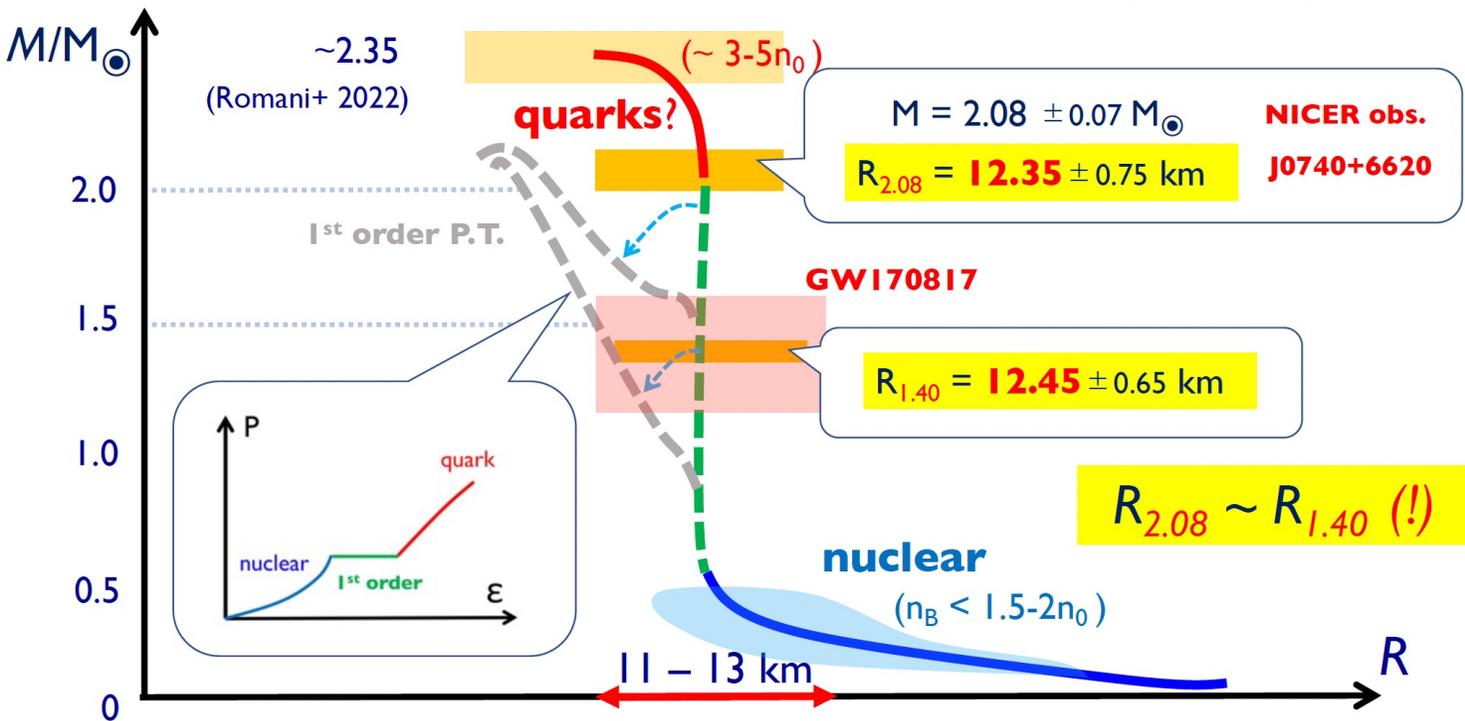


Brief review of sound velocity

Mamiya

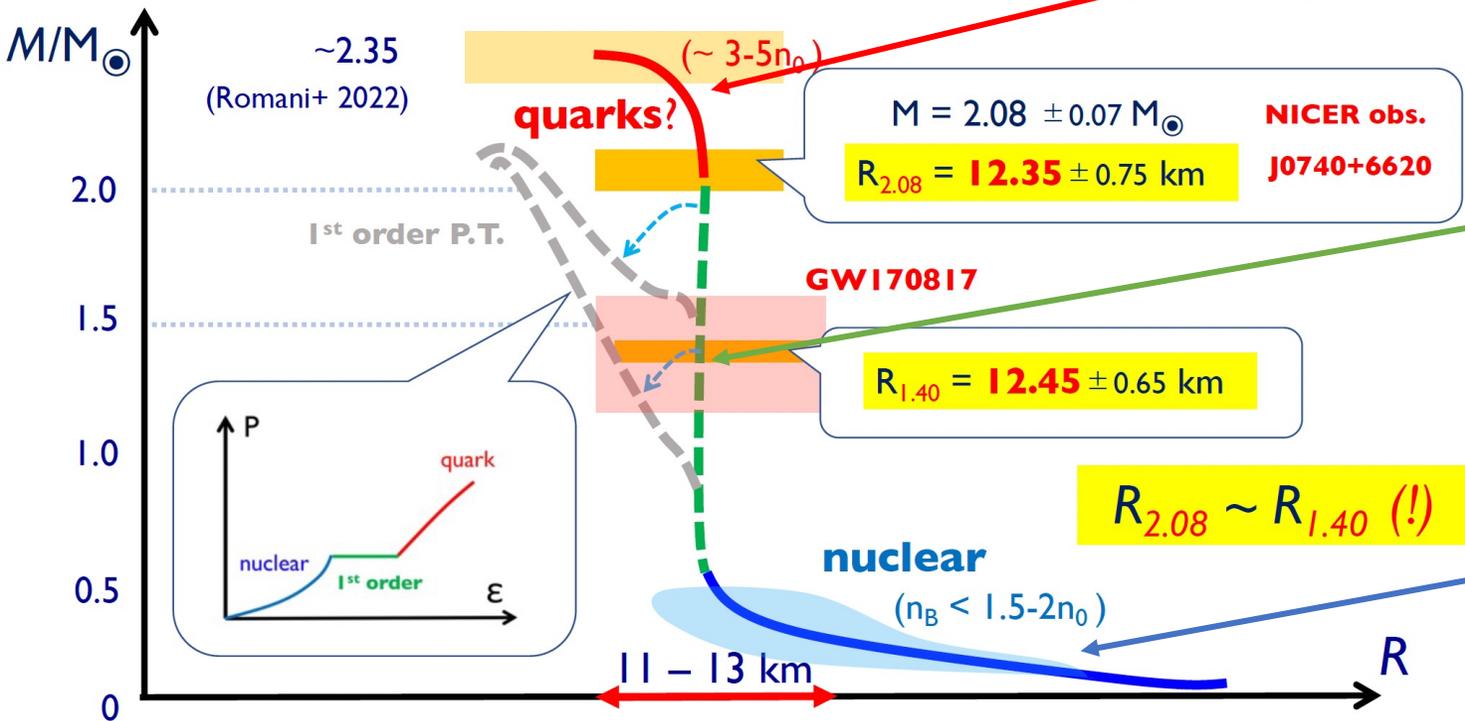
Mass and radius relation in neutron star (from Kojo's slide)

Observations: (NICER, GW170817, nuclear) [e.g., Miller+ '21]



Mass and radius relation in neutron star (from Kojo's slide)

Observations: (NICER, GW170817, nuclear) [e.g., Miller+ '21] 6/37

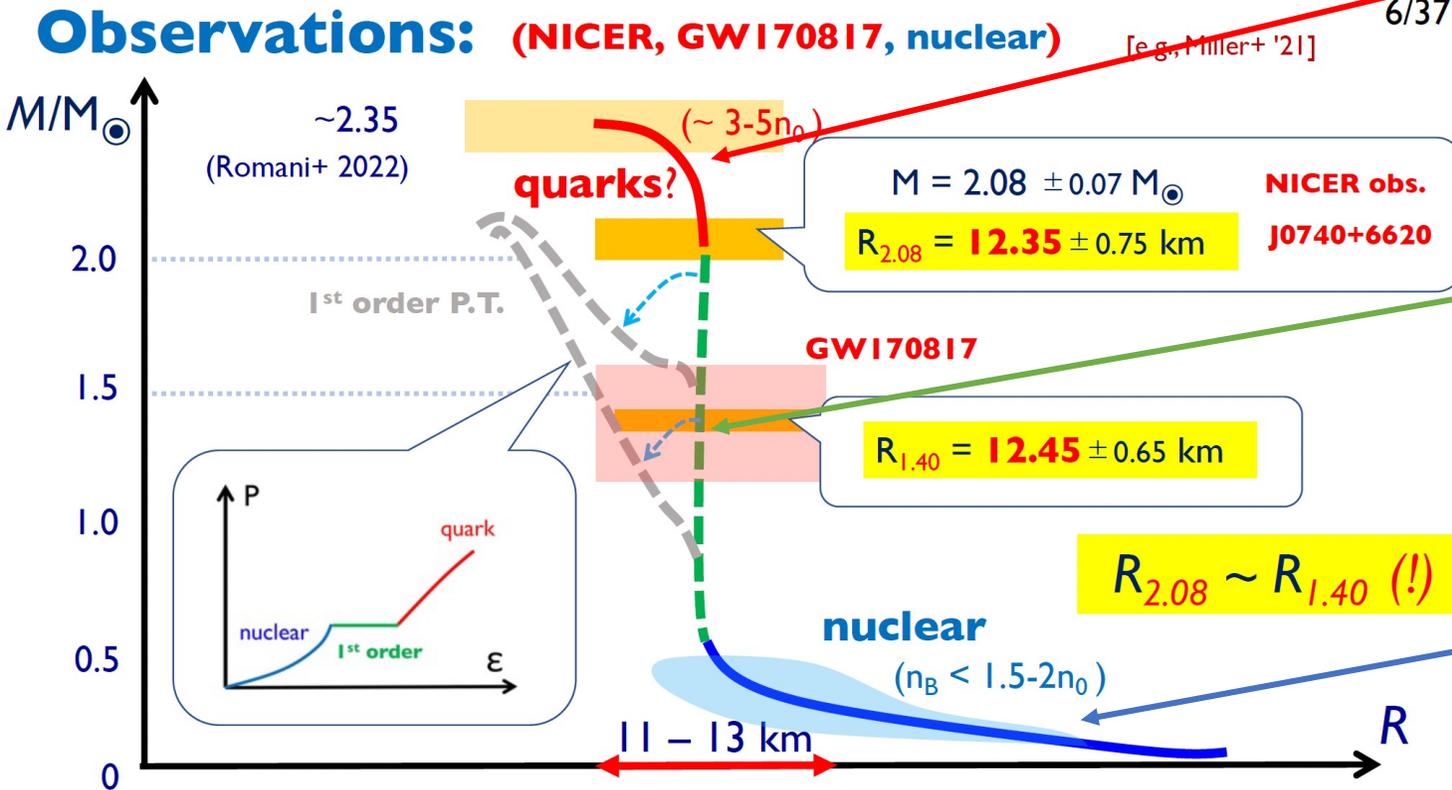


Quarks would be dominated in high density region.

Intermediate density region: phase transition would happen from nuclear phase to quark phase

QCD matter at low density regions should be described by nuclear.

Mass and radius relation in neutron star (from Kojo's slide)



Quarks would be dominated in high density region.

NJL model would work well.

Intermediate density region: phase transition would happen from nuclear phase to quark phase

We do not know relevant model... Instead, interpolating functions are used.

QCD matter at low density regions should be described by nuclear.

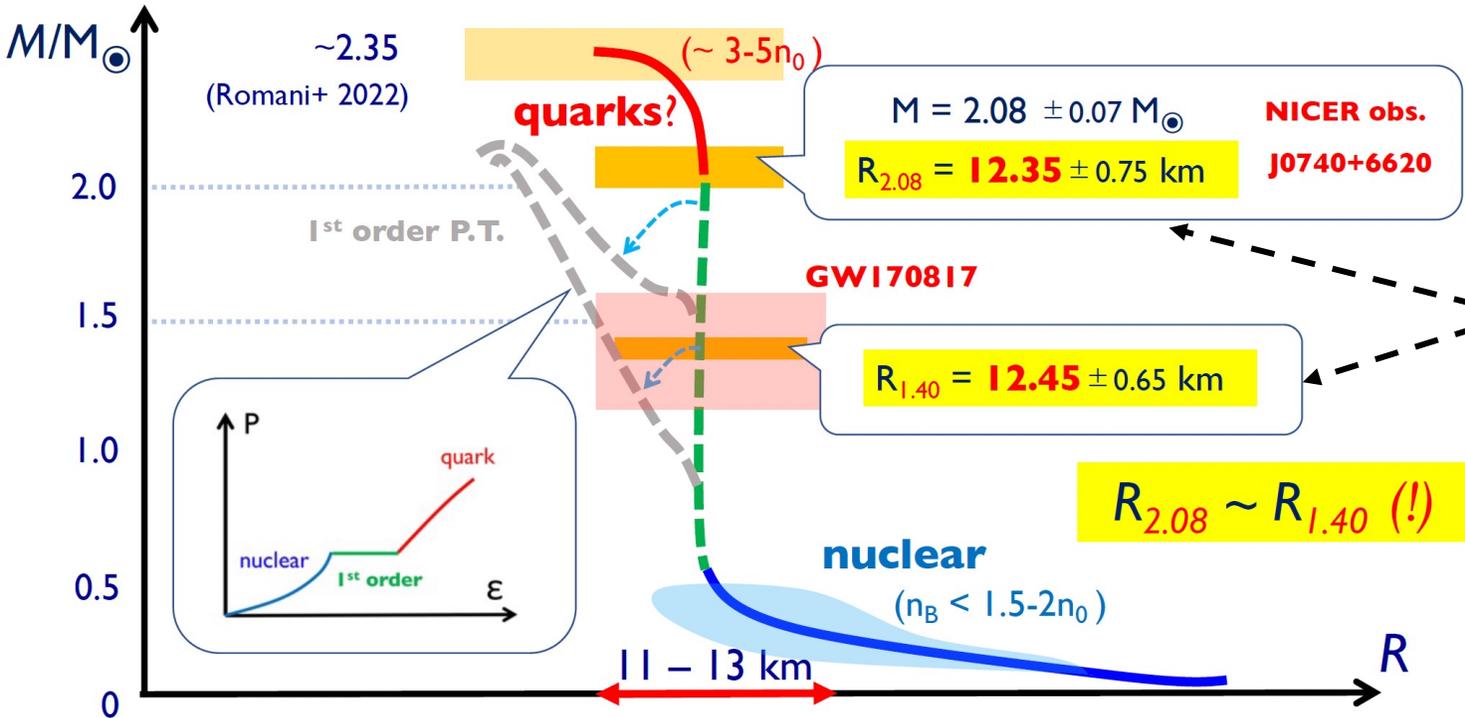
Nuclear model (Walecka Model) works.

Intermediate density region is unclear: nobody knows a relevant physical picture.
 → it is closely related to the QCD phase transition at finite density: nonperturbative property of QCD.

Mass and radius relation in neutron star (from Kojo's slide)

6/37

Observations: (NICER, GW170817, nuclear) [e.g., Miller+ '21]

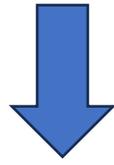


Intermediate density region:
phase transition would happen
from nuclear phase to quark phase



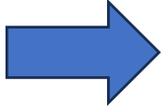
But...

There are experimental observations in intermediate density region

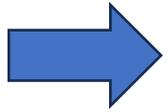


Model analysis should satisfy these observations.
→ Order of the phase transition is closely related to it.

Mass-Radius relation is crucially affected by order of the phase transition T. Kojo, arXiv:2011.10940

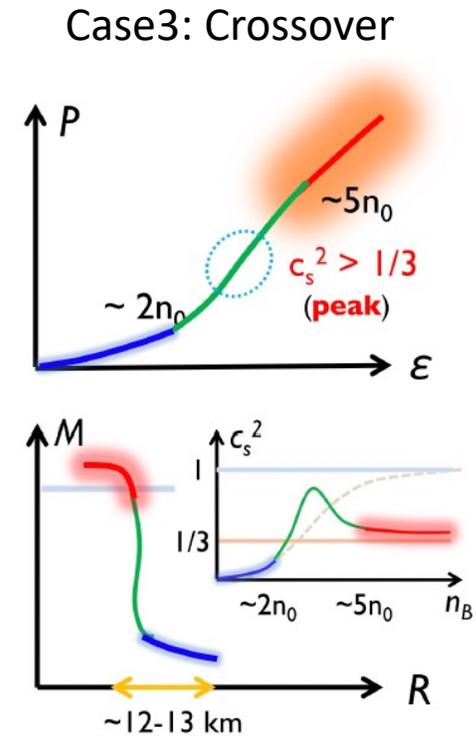
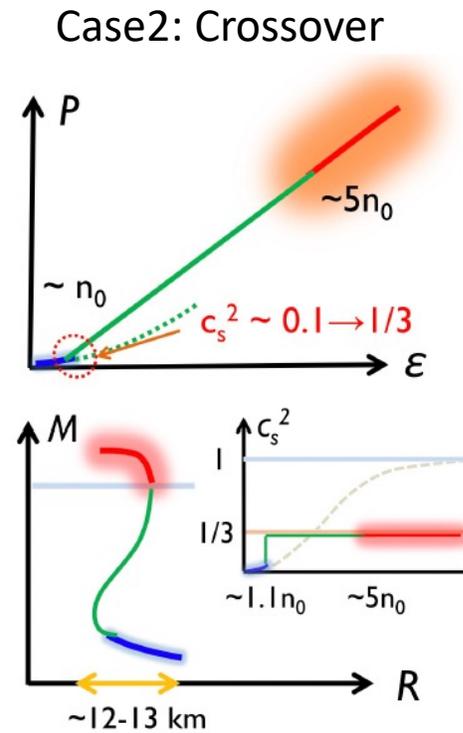
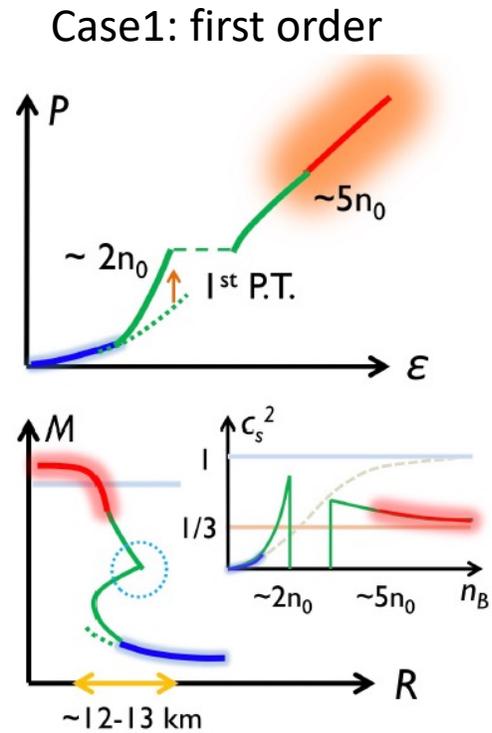


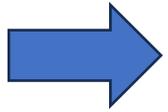
Order of phase transition is reflected to energy and pressure:
jump appears if first order phase transition happens. Otherwise smooth curve indicates the crossover



Order of phase transition is reflected to energy and pressure: jump appears if first order phase transition happens. Otherwise smooth curve indicates the crossover

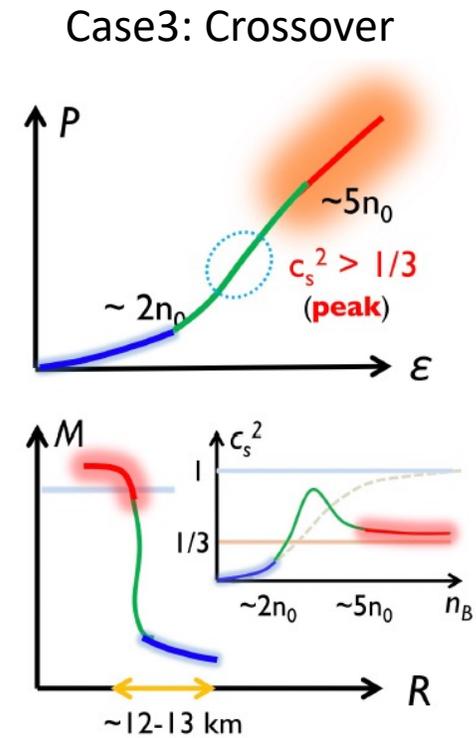
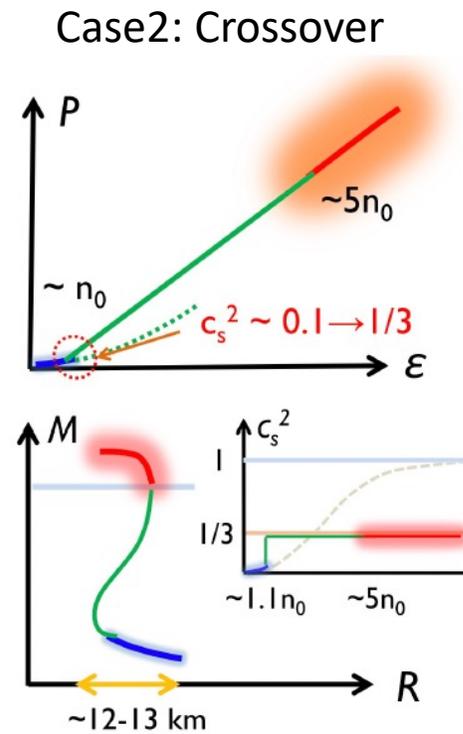
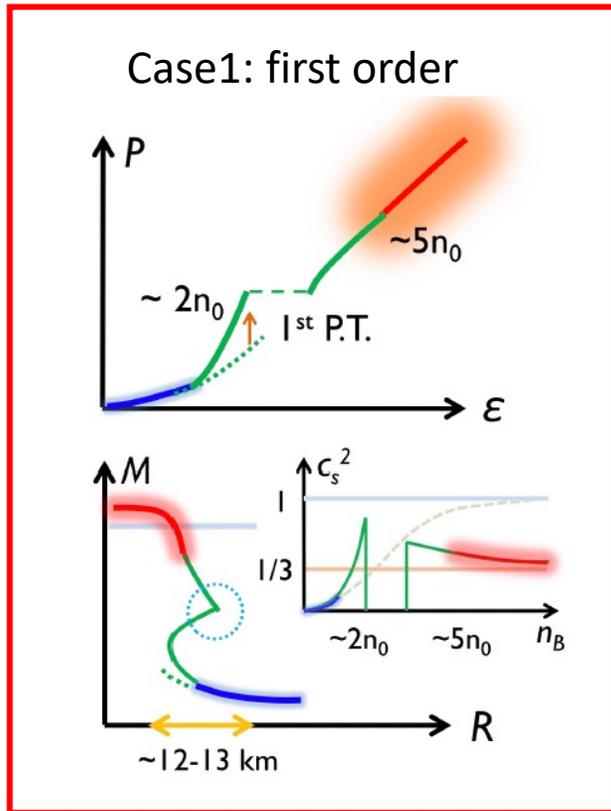
We can consider several cases.



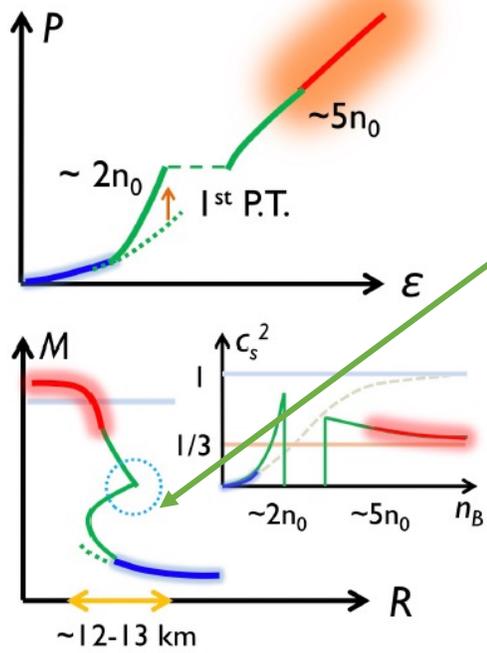


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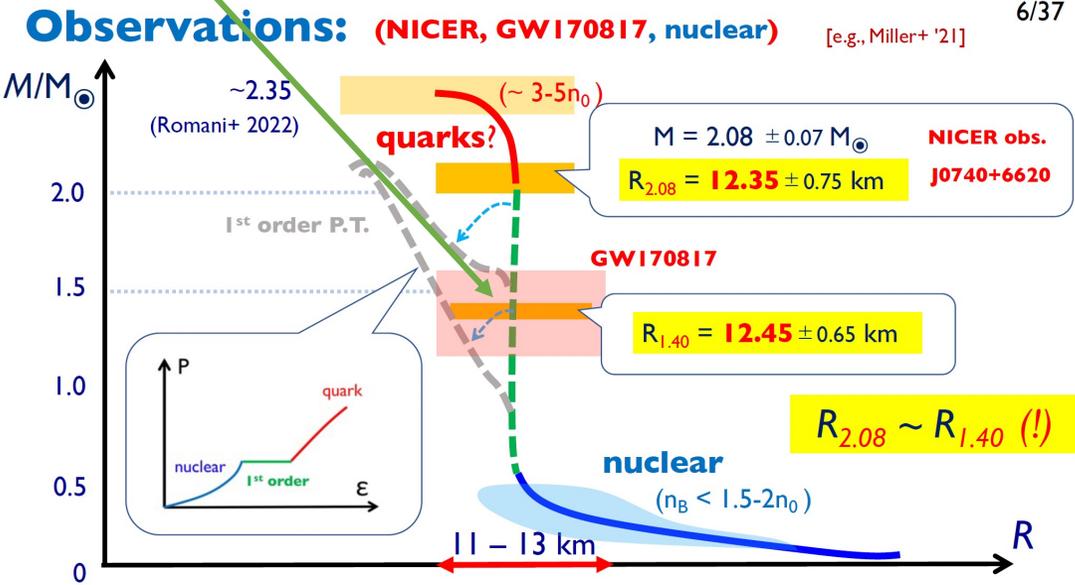


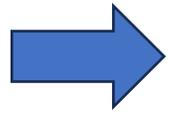
Case1: first order



Due to first order, mass and radius relation in intermediate density region becomes complicated.

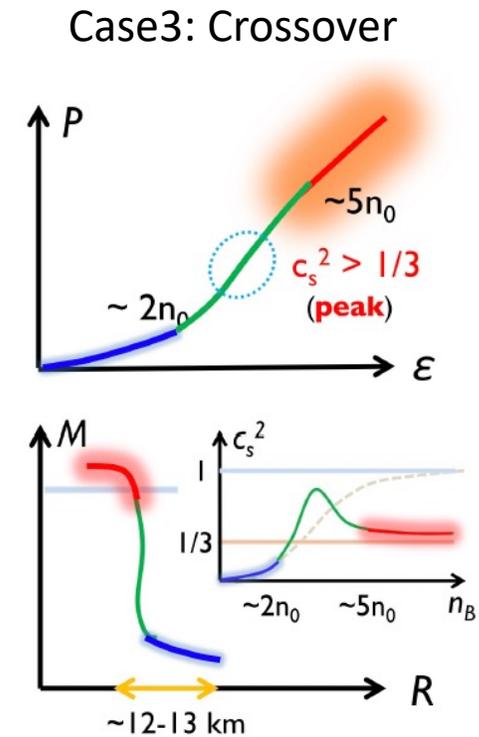
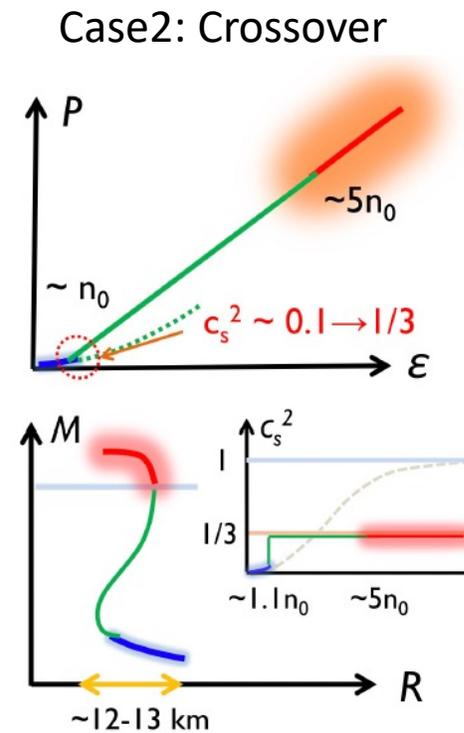
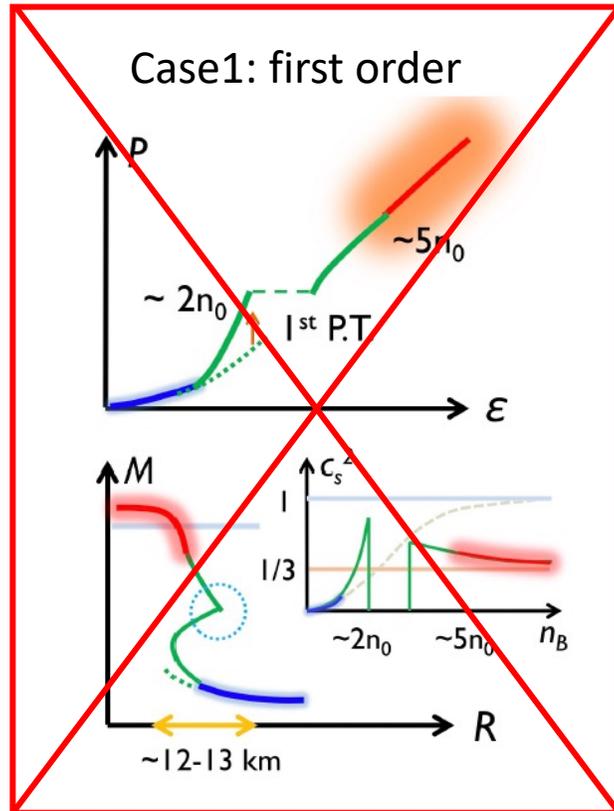
But... this complicated behavior is inconsistent with the experimental observations: **straight line** would be preferred.



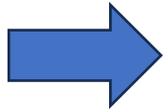


Order of phase transition is reflected to energy and pressure: jump appears if first order phase transition happens. Otherwise smooth curve indicates the crossover

We can consider several cases.

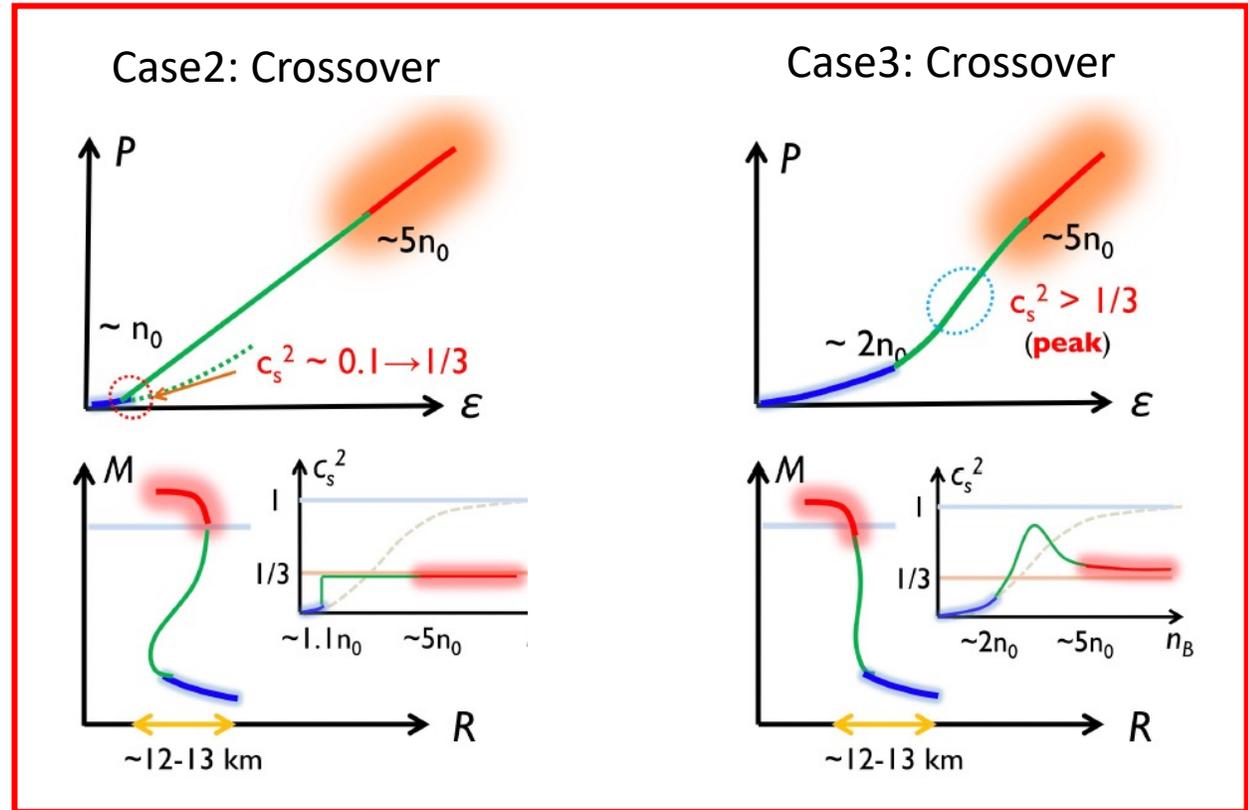
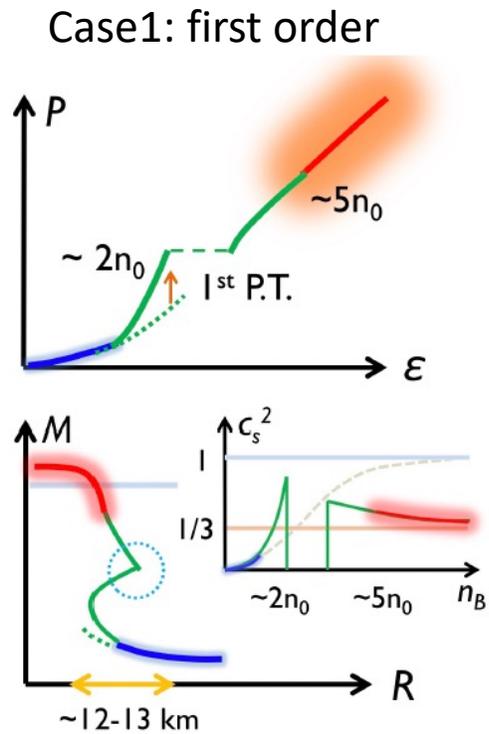


Case 1 would be discarded.



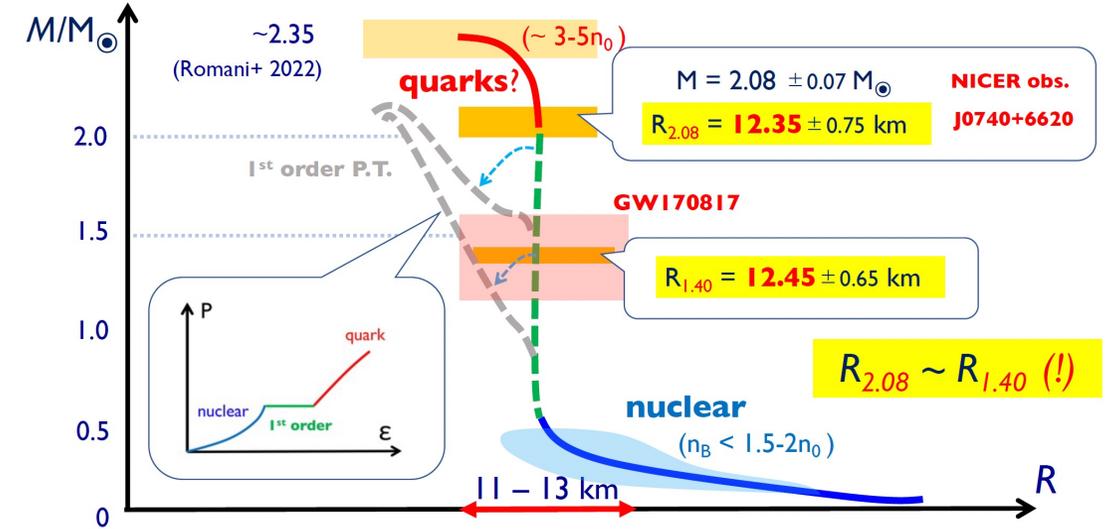
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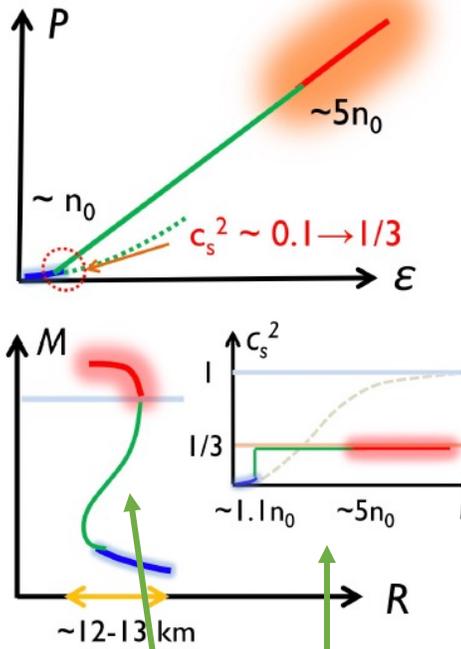
The difference comes from the behavior of the speed of sound.

Observations: (NICER, GW170817, nuclear) [e.g., Miller+ '21]

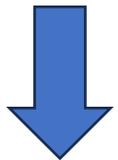


Experimental observations indicate that **straight line** would be preferred.

Case2: Crossover

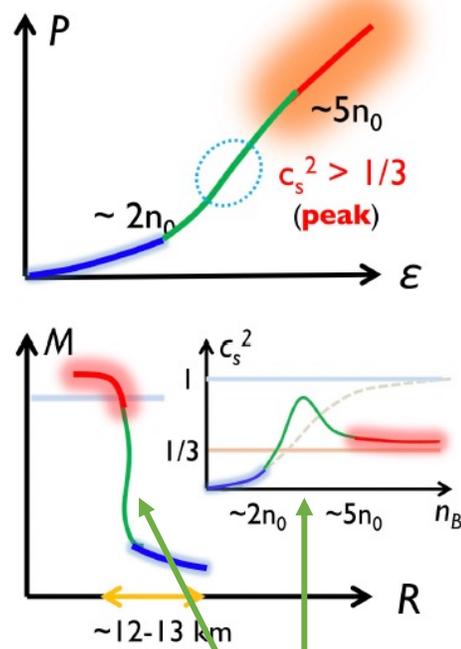


Sound velocity does not have peak.



Absence of peak makes it **curve**.

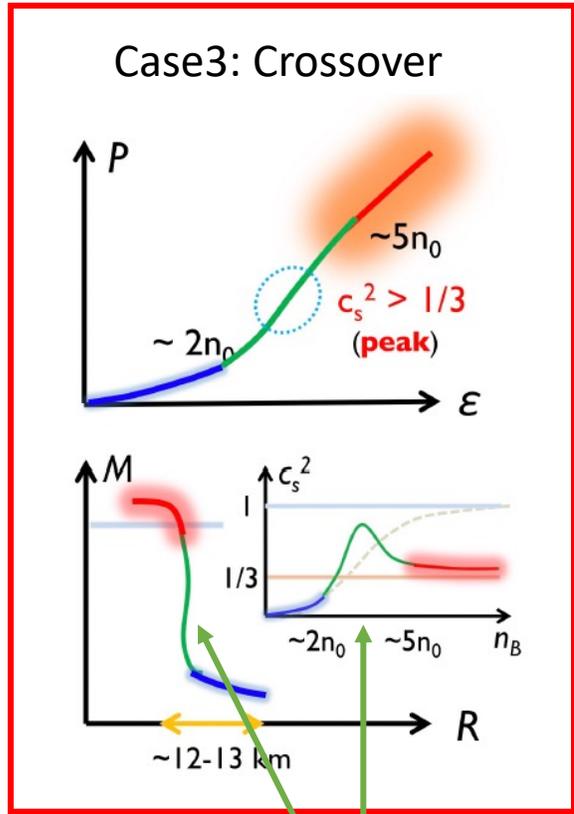
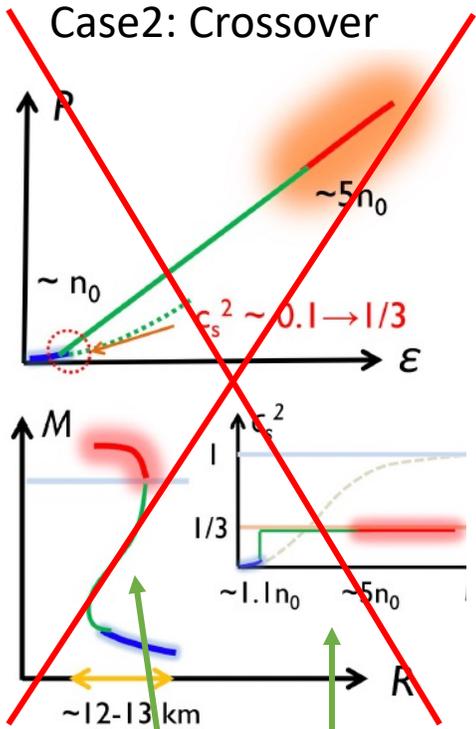
Case3: Crossover



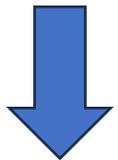
Sound velocity has peak.



Peak provides **straight line**

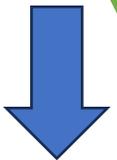


Sound velocity does not have peak.

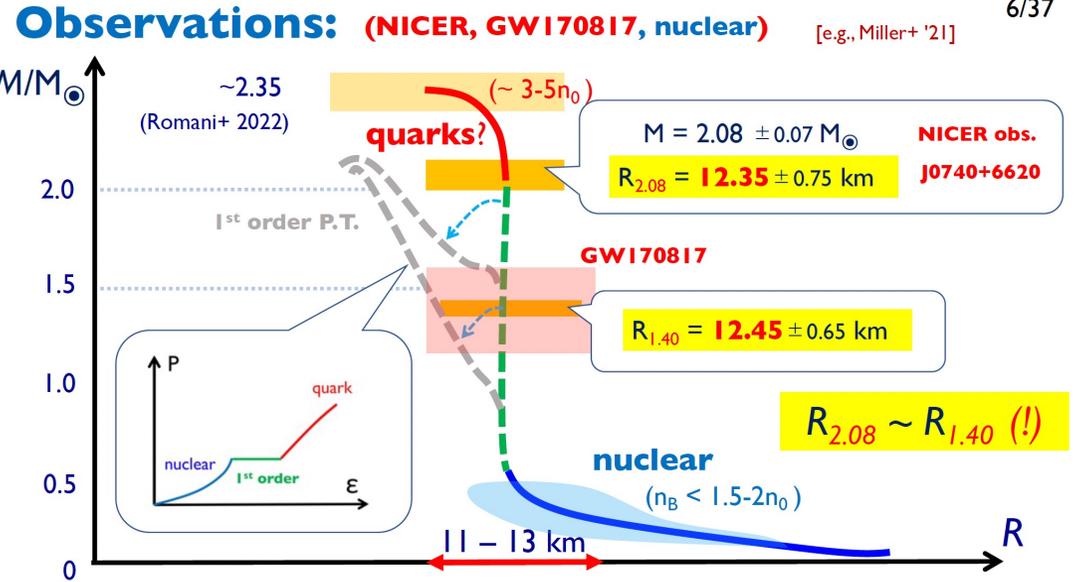


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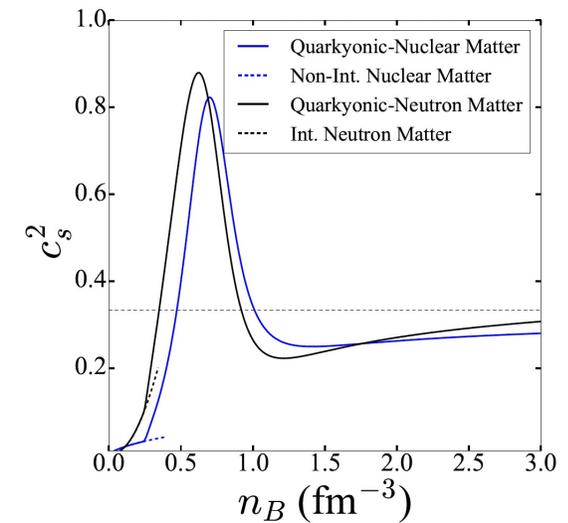
It is preferred that ...

- Energy and pressure is smooth (crossover).
- Sound velocity has peak structure.

*Note that this is one of possible scenarios. Maybe, there is a way for first order scenario to survive.

Sound velocity peak predicted by effective models

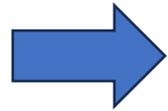
- K. Masuda, T. Hatsuda and T. Takatsuka, PTEP 2013, no.7, 073D01 (2013) (first study for sound velocity in neutron star)
 - They used the NJL model with vector interaction (three flavor analysis).
 - They discuss the correlation between the quark-hadron crossover picture observed neutron stars (M-R relation).
 - They have shown that the vector meson interaction enhances the peak structure of the sound velocity.
- L. McLerran and S. Reddy, PRL. 122, no.12, 122701 (2019)
 - They employed quarkyonic matter model.
 - Quarkyonic matter model also provides the peak.
 - Asymptotic behavior at high density has been pointed out:
“It reaches a maximum at relatively low density, decreases, and then increases again to its asymptotic value of 1/3.”
- T. Kojo and D. Suenaga, PRD 105, no.7, 076001 (2022)
 - Microscopic interpretation on the peak structure is given:
distribution of quark saturation is related to peak structure.



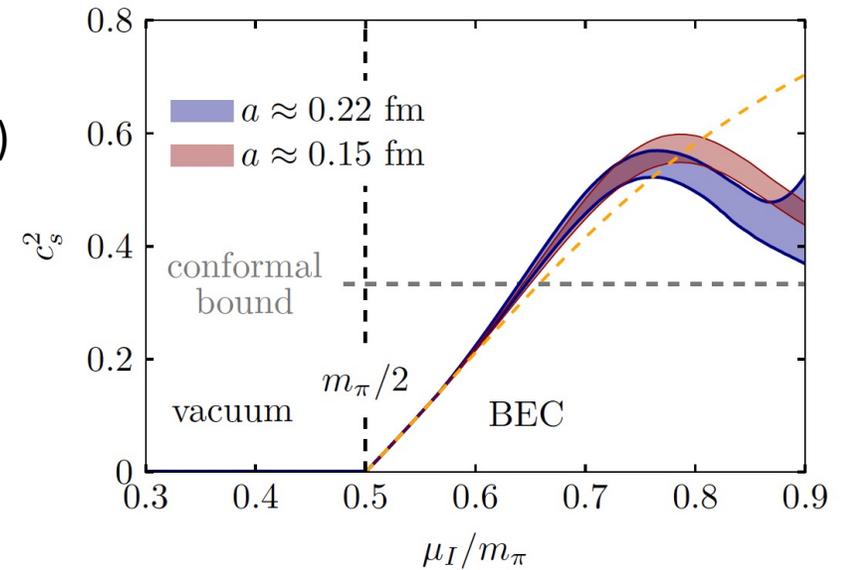
Almost analyses for the peak structure have been investigated by effective models based on quark pictures.
→ This indicates that hadron picture is irrelevant to the peak structure???
(still unclear)

Sound velocity in lattice QCD simulations (Recent hot topic)

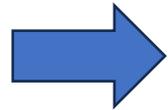
- B. B. Brandt, F. Cuteri and G. Endrodi, JHEP 07, 055 (2023)
3-color QCD at finite isospin chemical potential



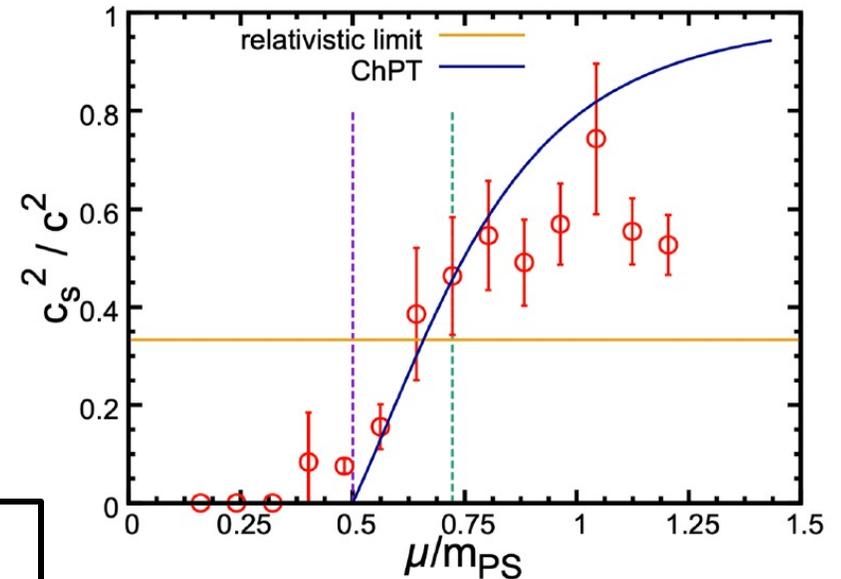
Peak appears.



- K. Iida and E. Itou, PTEP 2022, no.11, 111B01 (2022)
2-color QCD at finite baryon chemical potential



Peak appears.



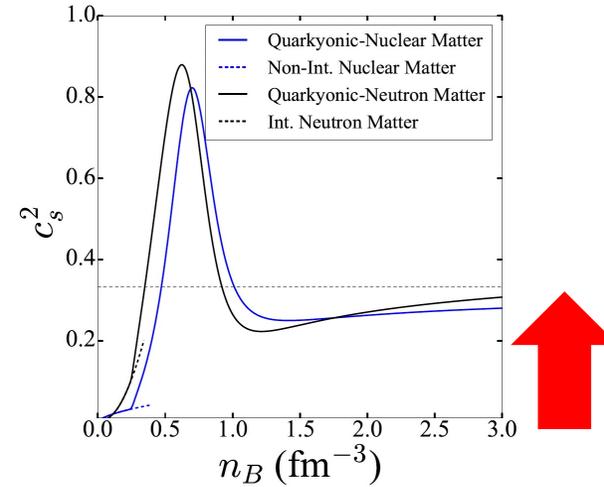
These results lead us to the following expectations:
3 color QCD at finite baryon chemical potential would have peak.

High density behavior of sound velocity (Recent hot topic)

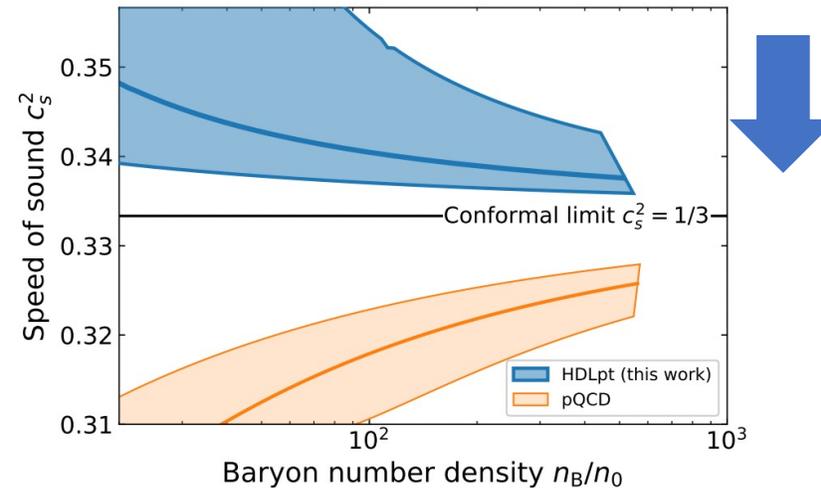
It is expected that the sound velocity approaches $1/3$.

Recently the detail of high density behavior has been extensively studied.

- L. McLerran and S. Reddy, PRL. 122, no.12, 122701 (2019)
In quarkyonic matter model,
the sound velocity **approaches $1/3$ from below.**

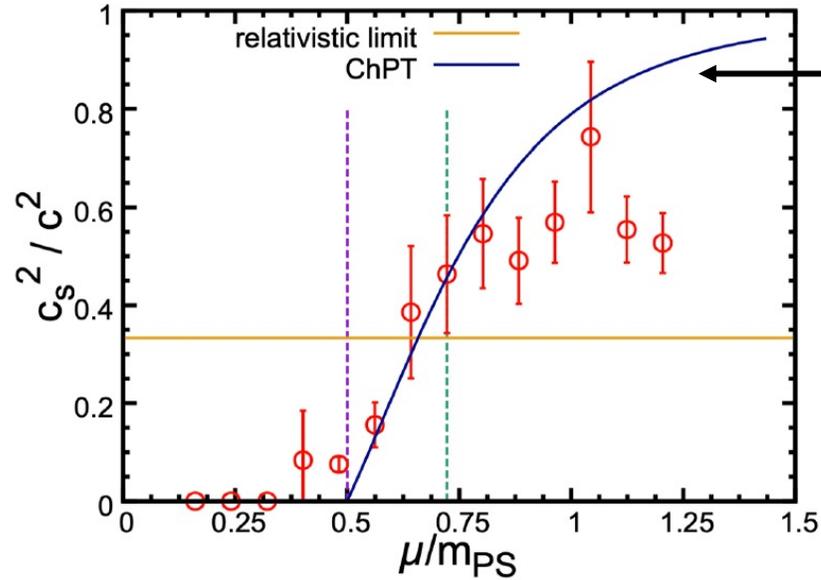


- Y. Fujimoto and K. Fukushima, PRD 105, no.1, 014025 (2022)
In hard Dense Loop resummation,
the sound velocity **approaches $1/3$ from above.**



My recent work in Riken

I focus on the two color QCD system



ChPT provides the benchmark line: $c_s^2 = \frac{1 - \mu_c^4/\mu^4}{1 + 3\mu_c^4/\mu^4}$ $\mu_c = m_\pi^{\text{vac}}/2$

Expression is independent on model parameter.



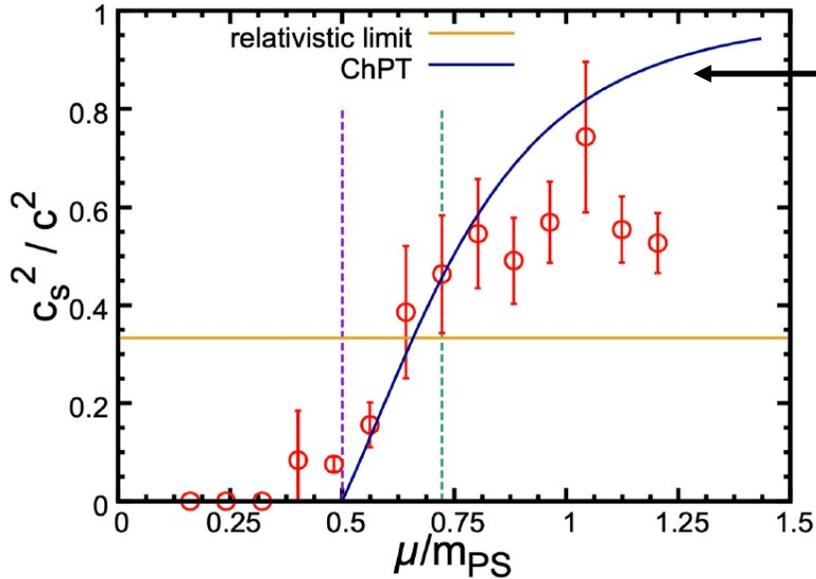
But...

ChPT result does not provide the peak structure.

→ ChPT line is inconsistent with lattice observation.

My recent work in Riken

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But...

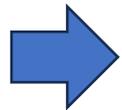
ChPT result does not provide the peak structure.
 → ChPT line is inconsistent with lattice observation.

In my work, I have found a new expression based on the linear sigma model.

$$(c_s^{\text{LSM}})^2 = \frac{n_{\text{ChPT}} + \delta n}{\mu_q (\chi_{\text{ChPT}} + \delta \chi)} = \frac{(1 - 1/\bar{\mu}^4) + 8(\bar{\mu}^2 - 1)/\delta \bar{m}_{\sigma-\pi}^2}{(1 + 3/\bar{\mu}^4) + 8(3\bar{\mu}^2 - 1)/\delta \bar{m}_{\sigma-\pi}^2},$$

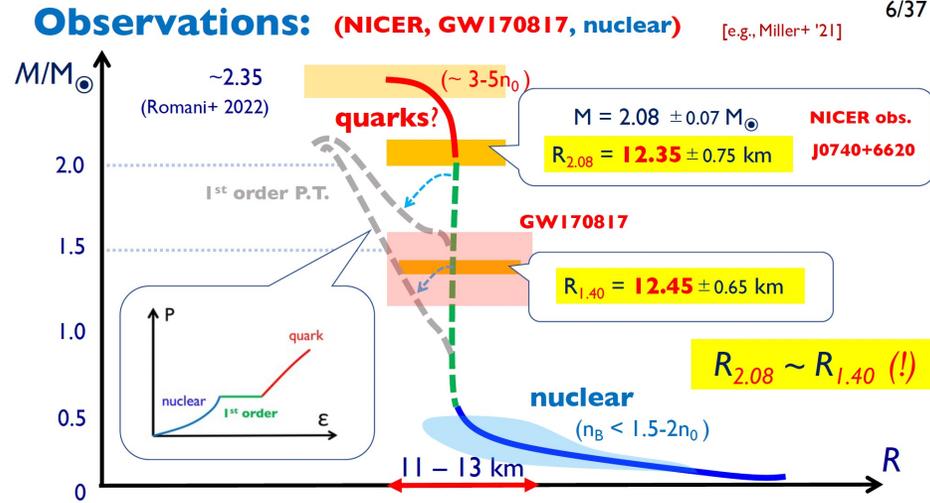
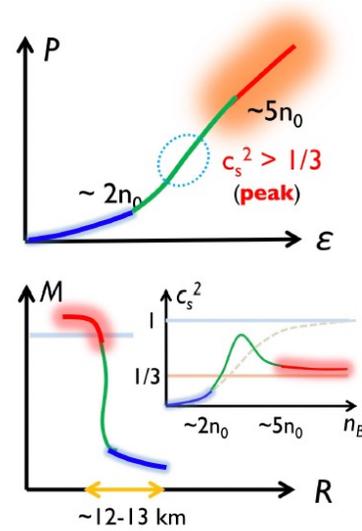
$$\delta \bar{m}_{\sigma-\pi}^2 = \{(m_\sigma^{\text{vac}})^2 - (m_\pi^{\text{vac}})^2\} / (\mu_q^{\text{cr}})^2$$

$$\bar{\mu} = \mu_q / \mu_q^{\text{cr}}$$



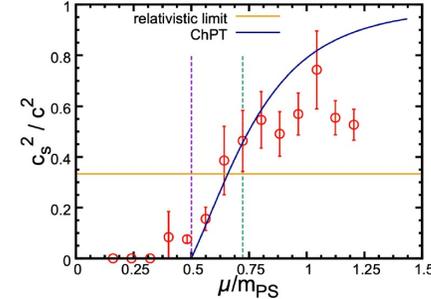
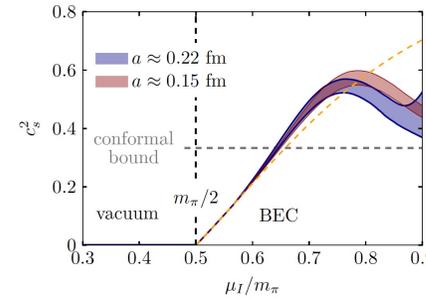
This is an expression extended from the ChPT: sigma meson mass is included.
 This is independent on the model parameters.
 Provides the peak: peak is driven by the sigma meson mass → peak is related to chiral structure.

Summary • Peak structure of sound velocity is necessary to explain the experimental observations:



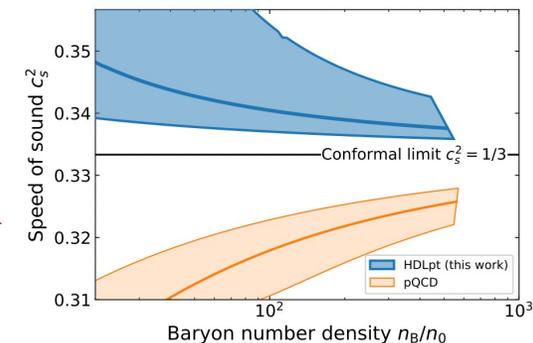
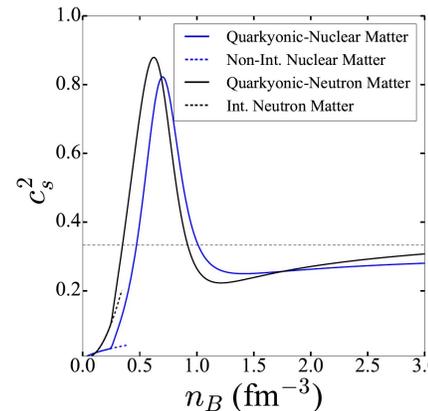
Its detail is still unclear, owing to the phase transition (non-perturbative property).

- Peak has been observed in lattices:
 - 3-color QCD at finite isospin chemical potential
 - 2-color QCD at finite baryon chemical potential
 (I focus on the 2-color QCD.)



- High density behavior of sound velocity

The sound velocity approaches 1/3 from below or above. (Fate of sound velocity is unknown.)



Thank you