

粲重子衰变过程中研究奇特强子态

王 恩 郑州大学

第三届强子与重味物理理论与实验联合研讨会

2024年4月5日-8日

Exotic states

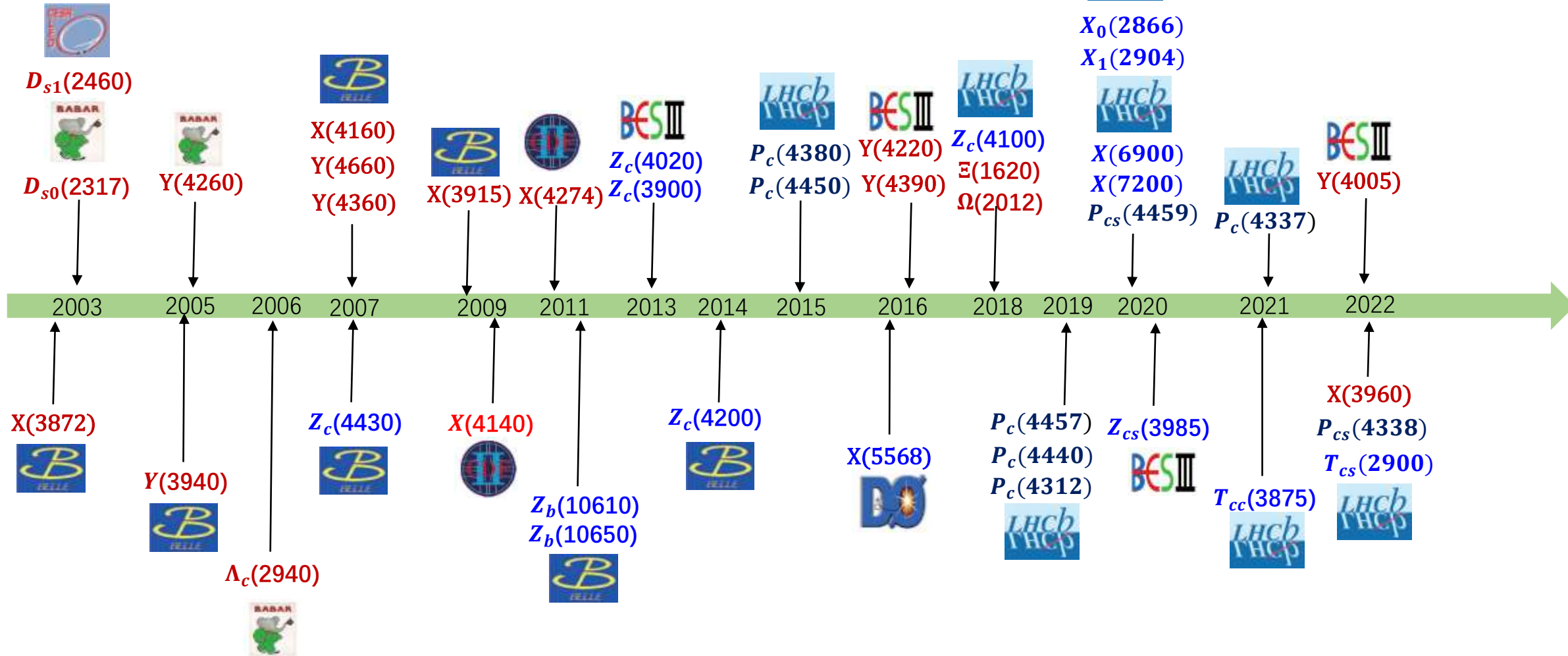
耿立升老师报告

Exotic mesons or baryons

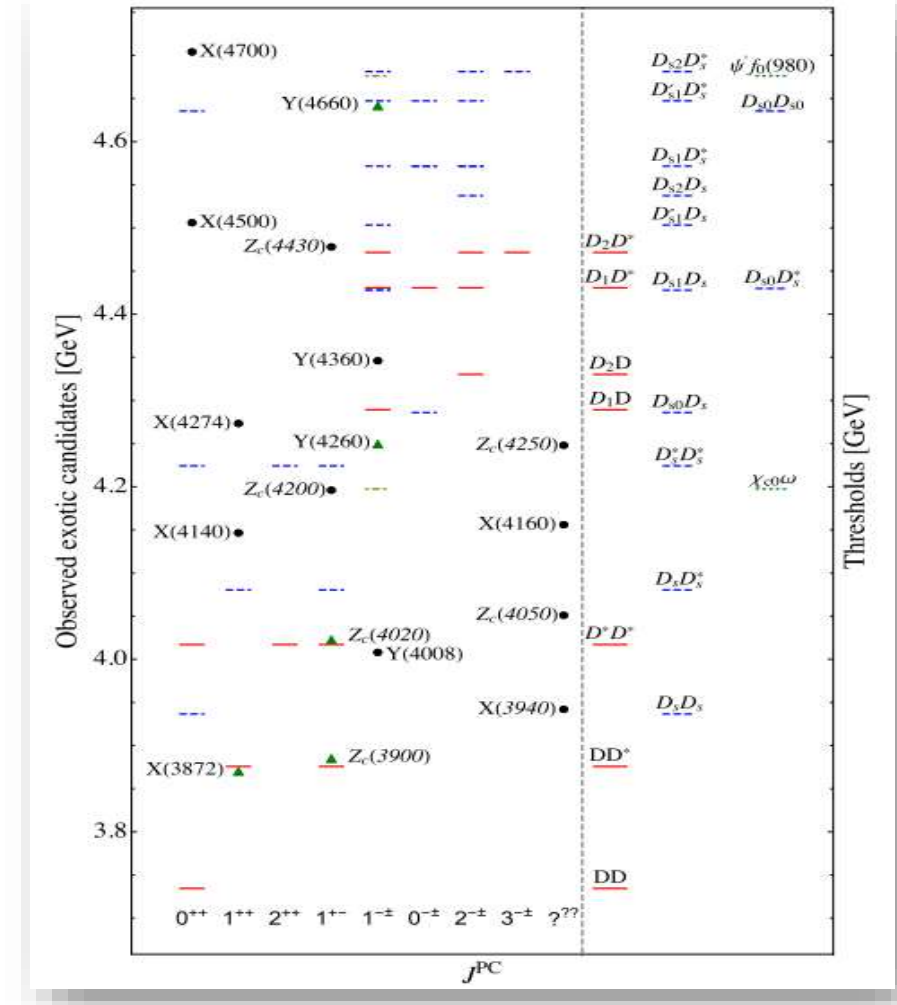
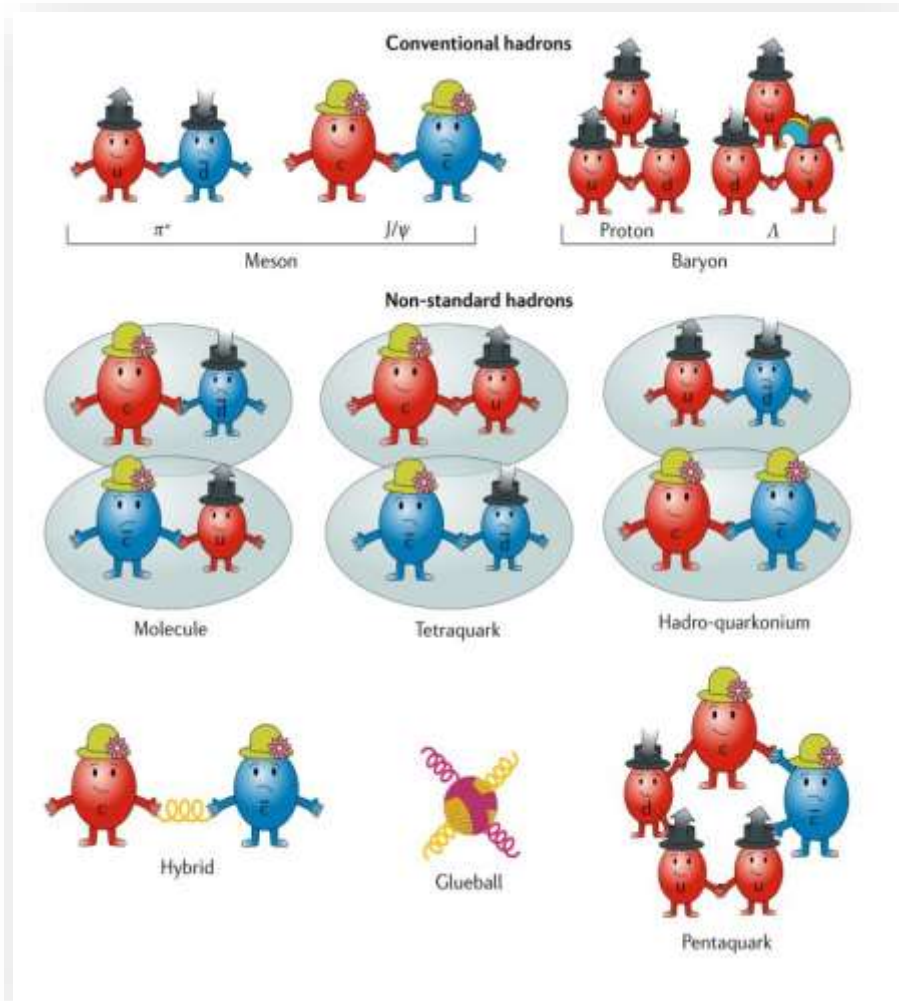
Tetraquark states



Pentaquark states

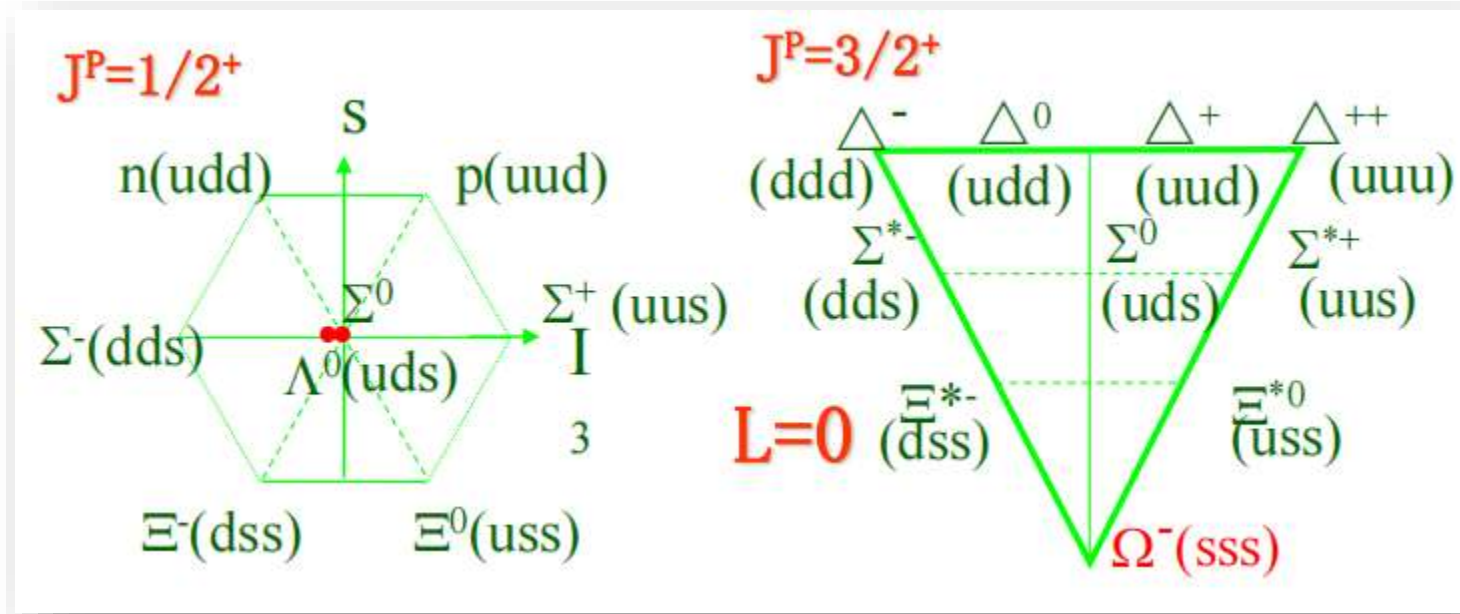


Hadrons



Ground light baryons

- SU(3) flavor multiplets of ground baryons



盖尔曼-大久保质量: $M = a + bY + c \left[I(I + 1) - \frac{1}{4}Y^2 \right]$

质量公式预言 $m_\Omega = 1670$ MeV
实验: $m_\Omega = 1672.45 \pm 0.29$ MeV



Low-lying baryons with $J^P=1/2^-$

$1/2^-$ baryon nonet with strangeness

Zou, EPJA 35 (2008) 325

- Mass pattern : quenched or unquenched ?

$$uds (L=1) 1/2^- \sim \Lambda^*(1670) \sim [us][ds] \bar{s}$$

$$uud (L=1) 1/2^- \sim N^*(1535) \sim [ud][us] \bar{s}$$

$$uds (L=1) 1/2^- \sim \Lambda^*(1405) \sim [ud][su] \bar{u}$$

$$uus (L=1) 1/2^- \sim \Sigma^*(1390) \sim [us][ud] \bar{d}$$

Zou et al, NPA835 (2010) 199 ; CLAS, PRC87(2013)035206

- Strange decays of $N^*(1535)$ and $\Lambda^*(1670)$:

$$N^*(1535) \text{ large couplings } g_{N^*N\eta}, g_{N^*K\Lambda}, g_{N^*N\eta'}, g_{N^*N\phi}$$

$$\Lambda^*(1670) \text{ large coupling } g_{\Lambda^*\Lambda\eta}$$

Citation: R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. 2022, 083C01 (2022)

$$\Sigma(1620) 1/2^-$$

$$I(J^P) = 1(\frac{1}{2}^-) \text{ Status: } *$$

OMITTED FROM SUMMARY TABLE

Citation: M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) and 2019 update

$$\Sigma(1480) \text{ Bumps}$$

$$I(J^P) = 1(?^?) \text{ Status: } *$$

OMITTED FROM SUMMARY TABLE

These are peaks seen in $\Lambda\pi$ and $\Sigma\pi$ spectra in the reaction $\pi^+ p \rightarrow (Y\pi)K^+$ at 1.7 GeV/c. Also, the Y polarization oscillates in the same region.

Exp. signals of $\Sigma(1480)$

$$\pi^+ p \rightarrow \pi^+ K^+ \Lambda$$

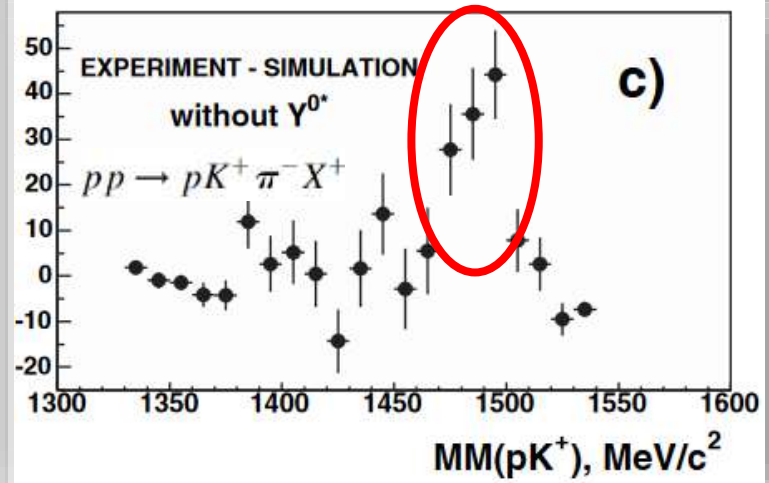
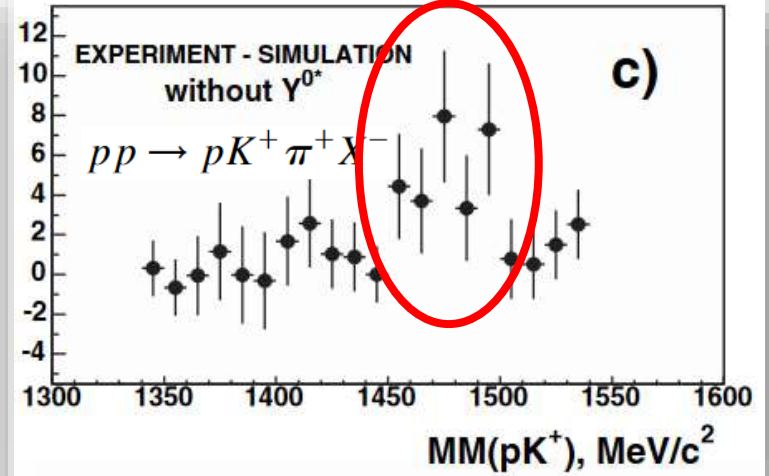
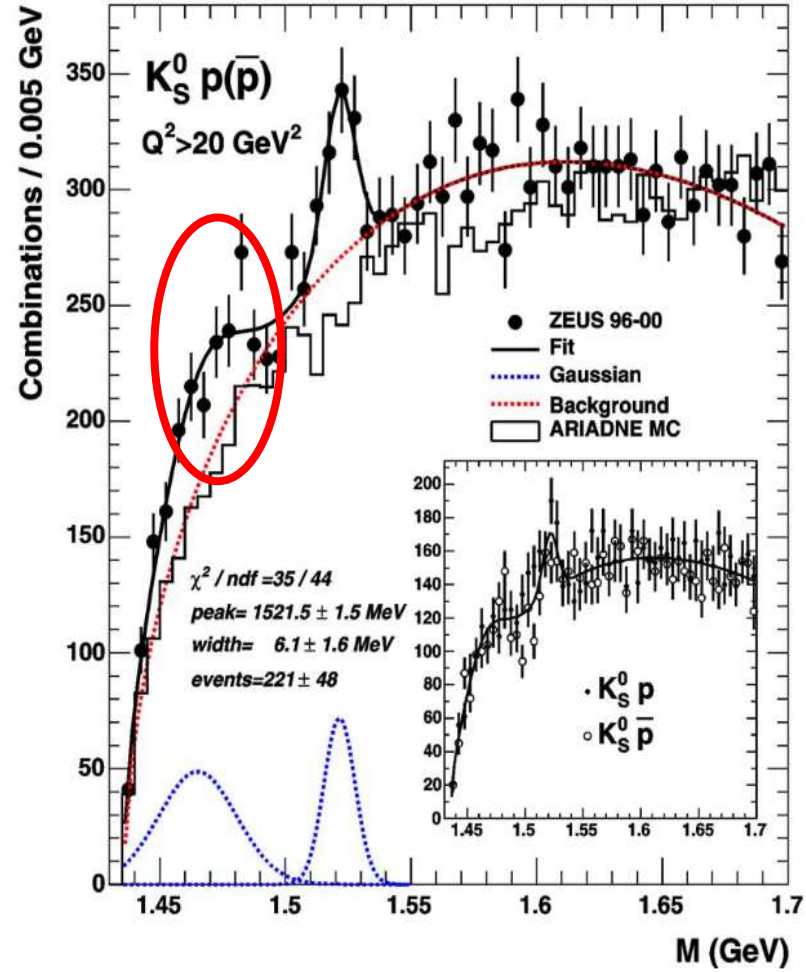
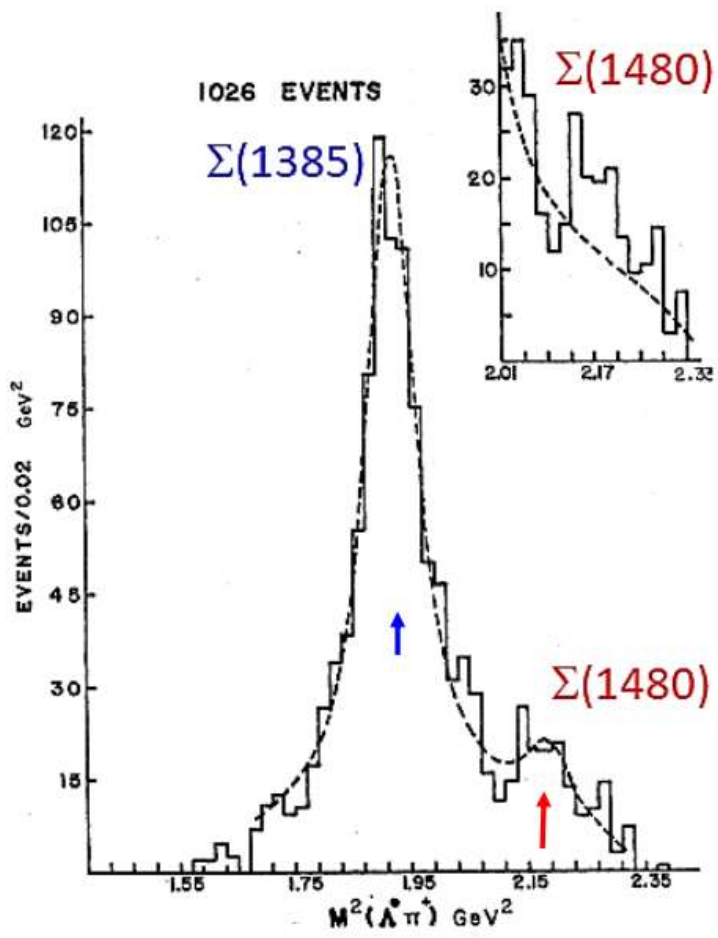
Yu-Li Pan *et al*, PRD 2, 449 (1970)

$$e^+ p \rightarrow e^+ K^0 p X$$

ZEUS PLB591 (2004) 7-22

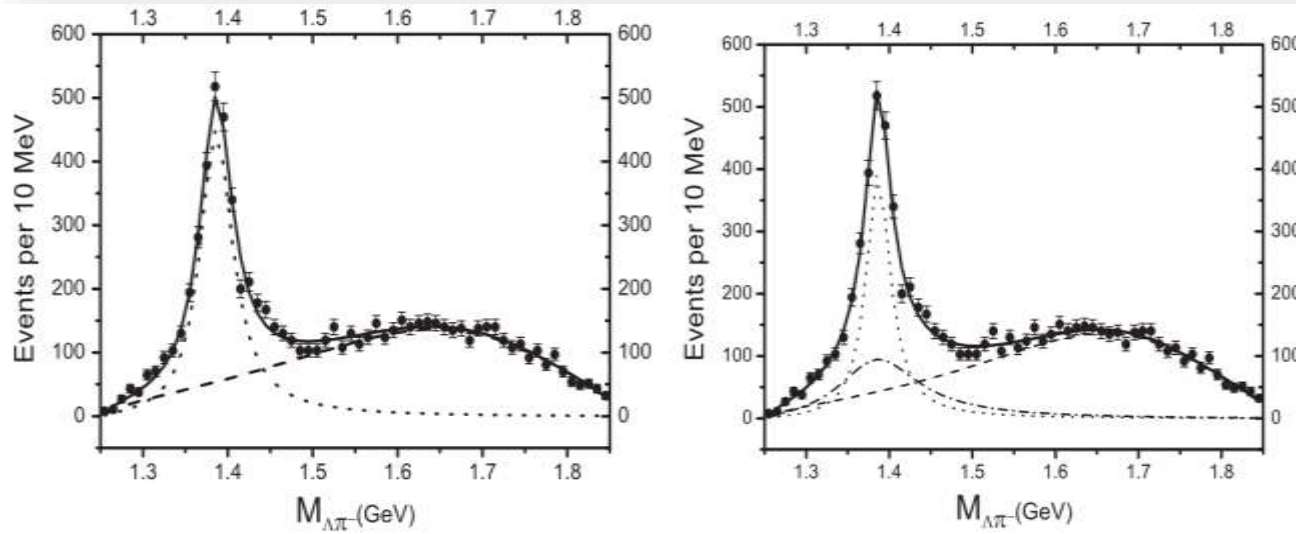
$$pp \rightarrow pK^+ Y^{0*}$$

COSY-Ju;ich PRL 96, 012002 (2006)



Evidence of $\Sigma(1/2^-)$

- $K^- p \rightarrow \Lambda \pi^+ \pi^-$, Wu-Dulat-Zou, PRD80(2009)017503

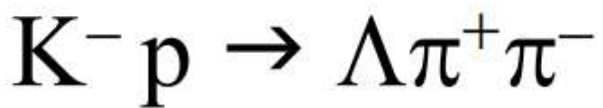


$$\frac{dN}{dm_{\Lambda\pi\pi^-}} \propto p_1 \times p_2 \times \sum_{i=1}^3 \frac{|a_i|}{(m_{\Lambda\pi\pi^-}^2 - m_i^2)^2 + m_i^2 \times \Gamma_i^2}$$

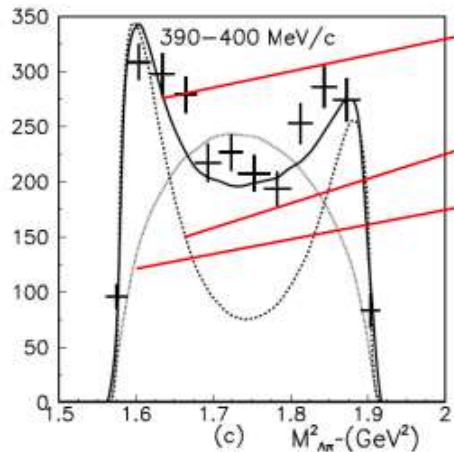
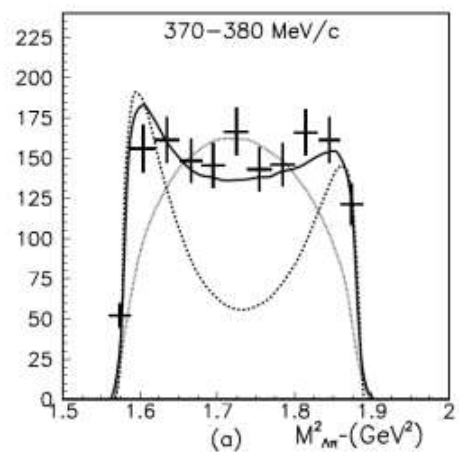
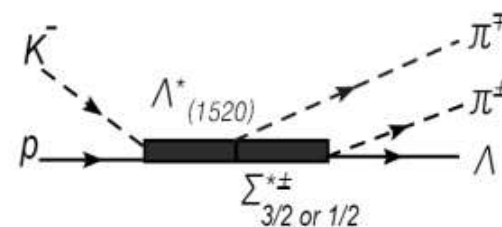
Here we reexamine some old data of the $K^- p \rightarrow \Lambda \pi^+ \pi^-$ reaction and find that besides the well-established $\Sigma^*(1385)$ with $J^P = 3/2^+$, there is indeed some evidence for the possible existence of a new Σ^* resonance with $J^P = 1/2^-$ around the same mass but with broader decay width. There are also indications for such a possibility in the $J/\psi \rightarrow \bar{\Sigma} \Lambda \pi$ and $\gamma n \rightarrow K^+ \Sigma^{*-}$ reactions. At present, the evidence is not strong. Therefore, high statistics studies

	$M_{\Sigma^*(3/2)}$	$\Gamma_{\Sigma^*(3/2)}$	$M_{\Sigma^*(1/2)}$	$\Gamma_{\Sigma^*(1/2)}$	χ^2/ndf (Fig. 1)	χ^2/ndf (Fig. 2)
Fit1	1385.3 ± 0.7	46.9 ± 2.5			68.5/54	10.1/9
Fit2	$1386.1^{+1.1}_{-0.9}$	$34.9^{+5.1}_{-4.9}$	$1381.3^{+4.9}_{-8.3}$	$118.6^{+55.2}_{-35.1}$	58.0/51	3.2/9

J.J.Wu's slide



$P_K=0.3-0.6$ GeV J. J. Wu, S. Dulat and B. S. Zou PRC 81,045210

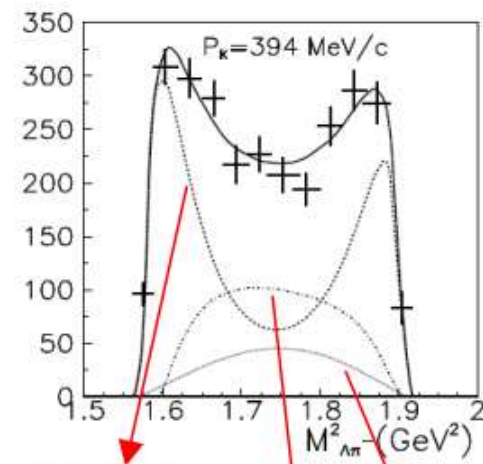


59% $\Sigma^*(3/2^+)$ + 41% $\Sigma^*(1/2^-)$

100% $\Sigma^*(3/2^+)$

Phase space

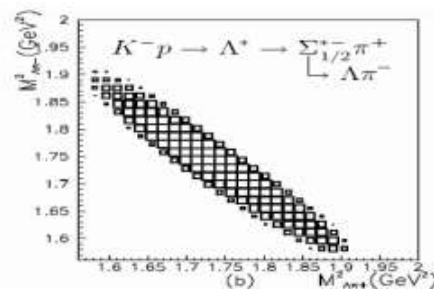
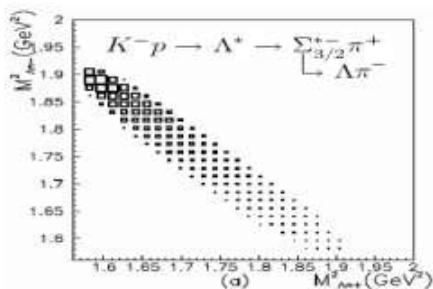
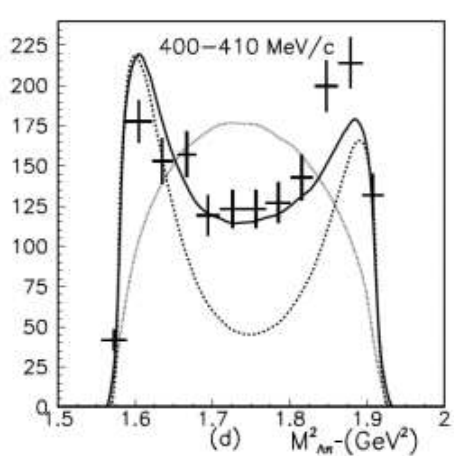
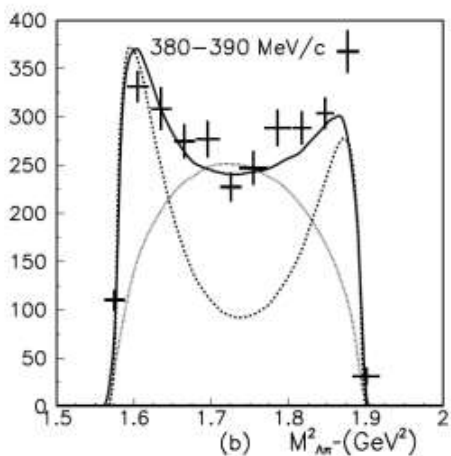
First reason: S-wave between the $\Sigma^*(3/2^+)$ and π^+ ; but P-wave between the $\Sigma^*(1/2^-)$ and π^+ .



59% $\Sigma^*(3/2^+)$

Interference

12.5% $\Sigma^*(1/2^-)$



Second reason: the width of $\Sigma^*(3/2^+)$ is 35.5MeV; but that of $\Sigma^*(1/2^-)$ is 118.6MeV from fit before.



Search for $\Sigma(1/2^-)$

- $\Lambda_c \rightarrow \Lambda \eta \pi$, **Xie-Geng, PRD95(2017) 074024**
- $\gamma n \rightarrow K \Sigma(1/2^-)$, **Lyu-EW-Xie-Wei, CPC47 (2023) 053108**
- $\chi_{c0} \rightarrow \bar{\Sigma} \Sigma \pi$, **EW-Xie-Oset, PLB753(2016)526**
- $\chi_{c0} \rightarrow \bar{\Lambda} \Sigma \pi$, **EW-Xie-Oset, PRD98(2018)114017**
- $\Lambda_c \rightarrow \Sigma^+ \pi^+ \pi^0 \pi^-$, **Xie-Oset, Phys.Lett.B 792 (2019) 450**
- $\gamma N \rightarrow \Sigma(1/2^-) N$, **Kim-Nam-Hosaka, PRD(2021)114017**

Low-lying baryons with $J^P=1/2^-$

• Chiral Lagrangian

$$L_1^{(B)} = \langle \bar{B} i \gamma^\mu \nabla_\mu B \rangle - M_B \langle \bar{B} B \rangle + \frac{1}{2} D \langle \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{1}{2} F \langle \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \quad B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

At lowest order in momentum

$$L_1^{(B)} = \langle \bar{B} i \gamma^\mu \frac{1}{4f^2} [(\Phi \partial_\mu \Phi - \partial_\mu \Phi \Phi) B - B(\Phi \partial_\mu \Phi - \partial_\mu \Phi \Phi)] \rangle,$$

$$\nabla_\mu B = \partial_\mu B + [\Gamma_\mu, B], \quad \text{Oset Ramos, NPA635(1998)99}$$

$$\Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger),$$

$$U = u^2 = \exp(i\sqrt{2}\Phi/f),$$

$$u_\mu = iu^\dagger \partial_\mu U u.$$

$$V_{ij} = -C_{ij} \frac{1}{4f^2} \bar{u}(p') \gamma^\mu u(p) (k_\mu + k'_\mu)$$



Neglect the spatial components at low energies

$$V_{ij} = -C_{ij} \frac{1}{4f^2} (k^0 + k'^0)$$

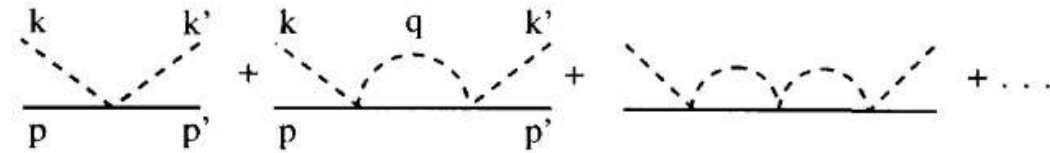
Low-lying baryons with $J^P=1/2^-$

$$V_{ij} = -C_{ij} \frac{1}{4f^2} (k^0 + k'^0)$$

Lippmann-Schwinger equations

$$t_{ij} = V_{ij} + V_{il} G_l T_{lj}$$

$$V_{il} G_l T_{lj} = i \int \frac{d^4 q}{(2\pi)^4} \frac{M_l}{E_l(q)} \frac{V_{il}(k, q) T_{lj}(q, k')}{k^0 + p^0 - q^0 - E_l(q) + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}$$



On-shell approximations

$$2iV_{\text{on}} \int \frac{d^3 q}{(2\pi)^3} \int \frac{dq^0}{2\pi} \frac{M}{E(q)} \frac{q^0 - k^0}{k^0 - q^0} \frac{1}{q^{02} - \omega(q)^2 + i\epsilon}$$

$I=0$	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$
$\bar{K}N$	3	$-\sqrt{\frac{3}{2}}$	$\frac{3}{\sqrt{2}}$	0
$\pi\Sigma$		4	0	$\sqrt{\frac{3}{2}}$
$\eta\Lambda$			0	$-\frac{3}{\sqrt{2}}$
$K\Xi$				3

$I=1$	$\bar{K}N$	$\pi\Sigma$	$\pi\Lambda$	$\eta\Sigma$	$K\Xi$
$\bar{K}N$	1	-1	$-\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{3}{2}}$	0
$\pi\Sigma$		2	0	0	1
$\pi\Lambda$			0	0	$-\sqrt{\frac{3}{2}}$
$\eta\Sigma$				0	$-\sqrt{\frac{3}{2}}$
$K\Xi$					1

Low-lying baryons with $J^P=1/2^-$

• Bethe-Salpter Equation

$$T = [1 - VG]^{-1}V$$

$$G_l = i \int \frac{d^4q}{(2\pi)^4} \frac{M_l}{E_l(\mathbf{q})} \frac{1}{k^0 + p^0 - q^0 - E_l(\mathbf{q}) + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}$$

$$= \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_l(q)} \frac{M_l}{E_l(\mathbf{q})} \frac{1}{p^0 + k^0 - \omega_l(\mathbf{q}) - E_l(\mathbf{q}) + i\epsilon},$$

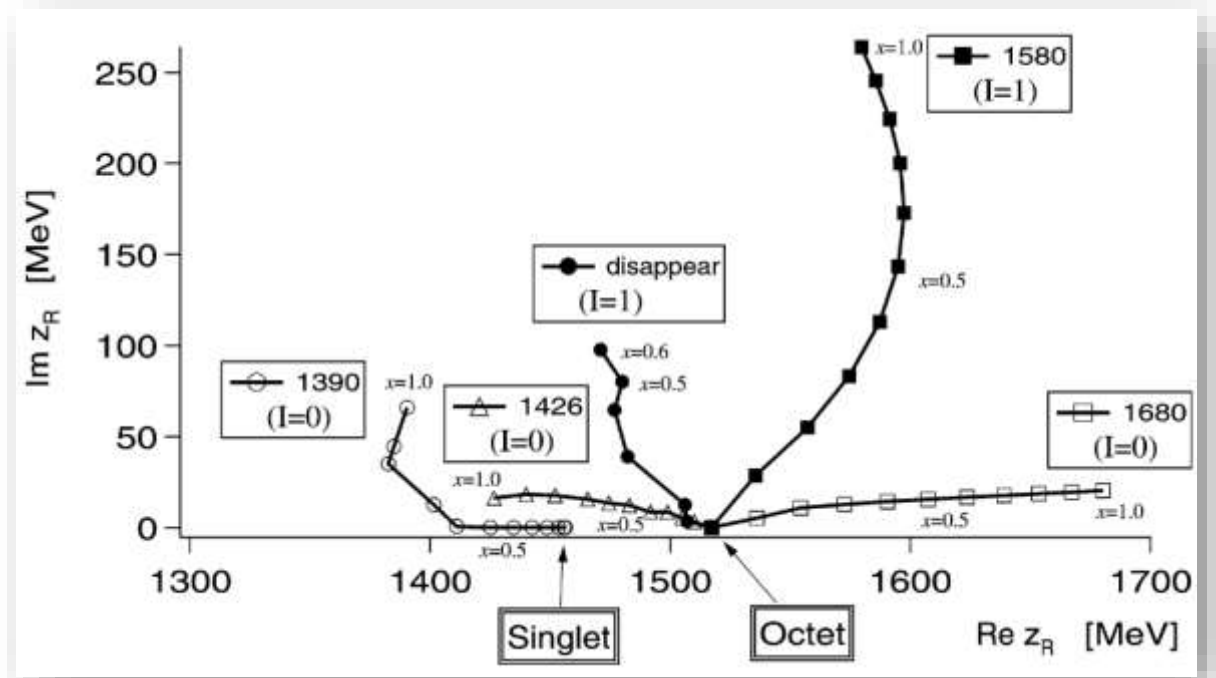
$$G_l = i2M_l \int \frac{d^4q}{(2\pi)^4} \frac{1}{(P-q)^2 - M_l^2 + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}$$

$$= \frac{2M_l}{16\pi^2} \left\{ a_l(\mu) + \ln \frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s} \ln \frac{m_l^2}{M_l^2} \right.$$

$$+ \frac{q_l}{\sqrt{s}} \left[\ln(s - (M_l^2 - m_l^2) + 2q_l\sqrt{s}) + \ln(s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) \right.$$

$$\left. \left. - \ln(-s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) - \ln(-s - (M_l^2 - m_l^2) + 2q_l\sqrt{s}) \right] \right\}$$

Jido Oller Oset Ramos Meissner, NPA725 (2003) 181



pole positions and couplings

$$T_{ij} = \frac{g_i g_j}{z - z_R}$$

$\Sigma(1/2^-)$

- $\pi\Sigma$ photoproduction, **Roca-Oset, PRC 88, 055206 (2013)**

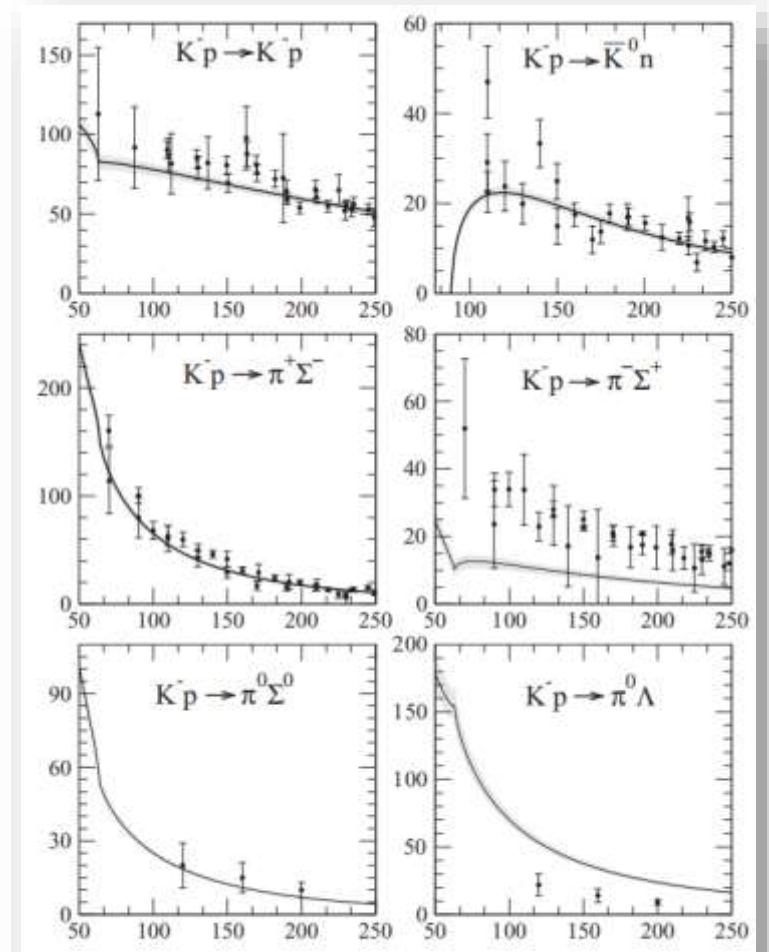
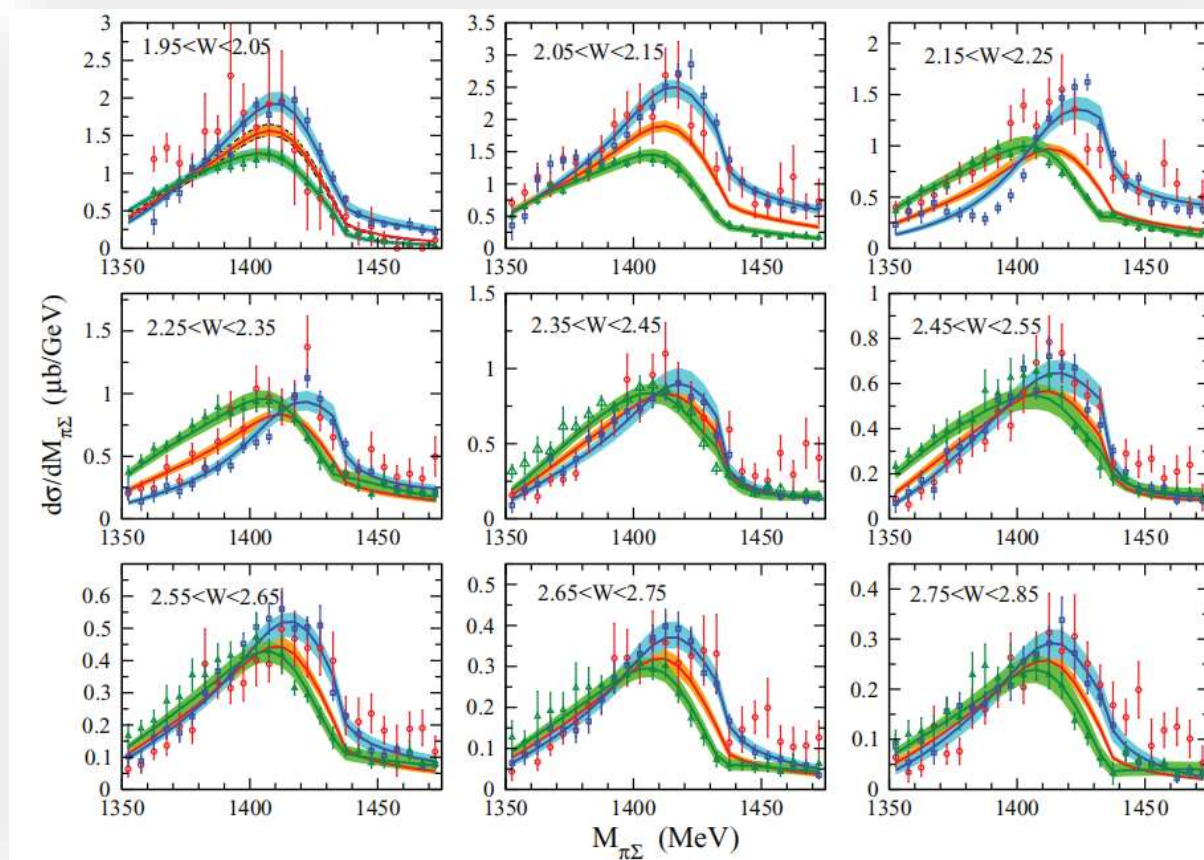
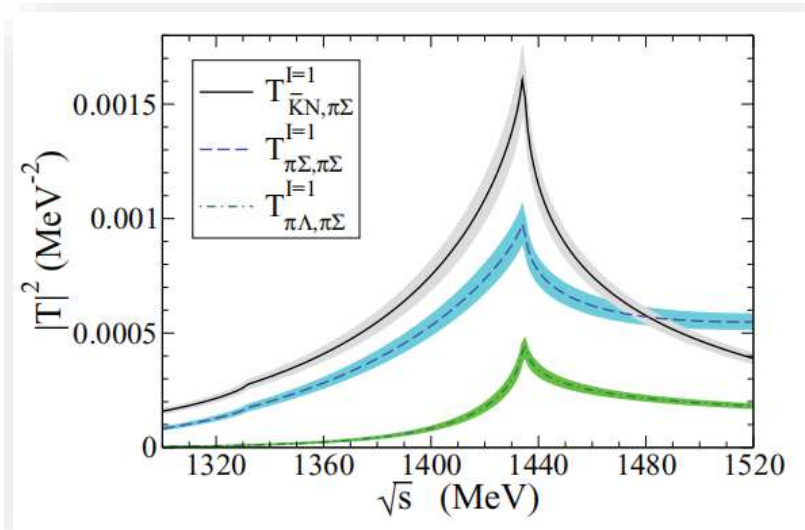


FIG. 6. Predicted K^-p cross sections (in millibarns). Experimental data are from Ref. [46].

$\Sigma(1430)$

- $\pi\Sigma$ photoproduction, **Roca-Oset, PRC 88, 055206 (2013)**



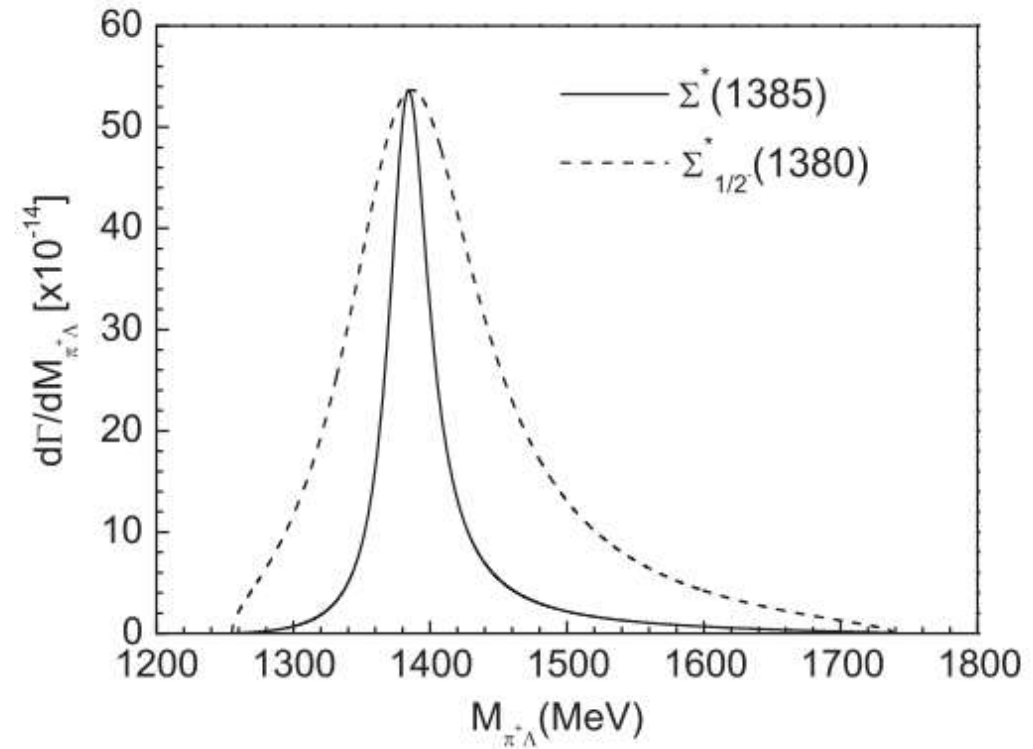
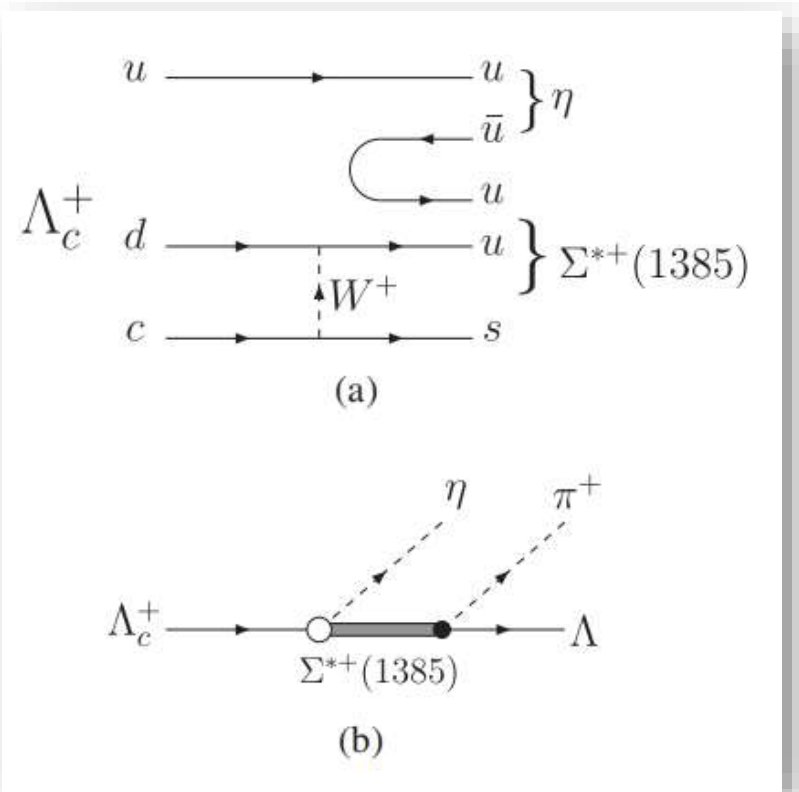
$$V_{ij} = -C_{ij} \frac{1}{4f^2} (k^0 + k'^0) \quad C_{ij}^1 = \begin{pmatrix} \alpha_{11}^1 & -\alpha_{12}^1 & -\sqrt{\frac{3}{2}}\alpha_{13}^1 \\ -\alpha_{12}^1 & 2\alpha_{22}^1 & 0 \\ -\sqrt{\frac{3}{2}}\alpha_{13}^1 & 0 & 0 \end{pmatrix}$$

α_{11}^0	α_{12}^0	α_{22}^0	α_{11}^1	α_{12}^1	α_{13}^1	α_{22}^1
1.037	1.466	1.668	0.85	0.93	1.056	0.77

- **Oset-Ramos, NPA635 (1998) 99 [nucl-th/9711022].**
- **PB,VB, Hosaka, PRD 85, 114020 (2012)**
- **Oller-Meißner, Phys. Lett. B 500 (2001) 263 [hep-ph/0011146]**

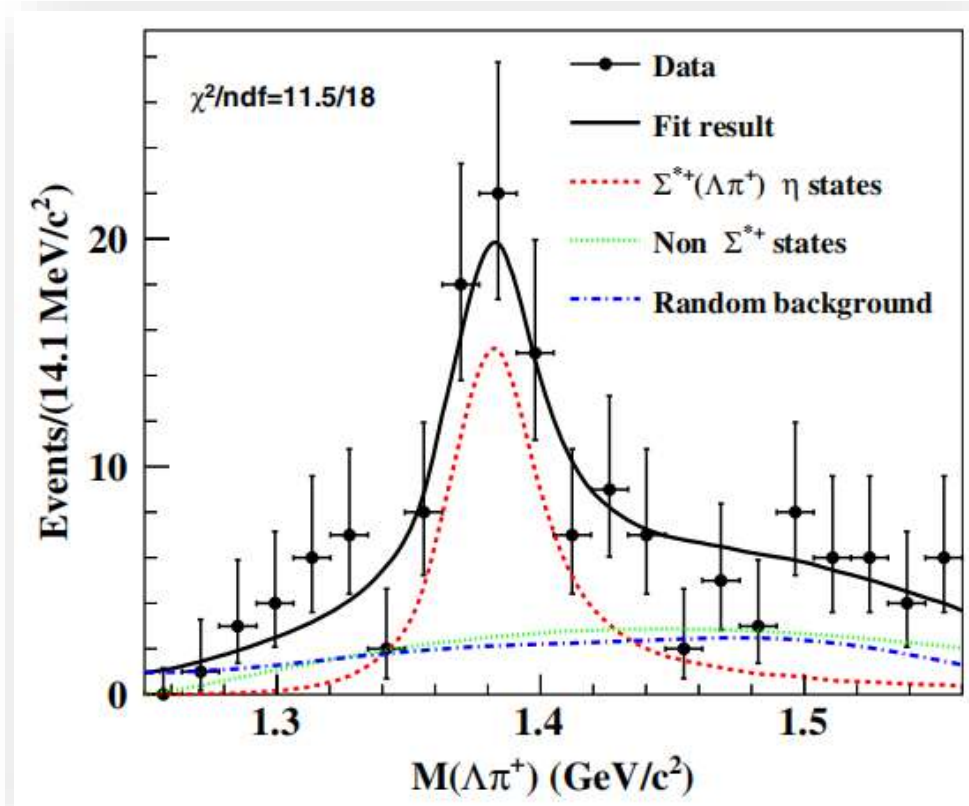
$\Sigma(1/2^-)$ in $\Lambda_c \rightarrow \Lambda \eta \pi$

□ J.J.Xie, L.S.Geng, EPJC76(2016) 496, PRD95(2017) 074024

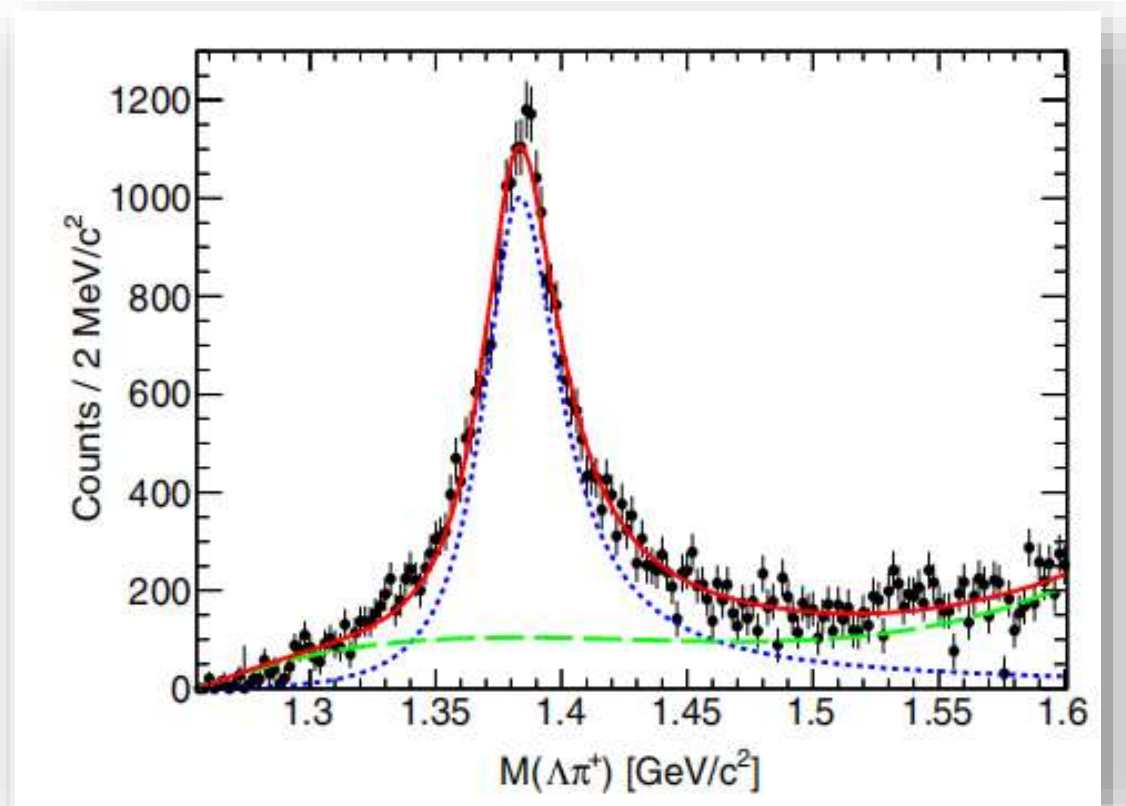


Belle and BESIII measurements

- $\Lambda_c \rightarrow \Lambda \eta \pi$

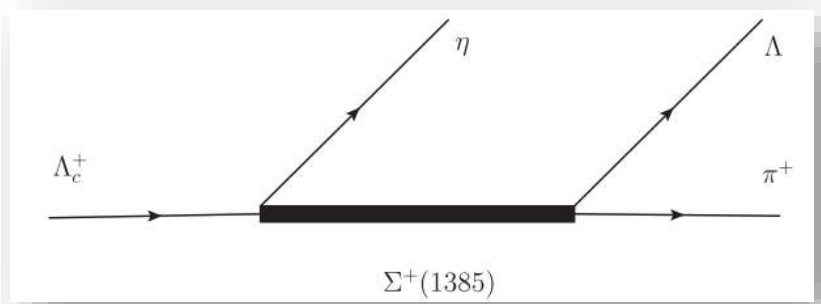
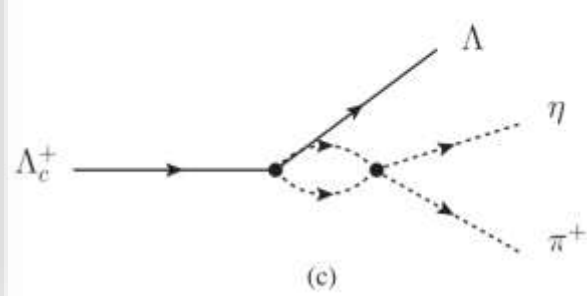
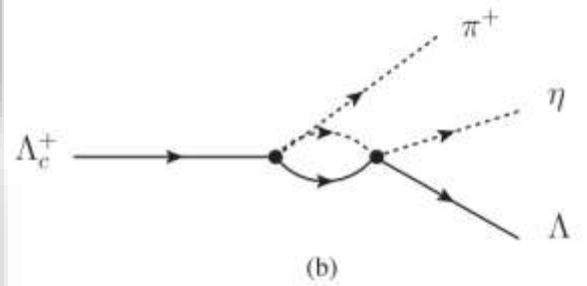
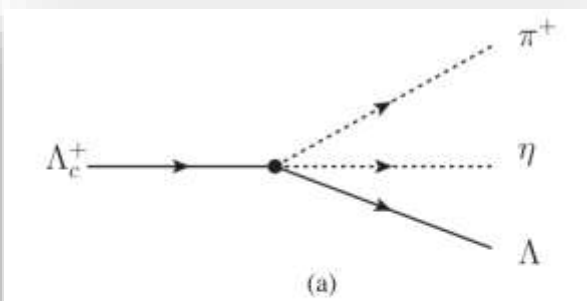
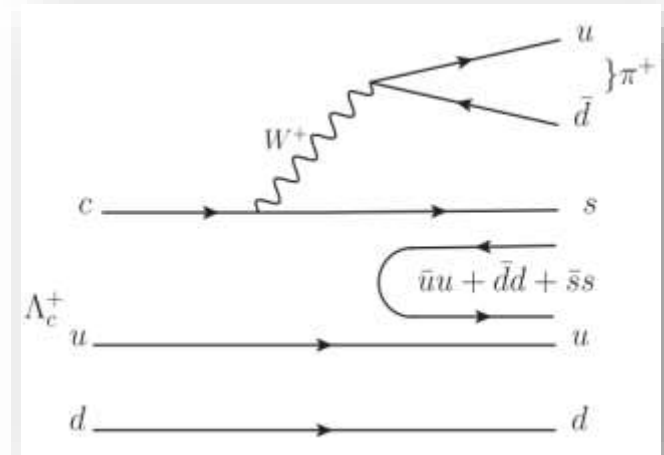
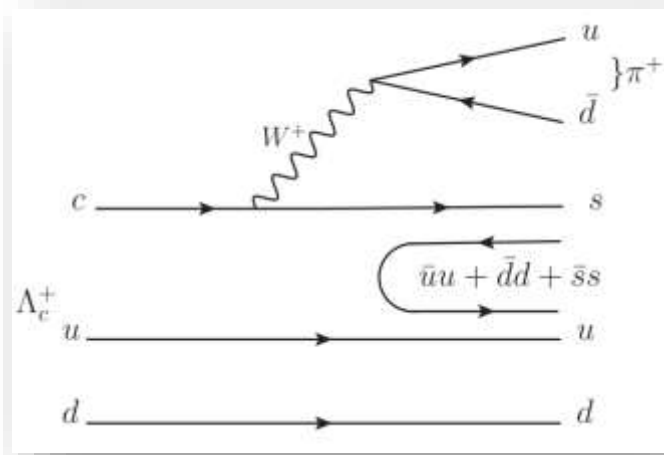


BESIII: PRD99, 032010 (2019)



Belle: PRD103(2021)052005

Mechanism of $\Lambda_c \rightarrow \eta\Lambda\pi$



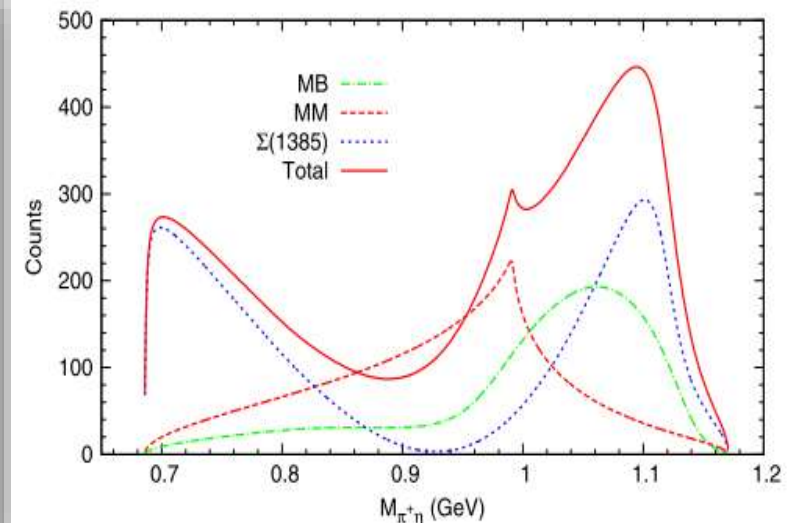
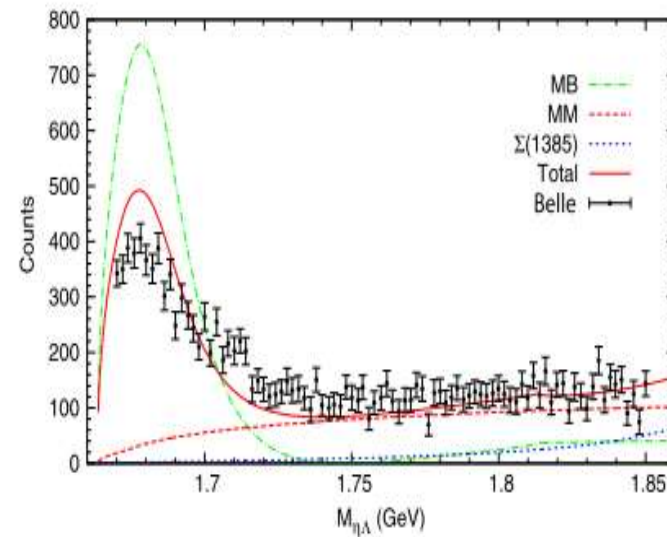
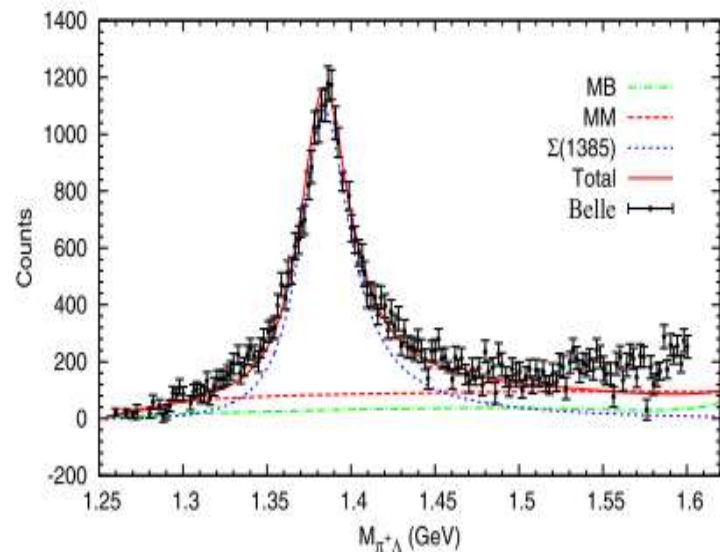
$$T^{\Sigma^*}(M_{\pi^+\Lambda}) = V_P'' \frac{|\vec{p}_\pi| \cdot |\vec{p}_\eta| \cdot \cos\theta}{M_{\pi^+\Lambda} - M_{\Sigma^*} + i\frac{\Gamma_{\Sigma^*}}{2}},$$

$$T^{\text{MB}}(M_{\eta\Lambda}) = V_P \left\{ -\frac{\sqrt{2}}{3} + G_{K^-p}(M_{\eta\Lambda})t_{K^-p \rightarrow \eta\Lambda}(M_{\eta\Lambda}) + G_{\bar{K}^0n}(M_{\eta\Lambda})t_{\bar{K}^0n \rightarrow \eta\Lambda}(M_{\eta\Lambda}) - \frac{\sqrt{2}}{3}G_{\eta\Lambda}(M_{\eta\Lambda})t_{\eta\Lambda \rightarrow \eta\Lambda}(M_{\eta\Lambda}) \right\},$$

$$T^{\text{MM}}(M_{\pi^+\eta}) = V_P' \frac{2\sqrt{2}}{3} \left\{ 1 + G_{\pi^+\eta}(M_{\pi^+\eta})t_{\pi^+\eta \rightarrow \pi^+\eta}(M_{\pi^+\eta}) + \frac{\sqrt{3}}{2}G_{K^+\bar{K}^0}(M_{\pi^+\eta})t_{K^+\bar{K}^0 \rightarrow \pi^+\eta}(M_{\pi^+\eta}) \right\}, \quad ($$

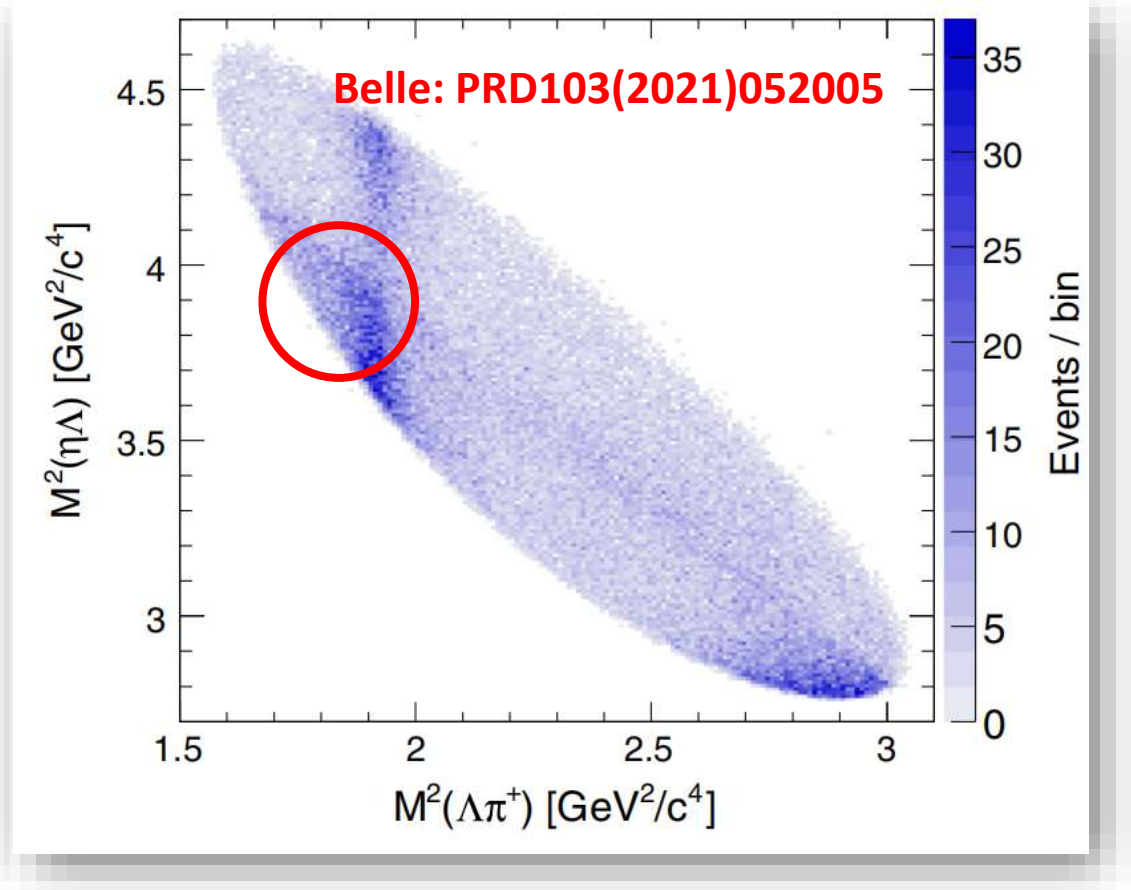
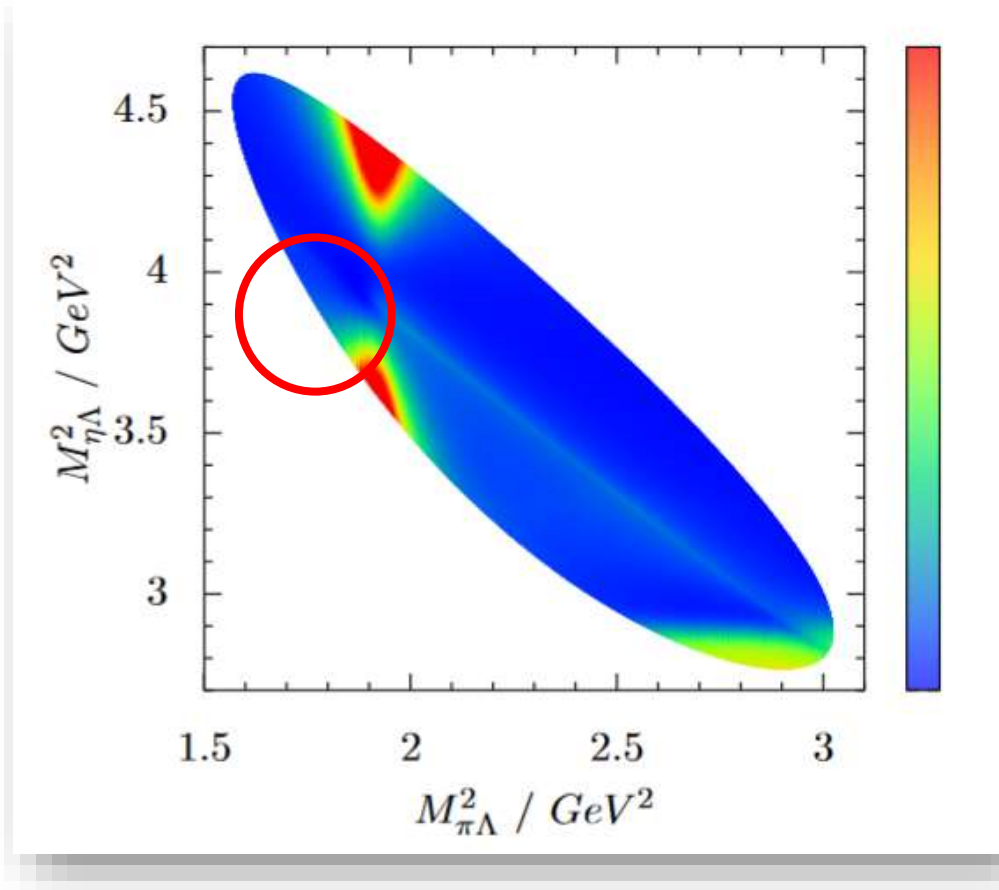
Analysis the Belle data

- $\Lambda_c \rightarrow \Lambda \eta \pi$, GYW-EW-Xie-Geng-Wei, PRD 106, 056001 (2022)



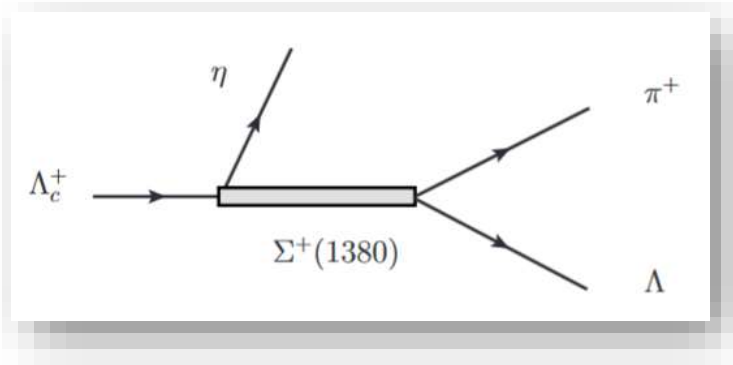
By regarding the $\Lambda(1670)$ as the molecule, we could well reproduce the Belle data of the mass distributions.

Dalitz plot of $\Lambda_c \rightarrow \eta\Lambda\pi$



$\Sigma(1380)$ in $\Lambda_c \rightarrow \eta\Lambda\pi$

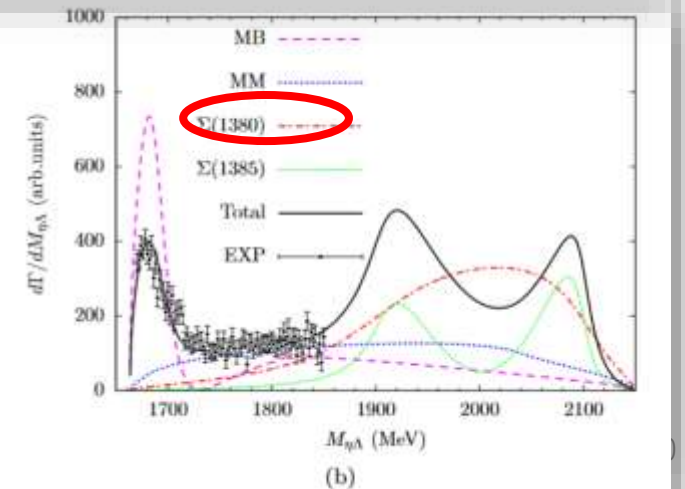
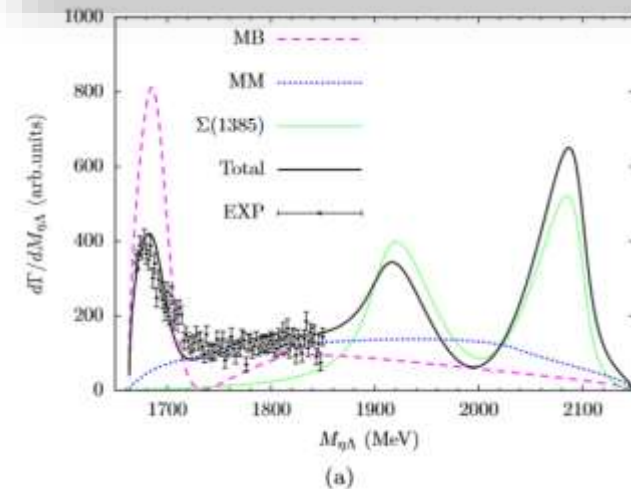
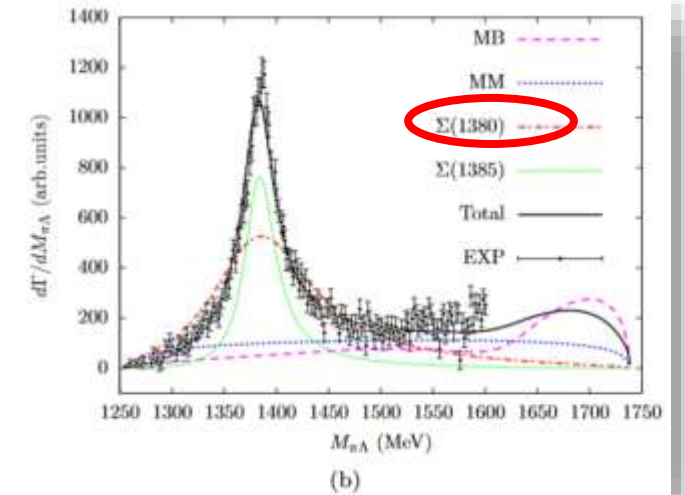
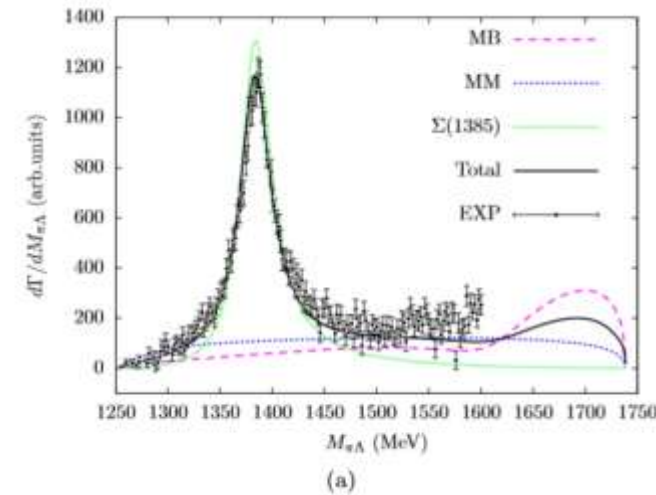
- Intermediate of $\Sigma(1380)$



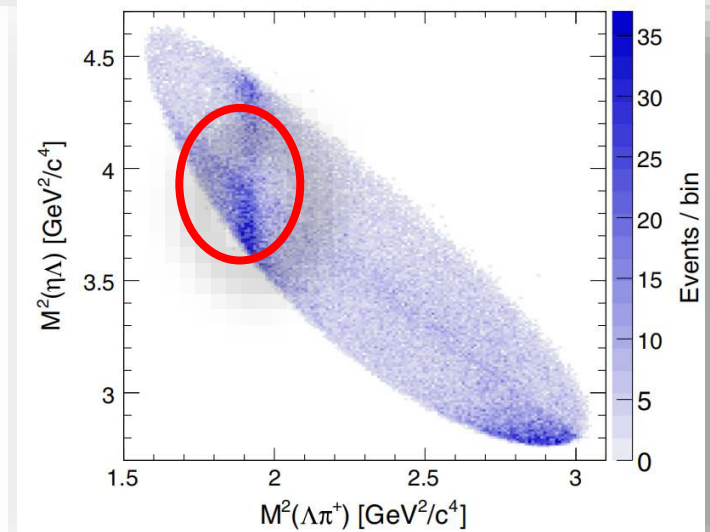
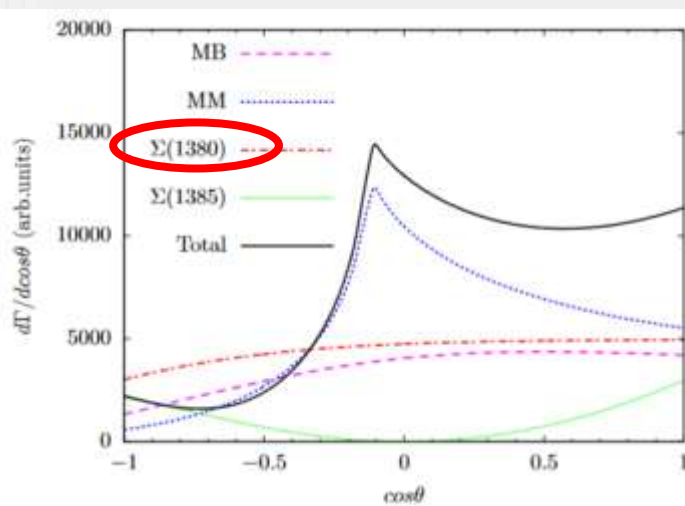
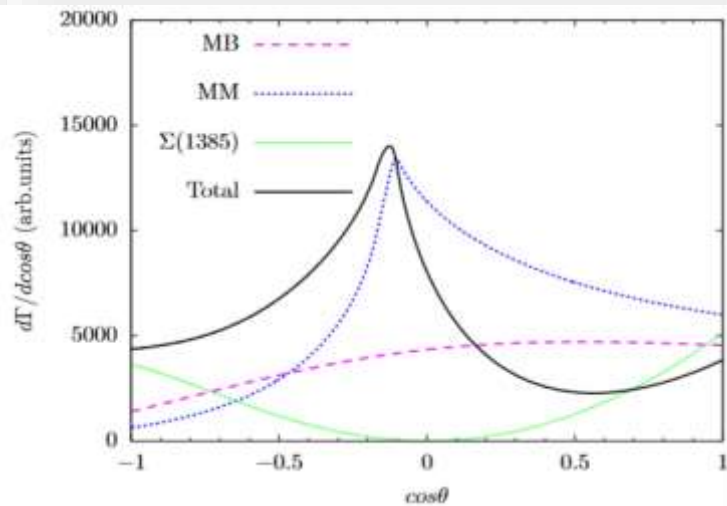
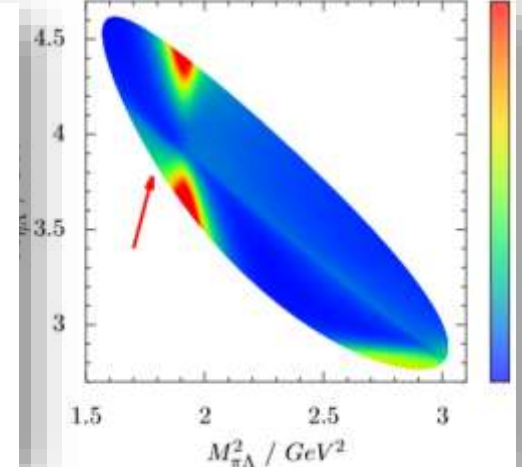
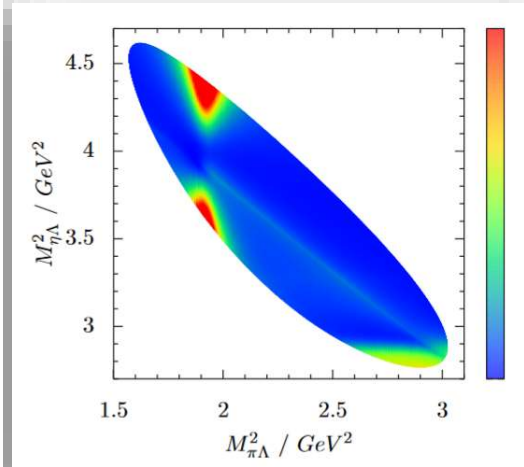
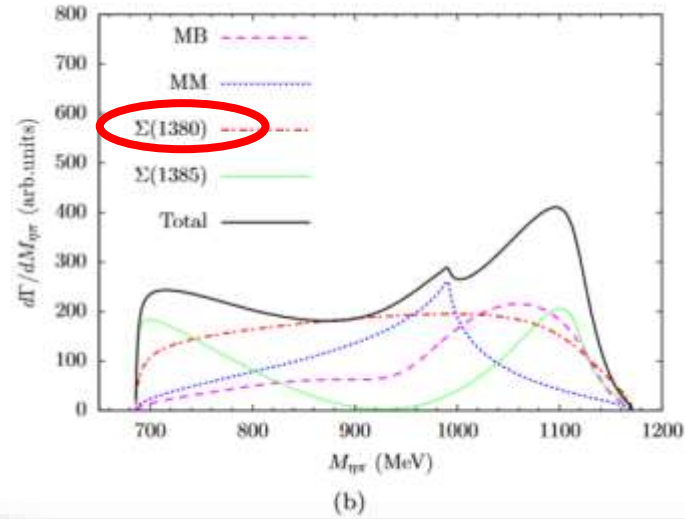
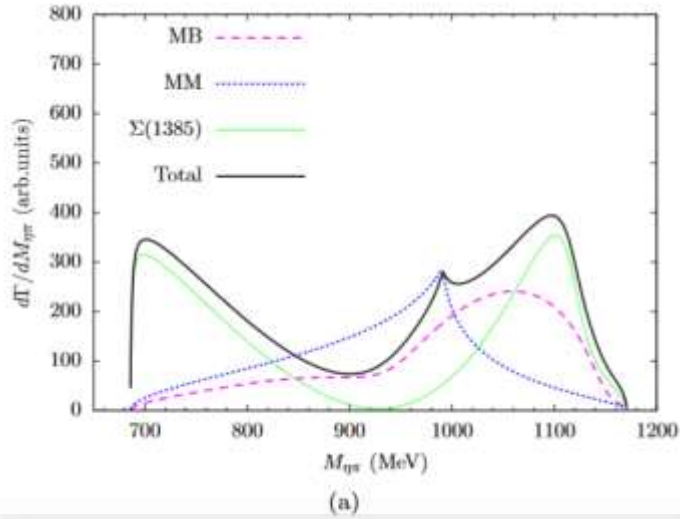
$$\mathcal{T}^{\Sigma(1380)} = \frac{V_P''' M_{\Sigma(1380)}}{M_{\pi+\Lambda} - M_{\Sigma(1380)} + i \frac{\Gamma_{\Sigma(1380)}}{2}},$$

'''

EW, JJWu, to be prepared



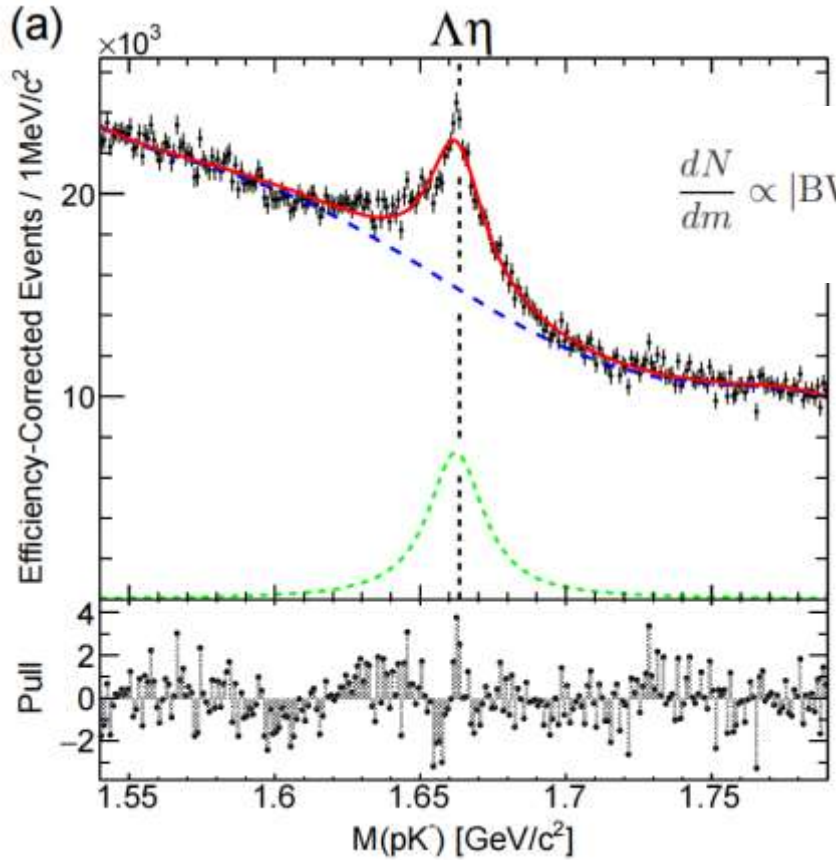
The results with/without $\Sigma(1380)$



$M_{\pi\Lambda} \geq 1450$ MeV and $M_{\eta\Lambda} \geq 1760$ MeV.

EW, JJWu, to be prepared

The cusp in the $\Lambda_c \rightarrow pK\pi$

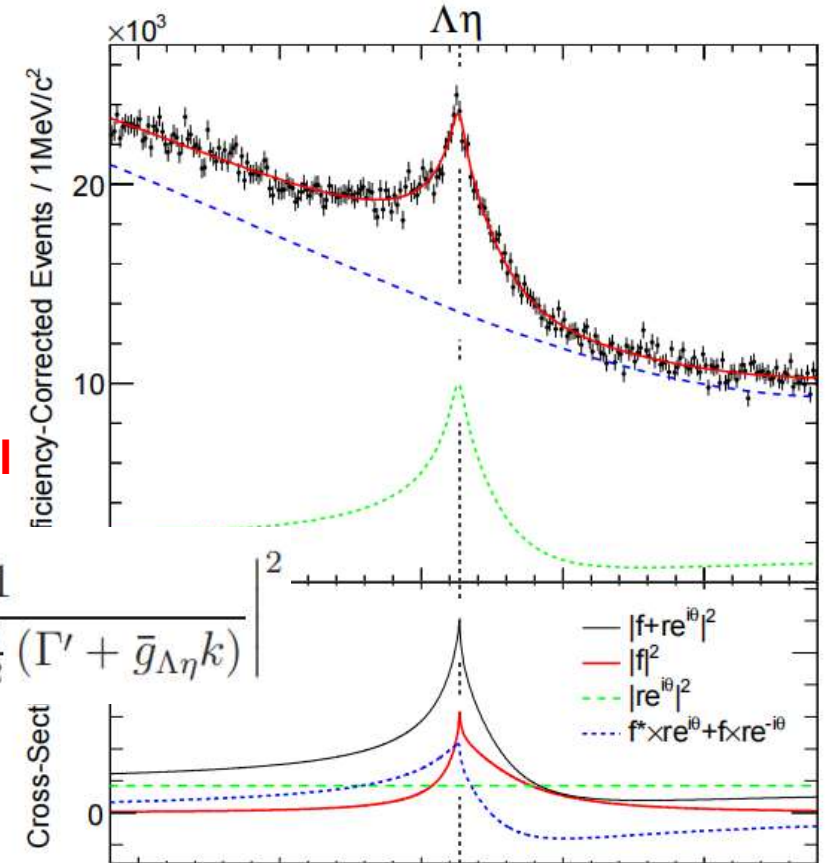


Belle: PRD108(2023)L031104

$$\frac{dN}{dm} \propto |\text{BW}(m)|^2 = \left| \frac{1}{(m - m_0) + i\frac{\Gamma_0}{2}} \right|^2$$

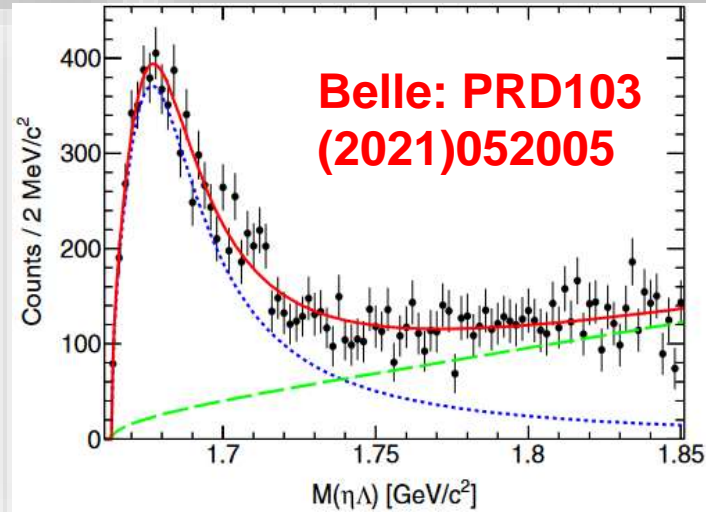
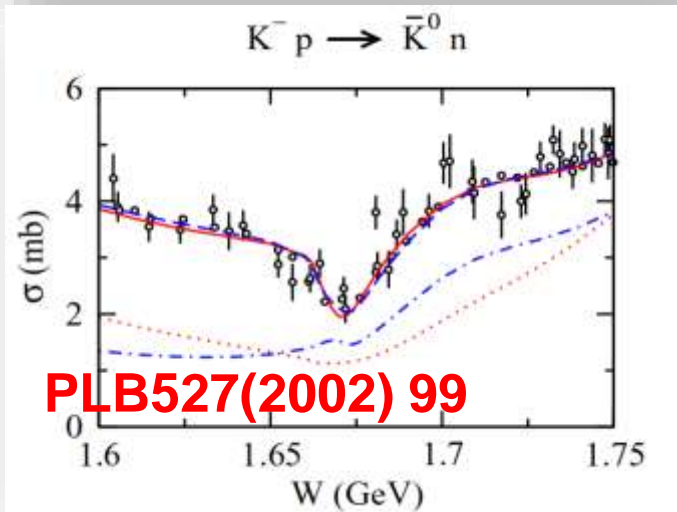
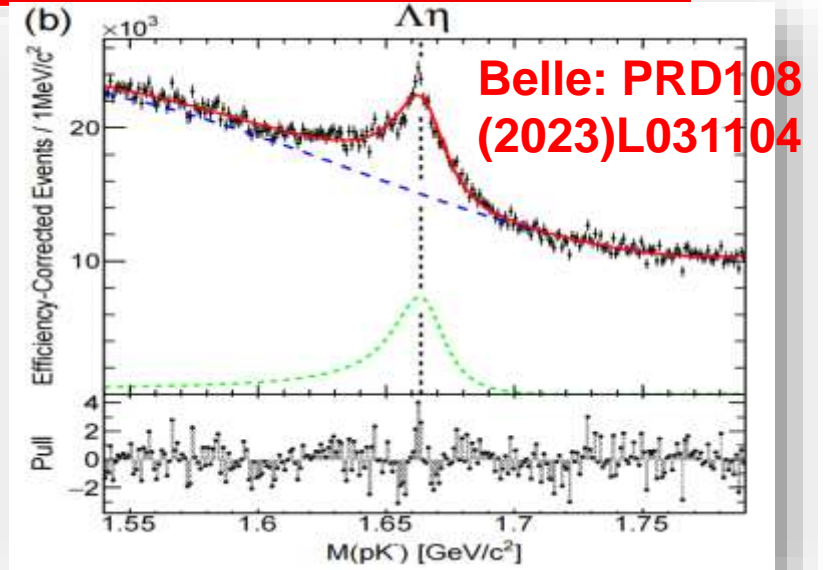
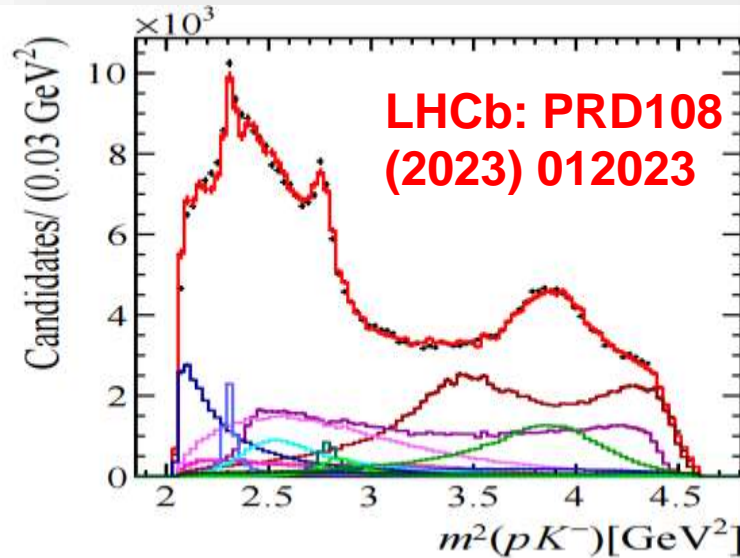
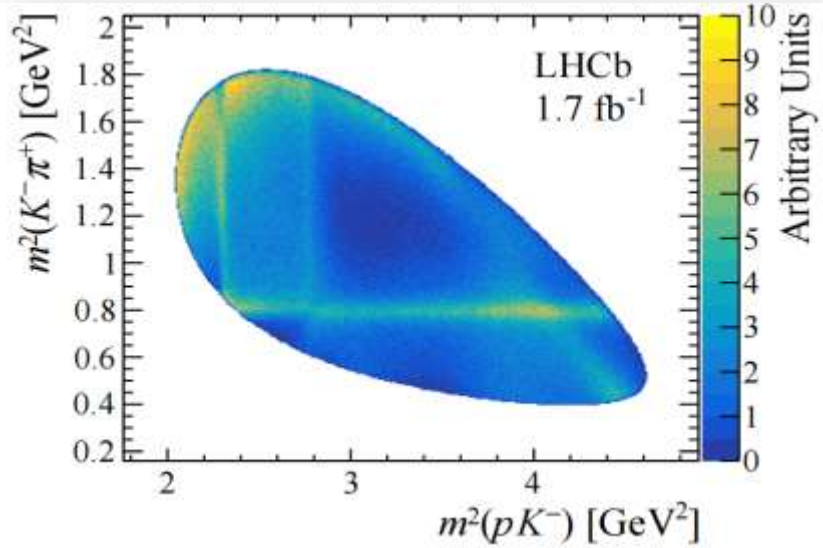
Flatte model

$$\frac{dN}{dm} \propto |f(m)|^2 = \left| \frac{1}{m - m_f + \frac{i}{2}(\Gamma' + \bar{g}_{\Lambda\eta}k)} \right|^2$$



point better than the BW function. These results show that the present peaking structure is explained better by a threshold cusp than to a new hadron resonance by more than 7σ .

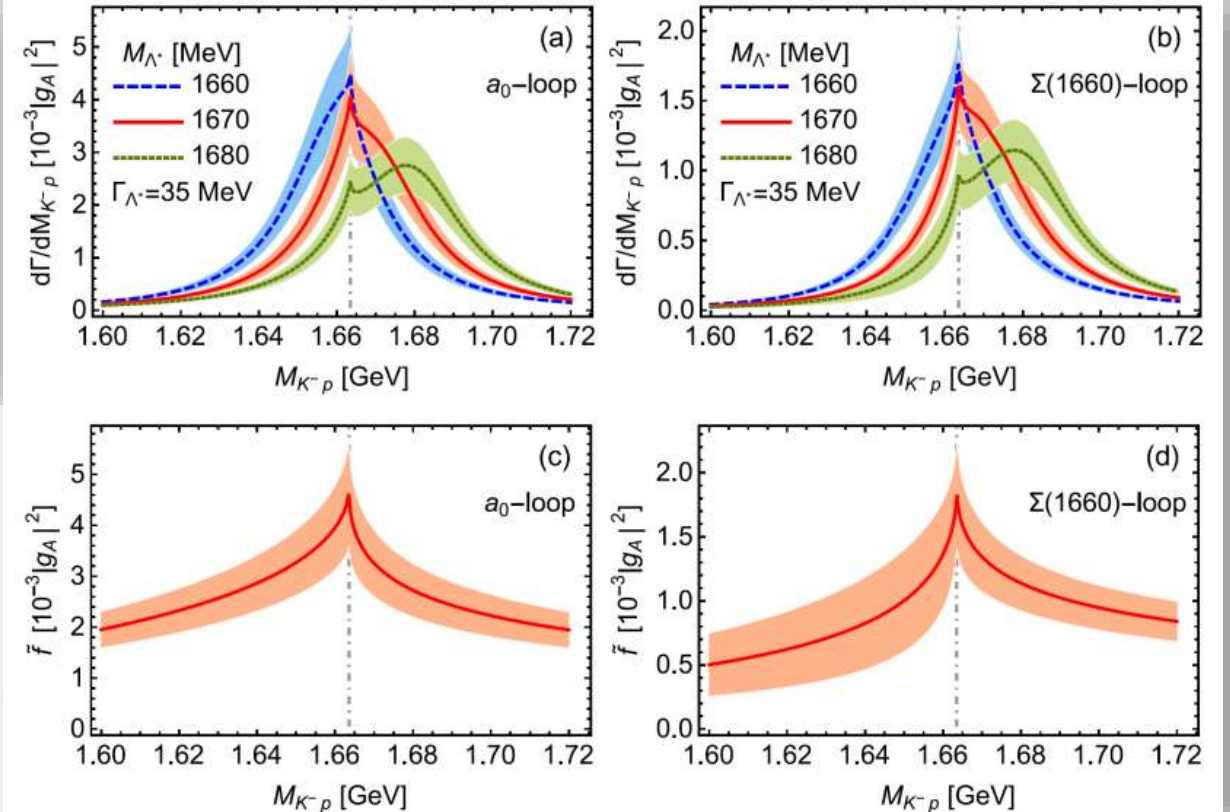
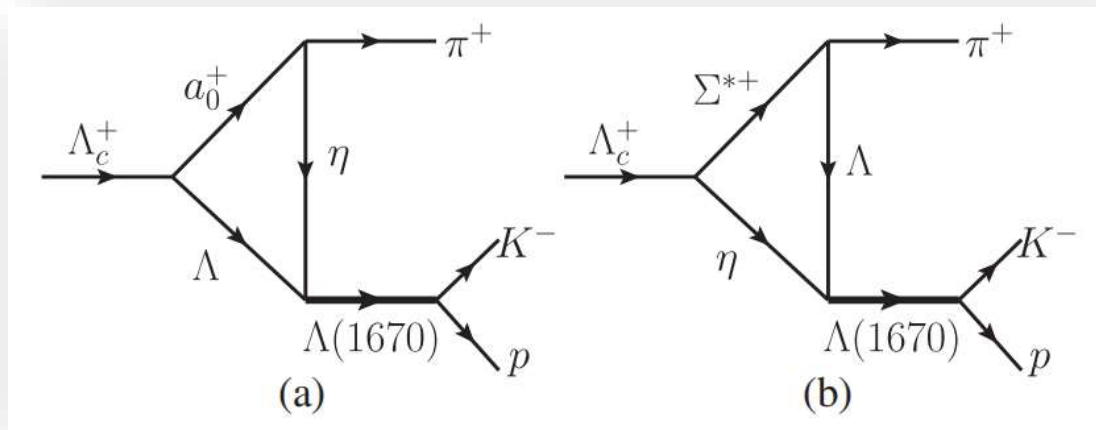
$\Lambda(1670)$ in $\Lambda_c \rightarrow pK^- \pi^+$



Resonances	Mass [MeV/c ²]	Width [MeV]
$\Lambda(1670)$	$1674.3 \pm 0.8 \pm 4.9$	$36.1 \pm 2.4 \pm 4.8$
$\Sigma(1385)^+$	$1384.8 \pm 0.3 \pm 1.4$	$38.1 \pm 1.5 \pm 2.1$

$\Lambda(1670)$: cusp? Dip? peak?

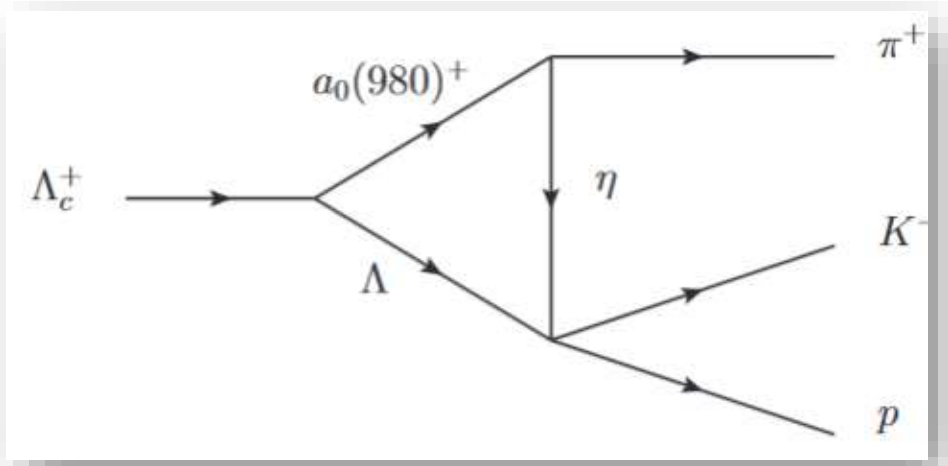
TS in $\Lambda_c \rightarrow pK\pi$



$$\mathcal{T} = \frac{1}{s - M_{\Lambda(1670)}^2 + iM_{\Lambda(1670)}\Gamma_{\Lambda(1670)}} \times \int \frac{d^4 q_1}{(2\pi)^4} \frac{\mathcal{A}}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)(q_3^2 - m_3^2)},$$

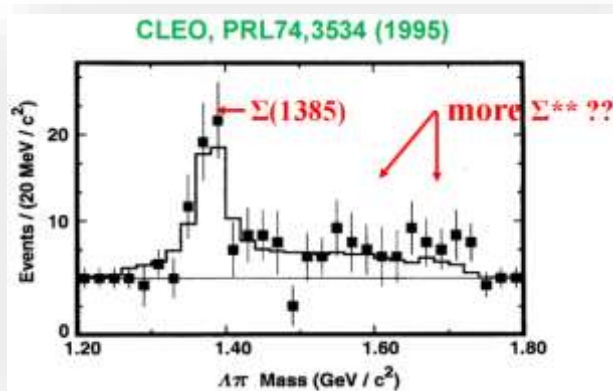
XHLiu, GLi, JJXie, and QZhao, PRD100(2019)054006

TS+ Coupled-channel effects

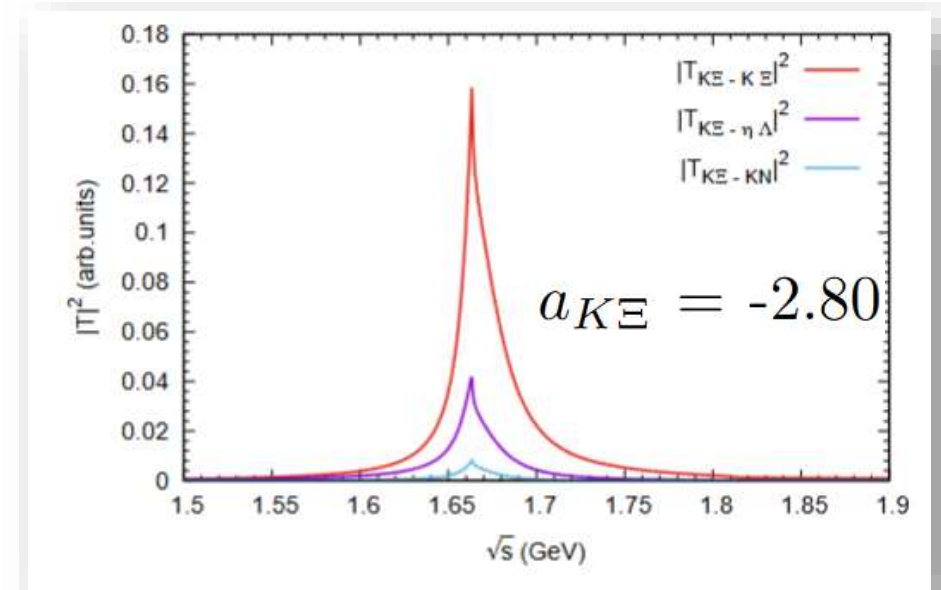
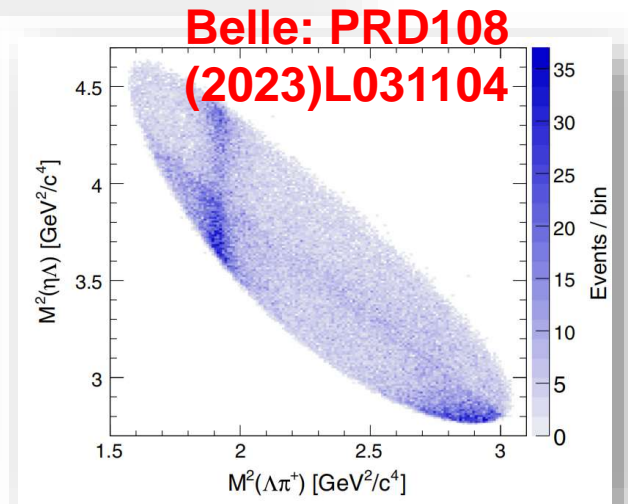


$$\mathcal{T}^{TS} = Q t^{TS} \times t_{\Lambda\eta \rightarrow K^- p},$$

$$t^{TS} = i \int \frac{d^4 q}{(2\pi)^4} \frac{2m_1}{q^2 - m_1^2 + i\epsilon} \frac{1}{(P - q)^2 - m_2^2 + im_2\Gamma_2} \times \frac{1}{(P - q - k)^2 - m_3^2 + i\epsilon}, \quad (10)$$

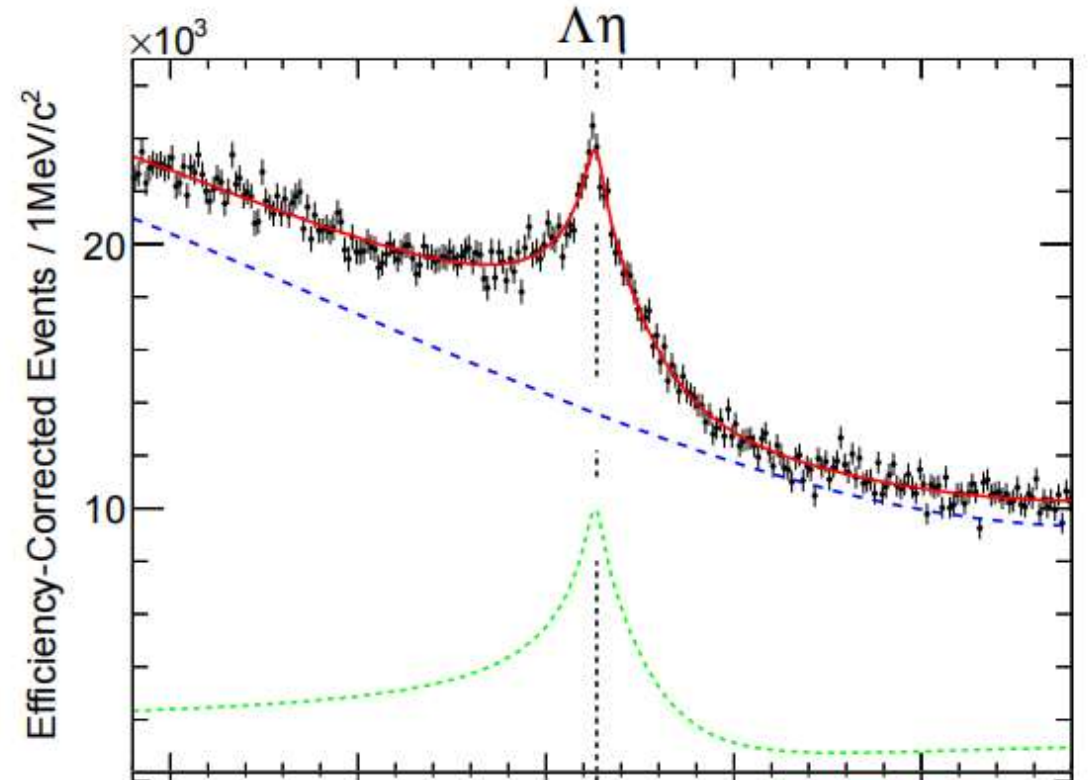
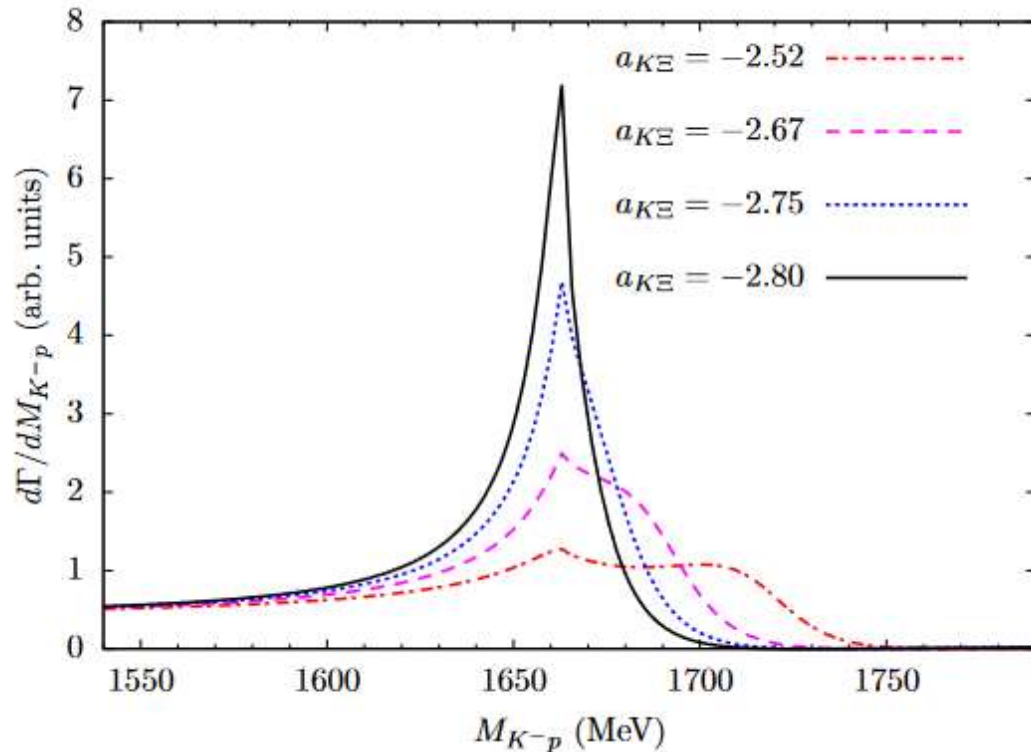


No signal of $\Sigma(1660)$



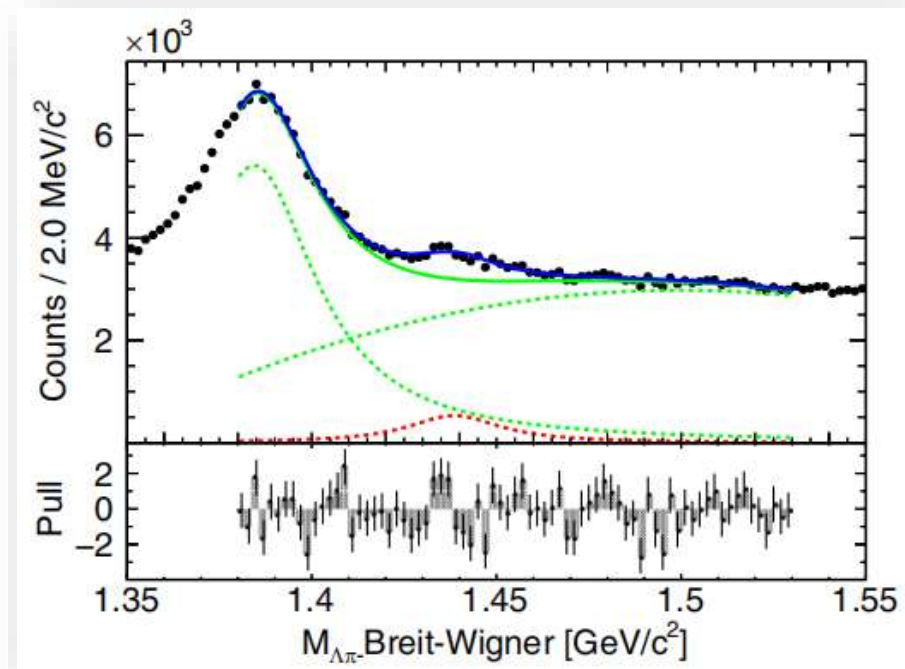
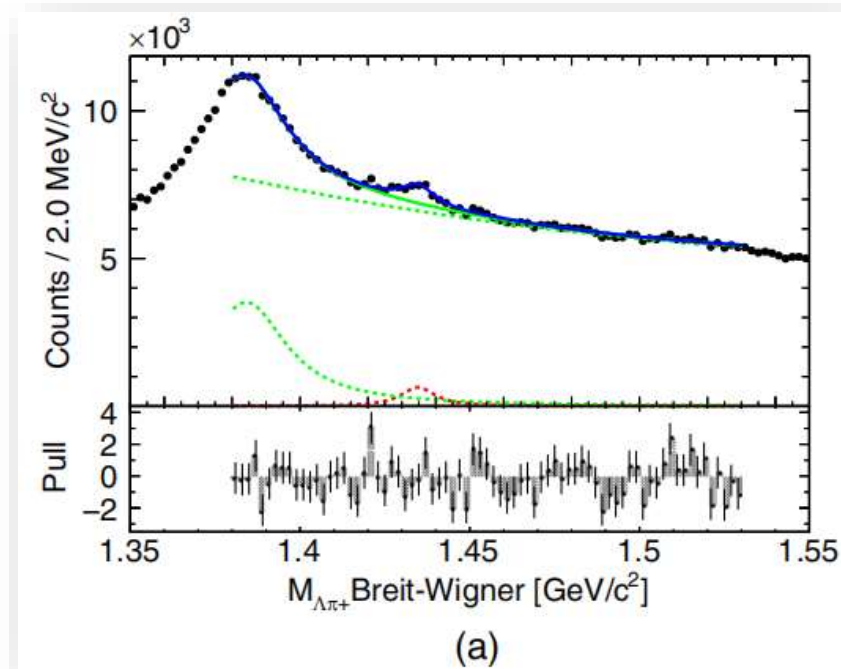
Results of the mass distribution

- EW-GYWang, to be prepared



Belle measurements

- $\Lambda_c \rightarrow \Lambda \pi^+ \pi^+ \pi^-$, Belle, PRL130, 151903 (2023)

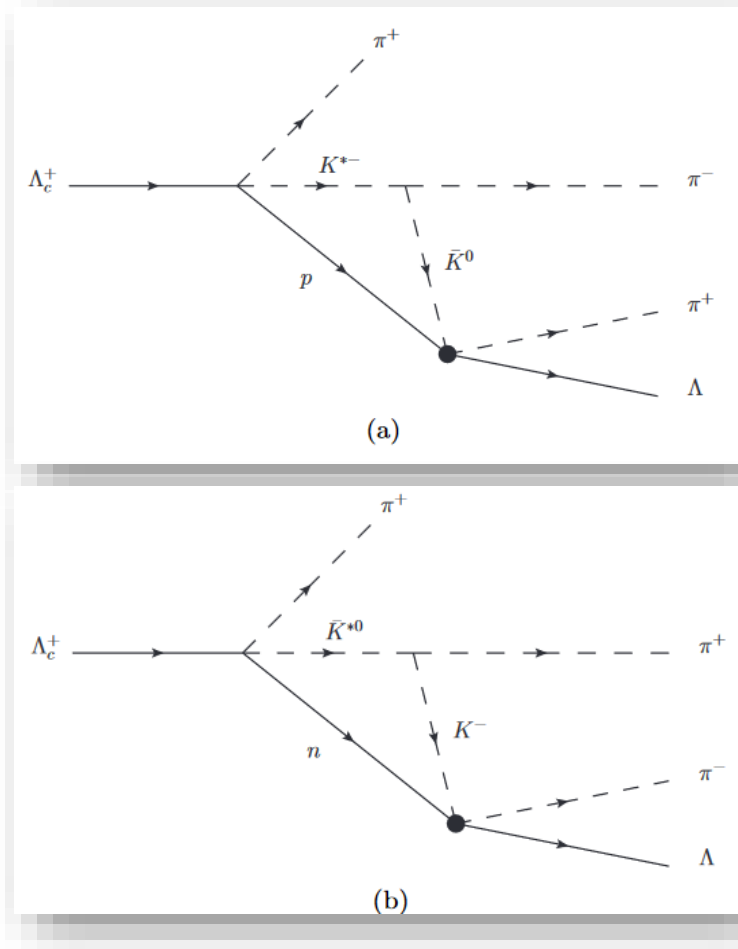


Mode	E_{BW} (MeV/ c^2)	Γ (MeV/ c^2)	χ^2/NDF
$\Lambda\pi^+$	1434.3 ± 0.6	11.5 ± 2.8	74.4/68
$\Lambda\pi^-$	1438.5 ± 0.9	33.0 ± 7.5	92.3/68

Evidence of $\Sigma(1430)$

- $\Lambda_c \rightarrow \Lambda \pi^+ \pi^+ \pi^-$, **TS**

Dai-Pavao-Sakai-Oset, PRD 97, 116004 (2018)
 Xie-Oset, PLB 792, 450-453 (2019)



$$t_{\Lambda_c^+ \rightarrow \pi^+ K^{*-} p} = A \vec{\sigma} \cdot \vec{\epsilon},$$

$$\frac{d\Gamma_{\Lambda_c^+ \rightarrow \pi^+ K^{*-} p}}{dM_{\text{inv}}(K^{*-} p)} = \frac{1}{(2\pi)^3} \frac{2M_{\Lambda_c^+} 2M_p}{4M_{\Lambda_c^+}^2} p_{\pi^+} \tilde{p}_{K^{*-}} \times \sum \sum |t_{\Lambda_c^+ \rightarrow \pi^+ K^{*-} p}|^2,$$

$$\mathcal{B}(\Lambda_c^+ \rightarrow \pi^+ \bar{K}^{*0} p) = (1.4 \pm 0.5) \times 10^{-2}$$

$$|A|^2 = (3.9 \pm 1.4) \times 10^{-16} \text{ MeV}^{-2}.$$

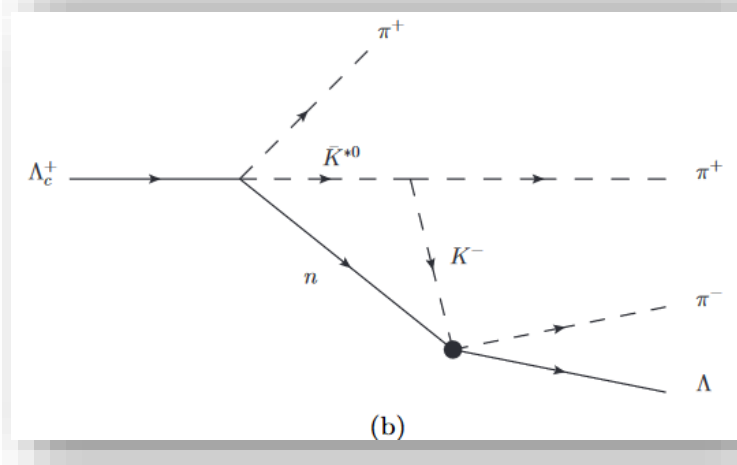
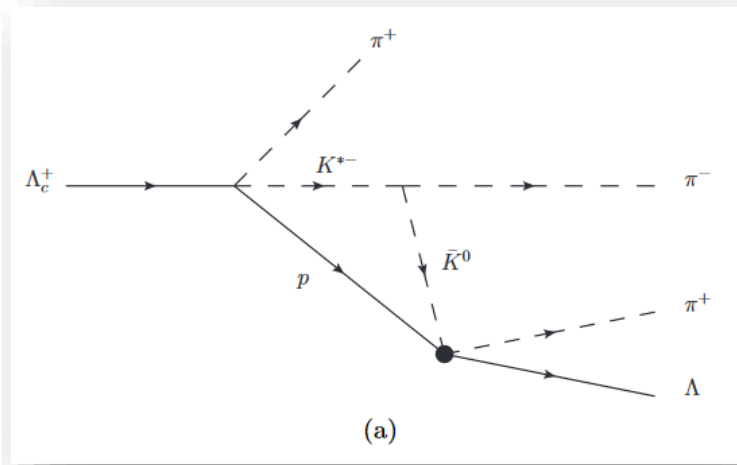
$$\mathcal{L}_{VPP} = -ig \langle V^\mu [P, \partial P] \rangle$$

$$\mathcal{L}_{\bar{K}^* \rightarrow \pi \bar{K}} = -ig (K^{*-})^\mu (\pi^- \partial_\mu \bar{K}^0 - \partial_\mu \pi^- \bar{K}^0).$$

Evidence of $\Sigma(1430)$

- $\Lambda_c \rightarrow \Lambda \pi^+ \pi^+ \pi^-$, **TS**

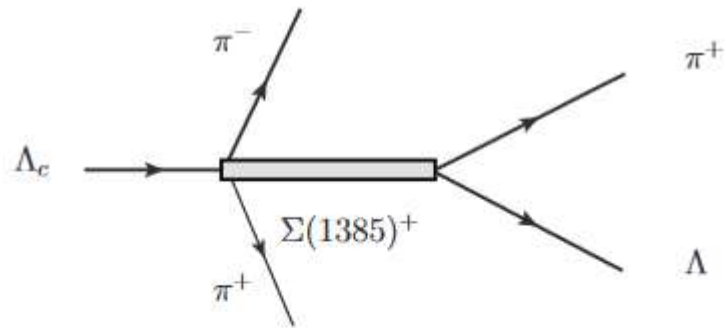
Dai-Pavao-Sakai-Oset, PRD 97, 116004 (2018)
 Xie-Oset, PLB 792, 450-453 (2019)



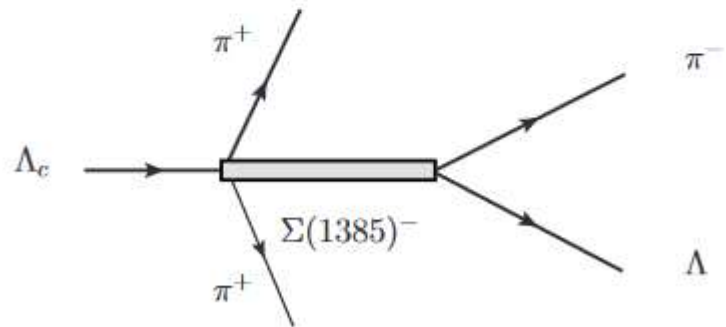
$$\begin{aligned}
 t_T^a = & \int \frac{d^3q}{(2\pi)^3} \frac{2M_p}{8\omega_p\omega_{K^{*-}}\omega_{\bar{K}^0}} \frac{1}{k_a^0 - \omega_{K^{*-}} - \omega_{\bar{K}^0} + i\frac{\Gamma_{K^{*-}}}{2}} \\
 & \times \frac{1}{P^0 + \omega_p + \omega_{\bar{K}^0} - k_a^0} \left(2 + \frac{\vec{q} \cdot \vec{k}}{|\vec{k}|^2}\right) \\
 & \times \frac{2P^0\omega_p + 2k_a^0\omega_{\bar{K}^0} - 2(\omega_p + \omega_{\bar{K}^0})(\omega_p + \omega_{\bar{K}^0} + \omega_{K^{*-}})}{P^0 - \omega_{K^{*-}} - \omega_p + i\frac{\Gamma_{K^{*-}}}{2}} \\
 & \times \frac{1}{P^0 - \omega_p - \omega_{\bar{K}^0} - k_a^0 + i\varepsilon}, \tag{19}
 \end{aligned}$$

Evidence of $\Sigma(1430)$

- $\Lambda_c \rightarrow \Lambda \pi^+ \pi^+ \pi^-$, **TS**



(a)



(b)

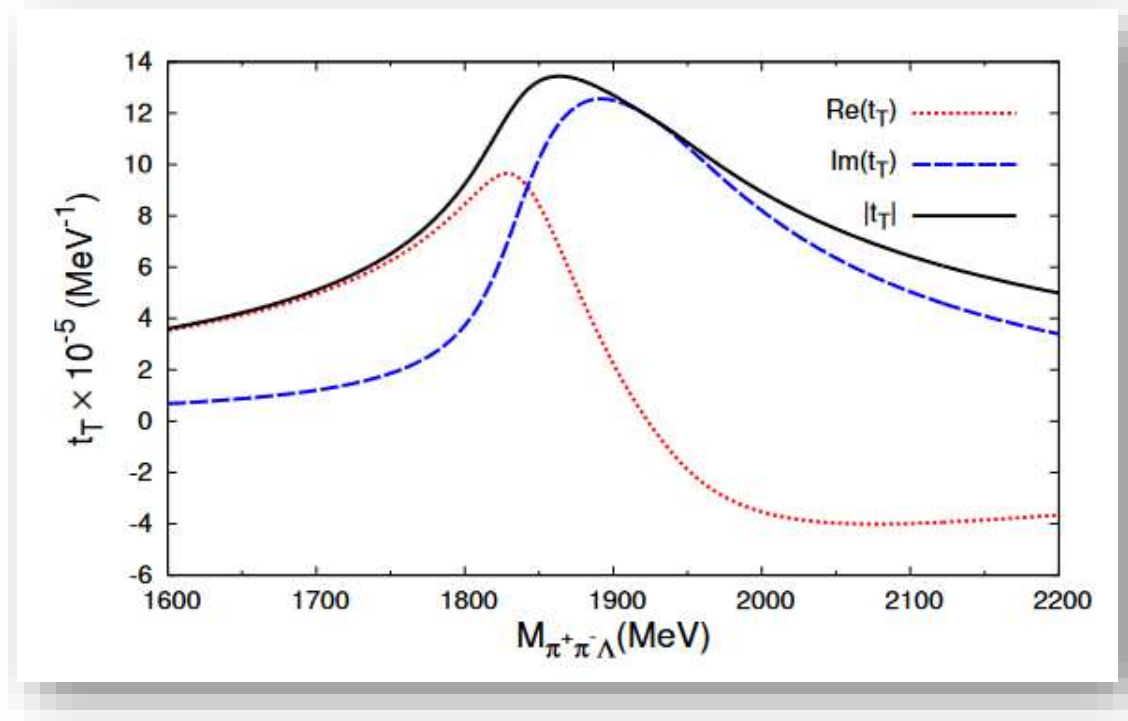
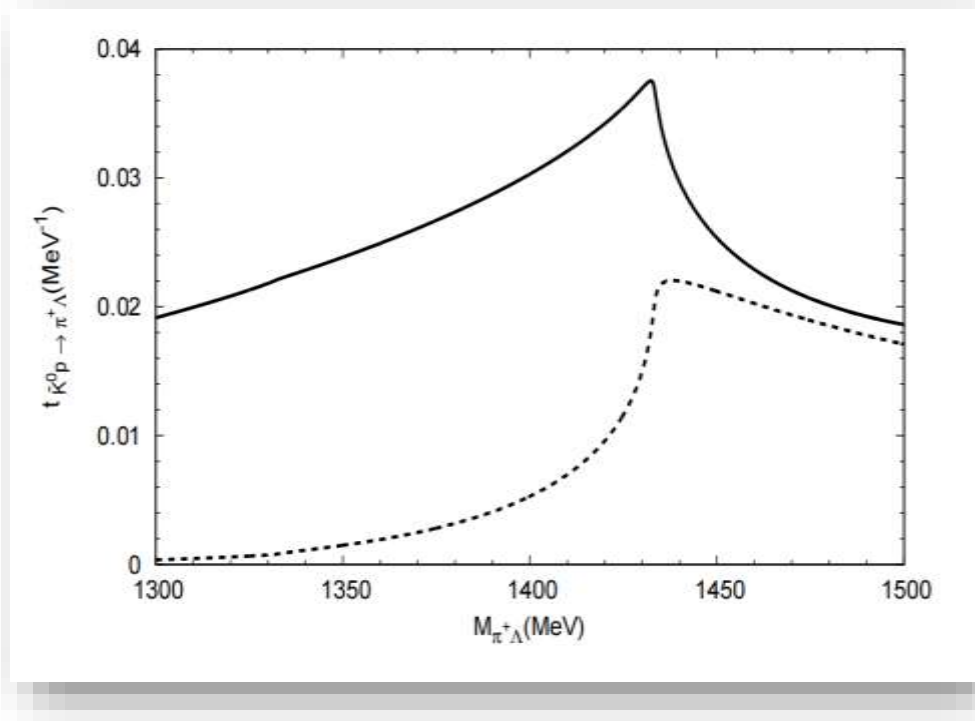
$$T^{\Sigma^{*+}(1385)} = \frac{V_p |p_{\pi^+}|}{M_{\pi^+\Lambda} - M_{\Sigma^{*+}} + i \frac{\Gamma_{\Sigma^{*+}}}{2}},$$

$$T^{\Sigma^{*-}(1385)} = \frac{V_p |p_{\pi^-}|}{M_{\pi^-\Lambda} - M_{\Sigma^{*-}} + i \frac{\Gamma_{\Sigma^{*-}}}{2}},$$

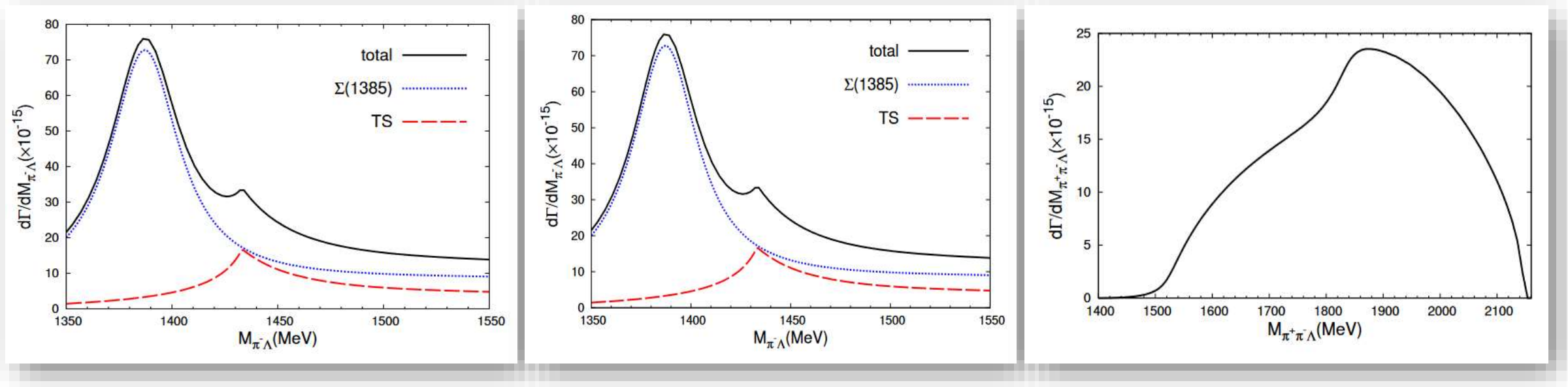
$$\frac{d^3\Gamma}{dM_{\pi^+\pi^-\Lambda} dM_{\pi^+\Lambda} dM_{\pi^-\Lambda}} = \frac{g^2 |A|^2 M_\Lambda}{64\pi^5 M_{\Lambda_c^+}} \tilde{p}_{\pi^+} \frac{M_{\pi^+\Lambda} M_{\pi^-\Lambda}}{M_{\pi^+\pi^-\Lambda}} \left\{ |\vec{k}_a|^2 |t_T^a \mathcal{M}^a|^2 + |\vec{k}_b|^2 |t_T^b \mathcal{M}^b|^2 + 2\text{Re}[t_T^a \mathcal{M}^a (t_T^b \mathcal{M}^b)^*] \right. \\ \left. \times \vec{k}_a \cdot \vec{k}_b + |T^{\Sigma^{*+}(1385)}|^2 + |T^{\Sigma^{*-}(1385)}|^2 \right\}, \quad (29)$$

Evidence of $\Sigma(1430)$

- $\Lambda_c \rightarrow \Lambda \pi^+ \pi^+ \pi^-$, **Lyu-GYW-EW-Xie-Geng, to prepare**



Results of $\Lambda_c \rightarrow \Lambda \pi^+ \pi^+ \pi^-$



Cusp signal of $\Sigma(1/2^-)$ around $\bar{K}N$ threshold!

Summary

- Belle measurements of $\Lambda_c \rightarrow \eta\Lambda\pi$ show some hints of the $\Sigma(1/2^-)$, and the more precise measurements could be used to test the existence of $\Sigma(1/2^-)$.
- The cusp observe in the pK^- mass distribution of $\Lambda_c \rightarrow pK^- \pi^+$ should be due to the TS and $\Lambda(1670)$, and the line-shape could be used to constrain the theoretical parameter.
- The cusp structure around 1430 MeV in $\Lambda_c \rightarrow \Lambda\pi\pi\pi$ could be associated with the $\Sigma(1430)$.

Thank you very much!