$B \to D\ell\bar{\nu}$衰变振幅的计算

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目录

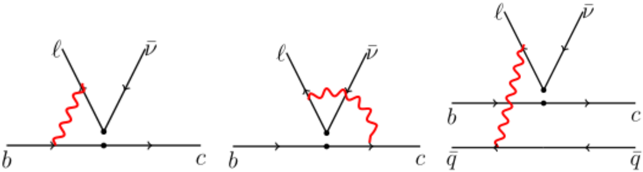
- 研究动机
- 工作进程
- 后续工作计划

研究动机

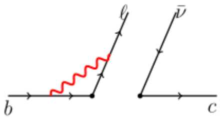
- R(D)反常是粒子物理学中一个重要研究领域
- $B \to D\ell\bar{\nu}$

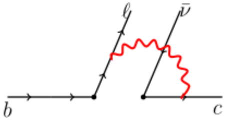
$$|\mathcal{M}|^2 = \frac{G_F^2|V_{cb}|^2}{2} H^{\mu\nu} L_{\mu\nu}$$

- 考虑交换光子的过程



工作进程

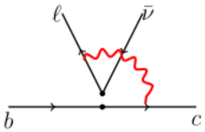




$$m_b \bar{c}\ \not{p}_\ell (1-\gamma_5)\nu \bar{\ell}(1+\gamma_5) b \Rightarrow \bar{c}(\not{p}_b - \not{p}_\nu - \not{p}_c)(1-\gamma_5)\nu\bar{\ell}(1+\gamma_5)\ \not{p}_b b$$

$$m_c \bar{c}\ \not{p}_\ell (1+\gamma_5)\nu \bar{\ell}(1-\gamma_5) b \Rightarrow \frac{m_c}{m_\ell}\bar{c}\ \not{p}_\ell(1+\gamma_5)\nu\bar{\ell}\ \not{p}_\ell(1-\gamma_5) b$$

工作进程



$$
\begin{aligned}
\mathcal{M}_b &= \int \frac{d^D k}{(2\pi)^D} \bar{c}(ieQ_c\gamma^a) \frac{i}{\not{p}_c - \not{k} - m_b + i\varepsilon} \gamma^\mu (1-\gamma_5)\, b \frac{-ig_{ab}}{k^2 + i\varepsilon} \\
&\times \bar{\ell}(ieQ_\ell \gamma_b) \frac{i}{\not{p}_\ell + \not{k} - m_\ell + i\varepsilon} \gamma_\mu (1-\gamma_5)\nu \\
&= \frac{2}{3} ie^2 \int \frac{d^D k}{(2\pi)^D} \frac{N}{k^2[(p_c-k)^2 - m_c^2][(p_\ell + k)^2 - m_\ell^2]}
\end{aligned}
$$

工作进程

费曼参数化：

$$\begin{aligned}
& \frac{1}{k^2[(p_c-k)^2-m_c^2][(p_\ell+k)^2-m_\ell^2]} \\
= \quad & \Gamma(3)\int \frac{\mathrm{d}x\mathrm{d}y}{\{k^2-2k\cdot(xp_c-yp_\ell)\}^3} \\
= \quad & \Gamma(3)\int \frac{\mathrm{d}x\mathrm{d}y}{\{[k-(xp_c-yp_\ell)]^2-(xp_c-yp_\ell)^2\}^3} \\
= \quad & \Gamma(3)\int \frac{\mathrm{d}x\mathrm{d}y}{\{\ell^2-\Delta\}^3}
\end{aligned}$$

工作进程

$$
\begin{aligned}
N &= \bar{c}\gamma^a(\not{p}_c - \not{k} + m_c)\gamma^\mu(1-\gamma_5)\, b\ell\, \gamma_a(\not{p}_\ell + \not{k} + m_\ell)\gamma_\mu(1-\gamma_5)\nu \\
&= \bar{c}\gamma^a(\bar{x}\not{p}_c + y\not{p}_\ell - \not{\ell} + m_c)\gamma^\mu(1-\gamma_5)b \\
&\times\ \bar{\ell}\gamma_a(\bar{y}\not{p}_\ell + x\not{p}_c + \not{\ell} + m_\ell)\gamma_\mu(1-\gamma_5)\nu \\
① &= \bar{c}\gamma^a\bar{x}\not{p}_c\gamma^\mu(1-\gamma_5)\, b\bar{\ell}\, \gamma_a\bar{y}\not{p}_\ell\gamma_\mu(1-\gamma_5)\nu \\
② &+ \bar{c}\gamma^a\bar{x}\not{p}_c\gamma^\mu(1-\gamma_5)\, b\bar{\ell}\, \gamma_a x\not{p}_c\gamma_\mu(1-\gamma_5)\nu \\
③ &+ \bar{c}\gamma^a\bar{x}\not{p}_c\gamma^\mu(1-\gamma_5)\, b\bar{\ell}\, \gamma_a m_\ell\gamma_\mu(1-\gamma_5)\nu \\
④ &+ \bar{c}\gamma^a y\not{p}_\ell\gamma^\mu(1-\gamma_5)\, b\bar{\ell}\, \gamma_a\bar{y}\not{p}_\ell\gamma_\mu(1-\gamma_5)\nu \\
⑤ &+ \bar{c}\gamma^a y\not{p}_\ell\gamma^\mu(1-\gamma_5)\, b\bar{\ell}\, \gamma_a x\not{p}_c\gamma_\mu(1-\gamma_5)\nu \\
⑥ &+ \bar{c}\gamma^a y\not{p}_\ell\gamma^\mu(1-\gamma_5)\, b\bar{\ell}\, \gamma_a m_\ell\gamma_\mu(1-\gamma_5)\nu \\
⑦ &- \bar{c}\gamma^a\not{\ell}\gamma^\mu(1-\gamma_5)\, b\bar{\ell}\, \gamma_a\not{\ell}\gamma_\mu(1-\gamma_5)\nu \\
⑧ &+ \bar{c}\gamma^a m_c\gamma^\mu(1-\gamma_5)\, b\bar{\ell}\, \gamma_a\bar{y}\not{p}_\ell\gamma_\mu(1-\gamma_5)\nu \\
⑨ &+ \bar{c}\gamma^a m_c\gamma^\mu(1-\gamma_5)\, b\bar{\ell}\, \gamma_a x\not{p}_c\gamma_\mu(1-\gamma_5)\nu \\
⑩ &+ \bar{c}\gamma^a m_c\gamma^\mu(1-\gamma_5)\, b\bar{\ell}\, \gamma_a m_\ell\gamma_\mu(1-\gamma_5)\nu
\end{aligned}
$$

工作进程

$$
\begin{aligned}
\bar{x}\bar{y}:① &= \bar{c}\gamma^a \not{p}_c\gamma^\mu(1-\gamma_5)b\bar{\ell}\gamma_a \not{p}_\ell\gamma_\mu(1-\gamma_5)\nu \\
&= \frac{1}{m_c m_\ell}\bar{c} \not{p}_c\gamma^a \not{p}_c\gamma^\mu(1-\gamma_5)b\bar{\ell} \not{p}_\ell\gamma_a \not{p}_\ell\gamma_\mu(1-\gamma_5)\nu \\
&= \frac{1}{m_c m_\ell}\frac{p_c^2 p_\ell^2}{D^2}\bar{c}\gamma^\alpha\gamma^a\gamma_\alpha\gamma^\mu(1-\gamma_5)\, b\bar{\ell}\,\gamma^\beta\gamma_a\gamma_\beta\gamma_\mu(1-\gamma_5)\nu \\
&= \frac{(D-2)^2}{D^2}m_c m_\ell\,\bar{c}\gamma^a\gamma^\mu(1-\gamma_5)b\bar{\ell}\gamma_a\gamma_\mu(1-\gamma_5)\nu \\
&= \frac{(D-2)^2}{D^2}m_c m_\ell\,\bar{c}(1-\gamma_5)\gamma^a\gamma^\mu\gamma_\lambda\gamma_a\gamma_\mu(1-\gamma_5)\nu \\
&= \frac{(D-2)^2}{D^2}m_c m_\ell\,\bar{c}(1-\gamma_5)\{4g_{\mu,\lambda}+(D-4)\gamma^\mu\gamma_\lambda\}\gamma_\mu(1-\gamma_5)\nu \\
&= 4\frac{(D-2)^2}{D^2}m_c m_\ell\,\bar{c}(1-\gamma_5)\gamma_\lambda(1-\gamma_5)\nu \\
&\quad+\frac{(D-2)^2(D-4)}{D^2}m_c m_\ell\,\bar{c}(1-\gamma_5)\gamma^\mu\gamma_\lambda\gamma_\mu(1-\gamma_5)\nu \\
&= -4\frac{D-2}{D^2}m_c m_\ell\,\bar{c}\gamma^\mu(1+\gamma_5)\gamma_\lambda\gamma_\mu(1-\gamma_5)\nu \\
&\quad+\frac{(D-2)^2(D-4)}{D^2}m_c m_\ell\,\bar{c}\gamma^\mu(1+\gamma_5)\gamma_\lambda\gamma_\mu(1-\gamma_5)\nu \\
&= -4\frac{D-2}{D^2}m_c m_\ell\,(\bar{c}b)_{V+A}(\bar{\ell}\nu)_{V-A} \\
&\quad+\frac{(D-2)^2(D-4)}{D^2}m_c m_\ell\,(\bar{c}b)_{V+A}(\bar{\ell}\nu)_{V-A} \\
&= \frac{(D^2-6D+4)(D-2)}{D^2}m_c m_\ell\,(\bar{c}b)_{V+A}(\bar{\ell}\nu)_{V-A}
\end{aligned}
$$

工作进程

$$
\begin{aligned}
\bar{x}\bar{y}:① &= \bar{c}\gamma^a \not{p}_c\gamma^\mu(1-\gamma_5)b\bar{\ell}\gamma_a \not{p}_\ell\gamma_\mu(1-\gamma_5)\nu \\
&= \bar{c}\{2p_{c,a}- \not{p}_c\gamma^a\}\gamma^\mu(1-\gamma_5)b\bar{\ell}\gamma_a \not{p}_\ell\gamma_\mu(1-\gamma_5)\nu \\
&= 2\bar{c}\gamma^\mu(1-\gamma_5)b\bar{\ell} \not{p}_c \not{p}_\ell\gamma_\mu(1-\gamma_5)\nu \\
&\quad -m_c\bar{c}\gamma^a\gamma^\mu(1-\gamma_5)b\bar{\ell}\{2p_{\ell,a}- \not{p}_\ell\gamma_a\}\gamma_\mu(1-\gamma_5)\nu \\
&= 2\bar{c}\gamma^\mu(1-\gamma_5)b\bar{\ell}\{2p_c\cdot p_\ell- \not{p}_\ell \not{p}_c\}\gamma_\mu(1-\gamma_5)\nu \\
&\quad -2m_c\,\bar{c} \not{p}_\ell\gamma^\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu(1-\gamma_5)\nu \\
&\quad +m_cm_\ell\,\bar{c}\gamma^a\gamma^\mu(1-\gamma_5)b\bar{\ell}\gamma_a\gamma_\mu(1-\gamma_5)\nu \\
&= 4p_c\cdot p_\ell\,\bar{c}\gamma^\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu(1-\gamma_5)\nu \\
&\quad -2m_\ell\,\bar{c}\gamma^\mu(1-\gamma_5)b\bar{\ell} \not{p}_c\gamma_\mu(1-\gamma_5)\nu \\
&\quad -2m_c\,\bar{c} \not{p}_\ell\gamma^\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu(1-\gamma_5)\nu \\
&\quad +m_cm_\ell\,\bar{c}\gamma^a\gamma^\mu(1-\gamma_5)b\bar{\ell}\gamma_a\gamma_\mu(1-\gamma_5)\nu \\
&= 4p_c\cdot p_\ell\,(\bar{c}b)_{V-A}(\bar{\ell}\nu)_{V-A} \\
&\quad +\frac{D-4}{D}m_cm_\ell\,\bar{c}\gamma^a\gamma^\mu(1-\gamma_5)b\bar{\ell}\gamma_a\gamma_\mu(1-\gamma_5)\nu \\
&= 4p_c\cdot p_\ell\,(\bar{c}b)_{V-A}(\bar{\ell}\nu)_{V-A} \\
&\quad +\frac{D-4}{D}[\frac{-4}{D-2}+(D-4)]m_cm_\ell\,\bar{c}\gamma^\mu(1+\gamma_5)b\bar{\ell}\gamma_\mu(1-\gamma_5)\nu \\
&= 4p_c\cdot p_\ell\,(\bar{c}b)_{V-A}(\bar{\ell}\nu)_{V-A} \\
&\quad +[\frac{(D-4)^2}{D}-\frac{4(D-4)}{D(D-2)}]m_cm_\ell\,(\bar{c}b)_{V+A}(\bar{\ell}\nu)_{V-A}
\end{aligned}
$$

工作进程

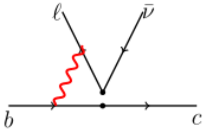
- 关系代换

$$p_c \cdot p_\ell \, (\bar{c}b)_{V-A}(\bar{\ell}\nu)_{V-A} = \frac{D^2-6D+4}{D^2(D-2)} m_c m_\ell \, (\bar{c}b)_{V+A}(\bar{\ell}\nu)_{V-A}$$

- 进行代换后分子为：

$$\begin{aligned} N \;=\; & \{\frac{D^2-10D+8}{D}(\ell^2+x^2m_c^2+y^2m_\ell^2) \\ & +[2(D-2)^2+8+\frac{8D}{D^2-6D+4}]xyp_c\cdot p_\ell \\ & +2(D-2)(x+y)p_c\cdot p_\ell \\ & +2xm_c^2+2ym_\ell^2+4p_c\cdot p_\ell\} \, (\bar{c}b)_{V-A}(\bar{\ell}\nu)_{V-A} \end{aligned}$$

工作进程



$$
\begin{aligned}
\mathcal{M}_a &= \int \frac{d^D k}{(2\pi)^D} \bar{c}\gamma^\mu (1-\gamma_5) \frac{i}{\not{p}_b - \not{k} - m_b + i\varepsilon} (ieQ_b \gamma^a) b \frac{-i g_{ab}}{k^2 + i\varepsilon} \\
&\times \ \bar{\ell}(ieQ_\ell \gamma_b) \frac{i}{\not{p}_\ell - \not{k} - m_\ell + i\varepsilon} \gamma_\mu (1-\gamma_5) \nu \\
&= -\frac{i}{3} e^2 \int \frac{d^D k}{(2\pi)^D} \frac{N}{k^2[(p_\ell - k)^2 - m_\ell^2][(p_b - k)^2 - m_b^2]}
\end{aligned}
$$

$$
\begin{aligned}
N &= \{ \frac{(D-2)^2}{D} (\ell^2 + x^2 m_\ell^2 + y^2 m_b^2 - 2x m_\ell^2 - 2y m_b^2) \\
&\quad - \frac{2(D-2)(D^2 - 2D + 2)}{D^2} xy \, m_b m_\ell + 2(D-2)(x+y) \, m_b m_\ell \\
&\quad - 4(D-2) \, m_b m_\ell \} (\bar{c} b)_{V-A} (\bar{\ell} \nu)_{V-A}
\end{aligned}
$$

工作进程

和分子中ℓ^2对应的项是发散的，取$D=4-2\varepsilon$

$$
\begin{aligned}
\mathcal{M}_{a,I} &= -\frac{i}{3}e^2\int\frac{d^D\ell}{(2\pi)^D}\Gamma(3)\frac{(D-2)^2}{D}\int dxdy\frac{\ell^2}{\{\ell^2-\Delta\}^3} \\
&= -\frac{i}{3}e^2\Gamma(3)\frac{(D-2)^2}{D}\frac{-iD}{2}\frac{(-1)^3}{(4\pi)^{2-\varepsilon}}\frac{\Gamma(\varepsilon)}{\Gamma(3)}\int dxdy\frac{1}{\Delta^\varepsilon} \\
&= \frac{1}{3}(\frac{e}{4\pi})^2(4\pi)^\varepsilon 2(1-\varepsilon)^2\Gamma(\varepsilon)\int\frac{dxdy}{(x^2m_\ell^2+y^2m_b^2+2xy\,p_\ell\cdot p_b)^\varepsilon} \\
&= \frac{1}{3}(\frac{e}{4\pi})^2(4\pi)^\varepsilon 2\{\frac{1}{\varepsilon}-(2+\gamma_E)+\cdots\}\int\frac{dxdy}{(x^2m_\ell^2+y^2m_b^2+2xy\,p_\ell\cdot p_b)^\varepsilon}
\end{aligned}
$$

$$
\begin{aligned}
&(x^2m_\ell^2+y^2m_b^2+2xy\,p_\ell\cdot p_b)^{-\varepsilon} \\
&= (m_b^2)^{-\varepsilon}(x^2\frac{m_\ell^2}{m_b^2}+y^2+2xy\frac{p_\ell\cdot p_b}{m_b^2})^{-\varepsilon} \\
&= (m_b^2)^{-\varepsilon}\{(x+y)^2+\frac{m_\ell^2-m_b^2}{m_b^2}x^2+2xy\frac{p_\ell\cdot p_b-m_b^2}{m_b^2}\}^{-\varepsilon} \\
&= (m_b^2)^{-\varepsilon}\{\frac{m_\ell^2-m_b^2}{m_b^2}x^2+2xy\frac{p_\ell\cdot p_b-m_b^2}{m_b^2}+1\}^{-\varepsilon} \\
&= (m_b^2)^{-\varepsilon}\{ax^2+bxy+1\}^{-\varepsilon}
\end{aligned}
$$

后续工作计划

- 积分计算
- 计算分支比

请各位老师同学批评指正