Global Hyperon Polarization and Spin Correlations in Relativistic Heavy Ion Collisions

Liang Zuo-tang
Shandong University

2024年4月14日，惠州

Outline

- Introduction: why polarization?
- Globally polarized QGP in relativistic heavy ion collisions: why global polarization?
- Quark spin correlations in QGP and polarizations of hadrons with different spins (1/2, 1, 3/2): why quark spin correlation?
- Summary and outlook

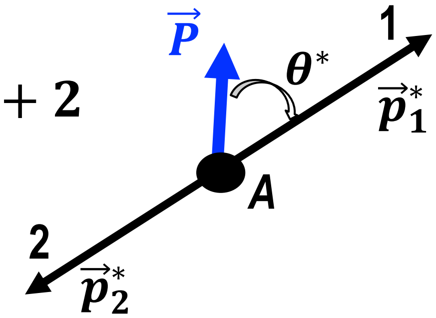
Why polarization?

Hyperon polarization can be measured via angular distribution of the "self spin analyzing parity violating decay":

$$H \to N + M$$

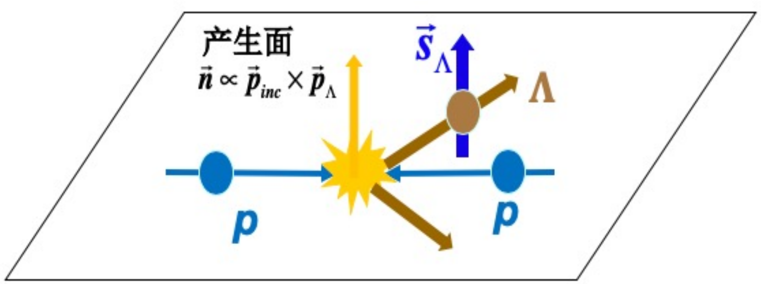
$$\frac{dN}{d\Omega} = \frac{1}{4\pi}(1 + \alpha P \cos\theta)$$

$$A \to 1 + 2$$



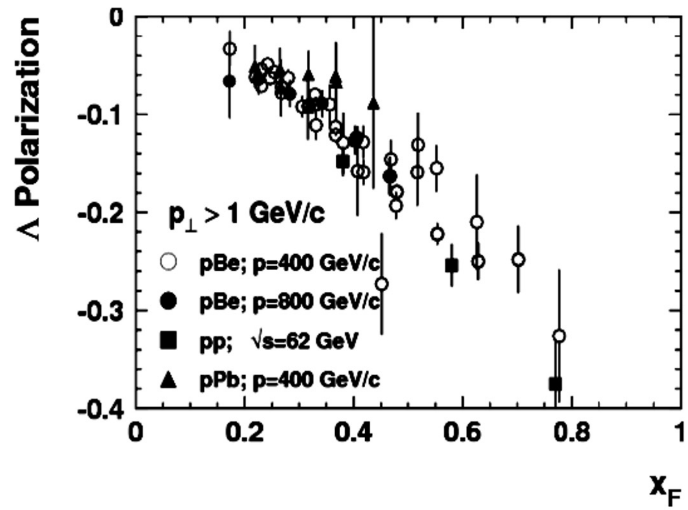
Striking hyperon polarization effect has been observed since 1970s

"Transverse polarization of hyperon in $pp \to \Lambda X$"



See e.g.,
A. Lesnik *et al.*, PRL35, 770 (1975);
G. Bunce *et al.*, PRL36, 1113 (1976);
S.A. Gourlay *et al.*, PRL56, 2244 (1986).

$$p + p/A \to \Lambda(\uparrow) + X$$



Why QCD high energy spin physics?

Striking spin effects have been observed in high energy reactions since 1970s

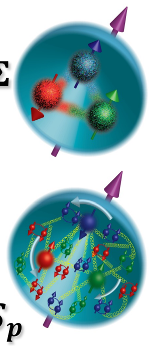
“Proton spin crisis” 质子自旋危机

夸克模型：

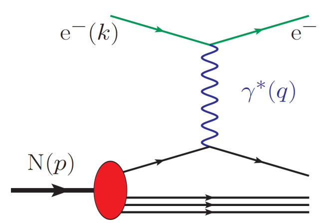
夸克自旋之和Σ
=质子自旋S_p

DIS实验：

89年: Σ~0
目前: Σ~20%S_p



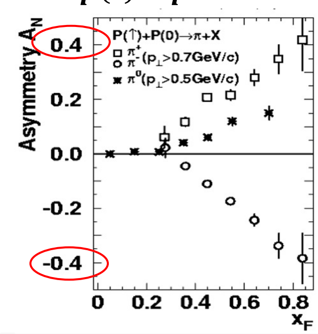
$$e + p \to e + X$$

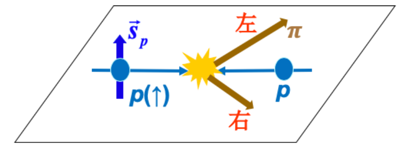


EMC, PLB 206,364 (1988)

“Single spin left-right asymmetry (SSA)”

$$p(\uparrow) + p \to \pi + X$$



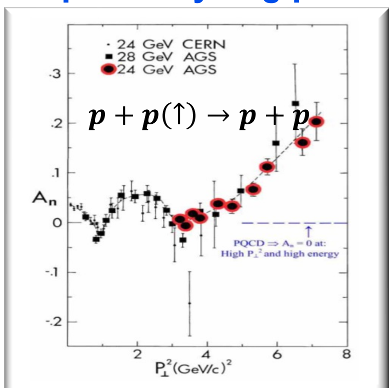


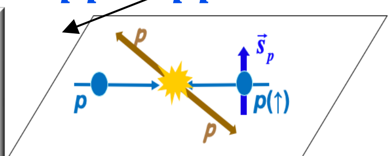
$$A_N \equiv \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$

e.g. FNAL E704,
PLB264, 462 (1991)

Predictions of pQCD ～ 0

“Spin analyzing power in $pp \to pp$”



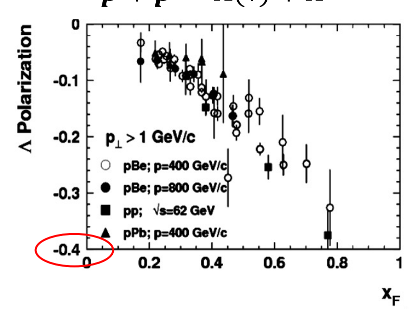


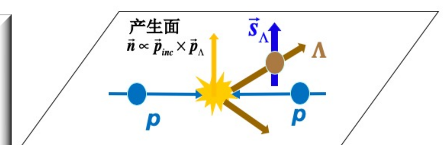
$$A_N \equiv -\frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}$$

e.g. D. Grab *et al.*,
PRL41, 1257 (1978)

“Transverse polarization of hyperon in $pp \to \Lambda X$”

$$p + p \to \Lambda(\uparrow) + X$$





$$P_\Lambda \equiv \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)}$$

e.g. S.A. Gourlay *et al.*,
PRL56, 2244 (1986)

Why QCD spin physics?

CONFERENCE KEYNOTE

QCD: Hard Collisions are Easy and Soft Collisions are Hard .

J. D. Bjorken

Proceedings of a NATO Advanced Research Workshop on
QCD Hard Hadronic Processes,
held October 8-13, 1987,
in St. Croix, US Virgin Islands

SLAC理论中心前主任

Polarization data has often been the graveyard of fashionable theories. If theorists had their way they might well ban such measurements altogether out of self-protection.[22] Nowadays the

“极化数据经常是流行理论的坟墓，如果理论家有办法，他们可能会为了自保而一起设法阻止这种测量。……”

那时理论家在面对这些数据时感到有些窘迫

Why QCD spin physics?

实验与理论严重冲突 ⇒ QCD理论研究的突破口 ⇒ QCD自旋物理
- 核子自旋结构（强子结构）
- 碎裂函数自旋依赖（强子产生）

Polarized deep inelastic scattering: The ultimate challenge to PQCD?

Giuliano Preparata (Milan U. and INFN, Milan) (Feb 6, 1989)

Published in: *Nuovo Cim.A* 102 (1989) 63, *AIP Conf.Proc.* 187 (2008) 754-763 • Contribution to: 8th Internationa High-energy Spin Physics, 754-763

DOI cite

Spin effects: A Challenge for perturbative QCD

Jacques Soffer (Marseille, CPT) (Jan, 1989)

Published in: *Nucl.Phys.B Proc.Suppl.* 11 (1989) 178-185 • Contribution to: 10th Autumn School: Physics Beyon

DOI cite

SPIN PHYSICS: A CHALLENGE TO THE GENERALLY ACCEPTED PICTURE OF QCD

Giuliano Preparata (Milan U. and INFN, Milan) (Jan, 1988)

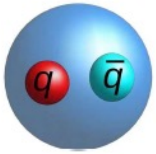
Published in: In *Trieste 1988, Proceedings, Spin and polarization dynamics in nuclear and particle physics* 128- Preparata, G. (88,rec.May) 17 p • Contribution to: Adriatico Research Conference: Spin and Polarization Dynamic Particle Physics, Adriatico Research Conference: Spin and Polarization Dynamics in Nuclear and Particle Physics,

Why Quark Orbital Angular Momentum (OAM)?

quark OAM was used to be neglected

夸克模型： Quark model used to be non-relativistic

baryon



meson

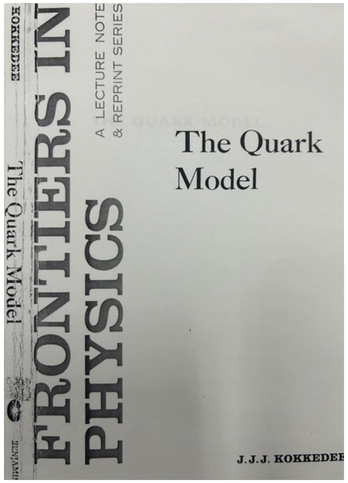
Physics Vol. 2, No. 2, pp. 95-105, 1965. Physics Publishing Co. Printed in Great Britain.

IS A NON-RELATIVISTIC APPROXIMATION POSSIBLE FOR THE INTERNAL DYNAMICS OF "ELEMENTARY" PARTICLES?*

G. MORPURGO

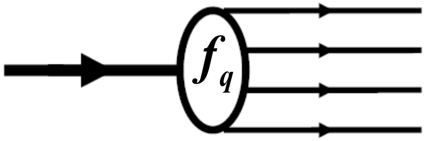
Istituto di Fisica dell'Universita di Genova
Sezione di Genova dell'Istituto Nazionale di Fisica Nucleare, Genova, Italy

(*Received* 28 *April* 1965)



on the depth of the potential well. For instance, for a quark antiquark model of the octet bosons with a quark mass of 5 GeV and a range of the binding force

部分子模型： Parton model used to be one-dimensional



Quark OAM should play an important role

Spin-orbit coupling is intrinsic in Relativistic Quantum Systems

Dirac equation: $i\partial_t\psi = \hat{H}\psi \quad \hat{H} = \vec{\alpha}\cdot\hat{\vec{p}} + \beta m \quad \psi = \begin{pmatrix}\varphi \\ \eta\end{pmatrix}$

Even for a free Dirac particle:

$$\left[\hat{H}, \hat{\vec{L}}\right] = -i\vec{\alpha}\times\hat{\vec{p}} \neq 0 \qquad \left[\hat{H}, \vec{\Sigma}\right] = 2i\vec{\alpha}\times\hat{\vec{p}} \neq 0 \qquad \left[\hat{H}, \hat{\vec{J}}\right] = 0 \qquad \hat{\vec{J}} = \hat{\vec{L}} + \frac{\vec{\Sigma}}{2}$$

If we have an external potential $V(r)$: $\hat{H} = \vec{\alpha}\cdot\hat{\vec{p}} + \beta m + V(r)$

$$\hat{H}_{eff}\varphi = E\varphi \qquad \hat{H}_{eff} \approx m + \frac{\hat{\vec{p}}^2}{2m} + V + \frac{1}{4m^2}\frac{dV}{rdr}\vec{\sigma}\cdot\hat{\vec{L}} + \cdots$$

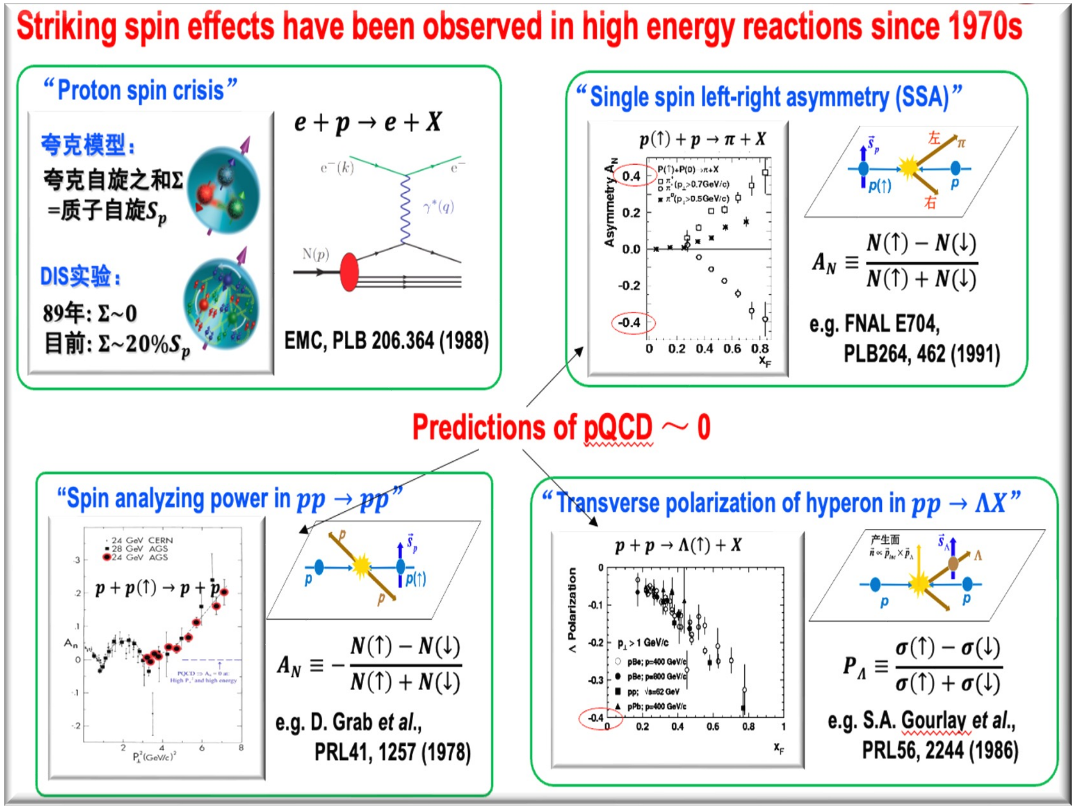
OAM is non-zero even if the quark is in the ground state:

$$\psi_0 \equiv \psi_{E_0\frac{1}{2}m+}(r,\theta,\varphi,S) = \begin{pmatrix} f_{00}(r)\Omega^0_{\frac{1}{2}m}(\theta,\varphi) \\ -g_{01}(r)\Omega^1_{\frac{1}{2}m}(\theta,\varphi)\end{pmatrix}$$

$$\left\langle\psi_0\middle|\hat{\vec{L}}^2\middle|\psi_0\right\rangle = 2\int dr\, r^2 g_{01}^2(r)$$

$$\left\langle\psi_0\middle|\hat{L}_z\middle|\psi_0\right\rangle = \frac{5m}{3}\int dr\, r^2 g_{01}^2(r)$$

Quark Orbital Angular Momentum (OAM) should play an important role!



The underline physics:

intuitively

systematic studies in 1990s

⟹ **quark OAM and spin-orbit coupling in QCD**

Original papers, e.g.,
D. W. Sivers, PRD 41, 83 (1990);
C. Boros, ZTL, Meng Ta-chung, PRL 70, 1751 (1993);
C. Boros, ZTL, PRL79, 3608 (1997);
S. Brodsky, D. Hwang, I. Schmidt, PLB 530, 99 (2002).

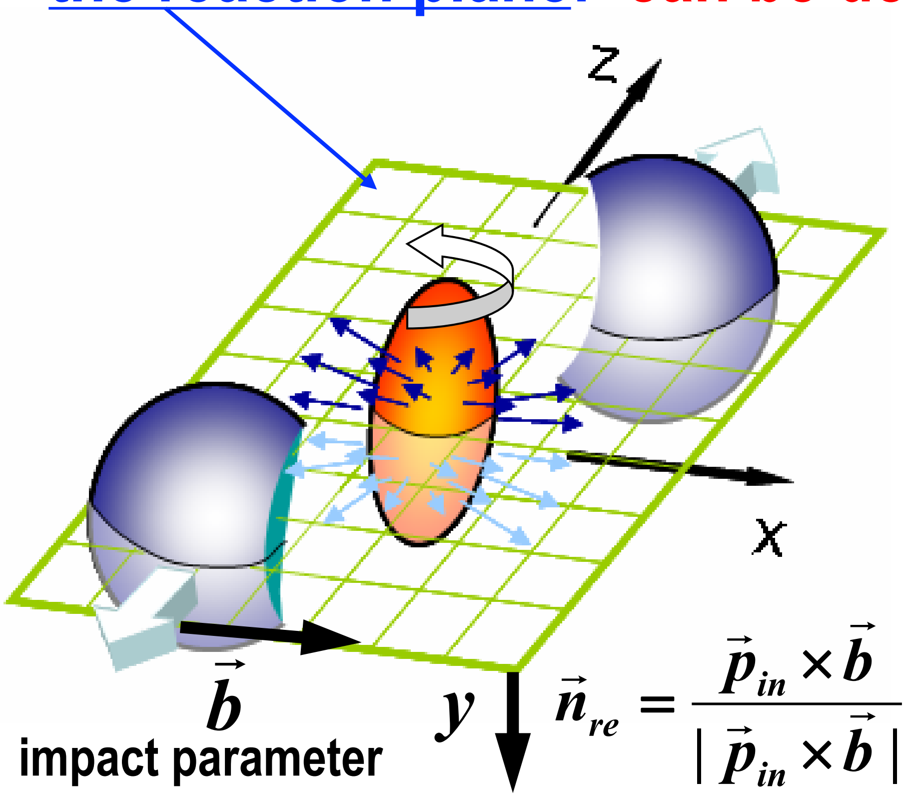
Reviews, e.g.,
S.B. Nurushev, Inter. J. Mod. Phys. A12, 3433 (1997);
G. P. Ramsey, Prog. Part. Nucl. Phys. 39,599(1997);
C. Boros, ZTL, Inter. J. Mod. Phys. A15, 927 (2000);
U. D'Alesio, F. Murgia, Prog. Part. Nucl. Phys. 61, 394 (2008).

Spin-orbit interaction seems to be essential in QCD Spin physics

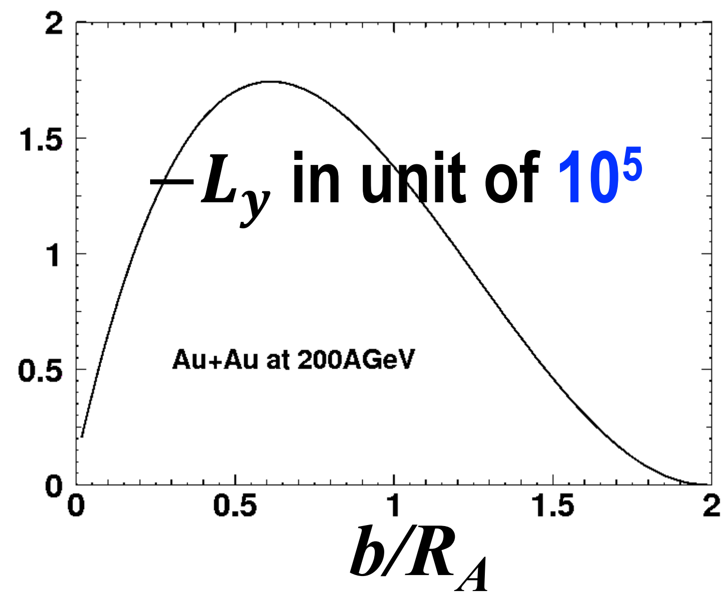
定量研究非常困难，进展缓慢........

Huge OAM of the colliding system in non-central HIC

the reaction plane: **can be determined experimentally !**

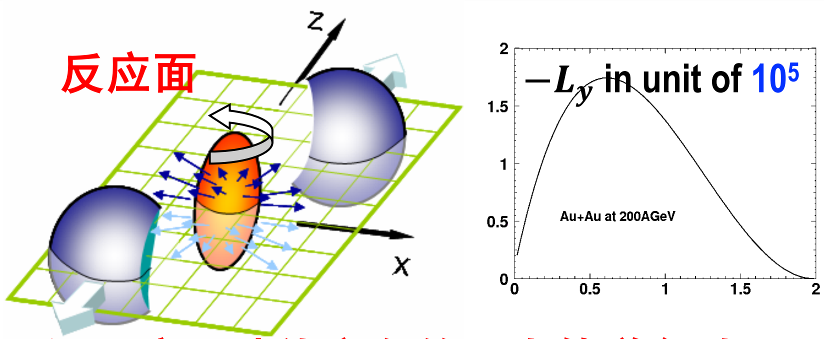


ZTL & Xin-Nian Wang, PRL 94, 102301 (2005)



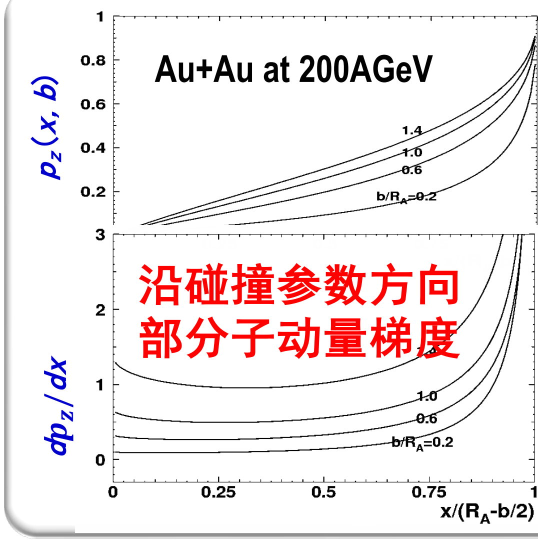
A unique place to study spin-orbit interaction in QCD!

Global polarization in heavy ion collisions



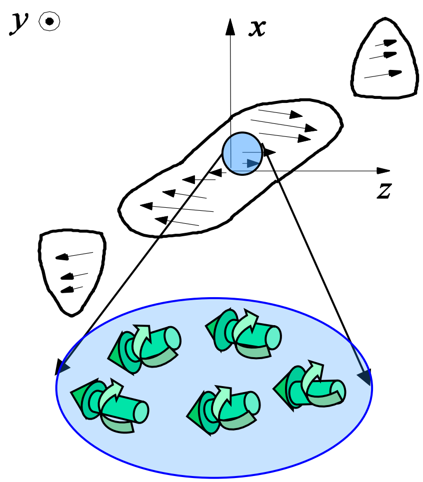
沿反应面法线方向的巨大轨道角动量

导致



自旋—轨道

相互作用导致



"global quark polarization"

强子化导致

（组合）

- 超子整体极化

$$P_H = P_{\bar{H}} = P_q = P_{\bar{q}}$$

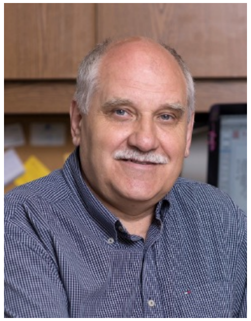
- 矢量介子整体自旋排列

(spin alignment)

$$\rho_{00} = \frac{1 - P_q^2}{3 + P_q^2}$$

ZTL & Xin-Nian Wang, PRL94, 102301(2005); PLB629, 20 (2005).

迅速得到同行响应



提交到arXiv网站仅3天，美国Wayne State大学Sergel A. Voloshin教授就试图把我们的思想推广到强子—强子碰撞过程，声称可以解释非极化强子—强子碰撞过程的超子极化

cent paper [1] discussing non-central nuclear collisions. I would totally concur with the results presented in this paper. Here, I discuss a few ideas beyond those already

In this short note I would like to point out that such a conversion of the orbital momentum into spin (and, in principle, wise versa) can be relevant not only for $A+A$ collisions but also could lead to important observable effects in hadron-hadron collisions. In particular I try

[1] Z.-T. Liang and X.-N. Wang, arXive:nucl-th/0410079, 2004.

ZTL & X.N. Wang 的文章2004年10月18日提交

arXiv.org > nucl-th > arXiv:nucl-th/0410079

Nuclear Theory

[Submitted on 18 Oct 2004 (v1), last revised 7 Dec 2005 (this version, v5)]

Globally Polarized Quark-gluon Plasma in Non-cer

Zuo-Tang Liang (Shandong U), Xin-Nian Wang (LBNL)

Sergei Voloshin于2004年10月21日提交

arXiv.org > nucl-th > arXiv:nucl-th/0410089

Nuclear Theory

[Submitted on 21 Oct 2004]

Polarized secondary particles in unpolarized higl

Sergei A. Voloshin

"In this short note I would like to point out that such a conversion of the orbital angular momentum into spin … can be relevant not only for A+A collisions but also could lead to important observable effects in hadron-hadron collisions (不仅对核—核 ... 而且 ... 强子—强子碰撞)"

迅速得到同行响应

➢ 美国哥伦比亚大学M. Gyulassy教授研究组将轨道角动量与QGP涡旋联系，研究了整体极化与涡旋的关系，并且强调“开启了一条新途径 （… opens a new avenue …）”

PHYSICAL REVIEW C **76**, 044901 (2007)

Polarization probes of vorticity in heavy ion collisions

Barbara Betz,[1,2] Miklos Gyulassy,[1,3,4] and Giorgio Torrieri[1,3]
[1]*Institut für Theoretische Physik, J. W. Goethe-Universität, Frankfurt, Germany*

and the observed spectra of Λ, Ξ, and Ω decay products. This opens a new avenue to investigate heavy ion collisions, which has been proposed both as a signal of a deconfined regime [3–6] and as a mark of global properties of the event [7–10].

[7] Z. T. Liang and X. N. Wang, Phys. Rev. Lett. **94**, 102301 (2005).
[8] Z. T. Liang and X. N. Wang, Phys. Lett. **B629**, 20 (2005).
[9] F. Becattini and L. Ferroni, arXiv:0707.0793 [nucl-th].
[10] Z. t. Liang, J. Phys. G **34** S323 (2007).

首次讨论
“vorticity”

迅速得到同行响应

➢ 意大利国家核物理所(INFN) F. Becattini教授研究组研究了把QGP看作平衡态的相对论理想气体，角动量守恒给出的极化与涡旋度的关系。

PHYSICAL REVIEW C **77**, 024906 (2008)

Angular momentum conservation in heavy ion collisions at very high energy

F. Becattini*
Dipartimento di Fisica, Università di Firenze, and INFN, Sezione di Firenze, Florence, Italy

The most distinctive signature of an intrinsic angular momentum would be the polarization of the emitted hadrons. This argument has been put forward in Refs. [6,7], where the authors take a QCD perturbative approach. Also, more recently, polarization has been related to the fluid vorticity [8], yet without the development of an explicit mathematical relation. In this paper, we take advantage of a very recent study of the ideal relativistic spinning gas [9] and present

[6] Z. T. Liang and X. N. Wang, Phys. Rev. Lett. **94**, 102301 (2005).
[7] J. H. Gao, S. W. Chen, W. T. Deng, Z. T. Liang, Q. Wang, and X. N. Wang, LBNL-63515, arXiv:0710.2943.

引入
“平衡态”
equilibrium

迅速得到同行响应

2006年，第18届“夸克物质大会” [The 18th International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions (Quark Matter 2006)]

- 邀请了梁作堂做大会报告 (plenary talks)，专门报告“Global polarization”整体极化理论工作。
- 并在随后的卫星会议“International Workshop On Hadron Physics at …”(2006年11月21-25日)上，组织了一个专门的session，对相关理论与实验进行针对性研讨。

24号下午日程，6个报告，包括：整体极化理论、实验测量、其它相关实验情况、未来实验计划等

Afternoon

Chairman: Prof. Qubing Xie

14:00-14:30	“Spin physics at RHIC STAR”, E.P. Sichtermann (LBL)
14:30-15:00	“Longitudinal polarization of Λ hyperons in DIS and the nucleon strangeness at COMPASS”, M. Sapoizhnikov (JINR)
15:00-15:30	“Global quark polarization in QGP in non-central AA collisions”, Jianhua Gao (SDU)
15:30-16:00	Coffee/Tea break
	Chairman: Prof. Zuotang Liang
16:00-16:30	“Global polarization measurements in Au+Au collisions”, Ilya Selyuzhenkov (Wayne State University, USA
16:30-17:00	“Spin alignment measurement of phi meson by STAR ” Jinhui Chen (SINAP)
17:00-17:30	“Spin alignment measurement of K* meson by STAR” Zibo Tang (USTC)

First measurements by the STAR Collaboration at 200GeV

STAR

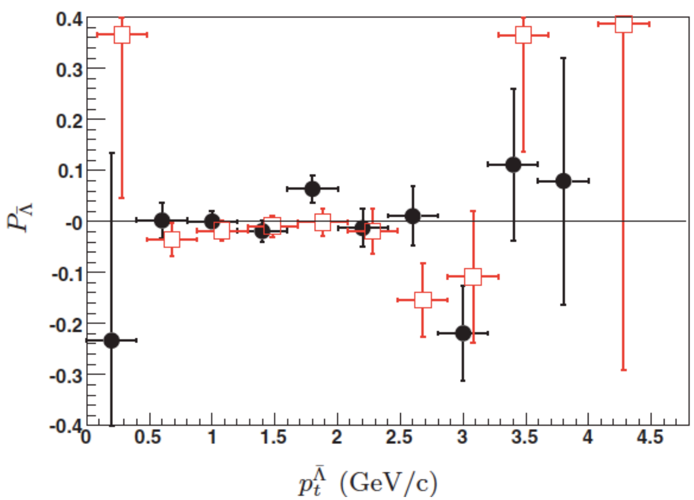
The STAR Collaboration

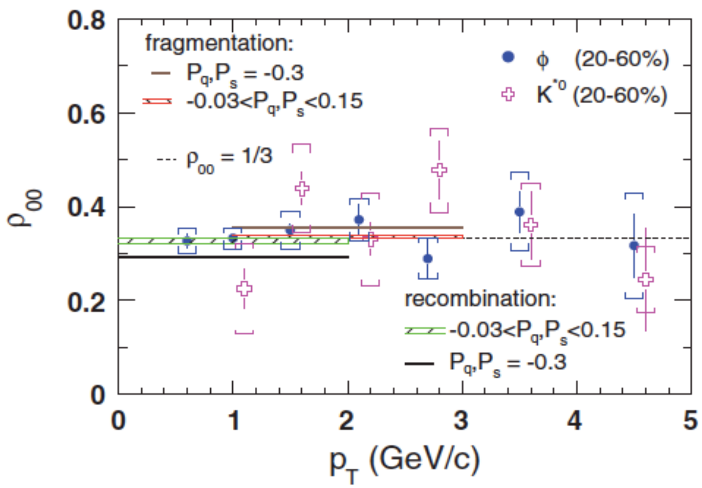
However, NOT observed at $\sqrt{s} = 200\text{GeV}$ with the statistics available at that time!

一盆冷水！

PHYSICAL REVIEW C **76**, 024915 (2007)

Global polarization measurement in Au+Au collisions





RAPID COMMUNICATIONS

PHYSICAL REVIEW C **77**, 061902(R) (2008)

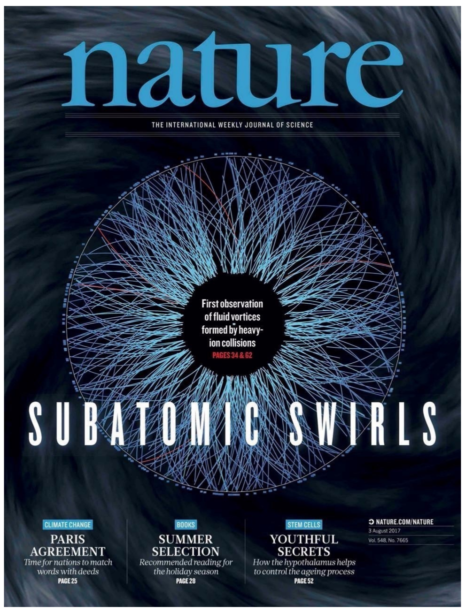
Spin alignment measurements of the $K^{*0}(892)$ and $\phi(1020)$ vector mesons in heavy ion collisions at $\sqrt{s_{NN}} = 200$ GeV

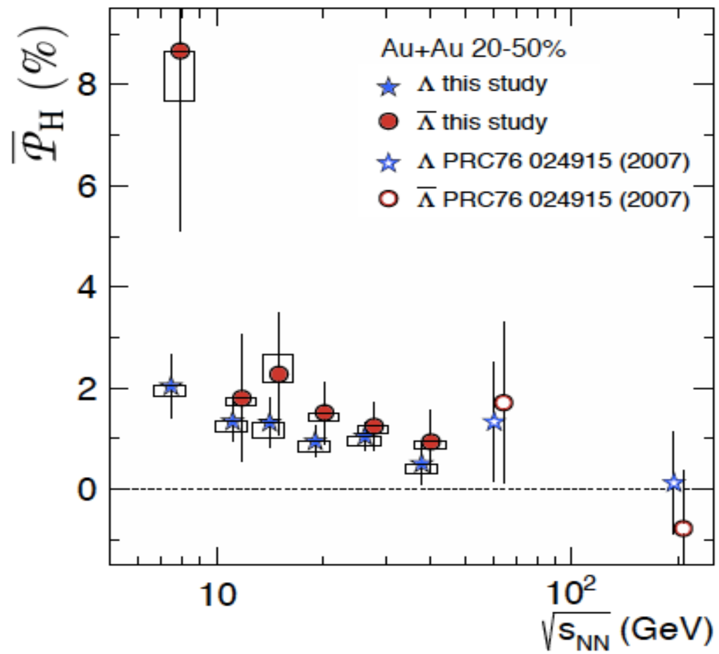
Results of STAR beam energy scan (BES I)

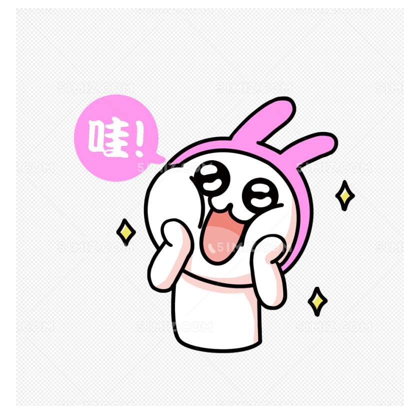
Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration, Nature 548, 62 (2017).

封面文章



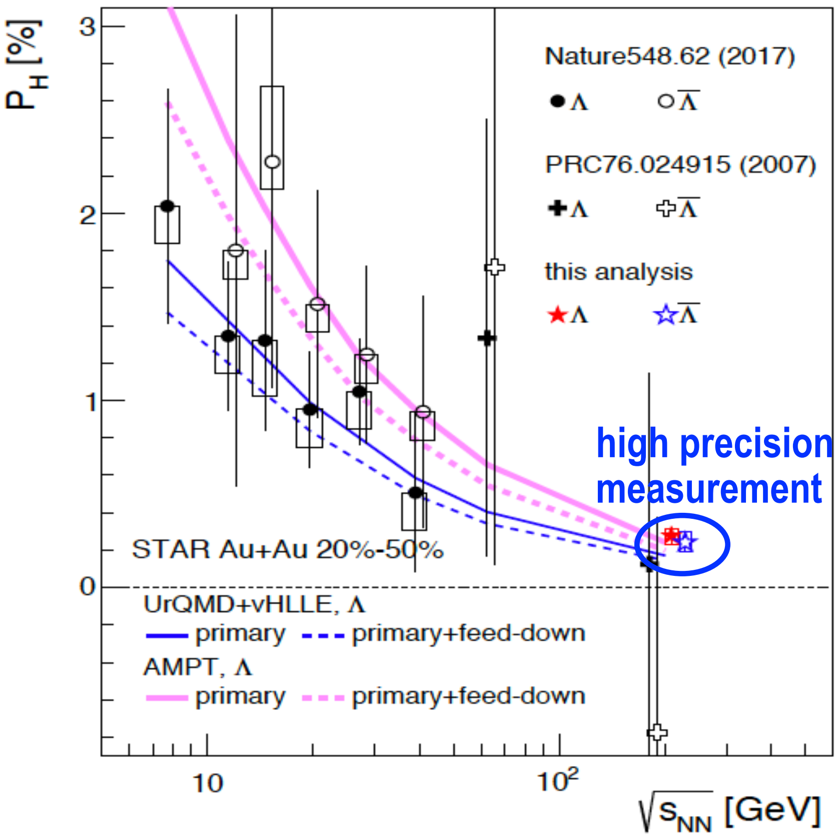




- At each energy, a polarization is observed at 1.1-3.6σ level
- The polarization decreases with increasing energy
- Averaged over energy $P_\Lambda = (1.08 \pm 0.15)\%$, $P_{\bar{\Lambda}} = (1.38 \pm 0.30)\%$

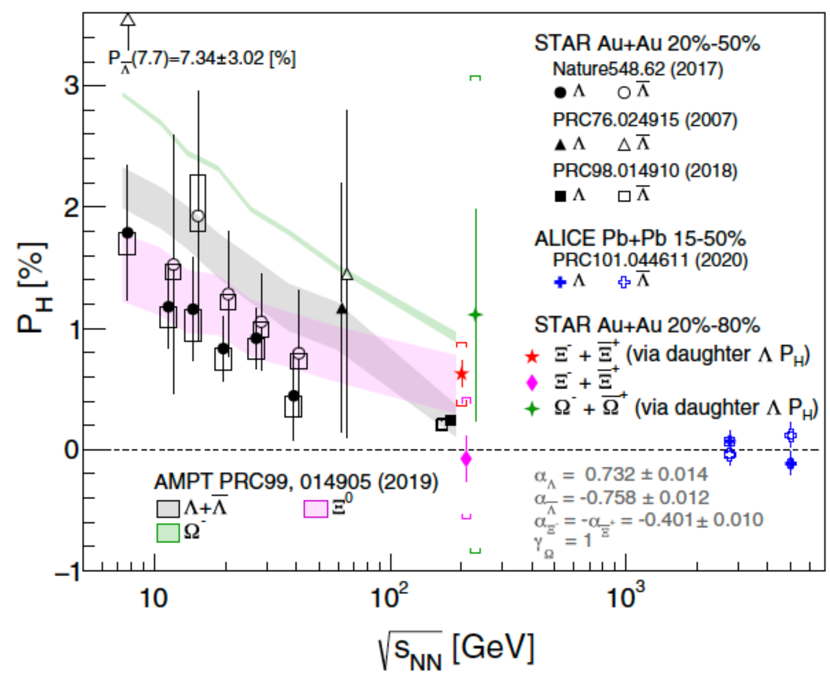
Intensive measurements by STAR at RHIC

Systematical studies at $\sqrt{s} = 200\text{GeV}$



J. Adam *et al.*, PRC 98,014910 (2018)

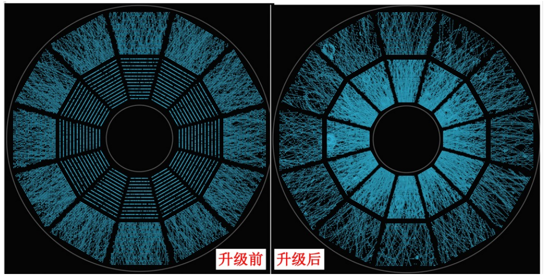
Other hyperons (Ξ, Ω)



J. Adam *et al.*, PRL 126, 162301 (2021)

Intensive measurements by STAR at RHIC

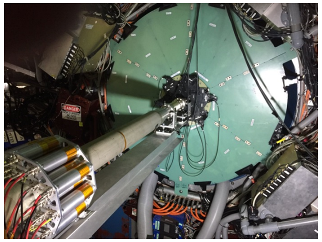
Beam energy scan (BES II)



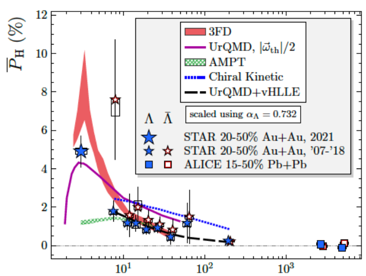
iTPC升级前后效果对比

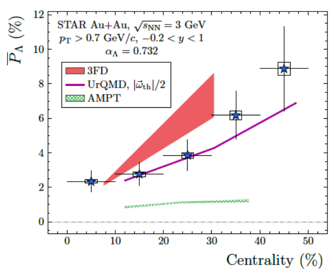
iTPC and EPD upgrades

更好的粒子分辨
（山大、科大、
上海应物所/复旦）

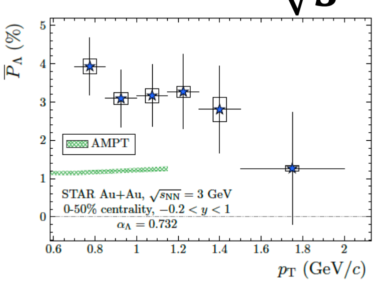


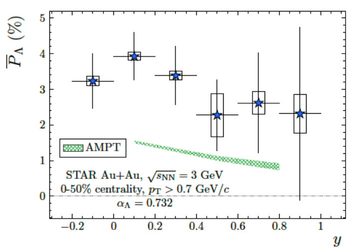
更好的平面确定（科大、清华）



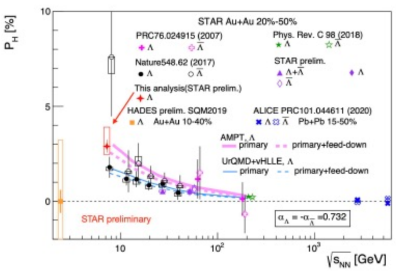


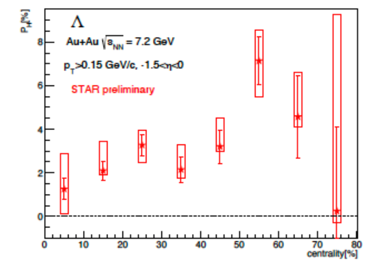
$\sqrt{s} = 3\text{GeV}$



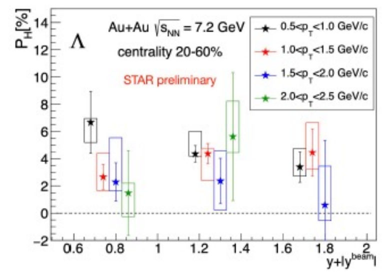


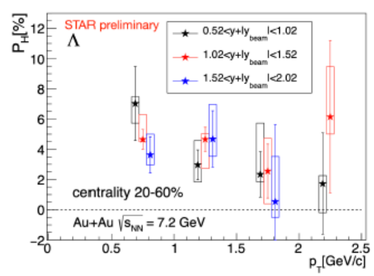
M.S. Abdallah *et al*., PRC 104, L061901 (2021)





$\sqrt{s} = 7.2\text{GeV}$

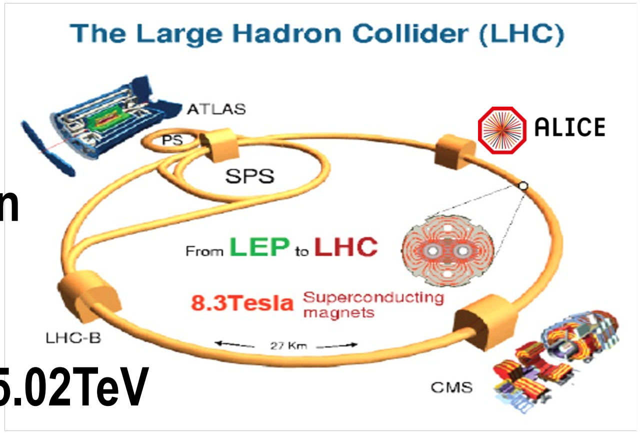


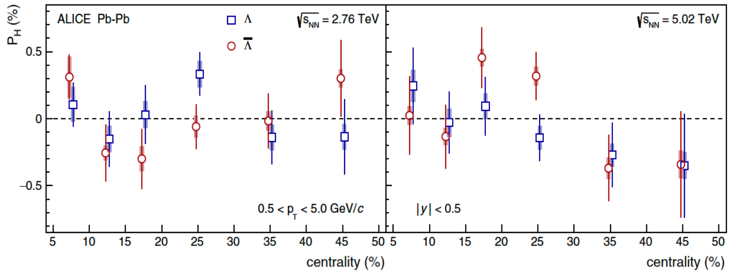


K. Okubo for STAR, 2108.10012 [nucl-ex]

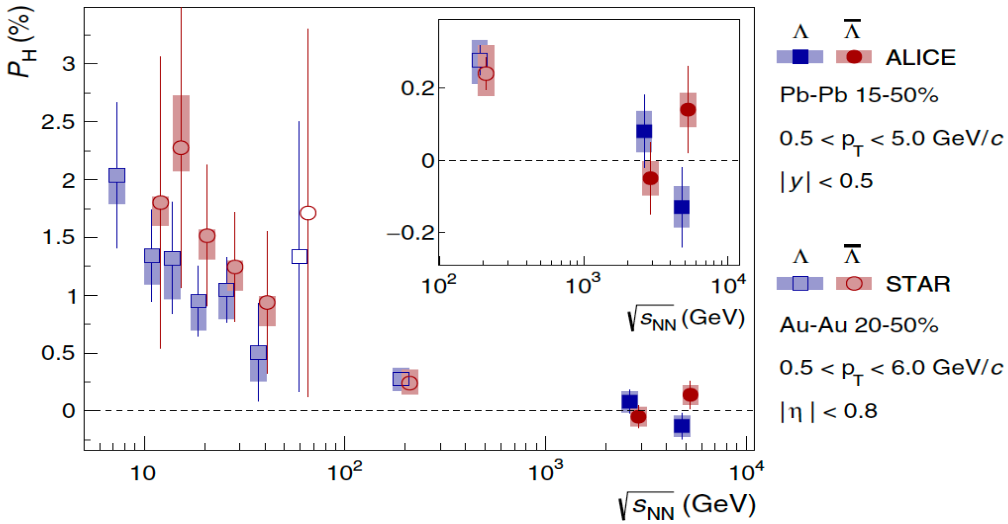
Further measurements by other experiments

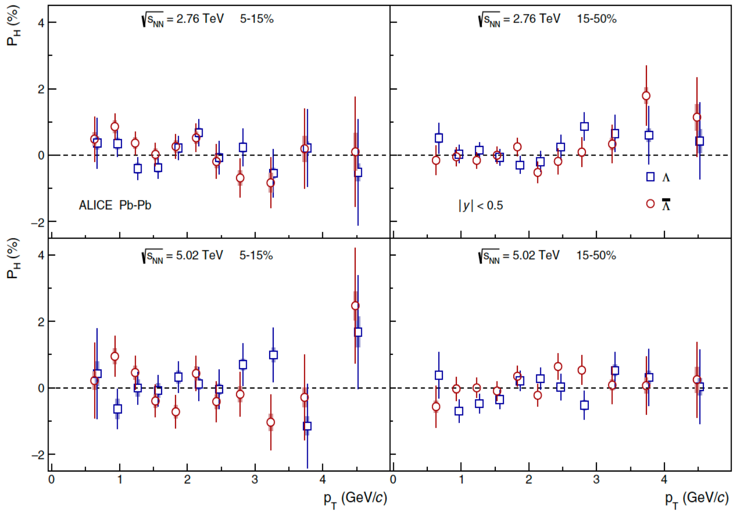
ALICE
Collaboration at LHC





Pb+Pb, $\sqrt{s}$ =2.76, 5.02TeV

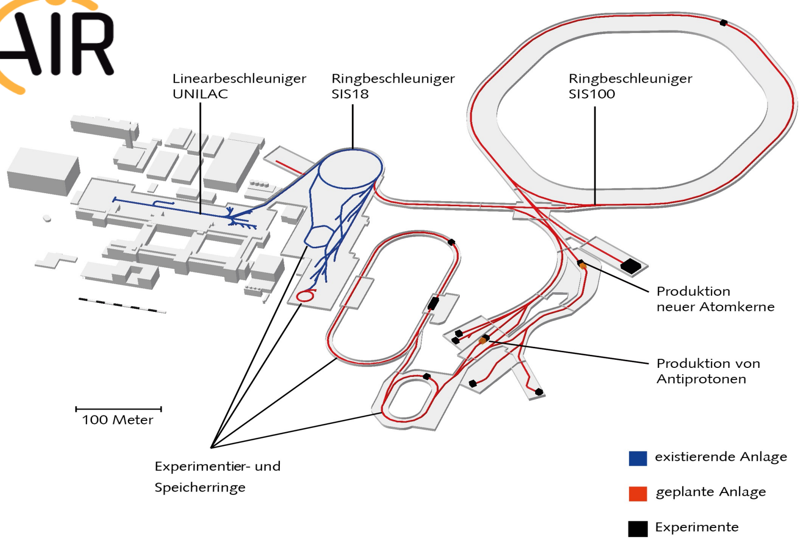


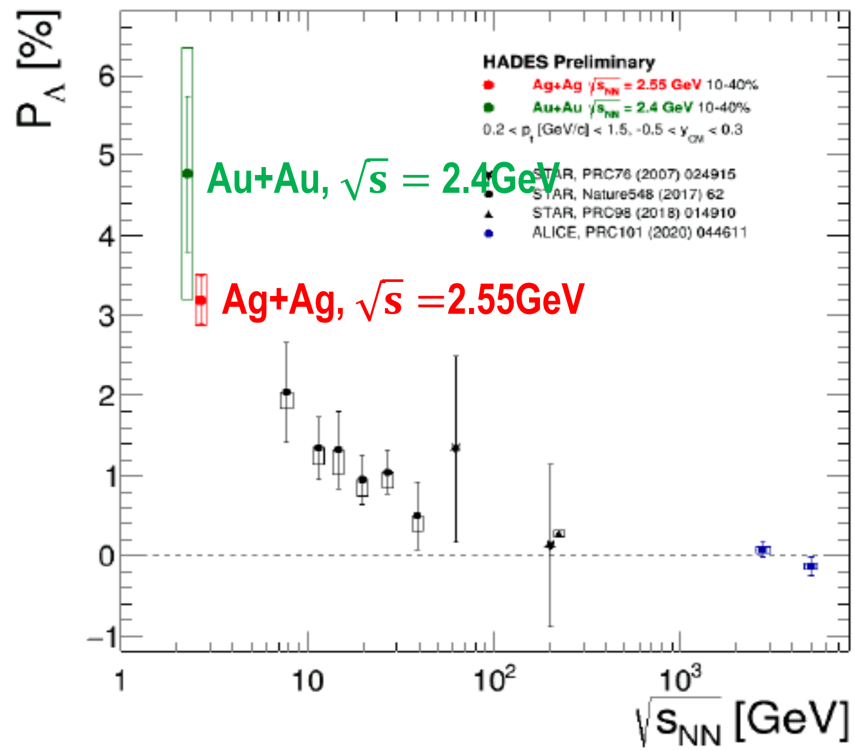


ALICE Collaboration, S. Acharya *et al.*, PRC 101, 044611 (2020)

Further measurements by other experiments

HADES at GSI





HADES Collaboration, R. Abou Yassine *et al.*, PLB 835, 137506 (2022)

Global polarization of Λ hyperon has been observed at different energies and decreases monotonically with increasing energies

Global vorticity and fit to the Global Λ Polarization

AMPT transport model

-- Li, Pang, Wang, Xia, PRC96, 054908(2017)

-- Wei, Deng, Huang, PRC99, 014905(2019)

UrQMD + vHLLE hydro

-- Karpenko, Becattini, EPJC 77, 213 (2017)

PICR hydro

-- Xie, Wang, Csernai, PRC 95, 031901 (2017)

Chiral Kinetic Equation + Collisions

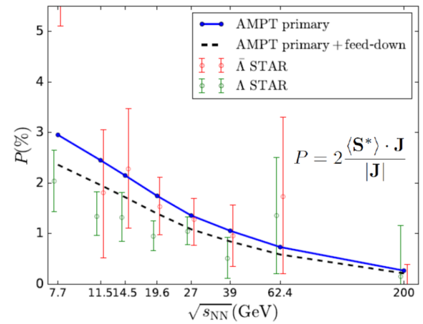
-- Sun, Ko, PRC96, 024906 (2017)

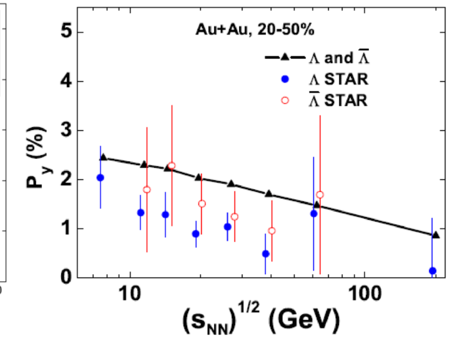
-- Liu, Sun, Ko, PRL125, 062301 (2020)

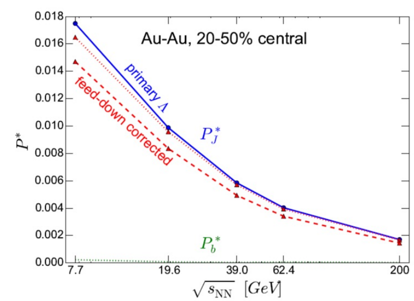
AVE+3FD

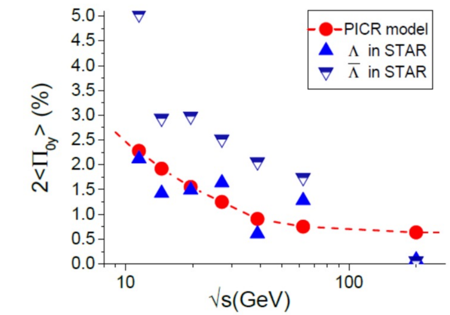
-- Ivanov, 2006.14328

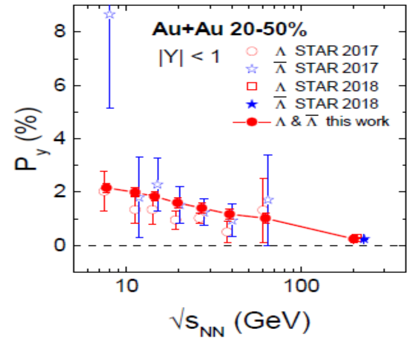
Other works ……

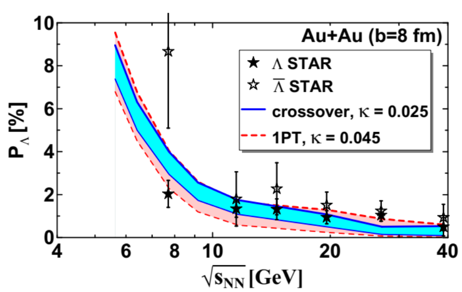








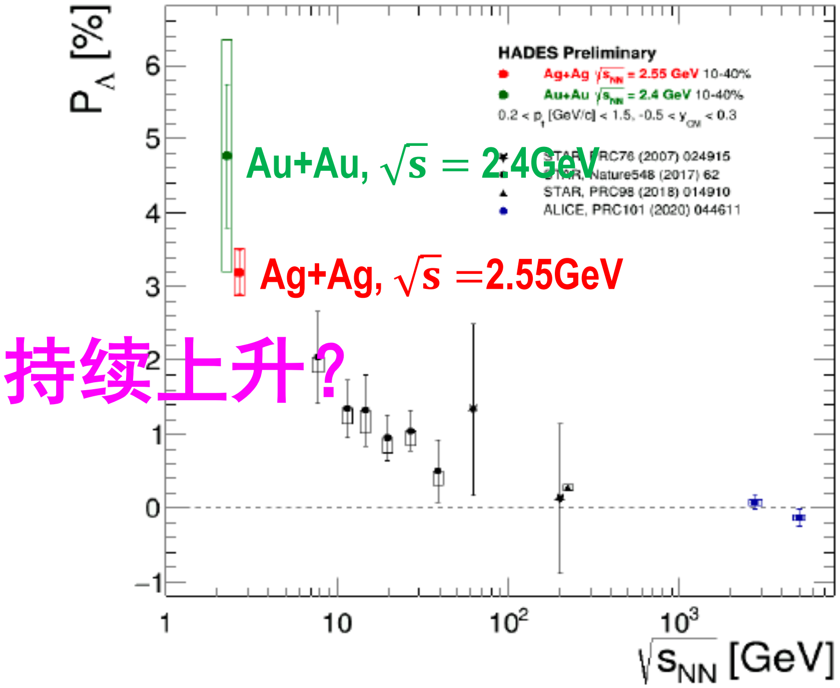


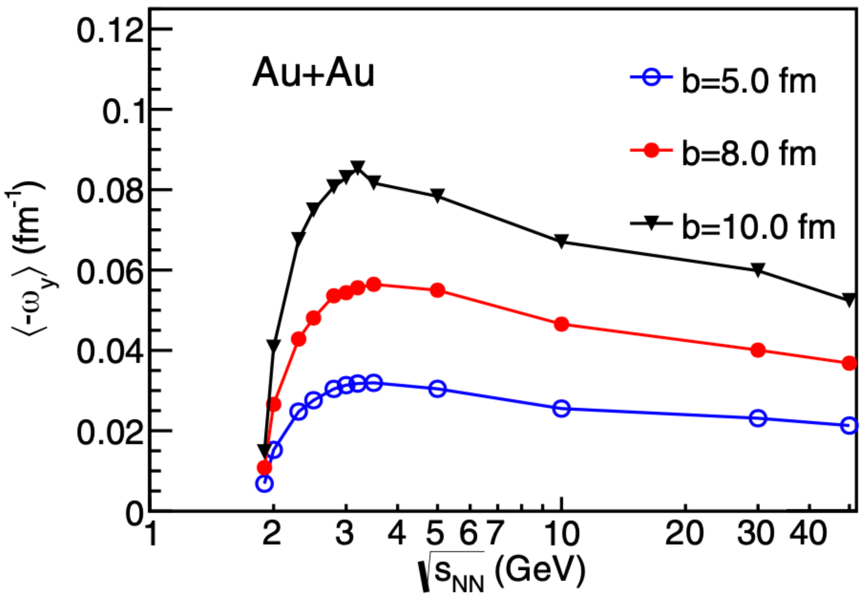


ppt from Huang Xu-guang, plenary talk at QM2019

热点问题一：低能区表现行为，持续上升？

超子整体极化效应被实验普遍证实，且随能量单调变化



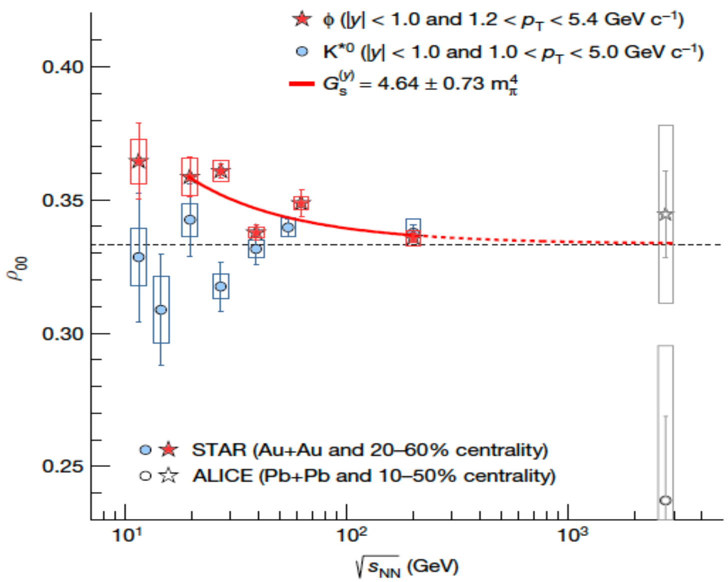


复旦大学组 **X. Deng, X. Huang, Y.G. Ma, S. Zhang, PRC 101, 064908 (2020);**
X. Deng, X. Huang, Y.G. Ma, PLB 835, 137560 (2022).

Global vector meson spin alignment —— experiments

STAR The STAR Collaboration

中国STAR组，来自复旦大学、
中国科学院近代物理研究所等
单位多位学者是主要作者



又一次在《Nature》发表！

M.S. Abdallah *et al*., Nature 614, 244 (2023)

Article

Pattern of global spin alignment of ϕ and K*0 mesons in heavy-ion collisions

- 确认矢量介子整体自旋排列
- $\left|\rho_{00}^{V} - \frac{1}{3}\right| \gg P_{\Lambda}^{2} \sim P_{q}^{2}$

Global vector meson spin alignment —— analysis

ZTL & Xin-Nian Wang, PRL 94, 102301 (2005).

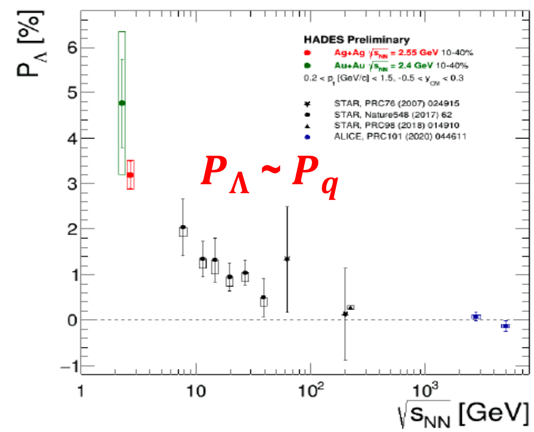
Hyperon polarization: $q_1^{\uparrow} + q_2^{\uparrow} + q_3^{\uparrow} \to H$

$$\hat{\rho}_{q_1 q_2 q_3} = \hat{\rho}_{q_1} \otimes \hat{\rho}_{q_2} \otimes \hat{\rho}_3$$

$$\hat{\rho}_q = \frac{1}{2}\begin{pmatrix} 1+P_q & 0 \\ 0 & 1-P_q \end{pmatrix}$$

$$P_H = P_{\bar{H}} = P_q$$

STAR experiments:

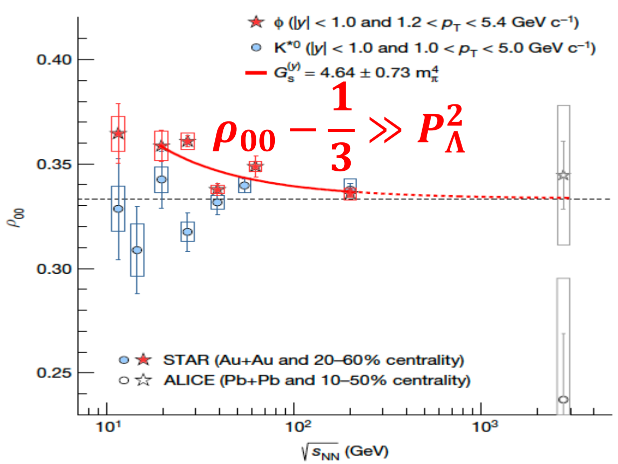


ZTL & Xin-Nian Wang, PLB 629, 20 (2005).

Vector meson spin alignment: $q_1^{\uparrow} + \bar{q}_2^{\uparrow} \to V$

$$\hat{\rho}_{q_1 \bar{q}_2} = \hat{\rho}_{q_1} \otimes \hat{\rho}_{\bar{q}_2}$$

$$\rho_{00}^V = \frac{1 - P_{q_1} P_{\bar{q}_2}}{3 + P_{q_1} P_{\bar{q}_2}} = \frac{1 - P_q^2}{3 + P_q^2} \sim \frac{1}{3}\left(1 - \frac{4}{3} P_q^2\right)$$



P_q was taken as a constant, no fluctuation, no other degree of freedom etc.

Global hyperon polarization

ZTL & Xin-Nian Wang, PRL 94, 102301 (2005).

Quark combination scenario $q_1^{\uparrow} + q_2^{\uparrow} + q_3^{\uparrow} \rightarrow H$

dominates at small and intermediate p_T

We took $\hat{\rho}_{q_1 q_2 q_3} = \hat{\rho}_{q_1} \otimes \hat{\rho}_{q_2} \otimes \hat{\rho}_{q_3}$ $\quad \hat{\rho}_q = \frac{1}{2}\begin{pmatrix} 1+P_q & 0 \\ 0 & 1-P_q \end{pmatrix}$

$$\rho_H(m, m') = \langle j_H, m' | \hat{\rho}_{q_1 q_2 q_3} | j_H, m \rangle$$

$$= \sum_{m_i, m'_i} \rho_{q_1 q_2 q_3}(m_i, m'_i) \langle j_H, m' | m'_1, m'_2, m'_3 \rangle \langle m_1, m_2, m_3 | j_H, m \rangle$$

C.G. coefficients

normalization

$$\rho_H(m, m') = \frac{\sum_{m_i, m'_i} \rho_{q_1 q_2 q_3}(m_i, m'_i) \langle j_H, m' | m'_1, m'_2, m'_3 \rangle \langle m_1, m_2, m_3 | j_H, m \rangle}{\sum_{m, m_i, m'_i} \rho_{q_1 q_2 q_3}(m_i, m'_i) \langle j_H, m | m'_1, m'_2, m'_3 \rangle \langle m_1, m_2, m_3 | j_H, m \rangle}$$

Global hyperon polarization

ZTL & Xin-Nian Wang, PRL 94, 102301 (2005).

Quark combination scenario $q_1^{\uparrow} + q_2^{\uparrow} + q_3^{\uparrow} \to H$

dominates at small and intermediate p_T

$$P_H = \rho_H\left(\frac{1}{2},\frac{1}{2}\right) - \rho_H\left(-\frac{1}{2},-\frac{1}{2}\right)$$

$$P_H = c_1 P_{q_1} + c_2 P_{q_2} + c_3 P_{q_3}$$

c_i's are constants determined by C.G. coefficients.

hyperon	Λ	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-
combination	P_s	$\frac{4P_u - P_s}{3}$	$\frac{2(P_u + P_d) - P_s}{3}$	$\frac{4P_d - P_s}{3}$	$\frac{4P_s - P_u}{3}$	$\frac{4P_s - P_d}{3}$

In the case that $P_u = P_d = P_s = P_{\bar{u}} = P_{\bar{d}} = P_{\bar{s}}$,

$P_H = P_{\bar{H}} = P_q$ for all H's and $\bar{H}'$s (global polarization)

Global vector meson spin alignment

ZTL & Xin-Nian Wang, PLB 629, 20 (2005).

Quark combination scenario $q_1^{\uparrow} + \bar{q}_2^{\uparrow} \to V$

$$\hat{\rho}_{q_1\bar{q}_2} = \hat{\rho}_{q_1} \otimes \hat{\rho}_{\bar{q}_2} \qquad \hat{\rho}_q = \frac{1}{2}\begin{pmatrix} 1+P_q & 0 \\ 0 & 1-P_q \end{pmatrix} \qquad \hat{\rho}_{\bar{q}} = \frac{1}{2}\begin{pmatrix} 1+P_{\bar{q}} & 0 \\ 0 & 1-P_{\bar{q}} \end{pmatrix}$$

$$\rho_V(m,m') = \frac{\sum_{m_i,m_i'} \rho_{q_1\bar{q}_2}(m_i,m_i') \langle j_V, m' | m_1', m_2' \rangle \langle m_1, m_2 | j_V, m \rangle}{\sum_{m,m_i,m_i'} \rho_{q_1\bar{q}_2}(m_i,m_i') \langle j_V, m | m_1', m_2' \rangle \langle m_1, m_2 | j_V, m \rangle}$$

$$\rho_{00}^V = \frac{1 - P_{q_1}P_{\bar{q}_2}}{3 + P_{q_1}P_{\bar{q}_2}} = \frac{1 - P_q^2}{3 + P_q^2}$$

spin alignment
自旋排列

$$\hat{\rho}^V = \begin{pmatrix} \rho_{11}^V & \rho_{10}^V & \rho_{1-1}^V \\ \rho_{01}^V & \rho_{00}^V & \rho_{0-1}^V \\ \rho_{-11}^V & \rho_{-10}^V & \rho_{-1-1}^V \end{pmatrix}$$

In both cases,

① took P_q as a constant, no fluctuation etc

② no quark spin correlations

③ considered only the spin degree of freedom

What does it change if we take other degrees of freedom into account?

Take other degrees of freedom into account

If we make a minimal step forward and consider other degrees of freedom denoted by α

The basis state for a quark: $|m, \alpha_q\rangle$

The element of the spin density matrix: $\langle m'_q, \alpha'_q|\hat{\rho}^{(q)}|m_q, \alpha_q\rangle \equiv \rho^{(q)}_{m_q,m'_q}(\alpha_q, \alpha'_q)$

We consider the simple case:

(1) $\hat{\rho}_q$ is diagonal w.r.t. α_q

$$\rho^{(q)}_{m_q,m'_q}(\alpha_q, \alpha'_q) = \rho^{(q)}_{m_q,m'_q}(\alpha_q)\delta_{\alpha_q,\alpha'_q} \qquad \hat{\rho}^{(q)}(\alpha_q) = \begin{pmatrix} \rho^{(q)}_{++}(\alpha_q) & \rho^{(q)}_{+-}(\alpha_q) \\ \rho^{(q)}_{-+}(\alpha_q) & \rho^{(q)}_{--}(\alpha_q) \end{pmatrix}$$

(2) $\hat{\rho}^{(q_1\bar{q}_2)}$ is taken as a direct product of $\hat{\rho}^{(q_1)}$ and $\hat{\rho}^{(\bar{q}_2)}$

$$\hat{\rho}^{(q_1\bar{q}_2)}(\alpha_{q_1}, \alpha_{\bar{q}_2}) = \hat{\rho}^{(q_1)}(\alpha_{q_1}) \otimes \hat{\rho}^{(\bar{q}_2)}(\alpha_{\bar{q}_2})$$

(3) the hadron wavefunction is factorized

$$\langle \alpha_{q_1}, m_{q_1}; \alpha_{\bar{q}_2}, m_{\bar{q}_2}|j_V, m_V, \alpha_V\rangle = \langle \alpha_{q_1}, \alpha_{\bar{q}_2}|\alpha_V\rangle \langle m_{q_1}, m_{\bar{q}_2}|j_V, m_V\rangle$$

$$\Rightarrow \quad \rho^V_{mm'}(\alpha_V) = \sum_{\alpha_{q_1},\alpha_{\bar{q}_2}\in V} \left|\langle \alpha_{q_1}, \alpha_{\bar{q}_2}|\alpha_V\rangle\right|^2 \rho^{V(l)}_{mm'}(\alpha_{q_1}, \alpha_{\bar{q}_2})$$

average inside V

Take other degrees of freedom into account

In this way, we obtain

$$\rho_{00}^{V}(\alpha_V) = \frac{1 - \langle P_{q_1} P_{\bar{q}_2} \rangle_V}{3 + \langle P_{q_1} P_{\bar{q}_2} \rangle_V}$$

average inside V

$$\langle P_{q_1} P_{\bar{q}_2} \rangle_V = \sum_{\alpha_{q_1}, \alpha_{\bar{q}_2} \in V} |\langle \alpha_{q_1}, \alpha_{\bar{q}_2} | \alpha_V \rangle|^2 P_{q_1}(\alpha_{q_1}) P_{\bar{q}_2}(\alpha_{\bar{q}_2})$$

Further average over α_V

$$\langle \rho_{00}^{V} \rangle = \frac{1 - \langle P_{q_1} P_{\bar{q}_2} \rangle}{3 + \langle P_{q_1} P_{\bar{q}_2} \rangle}$$

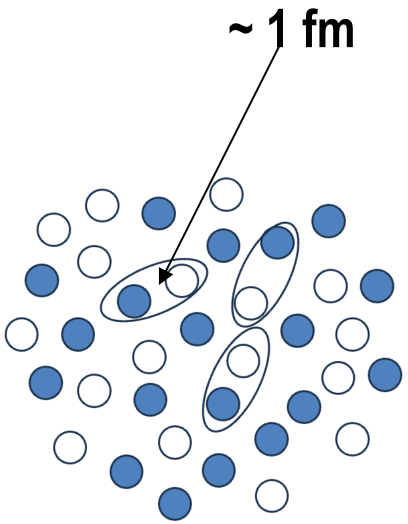
$$\langle P_{q_1} P_{\bar{q}_2} \rangle = \sum_{\alpha_V} f_V(\alpha_V) \langle P_{q_1} P_{\bar{q}_2} \rangle_V$$

The average is two folded:

$$\langle P_{q_1} P_{\bar{q}_2} \rangle = \langle \langle P_{q_1} P_{\bar{q}_2} \rangle_V \rangle_S$$

average inside the vector meson V

average over the system or a sub-system S



Hyperon polarization v.s. vector meson spin alignment

For $q_1^{\uparrow} + \bar{q}_2^{\uparrow} \to V$

$$\rho_{00}^{V} = \frac{1 - \langle P_{q_1} P_{\bar{q}_2} \rangle}{3 + \langle P_{q_1} P_{\bar{q}_2} \rangle}$$

For $q_1^{\uparrow} + q_2^{\uparrow} + q_3^{\uparrow} \to H$

$$P_H = \left\langle \left\langle c_1 P_{q_1} + c_2 P_{q_2} + c_3 P_{q_3} \right\rangle_H \right\rangle_S$$

$$= c_1 \langle P_{q_1} \rangle + c_2 \langle P_{q_1} \rangle + c_3 \langle P_{q_1} \rangle$$

The STAR data show that: $\langle P_q P_{\bar{q}} \rangle \neq \langle P_q \rangle \langle P_{\bar{q}} \rangle$ i.e. there is correlation between P_q and $P_{\bar{q}}$.

By studying P_H, we study the average of quark polarization P_q;
by studying ρ_{00}^{V}, we study the correlation between P_q and $P_{\bar{q}}$.

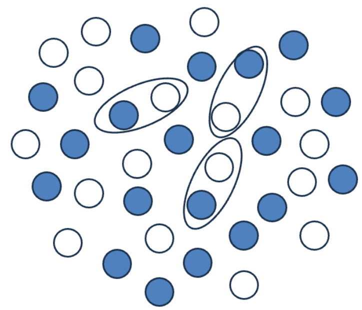
A window to study quark spin correlation in QGP

Local correlation or long range correlation

One has to take correlations into account, i.e.,: $\langle P_q P_{\bar{q}} \rangle \neq \langle P_q \rangle \langle P_{\bar{q}} \rangle$

(1) local correlation:

$$\langle P_q P_{\bar{q}} \rangle_V \neq \langle P_q \rangle_V \langle P_{\bar{q}} \rangle_V$$



$$\langle P_q P_{\bar{q}} \rangle = \left\langle \langle P_q P_{\bar{q}} \rangle_V \right\rangle_S$$

average inside the vector meson V

average over the system S

(2) long range correlation:

$$\langle P_q P_{\bar{q}} \rangle_V = \langle P_q \rangle_V \langle P_{\bar{q}} \rangle_V \qquad \left\langle \langle P_q \rangle_V \langle P_{\bar{q}} \rangle_V \right\rangle_S \neq \left\langle \langle P_q \rangle_V \right\rangle_S \left\langle \langle P_{\bar{q}} \rangle_V \right\rangle_S$$

One needs also take the off-diagonal components into account

$$\hat{\rho}_q = \frac{1}{2} \begin{pmatrix} 1 + P_{qy} & P_{qz} - iP_{qx} \\ P_{qz} + iP_{qx} & 1 - P_{qy} \end{pmatrix}$$

Quark spin correlations in QGP

Systematic studies:

① Systematic description of quark spin correlations in QGP

② Relationships between measurable quantities and those describing quark spin correlations

③ Numerical estimations?

Ji-peng Lv, Zi-han Yu, ZTL, Qun Wang, Xin-Nian Wang, e-Print: 2402.13721 [hep-ph]

Description of quark spin correlations in QGP

For single particle, the complete set of 2x2 Hermitian matrices: $\mathbb{I}, \hat{\sigma}_i$

$$\hat{\rho}^{(1)} = \frac{1}{2}(1 + P_{1i}\hat{\sigma}_{1i}) \qquad P_{1i} = \langle \hat{\sigma}_{1i} \rangle = \mathrm{Tr}[\hat{\rho}^{(1)}\hat{\sigma}_{1i}]$$

For two particle system (12), the complete set: $\mathbb{I}_1 \otimes \mathbb{I}_2$, $\hat{\sigma}_{1i} \otimes \mathbb{I}_2$, $\mathbb{I}_1 \otimes \hat{\sigma}_{2i}$, $\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j}$

we used to decompose $\hat{\rho}^{(12)} = \frac{1}{2^2}\left(\mathbb{I}_1 \otimes \mathbb{I}_2 + P_{1i}\hat{\sigma}_{1i} \otimes \mathbb{I}_2 + P_{2i}\mathbb{I}_1 \otimes \hat{\sigma}_{2i} + t_{ij}^{(12)}\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j}\right)$

$t_{ij}^{(12)} = \langle \hat{\sigma}_{1i}\hat{\sigma}_{2j} \rangle$ shortage: $t_{ij}^{(12)} = P_{1i}P_{2j} \neq 0$ if $\hat{\rho}^{(12)} = \hat{\rho}^{(1)} \otimes \hat{\rho}^{(2)}$

we propose

$$\hat{\rho}^{(12)} = \hat{\rho}^{(1)} \otimes \hat{\rho}^{(2)} + \frac{1}{2^2}c_{ij}^{(12)}\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j}$$

$$c_{ij}^{(12)} = \langle \hat{\sigma}_{1i}\hat{\sigma}_{2j} \rangle - \langle \hat{\sigma}_{1i} \rangle \langle \hat{\sigma}_{2j} \rangle \qquad c_{ij}^{(12)} = 0 \text{ if } \hat{\rho}^{(12)} = \hat{\rho}^{(1)} \otimes \hat{\rho}^{(2)}$$

For three particle system (123)

$$\hat{\rho}^{(123)} = \hat{\rho}^{(1)} \otimes \hat{\rho}^{(2)} \otimes \hat{\rho}^{(3)} + \frac{1}{2^2}\left[c_{ij}^{(12)}\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j} \otimes \hat{\rho}^{(3)} + c_{jk}^{(23)}\hat{\rho}^{(1)} \otimes \hat{\sigma}_{2j} \otimes \hat{\sigma}_{3k} + c_{ik}^{(13)}\hat{\sigma}_{1i} \otimes \hat{\rho}^{(2)} \otimes \hat{\sigma}_{3k}\right] + \frac{1}{2^3}c_{ijk}^{(123)}\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j} \otimes \hat{\sigma}_{3k}$$

Description of quark spin correlations in QGP

Consider α-dependence

For single particle

$$\hat{\rho}^{(1)}(\alpha) = \frac{1}{2}[1 + P_{1i}(\alpha)\hat{\sigma}_{1i}] \qquad P_{1i} = \langle\hat{\sigma}_{1i}\rangle = \mathrm{Tr}[\hat{\rho}^{(1)}(\alpha)\hat{\sigma}_{1i}]$$

For two particle system (12)

$$\hat{\rho}^{(12)}(\alpha_1,\alpha_2) = \hat{\rho}^{(1)}(\alpha_1) \otimes \hat{\rho}^{(2)}(\alpha_2) + \frac{1}{2^2} c_{ij}^{(12)}(\alpha_1,\alpha_2)\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j}$$

For three particle system (123)

$$\hat{\rho}^{(123)}(\alpha_1,\alpha_2,\alpha_3) = \hat{\rho}^{(1)}(\alpha_1) \otimes \hat{\rho}^{(2)}(\alpha_2) \otimes \hat{\rho}^{(3)}(\alpha_3) + \frac{1}{2^2}\left[c_{ij}^{(12)}(\alpha_1,\alpha_2)\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j} \otimes \hat{\rho}^{(3)}(\alpha_3) + \cdots\right] + \frac{1}{2^3} c_{ijk}^{(123)}(\alpha_1,\alpha_2,\alpha_3)\hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j} \otimes \hat{\sigma}_{3k}$$

Description of quark spin correlations in QGP

Suppose a system A consisting of (12) at α_{12} with wave function $|\alpha_{12}\rangle$

The α_{12}-dependent spin density matrix for A=(12) is

$$\hat{\bar{\rho}}^{(12)}(\alpha_{12}) = \langle\alpha_{12}|\,\hat{\rho}^{(12)}(\alpha_1,\alpha_2)|\alpha_{12}\rangle = \sum_{\alpha_1\alpha_2} |\langle\alpha_1,\alpha_2|\alpha_{12}\rangle|^2\,\hat{\rho}^{(12)}(\alpha_1,\alpha_2)$$

average inside A=(12), denoted by $\langle\ldots\rangle_A$

We decompose $\hat{\bar{\rho}}^{(12)}(\alpha_{12}) = \hat{\bar{\rho}}^{(1)}(\alpha_{12}) \otimes \hat{\bar{\rho}}^{(2)}(\alpha_{12}) + \frac{1}{2^2}\bar{c}_{ij}^{(12)}(\alpha_{12})\,\hat{\sigma}_{1i}\otimes\hat{\sigma}_{2j}$

$$\hat{\bar{\rho}}^{(1)}(\alpha_{12}) = \langle\alpha_{12}|\hat{\rho}^{(1)}(\alpha_1)|\alpha_{12}\rangle = \frac{1}{2}[1+\bar{P}_{1i}(\alpha_{12})\hat{\sigma}_{1i}]$$

The polarization $\bar{P}_{1i}(\alpha_{12}) = \sum_{\alpha_1\alpha_2} |\langle\alpha_1,\alpha_2|\alpha_{12}\rangle|^2\, P_{1i}(\alpha_1) \equiv \langle P_{1i}(\alpha_1)\rangle$

just equals to P_{1i} averaged inside A=(12)

However $\bar{c}_{ij}^{(12)}(\alpha_{12}) = \left\langle c_{ij}^{(12)}(\alpha_1,\alpha_2) + P_{1i}(\alpha_1)P_{2j}(\alpha_2)\right\rangle - \langle P_{1i}(\alpha_1)\rangle\langle P_{1i}(\alpha_1)\rangle$

$$\neq \left\langle c_{ij}^{(12)}(\alpha_1,\alpha_2)\right\rangle$$

does not equal to $c_{ij}^{(12)}$ averaged inside A=(12)

Description of quark spin correlations in QGP

The initial α_i-dependent $\hat{\rho}^{(12)}(\alpha_1, \alpha_2)$

$$\hat{\rho}^{(12)}(\alpha_1, \alpha_2) = \hat{\rho}^{(1)}(\alpha_1) \otimes \hat{\rho}^{(2)}(\alpha_2) + \frac{1}{2^2} c_{ij}^{(12)}(\alpha_1, \alpha_2)\, \hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j}$$

genuine correlation

The α_{12}-dependent $\hat{\bar{\rho}}^{(12)}(\alpha_{12})$, averaged inside (12)

$$\hat{\bar{\rho}}^{(12)}(\alpha_{12}) = \hat{\bar{\rho}}^{(1)}(\alpha_{12}) \otimes \hat{\bar{\rho}}^{(2)}(\alpha_{12}) + \frac{1}{2^2} \bar{c}_{ij}^{(12)}(\alpha_{12})\, \hat{\sigma}_{1i} \otimes \hat{\sigma}_{2j}$$

effective correlation

$$\bar{c}_{ij}^{(12)}(\alpha_{12}) = \left\langle c_{ij}^{(12)}(\alpha_1, \alpha_2) \right\rangle + \langle P_{1i}(\alpha_1) P_{2j}(\alpha_2) \rangle - \langle P_{1i}(\alpha_1) \rangle \langle P_{1i}(\alpha_1) \rangle$$

If $c_{ij}^{(12)}(\alpha_1, \alpha_2) = 0$, i.e. $\hat{\rho}^{(12)}(\alpha_1, \alpha_2) = \hat{\rho}^{(1)}(\alpha_1) \otimes \hat{\rho}^{(2)}(\alpha_2)$

induced correlation

$$\bar{c}_{ij}^{(12)}(\alpha_{12}) = \bar{c}_{ij}^{(12;0)}(\alpha_{12}) = \langle P_{1i}(\alpha_1) P_{2j}(\alpha_2) \rangle - \langle P_{1i}(\alpha_1) \rangle \langle P_{1i}(\alpha_1) \rangle$$

$$\bar{c}_{ij}^{(12)}(\alpha_{12}) = \langle c_{ij}^{(12)}(\alpha_1, \alpha_2) \rangle + \bar{c}_{ij}^{(12;0)}(\alpha_{12})$$

The effective correlation = the genuine correlation averaged + the induced correlation

Spin density matrix for V from quark combination

For $q_1 + \bar{q}_2 \rightarrow V$ $\quad \hat{\rho}^V = \hat{\mathcal{M}}\hat{\rho}^{(q_1\bar{q}_2)}\hat{\mathcal{M}}^\dagger$ $\quad \hat{\mathcal{M}}$: transition matrix

If only spin degree of freedom is considered

$$\rho^V_{mm'} = \langle jm|\hat{\mathcal{M}}\hat{\rho}^{(q_1\bar{q}_2)}\hat{\mathcal{M}}^\dagger|jm'\rangle$$

$$= \sum_{m_1 m_2, m'_1 m'_2} \langle jm|\hat{\mathcal{M}}|m_1 m_2\rangle\langle m_1 m_2|\hat{\rho}^{(q_1\bar{q}_2)}|m'_1 m'_2\rangle\langle m'_1 m'_2|\hat{\mathcal{M}}^\dagger|jm'\rangle$$

$$= N \sum_{m_1 m_2, m'_1 m'_2} \langle jm|m_1 m_2\rangle\langle m_1 m_2|\hat{\rho}^{(q_1\bar{q}_2)}|m'_1 m'_2\rangle\langle m'_1 m'_2|jm'\rangle$$

$$\langle jm|\hat{\mathcal{M}}|m_1 m_2\rangle = \sum_{j'm'} \langle jm|\hat{\mathcal{M}}|j'm'\rangle\langle j'm'|m_1 m_2\rangle$$

$$= \langle jm|\hat{\mathcal{M}}|jm\rangle\langle jm|m_1 m_2\rangle$$

$$= N_j\langle jm|m_1 m_2\rangle$$

angular momentum conservation
$j = j', m = m'$

space rotation invariance demands
$\langle jm|\hat{\mathcal{M}}|jm\rangle$ is independent of m

Spin density matrix for V from quark combination

The vector meson spin alignment

$$\rho_{00}^{V}=\frac{1-c_{yy}^{(q_1\bar{q}_2)}+c_{zz}^{(q_1\bar{q}_2)}+c_{xx}^{(q_1\bar{q}_2)}+\vec{P}_{q_1}\cdot\vec{P}_{\bar{q}_2}-2P_{q_1y}P_{\bar{q}_2y}}{3+c_{ii}^{(q_1\bar{q}_2)}+\vec{P}_{q_1}\cdot\vec{P}_{\bar{q}_2}}$$

$$c_{ii}^{(q_1\bar{q}_2)}=c_{xx}^{(q_1\bar{q}_2)}+c_{yy}^{(q_1\bar{q}_2)}+c_{zz}^{(q_1\bar{q}_2)}$$

strongly depends on the quark-anti-quark spin correlations.

$$\rho_{00}^{V}\longrightarrow\frac{1-P_{qy}P_{\bar{q}y}}{3+P_{qy}P_{\bar{q}y}}\qquad \text{if } c_{xx}^{(q_1\bar{q}_2)}=c_{yy}^{(q_1\bar{q}_2)}=c_{zz}^{(q_1\bar{q}_2)}=0$$

Spin density matrix for V from quark combination

also the off-diagonal elements

$$\mathrm{Re}\ \rho^V_{10} = \frac{c^{(q_1\bar{q}_2)}_{yz} + c^{(q_1\bar{q}_2)}_{zy} + P_{q_1z}(1 + P_{\bar{q}_2y}) + (1 + P_{q_1y})P_{\bar{q}_2z}}{\sqrt{2}(3 + c^{(q_1\bar{q}_2)}_{ii} + \vec{P}_{q_1} \cdot \vec{P}_{\bar{q}_2})}$$

$$\mathrm{Im}\ \rho^V_{01} = \frac{c^{(q_1\bar{q}_2)}_{xy} + c^{(q_1\bar{q}_2)}_{yx} + P_{q_1x}\left(1 + P_{\bar{q}_2y}\right) + (1 + P_{q_1y})P_{\bar{q}_2x}}{\sqrt{2}(3 + c^{(q_1\bar{q}_2)}_{ii} + \vec{P}_{q_1} \cdot \vec{P}_{\bar{q}_2})}$$

$$\mathrm{Re}\ \rho^V_{0-1} = \frac{c^{(12)}_{yz} + c^{(12)}_{zy} + P_{qz}(1 - P_{\bar{q}y}) + (1 - P_{qy})P_{\bar{q}z}}{\sqrt{2}(3 + c^{(q_1\bar{q}_2)}_{ii} + \vec{P}_{q_1} \cdot \vec{P}_{\bar{q}_2})}$$

$$\mathrm{Im}\ \rho^V_{-10} = \frac{c^{(12)}_{xy} + c^{(12)}_{yx} + P_{qx}\left(1 - P_{\bar{q}y}\right) + (1 - P_{qy})P_{\bar{q}x}}{\sqrt{2}(3 + c^{(q_1\bar{q}_2)}_{ii} + \vec{P}_{q_1} \cdot \vec{P}_{\bar{q}_2})}$$

$$\mathrm{Re}\ \rho^V_{1-1} = \frac{c^{(12)}_{zz} - c^{(q_1\bar{q}_2)}_{xx} + P_{q_1z}P_{\bar{q}_2z} - P_{q_1x}P_{\bar{q}_2x}}{3 + c^{(q_1\bar{q}_2)}_{ii} + \vec{P}_{q_1} \cdot \vec{P}_{\bar{q}_2}}$$

$$\mathrm{Im}\ \rho^V_{-11} = \frac{c^{(q_1\bar{q}_2)}_{yz} + c^{(q_1\bar{q}_2)}_{zy} + P_{q_1z}P_{\bar{q}_2x} + P_{q_1x}P_{\bar{q}_2x}}{3 + c^{(q_1\bar{q}_2)}_{ii} + \vec{P}_{q_1} \cdot \vec{P}_{\bar{q}_2}}$$

Hyperon polarization from quark combination

Hyperon polarization

the simplest case: Lambda polarization

$$P_\Lambda = P_{sz} - \frac{c_{iiz}^{(uds)} + c_{iz}^{(us)} P_{di} + c_{iz}^{(ds)} P_{ui}}{1 - c_{ii}^{(ud)} - \vec{P}_u \cdot \vec{P}_d} \dashrightarrow P_{sz}$$

- there are contributions from quark-quark spin correlations
- these contributions are proportional to products of quark-quark spin correlation and quark polarization.

Hyperon polarization from quark combination

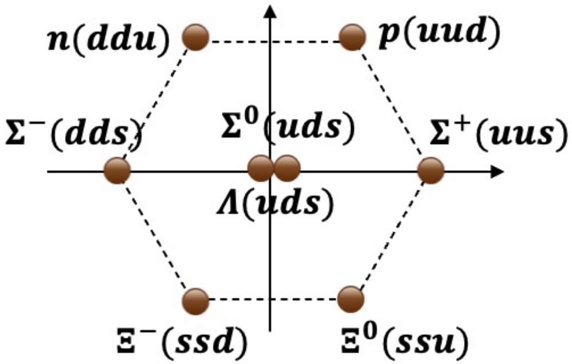
for other $J^P = \frac{1}{2}^+$ octet baryons

for flavor content of the type $H = (aab)$ baryons: $P_H = \frac{1}{3}(4P_{az} - P_{bz}) + \frac{\delta A_H}{B_H}$

$$\delta A_{H_{aab}} = -\frac{4}{3}(\vec{P}_a^2 - \vec{P}_a \cdot \vec{P}_b + c_{ii}^{(aa)} - c_{ii}^{(ab)})(P_{az} - P_{bz})$$

$$-4c_{iz}^{(aa)} P_{bi} + 2\left(c_{iz}^{(ab)} - 2c_{zi}^{(ab)}\right) P_{ai} + c_{iiz}^{(aab)} - 4c_{zii}^{(aab)}$$

$$B_{H_{aab}} = 3 + \vec{P}_a^2 - 4\vec{P}_a \cdot \vec{P}_b + c_{ii}^{(aa)} - 4c_{ii}^{(ab)}$$



$$P_{\Sigma^0} = \frac{1}{3}(2P_{uz} + 2P_{dz} - P_{sz}) + \frac{\delta A_{\Sigma^0}}{B_{\Sigma^0}}$$

$$\delta A_{\Sigma^0} = -\frac{2}{3}(\vec{P}_u \cdot \vec{P}_d + c_{ii}^{(ud)})(P_{uz} + P_{dz} - 2P_{sz})$$

$$+\frac{2}{3}(\vec{P}_u \cdot \vec{P}_s + c_{ii}^{(us)})(2P_{uz} - P_{dz} - P_{sz}) + (2c_{zi}^{(us)} - c_{iz}^{(us)})P_{di} + (u \leftrightarrow d)$$

$$-2(c_{zi}^{(ud)} + c_{iz}^{(us)})P_{si} + c_{iiz}^{(uds)} - 2c_{izi}^{(uds)} - 2c_{zii}^{(uds)}$$

$$B_{\Sigma^0} = 3 + \vec{P}_u \cdot \vec{P}_d + c_{ii}^{(ud)} - 2[\vec{P}_s \cdot \vec{P}_d + c_{ii}^{(sd)}] - 2[u \leftrightarrow d]$$

They all contain contributions from quark-quark correlations.

Spin density matrix for V from quark combination

If α-dependence is considered: $\hat{\rho}^V(\alpha_V) = \hat{\mathcal{M}}\hat{\rho}^{(q_1\bar{q}_2)}(\alpha_1,\alpha_2)\hat{\mathcal{M}}^\dagger$

$$\rho^V_{mm'}(\alpha_V) = \langle jm,\alpha_V|\hat{\mathcal{M}}\hat{\rho}^{(q_1\bar{q}_2)}(\alpha_1,\alpha_2)\hat{\mathcal{M}}^\dagger|jm',\alpha_V\rangle$$

$$= N\sum_{m_1m_2m'_1m'_2}\langle jm|m_1m_2\rangle\langle m_1m_2|\hat{\bar{\rho}}^{(q_1\bar{q}_2)}(\alpha_V)|m'_1m'_2\rangle\langle m'_1m'_2|jm'\rangle$$

$$\hat{\bar{\rho}}^{(q_1\bar{q}_2)}(\alpha_V) = \sum_{\alpha_1\alpha_2\in V}\hat{\rho}^{(q_1\bar{q}_2)}(\alpha_1,\alpha_2)\,|\langle\alpha_1,\alpha_2|\alpha_V\rangle|^2$$

if the wavefunction is factorized, i.e., $|jm,\alpha_V\rangle = |jm\rangle|\alpha_V\rangle$

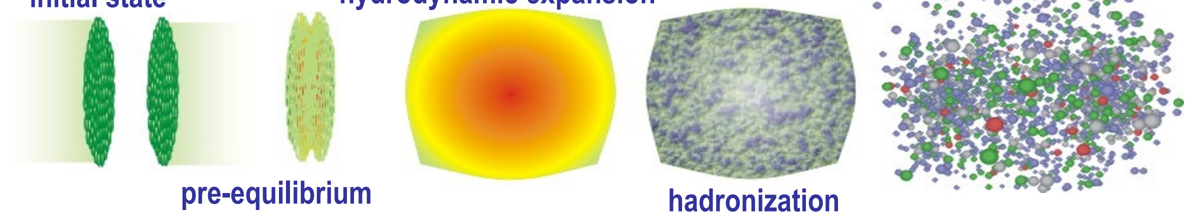
$$\rho^V_{00}(\alpha_V) = \frac{1+\bar{c}^{(q_1\bar{q}_2)}_{zz} + \bar{c}^{(q_1\bar{q}_2)}_{xx} - \bar{c}^{(q_1\bar{q}_2)}_{yy} + \vec{\bar{P}}_{q_1}\cdot\vec{\bar{P}}_{\bar{q}_2} - 2\bar{P}_{q_1y}\bar{P}_{\bar{q}_2y}}{3+\bar{c}^{(q_1\bar{q}_2)}_{ii} + \vec{\bar{P}}_{q_1}\cdot\vec{\bar{P}}_{\bar{q}_2}}$$

replaced by the corresponding effective quantities

Sensitive to the local correlations between q_1 and $\bar{q}_2$.

Vector meson spin alignment —— example

strong indication of phi-meson filed ⟹ strong local correlation



Strong interaction exhibits itself differently in different stages

$$P_s^{\mu}(x,p) \approx \frac{1}{4m_s}\varepsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} + \frac{g_\phi}{(u\cdot p)T_h}F^{\phi}_{\rho\sigma}\right)p_\nu$$

$$P_{\bar{s}}^{\mu}(x,p) \approx \frac{1}{4m_s}\varepsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} - \frac{g_\phi}{(u\cdot p)T_h}F^{\phi}_{\rho\sigma}\right)p_\nu$$

Strong phase space dependence leads to strong local correlation between P_s and $P_{\bar{s}}$

induced quark spin correlations !

[1] Xin-Li Sheng, Lucia Oliva, ZTL, Qun Wang and Xin-Nian Wang, PRL 131, 042304 (2023).
[2] Xin-Li Sheng, Lucia Oliva, ZTL, Qun Wang and Xin-Nian Wang, PRD 109 (2024) 3, 036004.

Hyperon spin correlation

Hyperon-anti-hyperon spin correlation

$$c_{NN}^{H_1\bar{H}_2} = \frac{N_{++}^{H_1\bar{H}_2} + N_{--}^{H_1\bar{H}_2} - N_{+-}^{H_1\bar{H}_2} - N_{-+}^{H_1\bar{H}_2}}{N_{++}^{H_1\bar{H}_2} + N_{--}^{H_1\bar{H}_2} + N_{+-}^{H_1\bar{H}_2} + N_{-+}^{H_1\bar{H}_2}}$$

$$\hat{\rho}^{H_1\bar{H}_2} = \widehat{\mathcal{M}}\hat{\rho}^{(q_1q_2q_3\bar{q}_4\bar{q}_5\bar{q}_6)}\widehat{\mathcal{M}}^{\dagger}$$

We need to consider the 6 body system

$$\hat{\rho}^{(1-6)} = \hat{\rho}^{(1)}\otimes\hat{\rho}^{(2)}\otimes\hat{\rho}^{(3)}\otimes\hat{\rho}^{(4)}\otimes\hat{\rho}^{(5)}\otimes\hat{\rho}^{(6)}$$

$$+\frac{1}{2^2}[c_{ij}^{(12)}\hat{\sigma}_{1i}\otimes\hat{\sigma}_{2j}\otimes\hat{\rho}^{(3)}\otimes\hat{\rho}^{(4)}\otimes\hat{\rho}^{(5)}\otimes\hat{\rho}^{(6)} \text{ + 14 exchange terms }]$$ 2-spin correlations

$$+\frac{1}{2^3}[c_{ijk}^{(123)}\hat{\sigma}_{1i}\otimes\hat{\sigma}_{2j}\otimes\hat{\sigma}_{3k}\otimes\hat{\rho}^{(4)}\otimes\hat{\rho}^{(5)}\otimes\hat{\rho}^{(6)} \text{ + 19 exchange terms }]$$ 3-spin correlations

$$+\frac{1}{2^4}[c_{ijkl}^{(1234)}\hat{\sigma}_{1i}\otimes\hat{\sigma}_{2j}\otimes\hat{\sigma}_{3k}\otimes\hat{\sigma}_{4l}\otimes\hat{\rho}^{(5)}\otimes\hat{\rho}^{(6)} \text{ + 14 exchange terms }]$$ 4-spin correlations

$$+\frac{1}{2^5}[c_{ijklm}^{(12345)}\hat{\sigma}_{1i}\otimes\hat{\sigma}_{2j}\otimes\hat{\sigma}_{3k}\otimes\hat{\sigma}_{4l}\otimes\hat{\sigma}_{5m}\otimes\hat{\rho}^{(6)} \text{ + 5 exchange terms }]$$ 5-spin correlations

$$+\frac{1}{2^6}c_{ijklmn}^{(123456)}\hat{\sigma}_{1i}\otimes\hat{\sigma}_{2j}\otimes\hat{\sigma}_{3k}\otimes\hat{\sigma}_{4l}\otimes\hat{\sigma}_{5m}\otimes\hat{\sigma}_{6n}$$ 6-spin correlations

Lambda-anti-Lambda spin correlation

Consider only spin degree of freedom, only two spin correlations

$$c_{zz}^{\Lambda\bar{\Lambda}} \approx P_{\Lambda z}P_{\bar{\Lambda} z} + c_{zz}^{(s\bar{s})} - \frac{P_{sz}}{C_\Lambda}\left[c_{iz}^{(d\bar{s})}P_{ui} + c_{iz}^{(u\bar{s})}P_{di}\right] - \frac{P_{\bar{s}z}}{C_{\bar{\Lambda}}}\left[c_{iz}^{(s\bar{d})}P_{\bar{u}i} + c_{iz}^{(s\bar{u})}P_{\bar{d}i}\right].$$

$$C_\Lambda = 1 - c_{ii}^{(ud)} - P_{ui}P_{di}.$$

Taking the α-dependence into account

consider the simple case that the wavefunction is factorized:

$$|\alpha_{H_1}, \alpha_{\bar{H}_2}; m_{H_1}, m_{\bar{H}_2}\rangle = |\alpha_{H_1}, \alpha_{\bar{H}_2}\rangle | m_{H_1}, m_{\bar{H}_2}\rangle$$

$$|\alpha_{H_1}, \alpha_{\bar{H}_2}\rangle = |\alpha_{H_1}\rangle|\alpha_{\bar{H}_2}\rangle \qquad |m_{H_1}, m_{\bar{H}_2}\rangle = |m_{H_1}\rangle|m_{\bar{H}_2}\rangle$$

$$c_{zz}^{\Lambda\bar{\Lambda}}(\alpha_\Lambda, \alpha_{\bar{\Lambda}}) \approx P_{\Lambda z}(\alpha_\Lambda)P_{\bar{\Lambda} z}(\alpha_{\bar{\Lambda}}) + \bar{c}_{zz}^{(s\bar{s})} - \frac{\bar{P}_{sz}}{\bar{C}_\Lambda}\left[\bar{c}_{iz}^{(d\bar{s})}\bar{P}_{ui} + \bar{c}_{iz}^{(u\bar{s})}\bar{P}_{di}\right] - \frac{\bar{P}_{\bar{s}z}}{\bar{C}_{\bar{\Lambda}}}\left[\bar{c}_{iz}^{(s\bar{d})}\bar{P}_{\bar{u}i} + \bar{c}_{iz}^{(s\bar{u})}\bar{P}_{\bar{d}i}\right].$$

q from Λ; $\bar{q}$ from $\bar{\Lambda}$

Sensitive to the long range correlation between q_1 and $\bar{q}_2$.

Description of polarization of particles with different spins

Spin 1/2 hadrons:

The spin density matrix is 2x2: $\hat{\rho} = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} = \frac{1}{2}(1 + \vec{S} \cdot \vec{\sigma})$

Vector polarization: $S^{\mu} = (0, \vec{S}_T, \lambda)$

Spin 1 hadrons:

See e.g. A. Bacchetta, & P.J. Mulders, PRD62, 114004 (2000)

The spin density matrix is 3x3: $\rho = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix} = \frac{1}{3}(1 + \frac{3}{2}\vec{S} \cdot \vec{\Sigma} + 3T^{ij}\Sigma^{ij})$

Vector polarization: $S^{\mu} = (0, \vec{S}_T, \lambda)$ — 3

Tensor polarization: S_{LL}, $S_{LT}^i = (S_{LT}^x, S_{LT}^y)$, $S_{TT}^{ij} = \begin{pmatrix} S_{TT}^{xx} & S_{TT}^{xy} \\ S_{TT}^{xy} & -S_{TT}^{xx} \end{pmatrix}$ — 5

3 + 5 = 8 independent components

Spin 3/2 hadrons:

See e.g. Jing Zhao, Zhe Zhang, ZTL, Tianbo Liu, Ya-jin Zhou, PRD106, 094006 (2022)

The spin density matrix is 4x4: $\hat{\rho} = \frac{1}{4}\left(1 + \frac{4}{5}S^i\Sigma^i + \frac{2}{3}T^{ij}\Sigma^{ij} + \frac{8}{9}R^{ijk}\Sigma^{ijk}\right)$

Vector polarization: $S^{\mu} = (0, \vec{S}_T, \lambda)$ — 3

Tensor polarization (rank 2): S_{LL}, $S_{LT}^i = (S_{LT}^x, S_{LT}^y)$, $S_{TT}^{ij} = \begin{pmatrix} S_{TT}^{xx} & S_{TT}^{xy} \\ S_{TT}^{xy} & -S_{TT}^{xx} \end{pmatrix}$ — 5

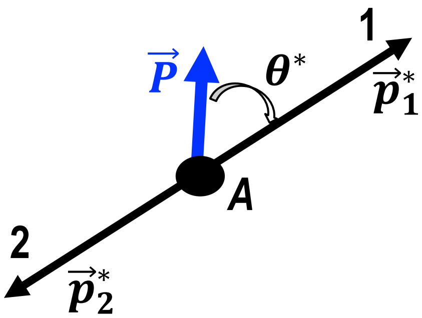
Tensor polarization (rank 3): S_{LLL}, $S_{LLT}^i = (S_{LLT}^x, S_{LLT}^y)$, $S_{LTT}^{ij} = \begin{pmatrix} S_{LTT}^{xx} & S_{LTT}^{xy} \\ S_{LTT}^{xy} & -S_{LTT}^{xx} \end{pmatrix}$ $S_{TTT}^{ijx} = \begin{pmatrix} S_{TTT}^{xxx} & S_{TTT}^{yxx} \\ S_{TTT}^{yxx} & -S_{TTT}^{xxx} \end{pmatrix}$ — 7

3 + 5 + 7 = 15 independent components

Measurements

For the strong decay (parity conserved) $B \to B_1 + M$,
where B is a $J^P = \left(\frac{3}{2}\right)^+$ baryon and B_1 is a $J^P = \left(\frac{1}{2}\right)^+$ baryon,
and M is a $J^P = 0^-$ meson, e.g., $\Delta \to N\pi$

$A \to 1 + 2$



$$W(\theta_N) \sim 1 + \frac{1}{2} S_{LL}(1 - 3\cos^2\theta_N)$$

For strong decay $B \to B_1 + M_1$, followed by the weak decay $B_1 \to B_2 + M_2$,
where B is a $J^P = \left(\frac{3}{2}\right)^+$ baryon and B_1 and B_2 are $J^P = \left(\frac{1}{2}\right)^+$ baryons
and M_1 and M_2 are $J^P = 0^-$ mesons, e.g., $\Sigma^* \to \Lambda\pi$, and $\Lambda \to p\pi^-$

$$W(\theta_\Lambda, \theta_p, \phi_p) \sim 1 + \frac{2}{5}\alpha_\Lambda S_L(\cos\theta_\Lambda \cos\theta_p - 2\sin\theta_\Lambda \sin\theta_p \cos\phi_p) - \frac{1}{4} S_{LL}(1 + 3\cos 2\theta_\Lambda)$$
$$- \frac{1}{4}\alpha_\Lambda S_{LLL}[(3\cos\theta_\Lambda + 5\cos 3\theta_\Lambda)\cos\theta_p - (\sin\theta_\Lambda + 5\sin 3\theta_\Lambda)\sin\theta_p \cos\phi_p]$$

Simple case

Consider the simplest case $\hat{\rho}^{(q)} = \frac{1}{2}\begin{pmatrix} 1+P_q & 0 \\ 0 & 1-P_q \end{pmatrix}$ as an example

$$S_L \approx \frac{5}{6}(P_{q_1} + P_{q_2} + P_{q_3})$$ quark polarization

$$S_{LL} \approx \frac{2}{3}(P_{q_1}P_{q_2} + P_{q_2}P_{q_3} + P_{q_3}P_{q_1})$$ two quark spin correlations

$$S_{LLL} \approx \frac{3}{5}P_{q_1}P_{q_2}P_{q_3}$$ three quark spin correlations

By studying S_L, we study the average of the polarization of quarks;

By studying S_{LL}, we study local two particle spin correlation between two quarks q_1 and q_2 ;

By studying S_{LLL}, we study local three particle spin correlation between three quarks q_1, q_2 and q_3.

Zhang Zhe *et al*., paper in preparation

未来实验测量

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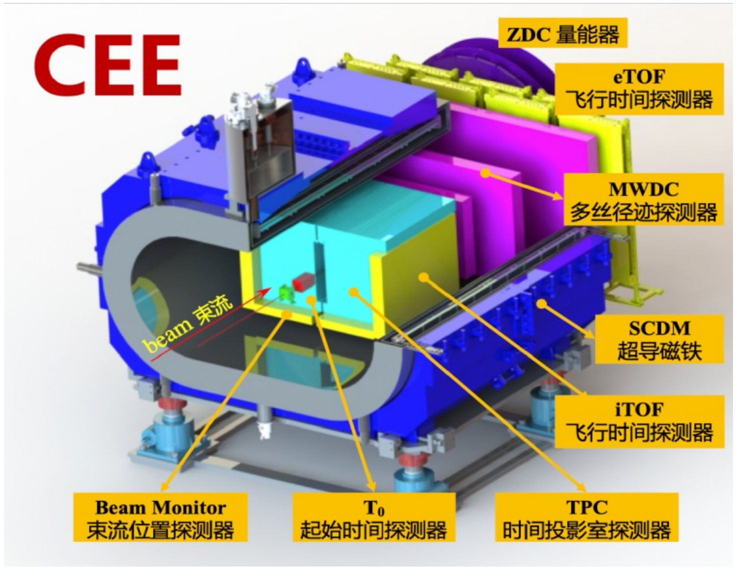

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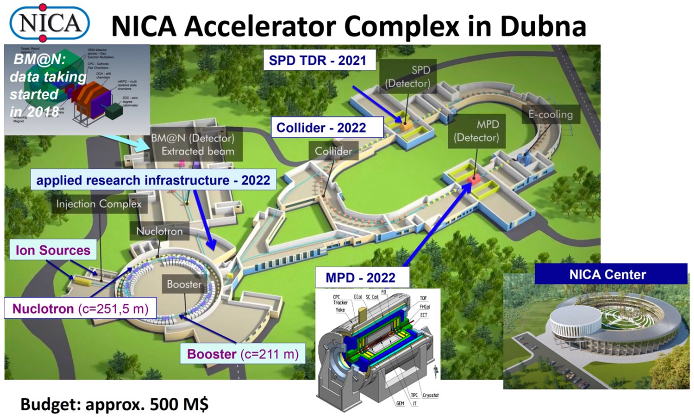
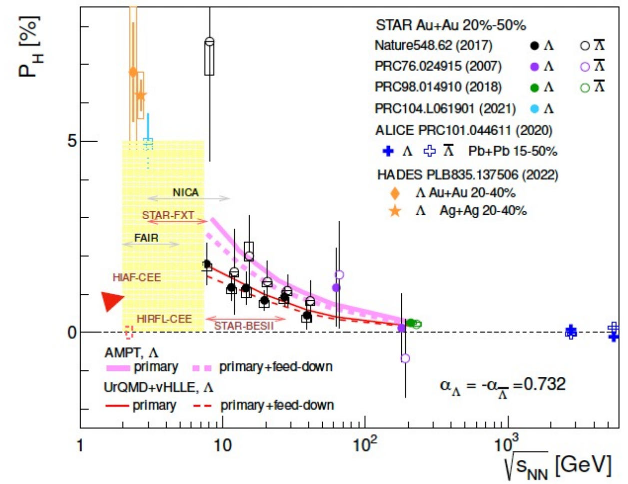
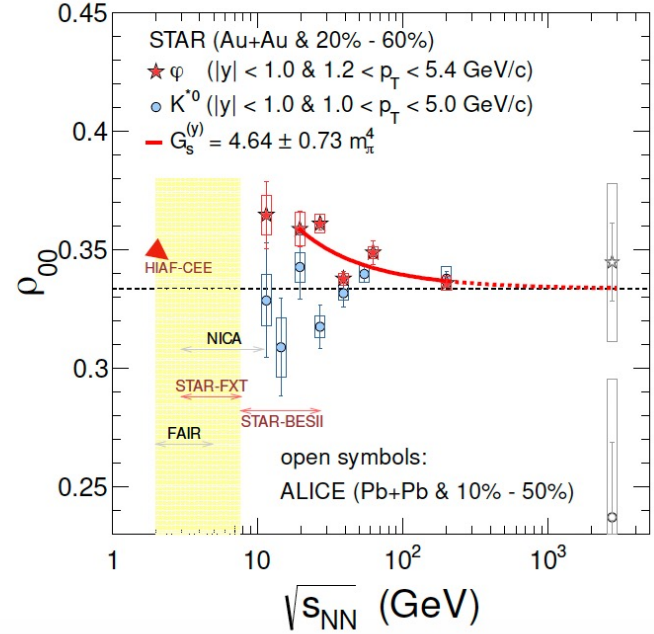






ppt from Sun Xu at Zhuhai workshop

Summary and Outlook

- Global polarization opens a new avenue to study properties of QGP.
- Measurements of the global vector meson spin alignment provide the opportunity to study quark spin correlations in QGP produced in heavy ion collisions.
- Quark spin correlations include two parts: the genuine correlation originated from the dynamical process and the induced correlation due to average over other degree of freedom.
- Quark spin correlations can be classified as local and long range correlations. Vector meson spin alignment and off diagonal elements contain both contributions, in particular the local correlations.
- It is also desirable to measure hyperon-hyperon and hyperon-anti-hyperon spin correlations. They should be more sensitive to the long range quark spin correlations.
- Polarization of spin 3/2 baryons can provide information on quark-quark spin correlations.

Thank you for your attention!

Measurements

For $V \to 1 + 2$, where 1 and 2 are two pseudoscalar mesons, e.g., $\rho \to \pi\pi$

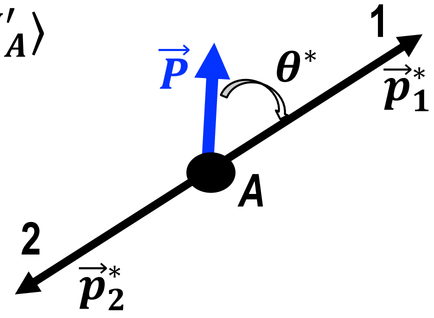
$$W(\theta, \varphi) = N \sum_{M_A, M'_A} |H_A|^2 D^{1*}_{M_A 0}(\varphi, \theta, -\varphi) D^{1}_{M'_A 0}(\varphi, \theta, -\varphi) \langle M_A | \hat{\rho}_A | M'_A \rangle$$

$$= \frac{3}{4\pi} \left\{ \frac{1}{2}(\rho_{11} + \rho_{-1-1}) \sin^2\theta + \rho_{00} \cos^2\theta \right.$$

$$- \frac{1}{\sqrt{2}} \sin 2\theta \left[\cos\varphi \left(\mathrm{Re}\rho_{10} - Re\rho_{-10}\right) - \sin\varphi \left(\mathrm{Im}\rho_{10} + \mathrm{Im}\rho_{-10}\right)\right]$$

$$\left. - \sin^2\theta \left(\cos 2\varphi \, \mathrm{Re}\rho_{1-1} - \sin 2\varphi \, \mathrm{Im}\rho_{1-1}\right) \right\}$$

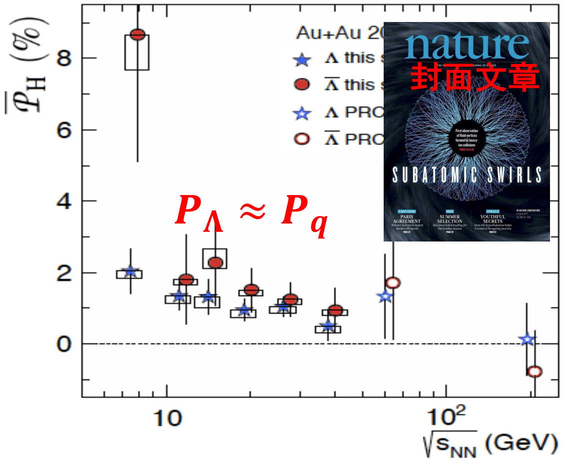
$A \to 1 + 2$



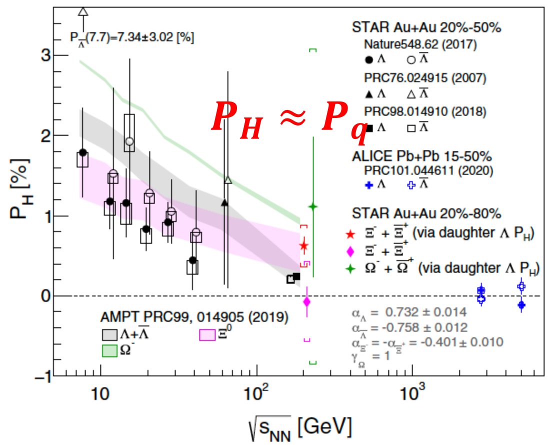
$$\int_0^{2\pi} d\varphi \, W(\theta, \varphi) = \frac{3}{4}\left[(1 - \rho_{00}) + (3\rho_{00} - 1)\cos^2\theta\right]$$

Great efforts of our experimental colleagues

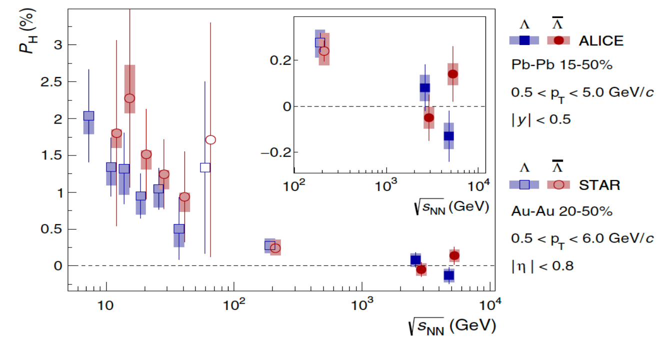
STAR, L. Adamczyk *et al.*,
Nature 548, 62 (2017).



STAR, J. Adam *et al.*,
PRL 126, 162301 (2021)

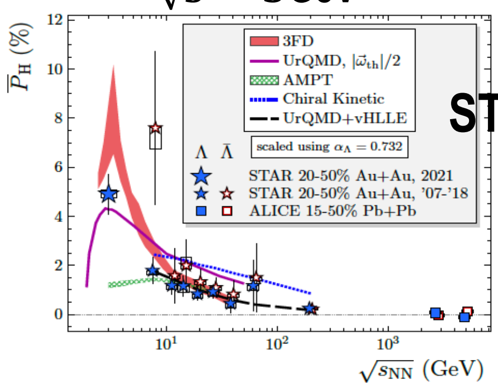


ALICE, S. Acharya *et al.*,
PRC 101, 044611 (2020)



STAR, M.S. Abdallah *et al.*,
PRC 104, L061901 (2021)

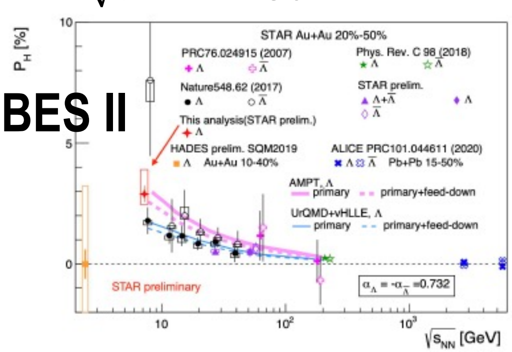
$\sqrt{s} = 3\text{GeV}$



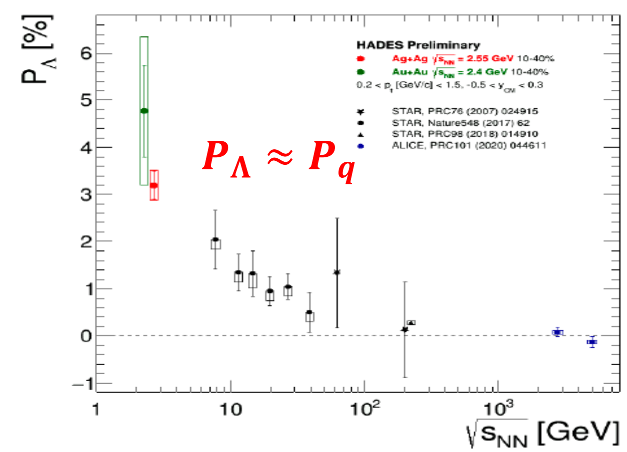
K. Okubo for STAR,
arXiv:2108.10012 [nucl-ex]

$\sqrt{s} = 7.2\text{GeV}$

STAR BES II



HADES, R. Yassine *et al.*,
PLB 835, 137506 (2022)



Off-diagonal elements of $\hat{\rho}^V$?

- ZTL & Xin-Nian Wang, PRL 94, 102301 (2005); PLB 629, 20 (2005).

considered the average $\langle\hat{\rho}_q\rangle = \frac{1}{2}\begin{pmatrix} 1+\langle P_q\rangle & 0 \\ 0 & 1-\langle P_q\rangle \end{pmatrix}$

i.e., $\langle P_{qy}\rangle = \langle P_q\rangle$, $\langle P_{qz}\rangle = \langle P_{qx}\rangle = 0$

- The STAR data show that: $\langle P_q P_{\bar{q}}\rangle \neq \langle P_q\rangle\langle P_{\bar{q}}\rangle$ $\quad \langle P_q P_{\bar{q}}\rangle \gg \langle P_q\rangle\langle P_{\bar{q}}\rangle$

indicates that $\Delta P_{qy}^2 \equiv \langle P_{qy}^2\rangle - \langle P_{qy}\rangle^2 \sim \langle P_{qy}^2\rangle \gg \langle P_{qy}\rangle^2$

Similar for the off-diagonal elements $\langle P_{qz}^2\rangle$ and $\langle P_{qx}^2\rangle$?

- take also the off-diagonal components into account

$$\hat{\rho}_q = \frac{1}{2}\begin{pmatrix} 1+P_{qy} & P_{qz}-iP_{qx} \\ P_{qz}+iP_{qx} & 1-P_{qy} \end{pmatrix}$$

Description of polarization of particles with different spins

Spin 3/2 hadrons:

$$\hat{\rho}=\frac{1}{4}\left(1+\frac{4}{5}S^i\Sigma^i+\frac{2}{3}T^{ij}\Sigma^{ij}+\frac{8}{9}R^{ijk}\Sigma^{ijk}\right)$$

$$\Sigma^i=(\Sigma^x,\Sigma^y,\Sigma^z)$$

$$\Sigma^x=\frac{1}{2}\begin{pmatrix}0&\sqrt{3}&0&0\\\sqrt{3}&0&0&0\\0&0&0&\sqrt{3}\\0&0&\sqrt{3}&0\end{pmatrix}\quad \Sigma^y=\frac{1}{2}\begin{pmatrix}0&-i\sqrt{3}&0&0\\i\sqrt{3}&0&-i&0\\0&i&0&-i\sqrt{3}\\0&0&i\sqrt{3}&0\end{pmatrix}\quad \Sigma^z=\frac{1}{2}\begin{pmatrix}3&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-3\end{pmatrix}$$

$$S^i=(S_T^x,S_T^y,S_L)$$

$$\Sigma^{ij}=\frac{1}{2}\left(\Sigma^i\Sigma^j+\Sigma^j\Sigma^i\right)-\frac{5}{4}\delta^{ij}1$$

$$T^{ij}=\begin{pmatrix}-S_{LL}+S_{TT}^{xx}&S_{TT}^{xy}&S_{LT}^{x}\\S_{TT}^{xy}&-S_{LL}-S_{TT}^{xx}&S_{LT}^{y}\\S_{LT}^{x}&S_{LT}^{y}&2S_{LL}\end{pmatrix}$$

$$\Sigma^{ijk}=\frac{1}{3}\left(\Sigma^{ij}\Sigma^k+\Sigma^{jk}\Sigma^i+\Sigma^{ki}\Sigma^j\right)-\frac{4}{15}\left(\delta^{ij}\Sigma^k+\delta^{jk}\Sigma^i+\delta^{ki}\Sigma^j\right)$$

$$R^{ijk}=\frac{1}{4}\begin{pmatrix}\begin{pmatrix}-3S_{LLT}^{x}+S_{TTT}^{xxx}&-S_{LLT}^{y}+S_{TTT}^{yxx}&-2S_{LLL}+S_{LTT}^{xx}\\-S_{LLT}^{y}+S_{TTT}^{yxx}&-S_{LLT}^{x}-S_{TTT}^{xxx}&S_{LTT}^{xy}\\-2S_{LLL}+S_{LTT}^{xx}&S_{LTT}^{xy}&2S_{LL}\end{pmatrix}\\\begin{pmatrix}-S_{LLT}^{y}+S_{TTT}^{yxx}&-S_{LLT}^{x}-S_{TTT}^{xxx}&S_{LTT}^{xy}\\-S_{LLT}^{x}-S_{TTT}^{xxx}&-3S_{LLT}^{y}-S_{TTT}^{yxx}&-2S_{LLL}-S_{LTT}^{xx}\\S_{LTT}^{xy}&-2S_{LLL}-S_{LTT}^{xx}&4S_{LLT}^{y}\end{pmatrix}\\\begin{pmatrix}-2S_{LLL}+S_{LTT}^{xx}&S_{LTT}^{xy}&4S_{LLT}^{x}\\S_{LTT}^{xy}&-2S_{LLL}-S_{LTT}^{xx}&4S_{LLT}^{y}\\4S_{LLT}^{x}&4S_{LLT}^{y}&4S_{LLL}\end{pmatrix}\end{pmatrix}$$