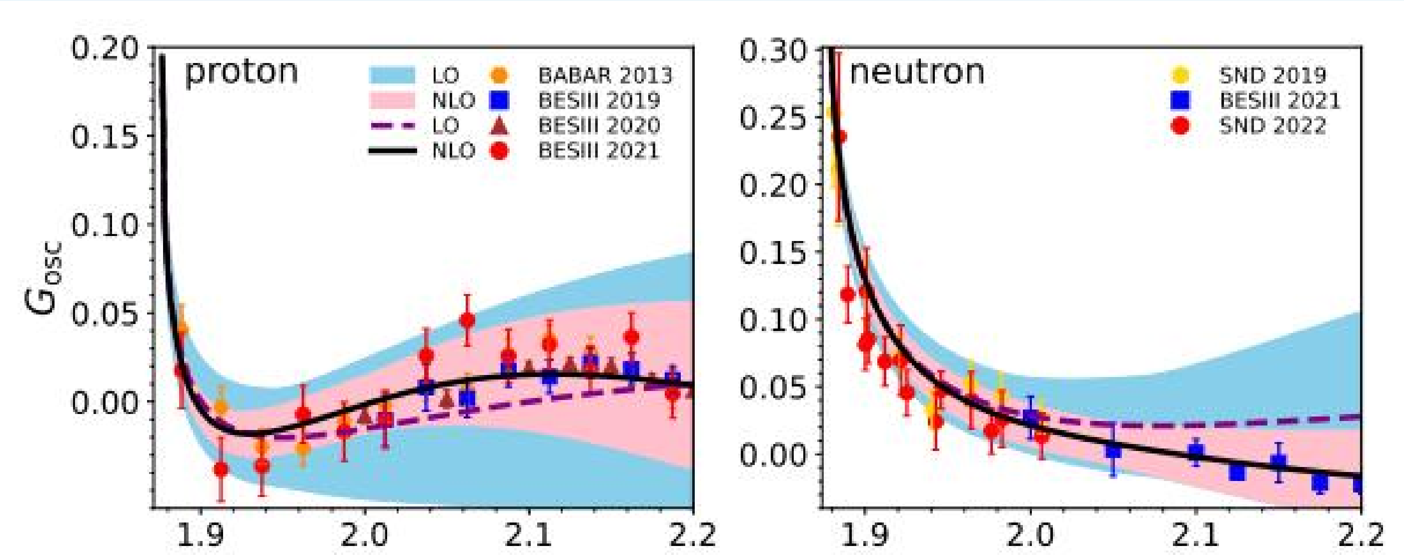
Oscillation

- Effective EMFFs

$$|G_{\text{eff}}(s)| = \sqrt{\frac{\sigma_{e^+e^-\to\bar{N}N}(s)}{\frac{4\pi\alpha^2\beta}{3s}C(s)[1+\frac{2M_N^2}{s}]}}$$

- Subtracted form factors

$$G_{\text{osc}}(s) = |G_{\text{eff}}| - G_D(s), \quad G_D^p(s) = \frac{\mathcal{A}_p}{(1+s/m_a^2)[1-s/q_0^2]^2}, \quad G_D^n(s) = \frac{\mathcal{A}_n}{[1-s/q_0^2]^2}$$



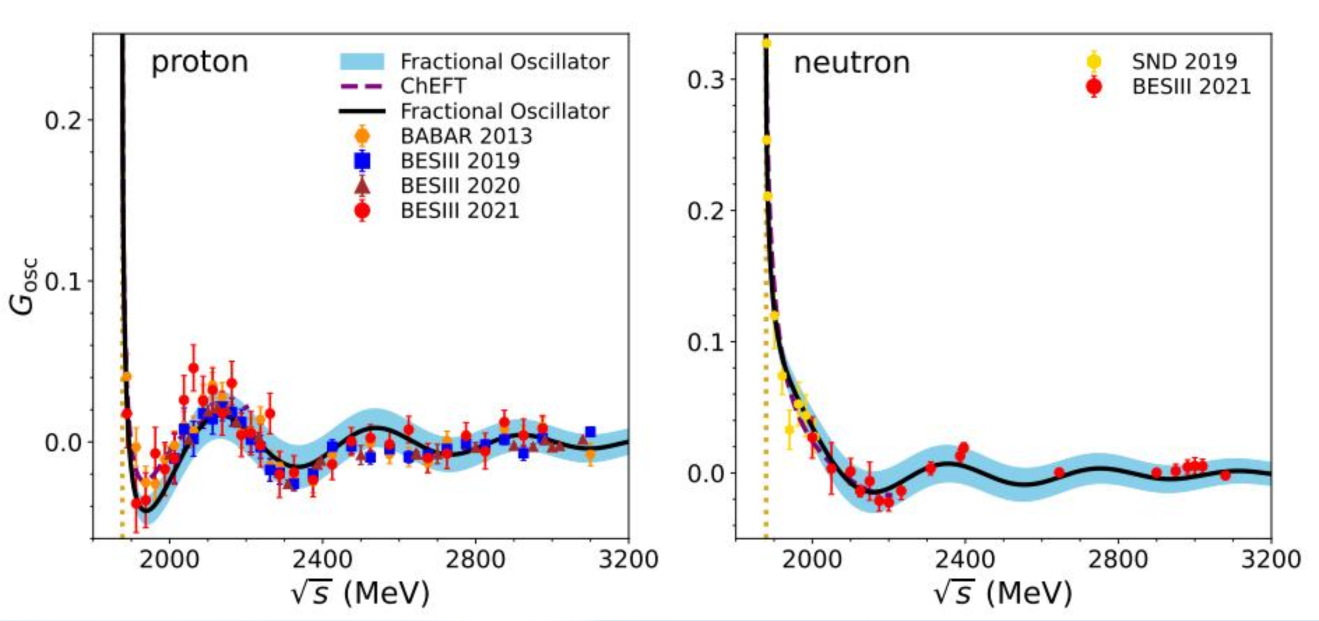
Oscillation

- We propose a fractional oscillation model

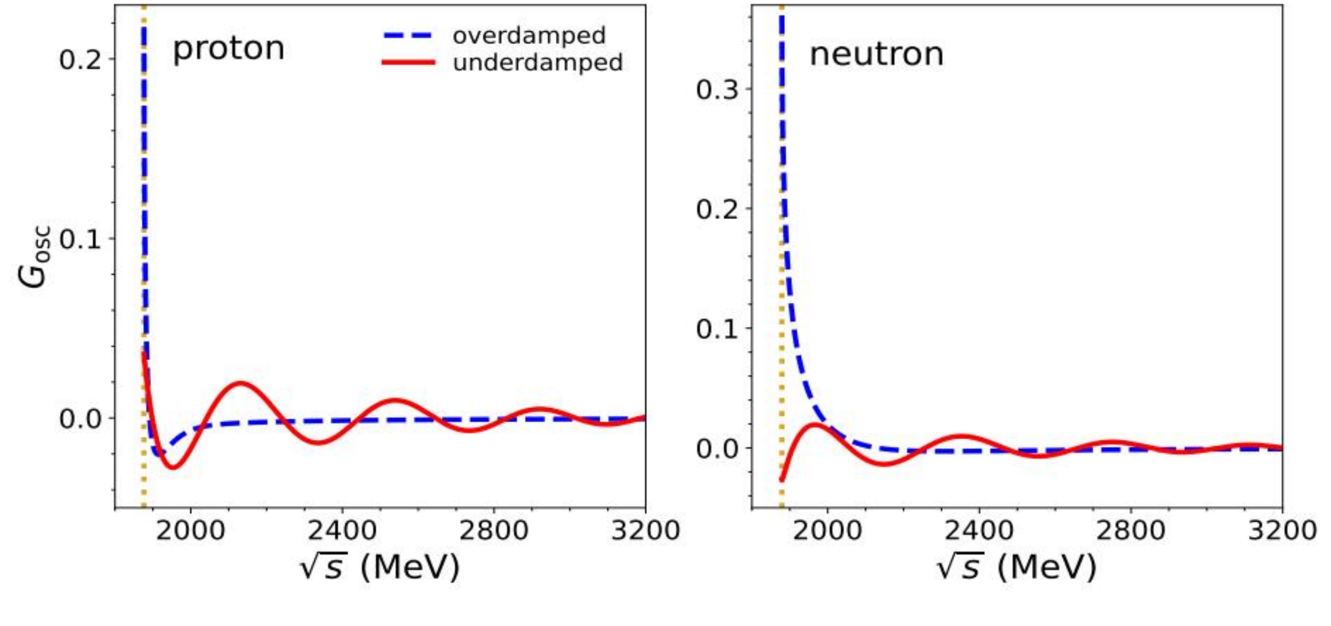
$$G^N_{\text{osc}}(\tilde{p}) = G^N_{\text{osc},1}(\tilde{p}) + G^N_{\text{osc},2}(\tilde{p}),$$

$$G^N_{\text{osc},j}(\tilde{p}) = G^{0,N}_{\text{osc},j} - \frac{\omega_j^2}{\Gamma(\alpha_j^N)} \int_0^{\tilde{p}+p_0^N} (\tilde{p} + p_0^N - t)^{\alpha_j^N - 1} G^N_{\text{osc},j}(t) dt$$

- Oscillation behavior of SFFs

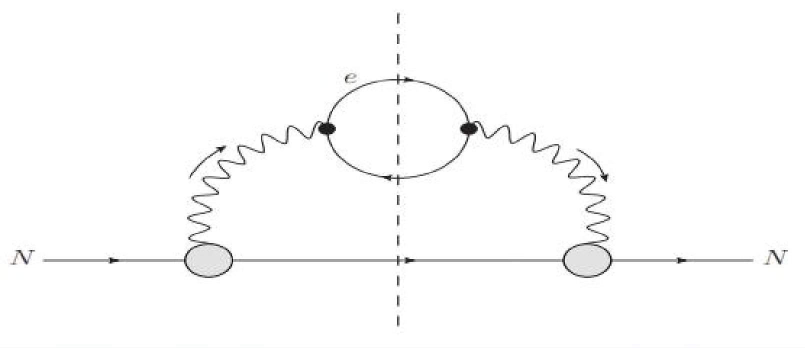


Oscillation



- The ‘overdamped’ oscillator dominates near the threshold. It reveals the enhancement near threshold.
- The ‘underdamped’ oscillator dominates in the high energy region. The proton’s and neutron’s has a ‘phase delay’.
- Other dynamics? → Cao, J.P. Dai, Lenske, PRD 105 (2022) 7, L071503, etc
  Qian, Liu, Cao, Liu, PRD 107 (2023) 9, L091502;
  Yan, Chen, Xie, PRD 107 (2023) 7, 076008

Underlying physics?



overdamped oscillator
octupole?

underdamped oscillator
quadrupole?

- Vacuum polarization around the nucleon

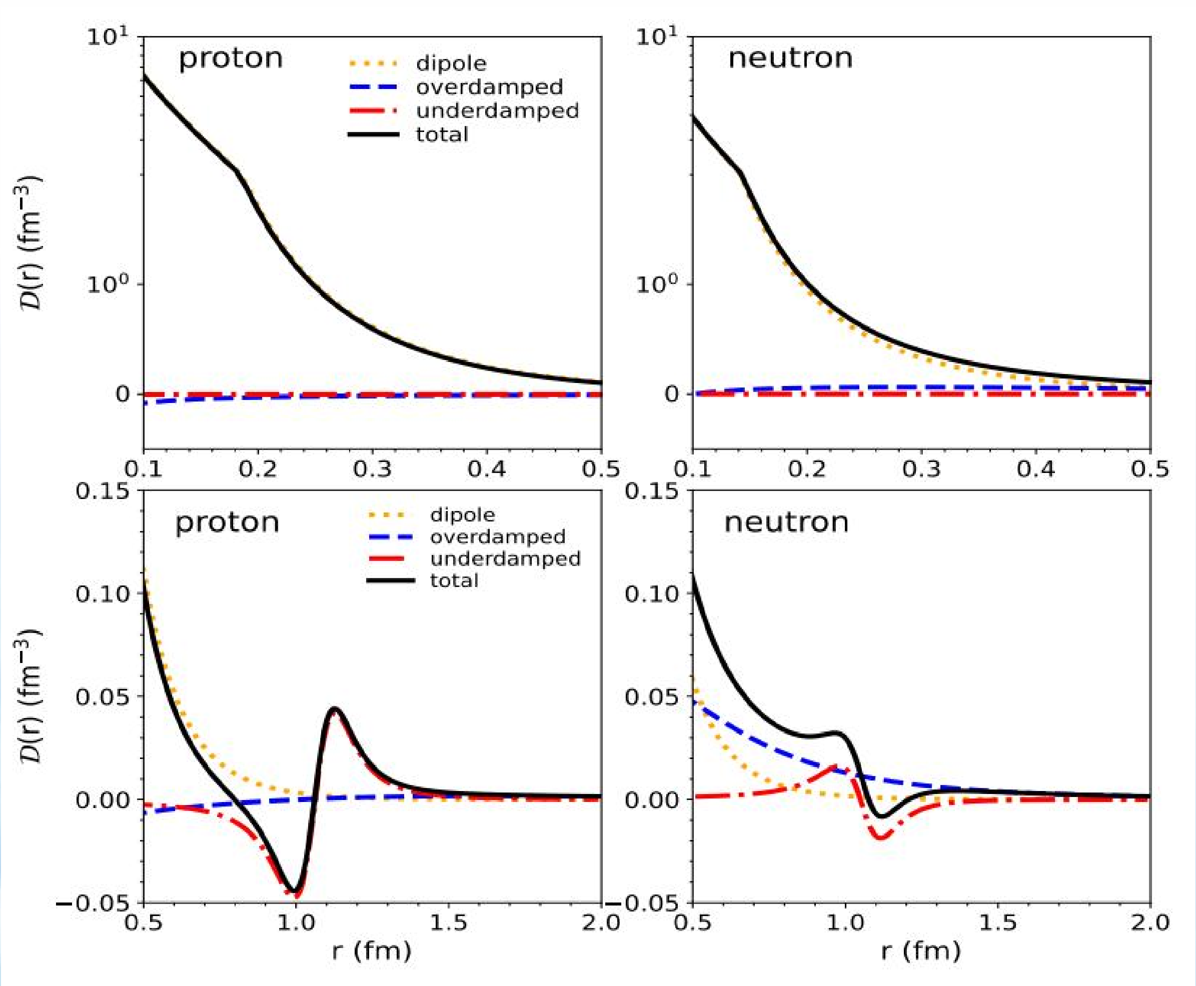
$1 < \alpha_j^N < 2$ $\alpha_1^p = 1.23$ $\alpha_1^n = 1.04$ $\alpha_2^p = \alpha_2^n = 1.87$

- Two limits of fractional oscillators： 1 for diffusion and 2 for wave equations of motions.

- Distributions of higher order polarized charges.

Yang, Guo, Dai, et. al., Sci.Bull. 68 (2023) 2729;

Underlying physics?



- Proton: valence quarks of uud; Neutron: udd
- negative polarization electric charges for the proton, when not very faraway from the nucleon.
- positive polarization for the neutron
- It explains the phase difference!

4、EMFFs of other baryons

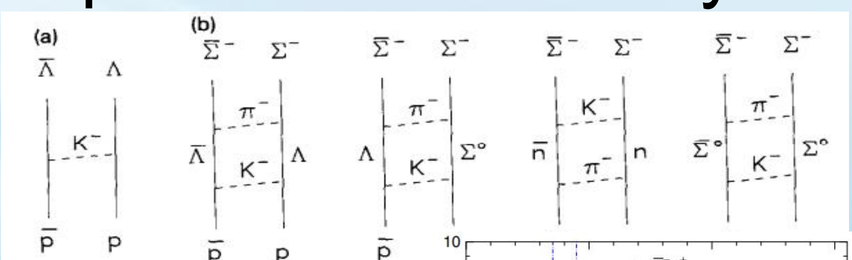
- NN-YY potentials given by Juelich model

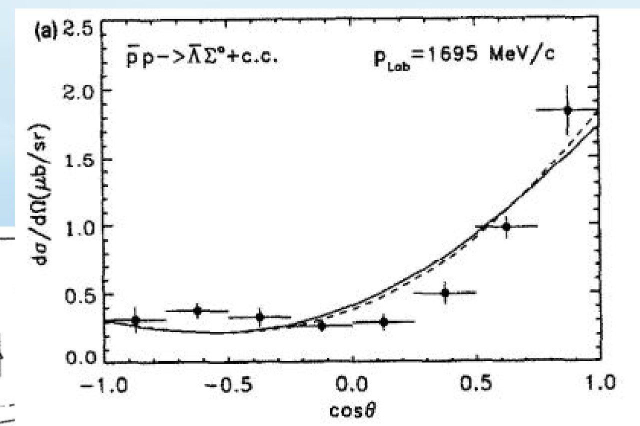
$$V^0 = \begin{pmatrix} V^0_{\bar{N}N,\bar{N}N} & V^0_{\bar{N}N,\bar{\Lambda}\Lambda} & V^0_{\bar{N}N,\bar{\Sigma}\Sigma} \\ V^0_{\bar{\Lambda}\Lambda,\bar{N}N} & V^0_{\bar{\Lambda}\Lambda,\bar{\Lambda}\Lambda} & V^0_{\bar{\Lambda}\Lambda,\bar{\Sigma}\Sigma} \\ V^0_{\bar{\Sigma}\Sigma,\bar{N}N} & V^0_{\bar{\Sigma}\Sigma,\bar{\Lambda}\Lambda} & V^0_{\bar{\Sigma}\Sigma,\bar{\Sigma}\Sigma} \end{pmatrix}$$

$$V^1 = \begin{pmatrix} V^1_{\bar{N}N,\bar{N}N} & V^1_{\bar{N}N,\bar{\Lambda}\Sigma} & V^1_{\bar{N}N,\bar{\Sigma}\Lambda} & V^1_{\bar{N}N,\bar{\Sigma}\Sigma} \\ V^1_{\bar{\Lambda}\Sigma,\bar{N}N} & V^1_{\bar{\Lambda}\Sigma,\bar{\Lambda}\Sigma} & V^1_{\bar{\Lambda}\Sigma,\bar{\Sigma}\Lambda} & V^1_{\bar{\Lambda}\Sigma,\bar{\Sigma}\Sigma} \\ V^1_{\bar{\Sigma}\Lambda,\bar{N}N} & V^1_{\bar{\Sigma}\Lambda,\bar{\Lambda}\Sigma} & V^1_{\bar{\Sigma}\Lambda,\bar{\Sigma}\Lambda} & V^1_{\bar{\Sigma}\Lambda,\bar{\Sigma}\Sigma} \\ V^1_{\bar{\Sigma}\Sigma,\bar{N}N} & V^1_{\bar{\Sigma}\Sigma,\bar{\Lambda}\Sigma} & V^1_{\bar{\Sigma}\Sigma,\bar{\Sigma}\Lambda} & V^1_{\bar{\Sigma}\Sigma,\bar{\Sigma}\Sigma} \end{pmatrix}$$

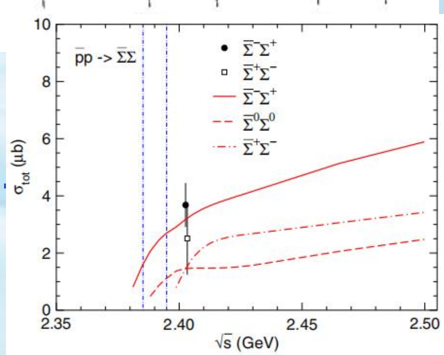
$$V^2 = \left(V^2_{\bar{\Sigma}\Sigma,\bar{\Sigma}\Sigma}\right)$$

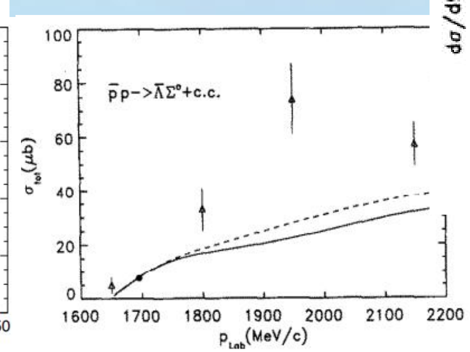
- FSI described by LS equation
- parameters fixed by fitting to the pp-->YY data





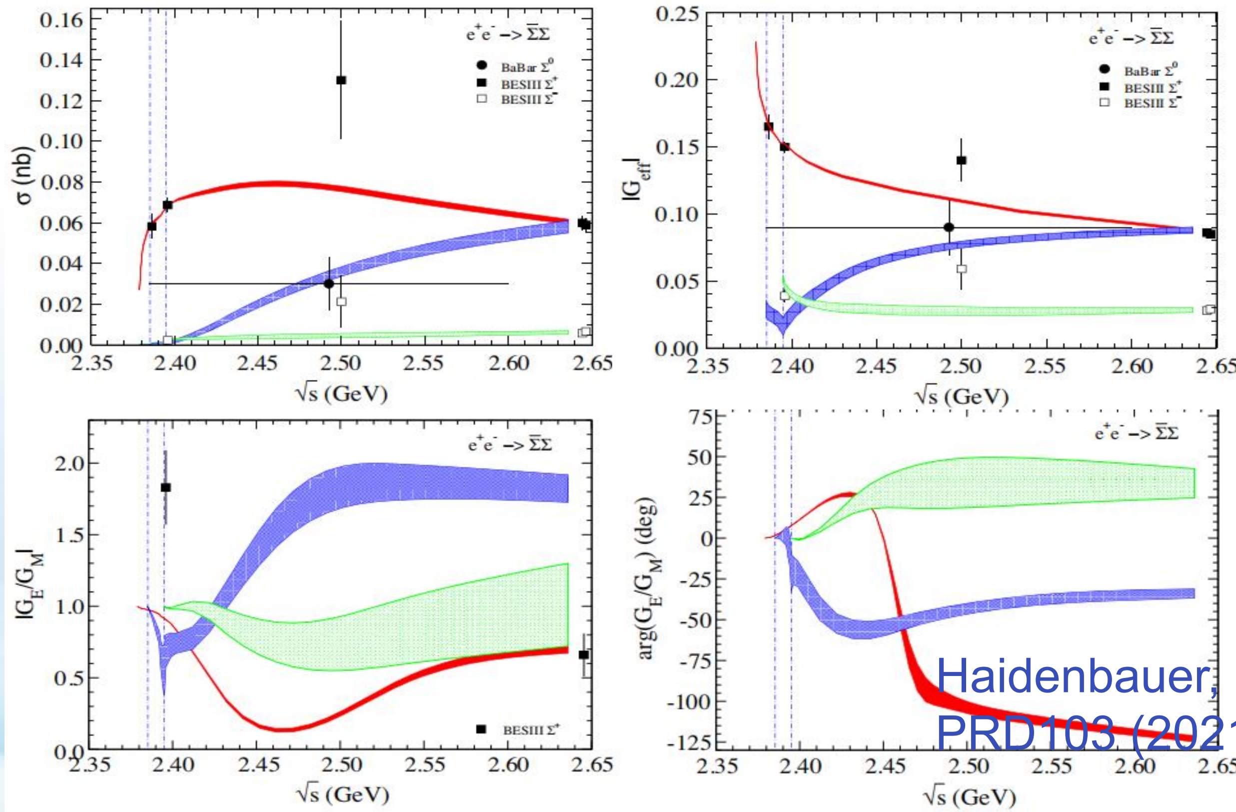
Haidenbauer et.al.,
NPA562 (1993) 317.





ee→ΣΣ

- red bands ΣΣ$^+$, blue:ΣΣ^0, green:ΣΣ$^-$



Haidenbauer, Meissner, Dai, PRD103 (2021) 014028.

- $|G_E/G_M|$: a cusp effect is found in ee-->ΣΣ^0 near ΣΣ$^-$ threshold, but not in the ΣΣ$^+$ channel
- A more profound derstandingu in pp → ΣΣ

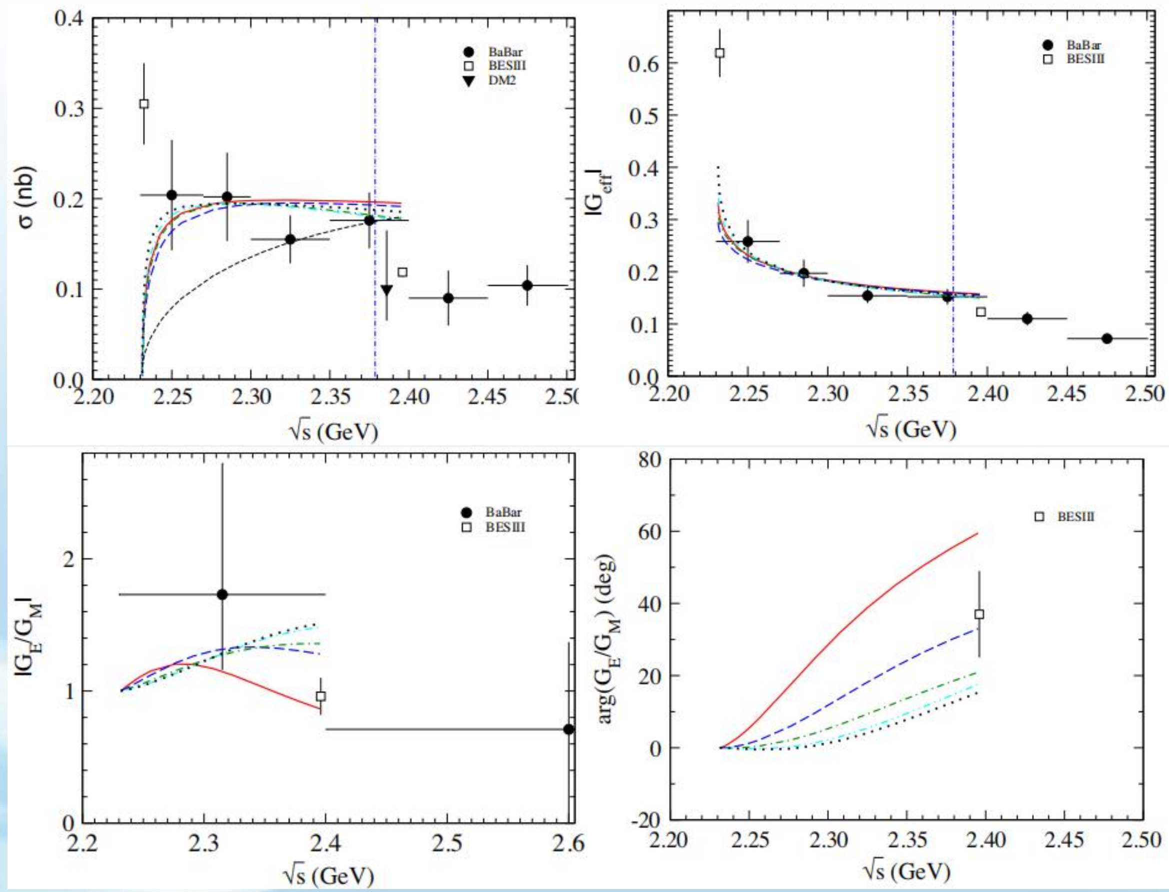
ee→ΛΛ

- Ratio and phase by BESIII
- The enhancement around threshold?
- Above 2.4GeV?

Haidenbauer et.al.,
PRD103 (2021) 014028,
PLB761 (2016) 456

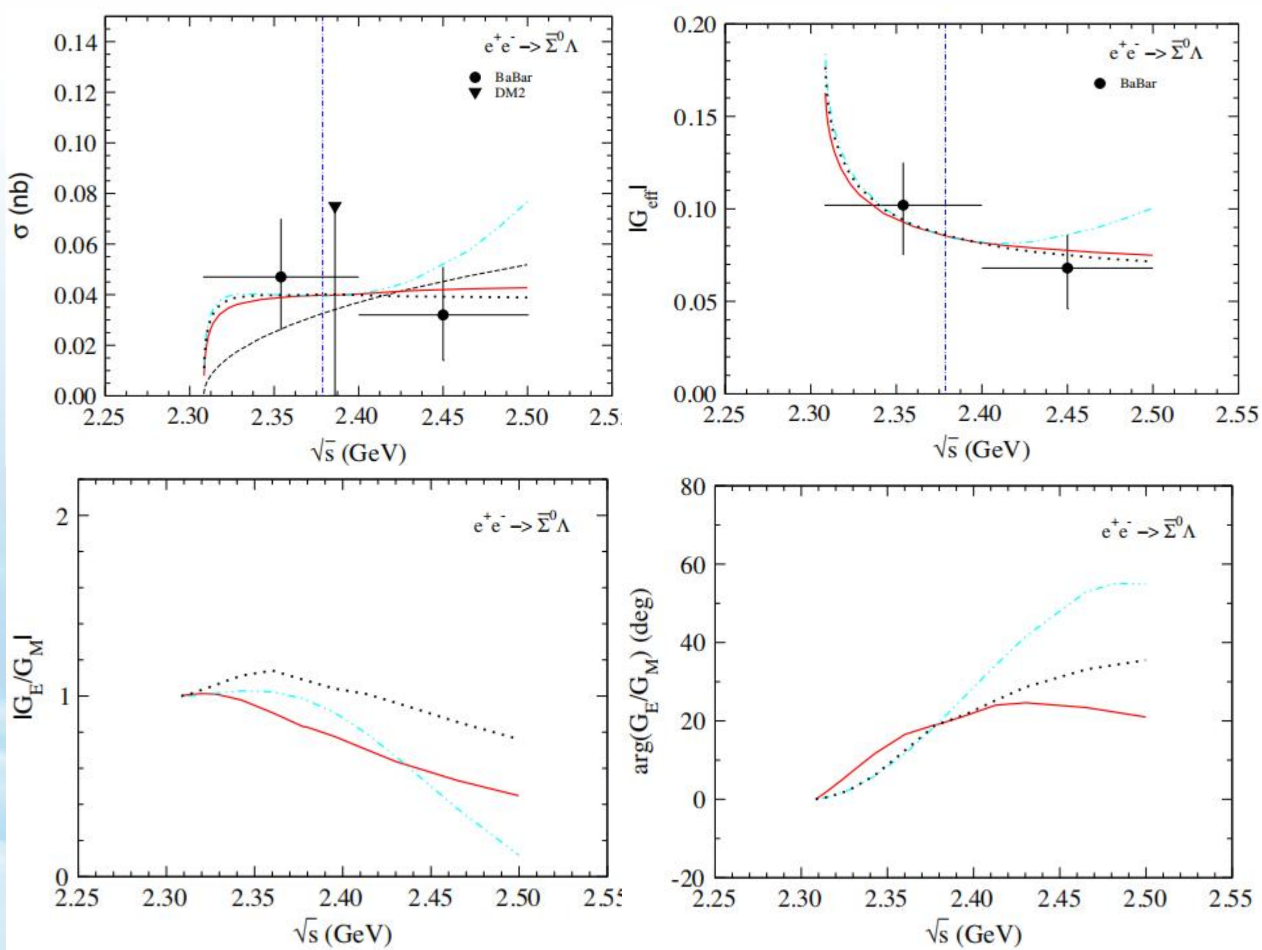
- Oscillation?

A.X. Dai, Li, Chang, Xie,
CPC 46 (2022) 073104;



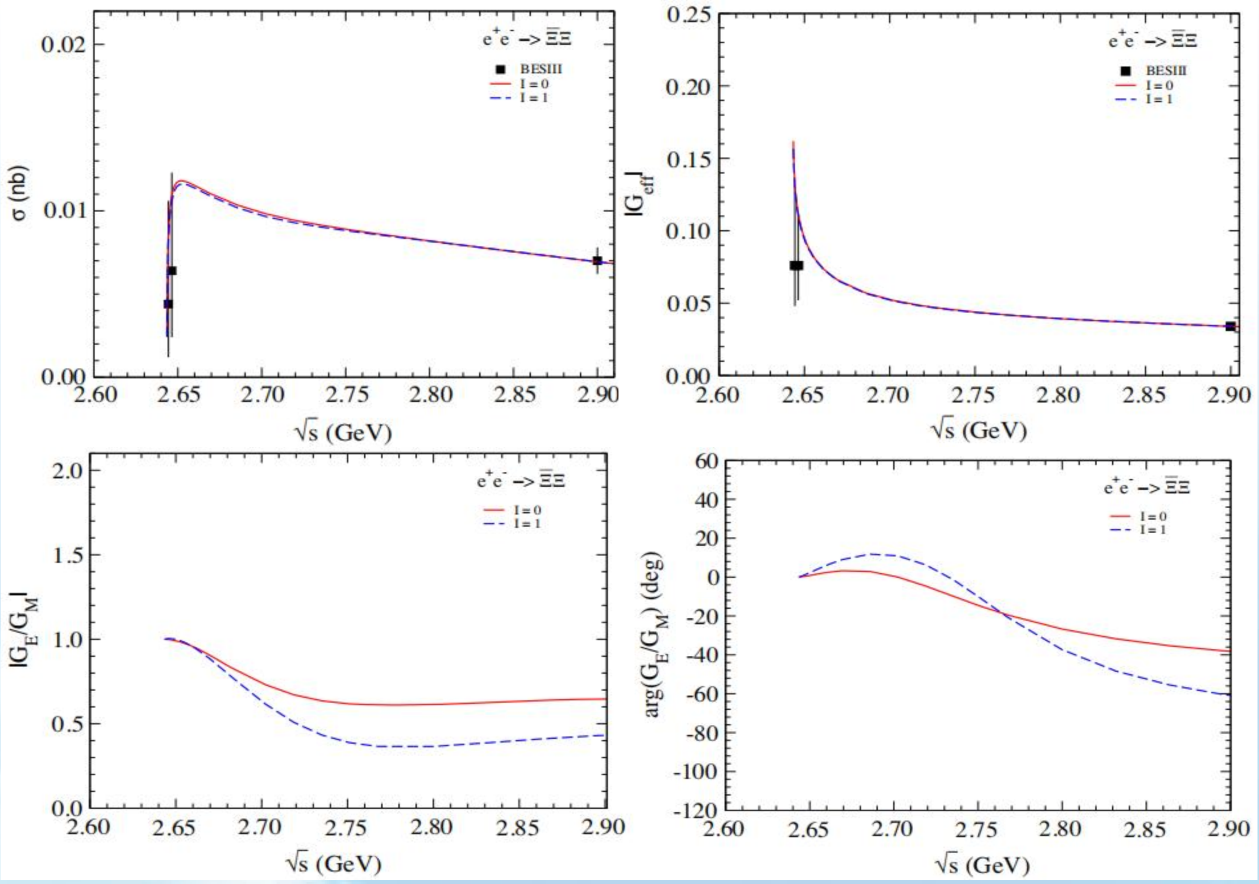
ee→ΣΛ

- No enhancement around 2.4GeV
- EMFF: qualitative similar to the ΛΛ channel's



ee⟶ΞΞ

- pp-->ΞΞ only has data of upper limit
- FSI effects for ΞΞ too strong?
- Exp measurement in 2.65-2.8GeV could help a lot.

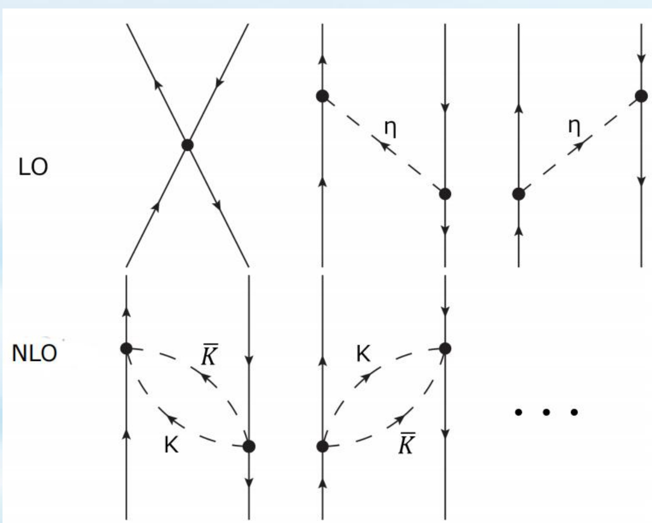


Individual EMFFs of Λ_c

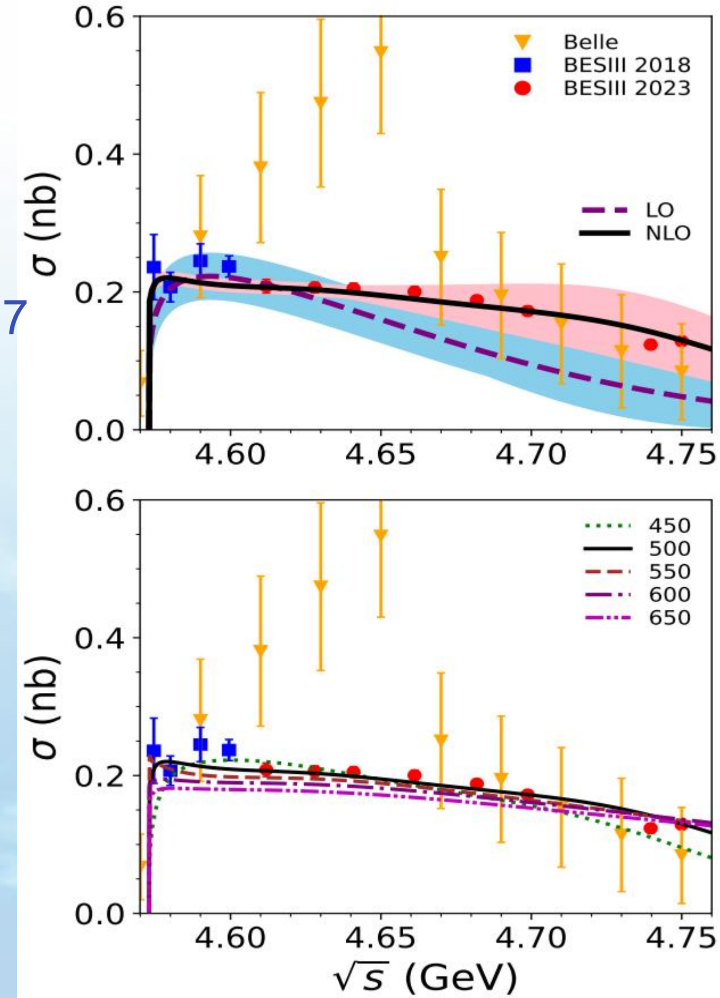
- Effective form factors for LO, NLO from ChEFT
- Cutoff independent.

$$B_{\bar{3}} = \begin{pmatrix} 0 & \Lambda_c & \Xi_c^+ \\ -\Lambda_c & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix} \quad B_6 = \begin{pmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^+ & \frac{1}{\sqrt{2}}\Xi_c'^+ \\ \frac{1}{\sqrt{2}}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}}\Xi_c'^0 \\ \frac{1}{\sqrt{2}}\Xi_c'^+ & \frac{1}{\sqrt{2}}\Xi_c'^0 & \Omega_c \end{pmatrix}$$

Yan, Cheng, et.al., PRD46 (1992) 1148
Zou, Liu, Liu, Jiang,PRD108 (2023) 014027

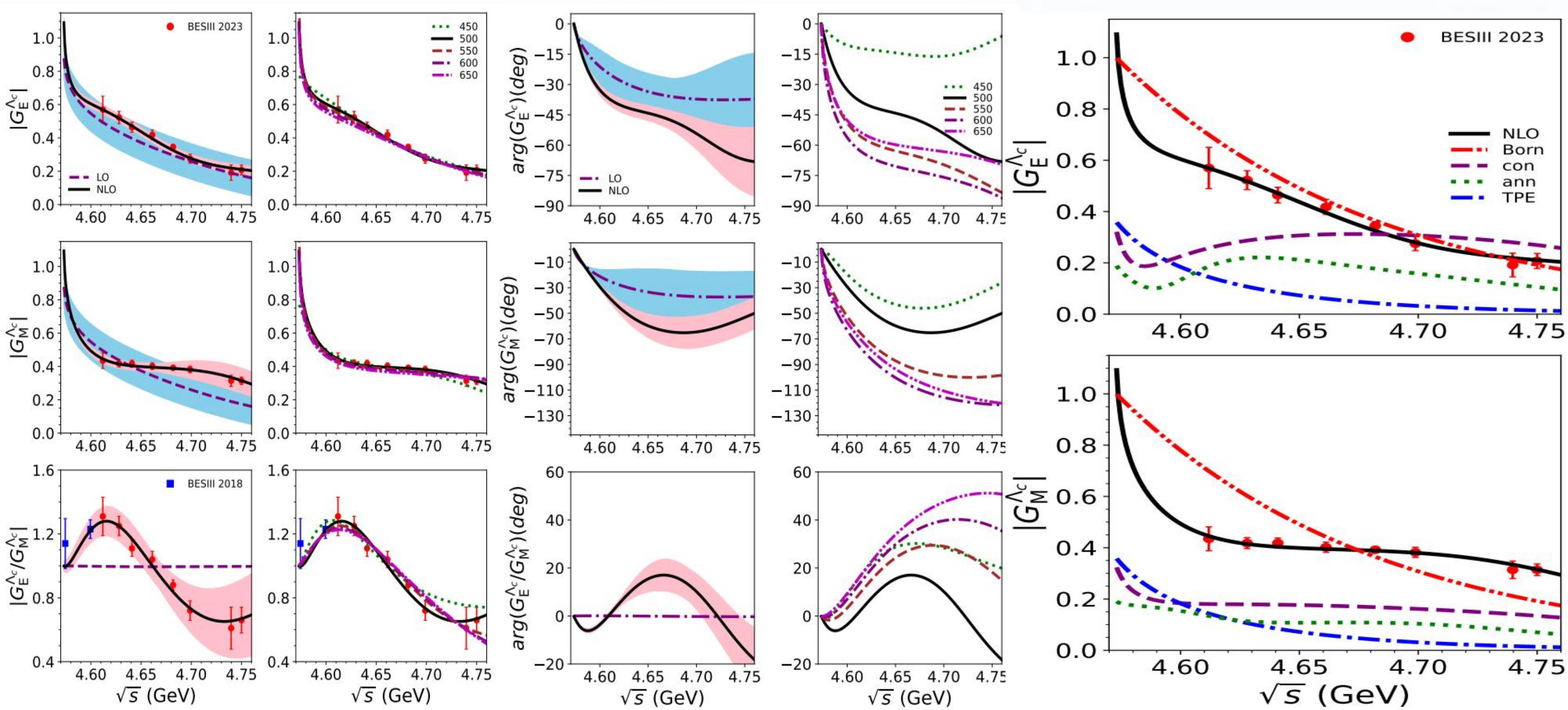


Guo, Yang, Dai, arxiv:2404.06191



Separated contributions

- Contact term: essential for threshold enhancement
  - S-wave contribution is significant!
- Annihilation term: crucial for fluctuation



5. EMFF summary

NN Amplitude

SU(3), up to N^3LO, ChEFT works well at P_{Lab}<300 MeV. Consistent with the exp's. SU(3), include other Baryons, but need more measurements on hyperons.

EMFFs of N

We study the oscillation of EMFFs of nucleons within SU(3) ChEFT. A fractional oscillation model is proposed, polarized charge density distributions?

EMFFs of Y

We study e+e− →YY processes close to the theshold.The EMFFs are predicted, it still lacks of measurements. BESIII? Also EMFFs of Λ_c .

Prospects?

ChEFT + OGE to study NN scatterings?
EMFFs of other baryons and resonances within ChEFT?

Thank You For your patience !