Presentation of some work

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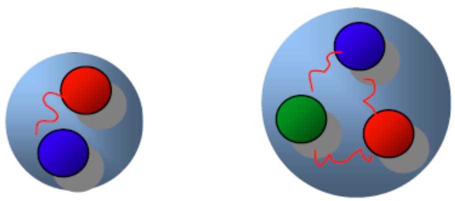
Outline

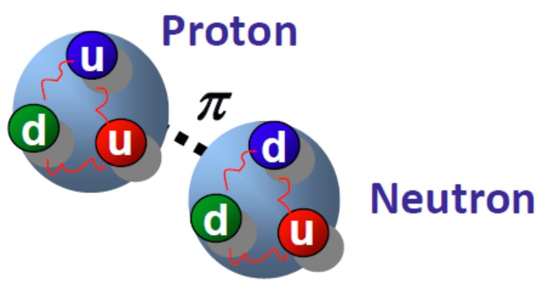
1. Background
2. Framework
3. Discussion on some work
4. Summary

Background

convential hadron



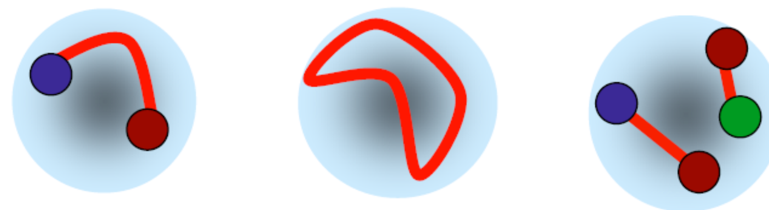
(q $\bar{q}$) (qqq)



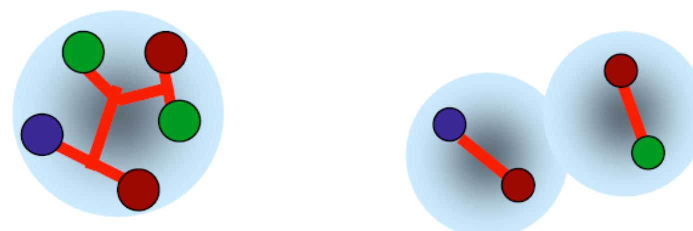
Deuteron: p-n molecule

Exotic hadrons

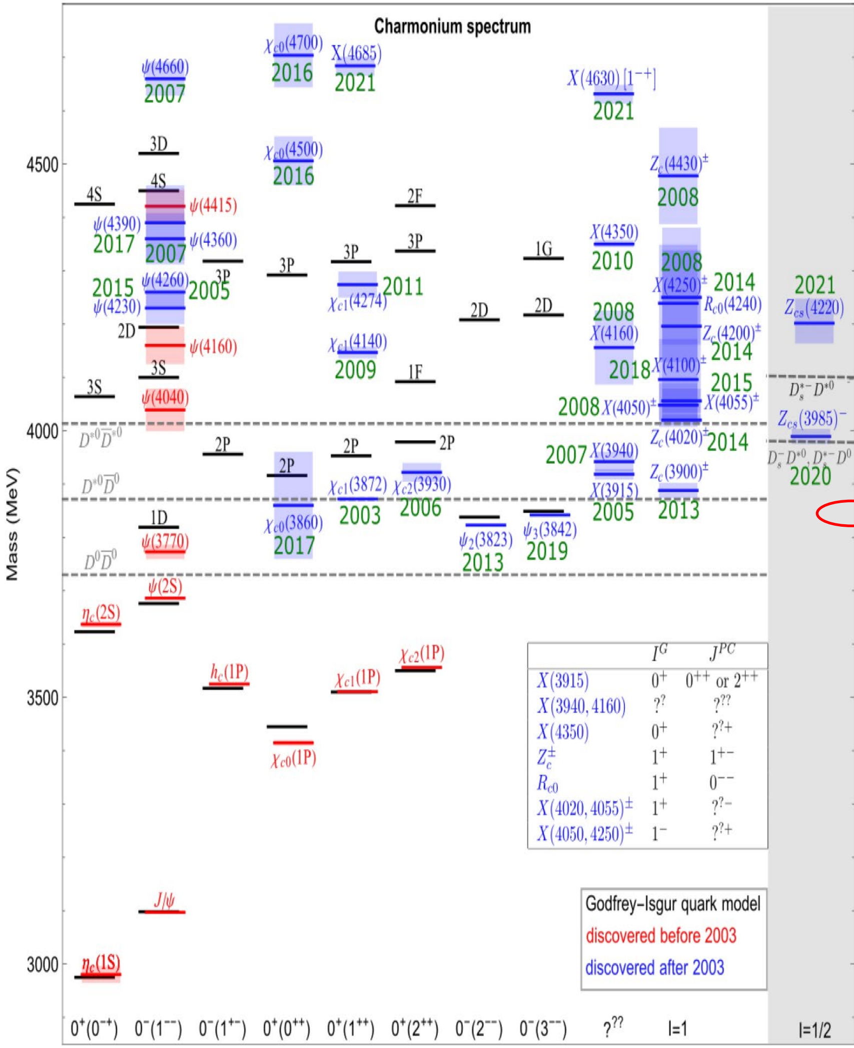
Hybrid Glueball Tetraquark



Pentaquark Hadronic molecule



Evidence for QCD exotic states is a missing piece of knowledge about the Nature of strong QCD.



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Prominent features of exotic hadrons:

(Zc(3900), Zcs(3985), X(4020), X(6900), Pc, Pcs …)

Most of them lie close to some underlying thresholds :

$Z_c(3900)$	$\bar{D}D^*$
$X(4020)$	$\bar{D}^*D^*$
$Z_{cs}(3985)$	$D_s^- D^*$

❖ Important question that follows:

Kinematical effects ? Or Hadron molecular ? Or Elementary/Compact state ?

Remain controversial

$P_c(4457)$, LHCb, PRL 115, 072001(2015); T_{cc}, arXiv: 2109.01038

Our research content :

The nature and structure of these exotic states are studied at the quark level by constructing multi-quark systems!

Frame work

◆ **The chiral quark model**

$$
\begin{aligned}
H &= \sum_{i=1}^{6}(m_i + \frac{p_i^2}{2m_i}) - T_c + \sum_{i<j} V(r_{ij}) \\
V(r_{ij}) &= V_{OGE}(r_{ij}) + V_{OPE}(r_{ij}) + V_{OSE}(r_{ij}) + V_{CON}(r_{ij}) \\
V_{OGE}(r_{ij}) &= \frac{1}{4}\alpha_s \vec{\lambda}_i^c \cdot \vec{\lambda}_j^c \{ \frac{1}{r_{ij}} - \frac{\pi}{2}\delta(\vec{r}_{ij})(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4\vec{\sigma}_i \cdot \vec{\sigma}_j}{3m_i m_j}) - \frac{3}{4m_i m_j r_{ij}^3} S_{ij} \} \\
V_{OPE}(r_{ij}) &= \frac{1}{3}\alpha_{ch} \frac{\Lambda^2}{\Lambda^2 - m_\pi^2} m_\pi \{ [Y(m_\pi r_{ij}) - \frac{\Lambda^3}{m_\pi^3} Y(\Lambda r_{ij})] \vec{\sigma}_i \cdot \vec{\sigma}_j \\
&\quad + [H(m_\pi r_{ij}) - \frac{\Lambda^3}{m_\pi^3} H(\Lambda r_{ij})] S_{ij} \} \vec{\tau}_i \cdot \vec{\tau}_j \\
V_{OSE}(r_{ij}) &= -\alpha_{ch} \frac{4m_q^2}{m_\pi^2} \frac{\Lambda^2}{\Lambda^2 - m_\sigma^2} m_\sigma [Y(m_\sigma r_{ij}) - \frac{\Lambda}{m_\sigma} Y(\Lambda r_{ij})] \\
V_{CON}(r_{ij}) &= -a_c \vec{\lambda}_i \cdot \vec{\lambda}_j r_{ij}^2 \\
\alpha_{ch} &= \frac{g_{ch}^2}{4\pi} \frac{m_\pi^2}{4m_u m_u}
\end{aligned}
$$

◆ Quark delocalization color screening model

- In the early 1990s, Professor Wang Fan and others from Nanjing University developed a new model based on the traditional constituent quark model - the quark delocalization color screening model (QDCSM)

➢ **Change the quark model wave function in the RGM variational wave function**

➢ **Made corrections to the quark confinement potential**

$$
\begin{aligned}
\psi_l(\vec{r}) &= (\phi_L + \epsilon\phi_R)/N(\epsilon), \\
\psi_r(\vec{r}) &= (\phi_R + \epsilon\phi_L)/N(\epsilon), \\
N(\epsilon) &= \sqrt{1+\epsilon^2+2\epsilon\langle\phi_R|\phi_L\rangle}.
\end{aligned}
$$

$$
V_{ij}^C = \begin{cases} -a_c\vec{\lambda}_i\cdot\vec{\lambda}_j r_{ij}^2 & \text{if } i, j \text{ occur in the same hadron orbit} \\ -a_c\vec{\lambda}_i\cdot\vec{\lambda}_j \dfrac{1-e^{\mu r_{ij}^2}}{\mu} & \text{if } i, j \text{ occur in a different hadron orbit.} \end{cases}
$$

➢ The success of the QDCSM lies in explaining nuclear forces and hadronic interactions using a quark model with minimal parameters. To date, this model is the only one that explains the similarity between nuclear and molecular forces, addressing why atomic nuclei can be approximated as nucleonic systems rather than multi-quark systems

◆ The resonance group method

- The conventional ansatz for two-cluster wave function is :

$$\Psi = A[[\phi_{B_1}(\vec{\rho}_1, \vec{\lambda}_1)\phi_{B_2}(\vec{\rho}_2, \vec{\lambda}_2)]^{[\sigma]IS}\chi(\vec{R})Z(\vec{R}_{cm})],$$

- Variational principle

$$\langle \delta\Psi'' | H - E | \Psi' \rangle = 0$$

$$\Psi'' = \int A[\phi_1(\rho_1,\lambda_1)\phi_2(\rho_2,\lambda_2)\delta(R-R'')]\chi(R'')dR'',$$

$$\Psi' = \int A[\phi_1(\rho_1,\lambda_1)\phi_2(\rho_2,\lambda_2)\delta(R-R')]\chi(R')dR',$$

$$\delta\Psi'' = \int A[\phi_1(\rho_1,\lambda_1)\phi_2(\rho_2,\lambda_2)\delta(R-R'')]\delta\chi(R'')dR''$$

- The RGM equation can be written :

$$\int H(R'',R')\chi(R')dR' = E\int N(R'',R')\chi(R')dR'$$

$$\left\{\begin{matrix} H(R'',R') \\ N(R'',R') \end{matrix}\right\} = \left\langle A[\phi_1\phi_2\delta(R-R'')] \middle| \left\{\begin{matrix} H \\ 1 \end{matrix}\right\} \middle| A[\phi_1\phi_2\delta(R-R')] \right\rangle$$

$$\xrightarrow{A=A^*A=1+A'}$$

$$H(R'',R') = H^D(R'',R')\delta(R''-R') + H^{EX}(R'',R')$$

$$N(R'',R') = N^D(R'',R')\delta(R''-R') + N^{EX}(R'',R')$$

$$\begin{pmatrix} N^D(R'',R') \\ H^D(R'',R') \end{pmatrix} \delta(R''-R') = \int \phi_1^*(\rho_1,\lambda_1)\phi_2^*(\rho_2,\lambda_2)\delta(R-R'') \begin{pmatrix} 1 \\ H \end{pmatrix}$$
$$\times \phi_1(\rho_1,\lambda_1)\phi_2(\rho_2,\lambda_2)\delta(R-R')d\rho_1 d\lambda_1 d\rho_2 d\lambda_2 dR$$
$$\begin{pmatrix} N^{EX}(R'',R') \\ H^{EX}(R'',R') \end{pmatrix} = \int \phi_1^*(\rho_1,\lambda_1)\phi_2^*(\rho_2,\lambda_2)\delta(R-R'') \begin{pmatrix} 1A'' \\ HA'' \end{pmatrix}$$
$$\times \phi_1(\rho_1,\lambda_1)\phi_2(\rho_2,\lambda_2)\delta(R-R')d\rho_1 d\lambda_1 d\rho_2 d\lambda_2 dR$$

- The RGM equation can also be written :

$$\int L(R'',R')\chi(R')dR' = 0$$

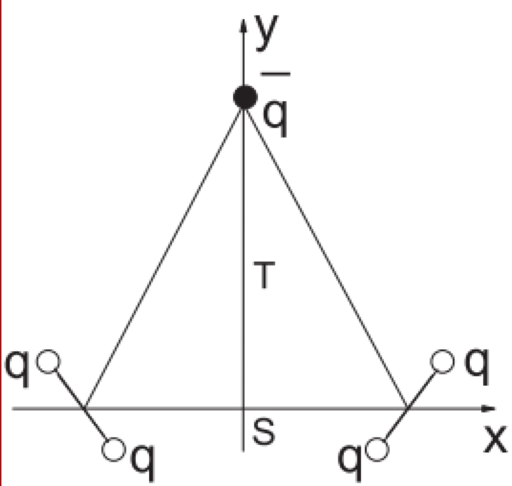
$$L(R'',R') = H(R'',R') - EN(R'',R') \quad = \quad L(R'',R') = [-\frac{\nabla_{R'}^2}{2\mu} + V_{rel}^D(R') - E_{rel}]\delta(R''-R') + H^{EX}(R'',R') - EN^{EX}(R'',R').$$

Group theory method

- Group chain

$$\begin{array}{ccccccccc} [1^n] & [\nu] & [\tilde{\nu}] & [c] & [\mu] & [f] & I & Y & J \end{array}$$

$$\mathrm{SU}_{36} \supset \mathrm{SU}_2^x \times \left\{\mathrm{SU}_{18} \supset \mathrm{SU}_3^c \times \left[\mathrm{SU}_6 \supset \left(\mathrm{SU}_3^f \supset \mathrm{SU}_2^\tau \times \mathrm{U}_1^Y\right) \times \mathrm{SU}_2^\sigma\right]\right\},$$



- The physical basis wave function of qq-qq-$\bar{q}$ structure.

$$\psi_2(q^2) = \left| \begin{array}{c} [\nu_2] W_{\nu_2} \\ [c_2]\, W_{c_2} [\mu_2][f_2] Y_2 I_2 M_{I_2} J_2 M_{J_2} \end{array} \right\rangle$$

Diquark-wave function

$$\Psi_{\alpha_4 k_4}(q^4) = \mathcal{A}\left[\psi_2(q_1 q_2)\psi_2(q_3 q_4)\right]^{[c_4] I_4 J_4}_{W_{c_4} M_{I_4} M_{J_4}},$$

Diquark-Diquark-wave function

$$\Psi_{\alpha k}(q^4 \bar{q}) = \left[\Psi_{\alpha_4 k_4}(q^4)\right)\psi_{[\bar{c}]\bar{I}\bar{J}}(\bar{q}_5)\right]^{[c] I J}_{W M_I M_J}.$$

- Symmetirc wave function of qq-qq-$\bar{q}$ structure.

$$\Phi_{\alpha_4 K_4}(q^4) = \left| \begin{array}{c} [\nu_4] W_{\nu_4} \\ [c_4]\, W_{c_4} [\mu_4][f_4] Y_4 I_4 M_{I_4} J_4 M_{J_4} \end{array} \right\rangle,$$

$$\Phi_{\alpha K}(q^4\bar{q}) = \left[\Phi_{\alpha_4 K_4}(q^4))\psi_{[\bar{c}]\bar{I}\bar{J}}(\bar{q}_5)\right]^{[c]IJ}_{WM_I M_J}.$$

- Physics basis and symmetric basis can be mutually converted.

$$\begin{aligned}\Psi_{\alpha k}(q^4\bar{q}) &= \sum_K C_{kK}\Phi_{\alpha K}(q^4\bar{q}) \\ &= \sum_{\tilde{\nu}_4 \mu_4 f_4} C^{[\tilde{\nu}_4][c_4][\mu_4]}_{[\tilde{\nu}_2][c_2][\mu_2],[\tilde{\nu}'_2][c'_2][\mu'_2]} C^{[\mu_4][f_4][J_4]}_{[\mu_2][f_2][J_2],[\mu'_2][f'_2][J'_2]} \\ &\quad C^{[f_4]Y_4 I_4}_{[f_2]Y_2 I_2,[f'_2]Y'_2 I'_2}\Phi_{\alpha K}(q^4\bar{q}),\end{aligned}$$

- Fractional parentage expansion technique
  - Quark-quark interaction

$$\langle \Phi_{\alpha K} | H_{34} | \Phi_{\alpha K'} \rangle = \sum_{1,2} C^{[1^4][\nu_4][\tilde{\nu}_4]}_{[1^2][\nu_1][\tilde{\nu}_1],[1^2][\nu_2][\tilde{\nu}_2]} C^{[\tilde{\nu}_4][c_4][\mu_4]}_{[\tilde{\nu}_1][c_1][\mu_1],[\tilde{\nu}_2][c_2][\mu_2]} C^{[\mu_4][f_4]J_4}_{[\mu_1][f_1]J_1,[\mu_2][f_2]J_2} C^{[f_4]Y_4I_4}_{[f_1]Y_1I_1,[f_2]Y_2I_2}$$
$$C^{[1^4][\nu_4][\tilde{\nu}_4]}_{[1^2][\nu_1'][\tilde{\nu}_1'],[1^2][\nu_2'][\tilde{\nu}_2']} C^{[\tilde{\nu}_4][c_4][\mu_4]}_{[\tilde{\nu}_1'][c_1'][\mu_1'],[\tilde{\nu}_2'][c_2'][\mu_2']} C^{[\mu_4][f_4]J_4}_{[\mu_1'][f_1']J_1',[\mu_2'][f_2']J_2'} C^{[f_4]Y_4I_4}_{[f_1']Y_1'I_1',[f_2']Y_2'I_2'}$$
$$C^{[\nu_4]W_{x_4}}_{[\nu_1]W_{x_1},[\nu_2]W_{x_2}} C^{[\nu_4]W_{x_4'}}_{[\nu_1']W_{x_1'},[\nu_2']W_{x_2'}} \langle \alpha_1 K_1 | \alpha_1' K_1' \rangle \langle \alpha_2 K_2 | H_{34} | \alpha_2' K_2' \rangle. \qquad (3\text{-}30)$$

$c_2^4 = 6$

  - Quark-antiquark interaction

$$\langle \Phi_{\alpha K} | H_{45} | \Phi_{\alpha K'} \rangle = \sum_{3,1} C^{[1^4][\nu_4][\tilde{\nu}_4]}_{[1^3][\nu_3][\tilde{\nu}_3],[1][\nu_1][\tilde{\nu}_1]} C^{[\tilde{\nu}_4][c_4][\mu_4]}_{[\tilde{\nu}_3][c_3][\mu_3],[\tilde{\nu}_1][c_1][\mu_1]} C^{[\mu_4][f_4]J_4}_{[\mu_3][f_3]J_3,[\mu_1][f_1]J_1} C^{[f_4]Y_4I_4}_{[f_3]Y_3I_3,[f_1]Y_1I_1}$$
$$C^{[1^4][\nu_4][\tilde{\nu}_4]}_{[1^3][\nu_3'][\tilde{\nu}_3'],[1][\nu_1'][\tilde{\nu}_1']} C^{[\tilde{\nu}_4][c_4][\mu_4]}_{[\tilde{\nu}_3'][c_3'][\mu_3'],[\tilde{\nu}_1'][c_1'][\mu_1']} C^{[\mu_4][f_4]J_4}_{[\mu_3'][f_3']J_3',[\mu_1'][f_1']J_1'} C^{[f_4]Y_4I_4}_{[f_3']Y_3'I_3',[f_1']Y_1'I_1'}$$
$$U(c_3c_1cc_{\bar{1}}; c_4c_2) U(I_3I_1II_{\bar{1}}; I_4I_2) U(J_3J_1JJ_{\bar{1}}; J_4J_2)$$
$$U(c_3'c_1'cc_{\bar{1}}'; c_4c_2') U(I_3'I_1'II_{\bar{1}}'; I_4I_2') U(J_3'J_1'JJ_{\bar{1}}'; J_4J_2')$$
$$C^{[\nu_4]W_{x_4}}_{[\nu_3]W_{x_3},[\nu_1]W_{x_1}} C^{[\nu_4]W_{x_4'}}_{[\nu_3']W_{x_3'},[\nu_1']W_{x_1'}} \langle \alpha_3 K_3 | \alpha_3' K_3' \rangle \langle \alpha_2 K_2 | H_{45} | \alpha_2' K_2' \rangle. \qquad (3\text{-}31)$$

$c_1^4 = 4$

  - The matrix elements of H on the symmetric basis can be obtained.

$$\langle \Phi_{\alpha K} | H_5 | \Phi_{\alpha K'} \rangle = 6 \langle \Phi_{\alpha K} | H_{34} | \Phi_{\alpha K'} \rangle + 4 \langle \Phi_{\alpha K} | H_{45} | \Phi_{\alpha K'} \rangle$$

Discussion on some work

- Hidden strange pentaquark states in constituent quark models.

PHYSICAL REVIEW C 98, 055203 (2018)

- The explanation of some exotic states in the $\bar{c}\bar{s}cs$ tetraquark system.

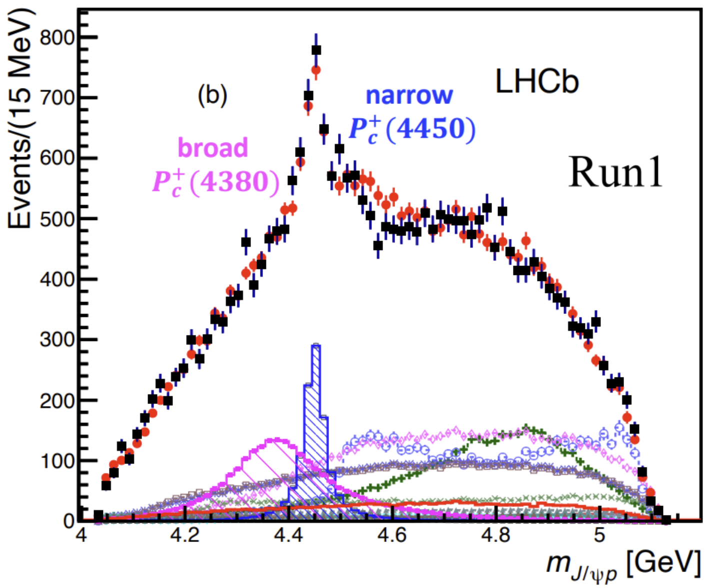
Eur. Phys. J. C (2021) 81:950

For the first work:

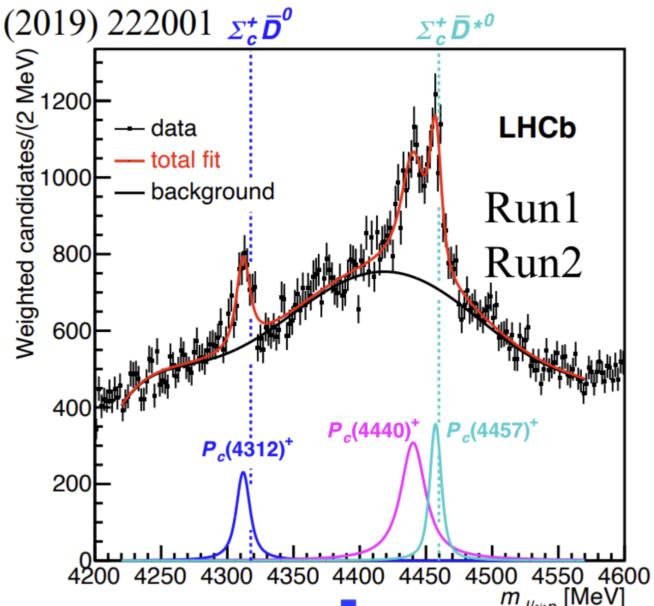
Observation of P_c^+ in the $\Lambda_b^0 \to J/\psi p K^-$

- $P_c^+(c\bar{c}uud)$ states were first observed in $\Lambda_b^0 \to J/\psi p K^-$ using Run1 data
- Analysis updated with ×9 more data
  - $P_c^+(4450)$ resolved into two peaks, $P_c^+(4440)$ and $P_c^+(4457)$
  - A new state $P_c^+(4312)$ observed

PRL 115 (2015) 072001



PRL 122 (2019) 222001



States close to $\Sigma_c^+\bar{D}^0$ and $\Sigma_c^+\bar{D}^{*0}$ mass thresholds consistent with molecular interpretation

The hideen charm pentaquark state: $\boldsymbol{P_c^+}$

?

The hideen strange pentaquark state: $\boldsymbol{P_s}$

- The evidence for a new baryon state with hidden strange was reported by SPHINX Collaboration in 1999

Eur. Phys.J.A 5, 409 (1999)

- NΦ bound state

Phys. Rev. C 63, 022201(R) (2001)
Phys. Rev. Lett. 64, 1011 (1990)
Phys. Lett. B 774, 108 (2017)
Phys. Rev. C 95, 055202 (2017)
Phys.Rev.C 73, 025207 (2006)
arXiv:1804.09383
Phys.Rev.D 91, 114503 (2015)

The work goals:

To investigate whether the NΦ states are bound or not in ChQM?

- Three kinds of chiral quark models:

CHQM1, σ meson exchange is used between any quark pair.
CHQM2, σ meson exchange is only valid for u/d quark pair.
CHQM3, the full SU(3) scalar nonet-meson exchange is employed.

TABLE III. The symmetries of colorful qqq and $q\bar{q}$ clusters.

	Δ'	$\Sigma^{*\prime}$	$\Xi^{*\prime}$	N''	Λ''	Σ''	Λ'_s
$[c]$	[21]	[21]	[21]	[21]	[21]	[21]	[21]
$[\sigma]$	[21]	[21]	[21]	[3]	[3]	[3]	[21]
$[f]$	[3]	[3]	[3]	[21]	[21]	[21]	[111]
I	$\frac{3}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	1	0
S	0	1	2	0	1	1	1
	η'	ϕ'	$K^{*\prime}$	$K_s^{*\prime}$			
$[c]$	[21]	[21]	[21]	[21]			
$[\sigma]$	[11]	[2]	[11]	[2]			
$[f]$	[21]	[21]	[21]	[21]			
I	0	0	$\frac{1}{2}$	$\frac{1}{2}$			
S	2	2	1	1			

- The color symmetries of qqq and q $\bar{q}$ cluster are all [111]
- The color symmetries of hidden color clusters are [21]

TABLE IV. Channels of the $N\phi$ system.

1	2	3	4	5	6	7	8
$N\phi$	ΛK^*	ΣK^*	$\Sigma^* K$	$\Sigma^* K^*$	$\Delta'\phi'$	$\Sigma^{*\prime} K^{*\prime}$	$\Xi^{*\prime} K^{*\prime}$
9	10	11	12	13	14	15	16
$N''\eta'$	$N''\phi'$	$\Lambda'' K'$	$\Lambda'' K^{*\prime}$	$\Sigma'' K'$	$\Sigma'' K^{*\prime}$	$\Sigma^{*\prime} K^{*\prime}$	$\Lambda'_s K^{*\prime}$

- the first five channels are color single channels
- Others are hidden-color channels

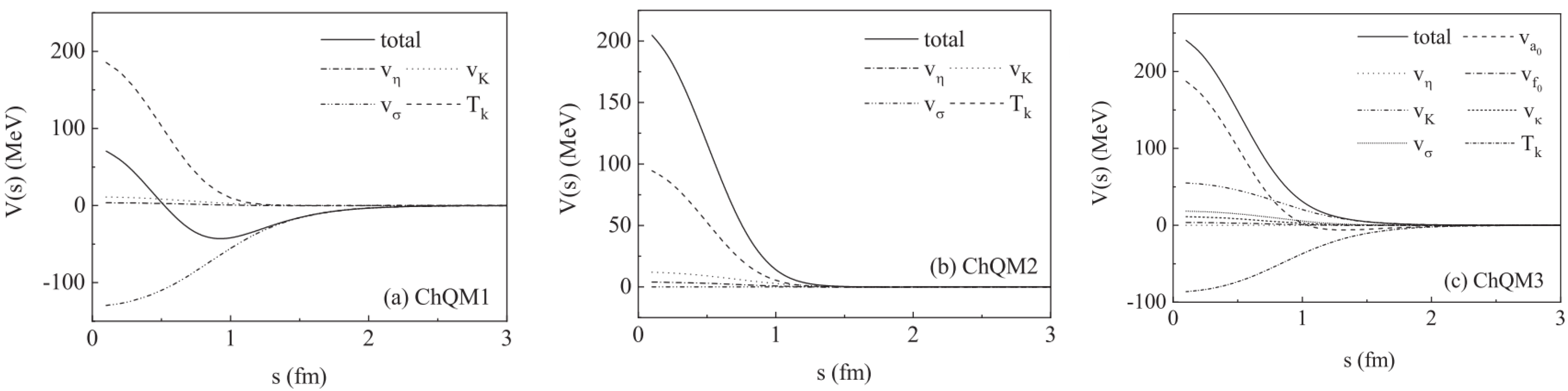
● The effective potential

➢ The definition of potential can be written as:

$$E(S_m) = \frac{\langle \Psi_{5q}(S_m)|H|\Psi_{5q}(S_m)\rangle}{\langle \Psi_{5q}(S_m)|\Psi_{5q}(S_m)\rangle},$$

➢ The effective potential between two colorless cluster is defined as:

$$V(S_m) = E(S_m) - E(\infty),$$



- The potential of NΦ state in both CHQM1 and CHQM3 are attractive
- The attraction in CHQM1 is much larger than one in CHQM3
- The potential in CHQM2 is repulsive

● The dynamic calculation

TABLE V. The binding energies B with channel coupling.

	B_{sc} (MeV)	B_{5cc} (MeV)	B_{16cc} (MeV)
ChQM1	ub	−5.70	−12.27
ChQM2	ub	ub	ub
ChQM3	ub	ub	ub

- The single channel NΦ is unbound in all three quark models
- The NΦ can be bound by the coupling of the color-single channels in CHQM1
- The NΦ can be bound by the coupling of all channels in CHQM1
- the mass of NΦ in CHQM1 by 16 channel coupling :

  M = 1946.7 MeV

The channel coupling has an important impact on the NΦ state !

● The relative motion wave function of NΦ state

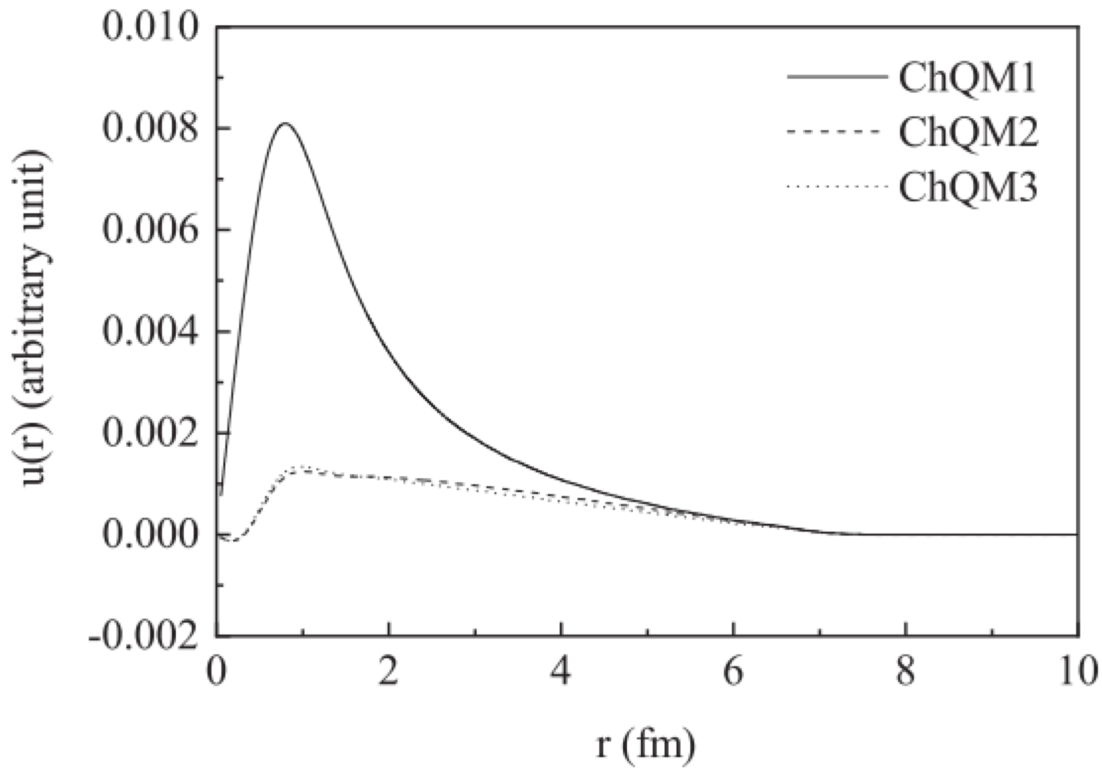


FIG. 2. The relative motion wave function of the $N\phi$ state in three quark models.

- The short summery

  - The NΦ state can be bound through the interaction of σ meson exchange plus the effect of channel coupling
  - The effect of channel coupling has an influence on the existence of this bound state.

Discussion on some work

- Hidden strange pentaquark states in constituent quark models.

PHYSICAL REVIEW C 98, 055203 (2018)

- The explanation of some exotic states in the $\bar{c}\bar{s}cs$ tetraquark system.

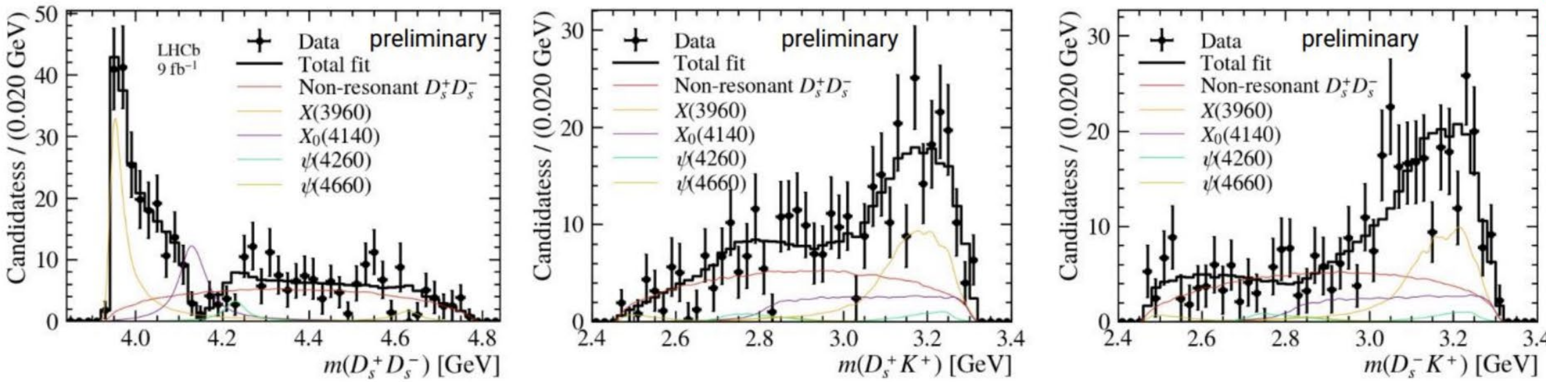
Eur. Phys. J. C (2021) 81:950

For the second work:

X states: New states in $D_s^+ D_s^-$

- 9 fb^{-1} at LHCb, near threshold structure $X(3960)$ in $B^+ \to D_s^+ D_s^- K^+$, 12σ, $J^P = 0^{++}$
- X(4140) accounts for the dip around 4.14 GeV
- If $X(3960)$ and $X(3915)$ the same particle? More measurement needed.

LHCb-PAPER-2022-018
LHCb-PAPER-2022-019



Component	J^{PC}	M_0 [MeV]	Γ_0 [MeV]	$\mathcal{F}$ [%]	$\mathcal{S}$ [σ]
$X(3960)$	0^{++}	$3955 \pm 6 \pm 12$	$48 \pm 17 \pm 10$	$24.2 \pm 7.6 \pm 7.9$	12.6 (14.3)
$X_0(4140)$	0^{++}	$4133 \pm 7 \pm 11$	$69 \pm 17 \pm 7$	$17.7 \pm 4.9 \pm 7.7$	3.7 (3.9)
$\psi(4260)$	1^{--}	4230	55	$3.7 \pm 0.4 \pm 3.0$	3.1 (3.3)
$\psi(4660)$	1^{--}	4633	64	$2.2 \pm 0.2 \pm 0.5$	2.9 (3.2)
NR	S-wave	-	-	$46.6 \pm 13.3 \pm 11.3$	3.1 (3.4)

- $X(3960)$: $M_0 = 3955 \pm 6 \pm 11$ MeV; $\Gamma_0 = 48 \pm 17 \pm 10$ MeV; $J^{PC} = 0^{++}$
- $\chi_{c0}(3930)$: $M_0 = 3924 \pm 2$ MeV; $\Gamma_0 = 17 \pm 5$ MeV; $J^{PC} = 0^{++}$

- Are they the same particle? If yes

$$\frac{\Gamma(X \to D^+D^-)}{\Gamma(X \to D_s^+D_s^-)} = \frac{\mathcal{B}(B^+ \to D^+D^-K^+)\mathcal{FF}^X_{B^+ \to D^+D^-K^+}}{\mathcal{B}(B^+ \to D_s^+D_s^-K^+)\mathcal{FF}^X_{B^+ \to D_s^+D_s^-K^+}}$$
$$= 0.29 \pm 0.09\,(\text{stat}) \pm 0.10\,(\text{syst}) \pm 0.08\,(\text{ext})$$

$\mathcal{FF}$: Fit fractions in the two B^+ decays

$\Gamma(X \to D^+D^-) < \Gamma(X \to D_s^+D_s^-)$ implies the exotic nature of the state

- Conventional charmonium predominantly decay into $D^{(*)}\overline{D}^{(*)}$
  - It is harder to excite an $s\bar{s}$ pair from vacuum compared with $u\bar{u}(d\bar{d})$

- The creation of an ***ss*** quark pair from the vacuum is suppressed relative to ***uu or dd*** pairs.
- X(3960) and χc0(3930) are either not the **same resonance**, or they are the same non-conventional charmonium-like state, for instance, a candidate containing the dominant *ccss* constituents

What is the nature of X(3960)?

- **Hybrid state** can be ruled out, due to too low mass for a QCD-hybrid candidate (the lightest 0^{++} charmonium hybrid around 4450 MeV)[1]. ✕

- **Lightest $c\bar{c}s\bar{s}$ tetraquark,** ~3920 MeV, is proposed by Lebed et al.[2]. The QCD sum rule[3] also favors $\chi_{c0}(3915)$ as a 0^{++} $cq\bar{c}\bar{q}$ or $cs\bar{c}\bar{s}$ tetraquark.
favour

- **Molecular $D_s^+ D_s^-$ (virtual) state,** is calculated in the quark delocalization color screening model [4]. The recent lattice QCD results[5] found a narrow 0^{++} $D_s^+ D_s^-$ bound state. Some phenomenological studies[6] regard it as the molecular (virtual) state.
favour

- **$c\bar{c}$ mixed with exotic components,** such as Ref.[7,8]
favour

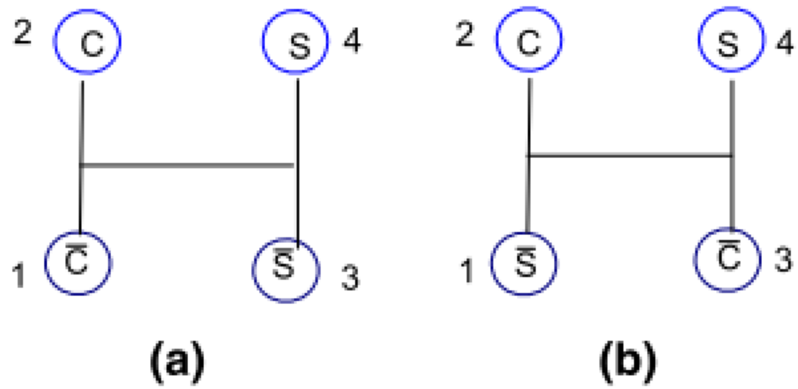
[1] arXiv:1204.5425. [2] arXiv:1602.08421, 2005.07100. [3]arXiv:1706.09731. [4] arXiv: 2103.12425.
[5] arXiv:2011.02542, 2111.02934. [6]arXiv: 1503.04431, 2101.01021. [7]arXiv: 2302.06278. [8]arXiv:2303.15388.

	QDCSM1	QDCSM2	QDCSM3
Quark masses			
m_u (MeV)	313	313	313
m_s (MeV)	536	536	536
m_c (MeV)	1728	1728	1728
Confinement			
b(fm)	0.29	0.3	0.315
a_c (MeV fm^{-2})	101	101	101
$V_{0_{uu}}$ (MeV)	-2.3928	-2.2543	-2.0689
$V_{0_{us}}$ (MeV)	-1.9137	-1.7984	-1.6429
$V_{0_{uc}}$ (MeV)	-1.4175	-1.3231	-1.2052
$V_{0_{ss}}$ (MeV)	-1.3448	-1.2826	-1.2745
$V_{0_{sc}}$ (MeV)	-0.7642	-0.6739	-0.5452
$V_{0_{cc}}$ (MeV)	0.6063	0.7555	0.9829
OGE			
α_s^{uu}	0.2292	0.2567	0.3019
α_s^{us}	0.2655	0.2970	0.3484
α_s^{uc}	0.3437	0.3805	0.4405
α_s^{ss}	0.3856	0.3604	0.3360
α_s^{sc}	0.5969	0.6608	0.7649
α_s^{cc}	1.5101	1.6717	1.9353

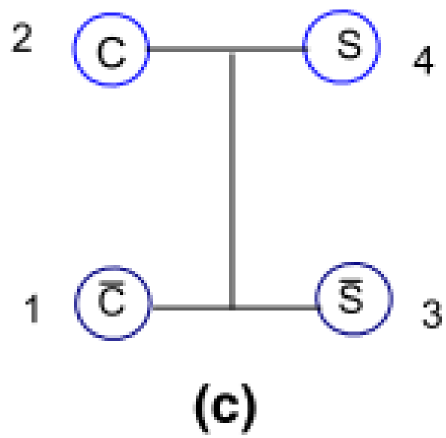
Table 2 The masses (in MeV) of the ground mesons. Experimental values are taken from the Particle Data Group (PDG) [45]

	K	K^*	π	ρ	ω	η	η'
Exp.	495	892	139	770	782	548	958
QDCSM1	495	892	139	770	585	535	958
QDCSM2	495	892	139	770	598	538	958
QDCSM3	495	892	139	770	615	527	958
	ϕ	D_s	D_s^*	η_c	J/ψ	D	D^*
Exp.	1020	1968	2112	2983	3096	1865	2007
QDCSM1	1020	1968	2112	2983	3096	1865	2007
QDCSM2	1020	1968	2112	2983	3096	1865	2007
QDCSM3	1020	1968	2112	2983	3096	1865	2007

● Two kinds of configurations



- Meson-meson configuration



- Diquark-antidiquark configuration

- **The wave function construction**

  - Four fundamental degrees of freedom

    Color, spin, flavor,orbit

  - The multiquark system’s wave function is an internal product of the color, spin, flavor, and orbit terms

    The total wave function=color*spin*flavor*orbit

✓ The color wave function

- The color wave function of a q $\bar{q}$ cluster

$$C^1_{[111]} = \sqrt{\frac{1}{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

$$C^2_{[21]} = r\bar{b},\ C^3_{[21]} = -r\bar{g}$$

$$C^4_{[21]} = g\bar{b},\ C^5_{[21]} = -b\bar{g}$$

$$C^6_{[21]} = g\bar{r},\ C^7_{[21]} = b\bar{r}$$

$$C^8_{[21]} = \sqrt{\frac{1}{2}}(r\bar{r} - g\bar{g})$$

$$C^9_{[21]} = \sqrt{\frac{1}{6}}(-r\bar{r} - g\bar{g} + 2b\bar{b})$$

[111]	color-single(1_c)
[21]	color-octer(8_c)

$$\chi^c_1 = C^1_{[111]}C^1_{[111]}$$

1_c* 1_c

$$\chi^c_2 = \sqrt{\frac{1}{8}}(C^2_{[21]}C^7_{[21]} - C^4_{[21]}C^5_{[21]} - C^3_{[21]}C^6_{[21]} + C^8_{[21]}C^8_{[21]} - C^6_{[21]}C^3_{[21]} + C^9_{[21]}C^9_{[21]} - C^5_{[21]}C^4_{[21]} + C^7_{[21]}C^2_{[21]})$$

8_c* 8_c

- The color wave function of a diquark cluster and antidiquark

$$C^1_{[2]} = rr,\ C^2_{[2]} = \sqrt{\frac{1}{2}}(rg + gr)$$

$$C^3_{[2]} = gg,\ C^4_{[2]} = \sqrt{\frac{1}{2}}(rb + br)$$

$$C^5_{[2]} = \sqrt{\frac{1}{2}}(gb + bg),\ C^6_{[2]} = bb$$

$$C^7_{[11]} = \sqrt{\frac{1}{2}}(rg - gr),\ C^8_{[11]} = \sqrt{\frac{1}{2}}(rb - br)$$

$$C^9_{[11]} = \sqrt{\frac{1}{2}}(gb - bg)$$

$$C^1_{[22]} = \bar{r}\bar{r},\ C^2_{[22]} = -\sqrt{\frac{1}{2}}(\bar{r}\bar{g} + \bar{g}\bar{r})$$

$$C^3_{[22]} = \bar{g}\bar{g},\ C^4_{[22]} = \sqrt{\frac{1}{2}}(\bar{r}\bar{b} + \bar{b}\bar{r})$$

$$C^5_{[22]} = -\sqrt{\frac{1}{2}}(\bar{g}\bar{b} + \bar{b}\bar{g}),\ C^6_{[22]} = \bar{b}\bar{b}$$

$$C^7_{[211]} = \sqrt{\frac{1}{2}}(\bar{r}\bar{g} - \bar{g}\bar{r}),\ C^8_{[211]} = -\sqrt{\frac{1}{2}}(\bar{r}\bar{b} - \bar{b}\bar{r})$$

$$C^9_{[211]} = \sqrt{\frac{1}{2}}(\bar{g}\bar{b} - \bar{b}\bar{g})$$

6_c*$\bar{6}_c$

$$\chi^c_3 = \sqrt{\frac{1}{6}}(C^1_{[2]}C^1_{[22]} - C^2_{[2]}C^{[2]}_{[22]} + C^3_{[2]}C^3_{[22]} + C^4_{[2]}C^4_{[22]} - C^5_{[2]}C^5_{[22]} + C^6_{2}C^6_{22})$$

$$\chi^c_4 = \sqrt{\frac{1}{3}}(C^7_{[11]}C^7_{[211]} - C^8_{[11]}C^8_{[211]} + C^9_{[11]}C^9_{[211]})$$

3_c*$\bar{3}_c$

✓ The flavor wave function

$$F_0^1 = c\bar{c}s\bar{s}$$
$$F_0^2 = c\bar{s}s\bar{c}$$
$$F_0^3 = cs\bar{c}\bar{s}$$

$$S_0^1 = \chi_{00}\chi_{00}$$
$$S_0^2 = \sqrt{\frac{1}{3}}(\chi_{11}\chi_{1-1} - \chi_{10}\chi_{10} + \chi_{1-1}\chi_{11})$$
$$S_1^3 = \sqrt{\frac{1}{2}}(\chi_{00}\chi_{11} + \chi_{11}\chi_{00})$$
$$S_1^4 = \sqrt{\frac{1}{2}}(\chi_{00}\chi_{11} - \chi_{11}\chi_{00})$$
$$S_1^5 = \chi_{00}\chi_{11}$$
$$S_1^6 = \chi_{11}\chi_{00}$$
$$S_1^7 = \sqrt{\frac{1}{2}}(\chi_{11}\chi_{10} - \chi_{10}\chi_{11})$$
$$S_2^8 = \chi_{11}\chi_{11}$$

✓ The spin wave function

- The wave function of two body clusters are:

$$\chi_{11} = \alpha\alpha,$$
$$\chi_{10} = \sqrt{\frac{1}{2}}(\alpha\beta + \beta\alpha)$$
$$\chi_{1-1} = \beta\beta$$
$$\chi_{00} = \sqrt{\frac{1}{2}}(\alpha\beta - \beta\alpha)$$

- The total spin wave functions:

✓ The orbital wave function

- The total orbital wave function can be constructed

$$\psi^L = \psi_1(R_1)\psi_2(R_2)\chi_L(R)$$

$$\chi_L(R) = \sqrt{\frac{1}{4\pi}}\left(\frac{3}{2\pi b^2}\right)\sum_{i=1}^{n} C_i \times \int exp\left[-\frac{3}{4b^2}(R - s_i)^2\right] Y_{LM}(\hat{s}_i)d\hat{s}_i$$

The complete wave function can be written as:

$$\psi = A[[\psi^L S_s^j]_{JM_J} F_I^i \chi_k^c]$$

Table 3 All possible channels for all quantum numbers

$J^{PC}=0^{++}$			$J^{PC}=1^{++}$			$J^{PC}=1^{+-}$			$J^{PC}=2^{++}$		
Index	$F_J^i;\, S_s^j;\, \chi_k^c$ [i;j;k]	Channels	Index	$F_J^i;\, S_s^j;\, \chi_k^c$ [i;j;k]	Channels	Index	$F_J^i;\, S_s^j;\, \chi_k^c$ [i;j;k]	Channels	Index	$F_J^i;\, S_s^j;\, \chi_k^c$ [i;j;k]	Channels
1	[1,1,1]	$\eta_c\eta'$	1	[1,7,1]	$J/\psi\phi$	1	[1,5,1]	$\eta_c\phi$	1	[1,8,1]	$J/\psi\phi$
2	[2,1,1]	$D_s^+D_s^-$	2	[2,4,1]	$D_s^+D_s^{*-}$	2	[1,6,1]	$J/\psi\eta'$	2	[2,8,1]	$D_s^{*+}D_s^{*-}$
3	[1,2,1]	$J/\psi\phi$	3	[3,3,3]	$(cs)(\bar{c}\bar{s})$	3	[2,3,1]	$D_s^+D_s^{*-}$	3	[3,8,3]	$(cs)(\bar{c}\bar{s})$
4	[2,2,1]	$D_s^{*+}D_s^{*-}$	4	[3,3,4]	$(cs)(\bar{c}\bar{s})$	4	[2,7,1]	$D_s^{*+}D_s^{*-}$	4	[3,8,4]	$(cs)(\bar{c}\bar{s})$
5	[3,1,3]	$(cs)(\bar{c}\bar{s})$				5	[3,4,3]	$(cs)(\bar{c}\bar{s})$			
6	[3,1,4]	$(cs)(\bar{c}\bar{s})$				6	[3,4,3]	$(cs)(\bar{c}\bar{s})$			
7	[3,2,3]	$(cs)(\bar{c}\bar{s})$				7	[3,7,3]	$(cs)(\bar{c}\bar{s})$			
8	[3,2,4]	$(cs)(\bar{c}\bar{s})$				8	[3,7,4]	$(cs)(\bar{c}\bar{s})$			

Table 4 The lowest-lying eigenenergies (in MeV) of $c\bar{c}s\bar{s}$ tetraquarks with $J^{PC} = 0^{++}$

Index	Channel	Threshold	QDCSM1			QDCSM2			QDCSM3		
			E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}
1	$\eta' \eta_c$	3942	3944	3938	3930	3944	3938	3930	3944	3938	3930
2	$D_s^+ D_s^-$	3936	3938			3938			3938		
3	$J/\psi\phi$	4117	4119			4119			4119		
4	$D_s^{*+} D_s^{*-}$	4224	4226			4226			4226		
5	$(cs)(\bar{c}\bar{s})$		4354	4168		4350	4163		4341	4138	
6	$(cs)(\bar{c}\bar{s})$		4553			4535			4499		
7	$(cs)(\bar{c}\bar{s})$		4469			4464			4453		
8	$(cs)(\bar{c}\bar{s})$		4310			4292			4252		

- The lowest energy of 3930 MeV is obtained by coupling all channels of two structures, which is 6 MeV lower than the threshold of the lowest channel Ds+Ds-. There is a bound state for the 0++ with the mass of 3930 MeV.
- For this bound state, the percentage of Ds+Ds+ is about 85%.
- X(3960) is a molecular state Ds+Ds-

This results is consistent with other work,(Lattice QCD, Bethe-Salpeter)

10.1007/JHEP06(2021)035

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Table 5 The lowest-lying eigenenergies (in MeV) of $c\bar{c}s\bar{s}$ tetraquarks with $J^{PC} = 1^{++}$

Index	Channel	Threshold	QDCSM1			QDCSM2			QDCSM3		
			E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}
1	$J/\psi\phi$	4117	4119	4082	4082	4119	4082	4082	4119	4082	4082
2	$D_s^+ D_s^{*-}$	4080	4082			4082			4082		
7	$(cs)(\bar{c}\bar{s})$		4341	4311		4360	4328		4393	4365	
8	$(cs)(\bar{c}\bar{s})$		4377			4397			4426		

Table 6 The lowest-lying eigenenergies (in MeV) of $c\bar{c}s\bar{s}$ tetraquarks with $J^{PC} = 1^{+-}$

Index	Channel	Threshold	QDCSM1			QDCSM2			QDCSM3		
			E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}
1	$\eta_c\phi$	4004	4006	4006	4006	4006	4006	4006	4006	4006	4006
2	$J/\psi\eta'$	4055	4057			4057			4057		
3	$D_s^+ D_s^{*-}$	4080	4082			4082			4082		
4	$D_s^{*+} D_s^{*-}$	4224	4226			4226			4226		
5	$(cs)(\bar{c}\bar{s})$		4434	4306		4431	4313		4424	4302	
6	$(cs)(\bar{c}\bar{s})$		4517			4504			4474		
7	$(cs)(\bar{c}\bar{s})$		4484			4480			4469		
8	$(cs)(\bar{c}\bar{s})$		4397			4379			4341		

Table 7 The lowest-lying eigenenergies (in MeV) of $c\bar{c}s\bar{s}$ tetraquarks with $J^{PC} = 2^{++}$

Index	Channel	Threshold	QDCSM1			QDCSM2			QDCSM3		
			E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}	E_{sc}	E_{cc}	E_{mix}
1	$J/\psi\phi$	4117	4122	4122	4119	4122	4121	4119	4122	4121	4119
2	$D_s^{*+}D_s^{*-}$	4224	4228			4229			4229		
3	$(cs)(\bar{c}\bar{s})$		4404	4399		4427	4421		4453	4444	
4	$(cs)(\bar{c}\bar{s})$		4409			4431			4454		

- The energy of each channel is above the threshold of the corresponding channel
- There is no bound state in 1++, 1+-, 2++ system

● The Stabilization method

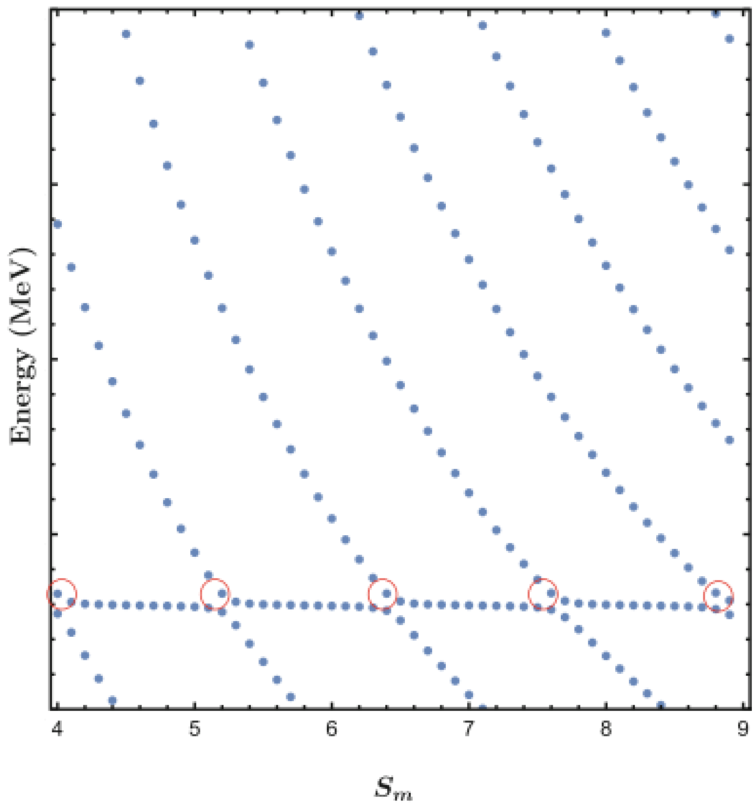
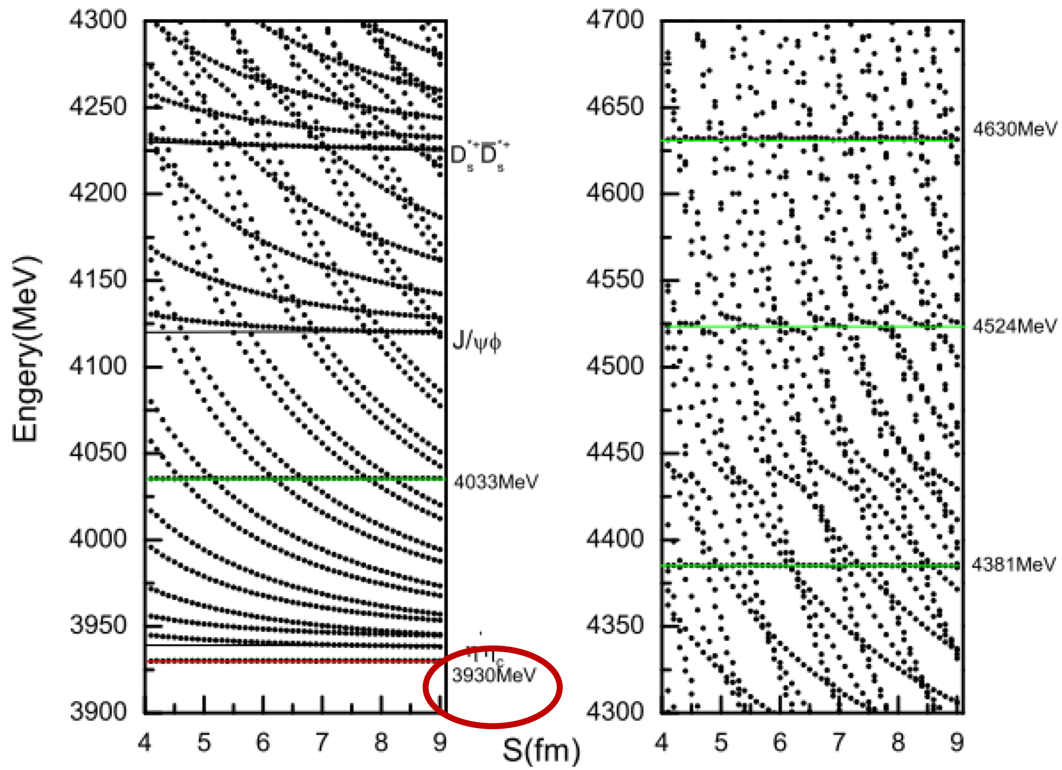


Figure 4 (Color online) Resonance shape in the real-scaling method.

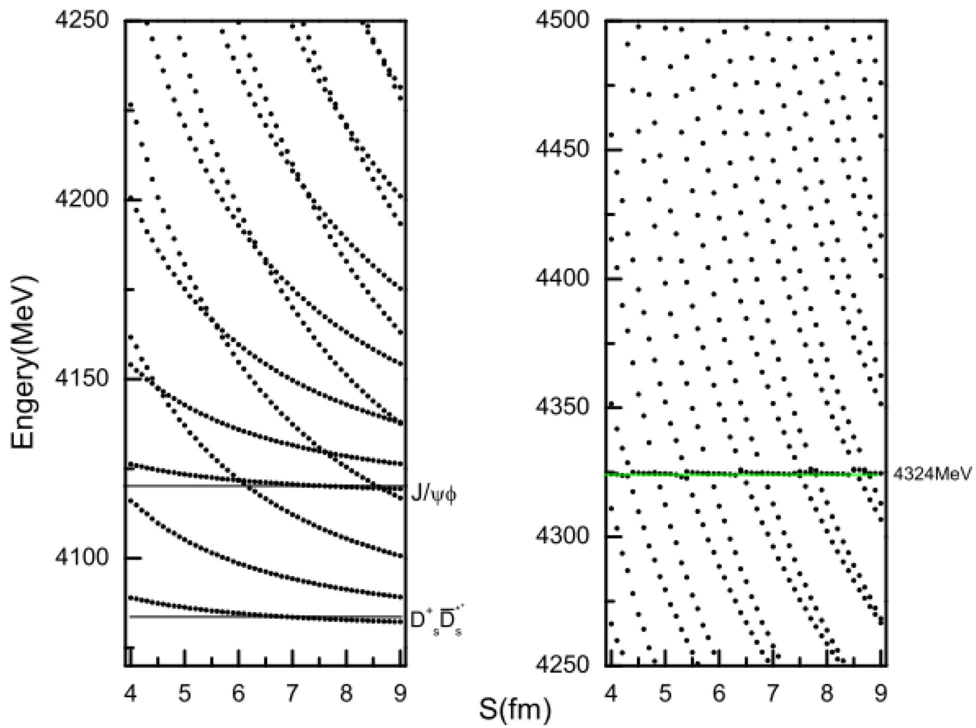
- A factor S, which is the distance between two cluster, is used to scale the finite volume.
- With the increase of the distance between two clusters, the continuum state will fall off toward its threshold.
- The bound states will remain unchanged
- A resonance state will tend to stable and act as an avoid-crossing structure.

➢ The stabilization plots of the energies of 0++



- A bound state and four resonance states
- X(4381) is close to the X(4350), so X(4350) can be explained as a compact tetraquark resonance state with 0++
- This result is agree with some work(Born–Oppenheimer approach X(4370))
  Phys. Rev. D 90, 014044 (2014)
- X(4524) is close to the X(4500), so it is possible to be a compact tetraquark with 0++
- X(4630) is close to X(4630), but the 0++ is different from the reported 0-. X(4630) may be X(4700).

- The stabilization plots of the energies of 1++

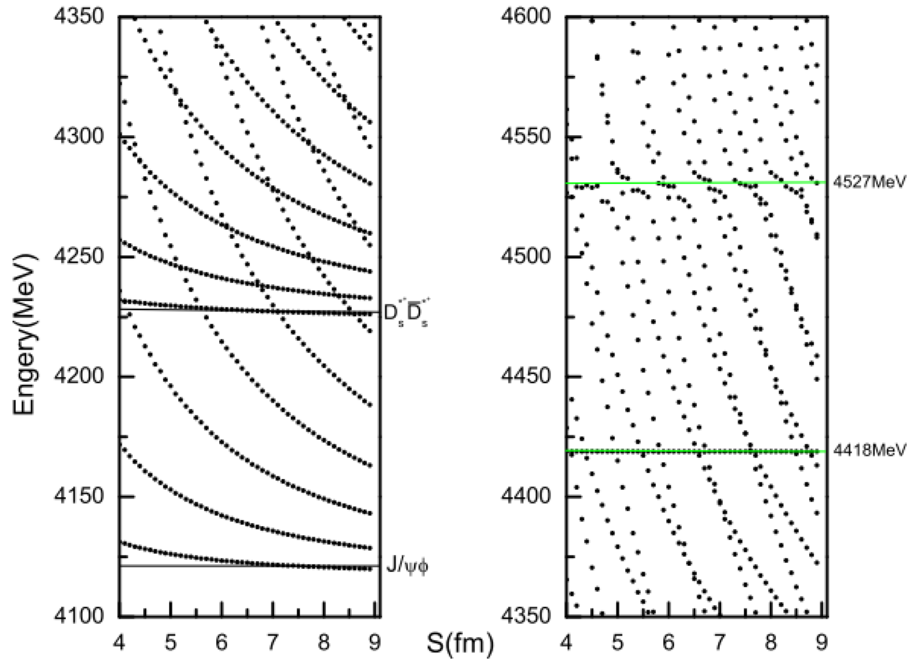


- A resonance states： X(4324)
- X(4324) can be explain as X(4274)
- This result is similar to the results of other work.

Phys. Rev. D 83, 034010 (2011)

(a resonance state with energy near 4.3 GeV is considered as X(4274))

➢ The stabilization plots of the energies of 2++



- Two resonance states

● The short summery

Table 8 The energies (in MeV) of the resonance states for the $c\bar{c}s\bar{s}$ system

QDCSM1					
$J^{PC} = 0^{++}$	3930*	4028	4401	4531	4632
$J^{PC} = 1^{++}$	4307				
$J^{PC} = 1^{+-}$	$--$				
$J^{PC} = 2^{++}$	4394	4536			
QDCSM2					
$J^{PC} = 0^{++}$	3930*	4033	4381	4524	4630
$J^{PC} = 1^{++}$	4324				
$J^{PC} = 1^{+-}$	$--$				
$J^{PC} = 2^{++}$	4418	4527			
QDCSM3					
$J^{PC} = 0^{++}$	3930*	4031	4378	4501	4648
$J^{PC} = 1^{++}$	4341				
$J^{PC} = 1^{+-}$	$--$				
$J^{PC} = 2^{++}$	4448	4526			

(* stands for bound state, $--$ means no resonance state exists)

- The molecular bound state Ds+Ds- with 0++ can be supposed to explain the X(3960)
- X(4350),X(4500),X(4700) can be explained as the compact tetraquark state with 0++
- X(4274) is possible to be a candidate of the compact tetraquark state with 1++

Thanks you!