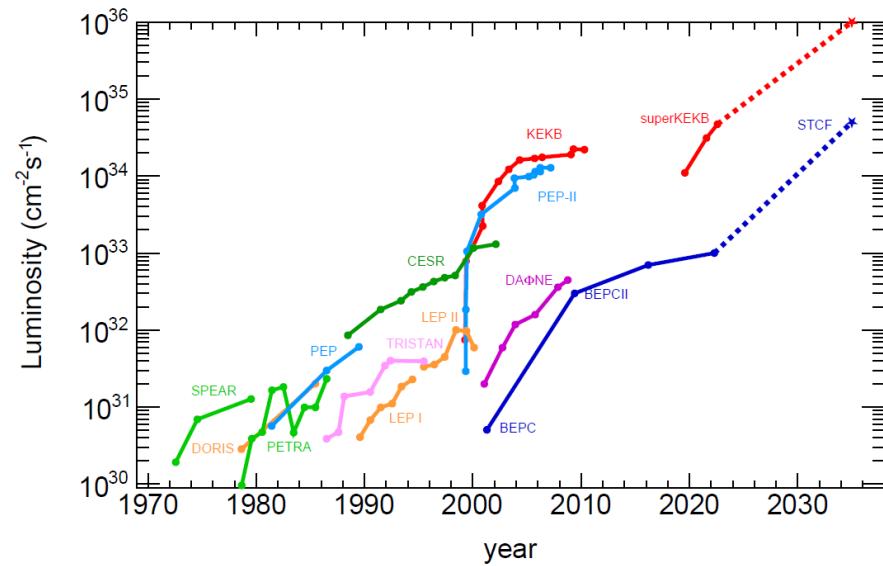


# Fragmentation function at low energy $e^+e^-$

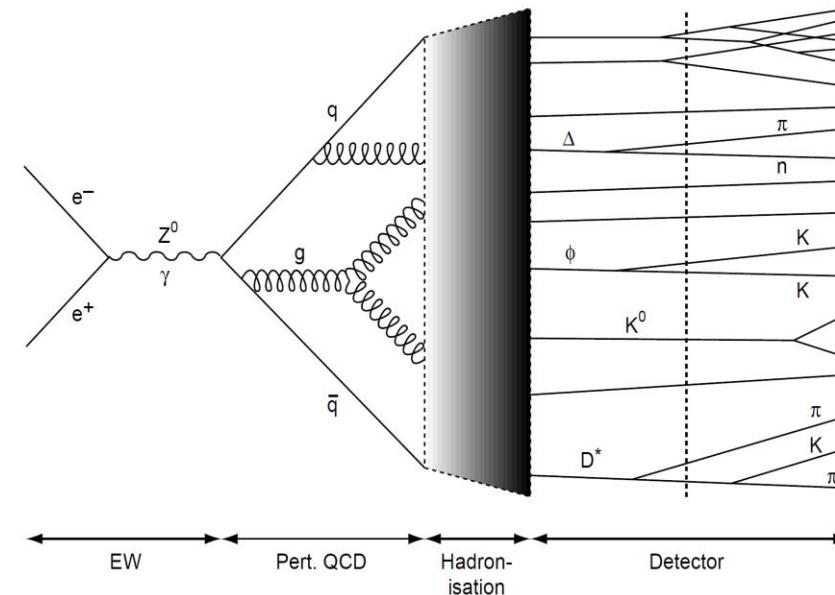
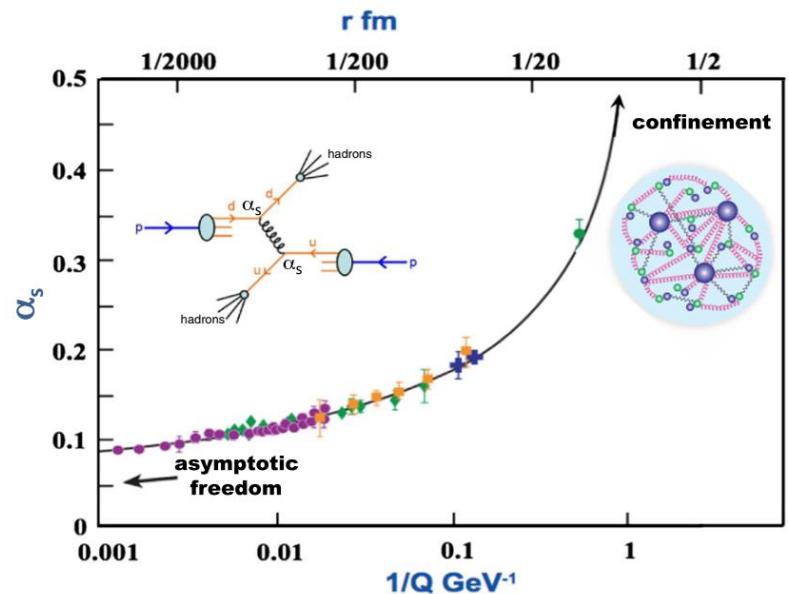
鄢文标(中国科学技术大学)



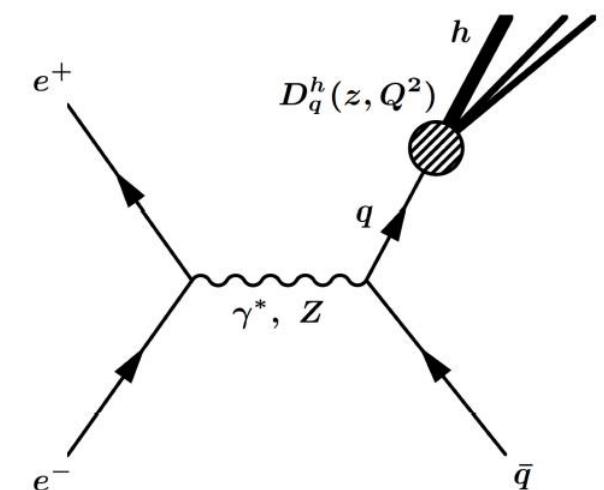
Leading Quark TMDFFs			
		Quark Polarization	
Unpolarized (or Spin 0) Hadrons		Un-Polarized (U)	Longitudinally Polarized (L)
Polarized Hadrons	L	$D_1 = \text{○} - \text{○}$ Unpolarized	$H_1^\perp = \text{○} - \text{○}$ Collins
	T	$G_1 = \text{○} \rightarrow - \text{○}$ Helicity	$H_{1L}^\perp = \text{○} \rightarrow - \text{○} \rightarrow$
		$D_{1T}^\perp = \text{○} \uparrow - \text{○} \downarrow$ Polarizing FF	$G_{1T}^\perp = \text{○} \uparrow - \text{○} \downarrow$ $H_1 = \text{○} \uparrow - \text{○} \downarrow$ Transversity

**arXiv:2304.03302**

# QCD: Asymptotic freedom & Confinement

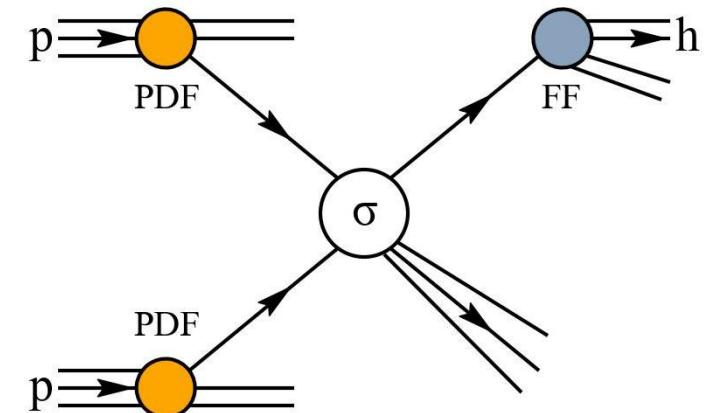
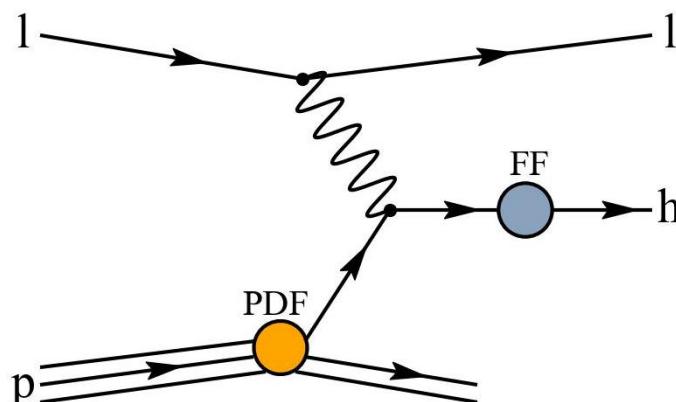
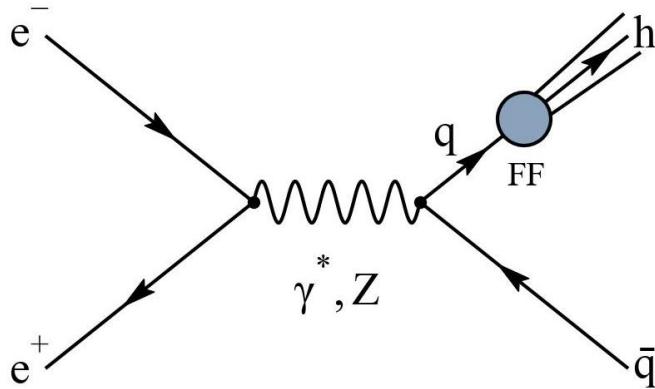
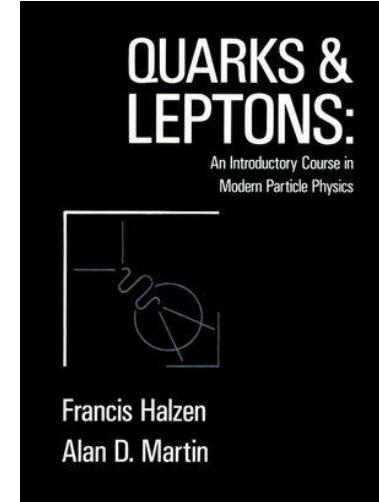


- QCD coupling constant  $\alpha_s(Q)$ 
  - ✓ High  $Q$ : asymptotic freedom, perturbative QCD
  - ✓ Low  $Q$ : **non-perturbative QCD**
- Confinement: partons do not exist as free particles, but are always confined in hadrons.
- Essence of confinement ?



# Fragmentation function: integrated $D_1^h(z)$

- Fragmentation function  $D_q^h(z)$ : probability that hadron  $h$  is found in the debris of a parton carrying a fraction  $z = 2E_h/\sqrt{s}$  of parton's energy
- Consequence of confinement
- FF: QCD first principle (NOT YET)
  - ✓ FF evolution function: DGLAP
  - ✓ Fitting: parametrization & experimental data
  - ✓ Universality:  $e^+e^-$ , DIS,  $p\bar{p}$ ,  $p\bar{p}$  data
- FFs contribute to virtually all processes



# FFs with quark/hadron polarization

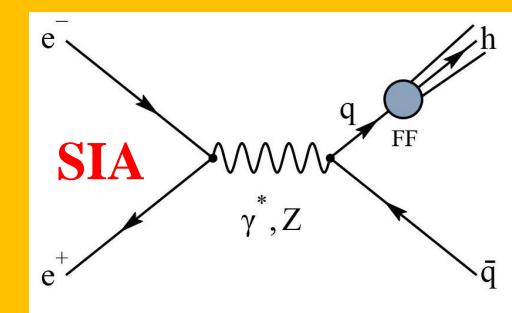
Hadron polarization	Quark polarization @ PPNP 91 136 (2016)		
	Unpolarized	Longitudinally	Transversely
Unpolarized	$D_1^h$		$H_1^{\perp h}$
Longitudinally		$G_1^h$	$H_{1L}^{\perp h}$
Transversely	$D_{1T}^{\perp h}$	$G_{1T}^h$	$H_1^h \ H_{1T}^{\perp h}$



$$D_{hq\uparrow}(z, P_{h\perp}) = D_1^q(z, P_{h\perp}^2) + H_1^{\perp q}(z, P_{h\perp}^2) \frac{(\hat{k} \times \mathbf{P}_{h\perp}) \cdot \mathbf{S}_q}{z M_h}$$

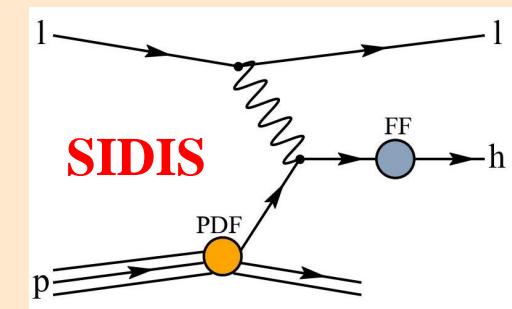
- Theoretically many more, in particular with **polarized hadrons** in the final state and **transverse momentum dependence (TMD)**

# Access FFs with QCD factorization theorem



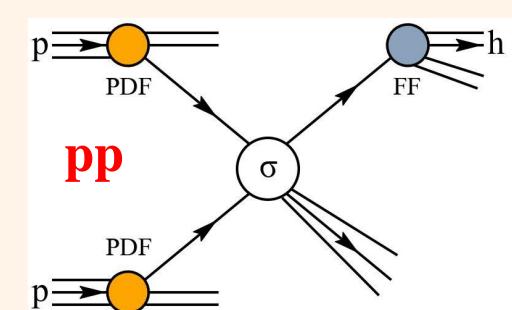
$$e^+e^- : \sigma = \sum_q \sigma(e^+e^- \rightarrow q\bar{q}) \otimes FF$$

- No PDFs necessary
- Calculations known at NNLO
- Flavor structure not directly accessible



$$SIDIS: \sigma = \sum_q PDF \otimes \sigma(eq \rightarrow e'q') \otimes FF$$

- Depend on unpolarized PDFs
- Flavor structure directly accessible
- FFs and PDFs

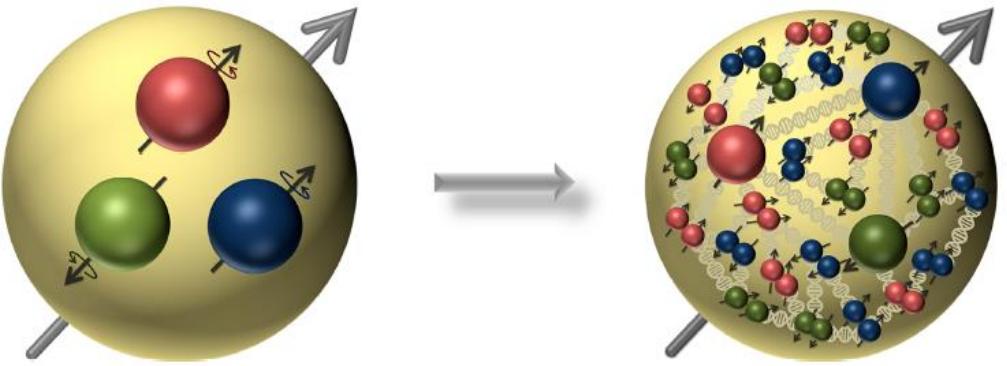


$$pp: \sigma = \sum_q PDF \otimes PDF \otimes \sigma(q_1 q_1 \rightarrow q'_1 q'_2) \otimes FF$$

- Depend on unpolarized PDFs
- Leading access to gluon FF
- Parton momenta not directly known

- SIA @  $e^+e^-$ : the **cleanest** input for FFs fitting

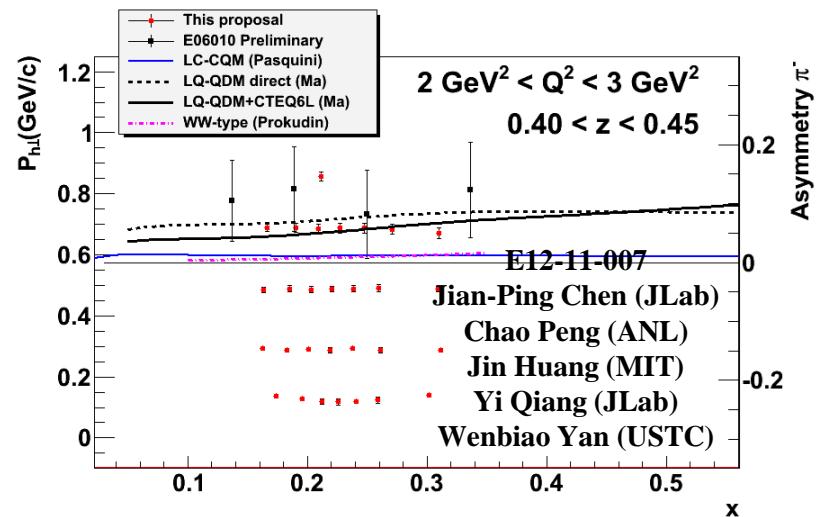
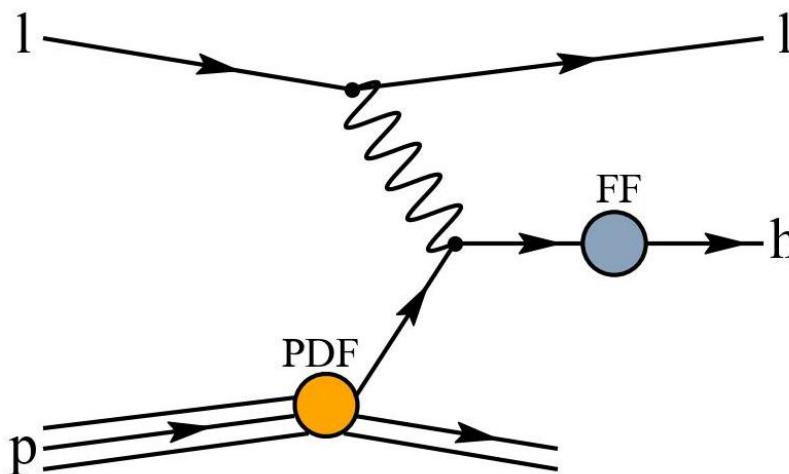
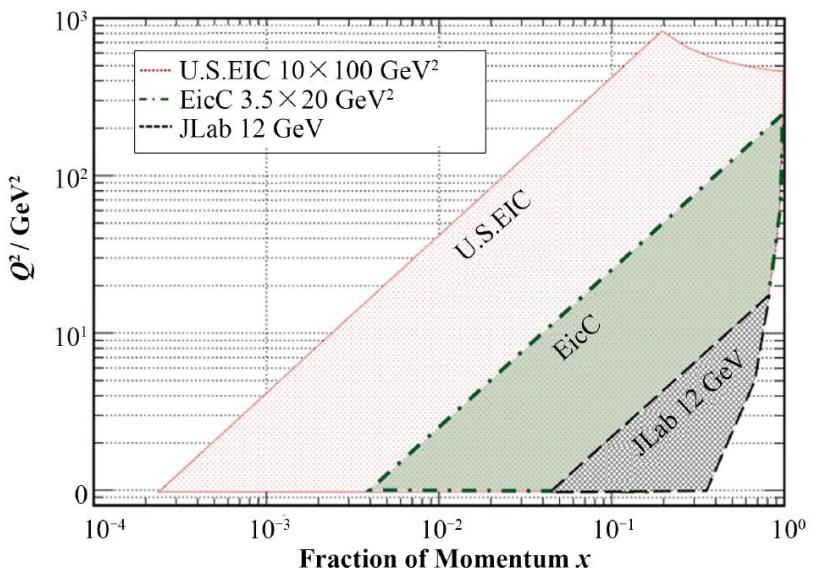
# FFs for EIC & EicC



Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

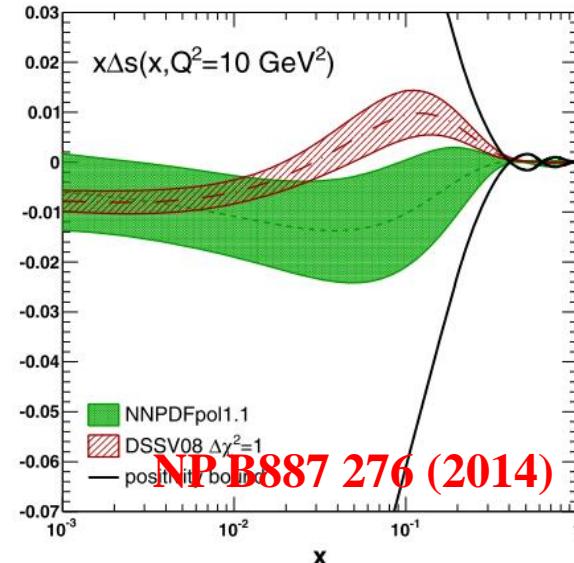
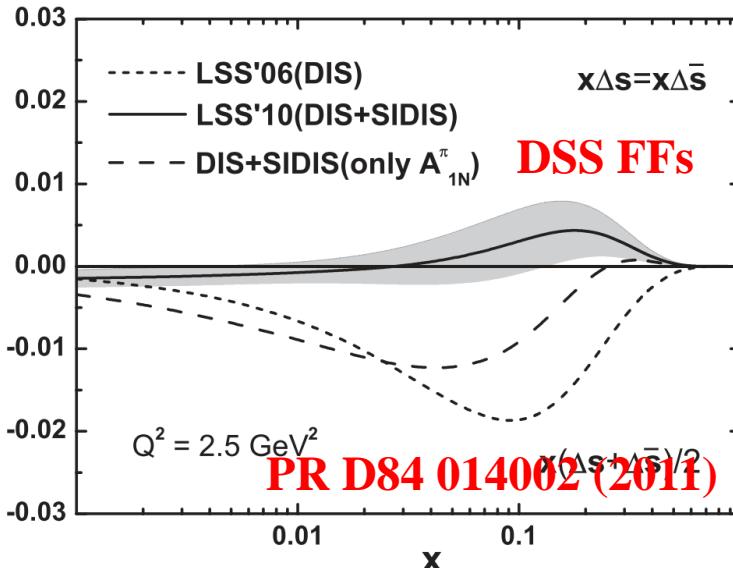
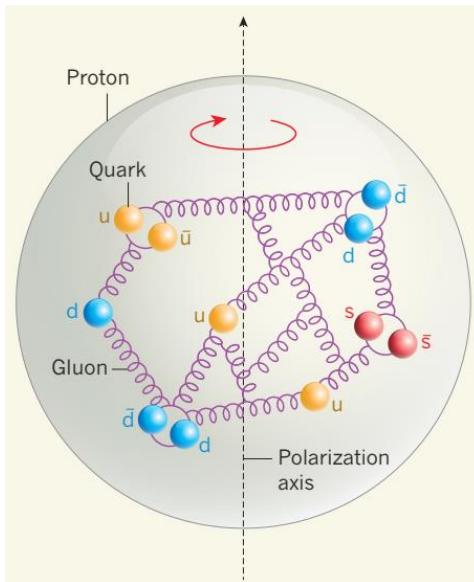
**arXiv:2304.03302**  
**TMD Handbook**

Renaud Boussarie<sup>1</sup>, Matthias Burkardt<sup>2</sup>, Martha Constantinou<sup>3</sup>, William Detmold<sup>4</sup>, Markus Ebert<sup>4,5</sup>, Michael Engelhardt<sup>2</sup>, Sean Fleming<sup>6</sup>, Leonard Gamberg<sup>7</sup>, Xiangdong Ji<sup>8</sup>, Zhong-Bo Kang<sup>9</sup>, Christopher Lee<sup>10</sup>, Keh-Fei Liu<sup>11</sup>, Simonetta Liuti<sup>12</sup>, Thomas Mehen<sup>13</sup>, Andreas Metz<sup>3</sup>, John Negele<sup>4</sup>, Daniel Pitonyak<sup>14</sup>, Alexei Prokudin<sup>7,16</sup>, Jian-Wei Qiu<sup>16,17</sup>, Abha Rajan<sup>12,18</sup>, Marc Schlegel<sup>2,19</sup>, Phiala Shanahan<sup>4</sup>, Peter Schweitzer<sup>20</sup>, Iain W. Stewart<sup>4</sup>, Andrey Tarasov<sup>21,22</sup>, Raju Venugopalan<sup>18</sup>, Ivan Vitev<sup>10</sup>, Feng Yuan<sup>23</sup>, Yong Zhao<sup>24,4,18</sup>

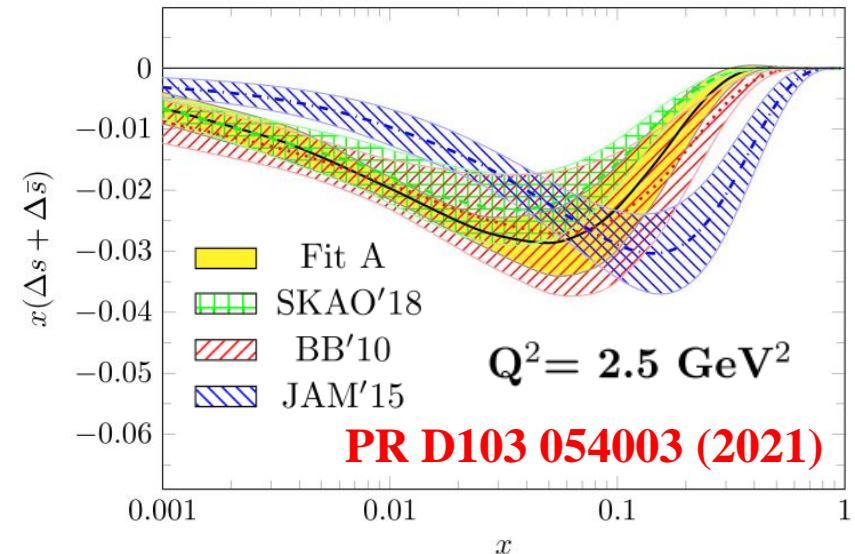


Precise knowledge of FFs will be crucial

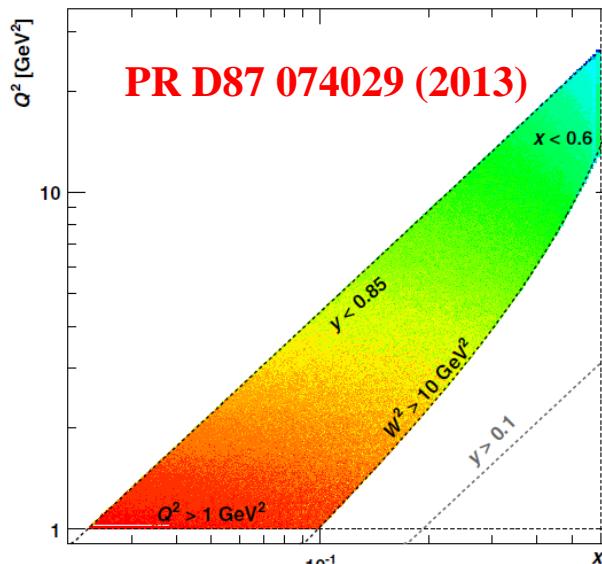
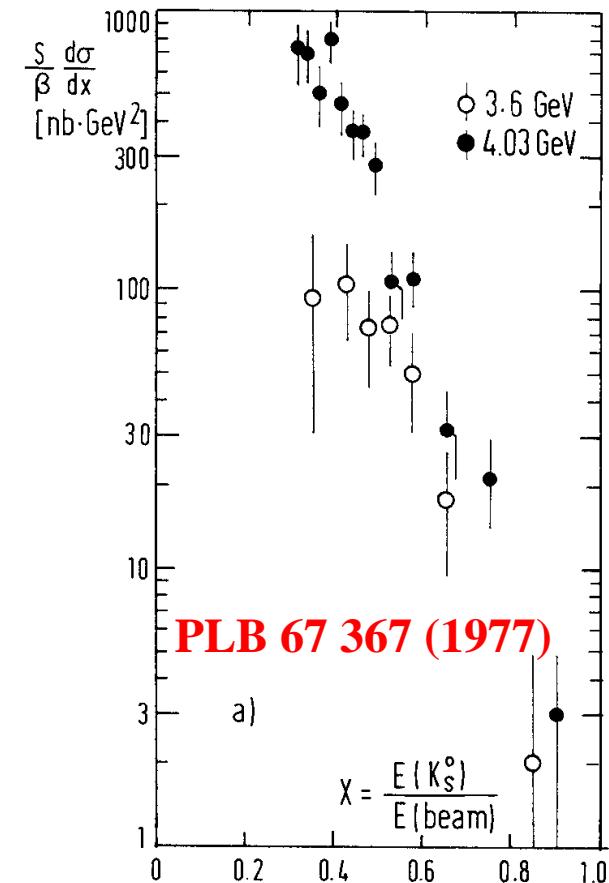
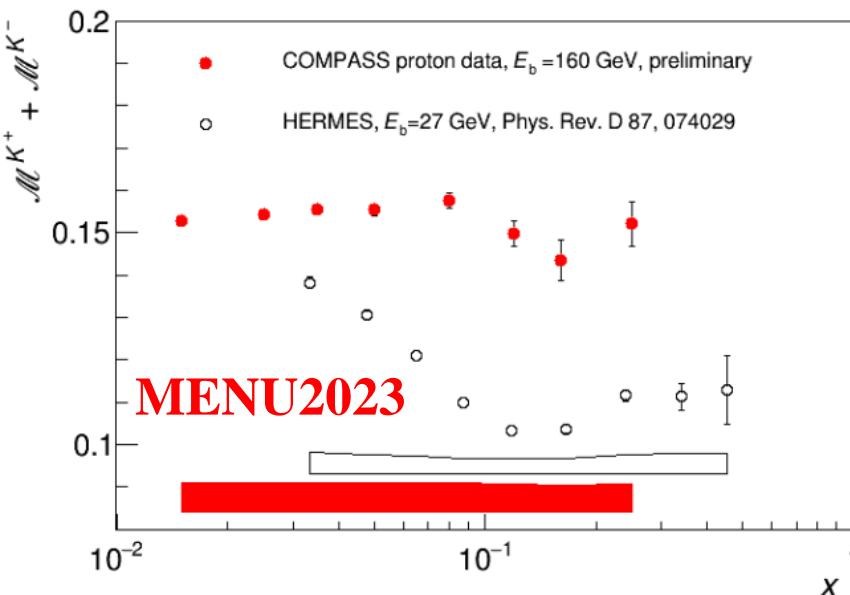
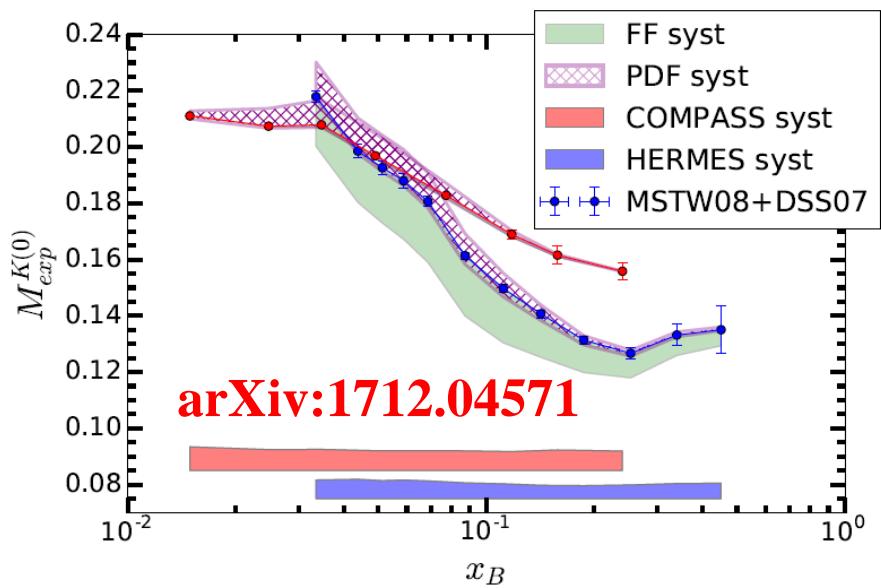
# Strange quark polarization puzzle



- Strange quark density function:  $\Delta s(x) + \Delta \bar{s}(x)$ 
  - ✓ Inclusive DIS: only proton PDF
    - a. **negative for all values of  $x$**
  - ✓ Semi-inclusive DIS: proton PDF & kaon FF
    - a. **DSS FFs: positive for most of measured  $x$**
    - b. **HKNS FF: negative**
    - c. **JAM FFs: negative**
- Reliable FFs knowledge ? Need more efforts



# Kaon multiplicity HERMES & COMPASS



- Hermes data vs. Compass data
  - ✓ Large discrepancies
  - ✓ Kinematic & binning issues
  - ✓ Hadron mass effect
- $e^+e^- \rightarrow K + X$  @ few GeV  $e^+e^-$  ?
  - ✓ Stat. uncertainty: **18-41%**
  - ✓ Precision data ? **Not yet**

# Global data fit on unpolarized FF

R.D. Field, R.P. Feynman, Phys.Rev.D 15, 2590 1977  
J.F. Owens, E. Reya, M. Gluck, Phys.Rev.D 18, 1501 1978  
R. Baier, J. Engels and B. Petersson, Z.Phys.C 2, 265 1979  
M. Anselmino, P. Kroll E. Leader, Z.Phys.C 18, 307 1983  
...  
...

“model estimates consistent with data”

LO groundbreaking

P. Chiappetta et al., Nucl.Phys.B 412, 3 1994  
J. Binneweis, B. Kniehl, G. Kramer, Z.Phys.C 65, 471 1995  
J. Binneweis, B. Kniehl, G. Kramer, Phys.Rev.D 52, 4947 1995  
J. Binneweis, B. Kniehl, G. Kramer, Phys.Rev.D 53, 3573 1996  
D. de Florian, M. Stratmann, W. Vogelsang, Phys.Rev.D 57, 5811 1998  
L. Bourhis et al. Eur.Phys.J.C 19, 89 2001  
B. Kniehl, G. Kramer, B. Potter, Nucl.Phys.B 582, 514 2000  
S. Kretzer, Phys.Rev.D 62, 054001 2000  
S. Albino, B. Kniehl, G. Kramer, Nucl.Phys.B 725 2005  
M. Hirai et al., Phys.Rev.D 75, 094009 2007  
... heavy flavors, hadron mass effects, resummations, ...

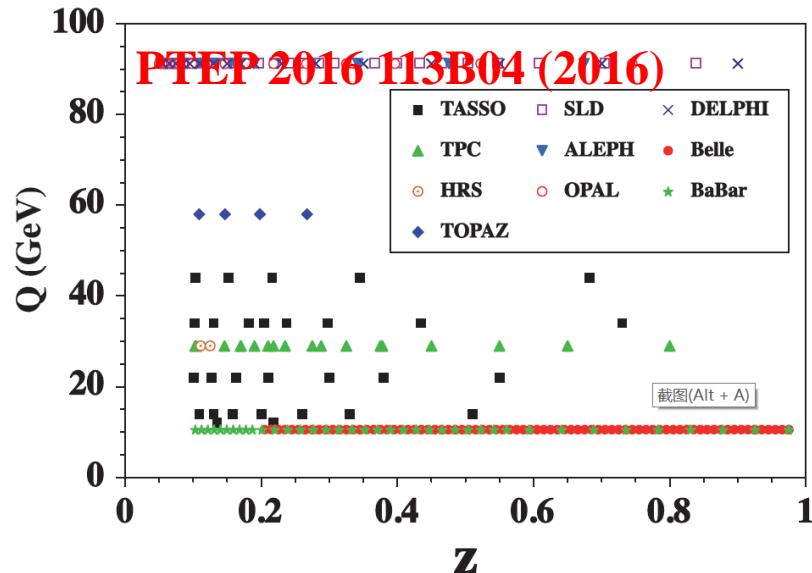
$\pi^0$	CGGRW94
$\pi^\pm, K^\pm$	BKK95
$\pi^\pm, K^\pm$ LEP	
$K^0$	
$\Lambda$	DSV97
$h^\pm$	BFGW00
$\pi^\pm, K^\pm, p/\bar{p}$	KKP00
Flavor tagging	KRE00
OPAL tagging	AKK95
uncertainties	HKNS07
NLO $e^+e^-$ parton... 截图(Alt + A)	

D. de Florian, R.S., M. Stratmann, Phys.Rev.D 75, 114010 2007  
S. Albino, B. Kniehl, G. Kramer, Nucl.Phys.B 803, 42 2008  
R.S., M. Stratmann, P. Zurita, Phys.Rev.D 81, 054001 2010  
C. Aidala, et al. Phys.Rev.D 83, 034002 2011  
E. Leader, A.V. Sidorov, D. Stamenov, arXiv:1312.5200  
M. Soleymaninia et al., Phys.Rev.D 88, 054019 2013  
D. de Florian et al., Phys.Rev.D 91, 4035 2015, D 95 094019 2017  
E. Leader, A.V. Sidorov, D. Stamenov, Phys.Rev.D 94, 096001 2016  
V. Bertone, et al., EPJC 77, 516 2017  
N. Sato, et al., Phys.Rev.D 101, 074020 2020  
R. A. Khalek, et al., Phys.Lett.B 834, 137456 2022

$e^+e^-, pp, SIDIS$	DSS07
$e^+e^-, pp$	AKK08
nFFs	SSZ10
$\eta$	AESS11
SIDIS only	LSS13
$e^+e^-, pSIDIS$	SKMNA13
$\pi^\pm, K^\pm$ update	DSS14/17
SIDIS only	LSS15
$h^\pm, e^+e^-$ only	NNFF1.0
$e^+e^-, SIDIS$	JAM19
$e^+e^-, SIDIS$	MAPFF1.0

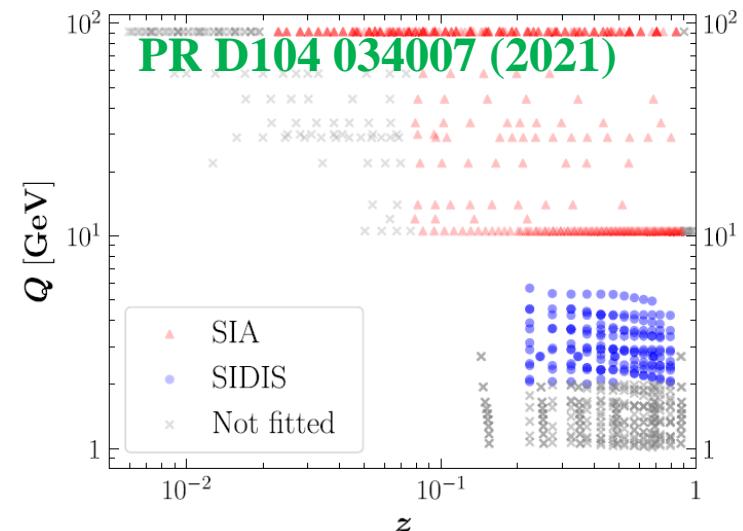
Global paradigm

# Used data set @ FFs fitting

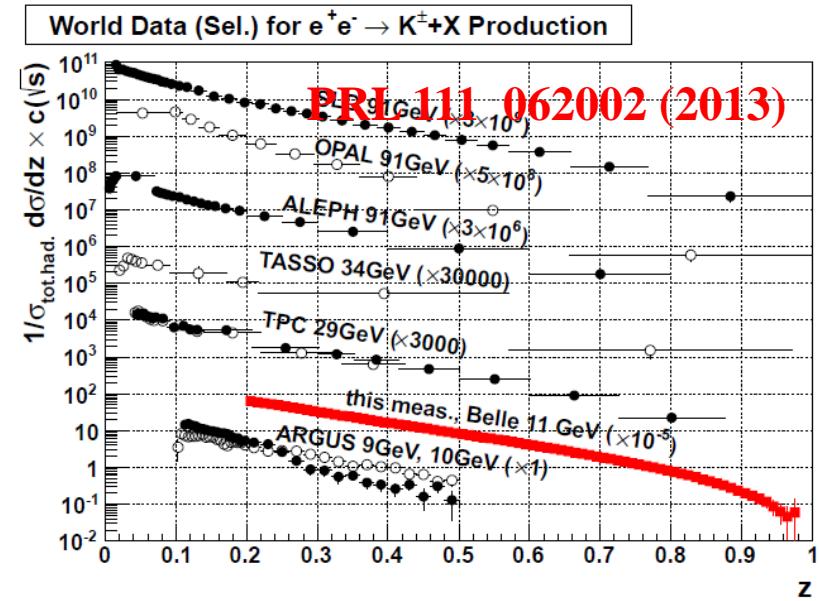
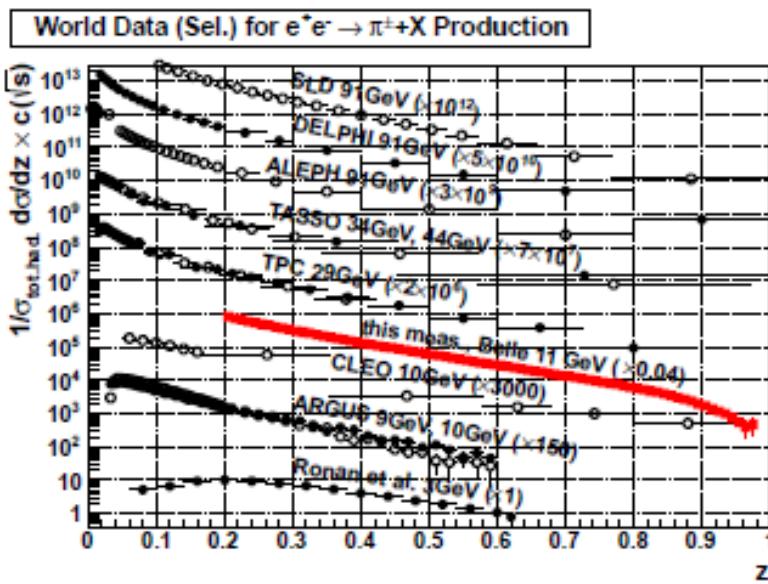
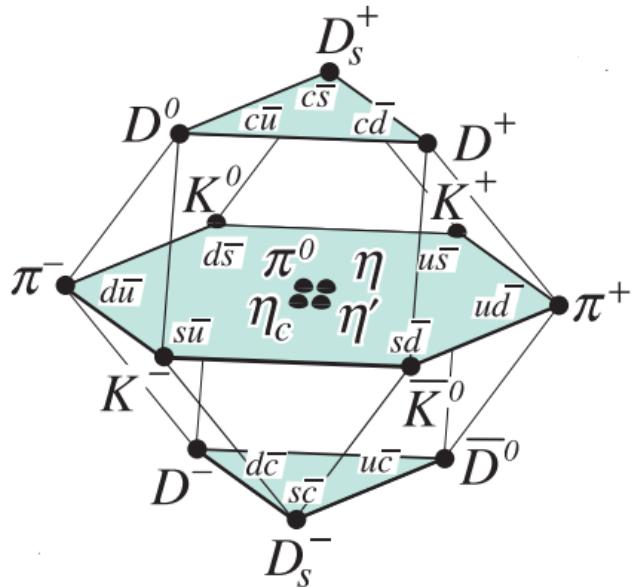


Experiment	Process	$\mathcal{L} [pb^{-1}]$	$Q^2 [GeV^2]$	Final states
PPNP91 136 (2016)				
TPC [288–291]	$e^+e^-$	240	24.44	$\pi^\pm, K^\pm, p/\bar{p}$
TASSO [292–294]	$e^+e^-$	34		$\pi^\pm, K^\pm, p/\bar{p}, K_S^0, \Lambda/\bar{\Lambda}$
[295–298]	$e^+e^-$	20	$M_Z$	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
SLD [299,300]	$e^+e^-$	800	$M_Z$	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
ALEPH [301,302]	$e^+e^-$	800	$M_Z$	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
DELPHI [303–306]	$e^+e^-$	800	$M_Z$	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
OPAL [307–310]	$e^+e^-$	800	$M_Z$	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
H1 [311–313]	$e+p$	500	$27.5(e) + 920(p)$	$h^\pm, K_0^0$
ZEUS [314–316]	$e+p$	500	$27.5(e) + 920(p)$	$h^\pm$
BELLE [317,318]	$e^+e^-$	$10^6$	Near 10.58	$\pi^\pm, K^\pm, p/\bar{p}$
BaBar [319,320]	$e^+e^-$	$557 \cdot 10^3$	Near 10.58	$\pi^\pm, K^\pm, \eta, p/\bar{p}$
HERMES [321,322]	$e+p(d)$	$272(p) 329(d)$	27.6 fixed target	$\pi^{\pm,0}, K^\pm$
COMPASS [323]	$\mu+p(d)$	775	160 GeV fixed target	$h^\pm$
PHENIX [324–326]	$pp$	$16 \times 10^{-3}$	62.4	
[327–329]	$pp$	2.5	200	
STAR [330–332]	$pp$	8	510	
[333–335]	$pp$		200	$\pi^{0,\pm}, \eta, p/\bar{p}, K_S^0, \Lambda/\bar{\Lambda}$
ALICE [336]	$pp$	$6 \times 10^{-3}$	$7 \times 10^3$	$\pi^0, \eta$
TOPAZ [337]	$e^+e^-$	278	52–61.4	$\pi^\pm, K^\pm, K_S^0,$
CDF [338,339]	$p+\bar{p}$	n/a	630 (1800)	$h^\pm, K_S^0 \Lambda^0$

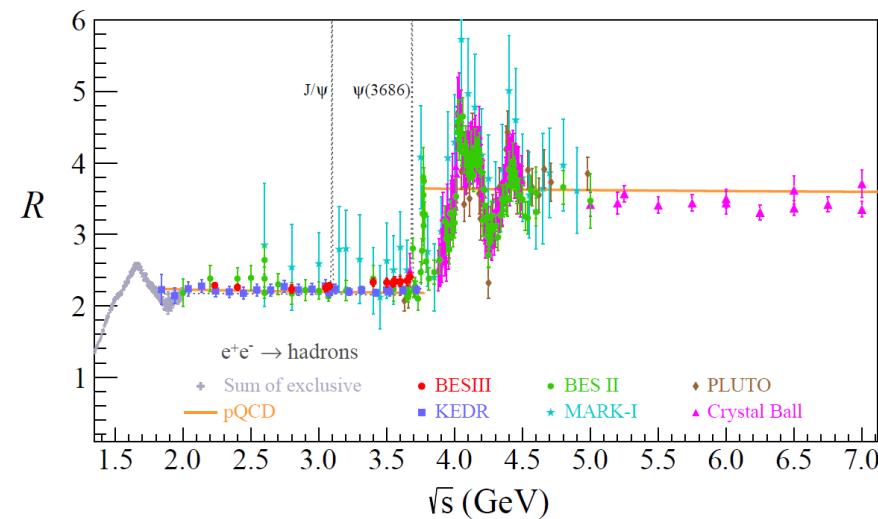
- Updated HKNS FFs @ 2016
  - ✓ only  $e^+e^- (\sqrt{s} > 10 \text{ GeV})$  data sets
- MAPFF1.0 FFs @ 2021
  - ✓  $e^+e^- (\sqrt{s} > 10 \text{ GeV})$  and SIDIS data sets
- Data set at  $\sqrt{s} < 10 \text{ GeV}$   $e^+e^-$  collision ?



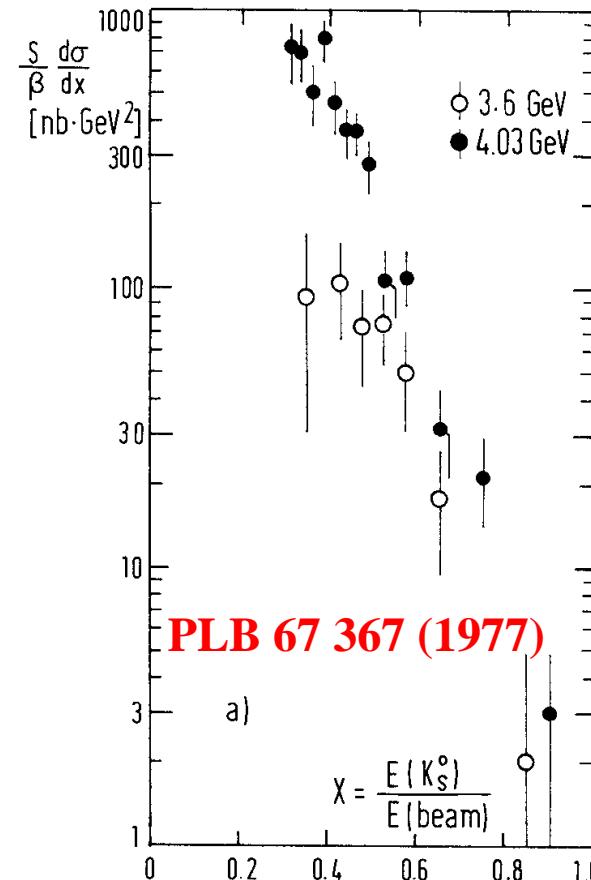
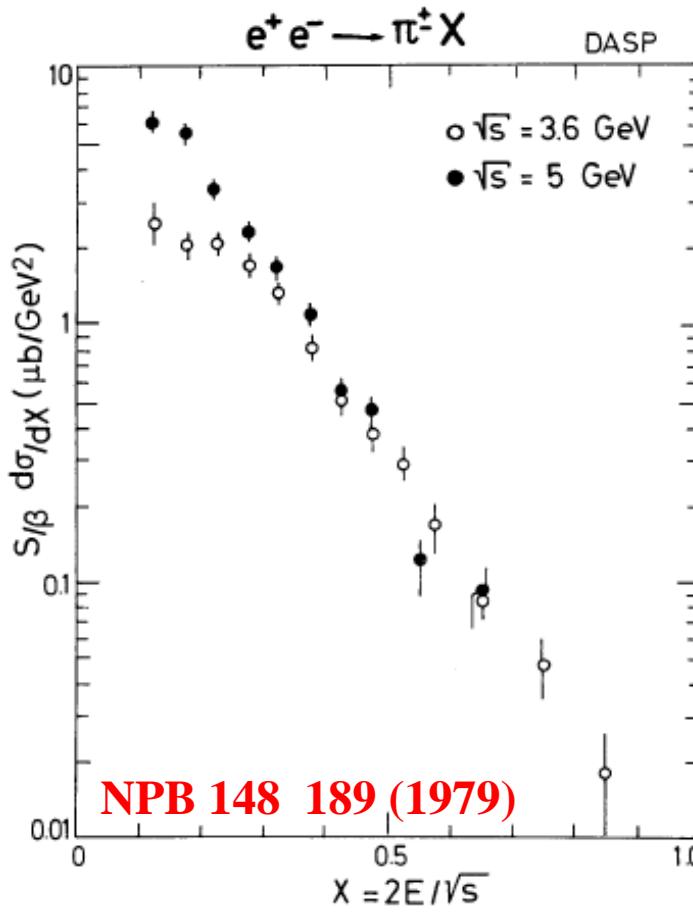
# World $\pi$ & K data on $e^+e^-$



- Precision data includes charged  $\pi$ , K
- Data sets at  $\sqrt{s} < 10$  GeV  $e^+e^-$  collision ?
  - ✓ high z data sets ?
- R scan data @ BESIII:  $\sim 10 \text{ pb}^{-1}$  @ each  $\sqrt{s}$

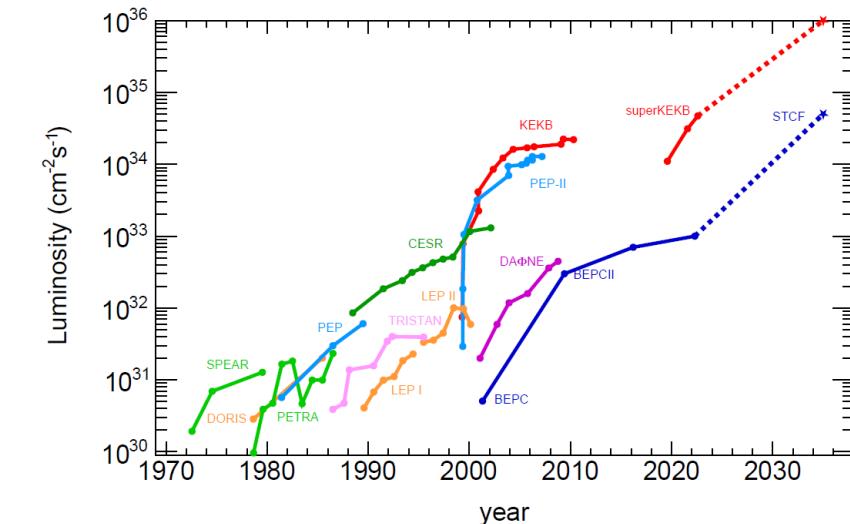


# World $\pi$ & K data on $e^+e^-$

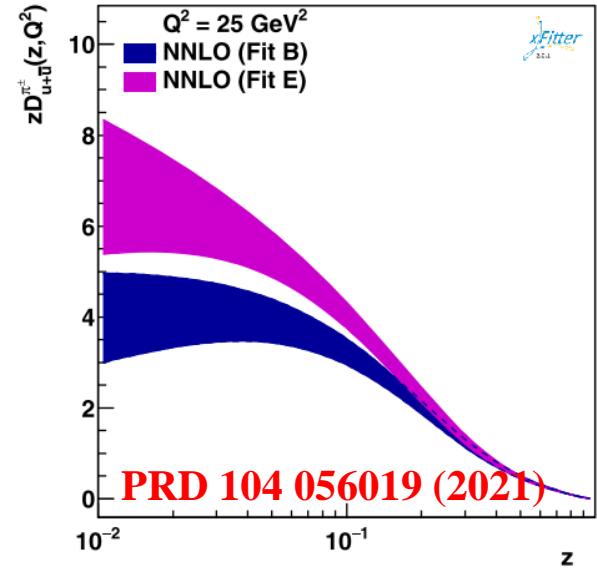
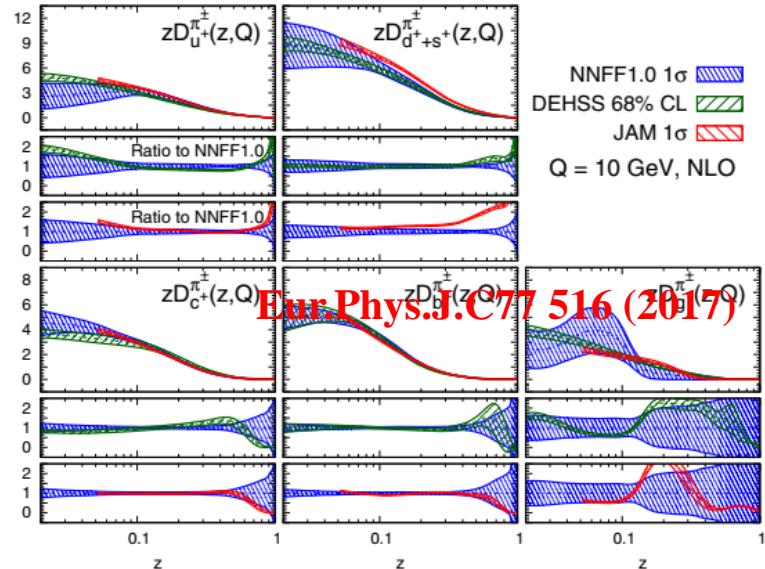
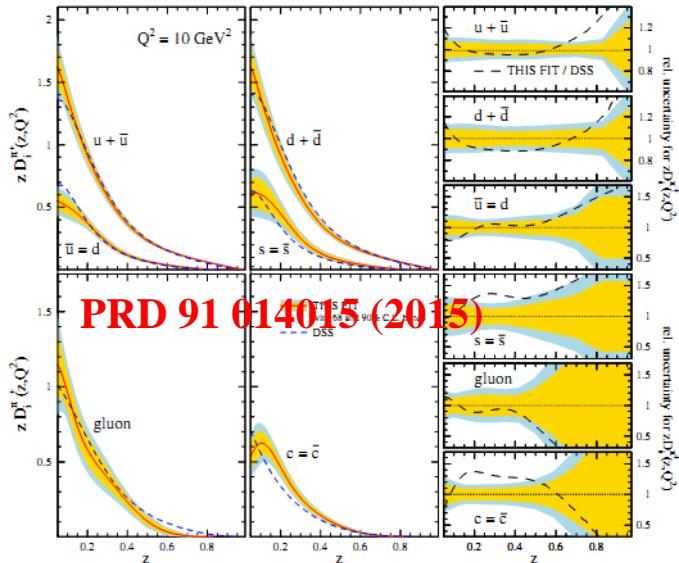


BESIII R scan data ⇒ Precision measurement ?

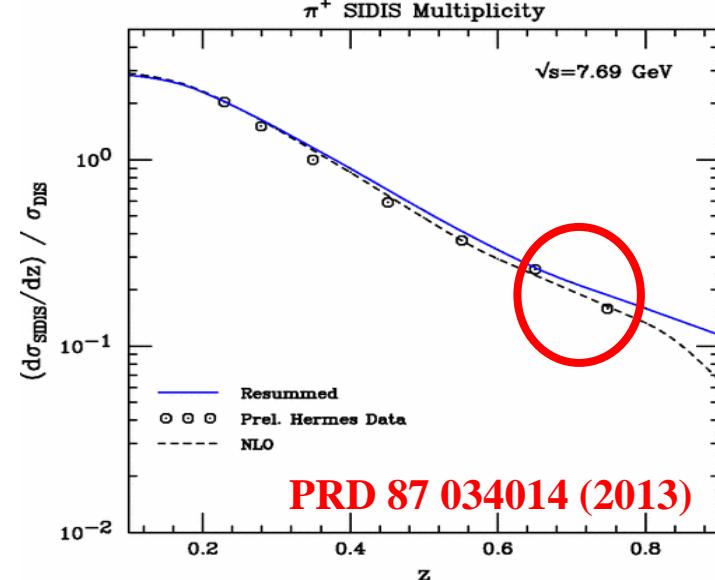
- Charged  $\pi$  @ DASP
  - ✓ about 44 years ago
  - ✓ stat. uncertainty: 18%
- $K_s^0$  @ PLUTO
  - ✓ about 46 years ago
  - ✓ stat. uncertainty: 18-41%
- Precision data ? TMD FFs ?



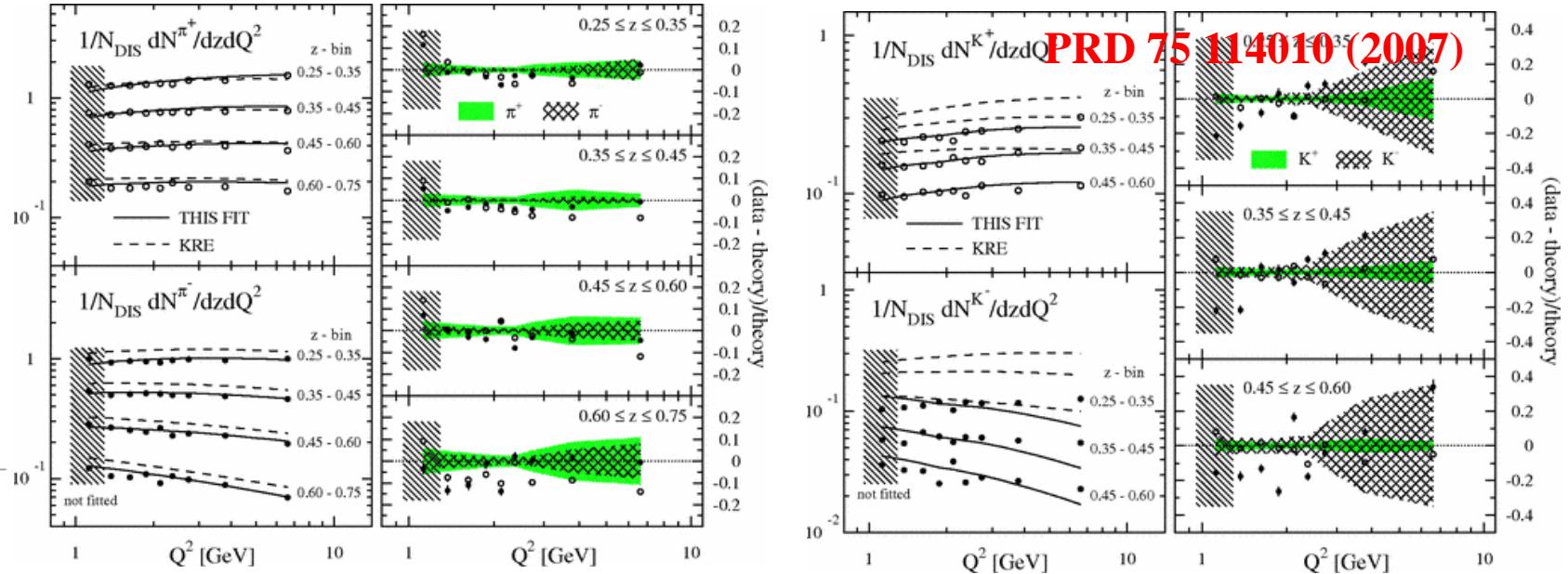
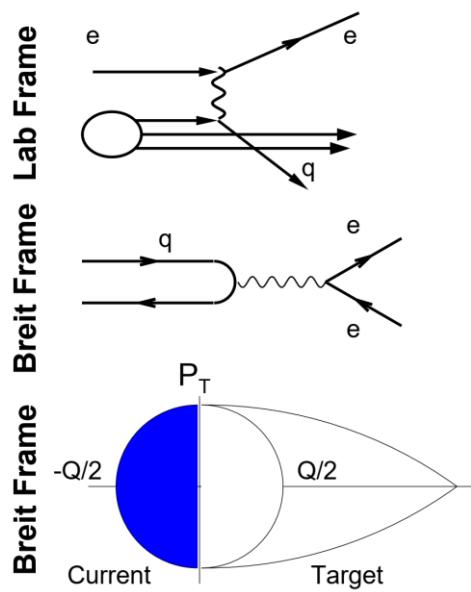
# Pion FF: Best known FF



- For  $z \geq 0.8$ , uncertainty rapidly increase because of the lack of experimental data
- Xfitter: data at  $\sqrt{s} > 10 \text{ GeV}$   $e^+e^-$ 
  - ✓ Low  $\sqrt{s}$   $e^+e^-$  data ?
- Large  $z$  re-summation
  - ✓ High  $z$  data ?

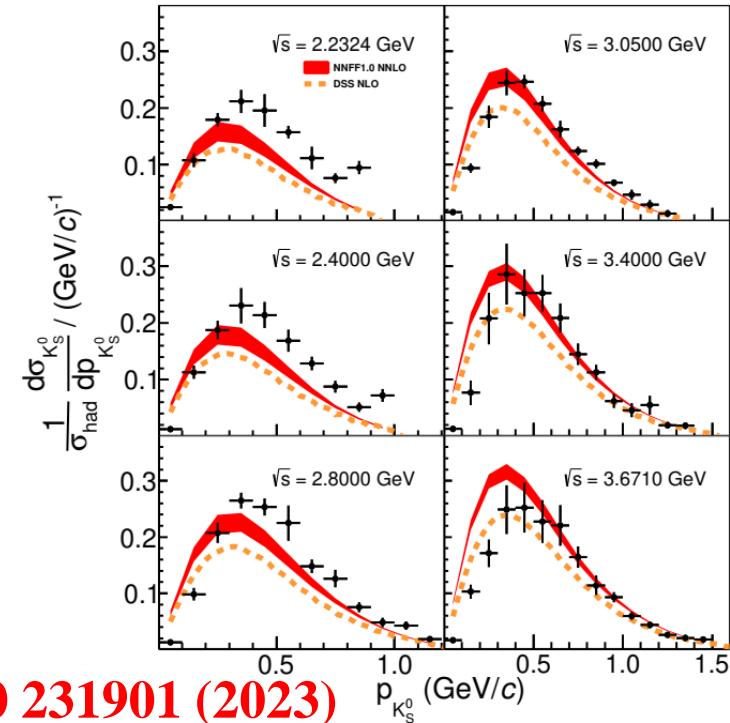
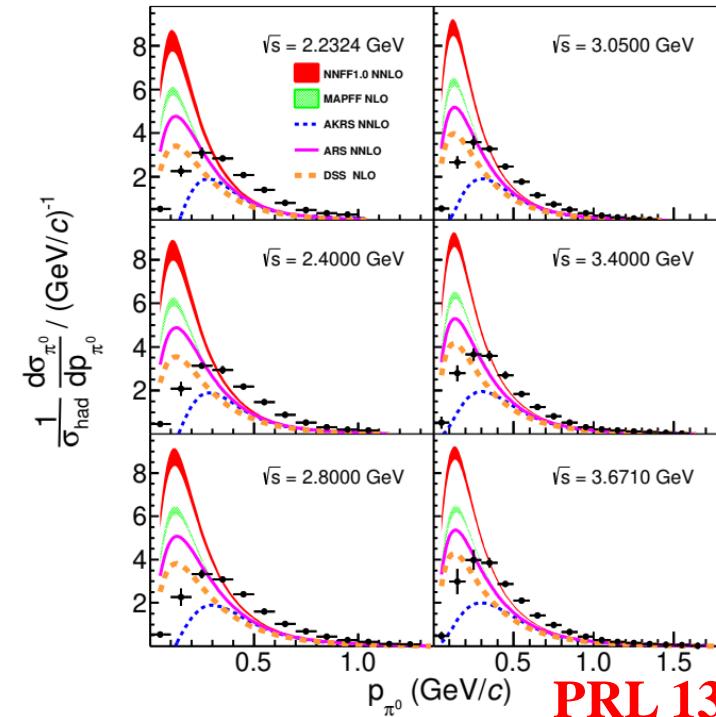
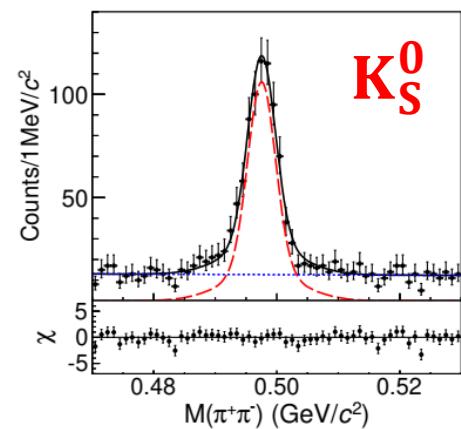
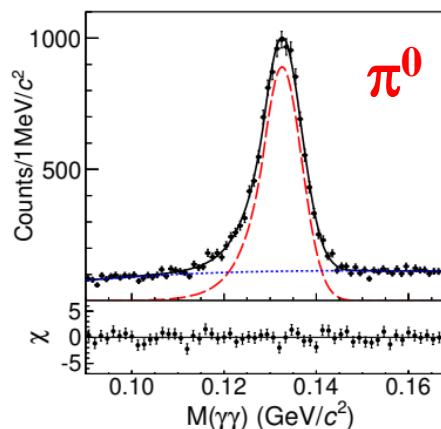
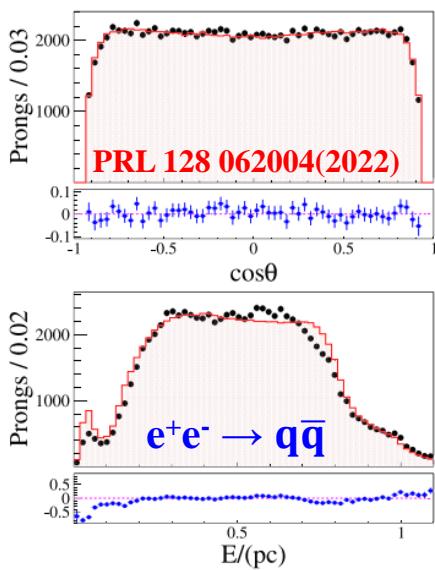
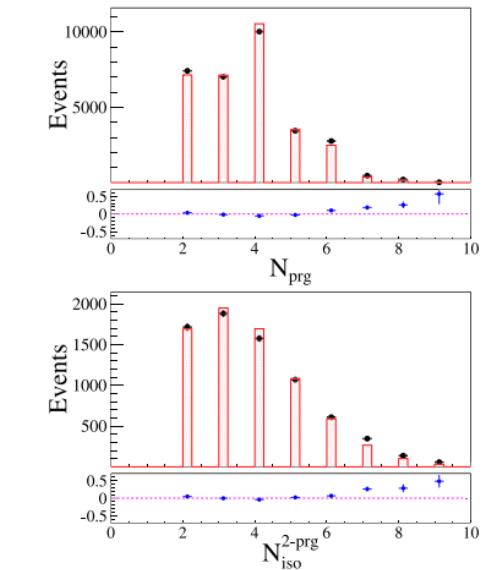


# Pion FFs



- DIS @ Breit frame
  - ✓ Incoming quark scatters off photon and returns along same axis
- Current region of Breit frame is analogous to  $e^+e^-$ 
  - ✓ Born level: DIS  $Q = e^+e^- \sqrt{s}$
- DSS FFs: HERMES ep pion data at **10% level**
- $e^+e^-$  data at low energy  $\sqrt{s}$ 
  - ✓ FFs packages could describe  $e^+e^-$  data at **?% level**

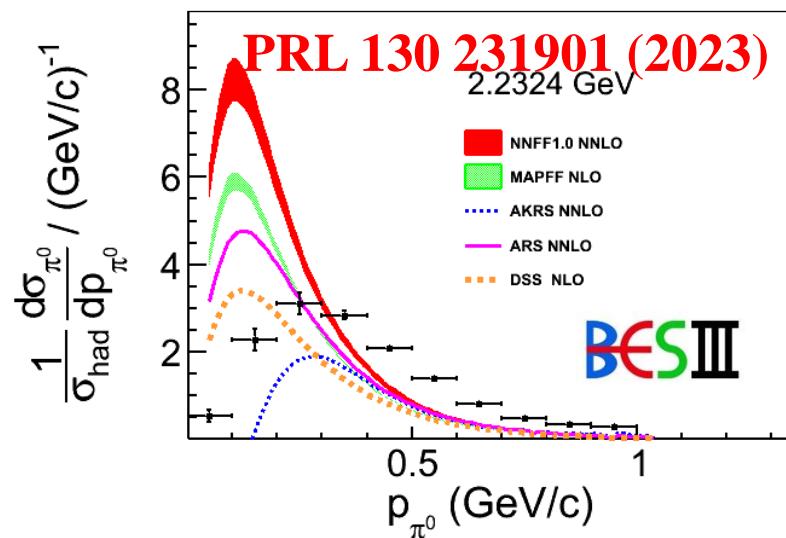
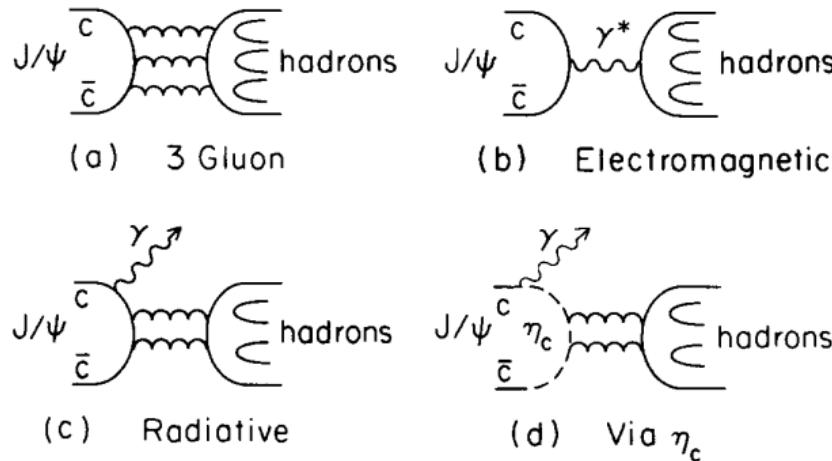
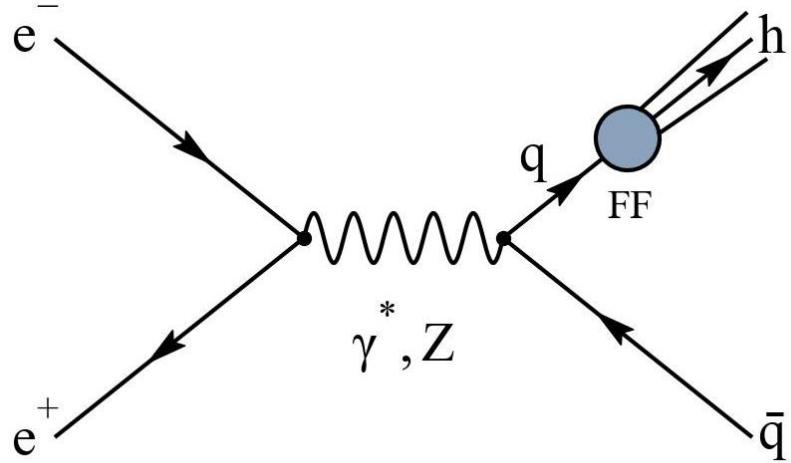
# $e^+e^- \rightarrow \pi^0/K_S^0 + X$ @ BESIII



PRL 130 231901 (2023)

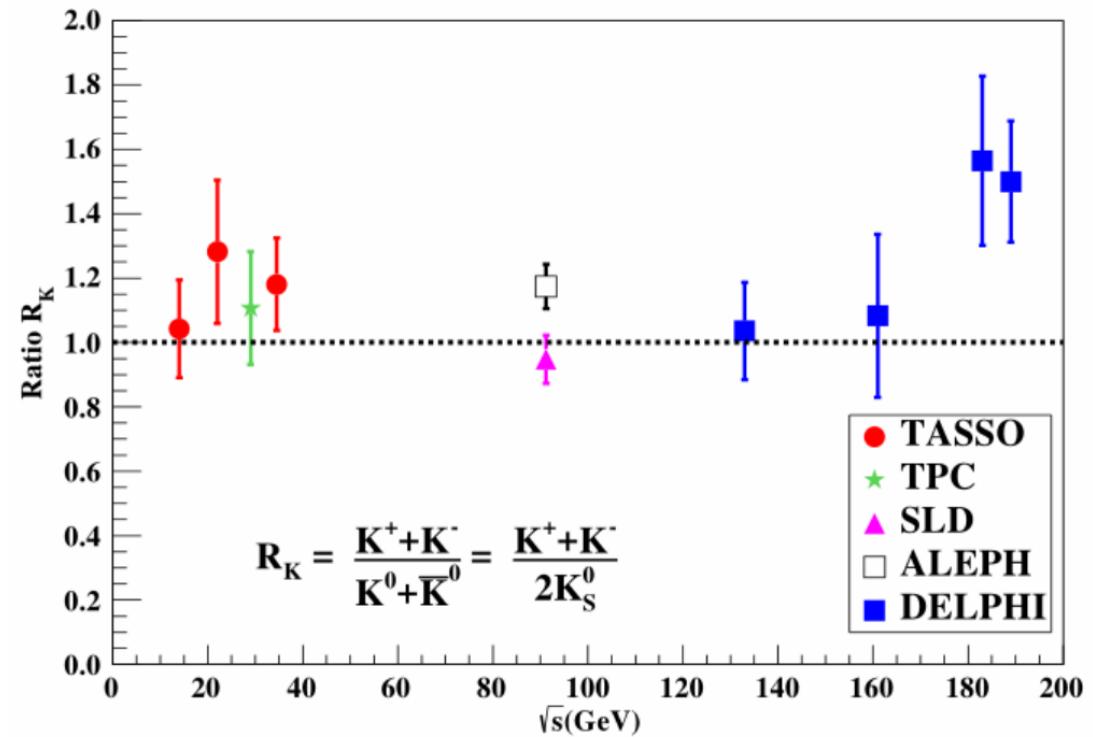
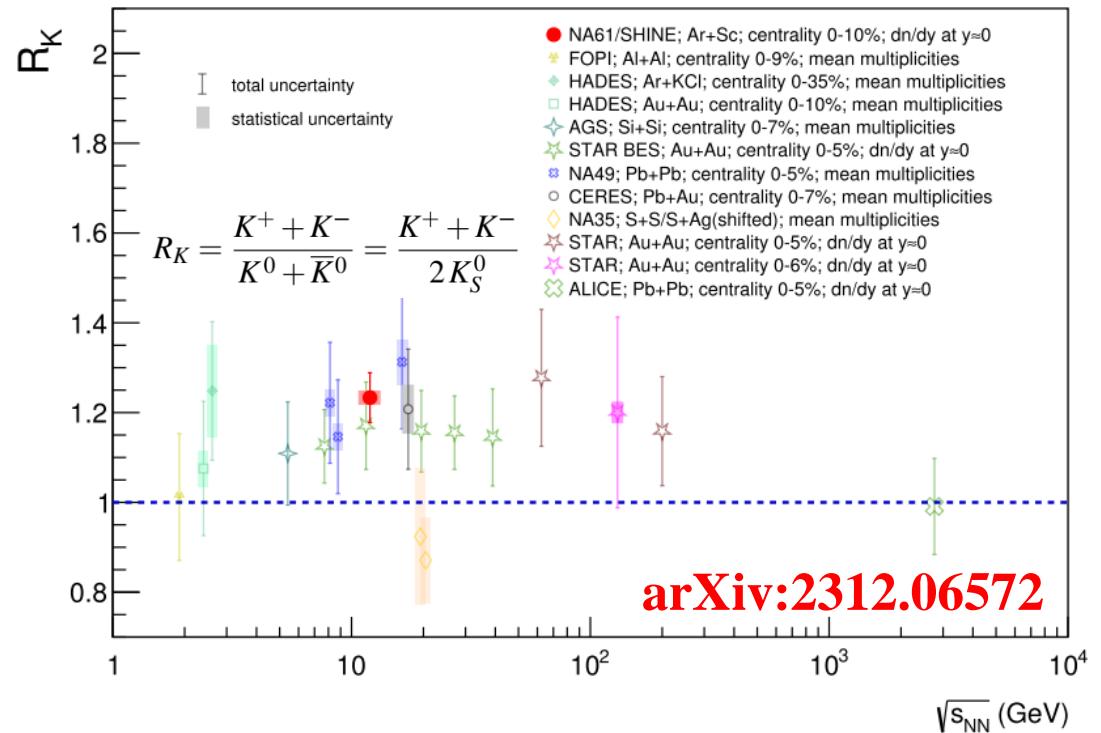
- Charged pion/kaon
  - ✓ Particle PID: right/wrong probability
  - ✓ Low  $p_t$ : curling track, not reconstructed
- inclusive  $\pi^0$  production: surprise
- inclusive  $K_S^0$  production: not so bad

# $e^+e^- \rightarrow \pi^0/K_S^0 + X @ \text{BESIII}$



- From theory side: fitting with BESIII data, hadron mass effect, large z re-summation, and so on
- From experimental side
  - ✓ Primary hadron vs from resonance decay  
⇒ measure  $e^+ e^- \rightarrow \rho(\omega, \phi) + X$ , and so on
  - ✓ Contribution of vector states  $\rho^*$ ,  $\omega^*$  and  $\phi^*$   
⇒  $e^+ e^- \rightarrow \rho^*/\omega^*/\phi^* \rightarrow h + X$

# $e^+e^- \rightarrow K + X$

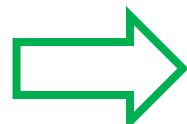


- Ratio  $R_K$  in nucleus-nucleus collisions

- ✓ Expected  $R_K < 1$ , opposite to observed results
- ✓ Large isospin breaking ?

- How about  $R_K$  in  $e^+e^-$  collision ?

- ✓ Precise measurement ?



$\pi$  ?  $K^*(892)$  ?

# $e^+ e^- \rightarrow K^+ K^- \pi^0$ vs. $e^+ e^- \rightarrow K_S^0 K_L^0 \pi^0$

Process	Fraction (%) (2.125 GeV)	Fraction (%) (2.396 GeV)
$\phi\pi^0$	$1.8 \pm 0.4$	$0.7 \pm 0.3$
$\rho(1450)\pi^0$	$3.8 \pm 0.7$	$0.2 \pm 0.2$
$\phi(1680)\pi^0$	$14.6 \pm 2.3$	$13.6 \pm 2.9$
$\rho(1900)\pi^0$	$2.1 \pm 0.3$	$3.0 \pm 1.0$
$\rho_3(2250)$	$0.9 \pm 0.5$	$0.9 \pm 0.6$
$K^*(892)K$	$2.8 \pm 0.3$	$9.3 \pm 1.2$
$K^*(1410)K$	$1.1 \pm 0.8$	$3.6 \pm 1.4$
$K_2^*(1430)K$	$73.0 \pm 3.7$	$64.6 \pm 3.2$
$K_3^*(1780)K$	$1.3 \pm 0.5$	$2.1 \pm 1.4$

Process	Fraction (%)		
	2.125 GeV	2.396 GeV	2.900 GeV
$\phi\pi^0$	$0.04 \pm 0.56$	$0.87 \pm 0.01$	$1.82 \pm 1.11$
$\phi(1680)\pi^0$	$2.39 \pm 1.23$	$5.96 \pm 2.10$	$5.22 \pm 1.50$
$K^*(892)^0 \bar{K}^0$	$79.89 \pm 1.12$	$86.01 \pm 1.38$	$72.65 \pm 2.11$
$K_2^*(1430)^0 \bar{K}^0$	$7.42 \pm 0.83$	$1.93 \pm 0.57$	$1.85 \pm 0.82$
$K(1680)^0 \bar{K}^0$	$3.00 \pm 1.11$	$6.73 \pm 1.91$	$5.82 \pm 1.96$

## ● KKπ system

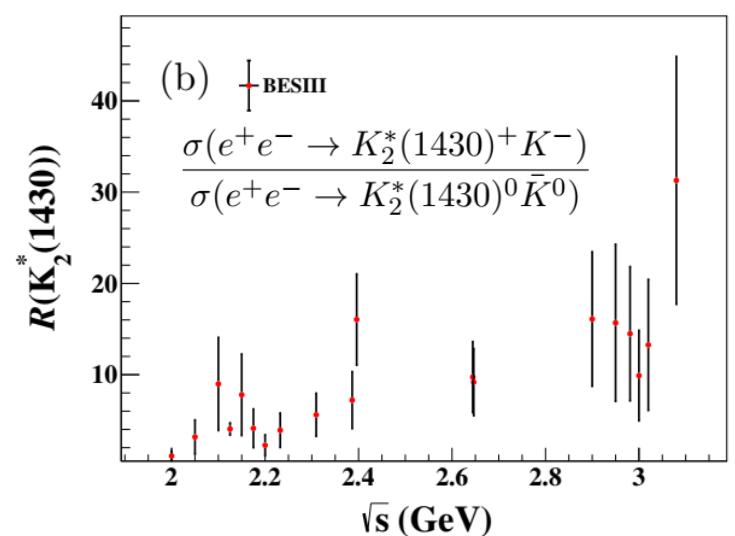
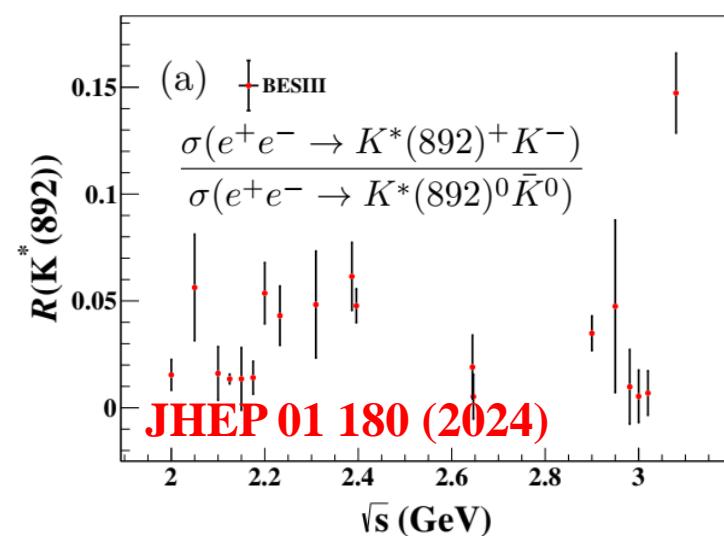
- ✓  $K_2^*(1430)K$  @  $K^+K^-\pi^0$
- ✓  $K^*(892)^0 \bar{K}^0$  @  $K_S^0 K_L^0 \pi^0$

## ● $\Gamma(K^+K^-) \approx 10 \Gamma(K_S K_L)$

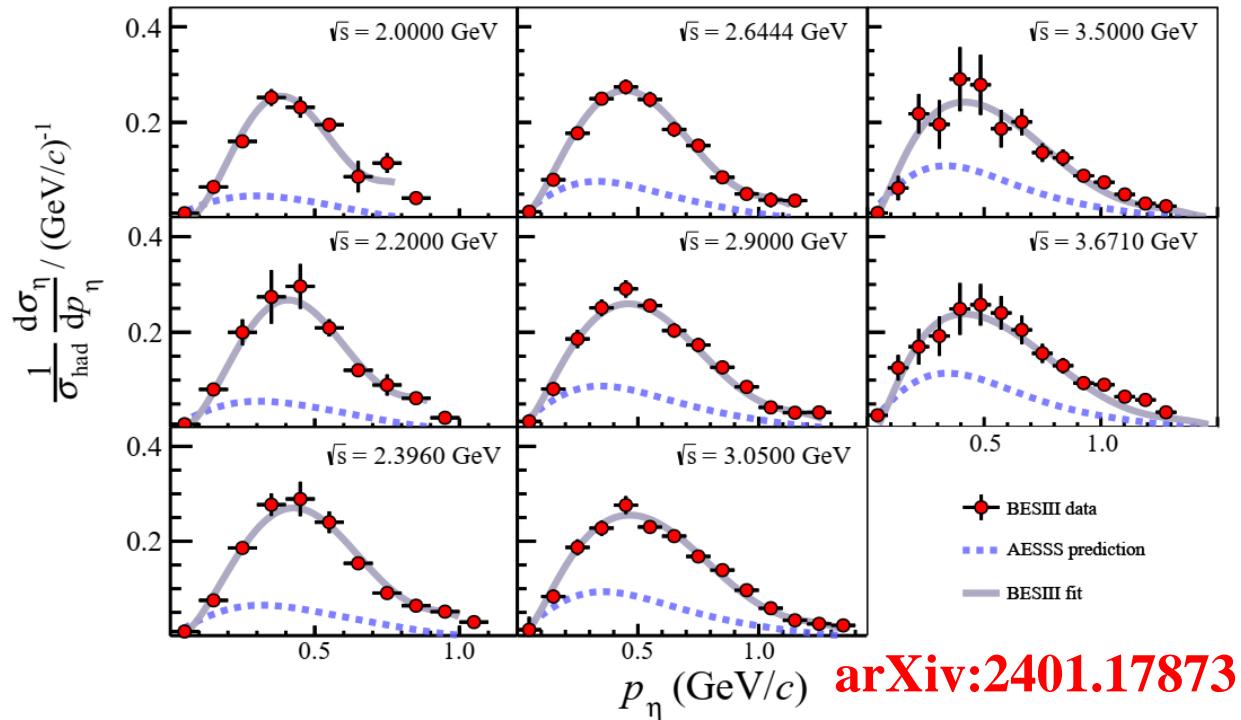
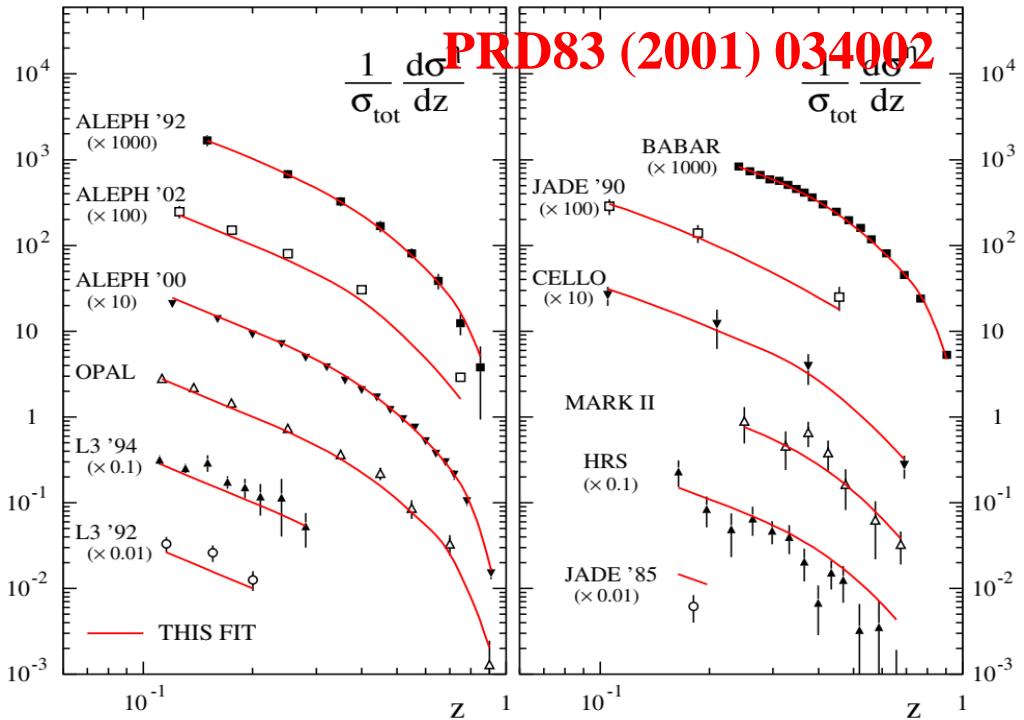
- ✓ PRD99 032001 (2019)
- ✓ PRD104 092014 (2021)
- ✓ valence quark charge

## ● $K^*K$ : isospin → one

- ✓ electromagnetic interaction



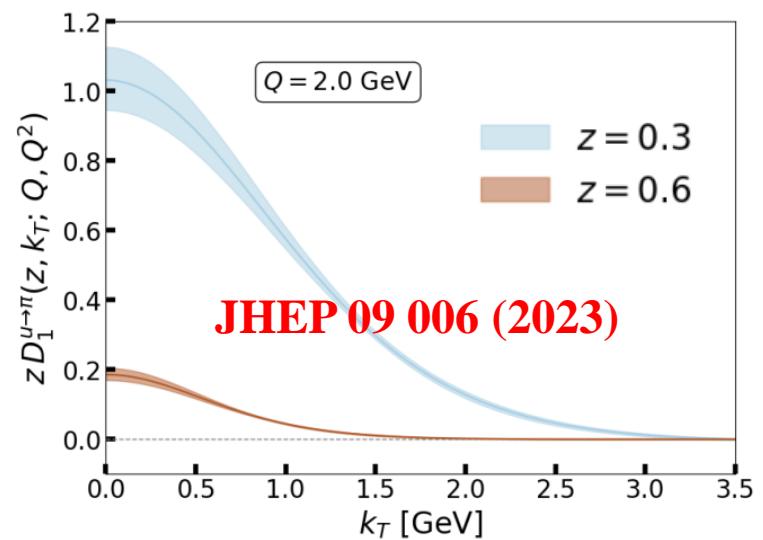
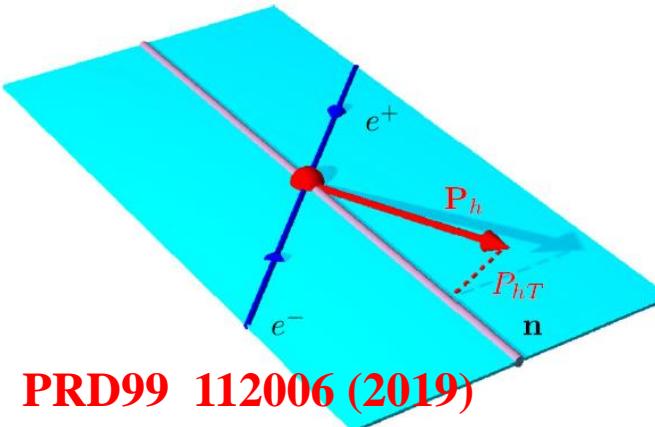
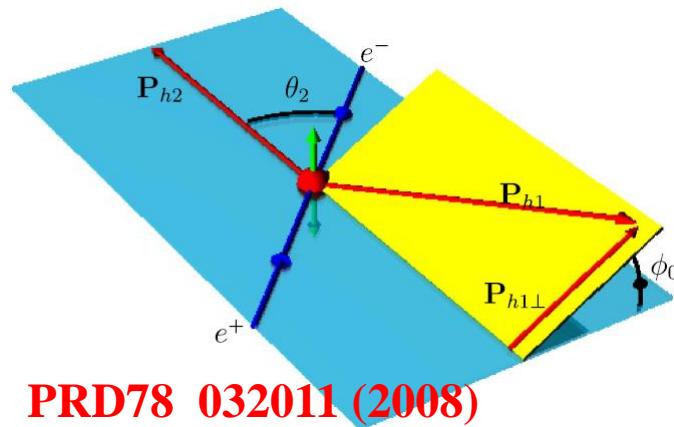
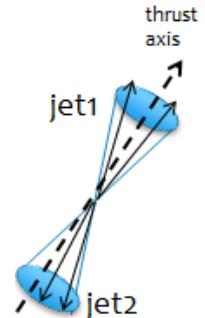
# $e^+e^- \rightarrow \eta + X$ @ BESIII



- $\eta$ : a Goldstone boson due to spontaneous breaking of QCD chiral symmetry
- $\eta$  FF @ NLO: with data at  $\sqrt{s} > 10\text{GeV}$   $e^+e^-$  collision
- BESIII fit: NNLO accuracy, hadron mass correction & higher twist contributions

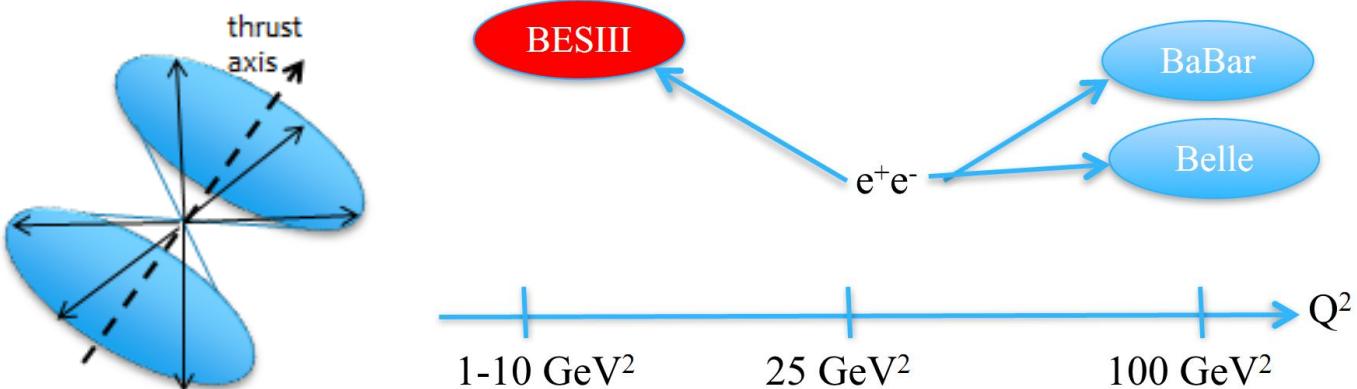
# TMD FFs: $D_1^h(z) \Rightarrow D_1^h(z, p_T)$

TMD FF $D_1(z, k_T)$		
$e^+e^- \rightarrow h_a h_b X$	$\sum_q e_q^2 D_1^{h_a/q}(z_a, k_{aT}) \otimes D_1^{h_b/\bar{q}}(z_b, k_{bT}) + \{q \leftrightarrow \bar{q}\}$	back-to-back production of hadron pair
$e^+e^- \rightarrow (h, \text{jet/thrust axis})X$	$\sum_q e_q^2 D_1^{h/q}(z, k_T)$	can access $z, k_T$

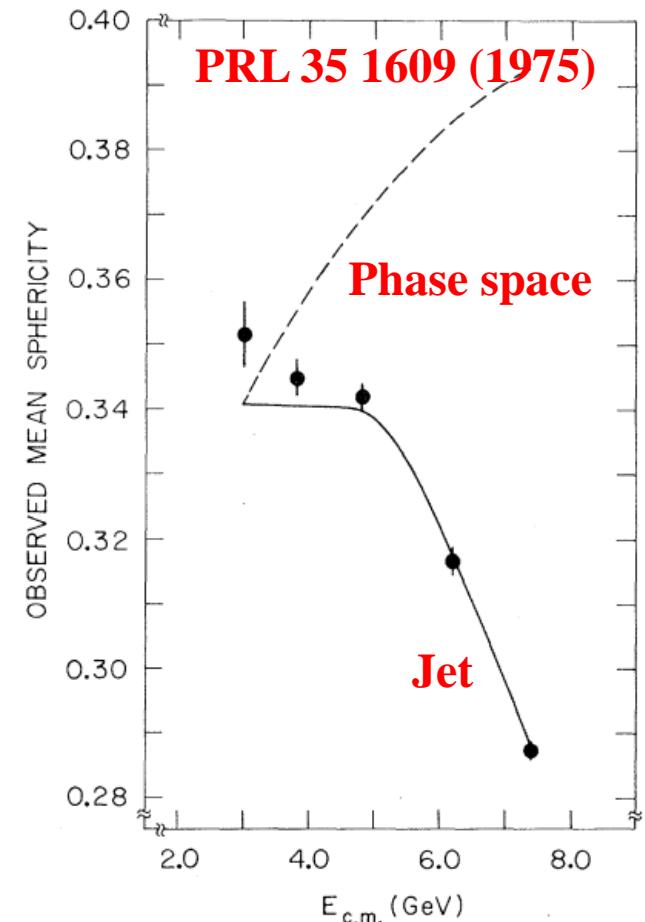
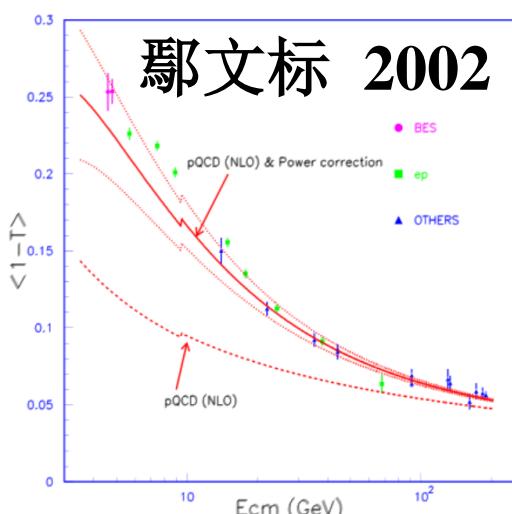


- Traditional 2-hadron FF
  - ✓ Use transverse momentum between two hadron
- Signal-hadron FF with Thrust or jet axis
  - ✓ Need  $q\bar{q}$  axis (quark or jet axis)

# TMD FFs: $D_1^h(z) \Rightarrow D_1^h(z, p_T)$



- Jet structure at BESIII & STCF
  - ✓ can reconstruct thrust axis correctly ?
- At lower  $\sqrt{s}$ : back-to-back pion events
  - ✓ Events with  $e^+e^- \rightarrow \pi\pi + X$ :  $\theta_{\pi\pi} > 120^\circ$
- At higher  $\sqrt{s}$ : jet @ [5, 7] GeV ?
  - ✓ Evidence for jet structure
  - ✓ Events with large thrust value ?



# STCF with polarized beam

一个左手极化或右手极化的入射轻子 $e_{L,R}^-$ 与一个无极化的 $e^+$ 的散射截面分别为：

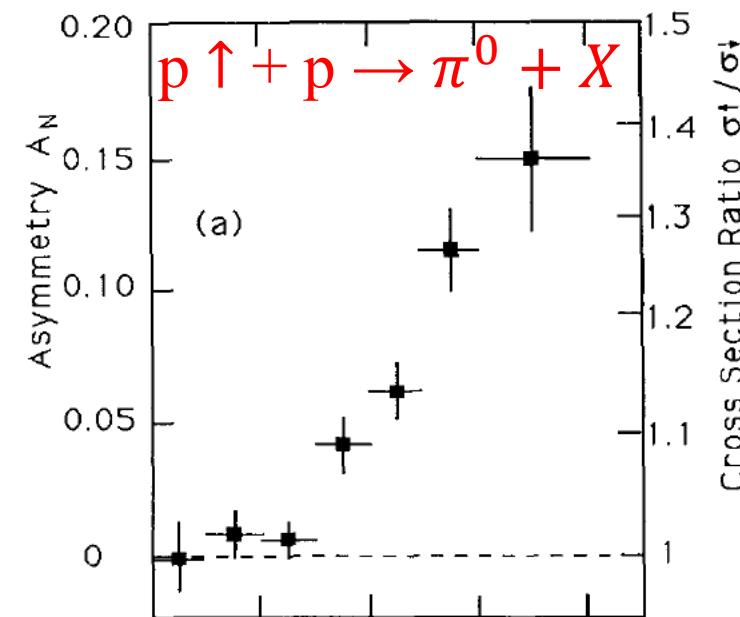
$$\frac{d\sigma_L}{dt} = \frac{1}{16\pi s^2} \{ u^2 |G_{LL}(s) + G_{LL}(t)|^2 + t^2 |G_{LR}(s)|^2 + s^2 |G_{LR}(t)|^2 \},$$
$$\frac{d\sigma_R}{dt} = \frac{1}{16\pi s^2} \{ u^2 |G_{RR}(s) + G_{RR}(t)|^2 + t^2 |G_{RL}(s)|^2 + s^2 |G_{RL}(t)|^2 \}.$$

$$A_L = \frac{d\sigma^\rightarrow - d\sigma^\leftarrow}{d\sigma^\rightarrow + d\sigma^\leftarrow}$$

- Longitudinally polarized  $e^-$  beam @ STCF
  - ✓ Effect on fragmentation function ?
- Single-spin asymmetry
  - ✓ What could we obtain ?
- We need theorist's help.

Large- $x_F$  spin asymmetry in  $\pi^0$  production  
by 200-GeV polarized protons\*

FNAL E704 Collaboration



# Summary

- The knowledge of FFs is an important ingredient in our understanding of non-perturbative QCD dynamics.
- Inclusive  $\pi^0$  &  $K_s$  production @ BESIII: Large discrepancy with theory calculation, need more study on FFs at low energy  $e^+e^-$  collision.
- Inclusive  $\eta$  production @ BESIII: fit with NNLO accuracy, hadron mass correction & higher twist contributions, could describe BESIII data
- $e^+e^-$  annihilation experiments such as STCF provide the cleanest environment in which to measure FFs.

