Light-cone distribution amplitudes of a light baryon in LaMET

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Outline

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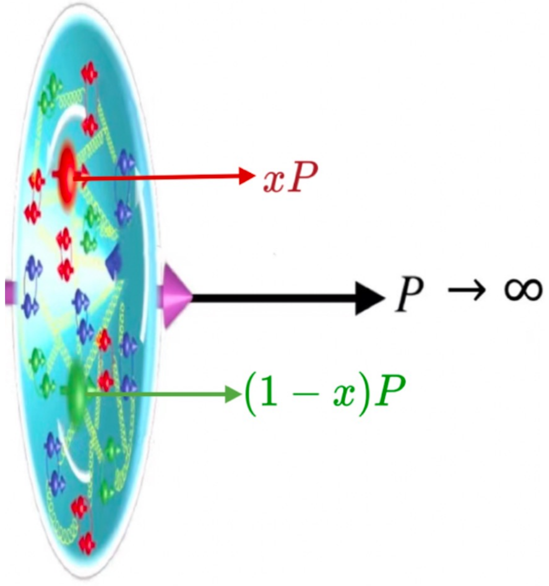
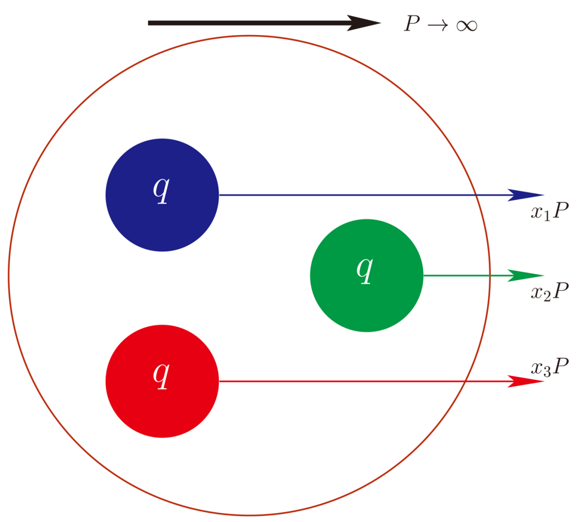
目 录

Background

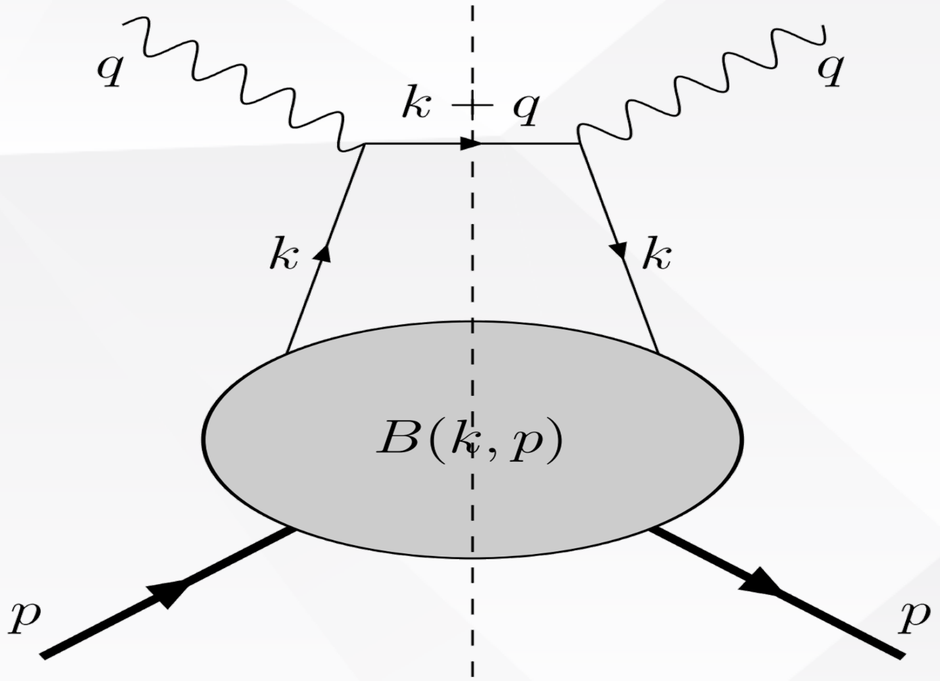
- Feynman proposed a parton model more than 50 years ago, and hadron structure information has been obtained by fitting a large number of high-energy experimental data.
- The hadron LCDA is a physical quantity that describes the momentum distribution of all parts of hadrons and reflects the internal structure of hadrons.
- The calculation of the hadron LCDA by the basic theory of the strong interaction has been slow for a long time.

Lack of first-principle results!

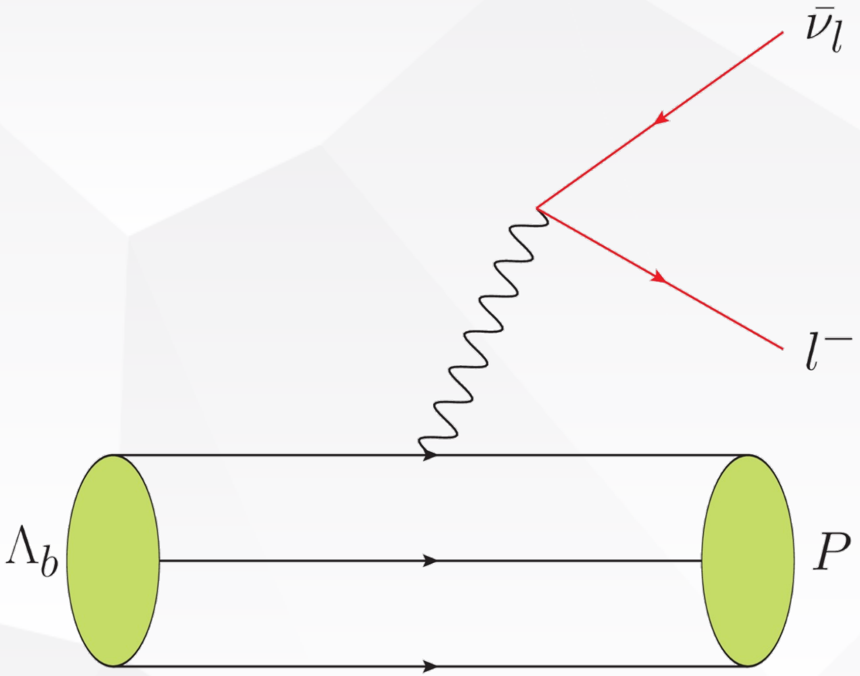




Baryon LCDA



Inclusive: PDF



Exclusive: WF/LCDAs

LCDAs provide a complementary role in understanding the structure of the hadron.

Baryon LCDA

Challenging:

1. Nonperturbative: cannot be calculated directly from first principles; must be extracted from experimental data or nonperturbative QCD approaches, such as lattice QCD simulations or phenomenological models.

2. Large number of degrees of freedom: The determination of LCDAs involves accounting for the contributions from all possible quark and gluon configurations within the baryon, leading to a large number of degrees of freedom and complicating the analysis.

3.Complexity of the operator product expansion (OPE): the OPE involves three-quark operators and gluonic operators, leading to a more complex and computationally challenging analysis compared to mesons.

4.Limited experimental data: limited experimental data for baryon structure observables, makes it challenging to constrain baryon light-cone distribution functions directly from the experiment.

Progress on baryon LCDA

Lattice QCD

1. Nucleon distribution amplitudes and proton decay matrix elements on the lattice(Vladimir M. Braun, 2009) **present a calculation of the first few moments of the leading-twist nucleon DA**
2. Light-cone distribution amplitudes of the nucleon and negative parity nucleon resonances from lattice QCD(Vladimir M. Braun, 2014) **calculate moments**
3. Light-cone distribution amplitudes of octet baryons from lattice QCD (RQCD Collaboration,2019)

Model

1. Modelling the Nucleon WF from Soft and hard processes(1996) parameterization
2. Nucleon distribution amplitude: The heterotic solution(1993)
   amalgamates features of the Chernyak-Ogloblin-Zhitnitsky model with those of the Cari-Stefanis model
3. Nucleon WF and Form Factors in QCD(1983)
   A model for the nucleon wave function is proposed based on a knowledge of these few first moments.
4. LCDA of the baryon(2021) Chiral quark-soliton model
5. Estimates of the isospin-violating Λb → Σ0φ;Σ0J=ψ decays and the Σ −Λ mixing(2023) COZ model

Progress on baryon LCDA

QCD sum rule

1. Nucleon WFs and QCD sum rules(1987)
2. Higher twist distribution amplitudes of the nucleon in QCD(2000)

   present the first systematic study of higher-twist LCDAs of the nucleon in QCD. Nonperturbative input parameters are estimated from QCD sum rules.
3. $\Lambda_b \to p$ transition form factors in perturbative QCD(2022)

Ligh-cone Sum Rule

1. Nucleon form factors and distribution amplitudes in QCD(2013)

   extracted from the comparison with the experimental data on form factors
2. Wave functions of octet baryons(1989)

   The model wave functions are proposed which fulfill the sum rules requirements.
3. Nucleon form factors in QCD(2006)

1. Introduction

- LCDA of a light baryon describe the momentum distributions of a quark/gluon in a baryonic system;
- a fundamental non-perturbative input in QCD factorization for an exclusive process with a large momentum transfer;
- valuable to extract the CKM matrix element in the standard model and to probe new physics beyond the standard model;
- many phenomenological analyses adopt model paramterizations resulting in uncontrollable errors in theoretical predictions for decay branching fractions of heavy baryons.

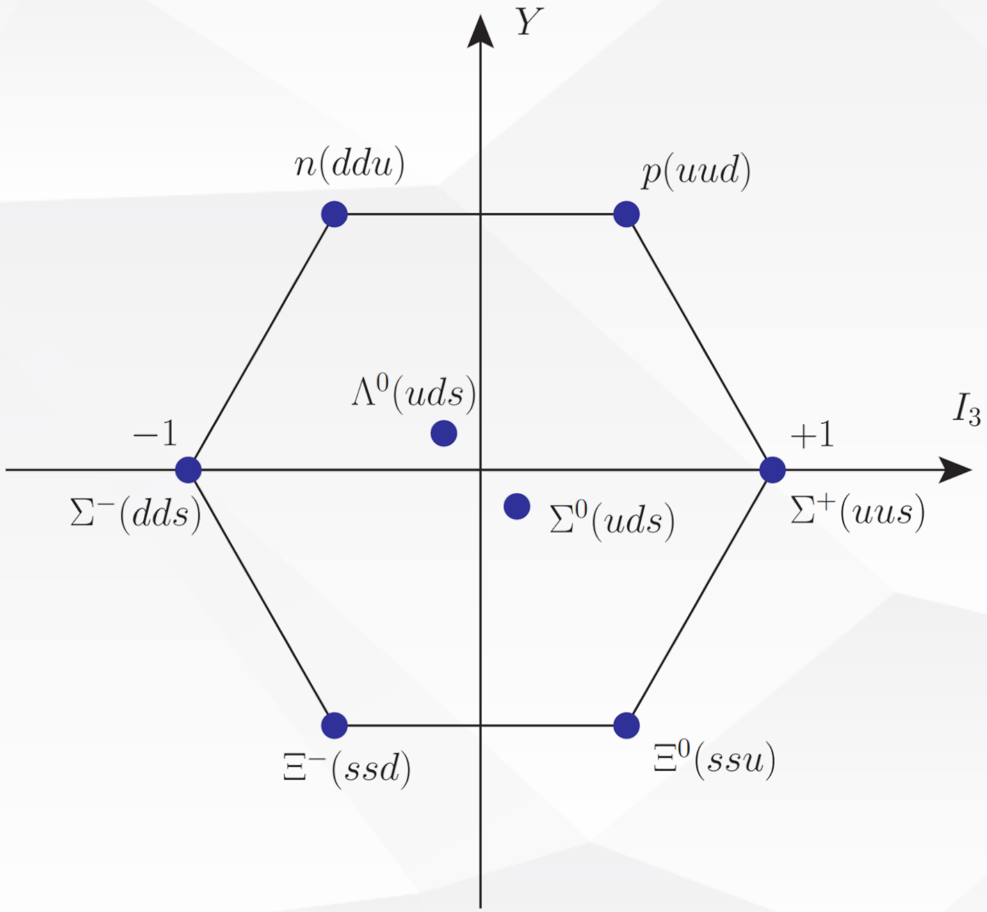
1. Introduction

- It is highly indispensable to develop a method to calculate the full shape of baryon LCDAs from the first principle of QCD.

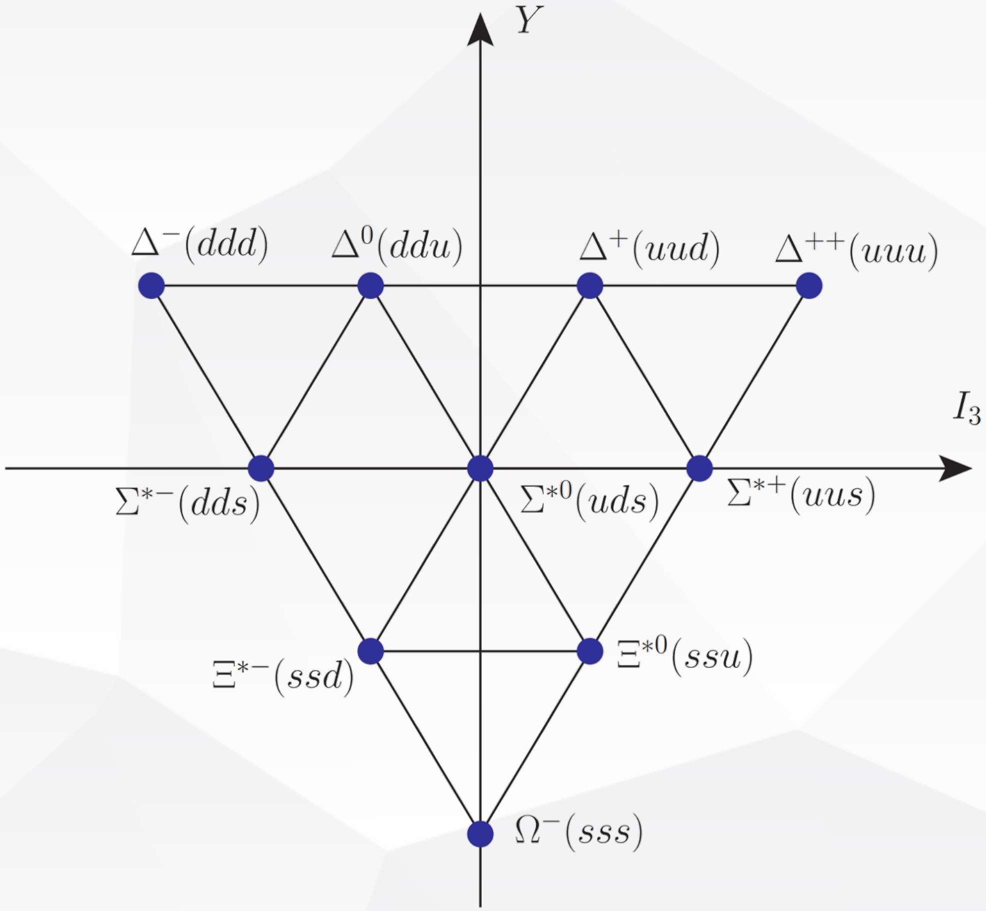
- LCDAs are defined as the correlation functions of light-cone operators inside a hadron, these quantities can not be directly evaluated on the lattice.

- A very inspiring approach was proposed to circumvent this problem and is now formulated as the large-momentum effective theory (LaMET).

1. Introduction

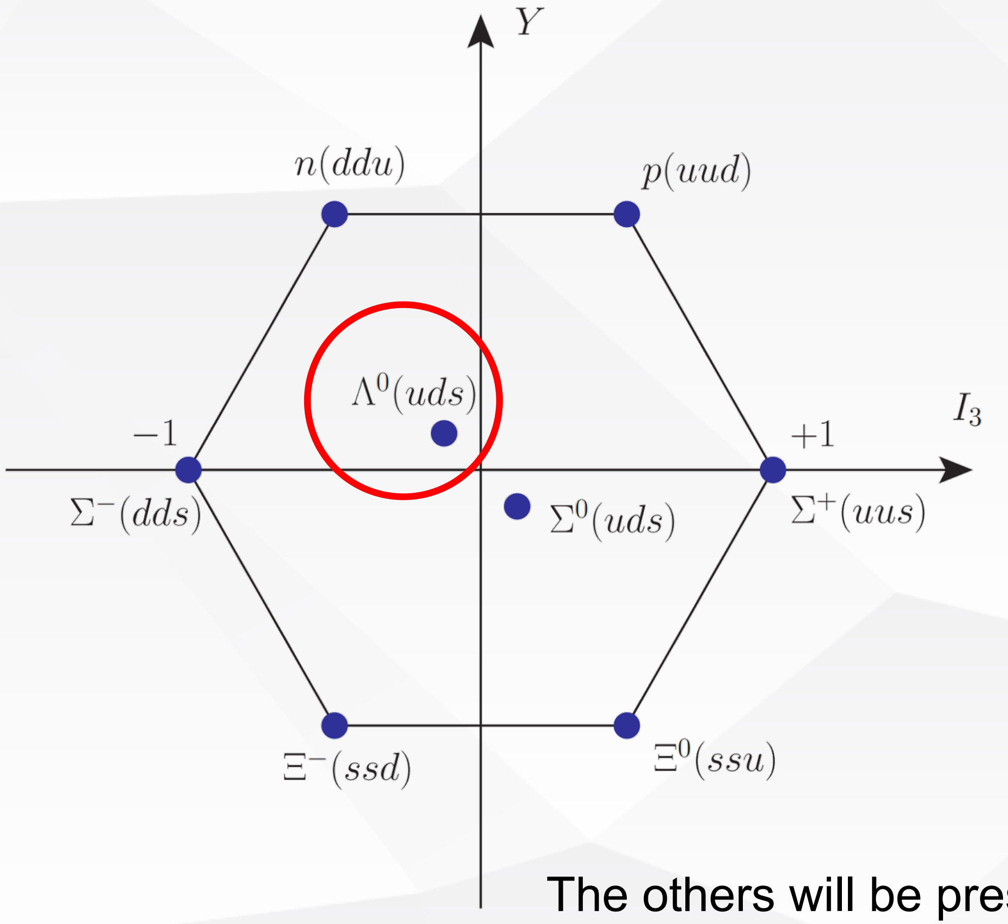


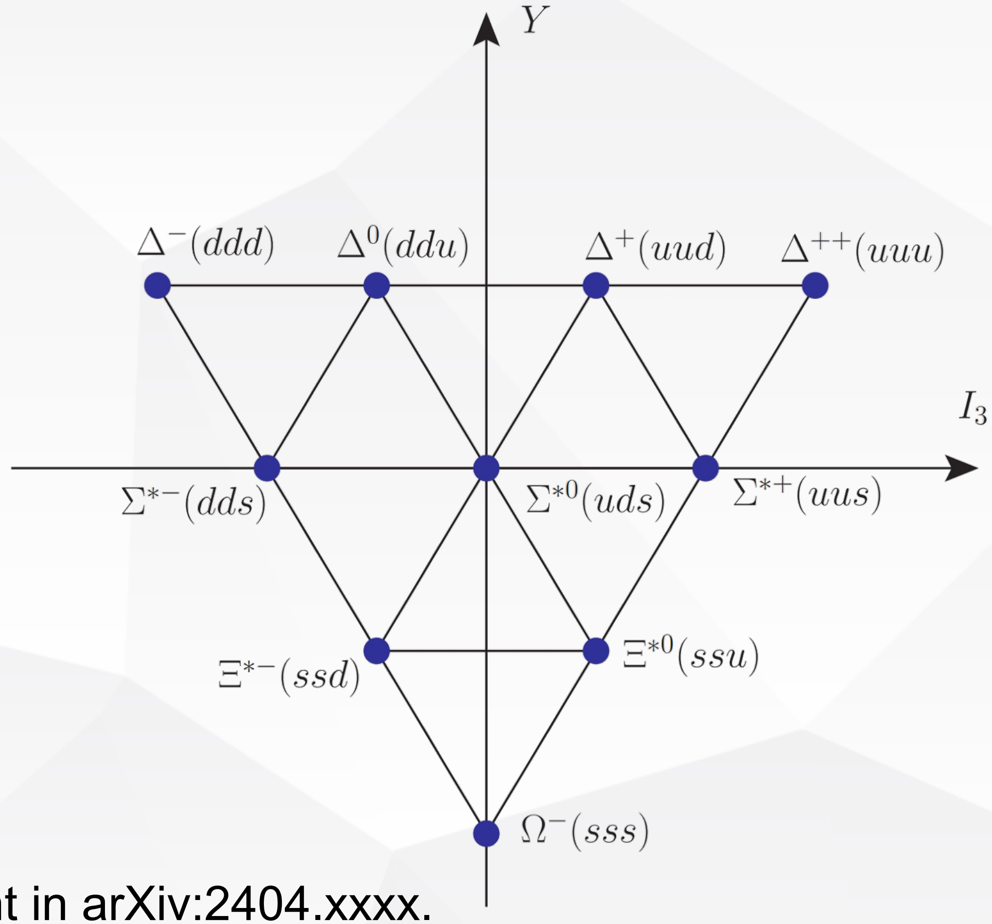
octet baryon



decuplet baryon

1. Introduction





The others will be present in arXiv:2404.xxxx.

octet baryon

decuplet baryon

1. Introduction

At leading twist, the LCDAs of an octet baryon can be decomposed into three terms as

$$
\begin{aligned}
&\langle 0\,|f_\alpha(z_1 n)\, g_\beta(z_2 n)\, h_\gamma(z_3 n)|\, B(P_B,\lambda)\rangle \\
&= \frac{1}{4} f_V \left[(\not{P}_B C)_{\alpha\beta} (\gamma_5 u_B)_\gamma \underline{V^B(z_i n\cdot P_B)} + (\not{P}_B \gamma_5 C)_{\alpha\beta} (u_B)_\gamma \underline{A^B(z_i n\cdot P_B)} \right] \\
&+ \frac{1}{4} f_T (i\sigma_{\mu\nu} P_B^\nu C)_{\alpha\beta} (\gamma^\mu \gamma_5 u_B)_\gamma \underline{T^B(z_i n\cdot P_B)},
\end{aligned}
$$

where $C \equiv i\gamma^2\gamma^0$ is the charge conjugation matrix, u_B stands for the spinor for the baryon and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. $f_{V/A/T}$ is the corresponding decay constant for each LCDA. For proton and neutron, $f_T = f_V$ due to the isospin symmetry.

$$
\begin{aligned}
\langle 0\,|f^T(z_1 n)\,(C\not{n}) g(z_2 n)\, h(z_3 n)|\, B\rangle &= -f_V V^B(z_i n\cdot P_B) P_B^+ \gamma_5 u_B, \\
\langle 0\,|f^T(z_1 n)\,(C\gamma_5\not{n}) g(z_2 n)\, h(z_3 n)|\, B\rangle &= f_V A^B(z_i n\cdot P_B) P_B^+ u_B, \\
\langle 0\,|f^T(z_1 n)\,(iC\sigma_{\mu\nu} n^\nu) g(z_2 n)\, \gamma^\mu h(z_3 n)|\, B\rangle &= 2 f_T T^B(z_i n\cdot P_B) P_B^+ \gamma_5 u_B,
\end{aligned}
$$

SJTU

1. Introduction

To fully exploit the benefits of SU(3) flavor symmetry it proves convenient to define the following set of DAs:

$$\Phi_{\pm}^{B\neq\Lambda}(x_{123}) = \frac{1}{2}\Big([V-A]^B(x_{123}) \pm [V-A]^B(x_{321})\Big),$$

$$\Pi^{B\neq\Lambda}(x_{123}) = T^B(x_{132}),$$

$$\Phi_{+}^{\Lambda}(x_{123}) = \sqrt{\frac{1}{6}}\Big([V-A]^\Lambda(x_{123}) + [V-A]^\Lambda(x_{321})\Big),$$

$$\Phi_{-}^{\Lambda}(x_{123}) = -\sqrt{\frac{3}{2}}\Big([V-A]^\Lambda(x_{123}) - [V-A]^\Lambda(x_{321})\Big),$$

$$\Pi^{\Lambda}(x_{123}) = \sqrt{6}\,T^\Lambda(x_{132}),$$

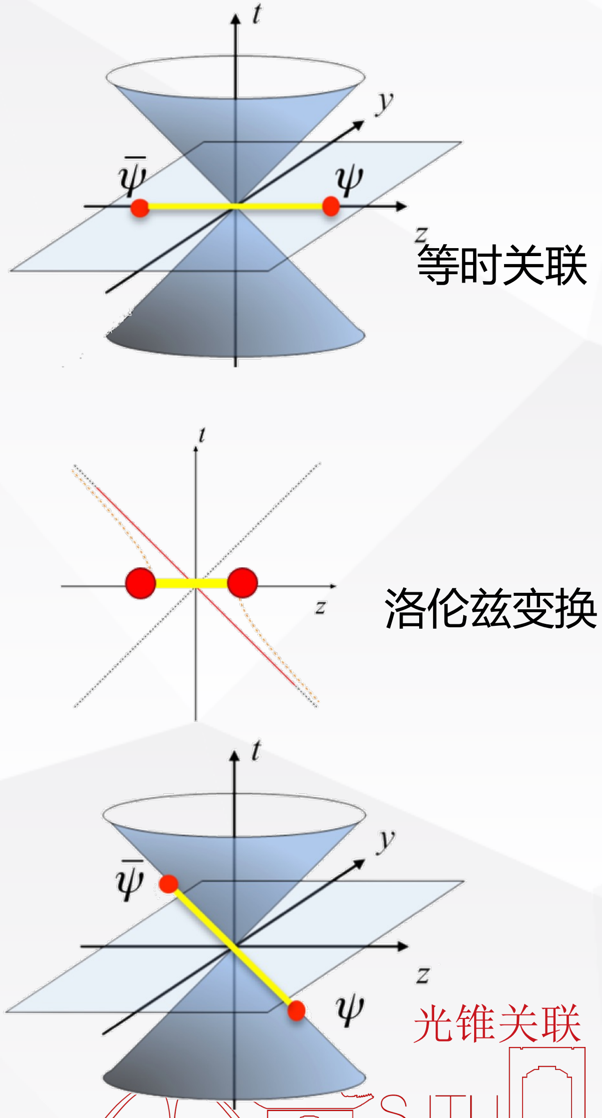
asymptotic form:

$$\begin{aligned}
\Phi_+^B &= 120x_1x_2x_3\left(\varphi_{00}^B\mathcal{P}_{00} + \varphi_{11}^B\mathcal{P}_{11} + \ldots\right),\\
\Phi_-^B &= 120x_1x_2x_3\left(\varphi_{10}^B\mathcal{P}_{10} + \ldots\right),\\
\Pi^{B\neq\Lambda} &= 120x_1x_2x_3\left(\pi_{00}^B\mathcal{P}_{00} + \pi_{11}^B\mathcal{P}_{11} + \ldots\right),\\
\Pi^{\Lambda} &= 120x_1x_2x_3\left(\pi_{10}^\Lambda\mathcal{P}_{10} + \ldots\right).
\end{aligned}$$

1. Introduction: large-momentum effective theory (LaMET)

- The LaMET provides a method to calculate the hadron LCDA from lattice QCD from first principles.

- The LCDA can be extracted by matching the quasi-DA to light cone direction by perturbation.



2. Baryon LCDA and quasi-DA

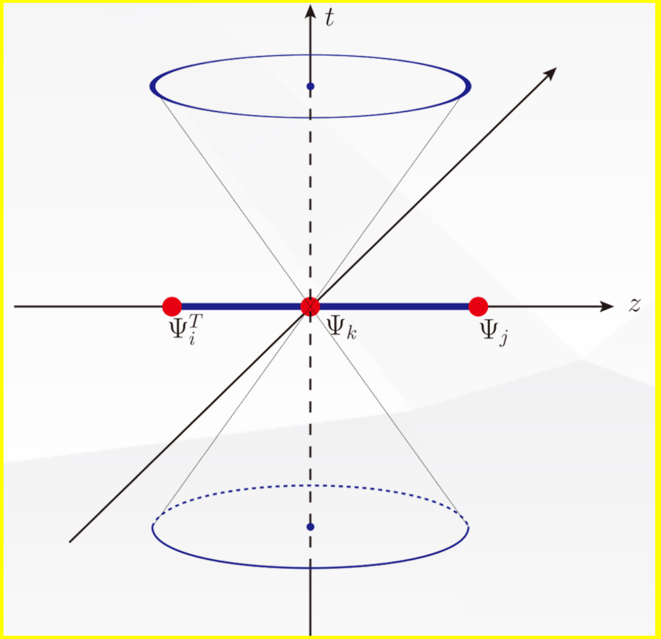
$$\boxed{\Phi(x_1,x_2)} f_\Lambda u_\Lambda(p) = \int \frac{dt_1 p^+}{2\pi} \int \frac{dt_2 p^+}{2\pi} e^{ix_1 p^+ t_1 + ix_2 p^+ t_2} \epsilon_{ijk} \langle 0 | U_i^T(t_1 n) \Gamma D_j(t_2 n) S_k(0) | \Lambda \rangle$$

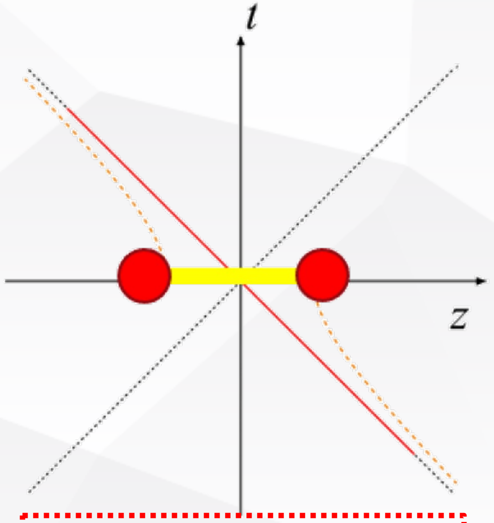
where $\Gamma = C\gamma^5 \not{n}$

$$\boxed{\tilde{\Phi}(x_1,x_2)} \tilde{f}_\Lambda u_\Lambda(p) = \int \frac{dt_1 p^z}{2\pi} \int \frac{dt_2 p^z}{2\pi} e^{ix_1 p^z t_1 + ix_1 p^z t_2} \epsilon_{ijk} \langle 0 | U_i^T(t_1 n_z) \tilde{\Gamma} D_j(t_2 n_z) S_k(0) | \Lambda \rangle$$

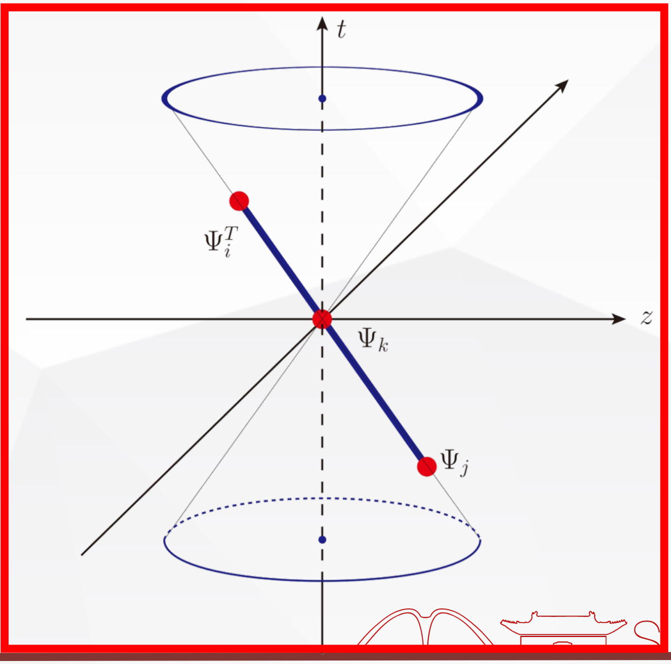
$$Q_i(x) = \mathcal{W}_{ii'}(\infty, x) q_{i'}(x)$$

$$\tilde{\Gamma} = C\gamma^5 \not{n}_\lambda \ (\lambda = t \text{ or } z)$$





Matching



2. Baryon LCDA and quasi-DA: Matching

$$\tilde{\Phi}(x_1,x_2,\mu) = \int dy_1 dy_2 \boxed{\mathcal{C}(x_1,x_2,y_1,y_2,\mu)} \Phi(y_1,y_2,\mu) + \mathcal{O}\Big(\frac{1}{x_1 p^z}, \frac{1}{x_2 p^z}, \frac{1}{(1-x_1-x_2)p^z}\Big)$$

Matching kernel is insensitive to the hadrons, in the calculation of TMDWFs one can replace the hadron by the partonic state.

$$|\Lambda\rangle \to \frac{\epsilon_{abc}}{6} |u_a(k_1) d_b(k_2) s_c(k_3)\rangle$$

the quark state is chosen to have the same J^{PC} with the Λ.

2. Baryon LCDA and quasi-DA: partonic LCDA

$$\boxed{\phi(x_1, x_2, \mu)} S = \int \frac{dt_1 p^+}{2\pi} \int \frac{dt_2 p^+}{2\pi} e^{ix_1 p^+ t_1 + ix_1 p^+ t_2} \frac{\epsilon_{ijk}\epsilon_{abc}}{6} \langle 0| U_i^T(t_1 n) \Gamma D_j(t_2 n) S_k(0) | u_a(k_1) d_b(k_2) s_c(k_3)\rangle$$

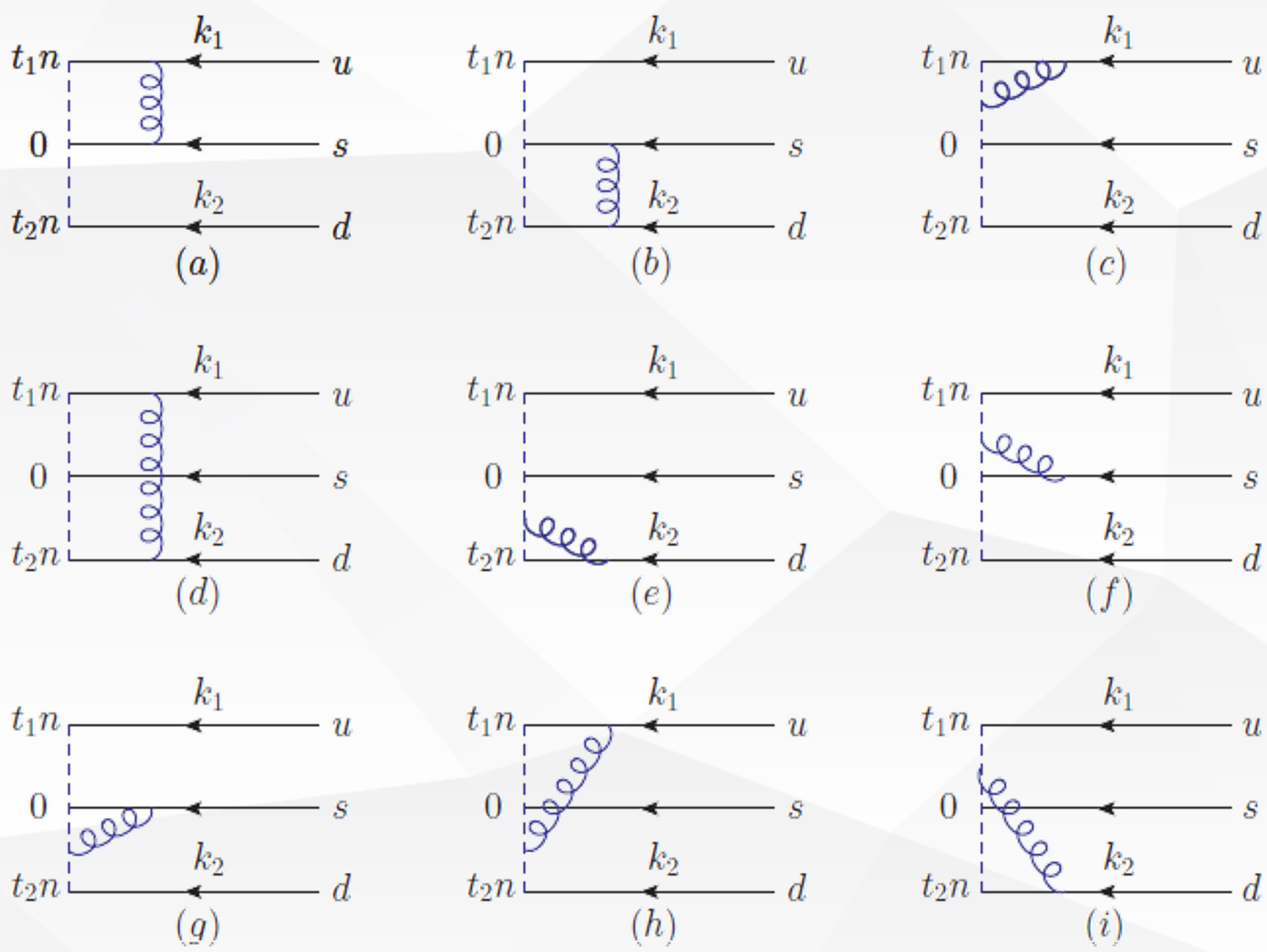
$$\boxed{\tilde{\phi}(x_1, x_2, \mu)} \tilde{S} = \int \frac{dt_1 p^z}{2\pi} \int \frac{dt_2 p^z}{2\pi} e^{ix_1 p^z t_1 + ix_1 p^z t_2} \frac{\epsilon_{ijk}\epsilon_{abc}}{6} \langle 0| (U_i)^T(t_1 n_z) \tilde{\Gamma} D_j(t_2 n_z) S_k(0) | u_a(k_1) d_b(k_2) s_c(k_3)\rangle$$

Normalization factor:

$$S = \frac{\epsilon_{ijk}\epsilon_{abc}}{6} \langle 0| (U_i)^T(0) \Gamma D_j(0) S_k(0) | u_a(k_1) d_b(k_2) s_c(k_3)\rangle$$

$$\tilde{S} = \frac{\epsilon_{ijk}\epsilon_{abc}}{6} \langle 0| (U_i)^T(0) \tilde{\Gamma} D_j(0) S_k(0) | u_a(k_1) d_b(k_2) s_c(k_3)\rangle$$

2. LCDA and Quasi-DA



$$
\begin{aligned}
\Phi(x_1, x_2) = \; & \delta(x_1 - x_{1,0})\delta(x_2 - x_{2,0}) - \frac{\alpha_s C_F}{8\pi}\frac{1}{\epsilon_{\rm IR}} \\
& \times \Bigg\{ \frac{\delta(x_1 - x_{1,0})\theta(x_1)\theta(\bar{x}_1)\theta(x_2)\theta(\bar{x}_2)}{\bar{x}_1\left(x_2 - x_{2,0}\right)} \\
& \times \Bigg[\frac{x_3\left(x_2 - x_{2,0} + 2\bar{x}_1\right)}{x_{3,0}}\theta\left(x_2 - x_{2,0}\right) \\
& + \frac{x_2\left(x_2 - x_{2,0} - 2\bar{x}_1\right)}{x_{2,0}}\theta\left(x_{2,0} - x_2\right) \Bigg] \\
& + \frac{\delta(x_3 - x_{3,0})2x_1\theta(x_1)\theta(\bar{x}_1)\theta(x_2)\theta(\bar{x}_2)}{x_{1,0}} \\
& \times \left[\frac{\theta\left(x_2 - x_{2,0}\right)}{x_1 + x_2} - \frac{\theta\left(x_{1,0} - x_1\right)}{x_1 - x_{1,0}} \right] \\
& + \{x_1 \leftrightarrow x_2, x_{1,0} \leftrightarrow x_{2,0}\} \Bigg\}_{\oplus}
\end{aligned}
$$

Diagram of DA. Self-energies of external lines are not shown.

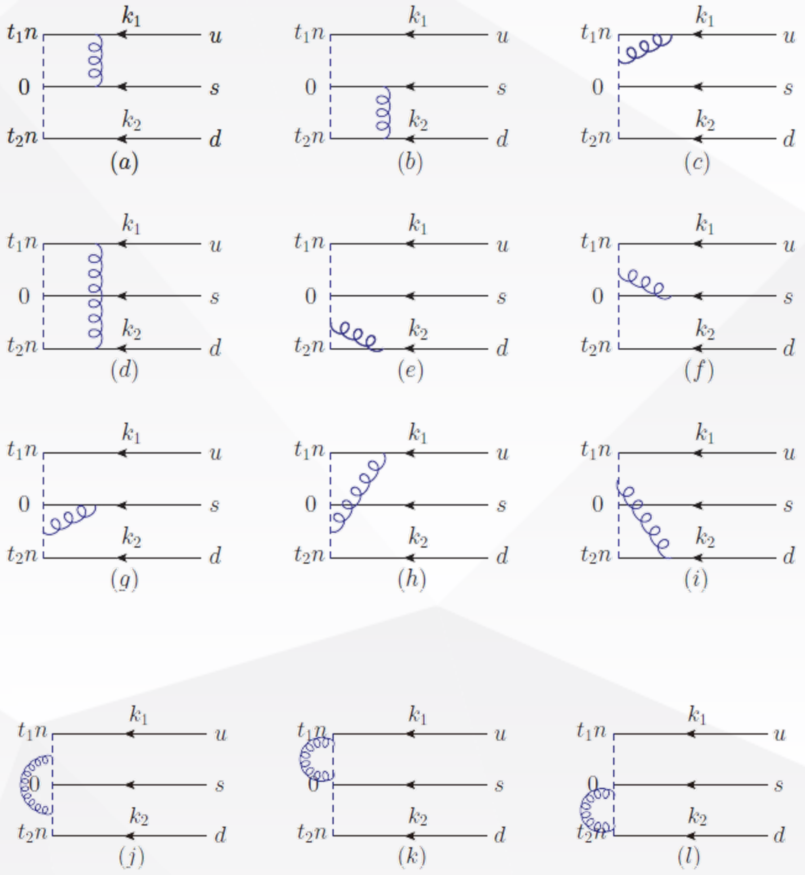
$$\frac{d\Phi(x_1, x_2, \mu)}{d\ln\mu} = \frac{\alpha_s C_F}{4\pi} \int dy_1 \int dy_2 V \Phi(y_1, y_2, \mu).$$

At the one-loop, the evolution kernel V is

$$V = f(x_1, x_2, y_1, y_2).$$

2. LCDA and Quasi-DA

$$\tilde{\Phi}(x_1, x_2, \mu) = \delta(x_1 - x_{1,0})\delta(x_2 - x_{2,0}) + \frac{\alpha_s C_F}{8\pi} \times \Big\{ \Big[g_2 \delta(x_2 - x_{2,0}) + g_3 \delta(x_3 - x_{3,0}) + \{x_2 \leftrightarrow x_1, x_{2,0} \leftrightarrow x_{1,0}\} \Big] \Big\}_{\oplus}$$



2. LCDA and Quasi DA

$$g_2=\begin{cases}\dfrac{(x_{1,0}+x_1)(x_{3,0}+x_3)\ln\left(-\frac{x_{3,0}-x_3}{x_3}\right)}{x_{1,0}x_{3,0}(x_{3,0}-x_3)}-\dfrac{x_1(2x_{1,0}+x_{3,0}+x_3)\ln\left(-\frac{x_1}{x_3}\right)}{x_{1,0}(x_{3,0}-x_3)(x_{1,0}+x_{3,0})}, & x_1<0\\[2ex] \dfrac{x_1(-x_{1,0}+2x_{2,0}+x_1-2)}{(x_1-x_{1,0})x_{1,0}(x_{2,0}-1)\epsilon_{\rm IR}}+\dfrac{2x_1\ln\left(\frac{4x_1(x_3-x_{3,0})(p^z)^2}{\mu^2}\right)}{x_{1,0}(x_3-x_{3,0})}+\dfrac{x_1\ln\left(\frac{4x_1x_3(p^z)^2}{\mu^2}\right)}{x_{1,0}(x_{1,0}+x_{3,0})} & \\ +\dfrac{x_1(-3x_{1,0}-2x_{3,0}+x_1)}{x_{1,0}(x_3-x_{3,0})(x_{1,0}+x_{3,0})}-\dfrac{\left((x_3-x_{3,0})^2-2x_3x_{1,0}\right)\ln\left(\frac{x_3-x_{3,0}}{x_3}\right)}{x_{1,0}(x_3-x_{3,0})x_{3,0}}, & 0<x_1<x_{1,0}\\[2ex] \dfrac{x_3(-x_{1,0}-2x_{2,0}+x_1+2)}{(x_1-x_{1,0})(x_{2,0}-1)x_{3,0}\epsilon_{\rm IR}}+\dfrac{2x_3\ln\left(\frac{4x_3(x_1-x_{1,0})(p^z)^2}{\mu^2}\right)}{(x_1-x_{1,0})x_{3,0}}+\dfrac{x_3\ln\left(\frac{4x_1x_3(p^z)^2}{\mu^2}\right)}{x_{3,0}(x_{1,0}+x_{3,0})} & \\ +\dfrac{x_3(-2x_{1,0}-3x_{3,0}+x_3)}{(x_1-x_{1,0})x_{3,0}(x_{1,0}+x_{3,0})}-\dfrac{\left((x_1-x_{1,0})^2-2x_1x_{3,0}\right)\ln\left(\frac{x_1-x_{1,0}}{x_1}\right)}{(x_1-x_{1,0})x_{1,0}x_{3,0}}, & x_{1,0}<x_1<x_{1,0}+x_{3,0}\\[2ex] \dfrac{(x_{1,0}+x_1)(x_{3,0}+x_3)\ln\left(-\frac{x_{3,0}-x_3}{x_3}\right)}{x_{1,0}x_{3,0}(x_{3,0}-x_3)}-\dfrac{x_1(2x_{1,0}+x_{3,0}+x_3)\ln\left(-\frac{x_1}{x_3}\right)}{x_{1,0}(x_{3,0}-x_3)(x_{1,0}+x_{3,0})}, & x_1>x_{1,0}+x_{3,0}\end{cases}$$

2. LCDA and Quasi DA

$$
g_3 = \begin{cases}
\dfrac{(x_{1,0}x_{2,0}+x_1x_2)\ln\left(\frac{x_2-x_{2,0}}{x_2}\right)}{x_{1,0}(x_2-x_{2,0})x_{2,0}} - \dfrac{x_1(x_{1,0}+x_2)\ln\left(-\frac{x_1}{x_2}\right)}{x_{1,0}(x_2-x_{2,0})(x_{1,0}+x_{2,0})}, & x_1<0 \\
\dfrac{2x_1(x_{1,0}+x_2)}{(x_1+x_2)(x_1-x_{1,0})x_{1,0}\epsilon_{\mathrm{IR}}} + \dfrac{x_1\ln\left(\frac{4x_1(x_2-x_{2,0})(p^z)^2}{\mu^2}\right)}{x_{1,0}(x_2-x_{2,0})} + \dfrac{x_1\ln\left(\frac{4x_1x_2(p^z)^2}{\mu^2}\right)}{x_{1,0}(x_{1,0}+x_{2,0})} & \\
+\dfrac{1}{x_1-x_{1,0}} + \dfrac{2x_1+x_2}{x_{1,0}(x_{1,0}+x_{2,0})} + \dfrac{\left(x_{1,0}(x_{2,0}+x_1)-x_1^2\right)\ln\left(\frac{x_2-x_{2,0}}{x_2}\right)}{x_{1,0}(x_2-x_{2,0})x_{2,0}}, & 0<x_1<x_{1,0} \\
\dfrac{2x_2(x_{2,0}+x_1)}{(x_1+x_2)(x_2-x_{2,0})x_{2,0}\epsilon_{\mathrm{IR}}} + \dfrac{x_2\ln\left(\frac{4x_2(x_1-x_{1,0})(p^z)^2}{\mu^2}\right)}{(x_1-x_{1,0})x_{2,0}} + \dfrac{x_2\ln\left(\frac{4x_1x_2(p^z)^2}{\mu^2}\right)}{x_{2,0}(x_{1,0}+x_{2,0})} & \\
+\dfrac{1}{x_2-x_{2,0}} + \dfrac{x_1+2x_2}{x_{2,0}(x_{1,0}+x_{2,0})} + \dfrac{\left((x_{1,0}+x_2)x_{2,0}-x_2^2\right)\ln\left(\frac{x_1-x_{1,0}}{x_1}\right)}{(x_1-x_{1,0})x_{1,0}x_{2,0}}, & x_{1,0}<x_1<x_{1,0}+x_{2,0} \\
\dfrac{(x_{1,0}x_{2,0}+x_1x_2)\ln\left(\frac{x_1-x_{1,0}}{x_1}\right)}{(x_1-x_{1,0})x_{1,0}x_{2,0}} - \dfrac{x_2(x_{2,0}+x_1)\ln\left(-\frac{x_2}{x_1}\right)}{(x_1-x_{1,0})x_{2,0}(x_{1,0}+x_{2,0})}, & x_1>x_{1,0}+x_{2,0}.
\end{cases}
$$

3. Macting kernel

$$\tilde{\Phi}(x_1, x_2, \mu) = \int dy_1 dy_2 \mathcal{C}(x_1, x_2, y_1, y_2, \mu)\Phi(y_1, y_2, \mu) + \mathcal{O}\left(\frac{1}{x_1 p^z}, \frac{1}{x_2 p^z}, \frac{1}{(1-x_1-x_2)p^z}\right)$$

$$\begin{aligned} \mathcal{C}(x_1, x_2, y_1, y_2, \mu) &= \delta(x_1 - y_1)\delta(x_2 - y_2) + \frac{\alpha_s C_F}{8\pi} \\ &\times \Big[C_2(x_1, x_2, y_1, y_2)\delta(x_2 - y_2) \\ &+ C_3(x_1, x_2, y_1, y_2)\delta(x_3 - y_3) \\ &+ \{x_1 \leftrightarrow x_2, y_1 \leftrightarrow y_2\} \Big], \end{aligned}$$

3. Macting kernel

$$C_2(x_1,x_2,y_1,y_2)=$$

$$\begin{cases}
\dfrac{(x_1+y_1)(x_3+y_3)\ln\frac{y_1-x_1}{-x_1}}{y_1(y_1-x_1)y_3}-\dfrac{x_3(x_1+y_1+2y_3)\ln\frac{x_3}{-x_1}}{(y_1-x_1)y_3(y_1+y_3)}, & x_1<0\\
\dfrac{(x_1-3y_1-2y_3)x_1}{y_1(x_3-y_3)(y_1+y_3)}-\dfrac{\left[(x_3-y_3)^2-2x_3y_1\right]\ln\frac{x_3-y_3}{x_3}}{y_1(x_3-y_3)y_3}+\dfrac{2x_1\ln\frac{4x_1(x_3-y_3)p_z^2}{\mu^2}}{y_1(x_3-y_3)}+\dfrac{x_1\ln\frac{4x_1x_3p_z^2}{\mu^2}}{y_1(y_1+y_3)}, & 0<x_1<y_1\\
\dfrac{(x_3-2y_1-3y_3)x_3}{y_3(x_1-y_1)(y_1+y_3)}-\dfrac{\left[(x_1-y_1)^2-2x_1y_3\right]\ln\frac{x_1-y_1}{x_1}}{(x_1-y_1)y_1y_3}+\dfrac{2x_3\ln\frac{4x_3(x_1-y_1)p_z^2}{\mu^2}}{(x_1-y_1)y_3}+\dfrac{x_3\ln\frac{4x_1x_3p_z^2}{\mu^2}}{y_3(y_1+y_3)}, & y_1<x_1<y_1+y_3\\
\dfrac{(x_1+y_1)(x_3+y_3)\ln\frac{y_3-x_3}{-x_3}}{y_1y_3(y_3-x_3)}-\dfrac{x_1(x_3+2y_1+y_3)\ln\frac{x_1}{-x_3}}{y_1(y_3-x_3)(y_1+y_3)}, & x_1>y_1+y_3
\end{cases}$$

$$C_3(x_1,x_2,y_1,y_2)=$$

$$\begin{cases}
\dfrac{(x_1x_2+y_1y_2)\ln\frac{x_2-y_2}{x_2}}{y_1(x_2-y_2)y_2}-\dfrac{x_1(x_2+y_1)\ln\frac{-x_1}{x_2}}{y_1(x_2-y_2)(y_1+y_2)}, & x_1<0\\
\dfrac{1}{x_1-y_1}+\dfrac{2x_1+x_2}{y_1(y_1+y_2)}+\dfrac{\left[(x_1+y_2)y_1-x_1{}^2\right]\ln\frac{x_2-y_2}{x_2}}{y_1(x_2-y_2)y_2}+\dfrac{x_1\ln\frac{4x_1(x_2-y_2)p_z^2}{\mu^2}}{y_1(x_2-y_2)}+\dfrac{x_1\ln\frac{4x_1x_2p_z^2}{\mu^2}}{y_1(y_1+y_2)}, & 0<x_1<y_1\\
\dfrac{1}{x_2-y_2}+\dfrac{x_1+2x_2}{y_2(y_1+y_2)}+\dfrac{\left[(x_2+y_1)y_2-x_2{}^2\right]\ln\frac{x_1-y_1}{x_1}}{(x_1-y_1)y_1y_2}+\dfrac{x_2\ln\frac{4x_2(x_1-y_1)p_z^2}{\mu^2}}{(x_1-y_1)y_2}+\dfrac{x_2\ln\frac{4x_1x_2p_z^2}{\mu^2}}{y_2(y_1+y_2)}, & y_1<x_1<y_1+y_2\\
\dfrac{(x_1x_2+y_1y_2)\ln\frac{x_1-y_1}{x_1}}{y_1(x_1-y_1)y_2}-\dfrac{x_2(x_1+y_2)\ln\frac{-x_2}{x_1}}{y_2(x_1-y_1)(y_1+y_2)}, & x_1>y_1+y_2.
\end{cases}$$

3. Renornalization: RI/MOM

in the space-like $p^2 = -\rho(p^z)^2 < 0$ kinematics

$$(a_1 + a_2 \not{p}_t \not{p}_z + a_3 \not{p}_\perp \not{p}_z + a_4 \not{p}_\perp \not{p}_t)\tilde{S}$$

In the on-shell limit, the third and the last term a_3, a_4 disappear after integrating out the momentum q, and the product $\not{p}_t \not{p}_z$ goes to a unit matrix. Therefore, the summation $a_1 + a_2$ captures all terms that lead to UV divergences in the on-shell limit.

$$\tilde{\phi}(x_1, x_2, \mu)_{\mathrm{RI/MOM}} = \frac{\tilde{\phi}(x_1, x_2, \mu)}{\tilde{\phi}(x_1, x_2, \mu)_{\mathrm{OF}}}$$

3. Renornalization: RI/MOM

$$
\begin{aligned}
\tilde{\phi}(x_1, x_2, \mu)_{\text{OF}} \;=\; & \delta(x_1 - x_{1,0})\delta(x_2 - x_{2,0}) + \frac{\alpha_s C_F}{8\pi} \\
& \times \left\{ \left[g_2' \delta(x_2 - x_{2,0}) + g_3' \delta(x_3 - x_{3,0}) \right. \right. \\
& \left. \left. + \{ x_2 \leftrightarrow x_1, x_{2,0} \leftrightarrow x_{1,0} \} \right] \right\}_{\oplus},
\end{aligned}
$$

3. Renornalization: RI/MOM

$$\begin{aligned}
\mathcal{C}^{\mathcal{R}}(x_1, x_2, y_1, y_2, \mu) &= \delta(x_1 - y_1)\delta(x_2 - y_2) + \frac{\alpha_s C_F}{8\pi} \\
&\times \Big[C_2'(x_1, x_2, y_1, y_2)\delta(x_2 - y_2) \\
&+ C_3'(x_1, x_2, y_1, y_2)\delta(x_3 - y_3) \\
&+ \{x_1 \leftrightarrow x_2, y_1 \leftrightarrow y_2\} \Big]_{\oplus}, \qquad (46)
\end{aligned}$$

where $C_2' = C_2 - g_2'|_{x_{1,0}\to y_1, x_{2,0}\to y_2}$ and $C_3' = C_3 - g_3'|_{x_{1,0}\to y_1, x_{2,0}\to y_2}$.

4. Expansion by regions

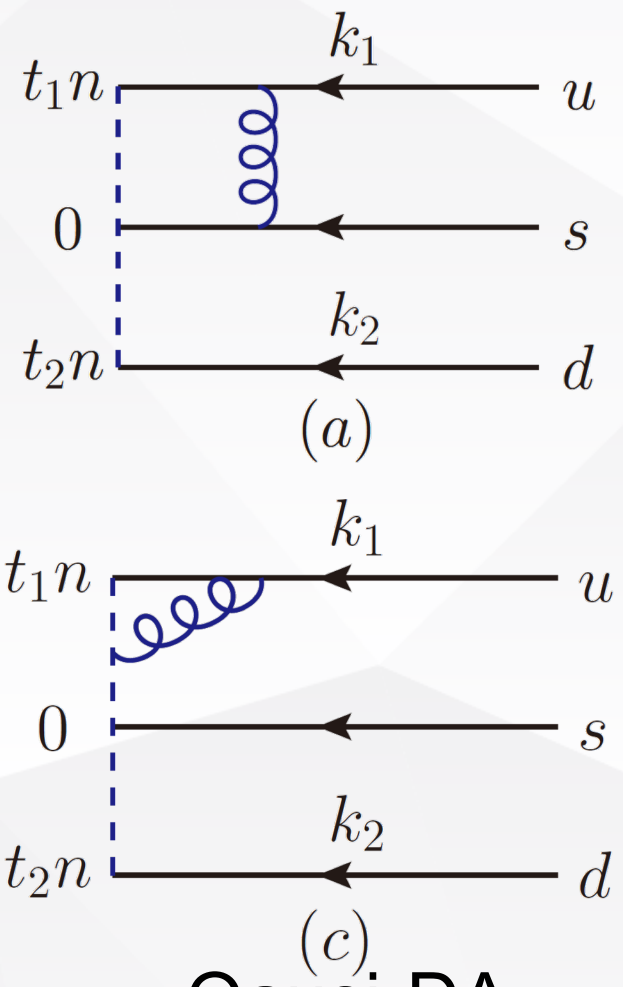
$$q^{\mu} = (q^{+}, q^{-}, q_{\perp})$$

Expansion by regions:

- ✓ **Hard:** $q^{\mu} \sim (Q, Q, Q)$
- ✓ **Collinear:** $q^{\mu} \sim (Q, \Lambda^2/Q, \Lambda)$
- ✓ **Soft:** $q^{\mu} \sim (\Lambda, \Lambda, \Lambda)$

$$\tilde{\phi}^{a}_{(1/0)} = ig^2 \frac{C_F}{2} p^z \delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\delta(x_1 p^z - q^z - k_1{}^z)}{(q+k_1)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{q_\perp^2}{(k_s - q)^2 + i\epsilon}$$

$$\tilde{\phi}^{c}_{(1/0)} = ig^2 C_F \mu_0^{2\epsilon} \int \frac{d^4 q}{(2\pi)^4} \frac{[(\not{k}_1 - \not{q})\gamma^{\mu} u_u(k_1)]^T \tilde{\Gamma} u_d(k_2) u_s(k_s)}{[(k_1 - q)^2 + i\epsilon](q^2 + i\epsilon)(-q^z)} \delta(x_1 p^z - q^z - k_1{}^z)\delta(x_1 p^z + q^z - k_2{}^z)$$



Qausi-DA

4. Expansion by regions

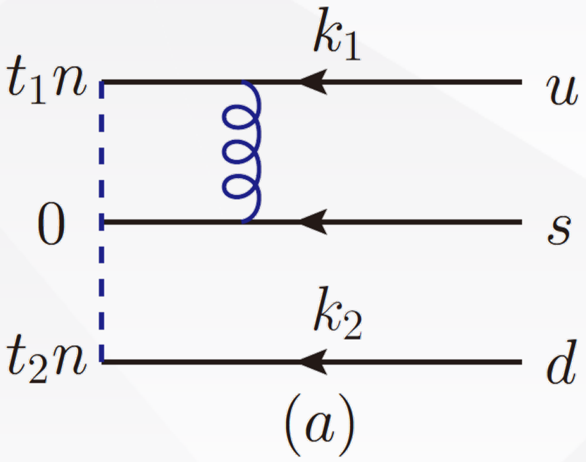
$$q^{\mu} = (q^{+}, q^{-}, q_{\perp})$$

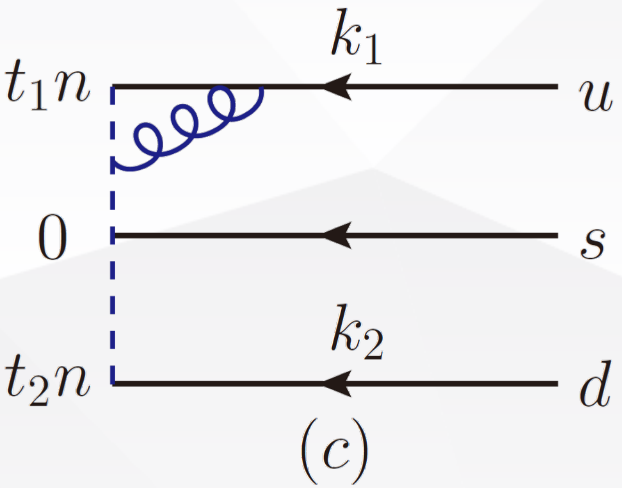
Expansion by regions:

- ✓ **Hard:** $q^{\mu} \sim (Q, Q, Q)$
- ✓ Collinear: $q^{\mu} \sim (Q, \Lambda^2/Q, \Lambda)$
- ✓ Soft: $q^{\mu} \sim (\Lambda, \Lambda, \Lambda)$

$$\tilde{\phi}^{c}_{(1/0)} = ig^2 C_F \mu_0^{2\epsilon} \int \frac{d^4 q}{(2\pi)^4} \frac{[(\not{k}_1 - \not{q})\gamma^{\mu} u_u(k_1)]^T \tilde{\Gamma} u_d(k_2) u_s(k_s)}{[(k_1 - q)^2 + i\epsilon](q^2 + i\epsilon)(-q^z)} \delta(x_1 p^z - q^z - k_1{}^z)\delta(x_1 p^z + q^z - k_2{}^z)$$

Leading Power!





4. Expansion by regions

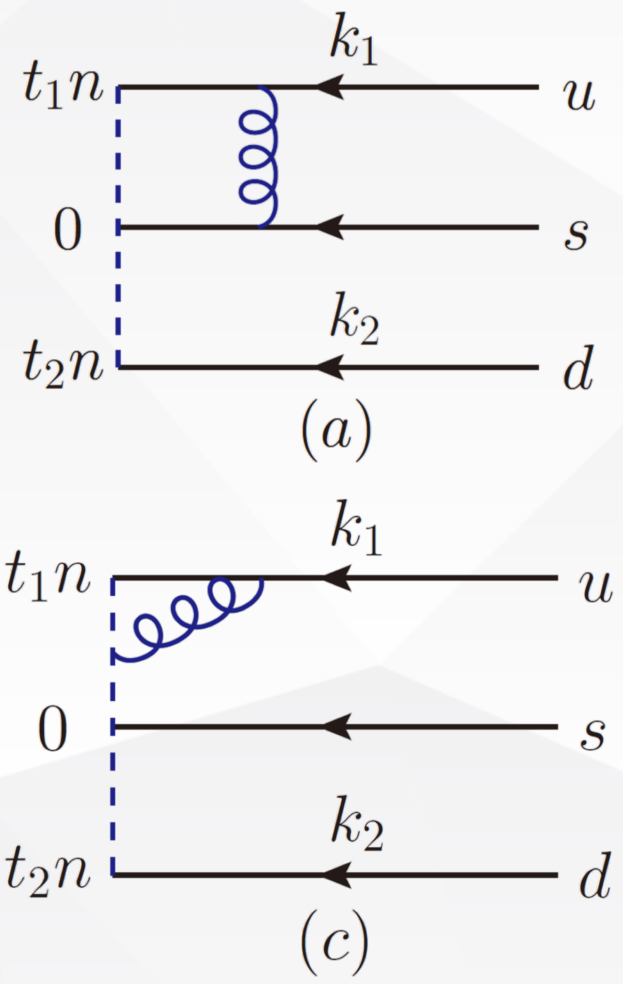
$$q^\mu = (q^+, q^-, q_\perp)$$

Expansion by regions:

- ✓ Hard: $q^\mu \sim (Q, Q, Q)$
- ✓ Collinear: $q^\mu \sim (Q, \Lambda^2/Q, \Lambda)$
- ✓ Soft: $q^\mu \sim (\Lambda, \Lambda, \Lambda)$

$$\tilde{\phi}^a_{(1/0)} = i g^2 \frac{C_F}{2} p^z \delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\delta(x_1 p^z - q^z - k_1{}^z)}{(q+k_1)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{q_\perp^2}{(k_s - q)^2 + i\epsilon}$$

Leading Power!



4. Expansion by regions

$$q^\mu = (q^+, q^-, q_\perp)$$

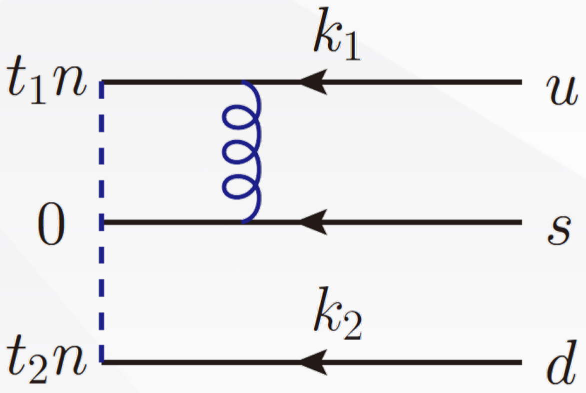
Expansion by regions:

- ✓ Hard: $q^\mu \sim (Q, Q, Q)$
- ✓ Collinear: $q^\mu \sim (Q, \Lambda^2/Q, \Lambda)$
- ✓ Soft: $q^\mu \sim (\Lambda, \Lambda, \Lambda)$

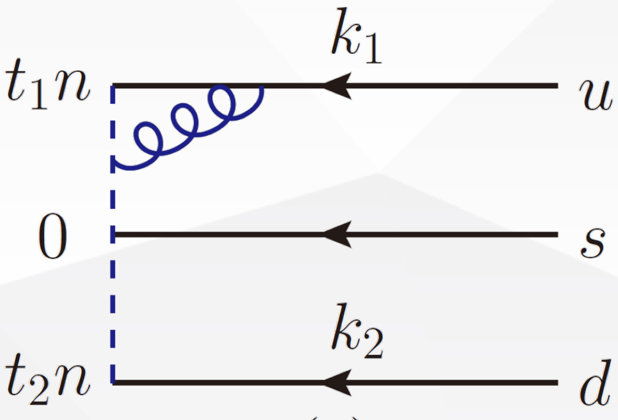
$$\tilde{\phi}^a_{(1/0)} = ig^2 \frac{C_F}{2} p^z \delta(x_2 - x_{2,0}) \int \frac{d^4q}{(2\pi)^4} \frac{\delta(x_1 p^z - q^z - k_1{}^z)}{(q+k_1)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{q_\perp^2}{(k_s - q)^2 + i\epsilon}$$

$$\tilde{\phi}^c_{(1/0)} = ig^2 C_F \mu_0^{2\epsilon} \int \frac{d^4q}{(2\pi)^4} \frac{[(\not{k}_1 - \not{q})\gamma^\mu u_u(k_1)]^T \tilde{\Gamma} u_d(k_2) u_s(k_s)}{[(k_1 - q)^2 + i\epsilon](q^2 + i\epsilon)(-q^z)} \delta(x_1 p^z - q^z - k_1{}^z)\delta(x_1 p^z + q^z - k_2{}^z)$$

Power suprresed!



(a)



(c)

4. Expansion by regions

- The one-loop LCDA and quasi-DA for baryon Λ does not contain the soft contributions.
- The one-loop quasi-DA contain the collinear and hard mode.
- QCD factorization shows that the hard and collinear modes in the quasi-DA can be factorized into a convolution of the hard matching coefficient and the LCDA which only contains collinear modes.

Factorization

Hard Mode + Collinear Mode

Collinear Mode

$$\boxed{\tilde{\Phi}(x_1, x_2, \mu)} = \int dy_1 dy_2 \boxed{\mathcal{C}(x_1, x_2, y_1, y_2, \mu)} \boxed{\Phi(y_1, y_2, \mu)} + \mathcal{O}\left(\frac{1}{x_1 p^z}, \frac{1}{x_2 p^z}, \frac{1}{(1-x_1-x_2)p^z}\right)$$

Hard Mode

5.Summary and outlook

- LCDAs of a light baryon can be obtained through a simulation of a quasi-DA calculable on lattice QCD under the LaMET.

- We have calculated the one-loop perturbative contributions to LCDA and quasi-DA and explicitly have demonstrated the factorization of quasi-DA at the one-loop level.

- Our result provides a first step to obtaining the baryon LCDA from first principle lattice QCD calculations in the future.

Thanks