



Nuclear Science
Computing Center at CCNU



Lattice QCD prediction of Pion & Kaon electromagnetic form factors at very large Q^2 :

Testing Factorization in Exclusive Processes

Heng-Tong Ding (丁亨通)

Central China Normal University (华中师范大学)

in collaboration with

X. Gao(高翔), A.D. Hanlon, S. Mukherjee, P. Petreczky,

Q. Shi(施岐), S. Syritsyn, R. Zhang and Y. Zhao

PoS LATTICE2023 (2024) 320, arXiv: 2404.xxxxxx

第三届强子与重味物理理论与实验联合研讨会

April 6-9, 2024

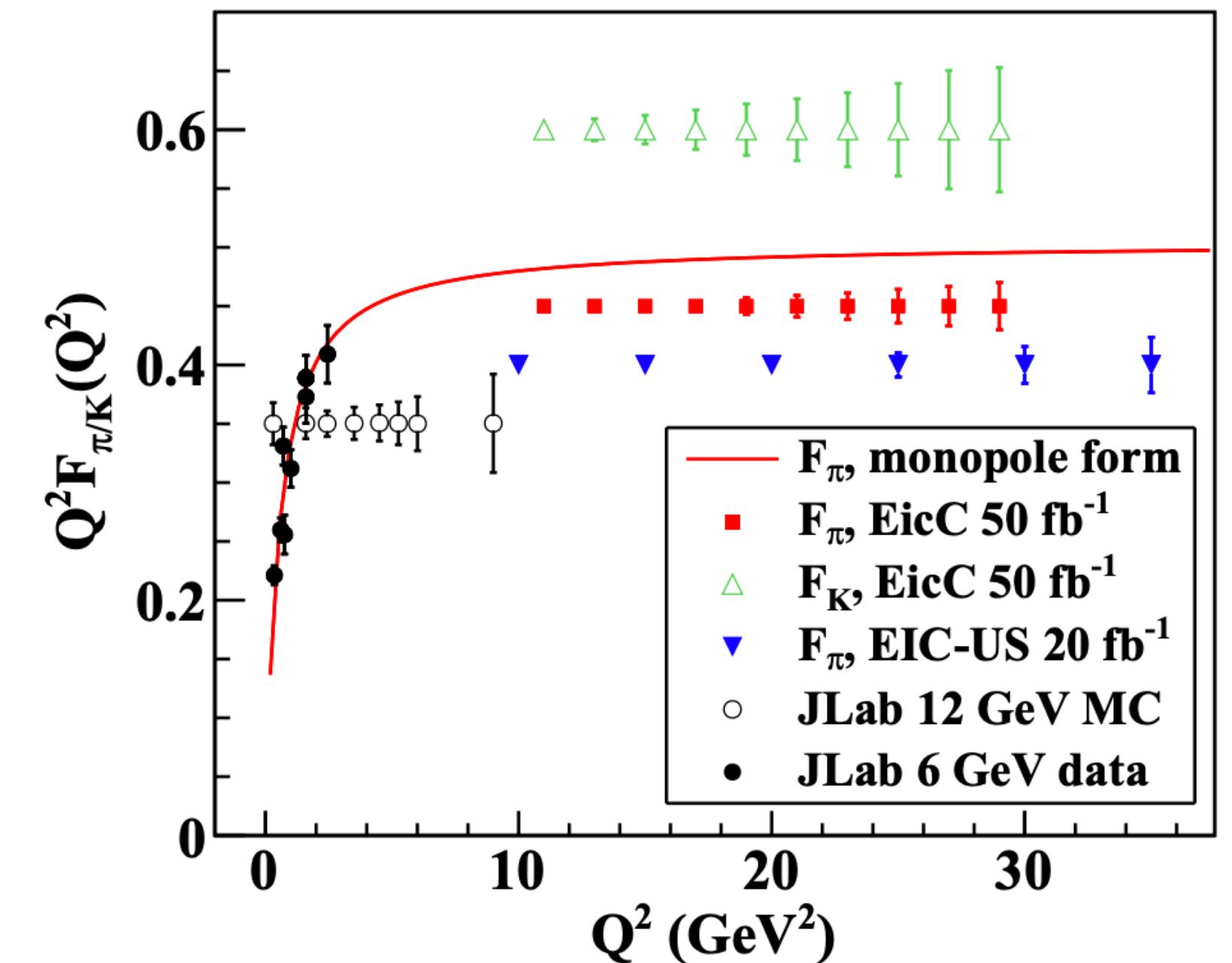
Hadron Electromagnetic (EM) form factor



Robert Hofstadter
Nobel Laureate, 1961

$$\langle H(P_1) | J_\mu | H(P_2) \rangle = (P_1 + P_2)_\mu F_H(Q^2)$$

- ✿ Insights into hadron structure, i.e. on the charge distribution
- ✿ Together with PDF produces General Parton Distribution (GPD), i.e. a 3-d image of hadron
- ★ Experiments: Jlab, EiC, EicC
- ★ pQCD, BSE, DSE, lattice QCD...



EicC white paper, arXiv:2102.09222

Pion/kaon EM form factors

Small Q^2 limit: hadronic picture

- Vector Meson Dominance \rightarrow Charge radius

$$r_{eff}^2(Q^2) = \frac{6(1/F_\pi(Q^2) - 1)}{Q^2}.$$

$$\langle r_\pi^2 \rangle = 0.42(2) \text{ fm}^2, \langle r_\pi^2 \rangle_{PDG} = 0.434(5) \text{ fm}^2$$

Large Q^2 limit: partonic picture

$$Q^2 F_M(Q^2) \approx 16\pi \alpha_s(Q^2) f_M^2 \omega_M^2(Q^2), \quad \omega_M^2(Q^2) = e_{\bar{q}} \omega_{\bar{q}}^2(Q^2) + e_u \omega_u^2(Q^2)$$

Lepage & Brodsky, 79', 80'
Efremov & Radyushkin 80'

$$\omega_f = \frac{1}{3} \int_0^1 dx q_f(x) \phi_M(x, Q^2)$$

leading-twist parton distribution amplitude (DA)

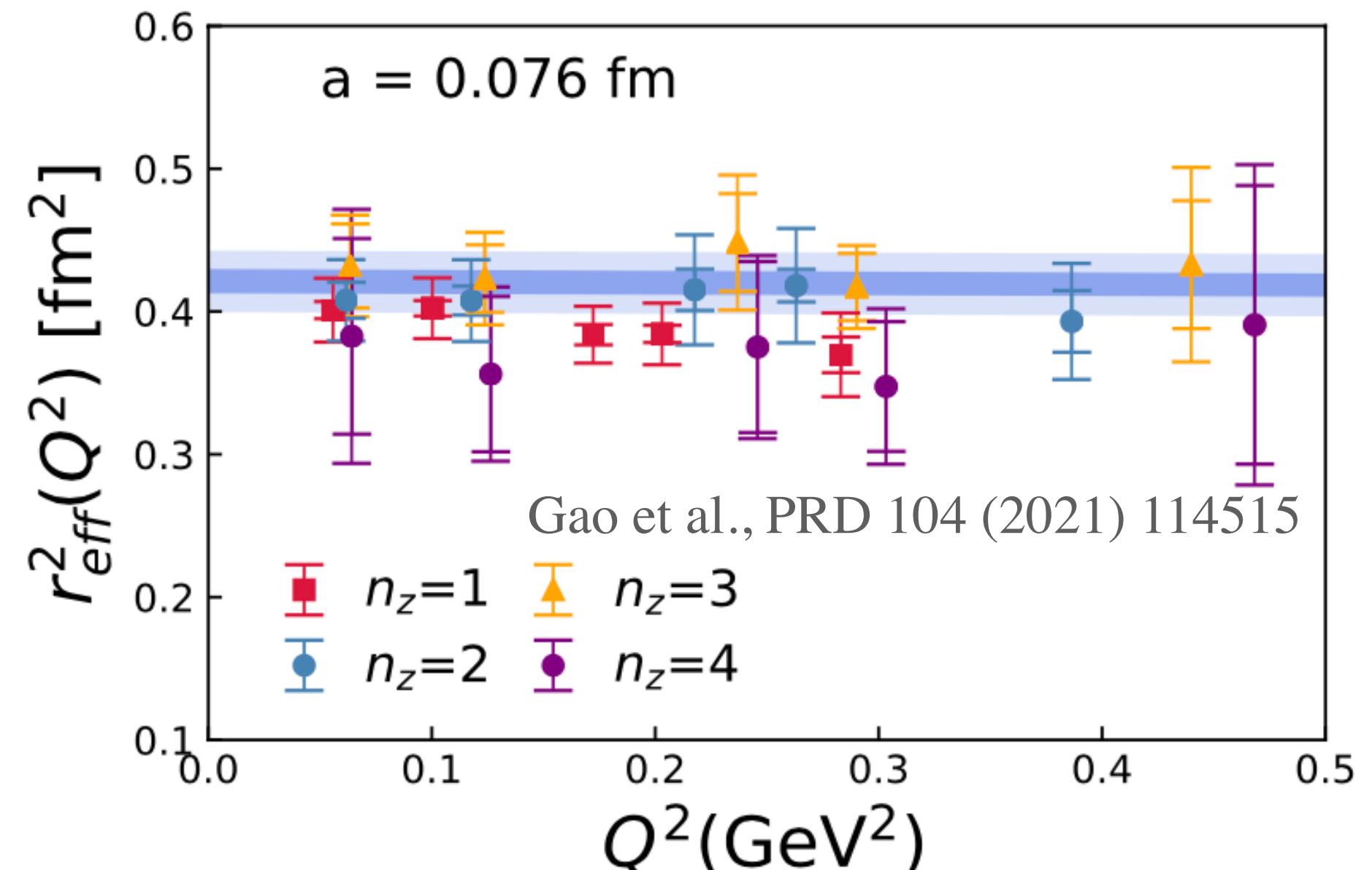
- Asymptotic DA: $\phi_M(x, Q^2 \rightarrow \infty) = 6x(1-x)$

张其安, 9:30, April 7

- DA from LQCD: pion & kaon etc.

J. Hua et al. [LPC], Phys.Rev.Lett. 129 (2022) 13

Gao et al., PRD 106 (2022) 074505, G. Bali et al., JHEP 08 (2019) 065, ...

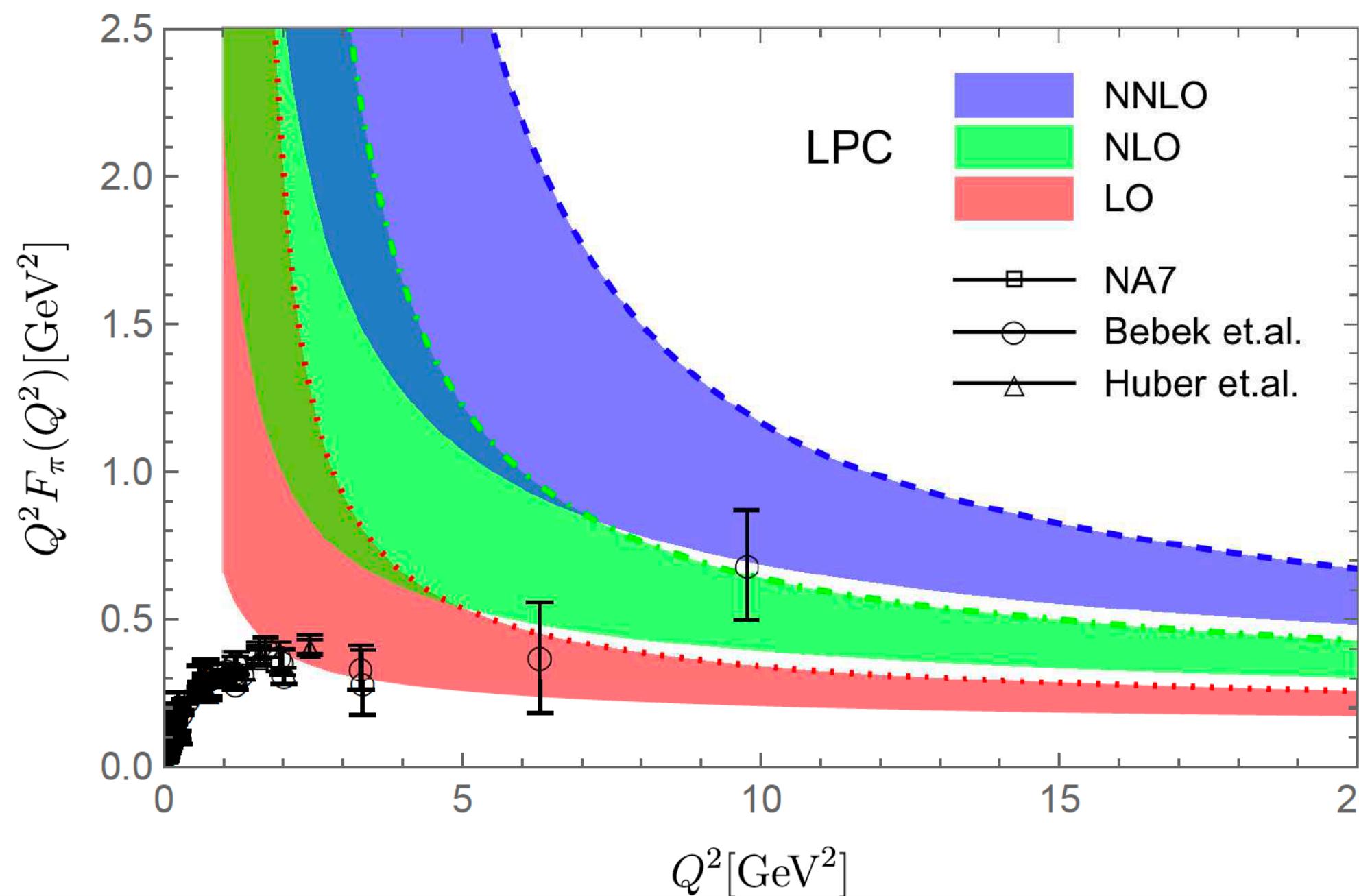


QCD factorization for hard exclusive processes

At leading twist, the collinear factorization of EM form factors

$$F_M(Q^2) = \int_0^1 \int_0^1 dx dy \phi_M^*(y, \mu_F^2) T_H(x, y, Q^2, \mu_R^2, \mu_F^2) \phi_M(x, \mu_F^2)$$

DA:
Non-perturbative
physics Hard-process kernel
 obtained in pQCD DA:
 Non-perturbative
 physics

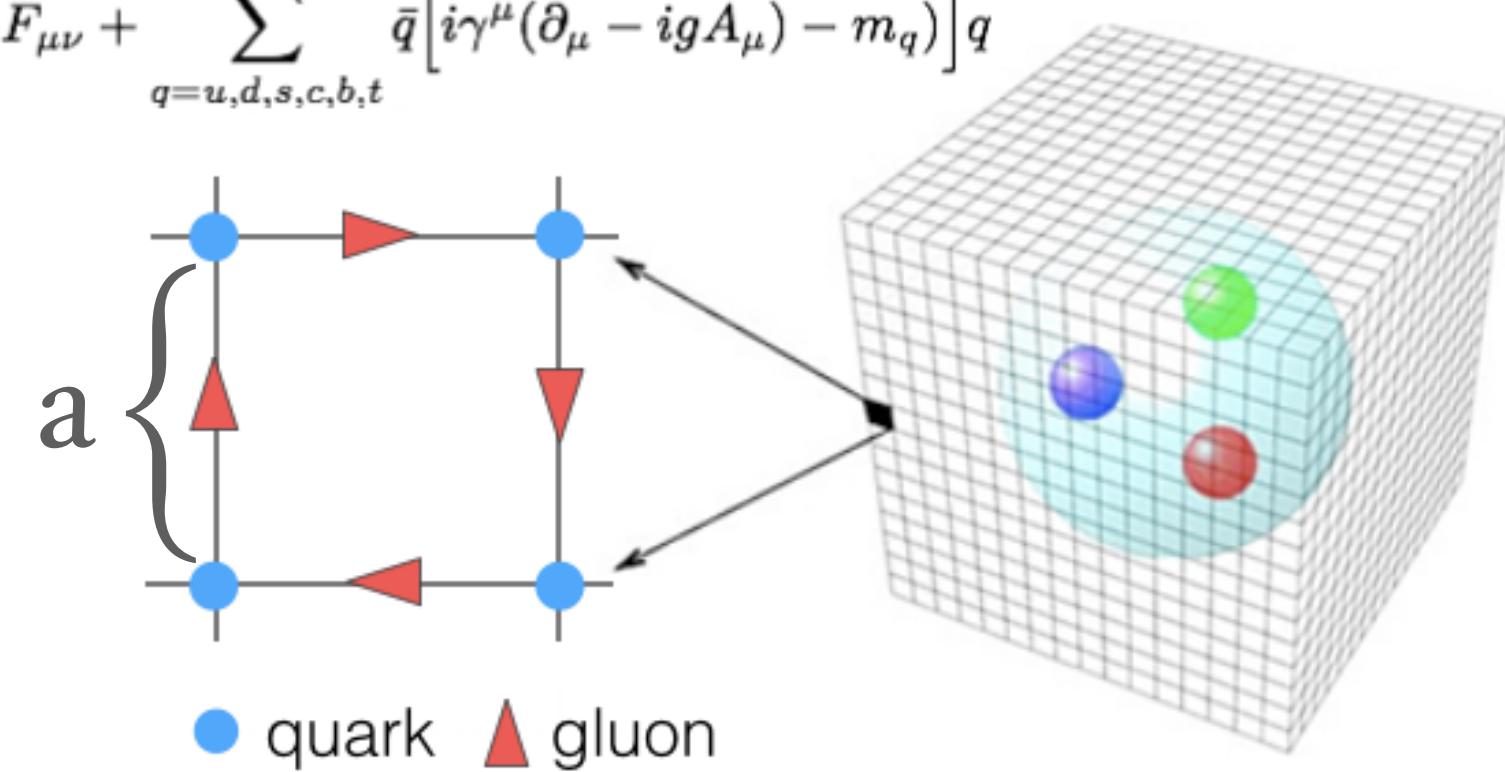


- NA7: $Q^2 \lesssim 0.25 \text{ GeV}^2$, elastic scattering of pion from atomic electron
NPB 277 (1986) 168
- Huber et al. (Jlab F_π collaboration): $Q^2 \lesssim 2.5 \text{ GeV}^2$,
PRC 78 (2008) 045203
- Bebek et al. (Cornell): $Q^2 \lesssim 10 \text{ GeV}^2$, large statistical and systematic uncertainties
PRD 17 (1978) 1693

Lattice QCD

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\left[i\gamma^\mu(\partial_\mu - igA_\mu) - m_q\right]q$$



- ➊ Lattice simulations of QCD give first principle results

- ➋ But need to have control of

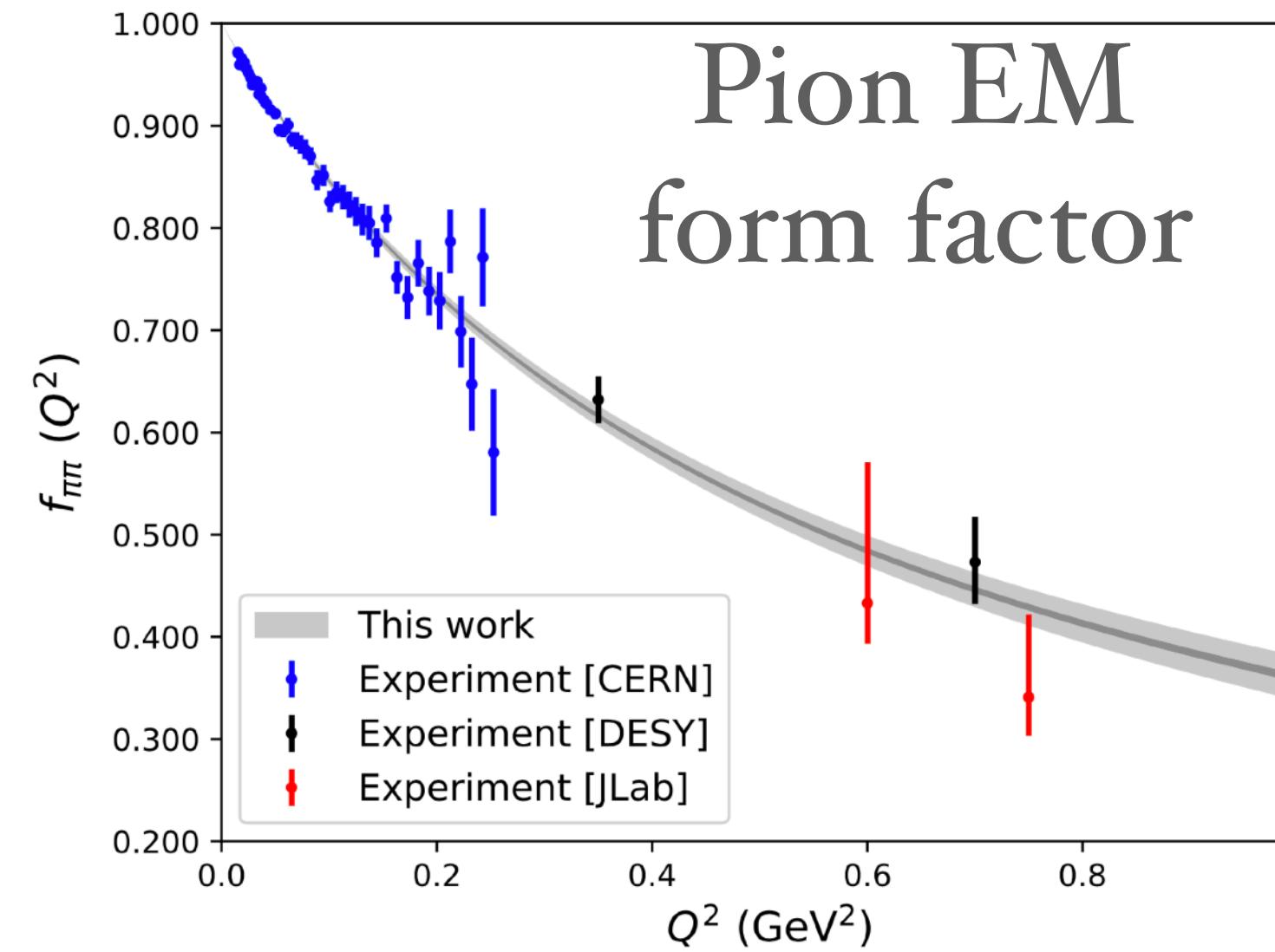
♣ Thermodynamic limit	$V = 2 \sim 4 \text{ fm}$	$V \rightarrow \infty$
♣ Continuum limit	$a = 0.1 \sim 0.04 \text{ fm}$	$a \rightarrow 0$
♣ Chiral extrapolation	$M_\pi \sim 500 \rightarrow 200 \text{ MeV}$	$M_\pi = 140 \text{ MeV}$ (Physical Point)
♣ Statistical errors	$N_{conf} \sim \mathcal{O}(1000)$	$N_{conf} \rightarrow \infty$

- ➌ Fast computers and algorithms are essential

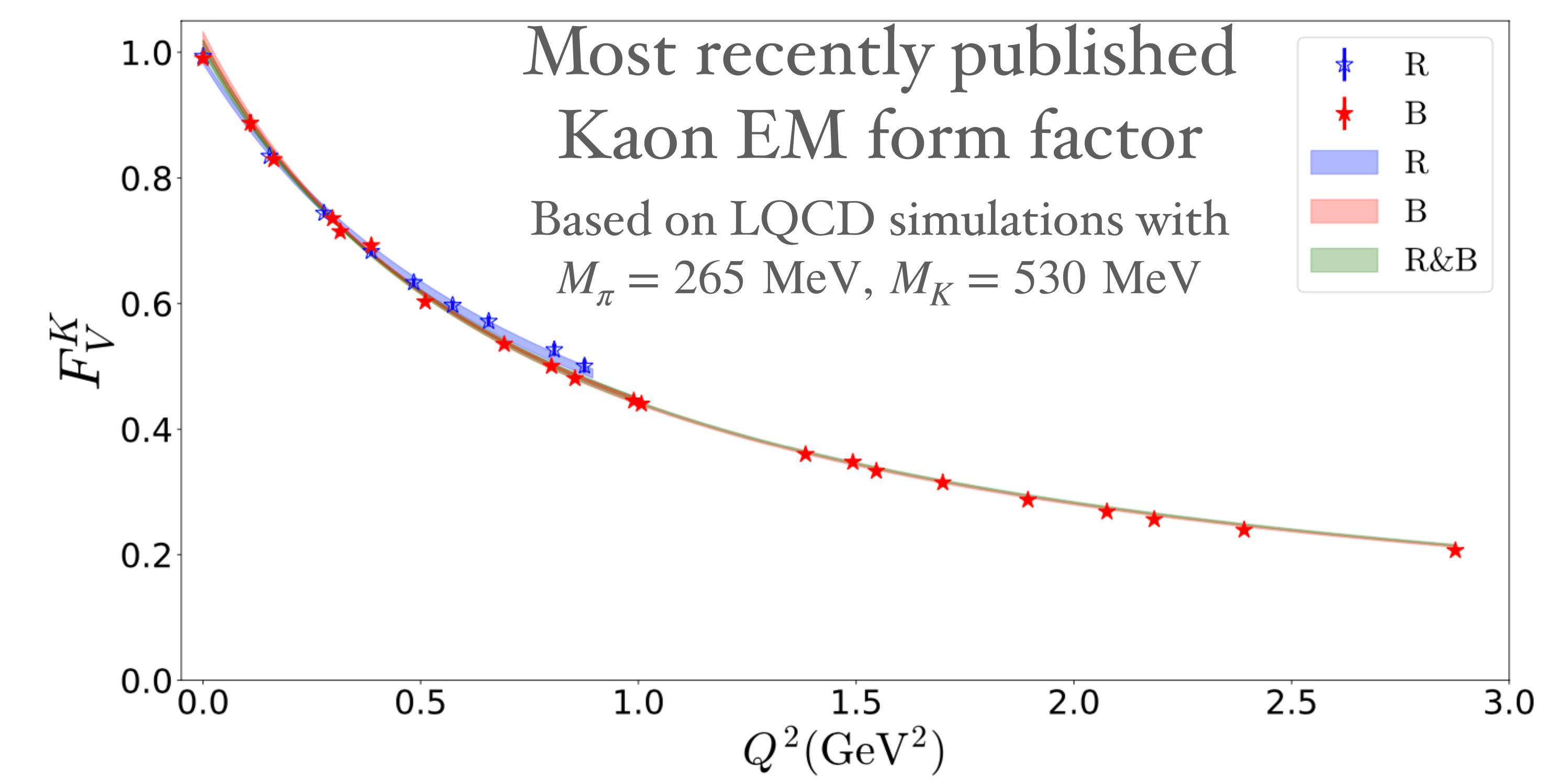
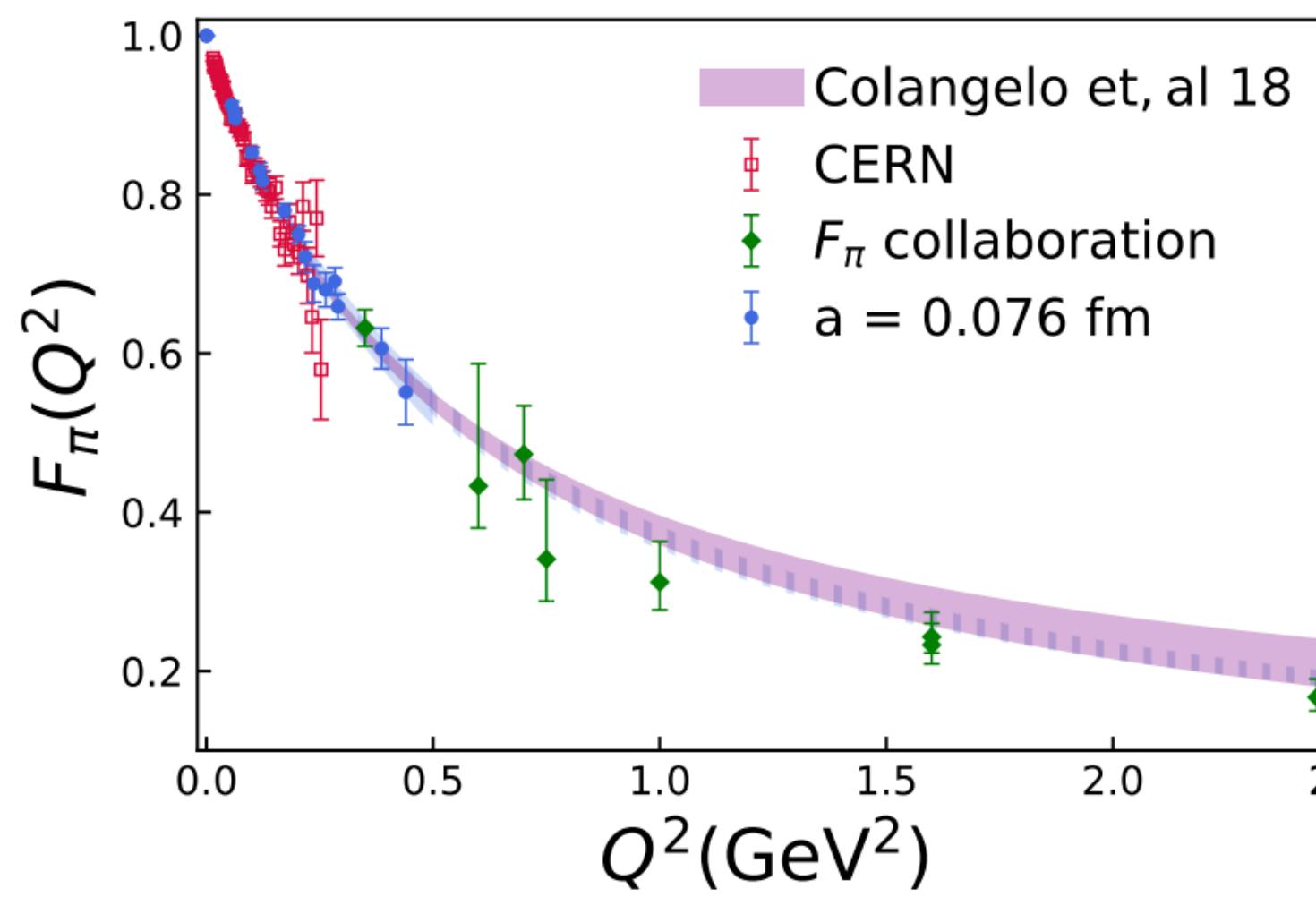


EM form factor of Kaon: $\langle K(P_1) | J_\mu | K(P_2) \rangle = (P_1 + P_2)_\mu F_K(Q^2)$

Current status: pion/kaon EM form factors from Lattice QCD



高翔 et al. Tsinghua-BNL-ANL, PRD 104 (2021) 114515

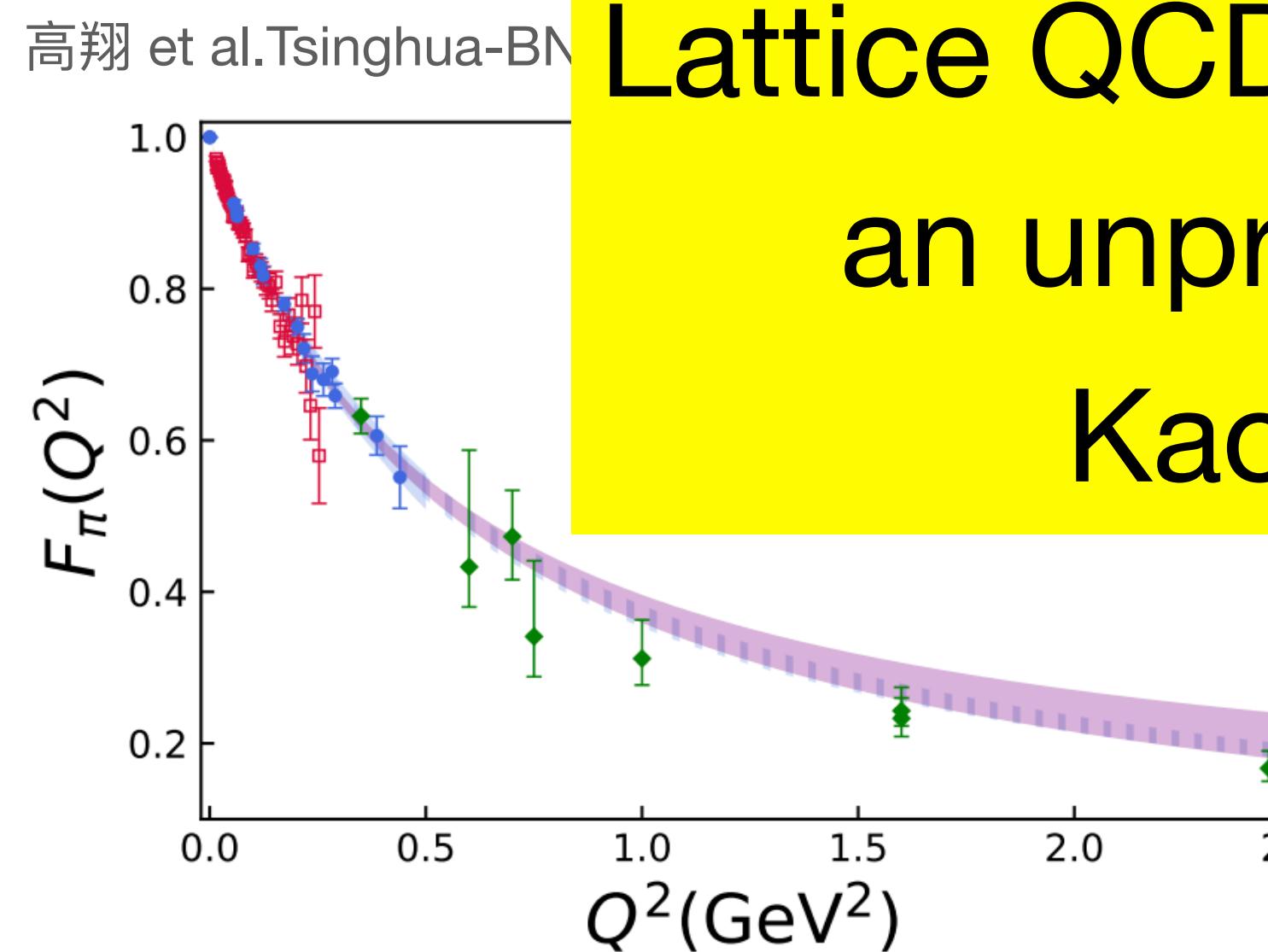
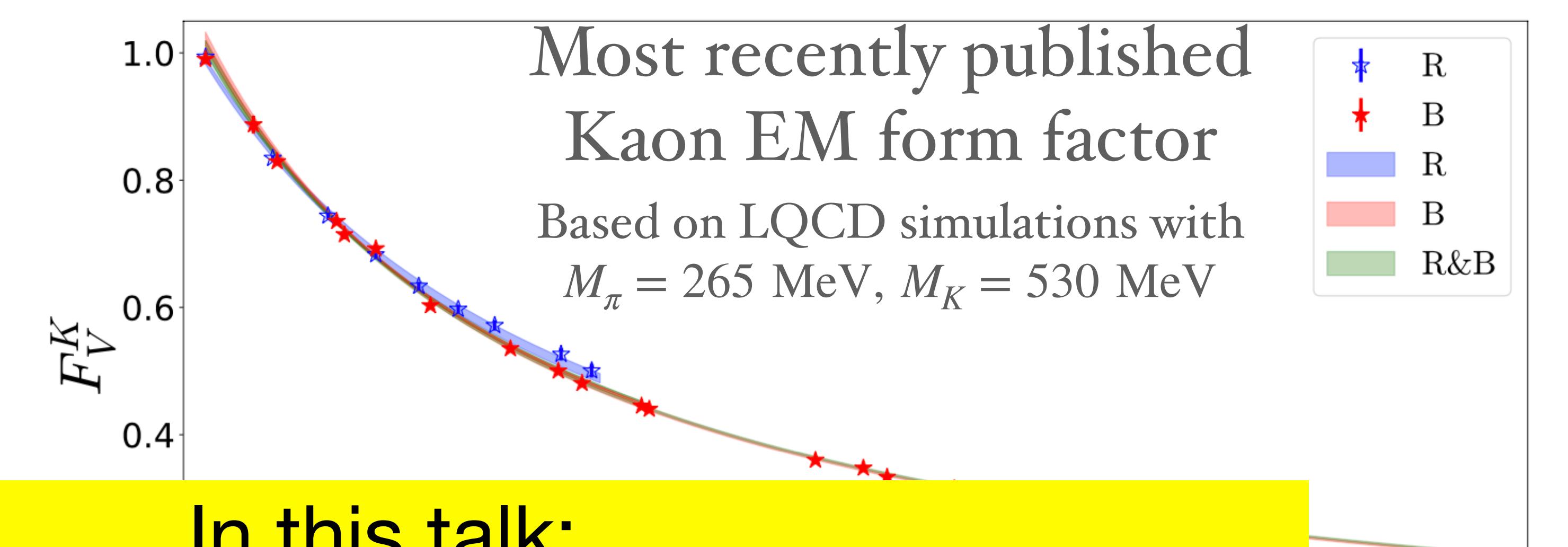
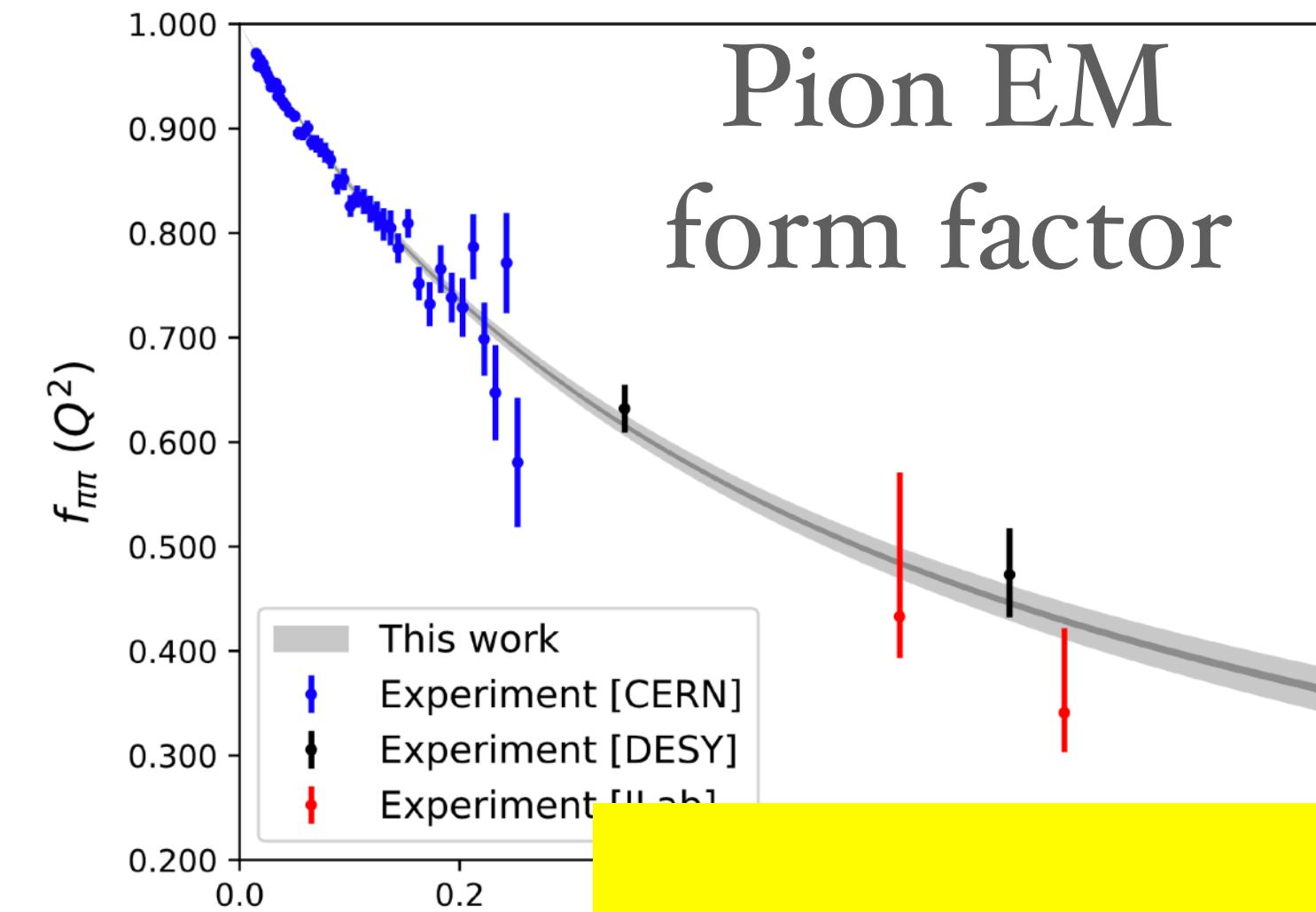


Alexandrou et al., [ETMC], Phys.Rev.D 105 (2022) 5, 054502

Many computations on the pion form factor,
but much less on kaon

Mostly restricted to $Q^2 \lesssim 3 \text{ GeV}^2$

Current status: pion/kaon EM form factors from Lattice QCD



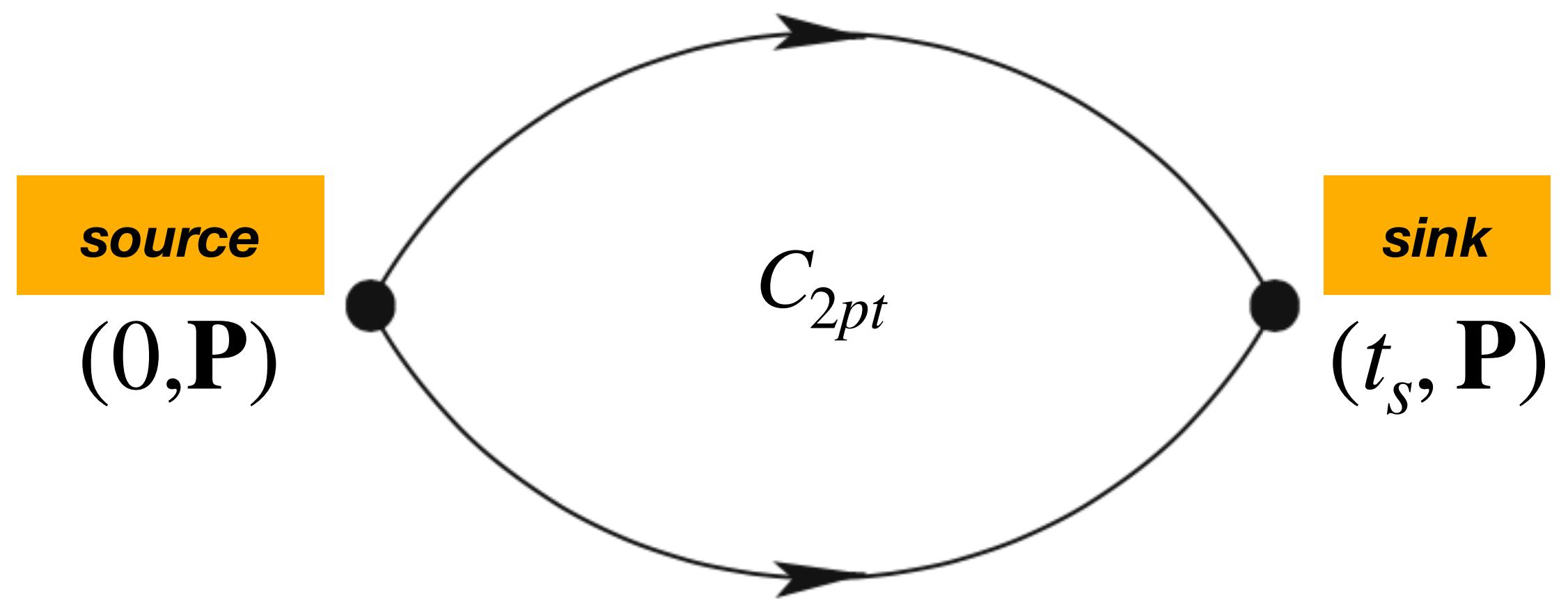
In this talk:
Lattice QCD prediction of EM form factors at up to
an unprecedented large (**intermediate**) Q^2 :
Kaon ~ 28 GeV 2 , Pion ~ 10 GeV 2

Many computations on the pion form factor,
but much less on kaon

Mostly restricted to $Q^2 \lesssim 3$ GeV 2

Kaon at nonzero momentum

- Two point kaon correlation function



$$C_{2\text{pt}}(\mathbf{P}, t_s) = \langle [K(\mathbf{P}, t_s)][K(\mathbf{P}, 0)]^\dagger \rangle$$

$$K(\mathbf{P}, t) = \sum_{\mathbf{x}} \bar{s}(\mathbf{x}, t) \gamma_5 u(\mathbf{x}, t) e^{-i\mathbf{P}\cdot\mathbf{x}}$$

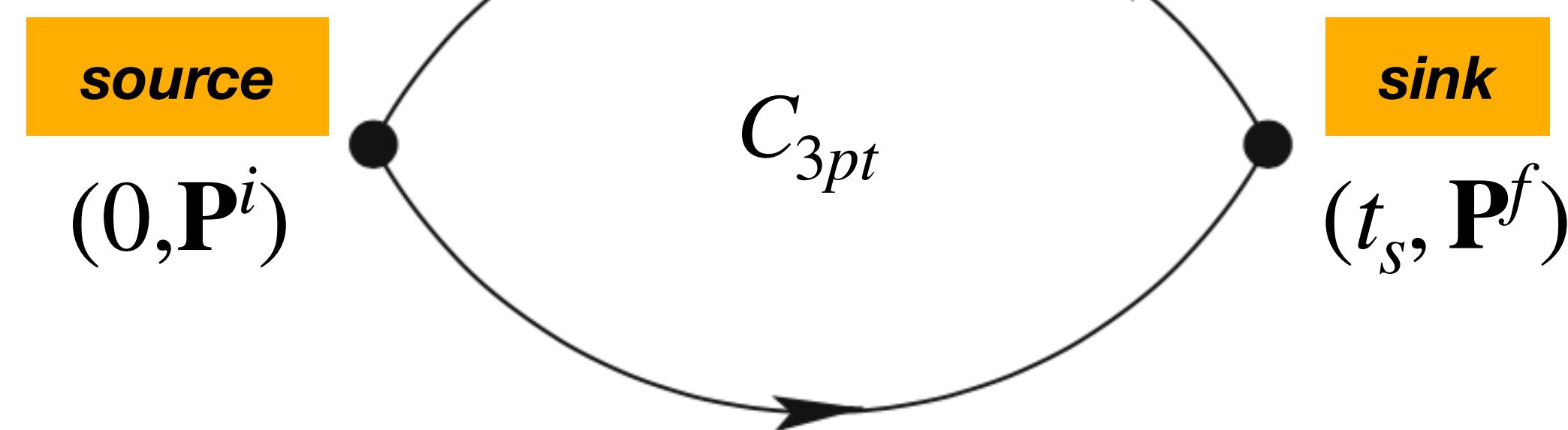
$$\mathbf{P} = \frac{2\pi}{N_\sigma} \mathbf{n} a^{-1}$$

- Determine energy of states from the energy decomposition:

$$C_{2\text{pt}}(\mathbf{P}, t_s) = \sum_{n=0}^{N_{\text{state}}-1} |\langle \Omega | K_S | n; \mathbf{P} \rangle|^2 (e^{-E_n t_s} + e^{-E_n (aL_t - t_s)})$$

Three point correlation function

$$O_\Gamma = \left(\frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{s} \gamma_\mu s \right)$$



$$C_{3pt}(\mathbf{P}^f, \mathbf{P}^i, \tau, t_s) = \langle [K_{S^f}(\mathbf{P}^f, t_s)] O_\Gamma(\mathbf{q}, \tau) [K_{S^i}^\dagger(\mathbf{P}^i, 0)]^\dagger \rangle$$

$$\mathbf{P}^i = \mathbf{P}^f - \mathbf{q}$$

$$Q^2 = -(\mathbf{P}^i - \mathbf{P}^f)^2$$

$$C_{3pt}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s) = \sum_{m,n} \langle \Omega | K_{S^f} | m; \mathbf{P}^f \rangle \langle m; \mathbf{P}^f | O_\Gamma | n; \mathbf{P}^i \rangle \langle n; \mathbf{P}^i | K_{S^i}^\dagger | \Omega \rangle \times e^{-(t_s - \tau) E_m^f} e^{-\tau E_n^i}$$

EM form factor:

Bare matrix element of kaon ground state $F^B(Q^2) = \langle 0; \mathbf{P}^f | O_\Gamma | 0; \mathbf{P}^i \rangle$

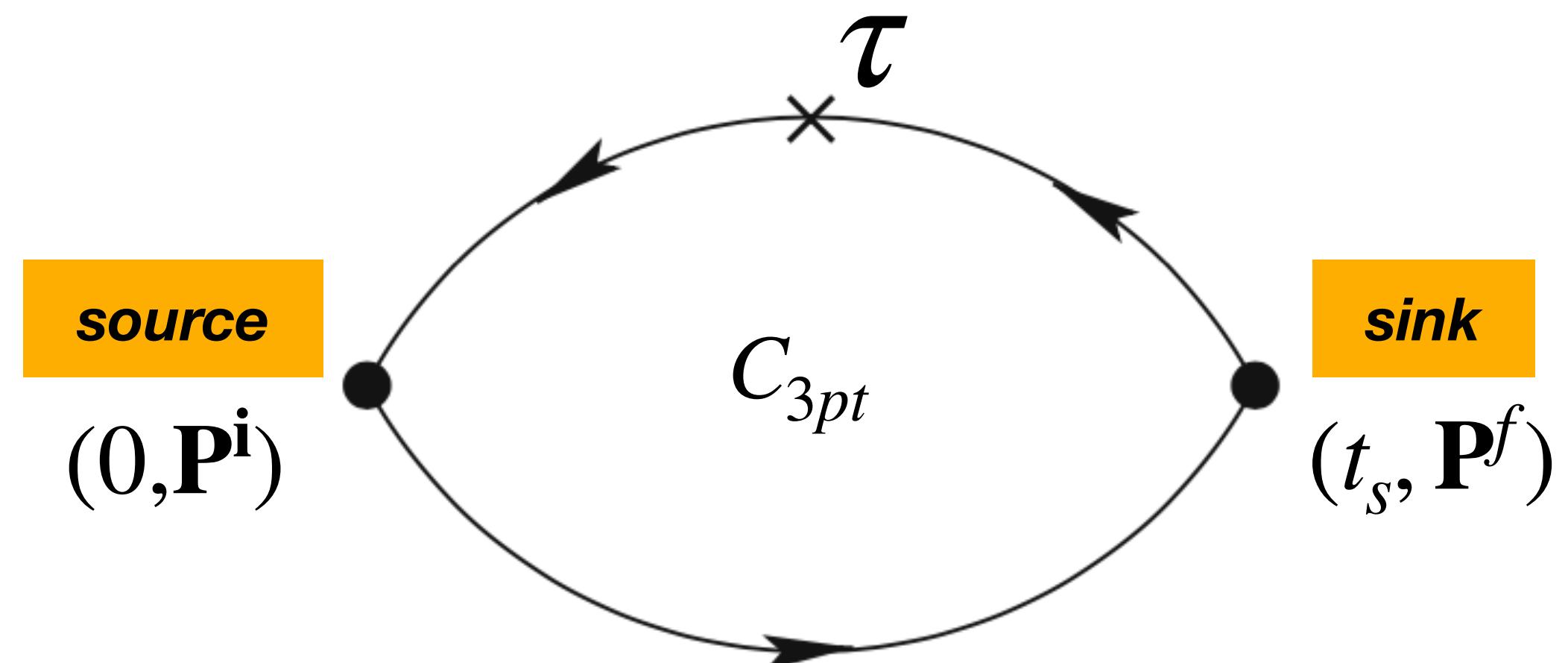
Extraction of bare form factor

- Construct the ratio between 3 and 2-pt corr.:

$$R^{fi}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s) \equiv \frac{2\sqrt{E_0^f E_0^i}}{E_0^f + E_0^i} \frac{C_{3pt}(\mathbf{P}^f, \mathbf{P}^i; \tau, t_s)}{C_{2pt}(t_s, \mathbf{P}^f)} \times \left[\frac{C_{2pt}(t_s - \tau, \mathbf{P}^i) C_{2pt}(\tau, \mathbf{P}^f) C_{2pt}(t_s, \mathbf{P}^f)}{C_{2pt}(t_s - \tau, \mathbf{P}^f) C_{2pt}(\tau, \mathbf{P}^i) C_{2pt}(t_s, \mathbf{P}^i)} \right]^{1/2}$$

- Bare form factor:

$$F^B(Q^2) = \lim_{\tau \rightarrow \infty, t_s \rightarrow \infty} R^{fi}(\mathbf{P}^f, \mathbf{P}^i, \tau, t_s)$$

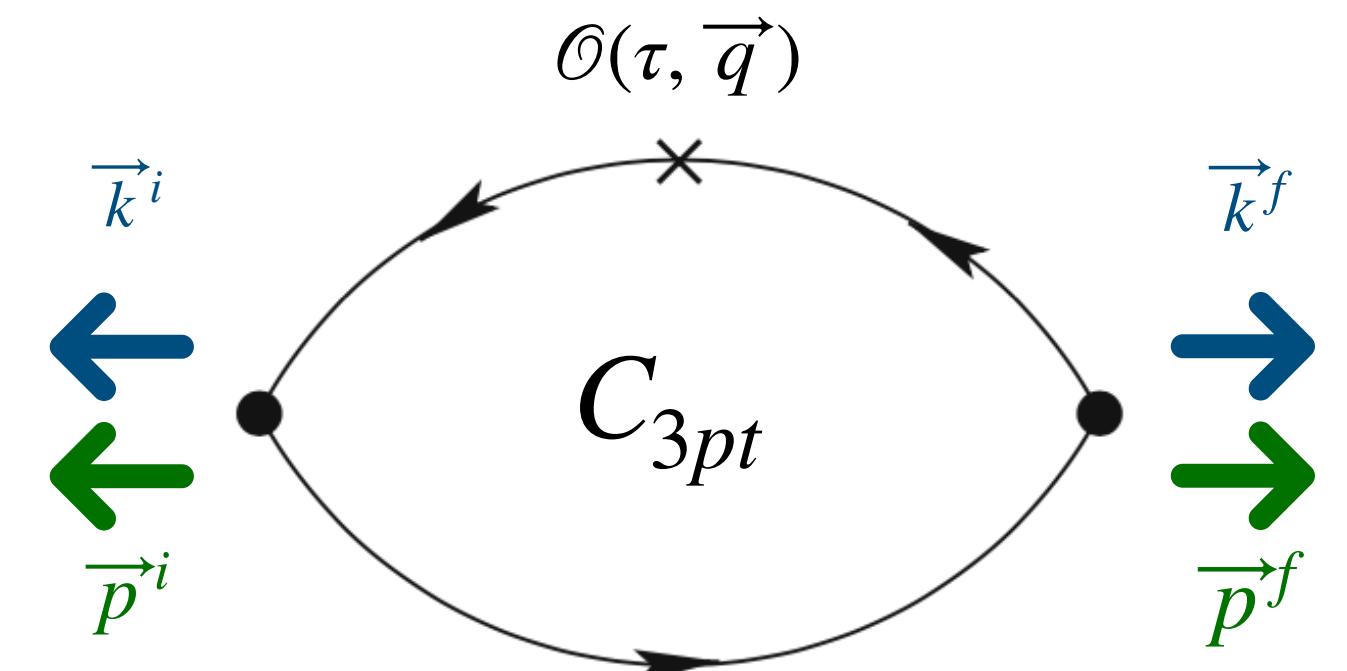


- Form factor: $F(Q^2) = F^B \times Z_V$

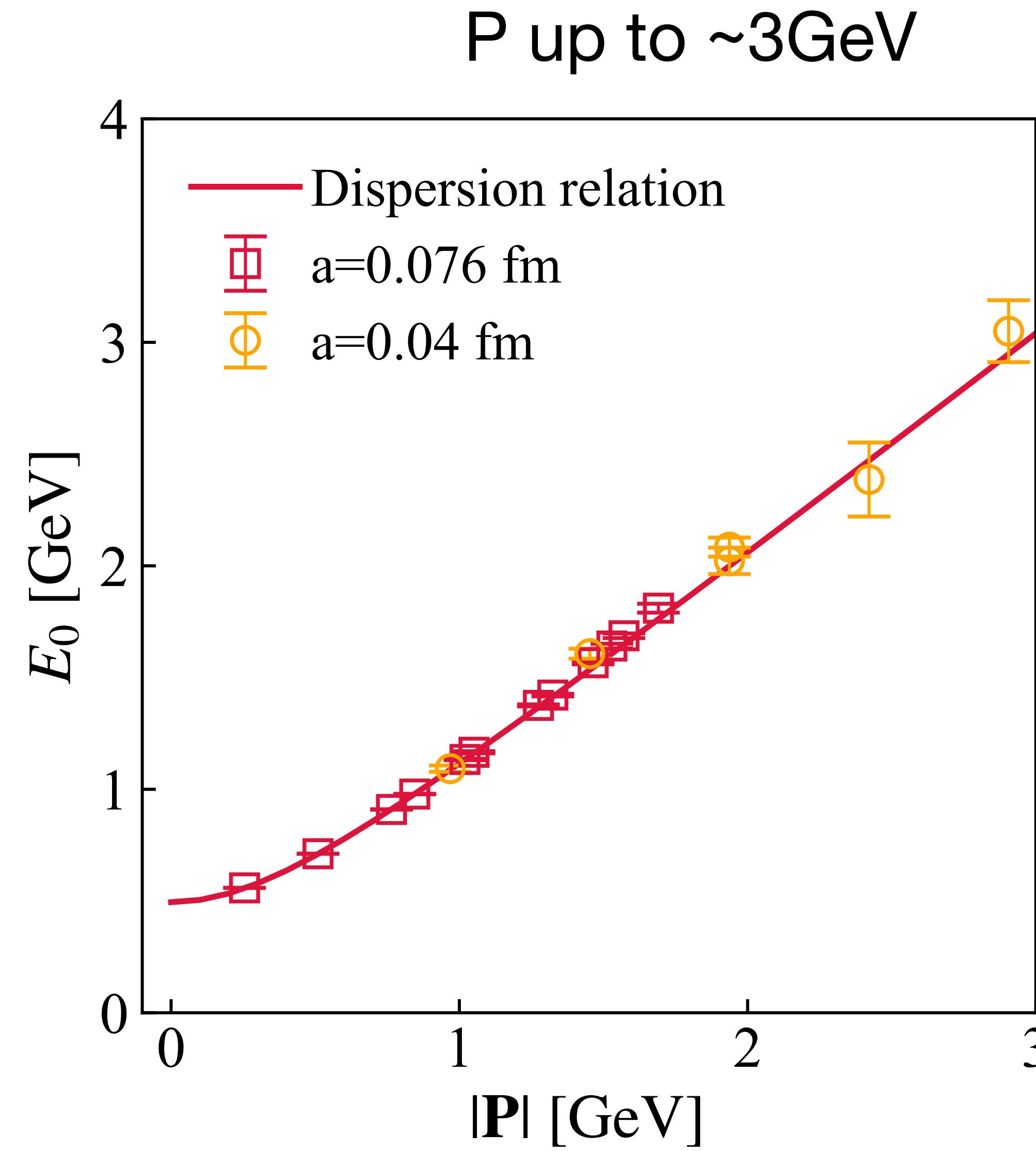
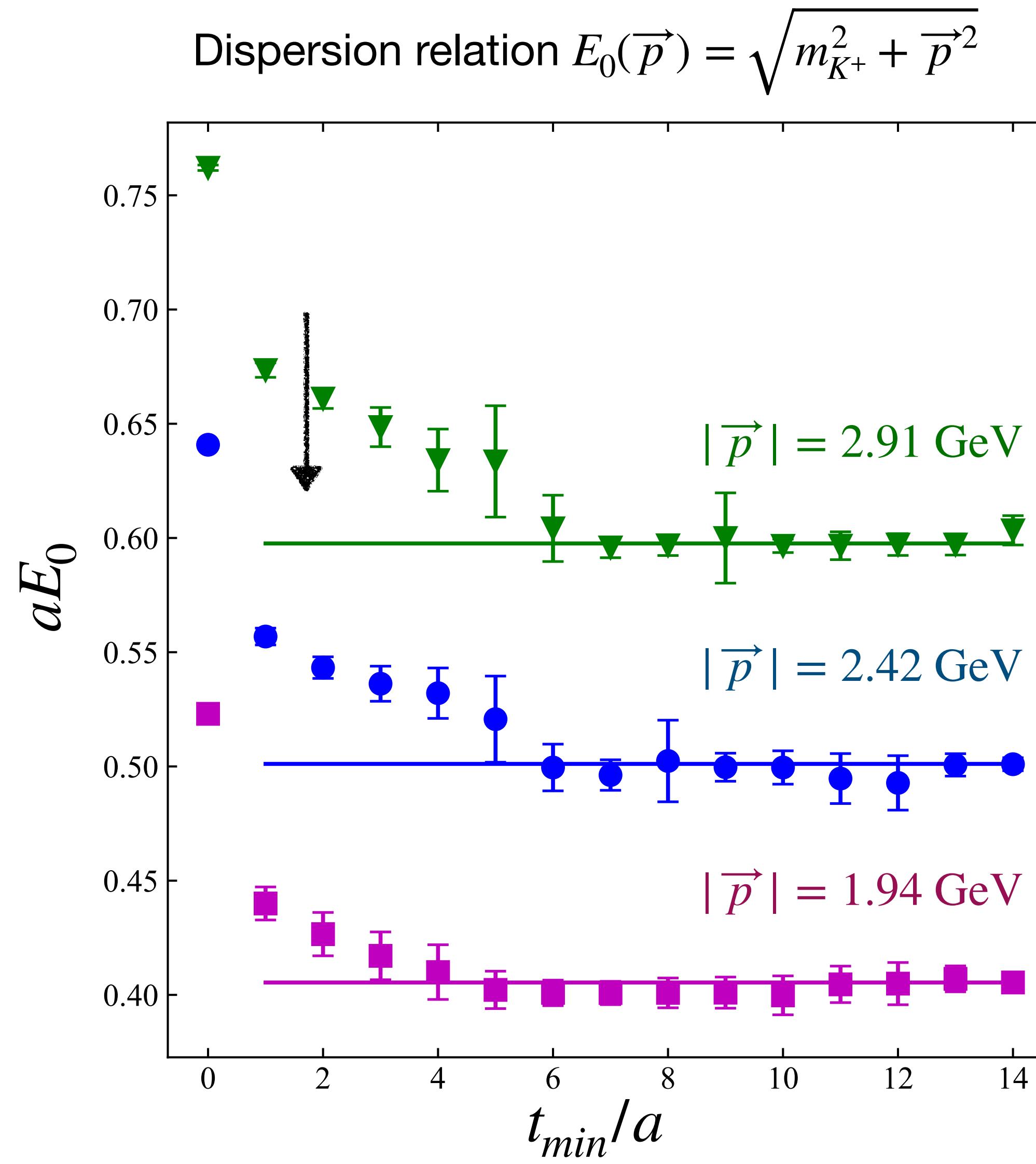
Lattice setup

- $N_f=2+1$ QCD on $64^3 \times 64$ lattices with $a=0.076$ & 0.04 fm ([HotQCD] configurations)
 - Sea quark: Highly Improved Staggered Quark (HISQ) action
 - Valence quark: Wilson-Clover action
- At the physical point: $M_{\pi^+} = 140$ MeV, $M_{K^+} = 497$ MeV
- Boost smearing with the corresponding signs of the quark momenta at source & sink

- Pion: up to 10 GeV 2 with $a = 0.076$ fm
- Kaon: up to 28 GeV 2 with $a = 0.076$ & 0.04 fm

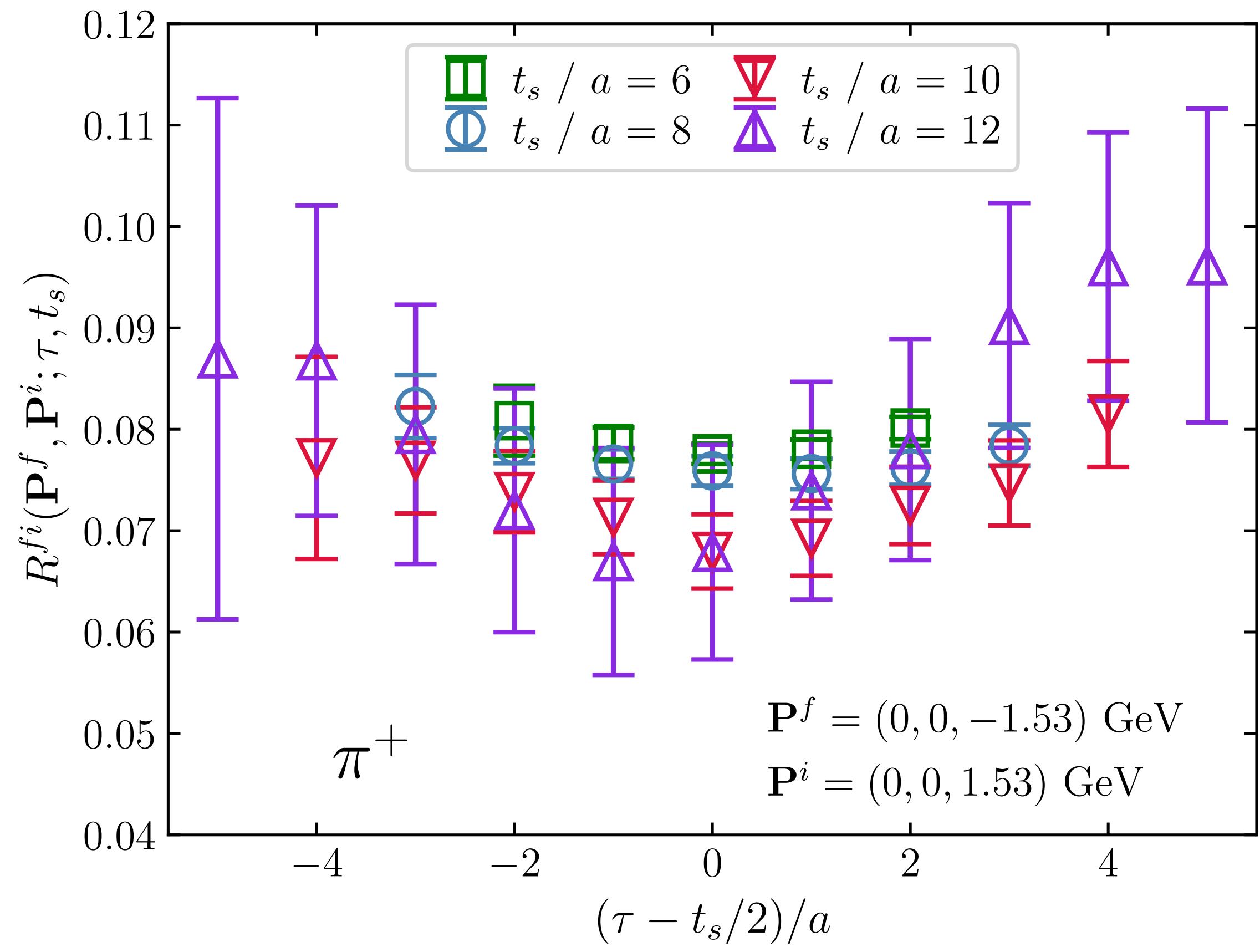


Kaon at large momentum

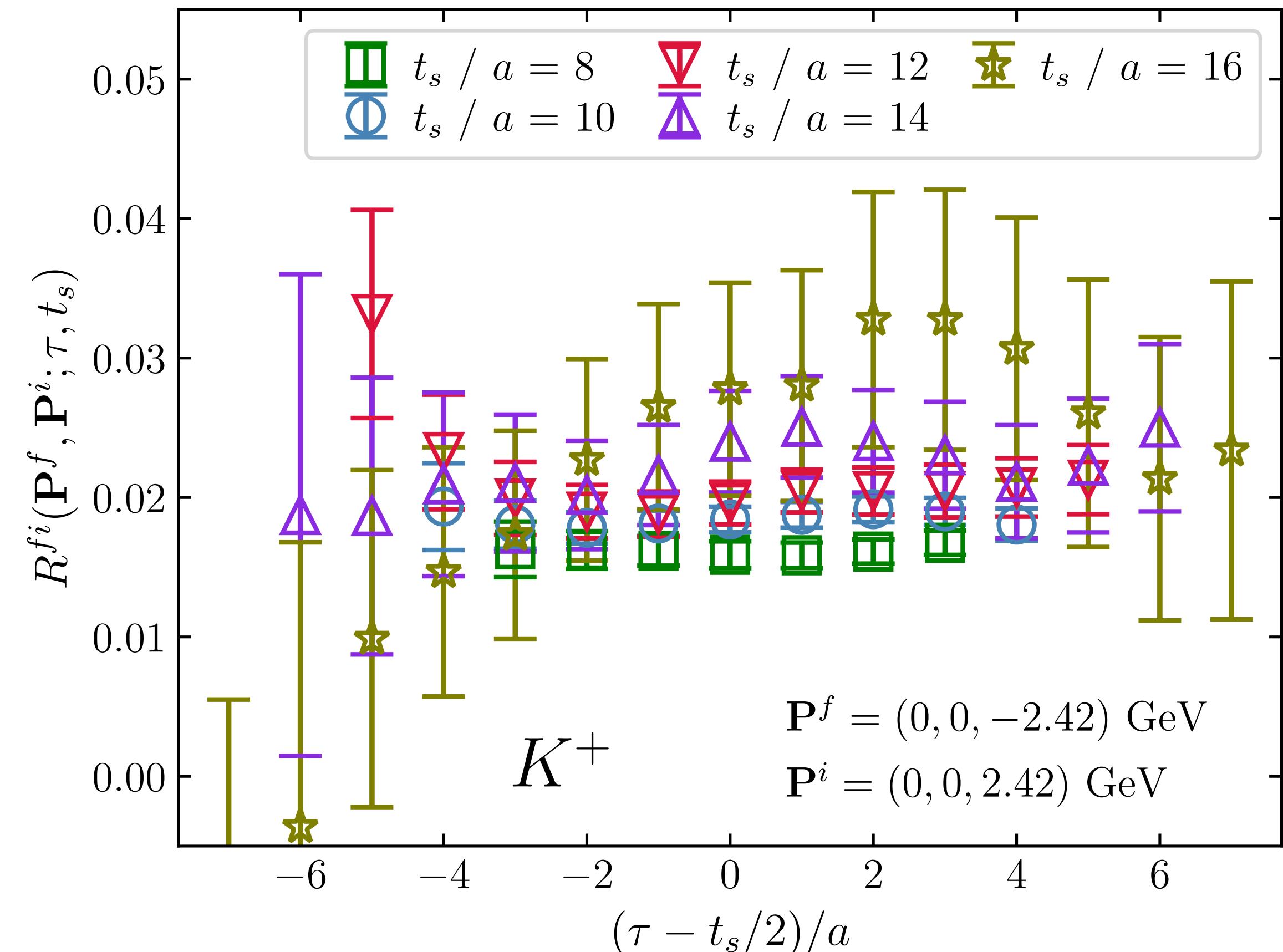


Lattice data of $R^{fi} \sim C_{3pt}/C_{2pt}$

$Q^2 = 9.4 \text{ GeV}^2$



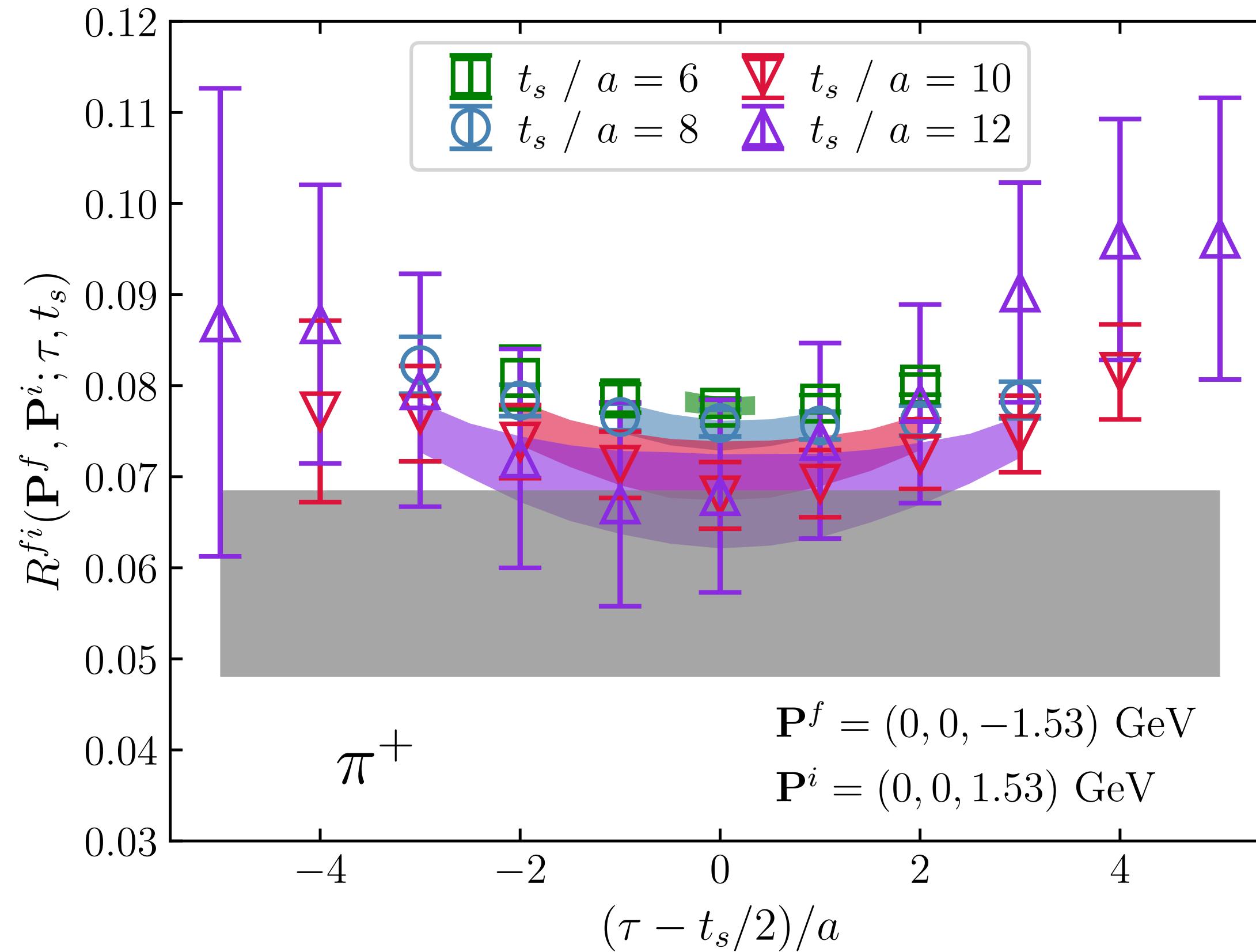
$Q^2 = 23.4 \text{ GeV}^2$



Extraction of the form factor

$$N_{state} = 2: R^{fi}(\tau, t_s) = \left(\frac{\mathcal{O}_{00}}{F^B} + \frac{A_1}{A_0} \mathcal{O}_{11} e^{-t_s \Delta E} + \sqrt{\frac{A_1}{A_0}} \mathcal{O}_{01} e^{-\tau \Delta E} + \sqrt{\frac{A_1}{A_0}} \mathcal{O}_{10} e^{-(t_s - \tau) \Delta E} \right) / \left(1 + \frac{A_1}{A_0} e^{-t_s \Delta E} \right), \Delta E = E_1 - E_0$$

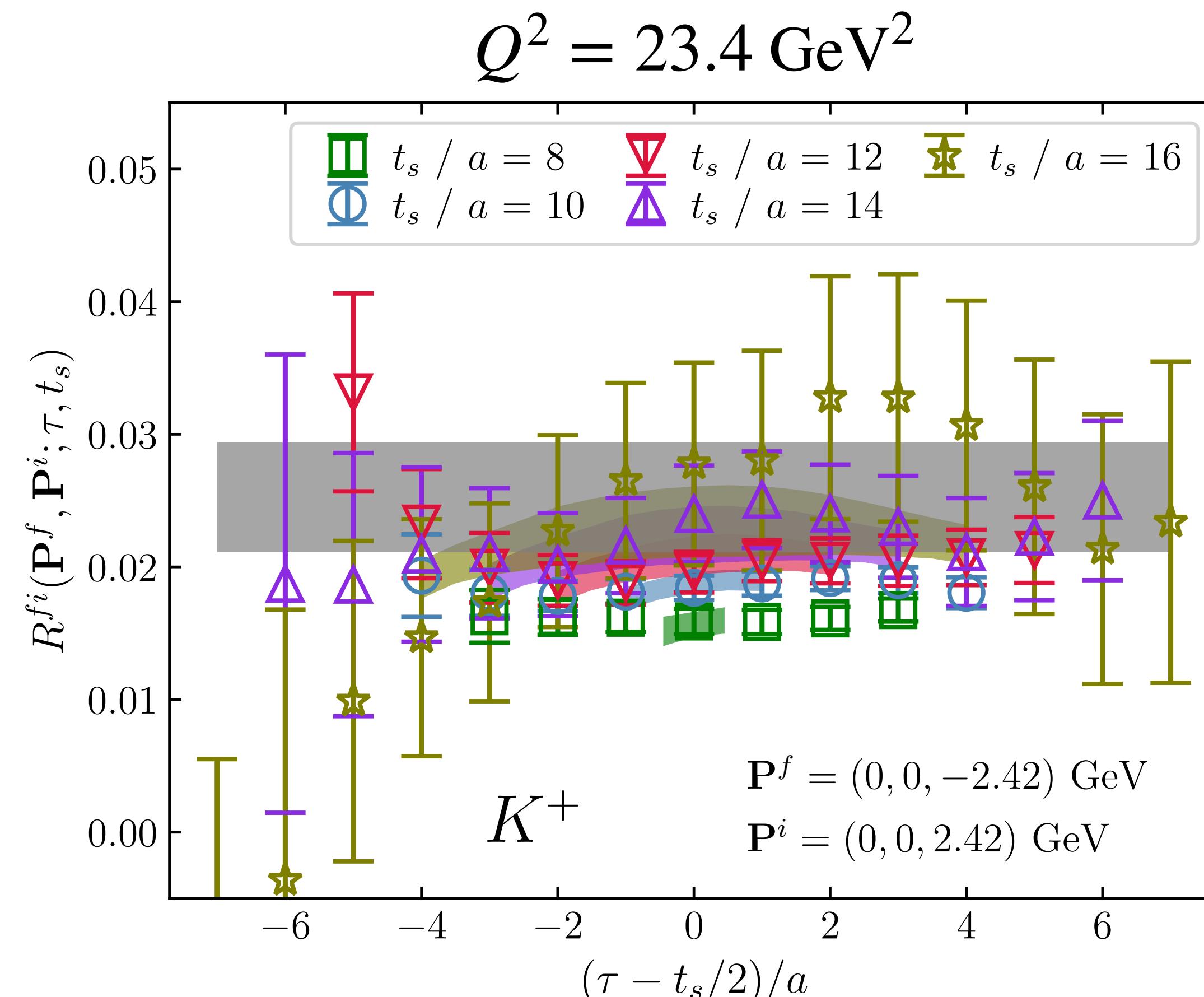
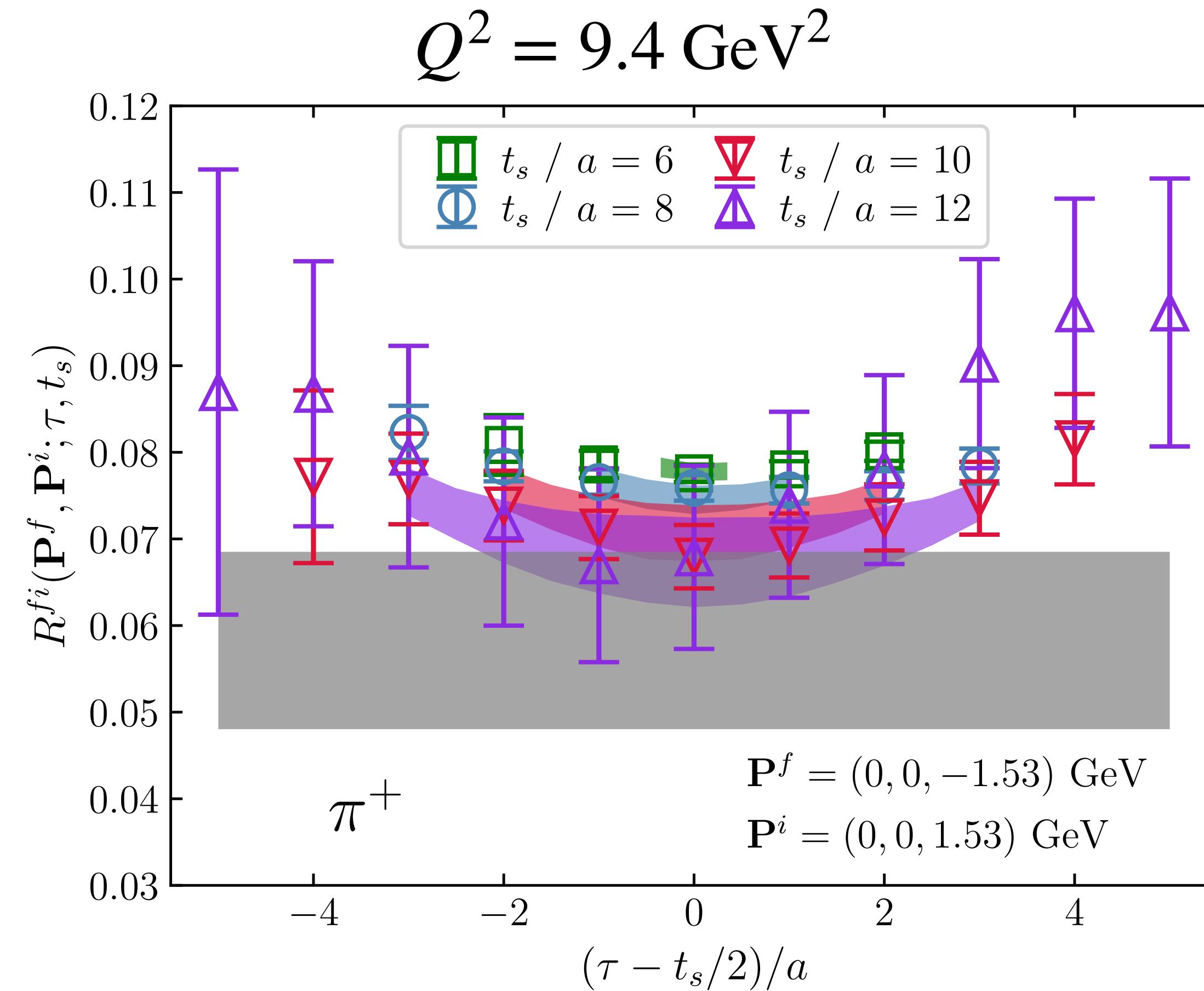
$$Q^2 = 9.4 \text{ GeV}^2$$



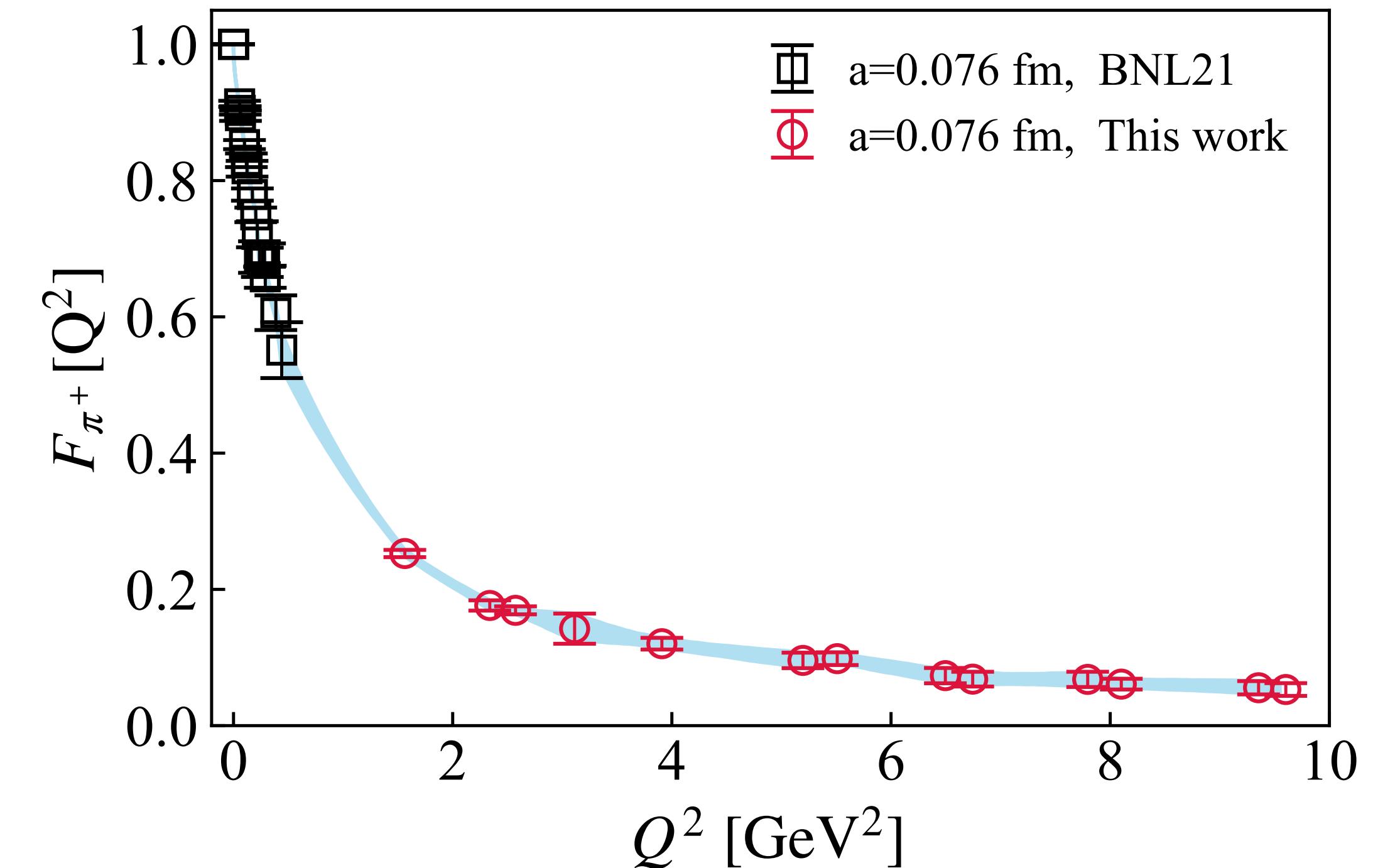
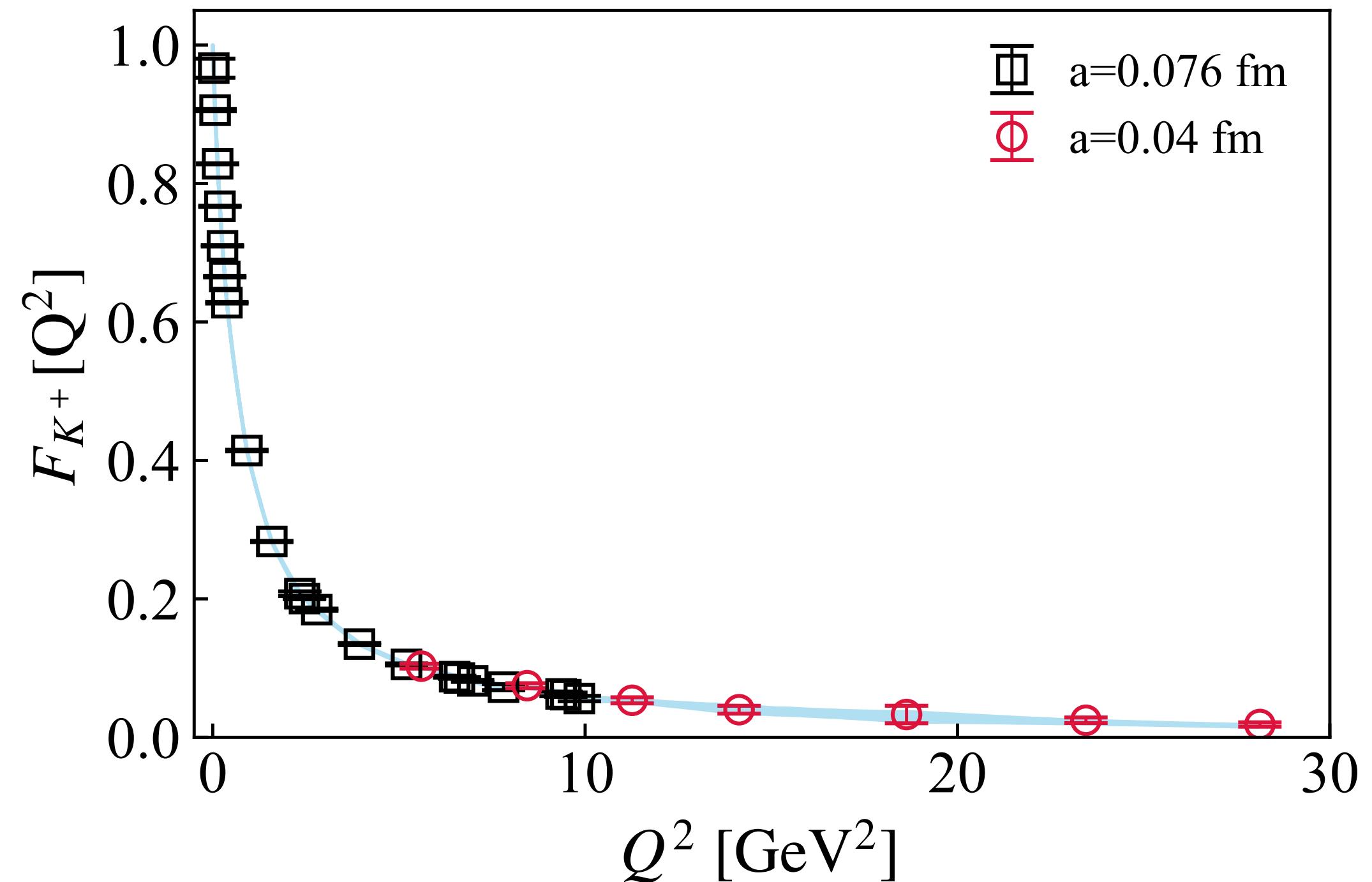
- Use the values of energy E_n and amplitude A_n extracted from C_{2pt}
- Perform a 4-parameter fit to the ratio R^{fi} to extract F^B

Extraction of the form factor

$$N_{state} = 2: R^{fi}(\tau, t_s) = \left(\frac{\mathcal{O}_{00}}{F^B} + \frac{A_1}{A_0} \mathcal{O}_{11} e^{-t_s \Delta E} + \sqrt{\frac{A_1}{A_0}} \mathcal{O}_{01} e^{-\tau \Delta E} + \sqrt{\frac{A_1}{A_0}} \mathcal{O}_{10} e^{-(t_s - \tau) \Delta E} \right) / \left(1 + \frac{A_1}{A_0} e^{-t_s \Delta E} \right), \Delta E = E_1 - E_0$$



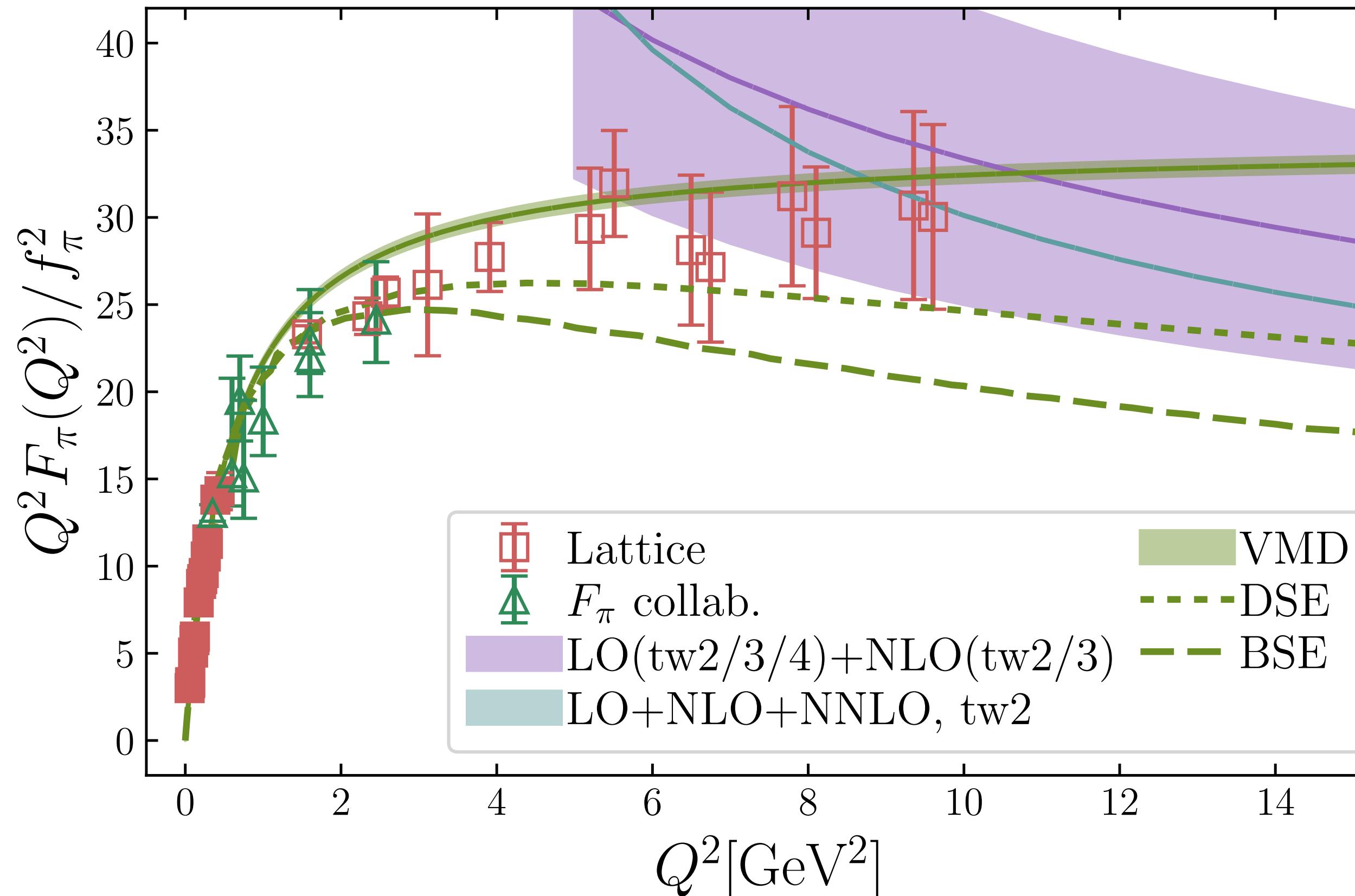
Renormalized Pion and Kaon form factors



$$F_M(Q^2 \rightarrow \infty) = 8\pi\alpha_s(Q^2)f_M^2/Q^2$$

$$Q^2 F_M(Q^2)/f_M^2 = 8\pi\alpha_s(Q^2)$$

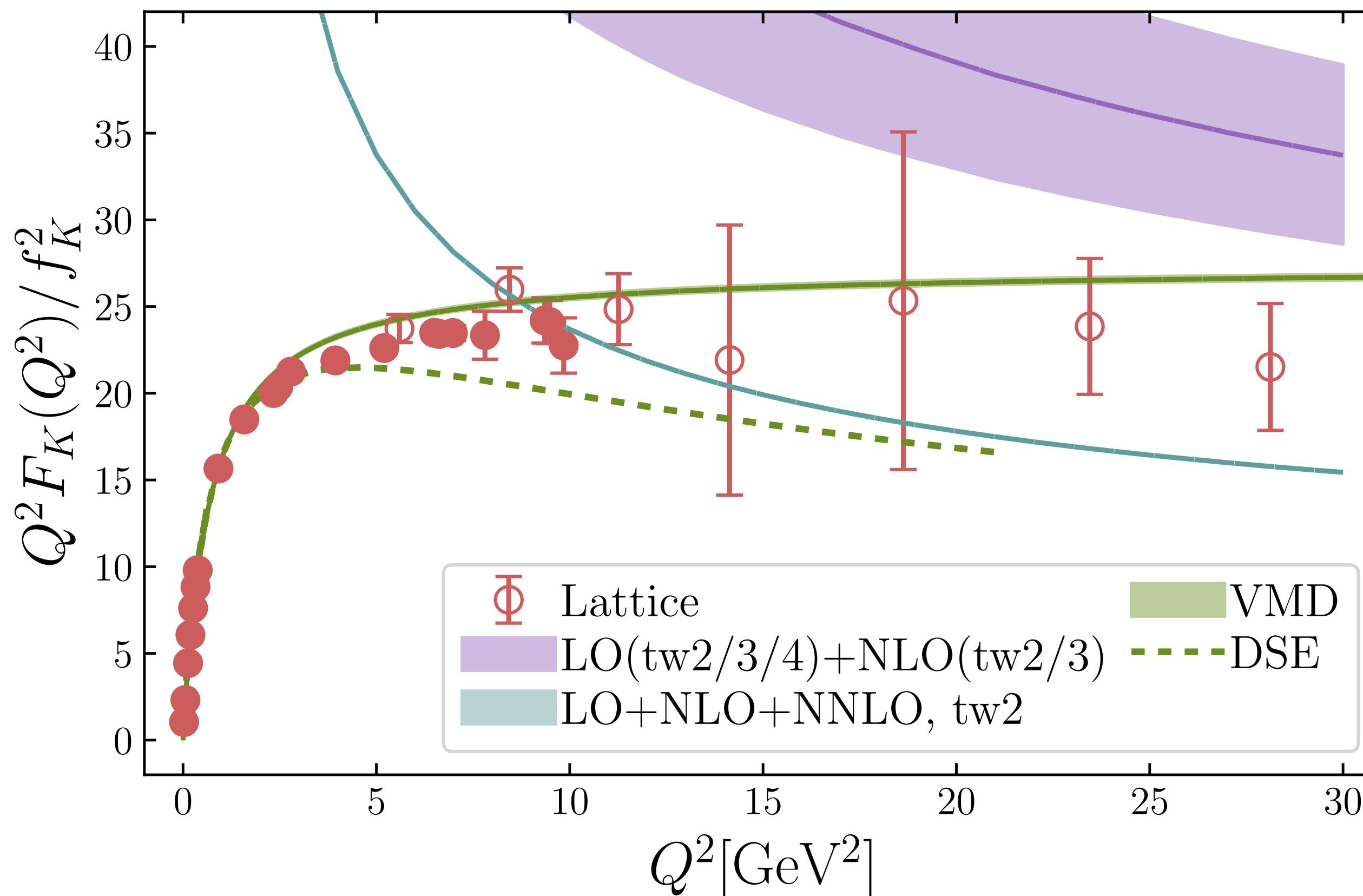
Pion form factor up to $Q^2 \sim 10 \text{ GeV}^2$



- Blue band: collinear factorization, Chen et al., 2312.17228
陈文, 17:00, April 7
- Purple band: k_T factorization, Cheng et al., PRD 19', EPJC23'
程山, 9:00, April 7
- DSE: Gao et al., PRD 96(2017)034024
- BSE: Ydrefors et al., PLB 820 (2021)136494
- VMD: $F_\pi(Q^2) = 1/(1 + Q^2/M^2)$, Gao et al., 2102.06047

LO asymptotic result: $Q^2 F_{\pi^+}(Q^2)/f_\pi^2 \simeq 8.6$

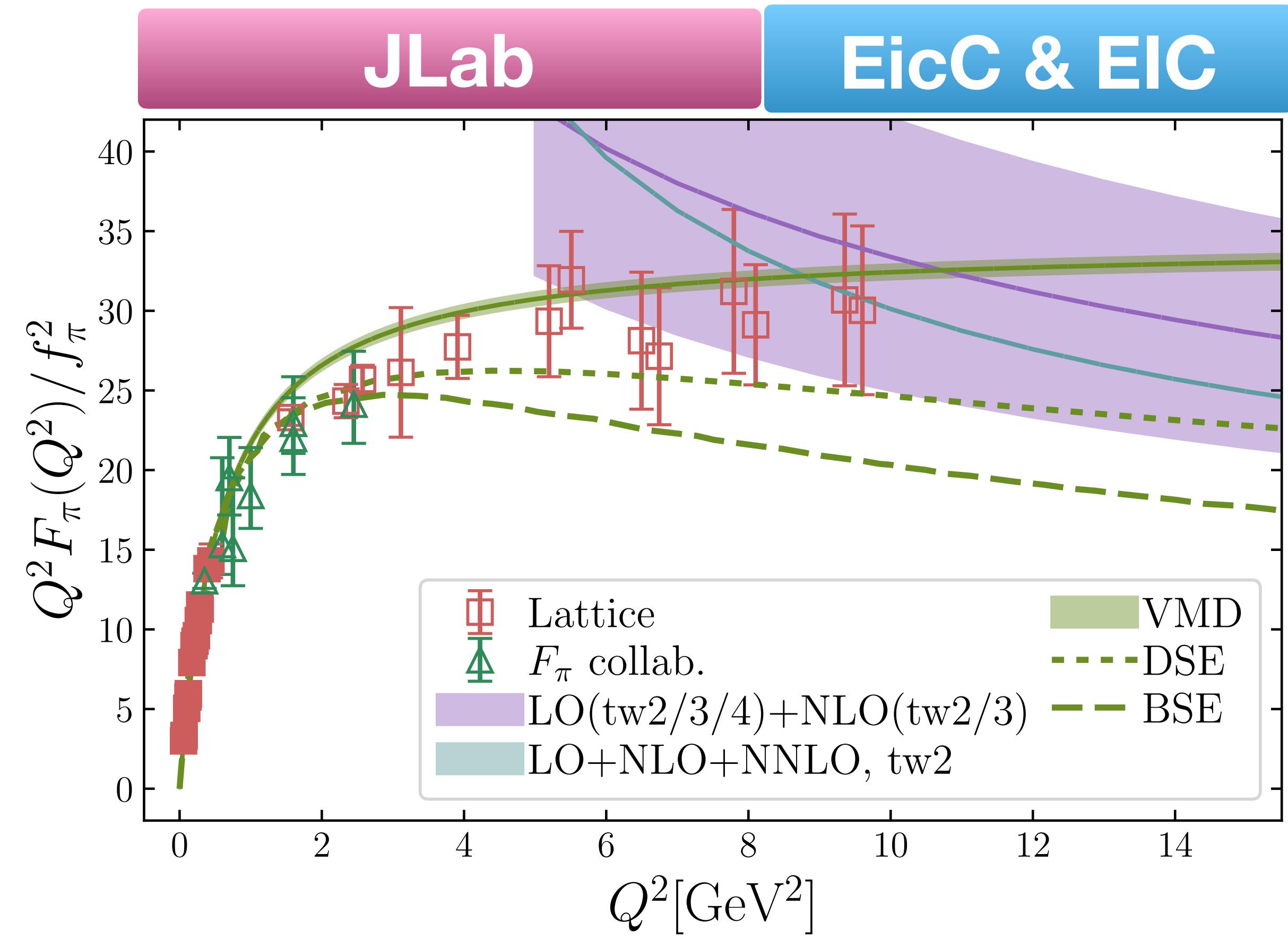
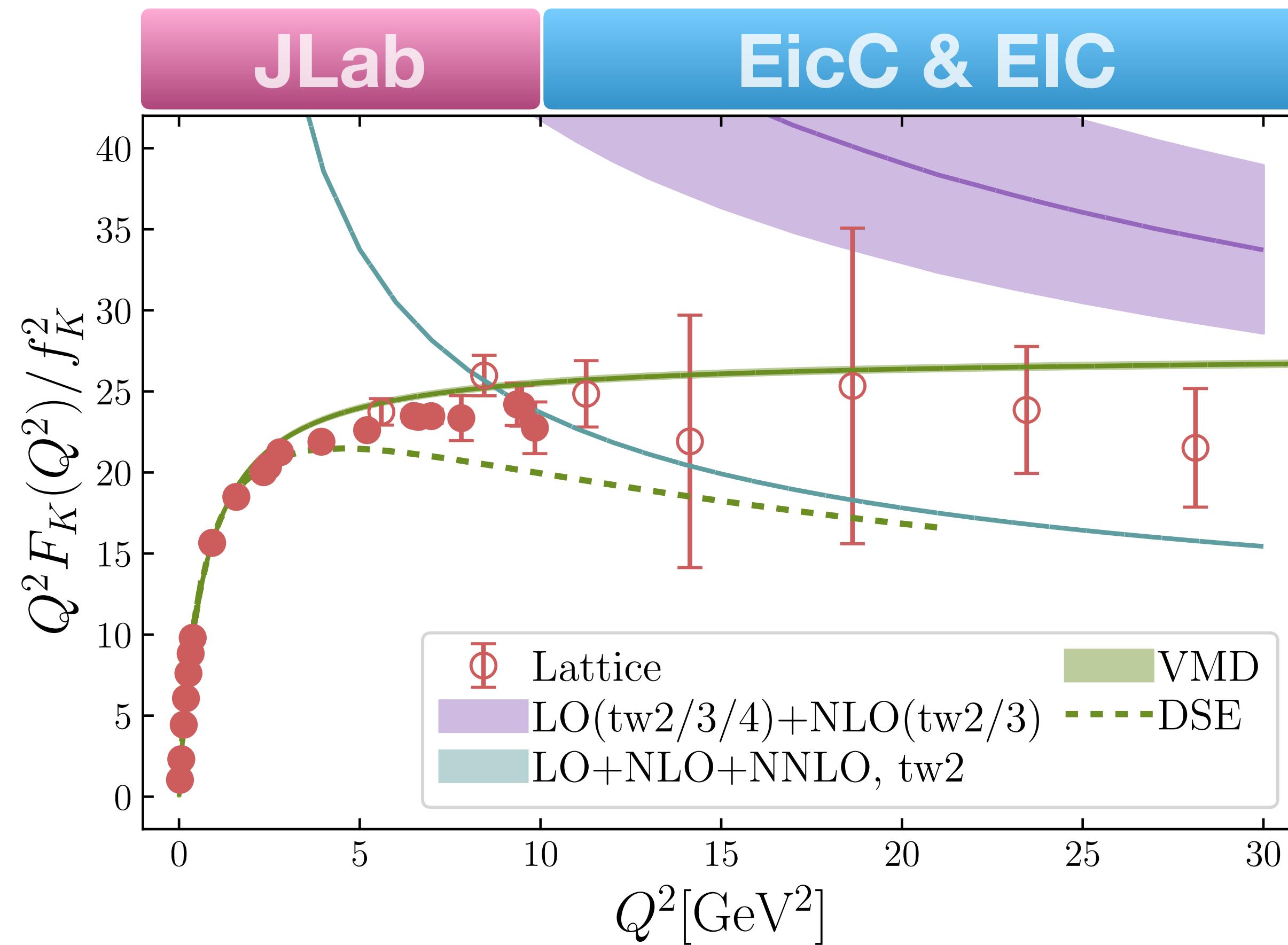
Kaon form factor up to $Q^2 \sim 28 \text{ GeV}^2$



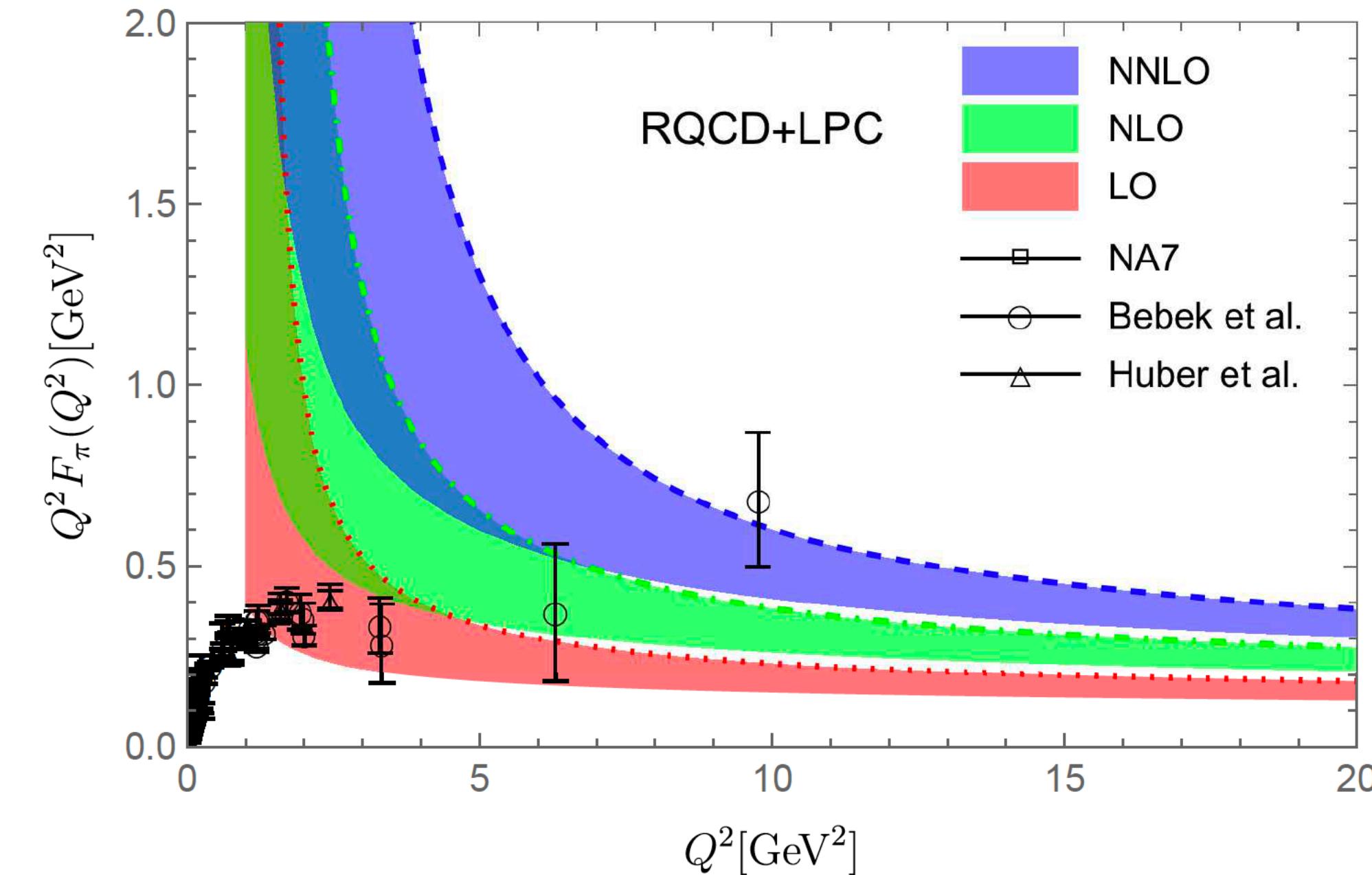
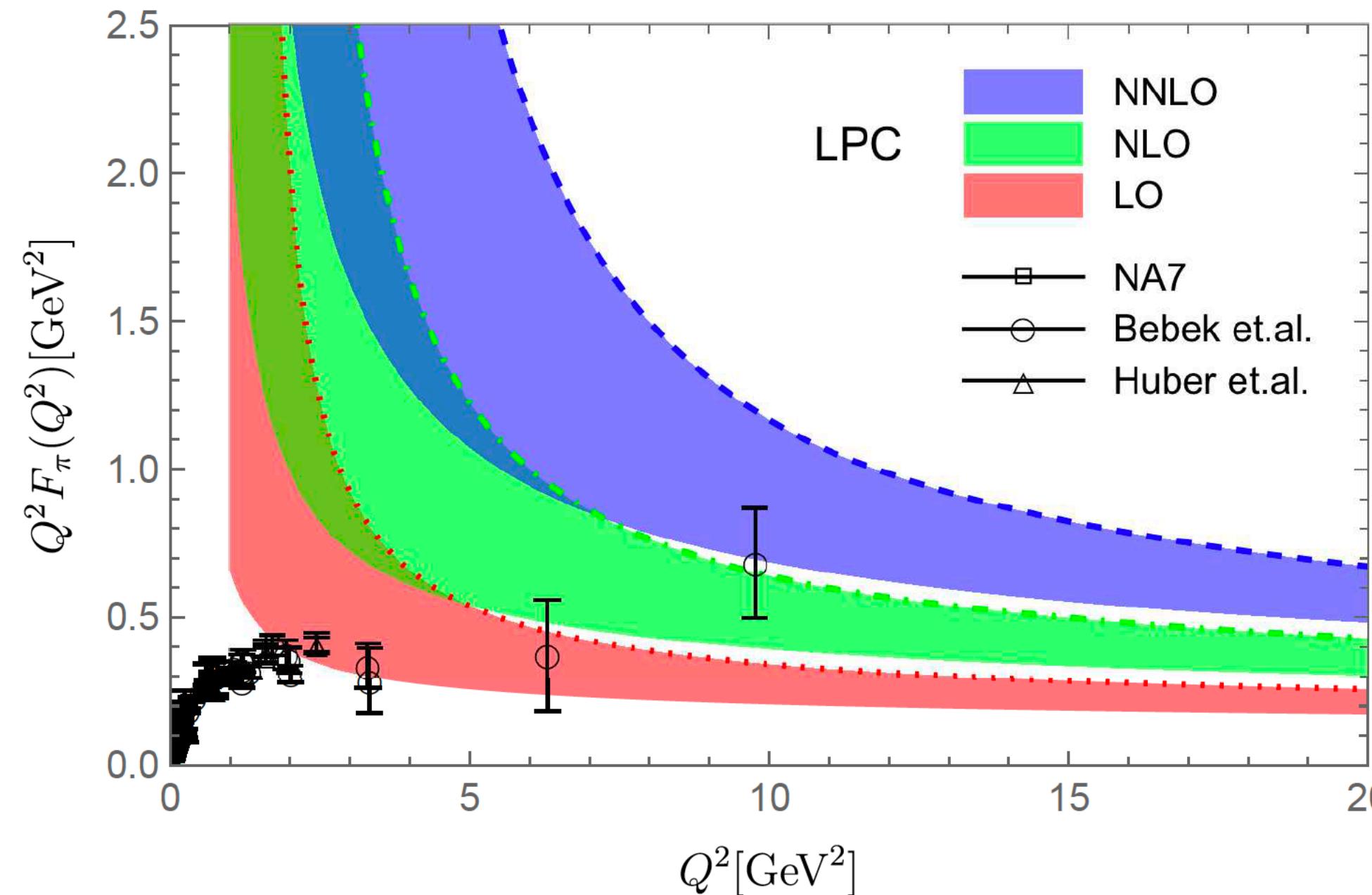
- Blue band: collinear factorization, Chen et al., 2312.17228
 - Purple band: k_T factorization, Cheng, priv. com.
 - DSE: Gao et al., PRD 96(2017)034024
 - VMD: $F_{K^+}(Q^2) = \sum_{\nu=\rho,\phi,\omega} c_\nu / (1 + Q^2/m_\nu^2)$
fit in low $Q^2 \lesssim 0.4 \text{ GeV}^2$, resulting $\langle r_K^2 \rangle = 0.360(2) \text{ fm}^2$
- Consistent with $\langle r_K^2 \rangle = 0.359(3) \text{ fm}^2$ Stamen et al., EPJC 82(2022)432

Summary

- ✓ A first LQCD prediction of Kaon and Pion electromagnetic form factors with Q^2 up to ~ 28 and 10 GeV^2 , respectively



Impact of Gegenbauer moments from DAs



Chen, Chen, Feng & Jia, arXiv:2312.17228

RQCD: $a_2(2 \text{ GeV}) = 0.116^{+0.019}_{-0.020}$

LPC: $a_2(2 \text{ GeV}) = 0.258 \pm 0.087, \quad a_4(2 \text{ GeV}) = 0.122 \pm 0.056, \quad a_6(2 \text{ GeV}) = 0.068 \pm 0.038.$

In this work, pion: $a_2 = 0.196(32), \quad a_4 = 0.085(26), \quad a_6 = 0.056(15)$