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第三届

强子与重味物理理论与实验

联合研讨会

河北大学 HEBEI UNIVERSITY

How to understand the charmed strange tetraquarks near 2.9 GeV?

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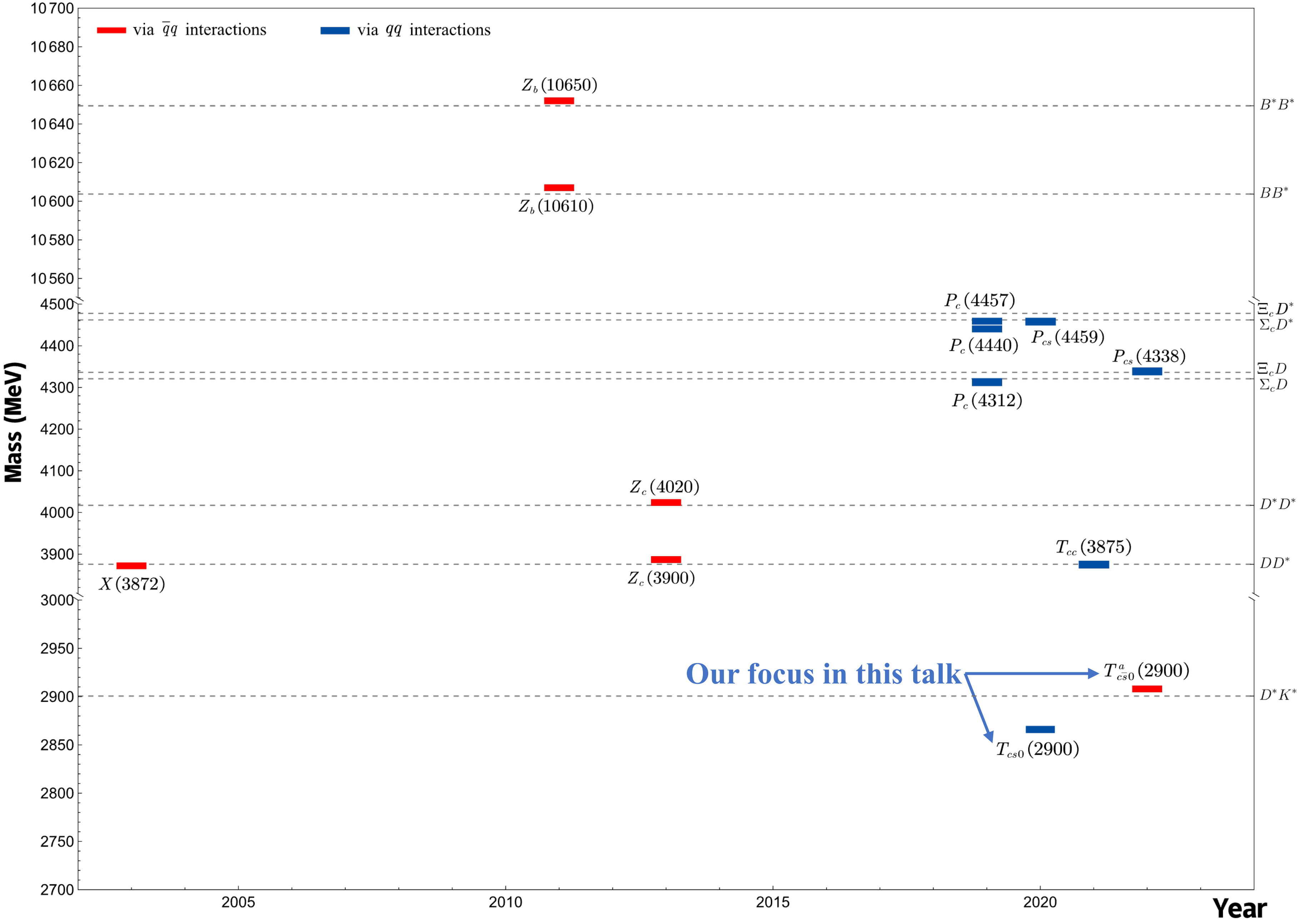
Outline

- **Background: spectrum of molecule candidates and implications**
- **An introduction to our framework**
- **Spectrum of the molecular tetraquarks:**
  1. **Understanding the $T_{cs0}(2900)$ and $T^{a}_{c\bar{s}0}(2900)$**
  2. **Highlights of our predictions**
- **Summary and outlook**

Background: spectrum of molecule candidates and implications



Two salient features:

1. Near-threshold
2. Strong isospin tropism

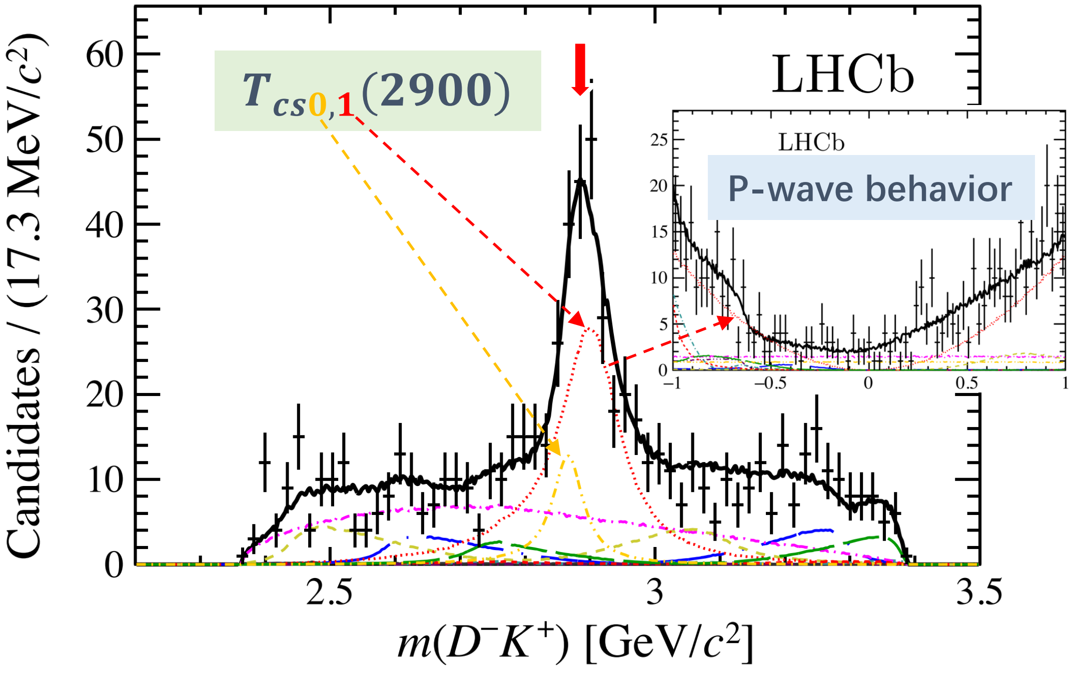
Are these states independent or correlated entities?

Recent reviews:

- H.-X. Chen et al, Phys. Rept. 639, 1 (2016)
- F.-K. Guo et al, Rev. Mod. Phys. 90, 015004 (2018)
- Y.-R. Liu et al, Prog. Part. Nucl. Phys. 107, 237 (2019)
- N. Brambilla et al, Phys. Rept. 873, 1 (2020)
- H.-X. Chen et al, Rept. Prog. Phys. 86, 026201 (2023)
- L. Meng et al, Phys. Rept. 1019, 1 (2023)

Background: status of the $T_{cs0}(2900)$

$B^+ \to D^+ D^- K^+$:Phys. Rev. D 102, 112003 (2020)



$T_{cs0}(2900)$, aka $X_0(2900)$

$M = 2.866 \pm 0.007 \pm 0.002\ \mathrm{GeV}/c^2$,

$\Gamma = 57 \pm 12 \pm 4\ \mathrm{MeV}$,

$m_{\bar{D}^* K^*} \simeq 2.9\ \mathrm{GeV}$

Explanations for $T_{cs}(2900)$

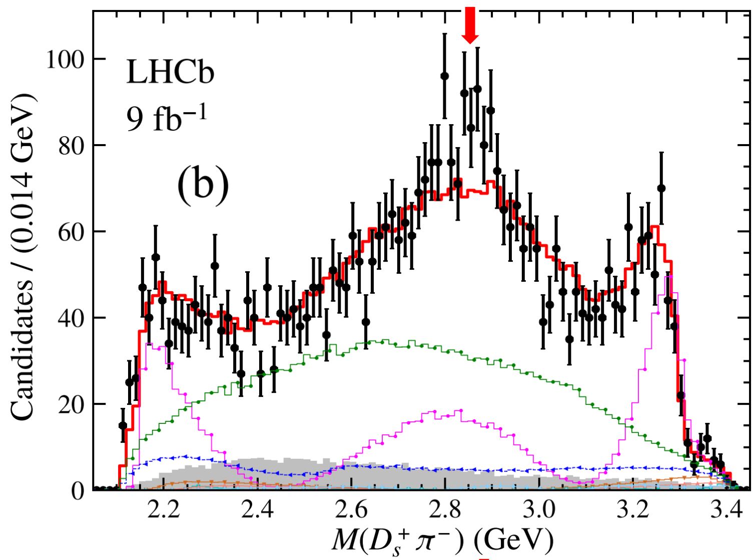
$\bar{D}^* K^*$ molecule	$\bar{c}\bar{s}ud$ tetraquark	Kinematic effect
Chen et al., Chin. Phys. Lett. 37, 101201 (2020)	Karliner et al., Phys. Rev. D 102, 094016(2020)	Liu et al., Eur. Phys. J. C 80, 1178 (2020)
He et al., Chin. Phys. C 45, 063102 (2021)	He et al., Eur. Phys. J. C 80, 1026 (2020)	Burns et al., Phys. Lett. B 813, 136057 (2021)
Liu et al., Phys. Rev. D 102, 091502 (2020)	Wang et al., Int. J. Mod. Phys. A 35, 2050187 (2020)	
Hu et al., Chin. Phys. C 45, 021003 (2021)	Zhang et al., Phys. Rev. D 103, 054019 (2021)	
B. Wang et al., Eur. Phys. J. C 82, 419 (2022)	Wang et al., Eur. Phys. J. C 81, 188 (2021)	
	Lü et al., Phys. Rev. D 102, 074021 (2020),	
	Tan et al., Chin. Phys. C 45, 093104 (2021)	

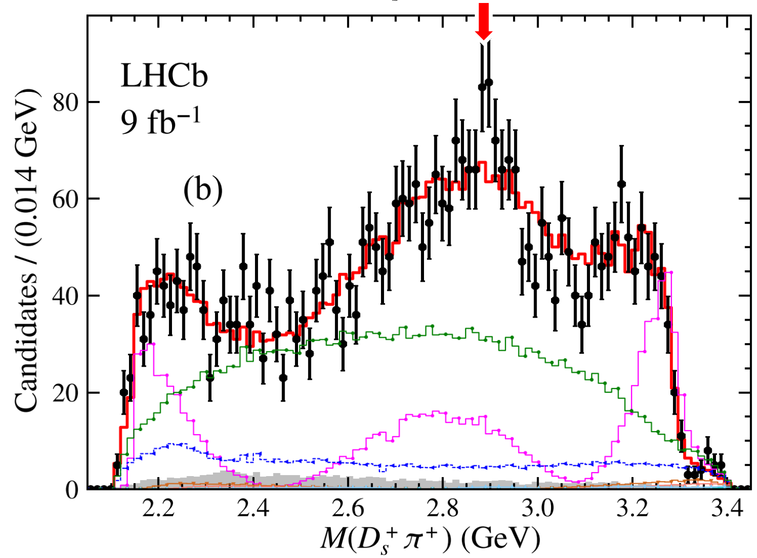
The calculations from quark model showed that

Compact tetraquark explanation appears incompatible with the current experimental data!

Background: status of the $T^a_{c\bar{s}0}(2900)$

$B^0 \to \overline{D}^0 D_s^+ \pi^-$ and $B^+ \to D^- D_s^+ \pi^+$:Phys. Rev. D 108, 012017 (2023)





$m_{D^*K^*} \simeq 2.9$ GeV

$$T^a_{c\bar{s}0}(2900)^0: \; M = (2.892 \pm 0.014 \pm 0.015)\ \text{GeV}, \quad \Gamma = (0.119 \pm 0.026 \pm 0.013)\ \text{GeV},$$

$$T^a_{c\bar{s}0}(2900)^{++}: \; M = (2.921 \pm 0.017 \pm 0.020)\ \text{GeV}, \quad \Gamma = (0.137 \pm 0.032 \pm 0.017)\ \text{GeV},$$

belong to an isospin triplet

with spin-parity 0^+

Explanations for $T_{c\bar{s}}(2900)$

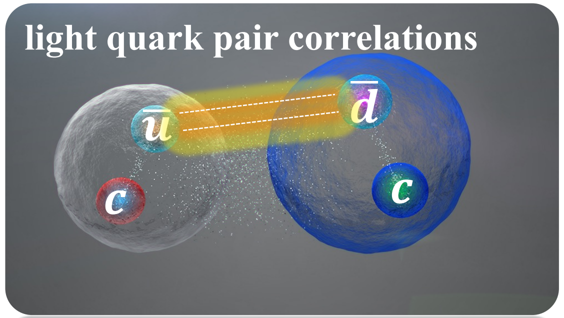
D^*K^* molecule: Chen et al., 2208.10196; Agaev et al., Phys. Rev. D 107, 094019 (2023); Yue et al., Phys. Rev. D 107, 034018 (2023); Duan et al., Phys. Rev. D 108, 074006 (2023) ; **B. Wang et al., 2309.02191**. [Ke et al., Phys. Rev. D 106, 114032 (2022) against a binding solution in the isovector D^*K^* system]

$c\bar{s}q\bar{q}$ tetraquark: Liu et al., Phys. Rev. D 107, 096020 (2023); Yang et al., Int. J. Mod. Phys. A 38, 2350056 (2023); Lian et al., 2302.01167; Wei et al., Phys. Rev. D 106, 096023 (2022); Ortega et al., 2305.14430.

For other related works, see PhysRevD.109.034027 and the references therein

Implications of the spectrum

Are these states independent or correlated entities?



2. Strong isospin tropism

1. Near-threshold → Highly plausible molecule candidates → Their existence is reliant on the light quark pair correlations

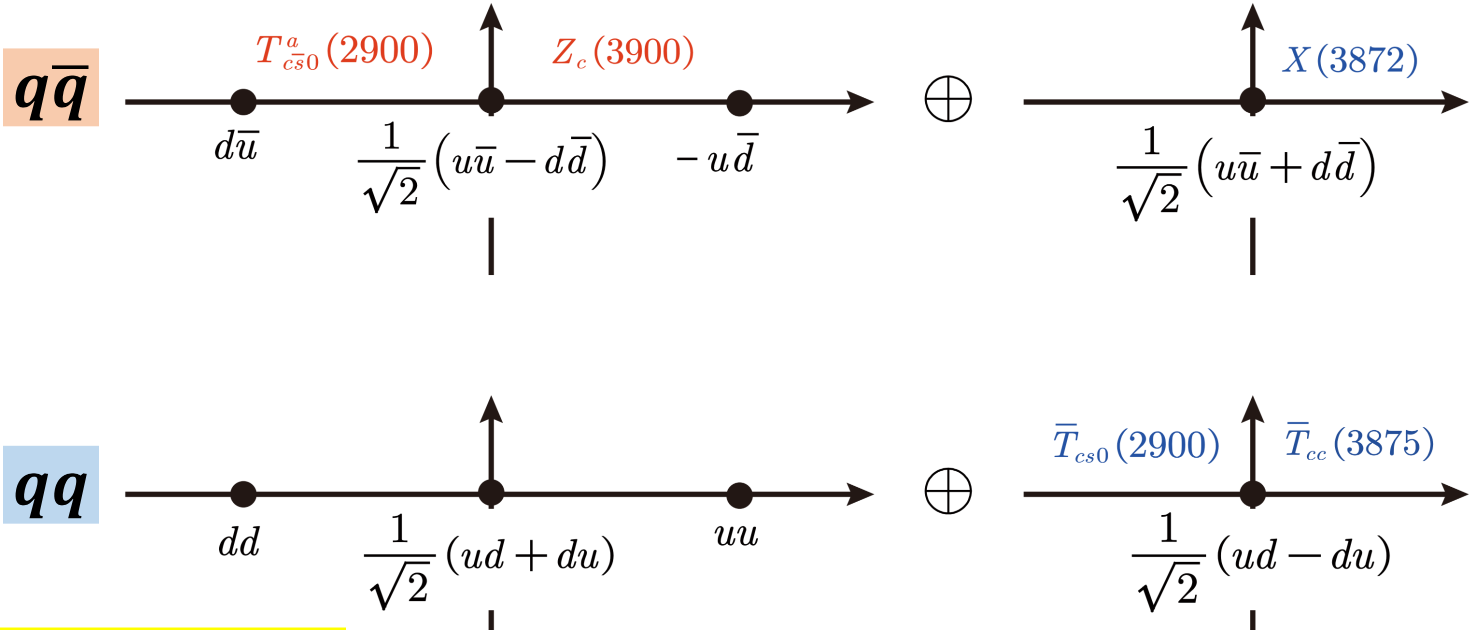
↓

These exotic states do not exist as independent entities

3. Flavor symmetry

Occam's Razor

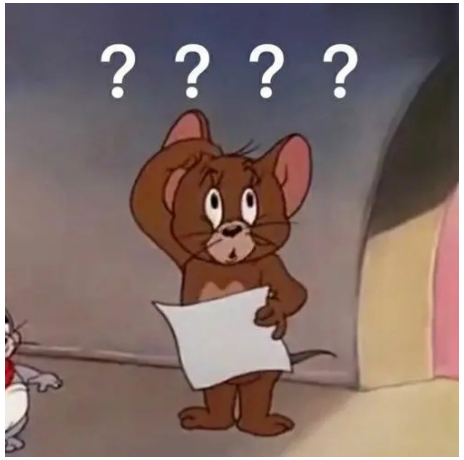
Light flavor wave functions



B. Wang et al., PhysRevD.109.034027

Are there underlying connections between

- $T^a_{c\bar{s}0}(2900)$ **and** $Z_c(3900)$, $X(3872)$
- $T_{cs0}(2900)$ **and** $T_{cc}(3875)$



An introduction to our framework

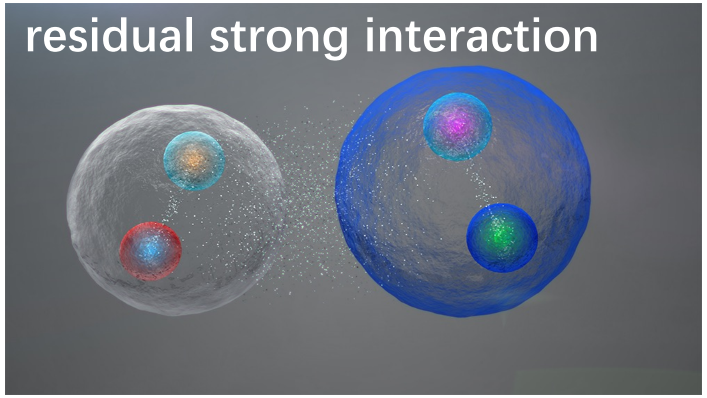
States	Nearest thresholds	Quark contents	V_{type}
$X(3872)$	$D\bar{D}^*$	$[c\bar{q}][\bar{c}q]$	
$Z_c(3900)$	$D\bar{D}^*$	$[c\bar{q}][\bar{c}q]$	$V_{\bar{q}q}$
$T^a_{c\bar{s}0}(2900)$	D^*K^*	$[c\bar{q}][\bar{s}q]$	
P_cs	$\Sigma_c\bar{D}^{(*)}$	$[cqq][\bar{c}q]$	
$\bar{T}_{cc}(3875)$	$\bar{D}\bar{D}^*$	$[\bar{c}q][\bar{c}q]$	V_{qq}
$\bar{T}_{cs0}(2900)$	$\bar{D}^*K^*$	$[\bar{c}q][\bar{s}q]$	

B. Wang et al., PhysRevD.109.034027

Strange quark: light or *heavy*?

$m_s < \Lambda_{\text{QCD}}$,
in contrast to the $m_{u,d} \ll \Lambda_{\text{QCD}}$
SU(3) symmetry is not good!

$m_s^{\text{QM}} \sim 500$ MeV
For shallow bound hadronic molecules:
$\gamma_b^{\text{ty}} = \sqrt{2\mu E_b} \lesssim 100$ MeV$\ll m_s^{\text{QM}}$
such as for the $DD^*/D\bar{D}^*$ systems:
$E_b \lesssim 10$ MeV

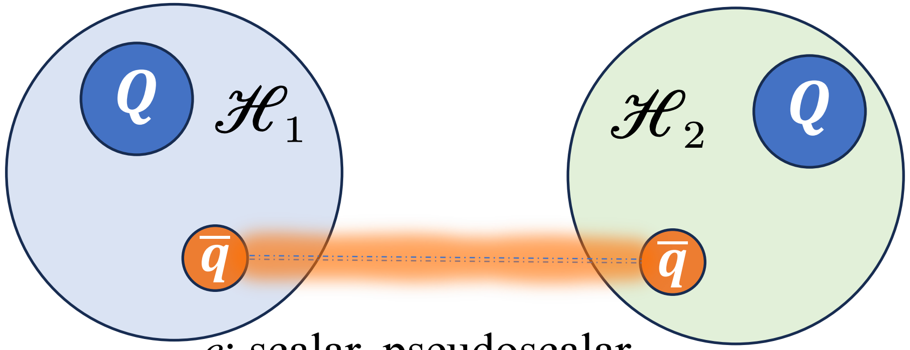


- ✓ The near-threshold interactions are too weak to excite the strange quarks inside the *heavy* hadrons.
- ✓ The strange quark will behave like an inert source (*heavy* quark) from the view of the residual strong interactions at the near-threshold energy scale.

An introduction to our framework

Constructing the effective potentials from a quark-level perspective

$$V_{\text{eff}} \sim \sum_e \frac{\{1, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}), \ldots\}}{\boldsymbol{q}^2 + M_e^2}$$

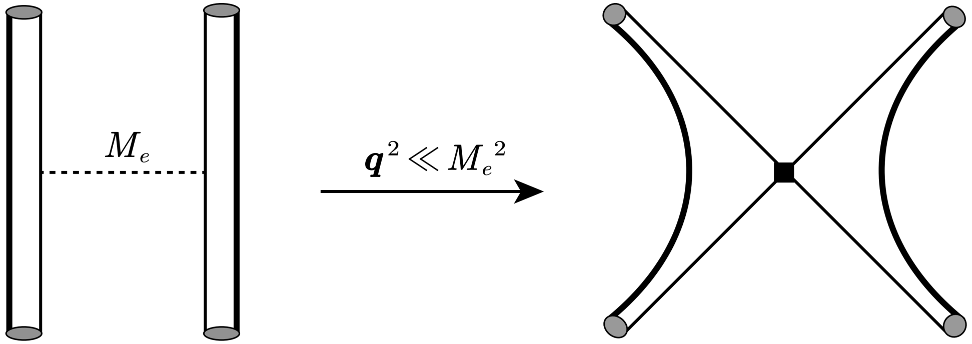


e: scalar, pseudoscalar, vector, axial-vector, tensor,…

For the near-threshold interactions: $\boldsymbol{q}^2 \ll M_e^2$

$$\frac{1}{\boldsymbol{q}^2 + M_e^2} = \frac{1}{M_e^2}\left(1 - \frac{\boldsymbol{q}^2}{M_e^2} + \ldots\right)$$

From "H" to a "butterfly"



Keep the leading order ($\boldsymbol{q}^0$) of the expansion

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An introduction to our framework

- ✓ Each exchanged meson fields contain both the isospin triplet and singlet:

$$\mathscr{S} = \mathscr{S}_i \tau_i + \frac{1}{\sqrt{2}} \mathscr{S}_1$$

- ✓ The net contributions from the exchanged particles will give rise to the non-relativistic effective potentials for the light qq and $q\bar{q}$ with the following forms, respectively,

$$V_{qq} = \left(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \frac{1}{2}\tau_{0,1} \cdot \tau_{0,2}\right)(c_s + c_a \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2),$$

$$V_{\bar{q}q} = \left(-\boldsymbol{\tau}_1^* \cdot \boldsymbol{\tau}_2 + \frac{1}{2}\tau_{0,1} \cdot \tau_{0,2}\right)(\tilde{c}_s + \tilde{c}_a \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2),$$

Pauli matrix in isospin space

Pauli matrix in spin space

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An introduction to our framework

✓ Translating the quark-level potential to the hadron-level:

$$V^{I,J}_{\mathscr{H}_1\mathscr{H}_2} = \left\langle [\mathscr{H}_1\mathscr{H}_2]^I_J \,|V_{qq}|\, [\mathscr{H}_1\mathscr{H}_2]^I_J \right\rangle$$

spin-flavor wave function of $\mathscr{H}_1\mathscr{H}_2$ di-hadron systems

✓ Equivalent to using a hadron-level contact Lagrangians:

$$\mathcal{L}_{\bar{D}^{(*)}K^*} = C_1(\bar{\tilde{\mathcal{H}}}\tilde{\mathcal{H}})(K^{*\mu\dagger}K^*_\mu) + C_2(\bar{\tilde{\mathcal{H}}}\tau_i\tilde{\mathcal{H}})(K^{*\mu\dagger}\tau_i K^*_\mu)$$
$$+iC_3(\bar{\tilde{\mathcal{H}}}\sigma^{\mu\nu}\tilde{\mathcal{H}})(K^{*\dagger}_\mu K^*_\nu)+iC_4(\bar{\tilde{\mathcal{H}}}\sigma^{\mu\nu}\tau_i\tilde{\mathcal{H}})(K^{*\dagger}_\mu\tau_i K^*_\nu)$$

B. Wang et al., PhysRevD.109.034027

An introduction to our framework

✓ To search for the bound/virtual state poles,

$$t = v + vGt, \longrightarrow t^{-1} = v^{-1} - G$$

$$G(E+i\epsilon) = \int_0^\Lambda \frac{k^2 dk}{(2\pi)^3}\frac{2\mu}{p^2-k^2+i\epsilon} = \frac{2\mu}{(2\pi)^3}\left[p\tanh^{-1}\left(\frac{p}{\Lambda}\right) - \Lambda - \frac{i\pi}{2}p\right]$$

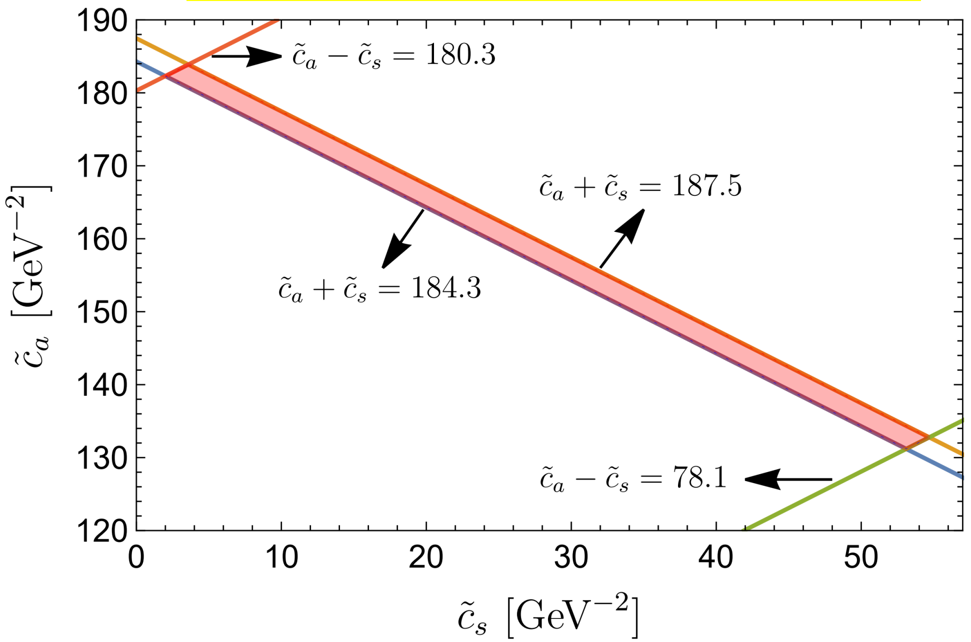
Bound state pole in Sheet-I (physical) : $G(E+i\epsilon)$

Virtual state pole in Sheet-II (unphysical) : $G(E+i\epsilon) + i\frac{\mu}{4\pi^2}p$

P_c and T_{cc} as inputs

$$c_s = 146.4 \pm 10.8\ \mathrm{GeV}^{-2},$$
$$c_a = -7.3 \pm 10.5\ \mathrm{GeV}^{-2}.$$

$X(3872)$ and $Z_c(3900)$ as inputs



Spectrum of the molecular tetraquarks: understanding the $T_{cs0}(2900)$

Systems	$I(J^P)$	$\langle\mathcal{O}_1\rangle$	$\langle\mathcal{O}_2\rangle$	$\langle\mathcal{O}_3\rangle$	$\langle\mathcal{O}_4\rangle$	$V_{\rm sys}$	E_B/E_V	States
DD	$1(0^+)$	$\frac{1}{2}$	1	0	0	$\frac{3}{2}c_s$	–	–
DD^*	$0(1^+)$	$\frac{1}{2}$	-3	$-\frac{1}{2}$	3	$-\frac{5}{2}c_s+\frac{5}{2}c_a$	$[3874.74\pm 0.04]_B$	$T_{cc}(3875)$ [Input]
	$1(1^+)$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}c_s+\frac{3}{2}c_a$	–	–
D^*D^*	$0(1^+)$	$\frac{1}{2}$	-3	$-\frac{1}{2}$	3	$-\frac{5}{2}c_s+\frac{5}{2}c_a$	$[4015.6^{+0.8}_{-2.3}]_B$	$\boldsymbol{T_{cc}(4015)}$
	$1(0^+)$	$\frac{1}{2}$	1	-1	-2	$\frac{3}{2}c_s-3c_a$	–	–
	$1(2^+)$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}c_s+\frac{3}{2}c_a$	–	–
$\bar{D}K^*$	$0(1^+)$	$\frac{1}{2}$	-3	0	0	$-\frac{5}{2}c_s$	$[2752.5^{+2.6}_{-3.6}]_V$	$\boldsymbol{T^f_{cs1}(2760)}$
	$1(1^+)$	$\frac{1}{2}$	1	0	0	$\frac{3}{2}c_s$	–	–
$\bar{D}^*K^*$	$0(0^+)$	$\frac{1}{2}$	-3	-1	6	$-\frac{5}{2}c_s+5c_a$	$[2897.8^{+2.6}_{-9.1}]_V$	$T^f_{cs0}(2900)$
	$0(1^+)$	$\frac{1}{2}$	-3	$-\frac{1}{2}$	3	$-\frac{5}{2}c_s+\frac{5}{2}c_a$	$[2896.6^{+3.0}_{-6.3}]_V$	$\boldsymbol{T^f_{cs1}(2900)}$
	$0(2^+)$	$\frac{1}{2}$	-3	$\frac{1}{2}$	-3	$-\frac{5}{2}c_s-\frac{5}{2}c_a$	$[2892.8^{+5.0}_{-9.8}]_V$	$\boldsymbol{T^f_{cs2}(2900)}$
	$1(0^+)$	$\frac{1}{2}$	1	-1	-2	$\frac{3}{2}c_s-3c_a$	–	–
	$1(1^+)$	$\frac{1}{2}$	1	$-\frac{1}{2}$	-1	$\frac{3}{2}c_s-\frac{3}{2}c_a$	–	–
	$1(2^+)$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}c_s+\frac{3}{2}c_a$	–	–

B. Wang et al., PhysRevD.109.034027

Spectrum of the molecular tetraquarks: understanding the $T^a_{c\bar{s}0}(2900)$

Systems	$I^{(G)}(J^{P(C)})$	$\langle\mathcal{O}_1\rangle$	$\langle\mathcal{O}_2\rangle$	$\langle\mathcal{O}_3\rangle$	$\langle\mathcal{O}_4\rangle$	$\tilde{V}_{\rm sys}$	E_B/E_V	States
$D\bar{D}$	$0^+(0^{++})$	$\frac{1}{2}$	-3	0	0	$-\frac{5}{2}\tilde{c}_s$	$[\sim 3696.5]_V$	$\boldsymbol{X(3700)}^\sharp$
	$1^-(0^{++})$	$\frac{1}{2}$	1	0	0	$\frac{3}{2}\tilde{c}_s$	–	–
$D\bar{D}^*$	$0^+(1^{++})$	$\frac{1}{2}$	-3	$\frac{1}{2}$	-3	$-\frac{5}{2}\tilde{c}_s-\frac{5}{2}\tilde{c}_a$	$[3871.2, 3871.6]_B$	$X(3872)$ [Input]
	$0^-(1^{+-})$	$\frac{1}{2}$	-3	$-\frac{1}{2}$	3	$-\frac{5}{2}\tilde{c}_s+\frac{5}{2}\tilde{c}_a$	–	–
	$1^+(1^{+-})$	$\frac{1}{2}$	1	$-\frac{1}{2}$	-1	$\frac{3}{2}\tilde{c}_s-\frac{3}{2}\tilde{c}_a$	$[3825.8, 3874.8]_V$	$Z_c(3900)$ [Input]
	$1^-(1^{++})$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}\tilde{c}_s+\frac{3}{2}\tilde{c}_a$	–	–
$D^*\bar{D}^*$	$0^+(0^{++})$	$\frac{1}{2}$	-3	-1	6	$-\frac{5}{2}\tilde{c}_s+5\tilde{c}_a$	–	–
	$0^-(1^{+-})$	$\frac{1}{2}$	-3	$-\frac{1}{2}$	3	$-\frac{5}{2}\tilde{c}_s+\frac{5}{2}\tilde{c}_a$	–	–
	$0^+(2^{++})$	$\frac{1}{2}$	-3	$\frac{1}{2}$	-3	$-\frac{5}{2}\tilde{c}_s-\frac{5}{2}\tilde{c}_a$	$[4011.8, 4012.2]_B$	$\boldsymbol{X(4012)}$
	$1^-(0^{++})$	$\frac{1}{2}$	1	-1	-2	$\frac{3}{2}\tilde{c}_s-3\tilde{c}_a$	$[4007.2, 4016.7]_B$	$\boldsymbol{Z_c(4010)}$
	$1^+(1^{+-})$	$\frac{1}{2}$	1	$-\frac{1}{2}$	-1	$\frac{3}{2}\tilde{c}_s-\frac{3}{2}\tilde{c}_a$	$[3973.6, 4014.5]_V$	$Z_c(4020)$
	$1^-(2^{++})$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}\tilde{c}_s+\frac{3}{2}\tilde{c}_a$	–	–
DK^*	$0(1^+)$	$\frac{1}{2}$	-3	0	0	$-\frac{5}{2}\tilde{c}_s$	$[\sim 2586.0]_V$	$\boldsymbol{T^f_{c\bar{s}0}(2760)}^\sharp$
	$1(1^+)$	$\frac{1}{2}$	1	0	0	$\frac{3}{2}\tilde{c}_s$	–	–
D^*K^*	$0(0^+)$	$\frac{1}{2}$	-3	-1	6	$-\frac{5}{2}\tilde{c}_s+5\tilde{c}_a$	–	–
	$0(1^+)$	$\frac{1}{2}$	-3	$-\frac{1}{2}$	3	$-\frac{5}{2}\tilde{c}_s+\frac{5}{2}\tilde{c}_a$	–	–
	$0(2^+)$	$\frac{1}{2}$	-3	$\frac{1}{2}$	-3	$-\frac{5}{2}\tilde{c}_s-\frac{5}{2}\tilde{c}_a$	$[2900.2, 2900.3]_V$	$\boldsymbol{T^f_{c\bar{s}2}(2900)}$
	$1(0^+)$	$\frac{1}{2}$	1	-1	-2	$\frac{3}{2}\tilde{c}_s-3\tilde{c}_a$	$[2887.6, 2900.5]_V$	$T^a_{c\bar{s}0}(2900)$
	$1(1^+)$	$\frac{1}{2}$	1	$-\frac{1}{2}$	-1	$\frac{3}{2}\tilde{c}_s-\frac{3}{2}\tilde{c}_a$	$[2676.2, 2876.3]_V$	$\boldsymbol{T^a_{c\bar{s}1}(2900)}$
	$1(2^+)$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}\tilde{c}_s+\frac{3}{2}\tilde{c}_a$	–	–

Spectrum of the molecular tetraquarks: understanding the $T^{a}_{c\bar{s}0}(2900)$

Line-shapes of $T^{a}_{c\bar{s}0}(2900)$ in the molecular scenario

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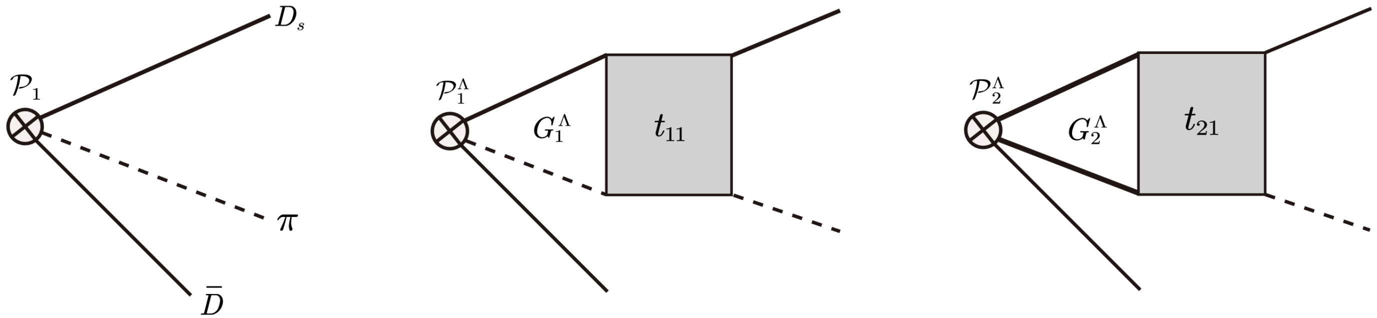
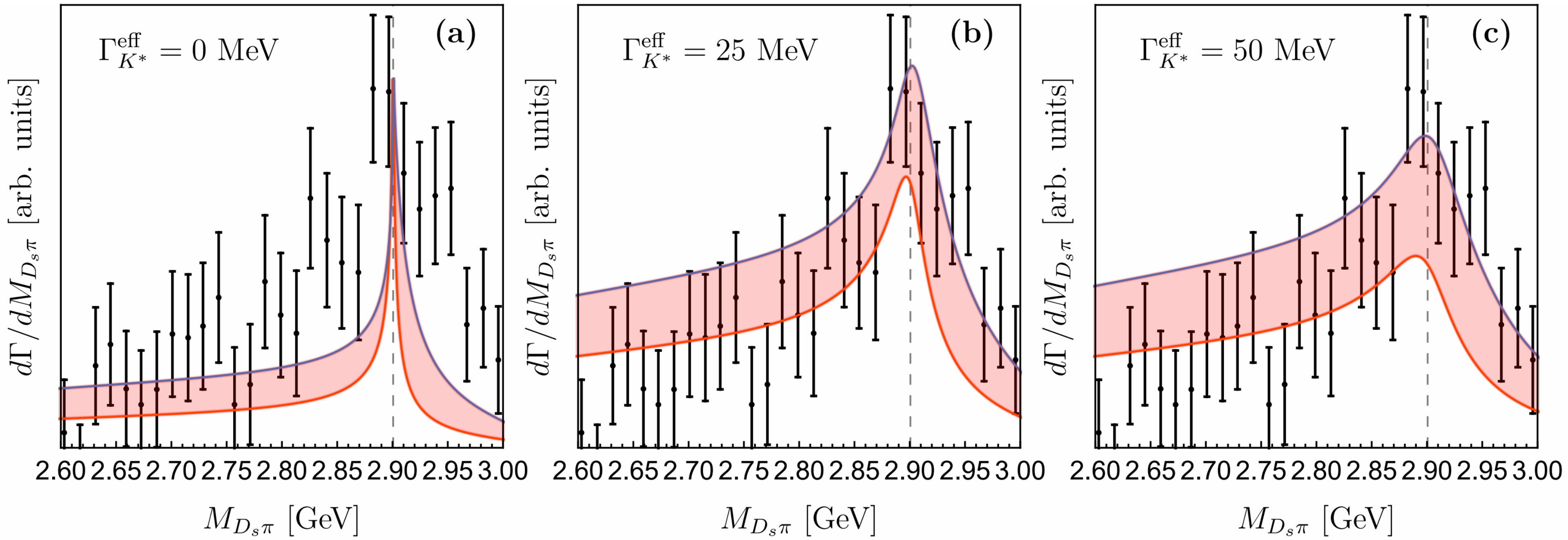
$$|1\rangle \equiv |D_s\pi\rangle$$

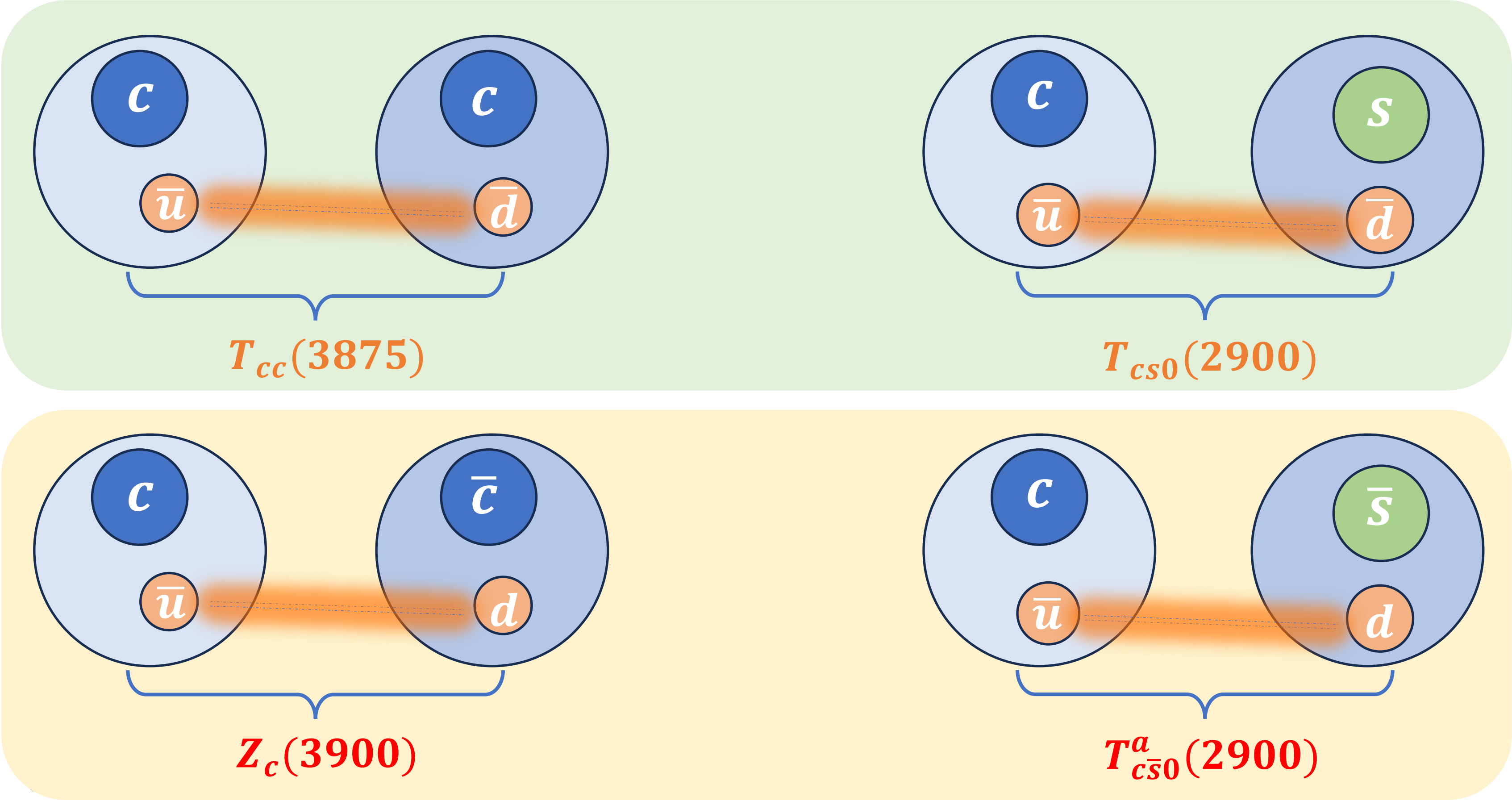
$$|2\rangle \equiv |D^* K^*\rangle$$





Unraveling the $T_{cs0}(2900)$ and $T^a_{c\bar{s}0}(2900)$

In our calculations, the recently discovered $T_{cs0}(2900)$ and $T^a_{c\bar{s}0}(2900)$ can be confidently identified as the charm-strange counterparts of $T_{cc}(3875)$ and $Z_c(3900)$, respectively.



$$Z_c(3900) \to J/\psi\pi$$
$$T_{c\bar{s}0}(2900) \to D_s\pi$$

Phys. Rev. Lett. **112**, 022001 (2014)

$$\frac{\Gamma[Z_c(3900) \to D\bar{D}^*]}{\Gamma[Z_c(3900) \to J/\psi\pi]} = 6.2 \pm 1.1 \pm 2.7$$

Conservative estimation

$$\frac{\Gamma[T_{c\bar{s}0}(2900) \to DK]}{\Gamma[T_{c\bar{s}0}(2900) \to D_s\pi]} \sim 10$$

Highlights of our predictions

Systems	$I(J^P)$	$\langle\mathcal{O}_1\rangle$	$\langle\mathcal{O}_2\rangle$	$\langle\mathcal{O}_3\rangle$	$\langle\mathcal{O}_4\rangle$	V_{sys}	E_B/E_V	States
DD	$1(0^+)$	$\frac{1}{2}$	1	0	0	$\frac{3}{2}c_s$	–	–
DD^*	$0(1^+)$	$\frac{1}{2}$	-3	$-\frac{1}{2}$	3	$-\frac{5}{2}c_s+\frac{5}{2}c_a$	$[3874.74\pm0.04]_B$	$T_{cc}(3875)$ [Input]
	$1(1^+)$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}c_s+\frac{3}{2}c_a$	–	–
D^*D^*	$0(1^+)$	$\frac{1}{2}$	-3	$-\frac{1}{2}$	3	$-\frac{5}{2}c_s+\frac{5}{2}c_a$	$\left[4015.6^{+0.8}_{-2.3}\right]_B$	$\boldsymbol{T_{cc}(4015)}$
	$1(0^+)$	$\frac{1}{2}$	1	-1	-2	$\frac{3}{2}c_s-3c_a$	–	–
	$1(2^+)$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}c_s+\frac{3}{2}c_a$	–	–
$\bar{D}K^*$	$0(1^+)$	$\frac{1}{2}$	-3	0	0	$-\frac{5}{2}c_s$	$\left[2752.5^{+2.6}_{-3.6}\right]_V$	$\boldsymbol{T^f_{cs1}(2760)}$
	$1(1^+)$	$\frac{1}{2}$	1	0	0	$\frac{3}{2}c_s$	–	–
$\bar{D}^*K^*$	$0(0^+)$	$\frac{1}{2}$	-3	-1	6	$-\frac{5}{2}c_s+5c_a$	$\left[2897.8^{+2.6}_{-9.1}\right]_V$	$T^f_{cs0}(2900)$
	$0(1^+)$	$\frac{1}{2}$	-3	$-\frac{1}{2}$	3	$-\frac{5}{2}c_s+\frac{5}{2}c_a$	$\left[2896.6^{+3.0}_{-6.3}\right]_V$	$\boldsymbol{T^f_{cs1}(2900)}$
	$0(2^+)$	$\frac{1}{2}$	-3	$\frac{1}{2}$	-3	$-\frac{5}{2}c_s-\frac{5}{2}c_a$	$\left[2892.8^{+5.0}_{-9.8}\right]_V$	$\boldsymbol{T^f_{cs2}(2900)}$
	$1(0^+)$	$\frac{1}{2}$	1	-1	-2	$\frac{3}{2}c_s-3c_a$	–	–
	$1(1^+)$	$\frac{1}{2}$	1	$-\frac{1}{2}$	-1	$\frac{3}{2}c_s-\frac{3}{2}c_a$	–	–
	$1(2^+)$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}c_s+\frac{3}{2}c_a$	–	–

Highlights of our predictions

$D^*D^*: 0(1^+)$

also expected in many works, such as Chen et al,. Eur. Phys. J. C 82, 581 (2022); Li et al., Phys. Rev. D 88, 114008 (2013); Liu et al., Phys. Rev. D 99, 094018 (2019); Ding et al., Eur. Phys. J. C 80, 1179 (2020); Dong et al., Commun. Theor. Phys. 73, 125201 (2021).

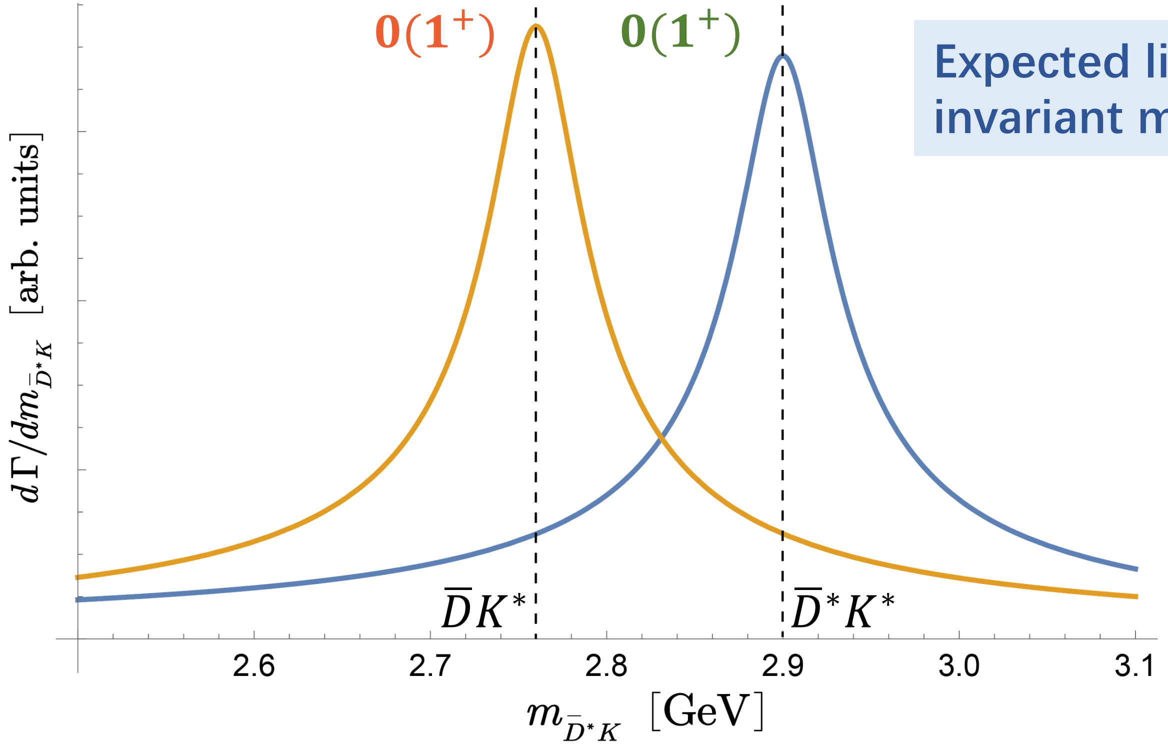
$\bar{D}K^*: 0(1^+)$ → $\bar{D}^*K$

It is expected that there should be a peak at the $\bar{D}K^*$ (2.76 GeV) and $\bar{D}^*K^*$ (2.9 GeV) thresholds in the $\bar{D}^*K$ channel, respectively.

$\bar{D}^*K^*: 0(1^+), 0(2^+)$

D-wave → $\bar{D}K$

D-wave → $\bar{D}^*K$



Expected line-shape of the $\bar{D}^*K$ invariant mass distribution

Highlights of our predictions

Systems	$I^{(G)}(J^{P(C)})$	$\langle\mathcal{O}_1\rangle$	$\langle\mathcal{O}_2\rangle$	$\langle\mathcal{O}_3\rangle$	$\langle\mathcal{O}_4\rangle$	$\tilde{V}_{\text{sys}}$	E_B/E_V	States
$D\bar{D}$	$0^+(0^{++})$	$\frac{1}{2}$	-3	0	0	$-\frac{5}{2}\tilde{c}_s$	$[\sim 3696.5]_V$	$\boldsymbol{X(3700)}^\sharp$ ✓
	$1^-(0^{++})$	$\frac{1}{2}$	1	0	0	$\frac{3}{2}\tilde{c}_s$	–	–
$D\bar{D}^*$	$0^+(1^{++})$	$\frac{1}{2}$	-3	$\frac{1}{2}$	-3	$-\frac{5}{2}\tilde{c}_s-\frac{5}{2}\tilde{c}_a$	$[3871.2, 3871.6]_B$	$X(3872)$ [Input]
	$0^-(1^{+-})$	$\frac{1}{2}$	-3	$-\frac{1}{2}$	3	$-\frac{5}{2}\tilde{c}_s+\frac{5}{2}\tilde{c}_a$	–	–
	$1^+(1^{+-})$	$\frac{1}{2}$	1	$-\frac{1}{2}$	-1	$\frac{3}{2}\tilde{c}_s-\frac{3}{2}\tilde{c}_a$	$[3825.8, 3874.8]_V$	$Z_c(3900)$ [Input]
	$1^-(1^{++})$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}\tilde{c}_s+\frac{3}{2}\tilde{c}_a$	–	–
$D^*\bar{D}^*$	$0^+(0^{++})$	$\frac{1}{2}$	-3	-1	6	$-\frac{5}{2}\tilde{c}_s+5\tilde{c}_a$	–	–
	$0^-(1^{+-})$	$\frac{1}{2}$	-3	$-\frac{1}{2}$	3	$-\frac{5}{2}\tilde{c}_s+\frac{5}{2}\tilde{c}_a$	–	–
	$0^+(2^{++})$	$\frac{1}{2}$	-3	$\frac{1}{2}$	-3	$-\frac{5}{2}\tilde{c}_s-\frac{5}{2}\tilde{c}_a$	$[4011.8, 4012.2]_B$	$\boldsymbol{X(4012)}$ ✓
	$1^-(0^{++})$	$\frac{1}{2}$	1	-1	-2	$\frac{3}{2}\tilde{c}_s-3\tilde{c}_a$	$[4007.2, 4016.7]_B$	$\boldsymbol{Z_c(4010)}$ ✓
	$1^+(1^{+-})$	$\frac{1}{2}$	1	$-\frac{1}{2}$	-1	$\frac{3}{2}\tilde{c}_s-\frac{3}{2}\tilde{c}_a$	$[3973.6, 4014.5]_V$	$Z_c(4020)$
	$1^-(2^{++})$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}\tilde{c}_s+\frac{3}{2}\tilde{c}_a$	–	–
DK^*	$0(1^+)$	$\frac{1}{2}$	-3	0	0	$-\frac{5}{2}\tilde{c}_s$	$[\sim 2586.0]_V$	$\boldsymbol{T^f_{c\bar{s}0}(2760)}^\sharp$ ✓
	$1(1^+)$	$\frac{1}{2}$	1	0	0	$\frac{3}{2}\tilde{c}_s$	–	–
D^*K^*	$0(0^+)$	$\frac{1}{2}$	-3	-1	6	$-\frac{5}{2}\tilde{c}_s+5\tilde{c}_a$	–	–
	$0(1^+)$	$\frac{1}{2}$	-3	$-\frac{1}{2}$	3	$-\frac{5}{2}\tilde{c}_s+\frac{5}{2}\tilde{c}_a$	–	–
	$0(2^+)$	$\frac{1}{2}$	-3	$\frac{1}{2}$	-3	$-\frac{5}{2}\tilde{c}_s-\frac{5}{2}\tilde{c}_a$	$[2900.2, 2900.3]_V$	$\boldsymbol{T^f_{c\bar{s}2}(2900)}$ ✓
	$1(0^+)$	$\frac{1}{2}$	1	-1	-2	$\frac{3}{2}\tilde{c}_s-3\tilde{c}_a$	$[2887.6, 2900.5]_V$	$T^a_{c\bar{s}0}(2900)$
	$1(1^+)$	$\frac{1}{2}$	1	$-\frac{1}{2}$	-1	$\frac{3}{2}\tilde{c}_s-\frac{3}{2}\tilde{c}_a$	$[2676.2, 2876.3]_V$	$\boldsymbol{T^a_{c\bar{s}1}(2900)}$ ✓
	$1(2^+)$	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}\tilde{c}_s+\frac{3}{2}\tilde{c}_a$	–	–

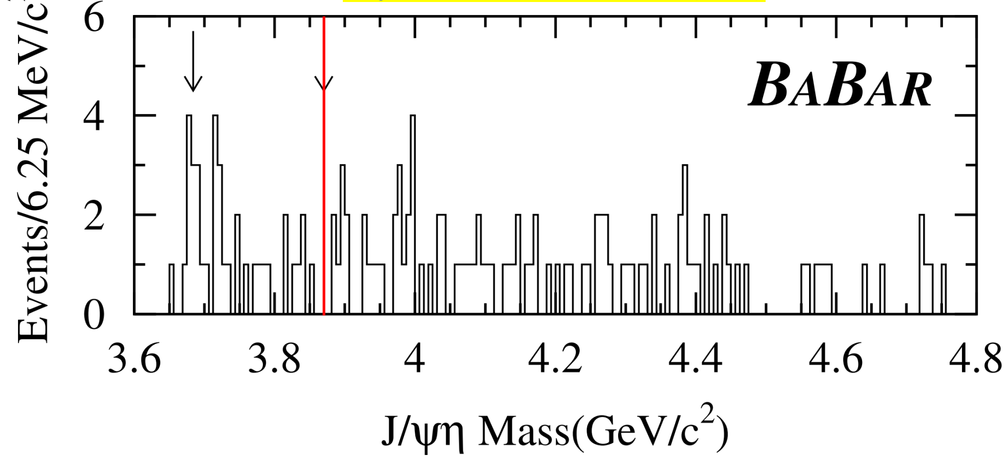
Highlights of our predictions

Does the $\tilde{X}$ (3872), the C-parity odd partner of $X(3872)$ exist?

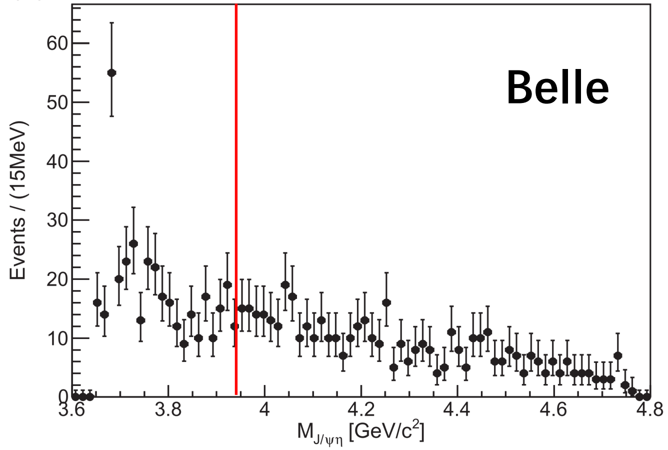
In our framework, $V[D\bar{D}^*, 0^-(1^{+-})] > 0$ ⟶ $\tilde{X}$ **(3872) does not exist!**

$\tilde{X}(3872)[0^-(1^{+-})] \to J/\psi[0^-(1^{--})] + \eta[0^+(0^{-+})]$

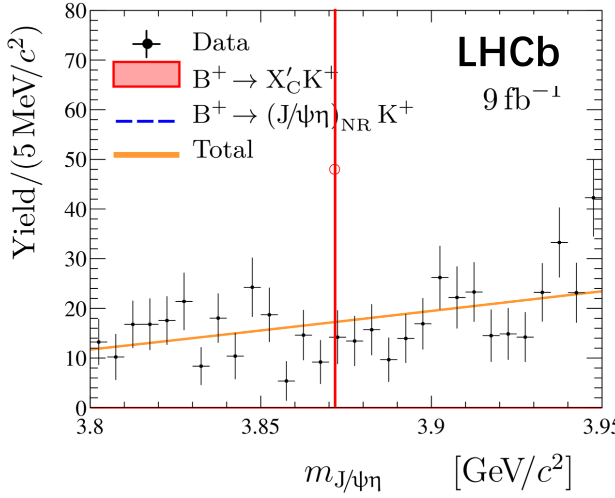
Phys. Rev. Lett. 93, 041801 (2004)



Prog. Theor. Exp. Phys. 2014, 43C01 (2014)



J. High Energy Phys.04, 046 (2022)



2403.16811

BESIII recently searched for the $e^+e^- \to \eta\tilde{X}$ **(3872): "*We do not find any evident signal for the* $\tilde{X}$ *(3872)......*"**

Highlights of our predictions

$D\bar{D}: 0^+(0^{++})$ — virtual state/nonexistent

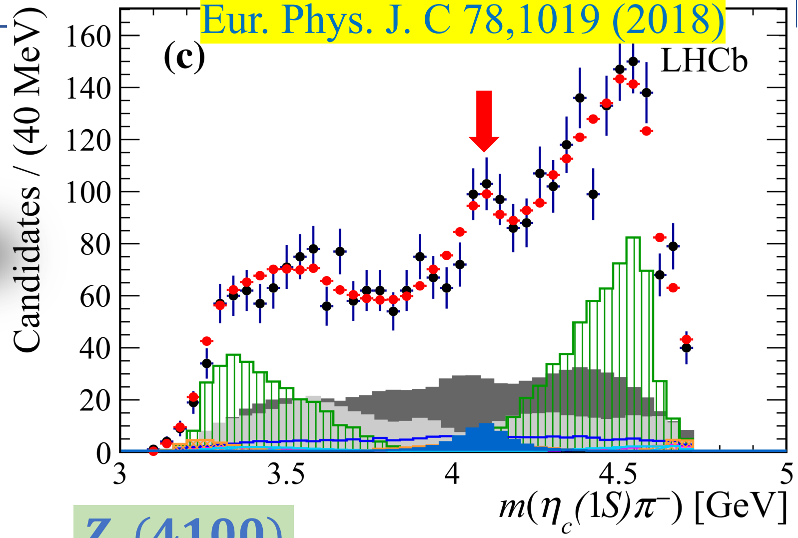
investigated in Phys. Rev. D 86, 056004 (2012); Phys. Rev. D 87, 076006 (2013) ; Phys. Rev. D 74, 014013 (2006) ; Phys. Rev. D 76, 074016 (2007); JHEP 06, 035 (2021).

$D^*\bar{D}^*: 0^+(2^{++}), 1^-(0^{++})$

- $0^+(2^{++})$: tensor partner of X(3872); **see the investigations** Phys. Rev. D 86, 056004 (2012) ; Phys.Rev. D 87, 076006 (2013) ; Chin. Phys. C 36, 194 (2012) ; Phys. Rev. D 88, 054007 (2013) ; Eur. Phys. J. C 75, 547 (2015) ; Phys. Lett. B 763, 20 (2016) .
- $1^-(0^{++})$: $\eta_c\pi, D\bar{D}, J/\psi\rho, \chi_{c1}\pi$ (P wave)

Evidence for an $\eta_c(1S)\pi^-$ resonance in $B^0 \to \eta_c(1S)K^+\pi^-$ decays

Eur. Phys. J. C 78,1019 (2018)



$Z_c(4100)$

$m = 4096 \pm 20^{+18}_{-22}$ MeV

$\Gamma = 152 \pm 58^{+60}_{-35}$ MeV

$DK^*: 0(1^+)$ — virtual state/nonexistent

can be used to infer the existence of the $0^+(0^{++})$ $D\bar{D}$

$D^*K^*: 0(2^+), 1(1^+)$ → DK, D^*K (D-wave), $D_s^*\pi$

Conservative estimation

$$\frac{\Gamma[T^a_{c\bar{s}1}(2900) \to D^*K]}{\Gamma[T^a_{c\bar{s}1}(2900) \to D_s^*\pi]} \sim 10$$

Summary and outlook

- The hadronic molecules do not exist as independent entities.
- Their formation crucially depends on the correlations between light quark pairs within separate hadrons.
- We have successfully constructed the complete mass spectrum of molecular tetraquarks, involving the $D^{(*)}D^{(*)}$, $D^{(*)}\bar{D}^{(*)}$, $D^{(*)}K^*$, and $\bar{D}^{(*)}K^*$ systems.
- The experimental search for the predicted states in various decay channels continues to be of great importance in advancing our understanding of the underlying physics.