Study of X(3872)’s structure

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第三届强子与重味物理理论与实验研讨会，
2024年4月5日-9日

PRD2016，EPJC2017，PRD2022，EPJC2022, PLB2024

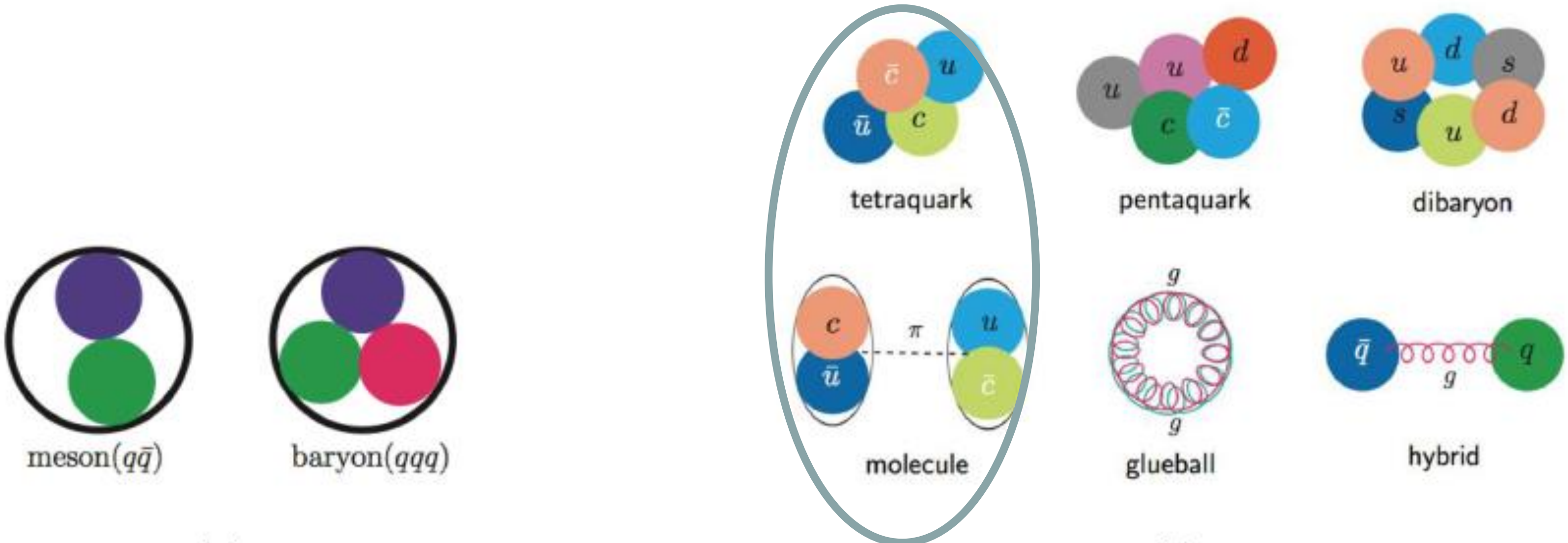
Outline

- Line shape of X(3872)
- Radiative decay of X(3872)

Introduction

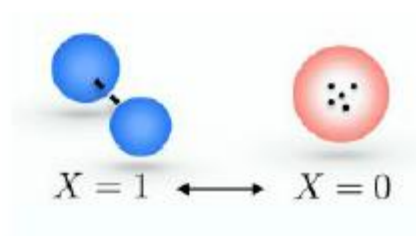
Two concepts: compositeness and CDD pole

- A series of exotic hadron candidate XYZ were and are observed, cannot be accommodated by potential model — kinematical effects, molecular, quark-gluon hybrid, et al..
- Typically different scenarios predicts different constituents, in practice, may involve several mechanism
  $\longrightarrow$ compositeness
- Stay close to threshold of meson pairs: only 2-3 MeV above meson pair threshold
  $\longrightarrow$ effective range expansion (ERE)



(a) (b)

Compositeness X: weight of two-meson components in the configuration of configuration



Compositeness for resonance

- Weinberg compositeness condition: wave function renormalization constant Z=0. In fact, $Z = 1 - X$, where $X = -\gamma^2 \frac{dG(s_R)}{ds_R}$ quantifies the weight of constituents; γ is the residue for $t(s)$ in the 1st sheet at the pole, and G is the two-point loop function.
- only applied to bound state — model-independent relation for deuteron [Weinberg 1963; 1965]
- For resonance case, as long as $\sqrt{\mathrm{Re} E_R^2}$ larger than the lightest threshold, $X = |\gamma^2 \frac{dG(s_R)}{s_R}|$, γ residue in the 2nd sheet [Guo and Oller, PRD2015]
- Adapted to non-relativistic case, criterion: $M_R > M_{\rm th}$, applied to Z_b and Z_c states [Kang, Guo and Oller, PRD2016]

PHYSICAL REVIEW VOLUME 101, NUMBER 1 JANUARY 1, 1956

Low's Scattering Equation for the Charged and Neutral Scalar Theories*

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AND

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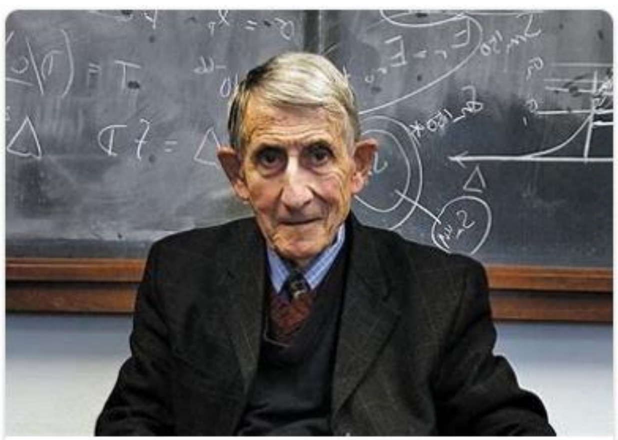
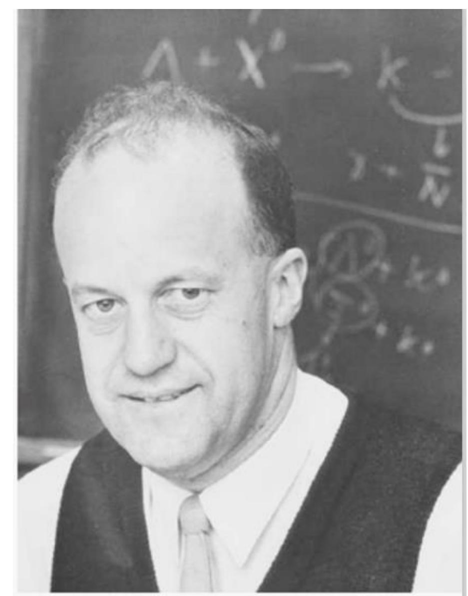
CDD poles

(Received August 3, 1955)

The Low scattering equation is studied in the one-meson approximation with both charged and neutral scalar meson theories. The general solution is found for each of these cases. It has the general character of a Wigner-Eisenbud dispersion formula and contains an infinite number of adjustable parameters. It follows that the Low equation, in this approximation at least, does not determine the scattering, but only expresses a property of the scattering which is independent of the internal structure of the scatterer.



F. J. Dyson



R. Dalitz

N/D方法简述

分波振幅$T_L(s)$: $$T_L(s)=\frac{N_L(s)}{D_L(s)}$$

$D_L(s)$：包含右手割线；$N_L(s)$：左手割线

仅考虑右手割线，从幺正关系得到：

$$\mathrm{Im}D_L=\mathrm{Im}T_L^{-1}N_L=-\rho(s)N_L,\quad s>s_{\mathrm{th}}$$

$$\mathrm{Im}D_L=0,\quad s<s_{\mathrm{th}}$$

$$D_L(s)=-\frac{(s-s_0)^{L+1}}{\pi}\int_{s_{\mathrm{th}}}^{\infty}ds'\frac{\rho(s')}{(s'-s)(s'-s_0)^{L+1}}+\sum_{m=0}^{L}a_m s^m+\sum_{i}^{M_L}\frac{\gamma_i}{s-s_i}$$

- 其中：M_L是CDD极点的数目，该项来自函数中CDD极点的出现，求和项与CDD极点一一对应。

(1）添加这些CDD极点不违背任何解析性、幺正性的要求

(2）CDD极点是T振幅的零点

(3）CDD极点的出现：与分波振幅有相同量子数的基本粒子相关联

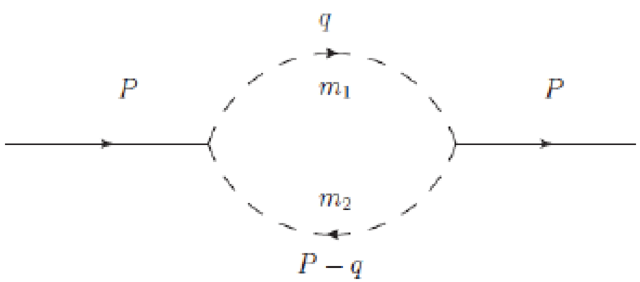
- $M_{\rm CDD}$ close to $M_{\rm th}$, then small X, i.e., containing also other important components, e.g., compact quark-gluon states; $M_{\rm CDD}$ far from $M_{\rm th}$, then the two meson constitute dominates

$$T(s)=\left[\sum_i \frac{\gamma_i}{s-s_i}+G(s)\right]^{-1}$$

函数$G(s)$

积分部分：一次减除色散关系

- 两介子圈函数



$$G(s)=a(\mu^2)+\frac{1}{(4\pi)^2}\left(\log\frac{m_2^2}{\mu^2}-\varkappa_+\log\frac{\varkappa_+-1}{\varkappa_+}-\varkappa_-\log\frac{\varkappa_--1}{\varkappa_-}\right)$$

$$\varkappa_\pm=\frac{s+m_1^2-m_2^2}{2s}\pm\frac{k}{\sqrt{s}},$$

$$k=\frac{\sqrt{(s-(m_1-m_2)^2)(s-(m_1+m_2)^2)}}{2\sqrt{s}}$$

Inclusion of CDD pole and ERE

- Only right-hand cut without crossed-channel effect [Oller and Oset, PRD1999] $t(E) = \left[\sum_i \frac{g_i}{E - M_{\text{CDD},i}} + \beta - ik\right]^{-1}$
- ERE: $t(E) = \left[-1/a + 1/2\, r\, k^2 - ik\right]^{-1}$
- Expansion of Re $t(E)^{-1}$ in powers of k^2 is equivalent to ERE, but worry for the small scale $[M_{\text{CDD}} - M_{\text{th}}]$, which restricts the validity range.
- M_{CDD} far away from M_{th}, then modulu of r is around 1 fm, otherwise r is very large.

$$1/a = \frac{g_i}{M_{\text{CDD}} - M_{\text{th}}} - \beta, \quad r = -\frac{g_i}{\mu(M_{\text{th}} - M_{\text{CDD}})^2}$$

- X(3872) state

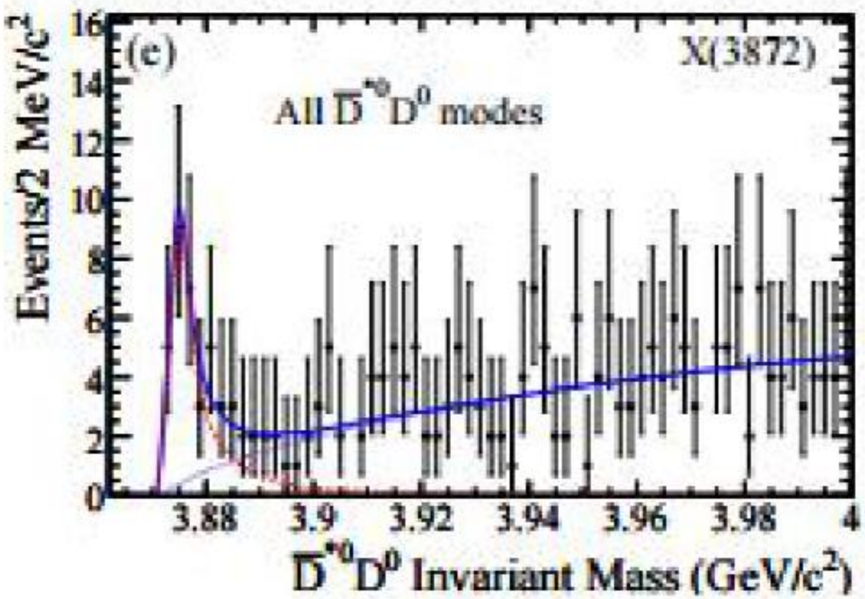
Quantum number of X(3872) state

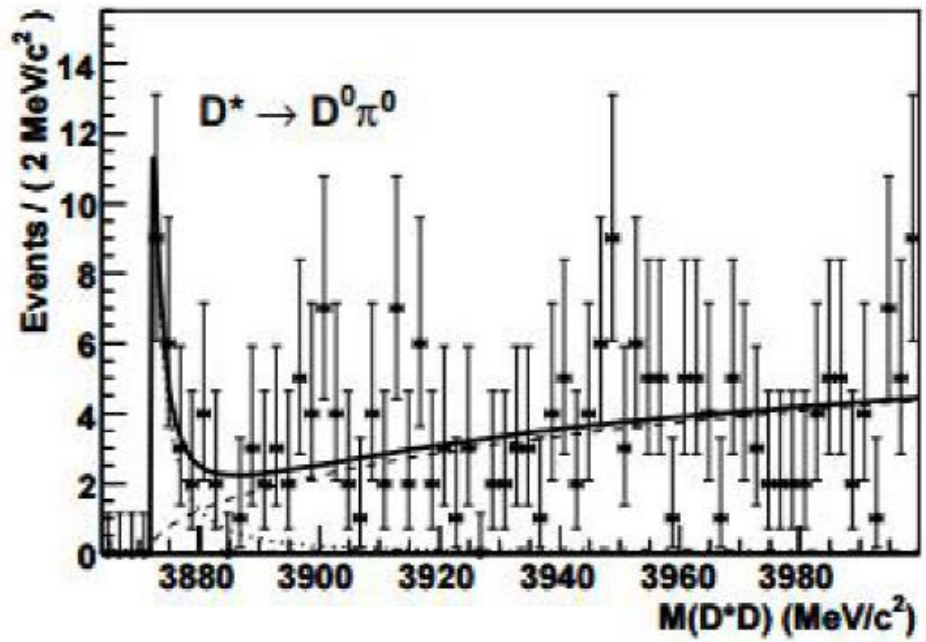
- First observation from Belle, PRL2003, triggering voluminous amount of papers
- PDG determination:

$$I^G(J^{PC}) = 0^+(1^{++}),$$
$$M = 3871.69 \pm 0.17 \text{ MeV}, \Gamma < 1.2 \text{ MeV}, \text{ CL} = 90\%$$
$$\bar{D}D^* : C = + \text{ combination } (D\bar{D}^* + \bar{D}D^*)/\sqrt{2}$$
$$\text{threshold } M_{\text{th}} = 3871.81 \text{ MeV}$$

- From now on, all the energy and M_{CDD} are measured respective to M_{th}. X(3872) mass: -0.11 ± 0.17 MeV
- Nature: molecular like virtual state (V) and bound state (B), or preexisting state, etc.

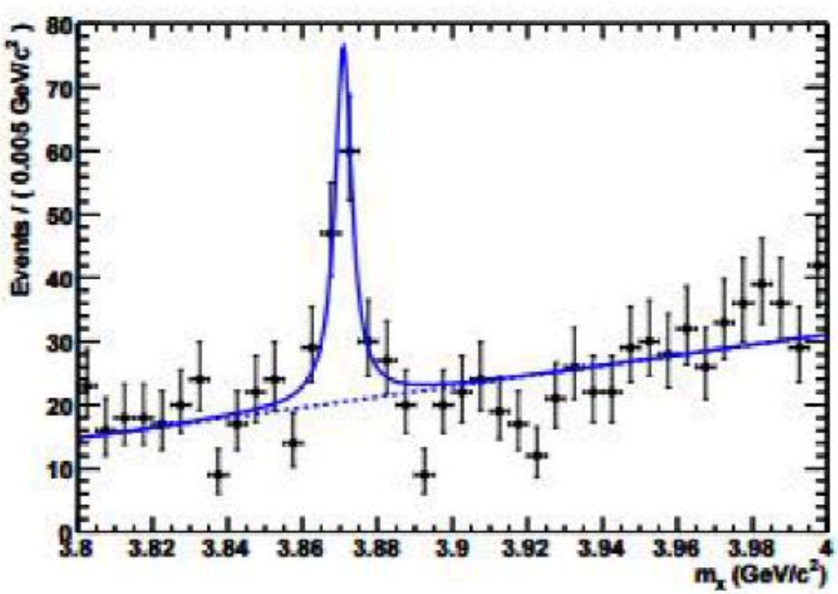
Experimental situation: $\bar{D}^0 D^{*0}$ channel

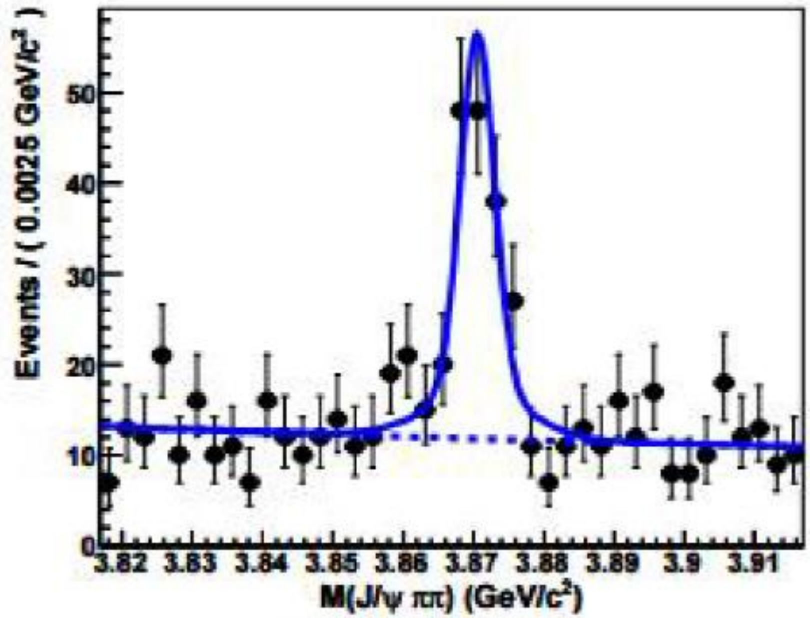




- the decay chain: $B \to X(3872)K \to \bar{D}^0 D^{*0} K$
- Left: BaBar2008, Right: Belle2010
- BaBar has total number of $B\bar{B}$ pairs, $N_{B\bar{B}}^{\text{BaBar}} = 3.83 \cdot 10^8$, while $N_{B\bar{B}}^{\text{Belle}} / N_{B\bar{B}}^{\text{BaBar}}$=1.75

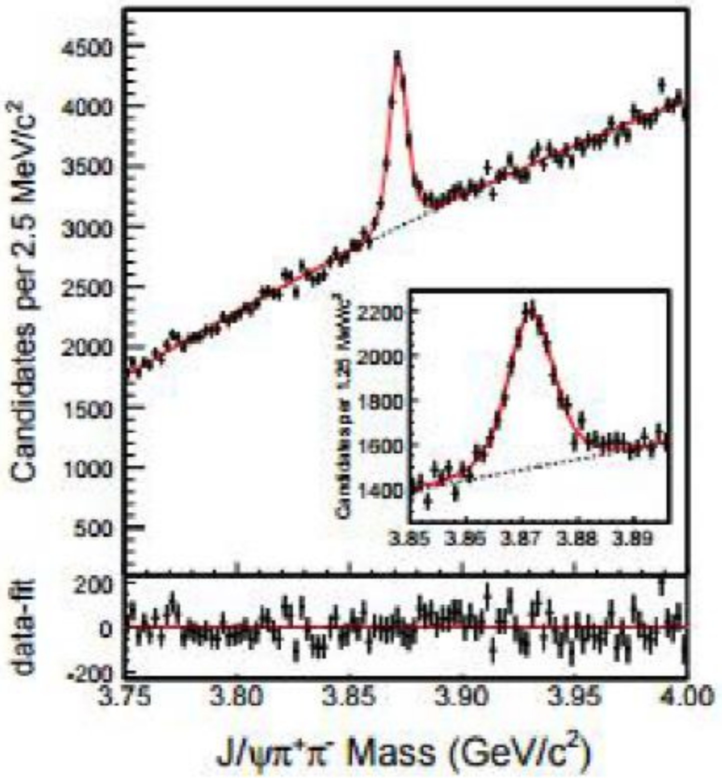
Experimental situation: $J/\psi\pi\pi$ channel





- the decay chain: $B \to X(3872)K \to J/\psi\pi^+\pi^-K$
- Left: BaBar2008, Right: Belle2008
- Data are compatible with each other.

Experimental situation: $J/\psi\pi\pi$ channel continued



- the decay chain: $p\bar{p} \to X(3872) +$ all with $X(3872) \to J/\psi\pi^+\pi^-$
- The inset shows an enlargement of the region around the $X(3872)$ peak, with very small bin width of 1.25 MeV.
- *"Precision Measurement of the $X(3872)$ in $J/\psi\pi\pi$ Decays"* from CDF2009.

Formalism (1)

- Exp summary: Belle $D\bar{D}\pi$ + BaBar $J/\psi\pi\pi$ + Belle $J/\psi\pi\pi$ + CDF $J/\psi\pi\pi$
- As introduced, scattering amplitude

$$t(E) = \left(\frac{\lambda}{E - M_{\text{CDD}}} + \beta - ik(E) \right)^{-1},$$

more general than ERE

- Removing the extra zeros due to the CDD pole, one ends with the final-state interaction

$$d(E) = \left(1 + \frac{E - M_{\text{CDD}}}{\lambda} (\beta - ik) \right)^{-1}$$

[Oller PLB2000, Bugg PLB2003]

- When M_{CDD} far, $M_{\text{CDD}} \to \infty$ keeping λ / M_{CDD} fixed, one recovers the scattering length approximation

$$t(E) \Longrightarrow f(E) = \frac{1}{-\lambda/M_{\text{CDD}} + \beta - ik} = \frac{1}{-\gamma - ik}$$

Formalism (2)

- The normalized standard non-relativistic mass distribution for a narrow resonance or bound state ($\Gamma_X \to 0$)

$$\frac{d\hat{M}}{dE} = \frac{\Gamma_X |d(E)|^2}{2\pi|\alpha|^2}$$

- α is a constant, obtained by singling out the pole contribution, in fact, the residue of $d(E)$, $d(E) \sim \frac{\alpha}{E-E_p}$, E_p pole position.
- Normalization integral $\mathcal{N} = \int_{-\infty}^{\infty} dE \frac{d\hat{M}}{dE}$
- For a narrow resonance (including bound state), $\mathcal{N} \approx 1$, but not so when $d(E)$ has a shape strongly departs from a non-relativistic Breit-Wigner, e.g., for a virtual state
- For $f(E)$ (ERE), the integral does not converge, just integrate in the signal region.

Formalism (3): event distribution for $J/\psi\pi\pi$ channel

- For $B \to KJ/\psi\pi\pi$ channel [simpler]:

$$N_i = 2N_{B\bar{B}} \left[\mathcal{B}_J \int_{E_i-\Delta/2}^{E_i+\Delta/2} dE' \int_{-\infty}^{\infty} dE R(E', E) \frac{d\hat{M}}{dE} + \text{cbg}_J \Delta \right]$$

- For $p\bar{p}$ to $J/\psi\pi\pi$ channel: just replace $2N_{B\bar{B}}$ by $\mathcal{L}\sigma_{p\bar{p}\to X\text{All}}$, with $\mathcal{L}$ luminosity, and total cross section σ for $p\bar{p} \to X + \text{All}$.
- $R(E', E)$ is the Gaussian, experimental resolution function
- $\int_{E_i-\Delta/2}^{E_i+\Delta/2} dE'$ indicates the integration in the bin width.

Formalism (4): event distribution for $\bar{D}^0 D^{*0}$ channel

- For $B \to K \bar{D}^0 D^{*0}$ channel [taking into account the small width of D^*, $\Gamma_* \approx 65$ KeV]

$$N_i = 2N_{B\bar{B}} \int_{E_i-\Delta/2}^{E_i+\Delta/2} dE' \int_0^\infty d\mathcal{E}' R(E', \mathcal{E}') \sqrt{\mathcal{E}'} \times \left[\frac{\mathcal{B}_D \Gamma_*}{\sqrt{2}\pi \left(\sqrt{E_X^2 + \Gamma_*^2/4} - E_X \right)^{1/2}} \int_{-\infty}^{\infty} dE \frac{d\hat{M}}{dE} \frac{1}{|\mathcal{E}' - E - i\Gamma_*/2|^2} + \text{cbg}_D \right]$$

- Pole position $E_X - i\Gamma_X/2$, with E_X relative to $\bar{D}^0 D^{*0}$ (reduced mass $\mu \approx 1$GeV) threshold, momentum at pole position k_X.

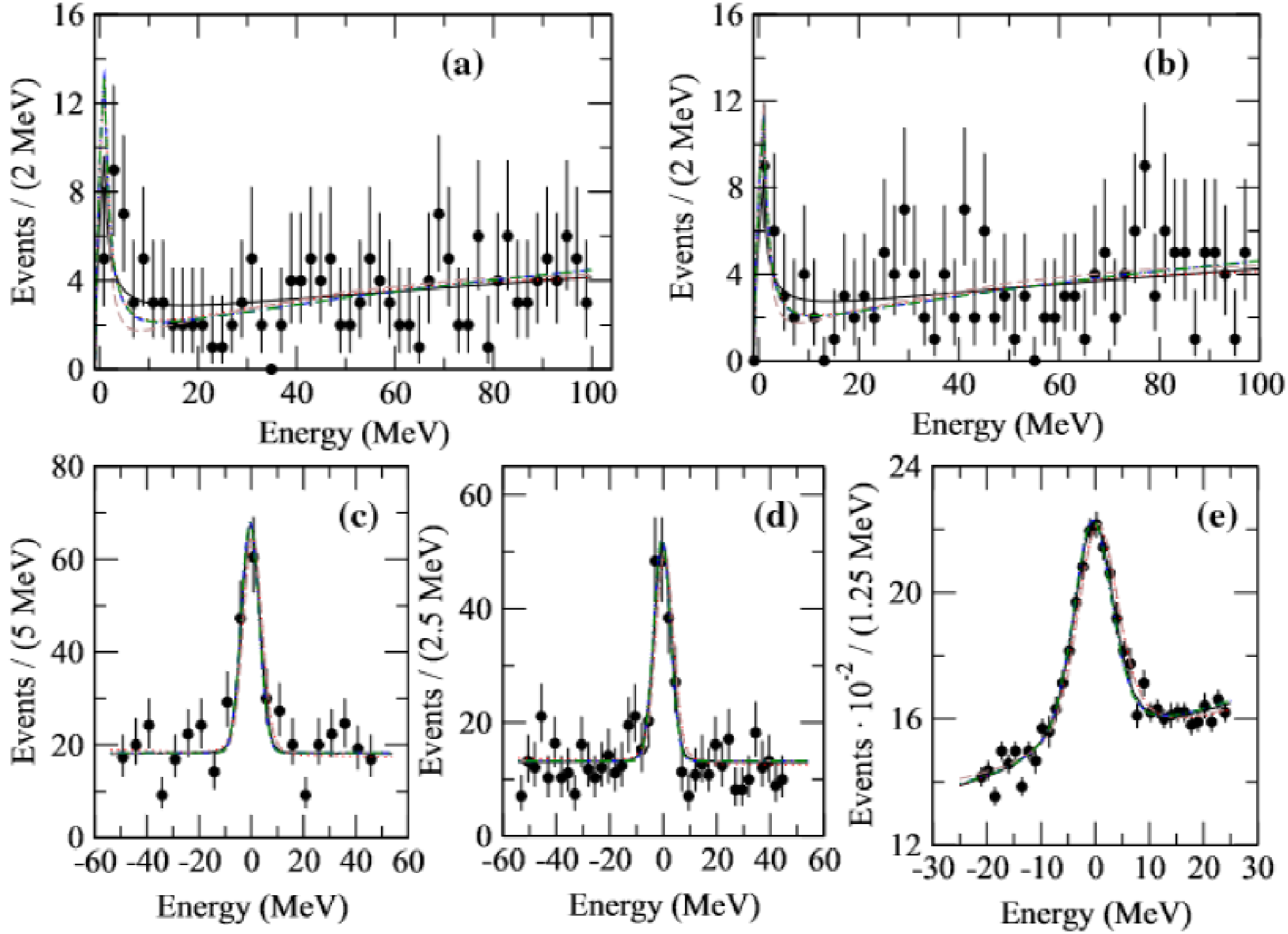
Fit strategy

- $\mathcal{B}_D$, $\mathcal{B}_J$, cbg_D, cbg_J, overall constants and background are always free parameters
- As mentioned, for bound state, $\mathcal{B}$ can be justified as branching ratio, otherwise not.
- M_{CDD}, λ, β characterize the line shape of $d(E)$.
- Braaten et al has also used ERE (only scattering length), but fit separately
- C. Hanhart et al used ERE including the effective range.
- One should use more general $d(E)$, other than ERE!
- Maximize likelihood fit, data errors are asymmetric.

Different scenarios

- Case i): using ERE, which is a pure bound state, making combined fits to all the existing data
- Case ii): imposing $t(E)$ has a virtual state, one can express λ and β from E_R and M_{CDD}.
- Case iii): taking into account the coupled channel effect, E_R determines the shape of $t(E)$:
  Quadratic equation: two solutions with case iii). I and case iii).II

Results



(a)
Events / (2 MeV)
Energy (MeV)
(b)
Events / (2 MeV)
Energy (MeV)
(c)
Events / (5 MeV)
Energy (MeV)
(d)
Events / (2.5 MeV)
Energy (MeV)
(e)
Events · 10^-2 / (1.25 MeV)
Energy (MeV)

Results

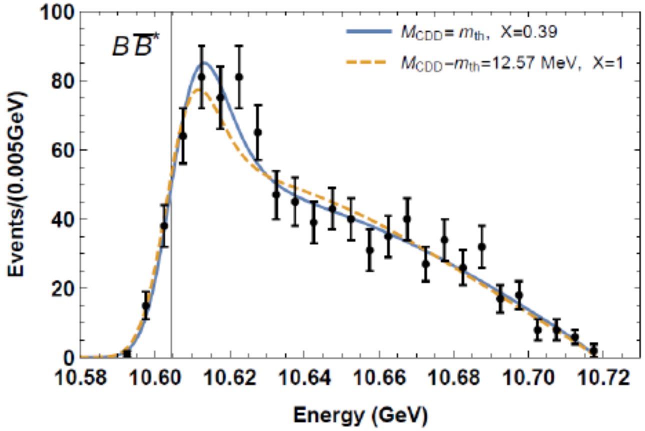
Cases	Pole position [MeV]	X	Residue [GeV^2]	Y_{D1} Y_{D2} Y_J $Y_J^{(p)}$
1	$-0.19^{+0.01}_{-0.01} - i\,0.0325$	1.0	$14.78^{+0.38}_{-0.14}$	$7.49^{+0.71}_{-0.41}$
				$6.45^{+0.32}_{-0.47}$
				$79.03^{+5.65}_{-6.11}$
				$5.23^{+0.07}_{-0.11} \times 10^3$
2.I	$-0.36^{+0.08}_{-0.10} - i\,0.18^{+0.01}_{-0.02}$		$-47.48^{+9.75}_{-12.40} - i\,66.06^{+10.87}_{-13.50}$	$83.13^{+22.42}_{-16.15}$
	$-0.70^{+0.11}_{-0.13} + i\,0.17^{+0.02}_{-0.01}$		$82.69^{+14.84}_{-11.88} + i\,66.03^{+13.50}_{-10.87}$	$40.13^{+11.86}_{-7.25}$
				$8.44^{+3.64}_{-2.59} \times 10^3$
				$5.78^{+2.29}_{-1.65} \times 10^5$
2.II	$-0.33^{+0.04}_{-0.03} - i\,0.31^{+0.02}_{-0.01}$		$-6.24^{+2.80}_{-2.20} - i\,1.41^{+0.14}_{-0.10} \times 10^2$	$79.75^{+22.46}_{-19.81}$
	$-0.84^{+0.07}_{-0.05} + i\,0.77^{+0.03}_{-0.04}$		$(2.32^{+0.16}_{-0.21} - i\,1.77^{+0.11}_{-0.08}) \times 10^2$	$42.20^{+9.18}_{-8.02}$
	$-1.67^{+0.10}_{-0.08} - i\,0.49^{+0.02}_{-0.02}$		$(-3.26^{+0.22}_{-0.16} + i\,3.18^{+0.18}_{-0.25}) \times 10^2$	$9.23^{+1.60}_{-1.57} \times 10^3$
				$6.23^{+0.71}_{-0.84} \times 10^5$
3.I	$-0.50^{+0.04}_{-0.03}$	$0.061^{+0.003}_{-0.002}$	$1.52^{+0.01}_{-0.01}$	$25.45^{+4.05}_{-4.15}$
	$-0.68^{+0.05}_{-0.03}$		$2.72^{+0.02}_{-0.04}$	$12.29^{+1.32}_{-1.89}$
				$80.14^{+5.67}_{-5.19}$
				$5.26^{+0.12}_{-0.08} \times 10^3$
3.II	$-0.51^{+0.03}_{-0.01}$	$0.158^{+0.001}_{-0.001}$	$3.96^{+0.03}_{-0.08}$	$22.90^{+2.94}_{-3.02}$
	$-1.06^{+0.05}_{-0.02}$		$7.56^{+0.08}_{-0.20}$	$11.03^{+1.40}_{-0.77}$
				$80.07^{+5.14}_{-5.36}$
				$5.28^{+0.05}_{-0.17} \times 10^3$

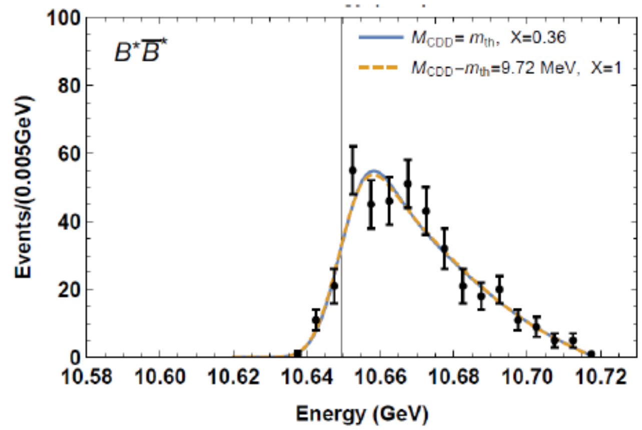
Higher-order poles of S-matrix

出现在相邻黎曼面，Morgan判据

Zb 数值结果

◆ 共振态不变质量分布的拟合图像





$M_{CDD}=m_{th}$：组分系数的值最小

$B\bar{B}^*$ 结构在$Z_b(10610)$共振态中，比例系数X=0.39；

$B^*\bar{B}^*$ 结构在$Z_b(10650)$共振态中，比例系数X=0.36；

X=1：对实验数据的描述十分近似

◆ 两个Z_b共振态组分系数的变化范围0.4~1.0

C. Y. Cui, Y. L. Liu and M. Q. Huang, PRD 85, 074014 (2012): both a $B^*\bar{B}$ molecular state and a $[bd][\bar{b}\bar{u}]$ tetraquark state coincide with Z_b(10610).

- Radiative decay of X(3872) in the covariant light-front quark model (CLFQM)

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Letter

Nature of $X(3872)$ from its radiative decay

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Overall blueprint: **Assuming X(3872) as a pure $c\bar{c}$ resonance, considering the decay $X(3872) \to \gamma J/\psi$ and $\gamma\psi'$ at the same footing**

Conclusion: their partial width can not reconcile with each other.

$\chi_{c1}(3872)$

$I^G(J^{PC}) = 0^+(1^{++})$

also known as $X(3872)$

Mass $m = 3871.65 \pm 0.06$ MeV
$m_{\chi_{c1}(3872)} - m_{J/\psi} = 775 \pm 4$ MeV
Full width $\Gamma = 1.19 \pm 0.21$ MeV (S = 1.1)

VALUE (MeV)	CL%	EVTS	DOCUMENT ID		TECN	COMMENT
$\mathbf{1.19 \pm 0.21}$	OUR AVERAGE Error includes scale factor of 1.1.					
$1.39 \pm 0.24 \pm 0.10$		15.6k	[1] AAIJ	2020AD	LHCB	$p\,p \to J/\psi\pi^+\pi^- X$
$0.96\,^{+0.19}_{-0.18} \pm 0.21$		4.2k	[2] AAIJ	2020S	LHCB	$B^+ \to J/\psi\pi^+\pi^- K^+$

Γ_{23}	$\gamma J/\psi$	$(8 \pm 4) \times 10^{-3}$	
Γ_{24}	$\gamma\chi_{c1}$	$< 9 \times 10^{-3}$	CL=90%
Γ_{25}	$\gamma\chi_{c2}$	$< 3.2\%$	CL=90%
Γ_{26}	$\gamma\psi(2S)$	$(4.5 \pm 2.0)\%$	

Feynman rule for light-front quark model

1. Momentum variables expressed in the light front coordinates

2. Wave function of the (axial-vector, vector, scalar, pseudo-scalar) meson encodes the bound state nature of $\bar{q}q$

3. Vertex comes from the SM, Melosh transformation:
The connection between *spin states in the rest frame and infinite momentum* frame Or between spin states in the conventional equal time dynamics and the light-front dynamics

4. Fermion internal line denotes a spin-1/2 propagator as usual

5. quark masses as parameters, but fixed for all calculations

$$\varphi' = \varphi'(x_2, p'_\perp) = 4\left(\frac{\pi}{\beta'^2}\right)^{\frac{3}{4}} \sqrt{\frac{dp'_z}{dx_2}} \exp\left(-\frac{p'^2_z + p'^2_\perp}{2\beta'^2}\right),$$

$$\varphi'_p = \varphi'_p(x_2, p'_\perp) = \sqrt{\frac{2}{\beta'^2}}\, \varphi', \qquad \frac{dp'_z}{dx_2} = \frac{e'_1 e_2}{x_1 x_2 M'_0}.$$

Parameter β characterizing the size of hadron, will be fixed by decay constant

However, the wave function can be solved by e.g., relativistic quark model in the quasi-potential approach.

Semileptonic decays of D and D_s mesons in the relativistic quark model #15

R.N. Faustov (Unlisted, RU), V.O. Galkin (Unlisted, RU), Xian-Wei Kang (Beijing Normal U.) (Nov 19, 2019)

Published in: *Phys.Rev.D* 101 (2020) 1, 013004 • e-Print: 1911.08209 [hep-ph]

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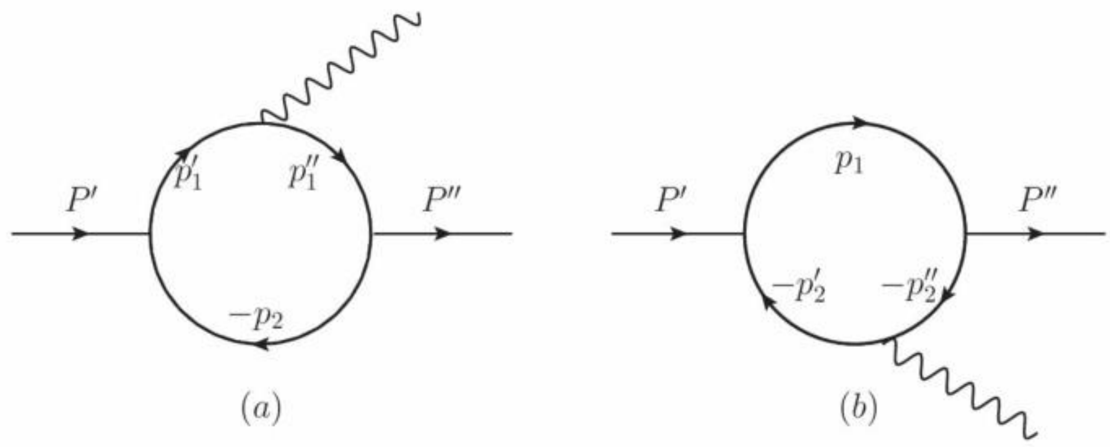

FIG. 1: Feynman diagrams for radiative transitions. The uppercase P' and P'' denote the four-momentum of the initial and final meson, respectively. The lower case $p_{1,2}$ is the momentum of the spectator quark, and other momenta with prime or double prime in the superscript correspond to the active quarks involving a photon emission shown by a wavy line.

The transition amplitude for $X(3872) \to J/\psi\,\gamma, \psi'\gamma$ can be written as

$$A(X(3872) \to \psi\gamma) = \epsilon^{*\alpha}(q)\epsilon'^{\mu}(P')\epsilon''^{*\nu}(P'')\mathcal{A}_{\alpha\mu\nu}$$

The polarization vectors in order are the ones for photon, X(3872), and J/ψ.

$$\mathcal{A}_{\alpha\mu\nu} = \varepsilon_{\alpha\nu\beta\eta}P^{\beta}q^{\eta}P_{\mu}f_m(q^2) + \varepsilon_{\alpha\mu\beta\eta}P^{\beta}q^{\eta}P_{\nu}f_p(q^2) + \varepsilon_{\alpha\mu\nu\rho}q^{\rho}f_6(q^2).$$

Independent from factors $f_m(q^2), f_p(q^2)$, $f_6(q^2)$

$$
\begin{aligned}
f_m^a(q^2) &= \frac{2e}{3}\frac{N_c}{16\pi^3}\int dx_2 d^2p'_\perp \frac{h'_A h''_V}{x_2\hat{N}'_1\hat{N}''_1}(-4)\left\{\frac{1}{w'_A}[(m'_1-m''_1)(A_3^{(2)}-A_4^{(2)})\right. \\
&+(m'_1+m''_1-2m_2)\times(A_2^{(2)}-A_3^{(2)})+m'_1(A_2^{(1)}-A_1^{(1)})]+A_2^{(2)}-A_3^{(2)} \\
&\left.-\frac{1}{w'_A w''_V}(2A_2^{(3)}-2A_1^{(3)})\right\}, \\
f_p^a(q^2) &= \frac{2e}{3}\frac{N_c}{16\pi^3}\int dx_2 d^2p'_\perp \frac{h'_A h''_V}{x_2\hat{N}'_1\hat{N}''_1}(-4)\left\{\frac{1}{w''_V}[(m'_1-m''_1)(A_3^{(2)}+A_4^{(2)}\right. \\
&-A_2^{(1)})+(m'_1+m''_1+2m_2)\times(A_2^{(2)}+A_3^{(2)}-A_1^{(1)})-m'_1(A_1^{(1)}+A_2^{(1)} \\
&\left.-1)]+A_1^{(1)}-A_2^{(2)}-A_3^{(2)}-\frac{1}{w'_A w''_V}(2A_1^{(3)}+2A_2^{(3)}-2A_1^{(2)})\right\}, \\
f_6^a(q^2) &= \frac{2e}{3}\frac{N_c}{16\pi^3}\int dx_2 d^2p'_\perp \frac{h'_A h''_V}{x_2\hat{N}'_1\hat{N}''_1}(-4)\left\{\frac{1}{w'_A}(m'_1+m''_1-2m_2)A_1^{(2)}\right. \\
&+\frac{1}{w''_V}(m'_1+m''_1+2m_2)A_1^{(2)}-\frac{1}{4}(1-2A_2^{(1)})[-q^2+\hat{N}'_1+\hat{N}''_1 \\
&\left.+(m'_1-m''_1)^2]-A_2^{(1)}(m''_1m_2-m'_1m_2)-m'_1m_2\right\}.
\end{aligned}
$$

Expressions of form factors. In fact, we only concern the values at $q^2=0$

The decay width can be calculated by

$$\begin{aligned}\Gamma &= \frac{|\boldsymbol{q}|}{8\pi M'^2}\left(|A_{+0-}|^2+|A_{-0+}|^2+|A_{0--}|^2+|A_{0++}|^2\right)\\ &= \frac{|\boldsymbol{q}|^3}{\pi}\left(\frac{f_6+2M'|\boldsymbol{q}|f_p}{4M''^2}+\frac{f_6^2}{4M'^2}+\frac{f_6 f_m|\boldsymbol{q}|}{M'}+f_m^2|\boldsymbol{q}|^2\right).\end{aligned}$$

In the helicity basis:

$A_{\lambda'\lambda''\lambda_\gamma}$, with λ', λ'', λ_γ denoting the helicity of $X(3872)$, J/ψ (or ψ'), the photon, respectively.

$$\begin{aligned}A_{+0-} &= -i\frac{M'|\boldsymbol{q}|}{M''}(f_6+2M'|\boldsymbol{q}|f_p),\\ A_{0++} &= i|\boldsymbol{q}|(f_6+2M'|\boldsymbol{q}|f_m),\\ A_{-0+} &= -A_{+0-},\quad A_{0--}=-A_{0++}.\end{aligned}$$

$|\boldsymbol{q}| = \frac{M'^2-M''^2}{2M'}$; M’ is the mass of X(3872), M’’ is the mass of J/ψ

Wave function of J/ψ and ψ'

$$\Gamma(V \to e^+e^-) = \frac{4\pi}{3}\frac{4}{9}\alpha^2\frac{f_V^2}{M'}, \tag{25}$$

where α is the fine structure constant, f_V is the decay constant, M' is the mass of vector meson. Taking $\mathrm{Br}(J/\psi \to e^+e^-) = (5.971 \pm 0.032)\%$, $\mathrm{Br}(\psi' \to e^+e^-) = (7.93 \pm 0.17) \times 10^{-3}$, $\Gamma(J/\psi) = (92.6 \pm 1.7)$ keV and $\Gamma(\psi(2S)) = (294 \pm 8)$ keV [22], we obtain the decay constants $f_{J/\psi} = 415.49$ MeV and $f_{\psi'} = 294.35$ MeV as our central values. The uncertainties are very small. The formula for the decay constant of vector mesons in the LFQM is given by [27]

$$f_V = \frac{N_c}{4\pi^3 M'}\int dx d^2p'_\perp \frac{h'_V}{x(1-x)(M'^2 - M_0'^2)}\left[xM_0'^2 - m_1'(m_1' - m_2) - p_\perp'^2 + \frac{m_1' + m_2}{M_0' + m_1' + m_2}p_\perp'^2\right], \tag{26}$$

from which we fix the parameters $\beta_{J/\psi} = 0.631$ GeV and $\beta_{\psi'} = 0.487$ GeV.

Experimental width $\Gamma(X(3872) \to J/\psi\gamma) = (3.2 \pm 1.6) \times 10^{-3}$ MeV

X(3872) as 2P state:
Exp width of $\Gamma(X(3872) \to \psi'\gamma) \Longrightarrow \beta_{X(3872)} = 0.56^{+0.04}_{-0.03}$GeV
$\Longrightarrow \Gamma(X(3872) \to J/\psi\gamma) = 0.91^{+0.17}_{-0.15}$ MeV

X(3872) as 1P state: excluded by its mass value

Consequently, the scenario of a pure charmonium assignment for X(3872) will encounter difficulty to reconcile the widths to J/ψ γ and ψ' γ.

Or stated differently, the probability that X(3872) is a pure $c\bar{c}$ resonance is rather tiny

Main points

- The current line shape data is not enough to pin down the structure of X(3872). Both the virtual state and bound state are possible scenarios.

- The radiative decay analysis basically excludes the pure ccbar scenarios.