形状因子和轻介子光锥分布振幅

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Overview

The emergent phenomena of QCD

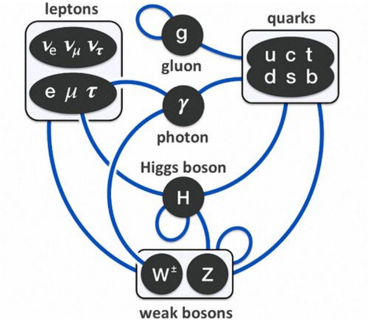
Take pion as the example

From ρ, f_0 to DiPion LCDAs

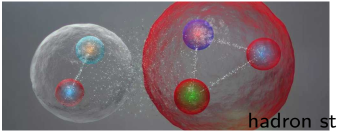
Conclusion

Emergent phenomena of QCD

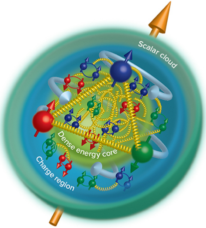
In philosophy, systems theory, science, and art, emergence occurs when an entity is observed to have properties its parts do not have on their own, properties or behaviors which emerge only when the parts interact in a wider whole.



Parts



hadron structure



hadron formation

Observed entity

Emergent phenomena of QCD

QCD is believed to confine, that is, its physical states are color singlets with internal quark and gluon degrees of freedom

- QCD allow us to study hadron structures in terms of partons
- Factorization theorem to separate the hard partonic physics out of the hadronic physics (soft, nonperturbative objects)
- Define hadron structures by quantum field theories
- Identify theoretical observables in factorizable formulism

$$\frac{d\sigma}{d\Omega} = \int_x^1 \frac{d\zeta}{\zeta} \mathcal{H}(\zeta) f(\frac{x}{\zeta})$$

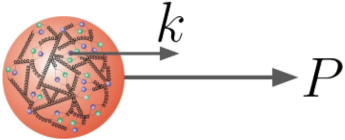
- The universal nonperturbative objects can be studied by QCD-based analytical (QCDSRs, χPT, instanton) and numerical approaches (LQCD)
- Also can be studied by performing global QCD analysis and fit, an inverse problem !
- CETQ, CT, MMHT, NNPDF, ABM, JAM, et.al.

This talk focus on mesons

Take pion as the example

Pion PDF,TMD,GPD

Definitions of pion distribution



One dimension PDF

$\triangle$ $f_i(\zeta) = \int \frac{dz^-}{4\pi} e^{-i\zeta P^+ z^-} \langle \pi | \bar{\psi}_i(0, z^-, 0_T)\gamma^+ \psi_i(0) | \pi \rangle$

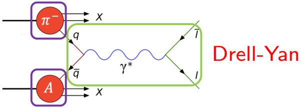
$\triangle$ $\zeta = \frac{k^+}{P^+}$, the parton momentum fraction

$\triangle$ $f_i(\zeta) \sim \sum_\alpha \int dk_T^2 \langle \pi | b^\dagger_{k,\alpha} b_{k,\alpha}(\zeta P^+, k_T, \alpha) | \pi \rangle$

number operator

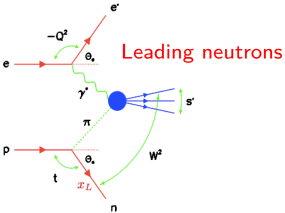
$\triangle$ transversal momentum distributions (TMD) $f(\zeta, k_T)$

$\triangle$ Generalized parton distributions (GPD) $f(\zeta, b_T)$



$$\sigma \propto \sum_{i,j} f_i^\pi(x_\pi, \mu) \otimes f_j^A(x_A, \mu) \otimes C_{i,j}(x_\pi, x_A, Q/\mu)$$

Extracted from **fixed target πA data**



Deeply virtuality meson production

$\triangle$ TDIS at 12GeV JLab, leading proton observable, fixed target instead of collider (HERA);

$\triangle$ EIC, EIcC, great integrated luminosity to reduce the systematics uncertainties;

$\triangle$ COMPASS++/AMBER give π-induced DY data.

Pion LCDAs

Colliders: Pion DAs in the light-cone dominated processes

- DA is expressed by the MEs of gauge invariant non-local operators

$$\langle 0|\bar{u}(x)\Gamma[x,-x]d(-x)|\pi^-(P)\rangle$$

- The path-ordered gauge factor along the straight line

$$[x,y] = \mathrm{P}\exp\left[ig\int_0^1 dt(x-y)_\mu A^\mu(tx+\bar{t}y)\right]$$

- Introduce light-like vectors p and z: $\quad P^2 = m_\pi^2,\ p^2 = 0,\ z^2 = 0$
- $P \to p$ in the limit $m_\pi^2 = 0$ and $x \to z$ for $x^2 = 0$

- Expansion in power of large momentum transfer is governed by contributions from small transversal separations x^2 between constituents

$$z_\mu = x_\mu - \frac{P_\mu}{m_\rho^2}\left[xP - \sqrt{(xP)^2 - x^2 m_\pi^2}\right] = x_\mu\left[1 - \frac{x^2 m_\pi^2}{4(z\cdot p)^2} - \frac{p_\mu}{2}\frac{x^2}{z\cdot p} + \mathcal{O}(x^4)\right]$$

$$p_\mu = P_\mu - \frac{z_\mu}{2}\frac{m_\pi^2}{p\cdot z} \quad \Rightarrow \quad z\cdot P = z\cdot p = \left[(xP)^2 - x^2 m_\pi^2\right]^{1/2}$$

Pion LCDAs

- Define the LCDAs with the Lorentz and gauge invariant ME

$\zeta = 2u - 1,\ m_0^\pi = \frac{m_\pi^2}{m_0 + m_d}$

$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(-x)|\pi^-(P)\rangle = f_\pi \int_0^1 du\, e^{i\zeta P\cdot x}\left[iP_\mu\left(\phi(u) + \frac{x^2}{4}g_1(u,\mu)\right) + \left(x_\mu - \frac{x^2 P_\mu}{2P\cdot x}\right)g_2(u,\mu)\right]$$

$$\langle 0|\bar{u}(x)i\gamma_5 d(-x)|\pi^-(P)\rangle = f_\pi m_0^\pi \int_0^1 du\, e^{i\zeta P\cdot x}\phi^p(u,\mu)$$

$$\langle 0|\bar{u}(x)i\sigma_{\mu\nu}\gamma_5 d(-x)|\pi^-(P)\rangle = -\frac{if_\pi m_0^\pi}{3}\left(P_\mu x_\nu - P_\nu x_\mu\right)\int_0^1 du\, e^{i\zeta P\cdot x}\phi^\sigma(u,\mu)$$

- LCDAs are dimensionless functions of u and renormalization scale μ

△ describe the probability amplitudes to find the π in a state with minimal number of constitutes and have small transversal separation of order $1/\mu$

△ nonlocal operators on the lhs are renormalized at scale μ

$$\phi_2(u,\mu) = Z_2(\mu)\int^{|k_\perp|<\mu} d^2k_\perp\, \phi_{\mathrm{BS}}(u,k_\perp)$$

△ decay constant $\langle 0|\bar{u}(0)\gamma_z\gamma_5 d(0)|\pi^-(P)\rangle = if_\pi p_\mu$ △ normalization $\int_0^1 du\,\Phi(u) = 1$

Pion LCDAs

Conformal spin and collinear twist definition

[Braun, Korchemsky, Müller 2003]

- A convenient tool to study DAs is provided by conformal expansion
- the underlying idea of *conformal expansion of LCDAs* is similar to *partial-wave expansion of wave function in quantum mechanism*
- *invariance of massless QCD under conformal trans. VS rotation symmetry*
- *longitudinal* $\otimes$ *transversal* dofs *VS* *angular* $\otimes$ *radial* dofs for spherically symmetry potential
- the transversal-momentum dependence (scale dependence of the relevant operators) is governed by the RGE
- the longitudinal-momentum dependence (orthogonal polynomials) is described in terms of irreducible representations of the corresponding symmetry group **collinear subgroup of conformal group** $SL(2, R) \cong SU(1, 1) \cong SO(2, 1)$

Pion LCDAs

$$\begin{aligned} & \langle 0|\bar{u}_i(0)\Gamma_{ij}d_j(z)|\pi^-(p)\rangle \\ = & -\frac{i}{4}\int_0^1 du e^{-iup\cdot z}\left[\not{p}\gamma_5\phi(u) + m_0^\pi(\mu)\gamma_5\phi^p(u) + m_0^\pi(\mu)\gamma_5(1-\not{n}\not{\bar{n}})\phi^t(u)\right]_{ji} \end{aligned}$$

$$\phi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(u)$$

$$\phi^\sigma(u) = 6u(1-u)\left[1 + 5\eta_{3\pi} C_2^{3/2}(u)\right]$$

$$\phi^p(u,\mu) = \left[1 + 30\eta_{3\pi} C_2^{1/2}(u) - 3\eta_{3\pi}\omega_{3\pi} C_4^{1/2}(u)\right]$$

- $\phi(x)$ and $\phi^{p,t}(u)$ are the twist two and twist three LCDAs
- $a_0^\pi = f_\pi$, $a_{n\geqslant 2}^\pi(\mu_0)$ and $m_0^\pi(\mu_0)$ are universal nonpertubative parameters
- μ dependences in a_n^π and others the integration over the **transversal dof**

  [Brodsky & Lepage1980, Balitsky & Braun1988]
- $C_n(u)$ are Gegenbauer polynomials $\sim$ Jacobi Polynomials $P_n^{j_1,j_2}\left(\frac{\overleftrightarrow{D}_+}{\overleftrightarrow{\partial}_+}\right)$ in the local collinear conformal expansion **longitudinal dof**

  [Lepage & Brodsky 1979, 80, Efremov & Radyushkin 1980, Braun & Filyanov 1990]

Pion LCDAs

$\phi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(u)$

- QCD definition $\quad a_n^\pi(\mu) = \langle \pi | q(z)\bar{q}(z) + z_\rho \partial_\rho q(z)\bar{q}(z) + \cdots |0\rangle$
- **LQCD**: 0.334 ± 0.129[UKQCD 2010], 0.135 ± 0.032[RQCD 2019], $0.258^{+0.079}_{-0.052}$[LPC 2022]

  △ default scale at 1 GeV $\quad$ scale running

$$a_n(\mu) = a_n(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{\gamma_n^{(0)}-\gamma_0^{(0)}}{2\beta_0}}, \quad \gamma_n^{\perp(\parallel),(0)} = 8C_F\left(\sum_{k=1}^{n+1}\frac{1}{k} - \frac{3}{4} - \frac{1}{2(n+1)(n+2)}\right)$$

  △ a_4^π is not available ← the growing number of derivatives in $q\bar{q}$ operator

- **QCDSR**: 0.19 ± 0.06[Chernyak 1984], $0.26^{+0.21}_{-0.09}$[Khodjamirian 2004], $0.28^{+0.08}_{-0.08}$[Ball 2006]

  △ nonlocal vacuum condensate is introduced and modeled for $a_{n>2}^\pi$ [Bakulev 2001]

- Dispersion relation as an **Inverse problem** [Li 2020, Yu 2022]

  quark-hadron duality → Laguerre Polynomials to construct spectral density

$$\{a_2, a_4, a_6, a_8\} = \{0.249, 0.134, 0.106, 0.096\}$$

Pion LCDAs

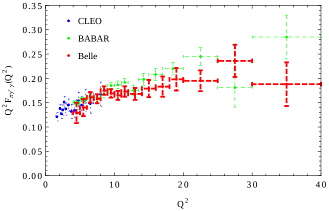
$\phi_\pi(u,\mu) = 6u(1-u)\sum_{n=0} a_n^\pi(\mu) C_n^{3/2}(u)$

- **Data-driven** with QCD calculations for the π involved exclusive processes

△ $F_{B\to\pi}$: 0.19 ± 0.19 [Ball 05], 0.16 [Khodjamirian 11], large error from B meson

△ $F_{\pi\gamma\gamma^*}$: 0.14 [Agaev 2010] BABAR+CLEO, 0.10 [Agaev 2012] Belle+CLEO

large uncertainty of $a^\pi_{n>2}$, discrepancy data at large Q^2



[SC 1901.06071]

DiPion LCDAs

Method	$a_2^\pi(2\text{ GeV})$	Refs.
LO QCDSR, CZ model	0.39	[30,31]
QCDSR	$0.18^{+0.15}_{-0.26}$	[32]
QCDSR	0.19 ± 0.06	[33]
QCDSR, NLC	0.13 ± 0.04	[34,35]
$F_{\pi\gamma\gamma^*}$, LCSRs	0.12 ± 0.04 (2.4 GeV)	[36]
$F_{\pi\gamma\gamma^*}$, LCSRs	0.21 (2.4 GeV)	[37]
$F_{\pi\gamma\gamma^*}$, LCSRs, R	0.19	[38]
$F_{\pi\gamma\gamma^*}$, LCSRs, R	0.31	[39]
$F_{\pi\gamma\gamma^*}$, LCSRs, NLO	0.096	[40]
$F_{\pi\gamma\gamma^*}$, LCSRs, NLO	0.068	[41]
F_π^{em}, LCSRs	$0.17 \pm 0.10 \pm 0.05$	[42]
F_π^{em}, LCSRs, R	0.14 ± 0.02	[43]
$F_{B\to\pi}$, LCSRs	0.13 ± 0.13	[44]
$F_{B\to\pi}$, LCSRs	0.11	[45,46]
LQCD, TWST, $N_f = 2$, CW	0.201 ± 0.114	[47]
LQCD, TWST, $N_f = 2+1$, DWF	0.233 ± 0.088	[48]
LQCD, MST, $N_f = 2$	0.136 ± 0.03	[27]
LQCD, MST, $N_f = 2+1$, CW	0.0762 ± 0.0127	[29]

△ F_π: 0.24 ± 0.17 [Bebek1978] Wilson Lab+NA7, 0.20 ± 0.03 [Agaev 2005] JLab

large uncertainty of $a^\pi_{n>2}$, available data only in small spacelike q^2

Pion LCDAs from F_π

- Spacelike data is available in the narrow region $q^2 \in [-2.5, 0]$ GeV^2
- Perturbative QCD calculations are valid in the intermediate/large $|q^2|$
  N^{2}LO calculation in collinear factorization $\sim$ NLO [Chen^2, Feng, Jia 2312.17228] see Wen Chen's talk
- The mismatch destroys the direct extracting programme from $F_\pi(q^2 < 0)$
- Timelike form factor $F_\pi(q^2 > 0)$ provides another opportunity
  - △ $e^+e^- \to \pi^+\pi^-(\gamma)$, $4m_\pi^2 \leqslant q^2 \lesssim 9$ GeV^2 [BABAR 2012]
  - △ $\tau \to \pi\pi\nu_\tau$, $4m_\pi^2 \leqslant q^2 \leqslant 3.125$ GeV^2 [Belle 2008]
  - △ $e^+e^-(\gamma) \to \pi^+\pi^-$, $0.6 \leqslant Q^2 \leqslant 0.9\,\mathrm{GeV}^2$ with ISR [BESIII 2016]
- TL measurement and SL predictions are related by dispersion relation
- The standard dispersion relation and The modulus representation

$$F_\pi(q^2 < s_0) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}F_\pi(s)}{s - q^2 - i\epsilon}$$

⇓ [**SC**, Khodjamirian, Rosov 2007.05550]

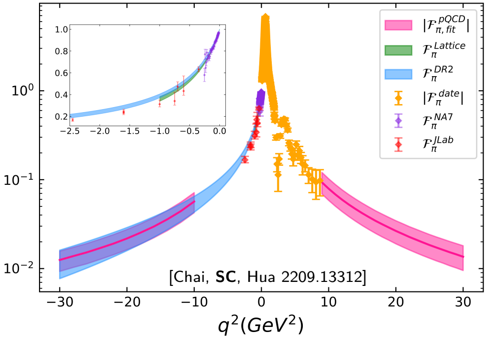
$$F_\pi(q^2 < s_0) = \exp\left[\frac{q^2\sqrt{s_0 - q^2}}{2\pi}\int_{s_0}^{\infty} \frac{ds \ln|F_\pi(s)|^2}{s\sqrt{s - s_0}\,(s - q^2)}\right]$$

$$|\mathcal{F}_\pi(s)|^2 = \Theta(s_{\max} - s)\,|\mathcal{F}^{\mathrm{data}}_{\pi,\mathrm{Inter.}}(s)|^2 + \Theta(s - s_{\max})\,|\mathcal{F}^{\mathrm{pQCD}}_{\pi}(s)|^2$$

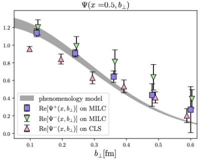
Pion LCDAs from F_π

- $a_2 = 0.275 \pm 0.055$, $a_4 = 0.185 \pm 0.065$, $m_0^\pi = 1.37^{+0.29}_{-0.32}$ GeV
  △ Pion deviates from the purely asymptotic one △ a_2^π is not enough
  △ $0.258^{+0.079}_{-0.052}$ [LPC 2201.09173[hep-lat]], $0.249^{+0.005}_{-0.006}$ [Li 2205.06746]

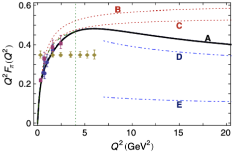
- a slight derivation in the small region



- intrinsic transverse momentum ?[LPC 2302.09961]



- dynamical chiral symmetry breaking ?[Chang et.al. 1307.0026]



Pion LCDAs from F_π

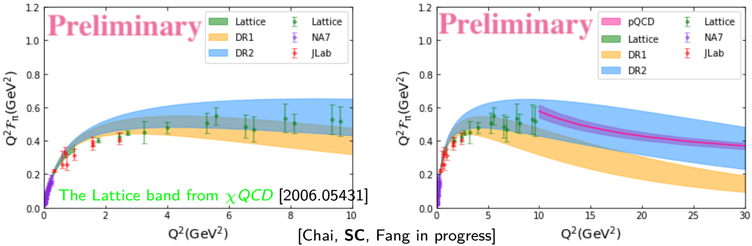
- Taking into account the contribution from the iTMD

$$\frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}}\phi^p(u,\mu) = \int \frac{d^2\vec{k}_T}{16\pi^3}\phi^p_{2p}(u,\vec{k}_T) + \int \frac{d^2\vec{k}_{T1}}{16\pi^3}\frac{d^2\vec{k}_{T2}}{4\pi^2}\phi^p_{3p}(u,\vec{k}_{T1},\vec{k}_{T2}).$$

$$\psi^p_{2p}(u,\vec{k}_T) = \frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}}\phi^p_{2p}(u,\mu)\Sigma(u,\vec{k}_T),$$

$$\psi^p_{3p}(u,\vec{k}_{1T},\vec{k}_{2T}) = \frac{f_\pi m_0^{\mathcal{P}}}{2\sqrt{6}}\eta_{3\pi}\phi^p_{3p}(u,\mu)\Sigma'(\alpha_i,\vec{k}_{1T},\vec{k}_{2T}).$$

- comparison with the impressive LQCD calculation see Heng-tong Ding's talk

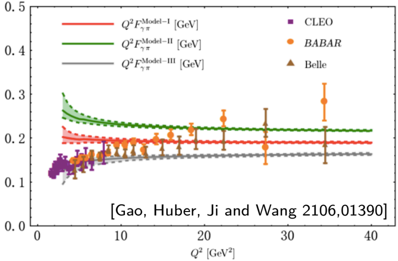


[Chai, **SC**, Fang in progress]

- the slight derivation is still there (not sensitive to iTMD)
- more result for electromagnetic form factor of Kaon meson

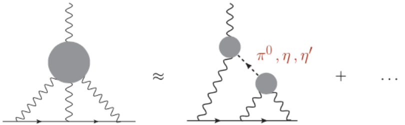
Pion LCDAs from $F_{\pi\gamma\gamma^*}$

- $F_{\pi\gamma\gamma^*}$ is the theoretically most clean observable $\propto a_n^\pi$
- Two-loop calculation of $F_{\pi\gamma\gamma^*}$ in hard-collinear factorization theorem
  $N^2LO \sim NLO$



[Gao, Huber, Ji and Wang 2106,01390]

Model-I [Brodsky, Teramond 0707.3859, RQCD 1903.08038]
Model-II [**SC**, Khodjamirian, Rosov 2007.05550]
Model-III [Mikhailov, Pimikov, Stefanis 1604.06391]



[Gérardin, Meyer, Nyffeler 1607.08174]
Hadronic light-by-light scattering (HLbL) contribution to $a_\mu^{HLbL;\pi^0}$

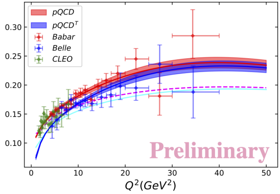
- pQCD calculation with taking into account the iTMD

† improve the pQCD power in the intermediate momentum transfers

† modification in the small and intermediate regions is significant

- more result of the $\eta^{(\prime)}$, η_q and η_s transition form factors

[Chai, **SC**, Fang in progress]

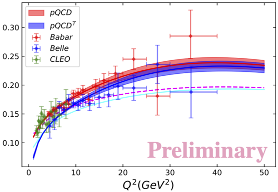


Pion LCDAs from $F_{\pi\gamma\gamma^*}$

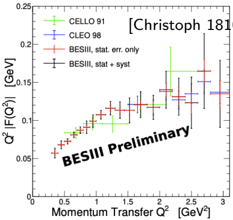
- pQCD calculation with taking into account the iTMD

† modification in the small and intermediate regions is significant (sensitive to the measurement)

[Chai, **SC**, Fang in progress]



- The measurement discrepancy starts from ~ 7 GeV^2
- BEPC up to 5.4 GeV (2023-2024), Belle-II(4+7 GeV) and future e^+e^- colliders, to settle down the "fat pion" issue ?



[Christoph 1810.00654[hep-ex]]

From Vector and Scalar mesons to DiPion LCDAs

K^* LCDAs

- For the vector mesons, the polarization vectors decompose into

$$\epsilon^\lambda_\mu = \frac{\epsilon^\lambda \cdot x}{p \cdot z} p_\mu + \frac{\epsilon^\lambda \cdot p}{p \cdot z} z_\mu + \epsilon^\lambda_{\perp\mu} = (\epsilon^\lambda \cdot x) \frac{P_\mu (P \cdot x) - x_\mu m_M^2}{(p \cdot x)^2 - x^2 m_M^2}$$

- Define the LCDAs with the Lorentz and gauge invariant ME

$$\langle K^*(p,\epsilon)|\bar{s}(x)\gamma_\mu s(0)|0\rangle = f^{\parallel}_{K^*} m_{K^*} \int_0^1 du e^{iup\cdot x} \Big\{ p_\mu \frac{\epsilon \cdot x}{p \cdot x} \Big[\phi_2^{\parallel}(u) - \phi_3^{\perp}(u) + \frac{m_{K^*}^2 x^2}{16} \Big(\phi_4^{\parallel}(u) - \phi_5^{\perp}(u) \Big) \Big]$$

$$- \frac{(\epsilon \cdot x) m_{K^*}^2}{2(p \cdot x)^2} x_\mu \Big[\psi_4^{\parallel}(u) - 2\phi_3^{\perp}(u) + \phi_2^{\parallel}(u) \Big] + \epsilon_\mu \Big[\phi_3^{\perp}(u) + \frac{m_{K^*}^2 x^2}{16} \phi_5^{\perp}(u) \Big] \Big\}$$

$$\langle K^*(p,\epsilon)|\bar{s}(x)\sigma_{\mu\nu} s(0)|0\rangle = -i f^{\perp}_{K^*} \int_0^1 du e^{iup\cdot x} \Big\{ (\epsilon_\mu p_\nu - \epsilon_\nu p_\mu) \Big[\phi_2^{\perp}(u) + \frac{m_{K^*}^2 x^2}{16} \phi_4^{\perp}(u) \Big] + \phi_3^{\parallel}(u) \cdots \Big\}$$

$$\langle K^*(p,\epsilon)|\bar{s}(x)\gamma_\mu\gamma_5 s(0)|0\rangle = - \frac{f^{\parallel}_{K^*} m_{K^*} \varepsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p^\rho x^\sigma}{4} \int_0^1 du e^{iup\cdot x} \Big[\tilde{\psi}_3^{\perp}(u) + \frac{m_{K^*}^2 x^2}{16} \tilde{\psi}_5^{\perp}(u) \Big]$$

$$\langle K^*(p,\epsilon)|\bar{s}(x)s(0)|0\rangle = -\frac{i}{2} f^{\perp}_{K^*} (\epsilon \cdot x) m_{K^*}^2 \int_0^1 du e^{iup\cdot x} \tilde{\psi}_3^{\parallel}(u) \,.$$

Twist anatomy of vector meson LCDAs [SC, Ju, Qin, Yu 2203,06797]

Notations	$\phi_2^{\parallel}$	$\phi_2^{\perp}$	$\phi_3^{\parallel}$	$\psi_3^{\parallel}$	$\phi_3^{\perp}$	$\psi_3^{\perp}$	$\phi_4^{\parallel}$	$\psi_4^{\parallel}$	$\phi_4^{\perp}$	$\psi_4^{\perp}$	$\phi_5^{\perp}$	$\psi_5^{\perp}$
Twist	2	2	3	3	3	3	4	4	4	4	5	5
Dirac	γ_μ	$\sigma_{\mu\nu}$	$\sigma_{\mu\nu}$	1	γ_μ	$\gamma_\mu\gamma_5$	γ_μ	γ_μ	$\sigma_{\mu\nu}$	$\sigma_{\mu\nu}$	γ_μ	$\gamma_\mu\gamma_5$

K^* LCDAs from Lattice QCD

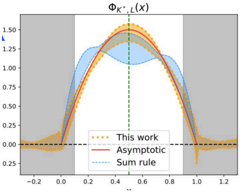
- Leading twist LCDAs $\phi_2^{\parallel(\perp)}(u) = 6u(1-u)\left[1 + a_1^{\parallel(\perp)} C_1^{3/2}(t) + a_2^{\parallel(\perp)} C_2^{3/2}(t)\right]$

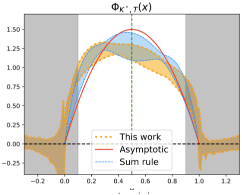
Parameters	$f_{K^*}^{\parallel}$ (GeV)	$f_{K^*}^{\perp}$ (GeV)	$a_1^{\parallel}$	$a_1^{\perp}$	$a_2^{\parallel}$	$a_2^{\perp}$
[1503.05534]	0.204(7)	0.159(6)	0.06(4)	0.04(3)	0.16(9)	0.10(8)

△ $f_{K^*}^{\parallel}$ obtained from $\tau \to K^* \nu_\tau$ data [Ball, Jones, Zwicky 0612081]

△ $f_{K^*}^{\perp}$ obtained by latice evaluation $r_{K^*} \equiv f_{K^*}^{\perp}/f_{K^*}^{\parallel} = 0.712(12)$ [RBC-UKQCD 1011.5906[hep-lat]]

- The entire distribution of K^*, ϕ based on the LaMET [LPC 2011.07988[hep-lat]]





Gegenbauer moments	a_1	a_2
K^*, L	-0.005(07)(07)	0.015(10)(08)
K^*, T	-0.074(06)(07)	0.181(07)(12)

$\Phi_{K^*,L}$ tend to be close to the asymptotic form

$\Phi_{K^*,T}$ have relatively large deviations from the asymptotic form

- K^* has tiny width, how to construct the width effect of ρ?
- The lowest moments of ρ LCDAs from Lattice QCD [RQCD 1612.02955[hep-lat]]

	f_ρ[MeV]	f_ρ^T[MeV]	f_ρ^T/f_ρ	$a_2^{\parallel}$	$a_2^{\perp}$
analysis 1	199(4)(1)	124(4)(1)	0.629(7)(4)	0.132(13)(24)	0.101(18)(12)
analysis 2	194(7)(1)	123(5)(1)	0.642(10)(4)	0.117(16)(24)	0.093(20)(11)

$\rho \to \pi\pi$ decay is not taken into account

$\leftarrow$ the data with the smallest pion mass

DiPion LCDAs

- **Introduce DiPion LCDAs to describe the width effects (resonance contribution and nonresonant background) in heavy flavor decays**
- Chiral-even LC expansion with gauge factor $[x, 0]$[Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1)\pi^b(k_2)|\overline{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

- Three independent kinematic variables

  △ momentum fraction u carried by anti-quark with respecting to the total momentum of DiPion state,

  △ longitudinal momentum fraction carried by one of the pions $\zeta = k_1^+/k^+$ △ k^2

- DiPion is not a tetraquark state, but a collinear two pion system with nonlocal $q\bar{q}, \cdots$ operators
- 2πDAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=0,\mathrm{even}}^{\infty}\sum_{l=1,\mathrm{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(z,\zeta,k^2,\mu) = 6z(1-z)\sum_{n=1,\mathrm{odd}}^{\infty}\sum_{l=0,\mathrm{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu)C_n^{3/2}(2z-1)C_\ell^{1/2}(2\zeta-1)$$

- $B_{n\ell}(k^2,\mu)$ have similar scale dependence as the a_n of ρ meson

DiPion LCDAs and B_{l4} decays

- $|V_{ub}|$ tension $\quad |V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$ [PDG 2022]

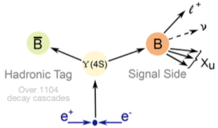
  2.5σ tension between $(4.13 \pm 0.25) \times 10^{-3}$ (incl.) and $(3.70 \pm 0.16) \times 10^{-3}$ (excl.)

- Continue to improve the measurements and predictions

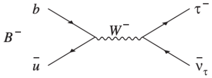
$$\mathscr{B}(B \to \pi^0 \ell \nu) + \mathscr{B}(B \to \pi^+ \ell \nu) + \mathscr{B}(B \to X_u^{\text{other}} \ell \nu) = \mathscr{B}(B \to X_u \ell \nu)$$

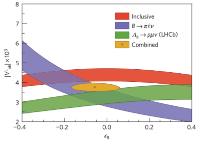
‡ new result from Belle collaboration with Simultaneous Determination in excl. and incl. processes [Belle PRL131, 211801 (2023)]

$3.78 \pm 0.23 \pm 0.16 \pm 0.14$ and
$3.88 \pm 0.20 \pm 0.31 \pm 0.09$, respectively



- Enlarge the set of exclusive processes independent measurement





‡ $|V_{ub}| f_B$ in pure leptonic decay

0.72 ± 0.09 MeV from Belle, 1.01 ± 0.14 MeV from BABAR,
0.77 ± 0.12 MeV [FLAG2021]

‡ $|V_{ub}|$ in baryon decay [Nature Physics 11, 743 (2015)]

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} = \frac{\mathcal{B}(\Lambda_b^0 \to p \mu^- \bar{\nu})}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ \mu^- \bar{\nu})} \quad R_{\rm FF} = 0.68 \pm 0.07 \Downarrow$$

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.06 \xrightarrow{|V_{cb}|} |V_{ub}| = (3.27 \pm 0.23) \times 10^{-3}$$

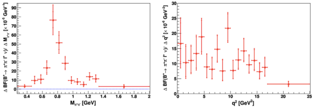
DiPion LCDAs and B_{l4} decays

- Continue to enlarge the set of exclusive processes ($B \to \rho l \bar{\nu}$) to carry out the independent measurement
- Comparison between $B \to \pi l \bar{\nu}$ and $B \to \rho l \bar{\nu}$ [Gao, Lü, Shen, Wang, Wei 1902.11092]

$$|V_{ub}| = \left(3.05^{+0.67}_{-0.52}\Big|_{\rm theo}\ {}^{+0.19}_{-0.20}\Big|_{\rm exp}\right) \times 10^{-3}, \quad \text{from B} \to \rho l \nu$$

$$|V_{ub}|_{\rm PDG} = (3.70 \pm 0.12|_{\rm theo} \pm 0.10|_{\rm exp}) \times 10^{-3} \quad \text{from B} \to \pi l \nu$$

- Propose to measure the $B \to \pi^+ \pi^0 l^- \bar{\nu}$ decay with the $B \to \pi^+ \pi^0$ form factor calculated from B meson LCSRs [**SC**, Khodjamirian, Virto 1701,01633]
- $B \to \pi \pi l \bar{\nu}_l$ has already been measured, mainly its resonant part $B \to \rho l \bar{\nu}_l$ $(1.58 \pm 0.11) \times 10^{-4}$ [CLEO 2000, BABAR 2011, Belle 2013]
- First measurement of the branching fraction of $B^+ \to \pi^+ \pi^- l^+ \bar{\nu}_l$ $(2.3 \pm 0.4) \times 10^{-4}$ [Belle 2005.07766]
  More data on the way from Belle II



- First Lattice QCD study of the $B \to \pi \pi l \bar{\nu}$ transition amplitude in the region of large q^2 and $\pi\pi$ invariant mass near the ρ resonance [Leskovec et.al. 2212.08833[hep-lat]]

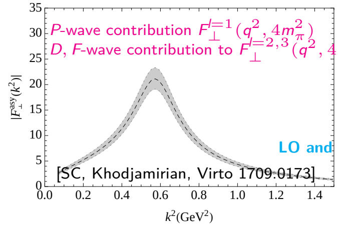
DiPion LCDAs and B_{l4} decays

$B \to \rho$ to $B \to \pi\pi$ **form factors** [Hambrock 1511.02509]

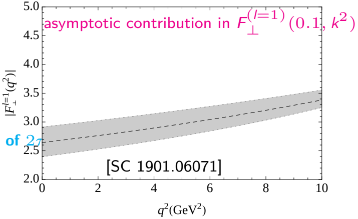
$$i\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\,i\epsilon_{\nu\alpha\beta\gamma}\,q^\alpha\,k^\beta\,\bar{k}^\gamma$$

$$+F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu - \frac{k\cdot q}{q^2}q_\nu\right) + F_\parallel(q^2,k^2,\zeta)\,[\cdots]$$

- Partial wave expansions $F_\perp(k^2,q^2,\zeta) = \sum_\ell \sqrt{2\ell+1}\,F^{(\ell)}_{\perp,\parallel}(k^2,q^2)\,\frac{P^{(1)}_\ell(\cos\theta_\pi)}{\sin\theta_\pi}$



LO and leading twist of 2π



- High partial waves give few percent contributions to $B \to \pi\pi$ form factors
- ρ', ρ'' and NR background contribute $\sim 20\% - 30\%$ to P-wave

f_0 LCDAs from QCDSRs

- Definition of the scalar meson LCDAs

$$\langle S(p_1)|\bar{q}_{2\beta}(z)q_{1\alpha}(0)|0\rangle = \frac{1}{4}\int_0^1 du\, e^{iup_1\cdot z}\left\{\not{p}_1\phi(u) + m_S\left[\phi^s(u) - \frac{\sigma_{\rho\delta}p_1^\rho z^\delta}{6}\phi^\sigma(u)\right]\right\}_{\beta\alpha}$$

- Scalar decay constant $\bar{f}_S = \mu_S f_S$, $\mu_S \equiv m_S/[m_{q_2}(\mu) - m_{q_1}(\mu)]$

[SC, Zhang 2307.02309] [CCY 0508104]

Sum rules	QCDSR	BFTSR	QCDSR [5]
$m_{f_0(980)}$(MeV)	990 ± 50 (input)		990 ± 50
$\bar{f}_{f_0(980)}$(MeV)	335^{+9}_{-12}	331^{+9}_{-12}	370 ± 20

- Leading twist LCDA $\phi(u,\mu) = \bar{f}_S 6u\bar{u}\sum_{n=0}^{\infty} a_n(\mu) C_n^{3/2}(2u-1)$

[CCY 0508104] [WAL 0804.2204] [WFZW 2211.05390]

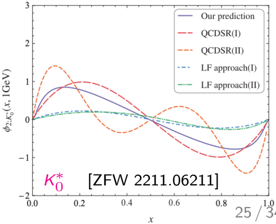
Sum rules	QCDSR	BFTSR	QCDSR [5]	QCDSR [62]	BFTSR [57]
$a_1(f_0(980))$	$-0.891^{+0.039+0.004}_{-0.033-0.004}$	$-0.855^{+0.039+0.004}_{-0.033-0.005}$	-0.78 ± 0.08	—	-0.51 ± 0.07
$a_1(f_0(1500))$	—	—	0.80 ± 0.40	-0.48 ± 0.11	—

- Subleading twist LCDA

$$\phi^{s(\sigma)}(u,\mu) = \bar{f}_S 6u\bar{u}\left[1 + \sum_{n=1}^{\infty} a_n^{s(\sigma)}(\mu) C_n^{3/2}(2u-1)\right]$$

[LWZ 0612210] [HWFZ 1301.3978]

Sum rules	QCDSR	BFTSR	QCDSR [55]	BFTSR [24]
$a_2^s(f_0(980))$	0.296 ± 0.044	-0.828 ± 0.065	—	—
$a_2^s(f_0(1500))$	—	—	$[-0.33, -0.18]$	$[-0.02, 0.05]$
$a_2^\sigma(f_0(980))$	0.169 ± 0.026	0.367 ± 0.039	—	—
$a_2^\sigma(f_0(1500))$	—	—	$[-0.15, -0.09]$	$[-0.03, 0.00]$



DiPion LCDAs and D_{l4} decays

Structures of scalar mesons

- $f_0(1370), f_0(1500), a_0(1450), K_0^*(1430)$ form a $SU(3)$ flavor nonet
  $q\bar{q}$ replenished with some possible gluon content
  i.e., $f_0(1370) \to 2\rho \to 4\pi$, $|n\bar{n}\rangle$, $f_0(1500) \to 4\pi, 2\pi$, gluon content
- $f_0(500)/\sigma, f_0(980), a_0(980), K_0^*(700)/\kappa$ form another nonet
  compact tetraquark and $K\bar{K}$ bound state
- the spectral analysis $q\bar{q}$ has one unit of orbital argular momentum which increases the masses, but f_0 and a_0 are mass degeneracy
- in B_s decays $q\bar{q}$ is dominated in the energetic $f_0(980)$
  $q^2\bar{q}^2$ is power suppressed, FSI is also weak [SC, J-M Shen 1907.08401]
- in D_s decays how about the energetic $q\bar{q}$ picture $f_0(980)$?

DiPion LCDAs and D_{l4} decays

- $D_{(s)} \to S l \nu$ decays provide clean environment to study the scalar meson
- $D_s \to f_0(\to \pi^0\pi^0, K_s K_s) e^+\nu$ [BESIII 22], $D_s \to f_0(\to \pi^+\pi^-) e^+\nu$ [BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0)e^+\nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-)e^+\nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$
$$f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

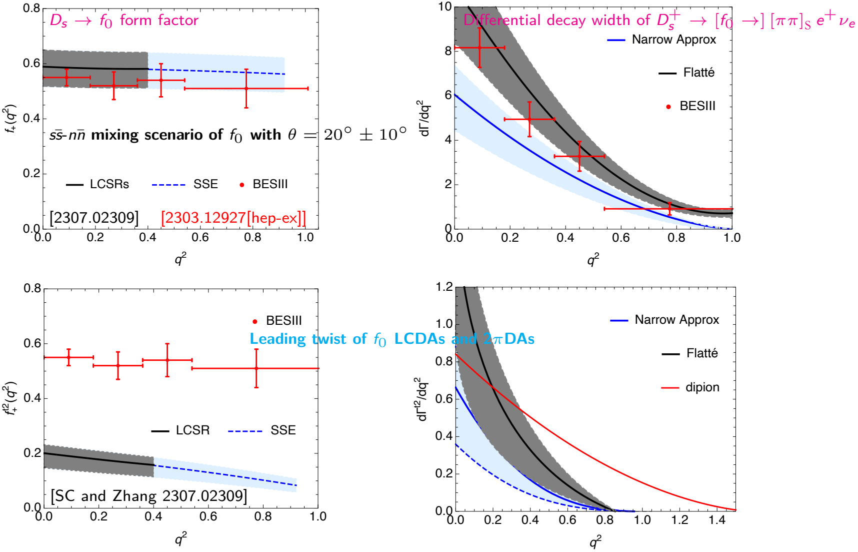
- Theoretical consideration $\frac{d\Gamma(D_s^+ \to f_0 l^+ \nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2$
- Improvement with the width effect ($\pi\pi$ invariant mass spectral)

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_S l^+\nu)}{ds dq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1^2 \beta_\pi(s)}{|m_S^2 - s + i\left(g_1^2 \beta_\pi(s)) + g_2^2 \beta_K(s)\right)|^2}$$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_S l^+\nu)}{dk^2 dq^2} = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} \frac{\beta_{\pi\pi}(k^2)\sqrt{\lambda_{D_s}} q^2}{16\pi} \sum_{\ell=0}^{\infty} 2|F_0^{(\ell)}(q^2, k^2)|^2$$

- $D_s \to f_0$ form factors $\langle f_0(p_1)|\bar{s}\gamma_\mu\gamma_5 c|D_s^+(p)\rangle = -i\left[f_+(q^2)\,(p+p_1)_\mu + f_-(q^2) q_\mu\right]$
- $D_s \to [\pi\pi]_S$ form factors
$\langle [\pi(k_1)\pi(k_2)]_S\, |\bar{s}\gamma_\mu(1-\gamma_5)c|D_s^+(p)\rangle = -iF_t(q^2, s, \zeta)k_\mu^t - iF_0(q^2, s, \zeta)k_\mu^0 - iF_\parallel(q^2, s, \zeta)k_\mu^\parallel$

DiPion LCDAs and D_{l4} decays



- Twist-3 LCDAs give dominate contribution in $D_s \to f_0, [\pi\pi]_S$ transitions
- further measurements would help us to understand the DiPion system

Conclusion

- In the light-cone dominated processes, hadron structure is studied in terms of **LCDAs**
- (Lattice) QCD practitioners have paid much more attention on LCDAs
- **DiPion LCDAs** are introduced to describe the width effect (resonant contribution and nonresonant background) in heavy flavor decays
- A new booster in the accurate calculation in flavor physics ? in cooperation with the high precise loop calculations

Thank you for your patience.

The modulus representation of DR

- Introducing an auxiliary function $g_\pi(q^2) \equiv \frac{\ln F_\pi(q^2)}{q^2\sqrt{s_0 - q^2}}$ [Geshkenbein 1998]
- Cauchy theorem and Schwartz reflection principle

$$g_\pi(q^2) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\, g_\pi(s)}{s - q^2 - i\epsilon}$$

- At $s > s_0$ on the real axis, the imaginary part of g_π reads as

$$\mathrm{Im}\, g_\pi(s) = \mathrm{Im}\left[\frac{\ln(|F_\pi(s)|e^{i\delta_\pi(s)})}{-is\sqrt{s - s_0}}\right] = \frac{\ln|F_\pi(s)|}{s\sqrt{s - s_0}},$$

- Substituting $g_\pi(q^2)$ and $\mathrm{Im}\, g_\pi(s)$ into the dispersion relation, for $q^2 < s_0$

$$frac\ln F_\pi(q^2) q^2 \sqrt{s_0 - q^2} = \frac{1}{2\pi}\int_{s_0}^{\infty} \frac{ds\ \ln|F_\pi(s)|^2}{s\sqrt{s - s_0}\,(s - q^2)}$$

- The only assumption: $F_\pi(q^2)$ is free from zeros in the complex q^2 plane, then $\ln F_\pi(q^2)$ does not diverge. [Leutwyler 2002] [Ananthanarayan 2011]
- If $F_\pi(q^2)$ has zeros in the complex q^2 plane, deserves a separate analysis [Dominguez 2001] [Ananthanarayan 2004]

△ $F_\pi(q^2)$ evaluated by the standard and modulus DR have a tiny difference, → the zeros of $F_\pi(q^2)$ are either absent or their influence is beyond our accuracy

Backup Slides $m_0^\pi(\mu)$

- QCD definition $\langle \pi^+ | \bar{u}(0)(-i\gamma_5) d(0) | 0 \rangle = f_\pi m_0^\pi(\mu)$
- $m_0^\pi(1\,\mathrm{GeV}) = 1.892$ GeV is obtained from $\chi\mathrm{PT}$[Leutwyler 1996]
- $\phi_\pi^{p/\sigma}$ (m_0^π) are not involved in F_π^{LCSRs} due to the chiral symmetry limit
- the dominant contribution in F_π^{pQCD} $\Leftarrow$ chiral enhancement $\mathcal{O}(m_0^\pi/Q^2)$

Table 1. Input of m_0^π in the previous pQCD calculations.

Physical quantity (Accuracy)	m_0^π	Refs
Pion EM FF (NLO, 2p, twist-3)	1.74	[12–15]
Pion EM FF (NLO, 3p, twist-3)	1.74	[16]
Pion EM FF (NLO, 3p, twist-4)	1.90(1 GeV)	[9]
$B \to \pi$ FF (LO, 3p)	1.4	[17]
$B \to \pi$ FF (twist-2 NLO, 2p)	$1.74^{+0.67}_{-0.38}$	[18]
$B \to \pi$ FF (twist-3 NLO, 2p)	1.4	[19]

△ usually chosen at a fixed value in the previous pQCD study
△ maybe the largest error source of pQCD predictions of F_π
△ the corresponding large uncertainty is formerly disregarded

DiPion LCDAs

- Chiral-even LC expansion with gauge factor $[x, 0]$[Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1)\pi^b(k_2)|\overline{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

△ $n^2 = 0$, △ index f, f' respects the (anti-)quark flavor, △ a, b indicates the electric charge

△ coefficient $\kappa_{+-/00} = 1$ and $\kappa_{+0} = \sqrt{2}$, △ $k = k_1 + k_2$ is the invariant mass of dipion state

△ $\tau = 1/2, \tau^3/2$ corresponds to the isoscalar and isovector 2πDAs,

△ higher twist $\propto 1, \gamma_\mu\gamma_5$ have not been discussed yet, γ_5 vanishes due to P-parity conservation

- Normalization conditions

$$\int_0^1 \Phi_{\parallel}^{I=1(0)}(u,\zeta,k^2) = (2\zeta - 1)F_\pi(k^2)$$

$$\int_0^1 dz\,(2z-1)\Phi_{\parallel}^{I=0}(z,\zeta,k^2) = -2M_2^{(\pi)}\zeta(1-\zeta)F_\pi^{\rm EMT}(k^2)$$

△ $F_\pi^{em}(0) = 1$, △ $F_\pi^{\rm EMT}(0) = 1$,

△ $M_2^{(\pi)}$ is the momentum fraction carried by quarks in the pion associated to the usual quark distribution

- $B_{n\ell}(k^2,\mu)$ have similar scale dependence as the a_n of π, ρ, f_0 mesons

$$B_{n\ell}(k^2,\mu) = B_{n\ell}(k^2,\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{\gamma_n^{(0)}-\gamma_0^{(0)}}{2\beta_0}}, \quad \gamma_n^{\perp(\parallel),(0)} = 8C_F\left(\sum_{k=1}^{n+1}\frac{1}{k} - \frac{3}{4} - \frac{1}{2(n+1)(n+2)}\right)$$

- Soft pion theorem relates the chirarlly even coefficients with a_n^π

$$\sum_{\ell=1}^{n+1} B_{n\ell}^{\|,I=1}(0) = a_n^\pi\,, \quad \sum_{\ell=0}^{n+1} B_{n\ell}^{\|,I=0}(0) = 0$$

- 2πDAs relate to the skewed parton distributions (SPDs) by crossing

△ express the moments of SPDs in terms of $B_{nl}(k^2)$ in the forward limit as

$$M^\pi_{N=\mathrm{odd}} = \frac{3}{2}\frac{N+1}{2N+1} B^{I=1}_{N-1,N}(0), \quad M^\pi_{N=\mathrm{even}} = 3\frac{N+1}{2N+1} B^{I=0}_{N-1,N}(0)$$

- In the vicinity of the resonance, 2πDAs reduce to the DAs of ρ/f_0

△ relation between the a_n^ρ and the coefficients $B_{n\ell}$

$$a_n^\rho = B_{n1}(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1} c_m^{n1}\, m_\rho^{2m}\right], \quad c_m^{(n1)} = \frac{1}{m!}\frac{d^m}{dk^{2m}}\left[\ln B_{n1}(0) - \ln B_{01}(0)\right]$$

△ f_ρ relates to the imaginary part of $B_{nl}(m_\rho^2)$ by $\quad \langle \pi(k_1)\pi(k_2)|\rho\rangle = g_{\rho\pi\pi}(k_1 - k_2)^\alpha \epsilon_\alpha$

$$f_\rho^{\|} = \frac{\sqrt{2}\,\Gamma_\rho\,\mathrm{Im}B_{01}^{\|}(m_\rho^2)}{g_{\rho\pi\pi}}, \quad f_\rho^{\perp} = \frac{\sqrt{2}\,\Gamma_\rho\, m_\rho\,\mathrm{Im}B_{01}^{\perp}(m_\rho^2)}{g_{\rho\pi\pi}\, f_{2\pi}^{\perp}}$$

- 2πDAs in a wide range of energies is given by δ_ℓ^I and a few subtraction constants

$$B^I_{n\ell}(k^2) = B^I_{n\ell}(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B^I_{n\ell}(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^{\infty} ds\,\frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

- The subtraction constants of $B_{n\ell}(s)$ [SC 1901.06071]

(nl)	$B^{\parallel}_{n\ell}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ln B^{\parallel}_{n\ell}(0)$	$B^{\perp}_{n\ell}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ln B^{\perp}_{n\ell}(0)$
(01)	1	0	1.46 $\to$ 1.80	1	0	0.68 $\to$ 0.60
(21)	-0.113 $\to$ 0.218	-0.340	0.481	0.113 $\to$ 0.185	-0.538	-0.153
(23)	0.147 $\to$ -0.038	0	0.368	0.113 $\to$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

- Firstly studied by low-energy EFT based on instanton vacuum [Polyakov 1999]
- Updated recently with the kinematical constraints [SC 2019, SC 2023]
  - △ Soft pion theorem relates the chirarlly even coefficients with a_n^π
  - △ 2πDAs relate to the skewed parton distributions (SPDs) by crossing
  - △ In the vicinity of the resonance, 2πDAs reduce to the DAs of ρ/f_0