Double-mixing CP Violation in B decays

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Based on arXiv: 2301.05848 and 2403.01904

Outline

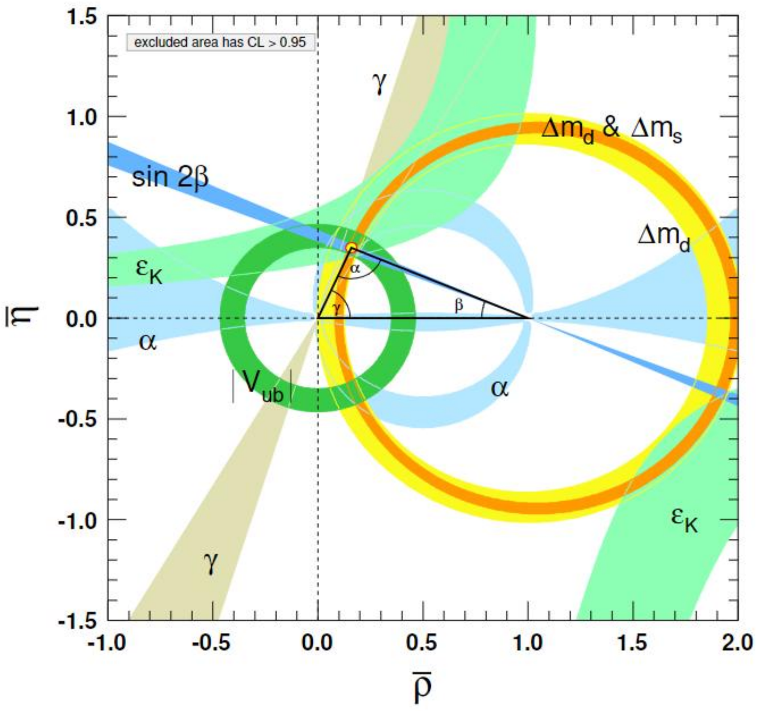
- Background and motivation
- Theoretical framework
- Numerical results
- CKM phase extraction
- Summary

CP violation

- SM precision test

Unitarity

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



[PDG, 2023]

- A necessary condition for baryogenesis, the process of dynamically generating the matter-antimatter asymmetry of the universe.

Search for new **CPV**

How?

- Baryon number violation
- C and CP violation
- Thermodynamic non equilibrium

Sakharov conditions

[A.D.Sakharov et al, Fiz.5,32-35 (1967)]

CP violation observables

- Common CPV observables

  ✓ Direct CP violation

$$\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| \neq 1 \quad \Longleftrightarrow \quad \Gamma(M \to f) \neq \Gamma(\bar{M} \to \bar{f})$$

  ✓ CPV in mixing (indirect CPV)

$$\left|\frac{q}{p}\right| \neq 1 \quad \Longleftrightarrow \quad \Gamma(M^0 \to \overline{M^0}) \neq \Gamma(\overline{M^0} \to M^0)$$

  ✓ CPV in interference between a decay with and without **initial** mixing

$$(M^0 \to f) + (M^0 \to \overline{M^0} \to f)$$

- CPV in interference between a decay with and without **final** mixing

$$(P \to M^0) + (P \to \overline{M^0} \to M^0)$$

[Wang, Yu, Li, *PRL* 119(2017)181802]

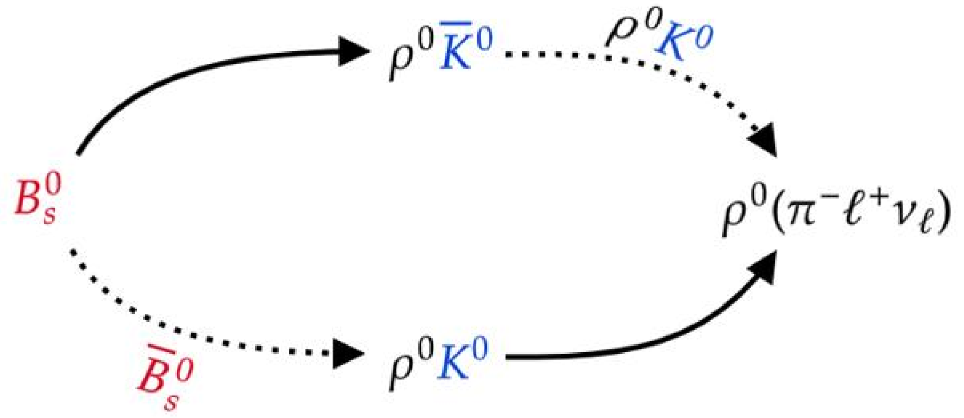
New type of CPV

Consider $B_s^0 \dashrightarrow \rho^0 K \dashrightarrow \rho^0(\pi^- \ell^+ \nu_\ell)$

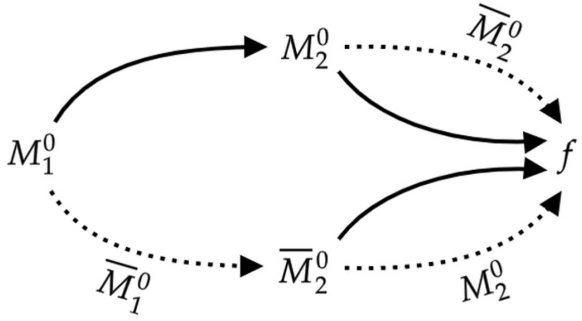
Upper path $\quad B_s^0 \to \rho^0 \overline{K}^0 \to \rho^0 K^0 \to \rho^0(\pi^- \ell^+ \nu_\ell)$

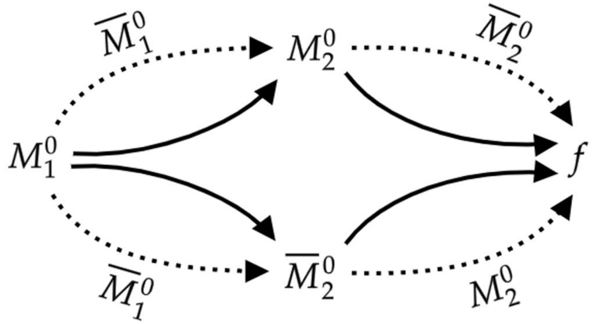
Lower path $\quad B_s^0 \to \overline{B}_s^0 \to \rho^0 K^0 \to \rho^0(\pi^- \ell^+ \nu_\ell)$

- Induced by interferences between two mixing processes in one cascade decay.



- More complicated cases





[Shen, **WJS**, Qin, 2301.05848]

Double-mixing CPV

Adv:

- The **double-mixing CPV** can be very significant and practically measurable by experiments.
- It does not require nonzero strong phases, providing opportunities to directly extract weak phases without strong pollution.
- Strong phases can be extracted from experimental data without theoretical input.
- The two-dimensional time-dependent CP asymmetry can be analyzed. ➡ $A_{CP}(t_1, t_2)$

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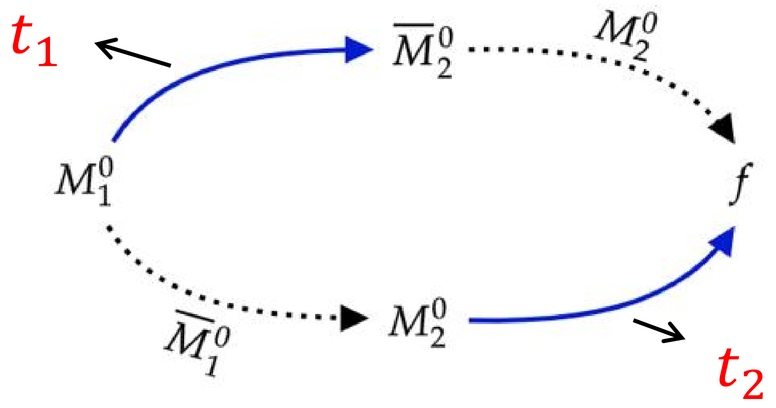
CP asymmetry

- The two-dimensional time-dependent CP asymmetry is defined as

$$A_{CP}(t_1,t_2) \equiv \frac{\Gamma_f(t_1,t_2) - \Gamma_{\bar{f}}(t_1,t_2)}{\Gamma_f(t_1,t_2) + \Gamma_{\bar{f}}(t_1,t_2)} = \frac{N(t_1,t_2)}{D(t_1,t_2)}$$



$$|M^0(t)\rangle = g_+(t)|M^0\rangle - \frac{q}{p} g_-(t)|\bar{M}^0\rangle$$
$$|\bar{M}^0(t)\rangle = g_+(t)|\bar{M}^0\rangle - \frac{p}{q} g_-(t)|M^0\rangle$$

$$A_{CP}(t_1,t_2) = \frac{\left|g_{1,+}(t_1)\right|^2 C_+(t_2) + \left|g_{1,-}(t_1)\right|^2 C_-(t_2) + e^{-\Gamma_1 t_1} \sinh\frac{\Delta\Gamma_1 t_1}{2} S_h(t_2) + e^{-\Gamma_1 t_1} \sin\Delta m_1 t_1\, S_n(t_2)}{\left|g_{1,+}(t_1)\right|^2 C'_+(t_2) + \left|g_{1,-}(t_1)\right|^2 C'_-(t_2) + e^{-\Gamma_1 t_1} \sinh\frac{\Delta\Gamma_1 t_1}{2} S'_h(t_2) + e^{-\Gamma_1 t_1} \sin\Delta m_1 t_1\, S'_n(t_2)}$$

✓ Take $B_d^0(t_1) \to J/\psi\, K(t_2) \to J/\psi\, (\pi^+\pi^-)$ as an example :

$$S_n(t_2) = \frac{e^{-\Gamma_{\rm K} t_2}}{2}\left[\sin \Delta m_{\rm K} t_2 \left(\left|\frac{q_K}{p_K}\right| - \left|\frac{p_K}{q_K}\right| \right) \cos(\omega_{B_d} - \phi_{B_d} + \phi_K) - \sinh\frac{\Delta\Gamma_{\rm K} t_2}{2} \left(\left|\frac{q_K}{p_K}\right| + \left|\frac{p_K}{q_K}\right| \right) \sin(\omega_{B_d} - \phi_{B_d} + \phi_K) \right]$$

$\omega_{B_d} - \phi_{B_d} + \phi_K \approx -2\beta$

Approximations

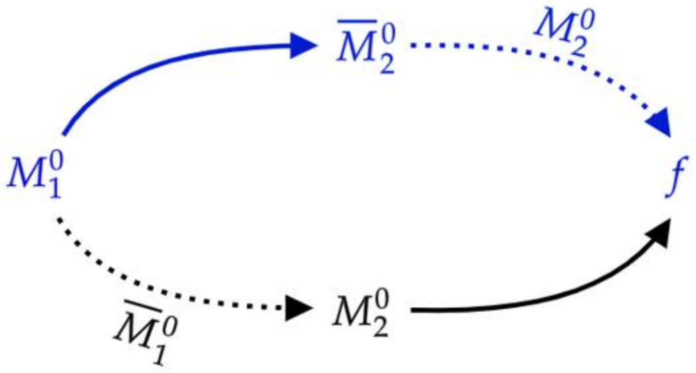
- $|q/p|_{B_d} \approx 1$ and $|q/p|_K \approx 1$
- $\cosh(\Delta\Gamma_{B_d} t_1/2) \approx 1$ and $\sinh(\Delta\Gamma_{B_d} t_1/2) \approx 0$

$$S_n(t_2) = \frac{1}{2} \sin 2\beta\, e^{-\Gamma_s t_2} \quad \Longrightarrow \quad A_{CP}(t_1, t_2) = \sin 2\beta \sin \Delta m_{B_d} t_1$$

which is consistent with the formulas (13.74) and (13.82) in the "CP Violation in the Quark Sector" review of the PDG.

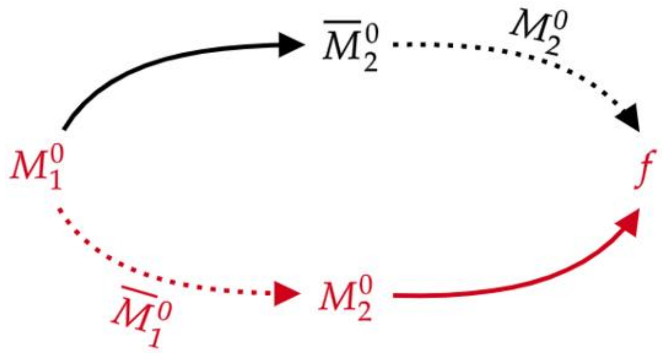
Theoretical framework

✓ Take $B_s^0(t_1) \to \rho^0 K(t_2) \to \rho^0(\pi^- \ell^+ \nu_\ell)$ as an example (penguin ≈ 0)



$$\propto \quad |g_{1,+}(t_1)|^2 |g_{2,-}(t_2)|^2 \left(\left|\frac{p_2}{q_2}\right|^2 - \left|\frac{q_2}{p_2}\right|^2 \right)$$

CP violation in $\boldsymbol{M_2^0}$ **mixing**

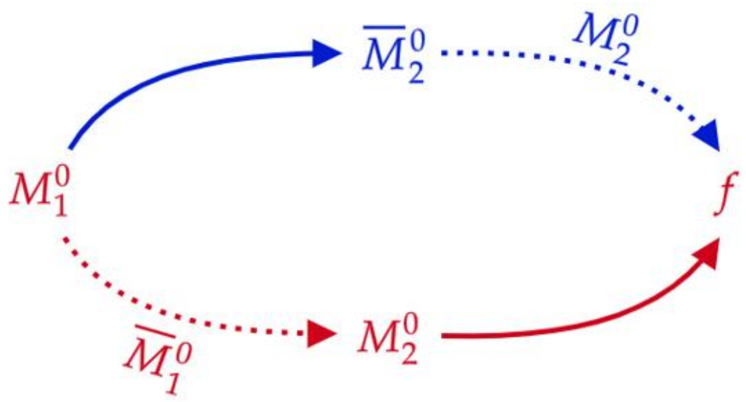


$$\propto \quad |g_{1,-}(t_1)|^2 |g_{2,+}(t_2)|^2 \left(\left|\frac{q_1}{p_1}\right|^2 - \left|\frac{p_1}{q_1}\right|^2 \right)$$

CP violation in $\boldsymbol{M_1^0}$ **mixing**

Double mixing CPV

✓ Take $B_s^0(t_1) \to \rho^0 K(t_2) \to \rho^0(\pi^- \ell^+ \nu_\ell)$ as an example (penguin ≈ 0)



$$A_h(t_1, t_2) \quad \propto \quad -e^{-\Gamma_1 t_1} \sinh \frac{\Delta\Gamma_1 t_1}{2} \, [2S_{n2}(t_2) \; \sin(\phi_1 + \phi_2 + 2\delta)]$$

$K_S - K_L$ interference

$$A_n(t_1, t_2) \quad \propto \quad e^{-\Gamma_1 t_1} \sin\Delta m_1 t_1 \, [2S_{h2}(t_2) \; \sin(\phi_1 + \phi_2 + 2\delta)]$$

$K_S + K_L$

$$q_1/p_1 = |q_1/p_1| \, e^{-i\phi_1}$$
$$q_2/p_2 = |q_2/p_2| \, e^{-i\phi_2}$$
$$\langle \rho^0 \overline{K}^0 | B_s^0 \rangle = \langle \rho^0 K^0 | \overline{B}_s^0 \rangle e^{2i\delta}$$

- With ϕ_1, ϕ_2, δ the relevant weak phases.

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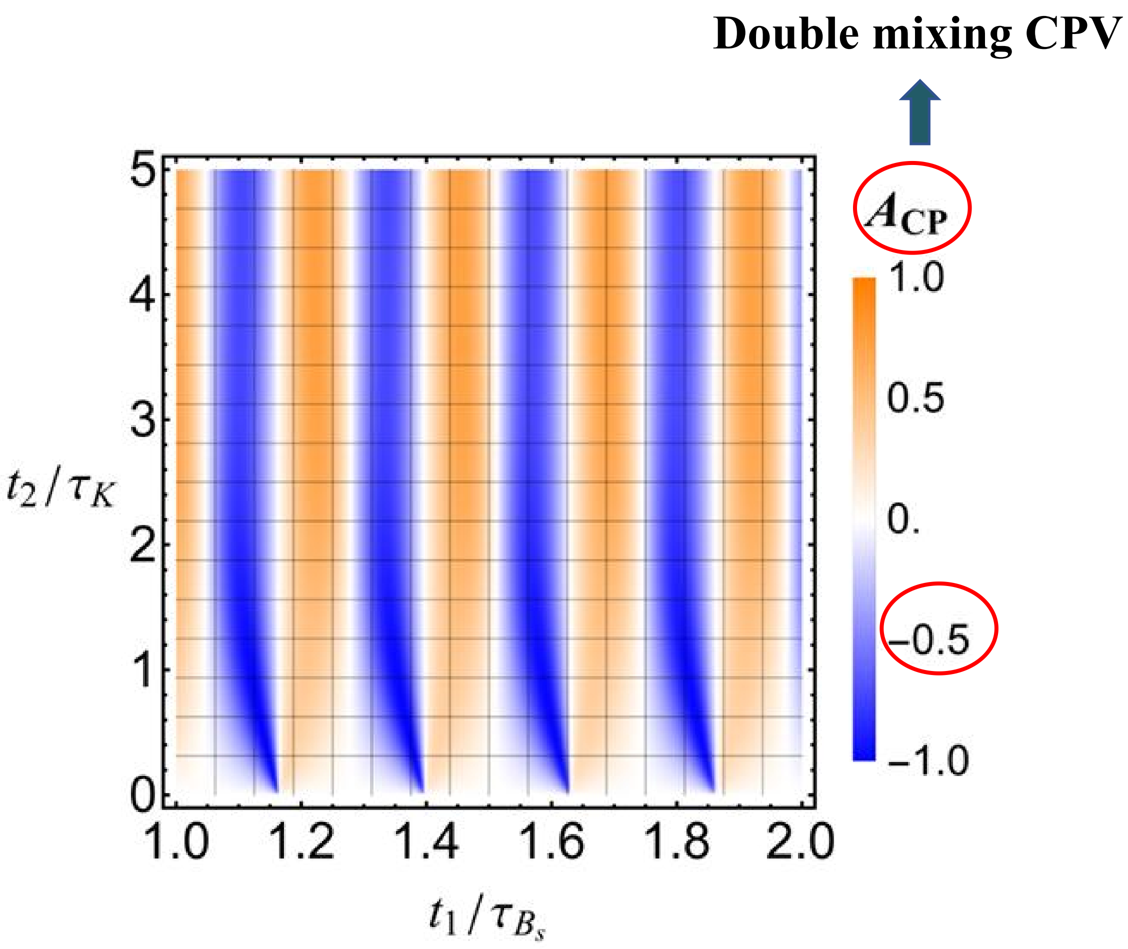
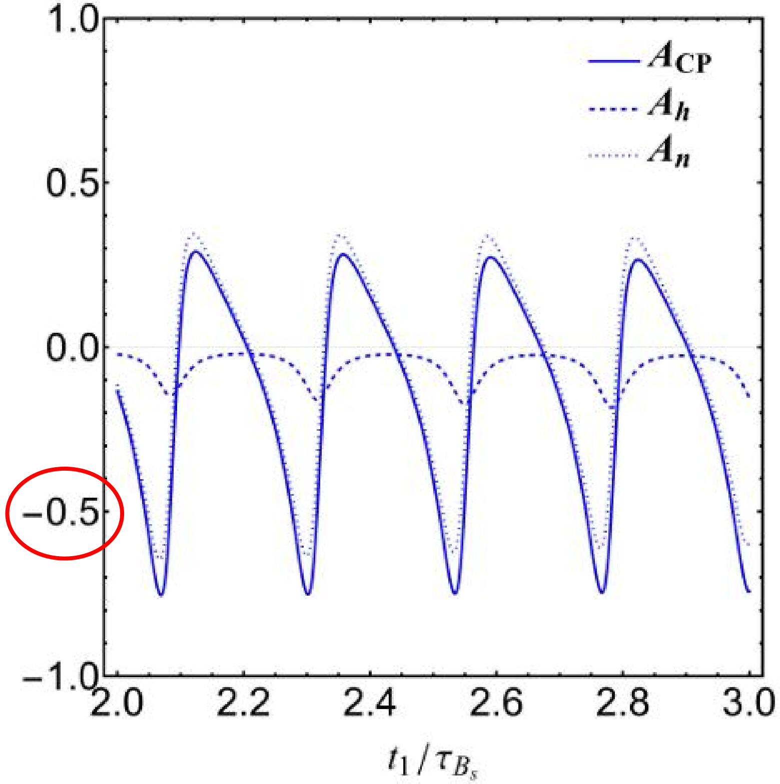
t_1, t_2 Dependence

- $B_s^0(t_1) \to \rho^0 K(t_2) \to \rho^0(\pi^- \ell^+ \nu_\ell)$
- The $A_{CP}(t_1, t_2)$ dependence on t_1 and t_2.
- The magnitude of the peak values can be larger than **50%**.

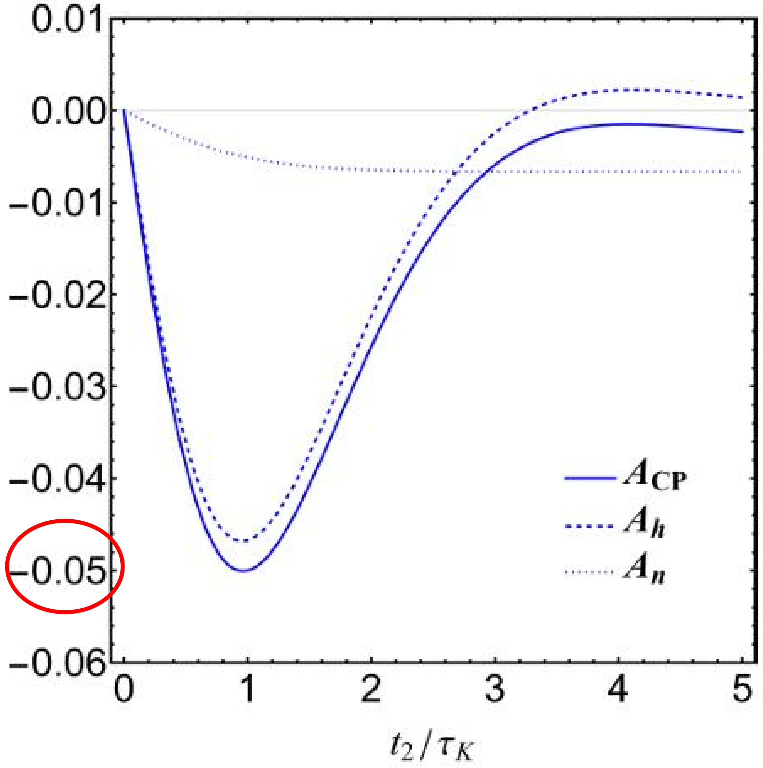


t_1 Dependence



- Integrate out the t_2 from 0 to τ_K

t_2 Dependence



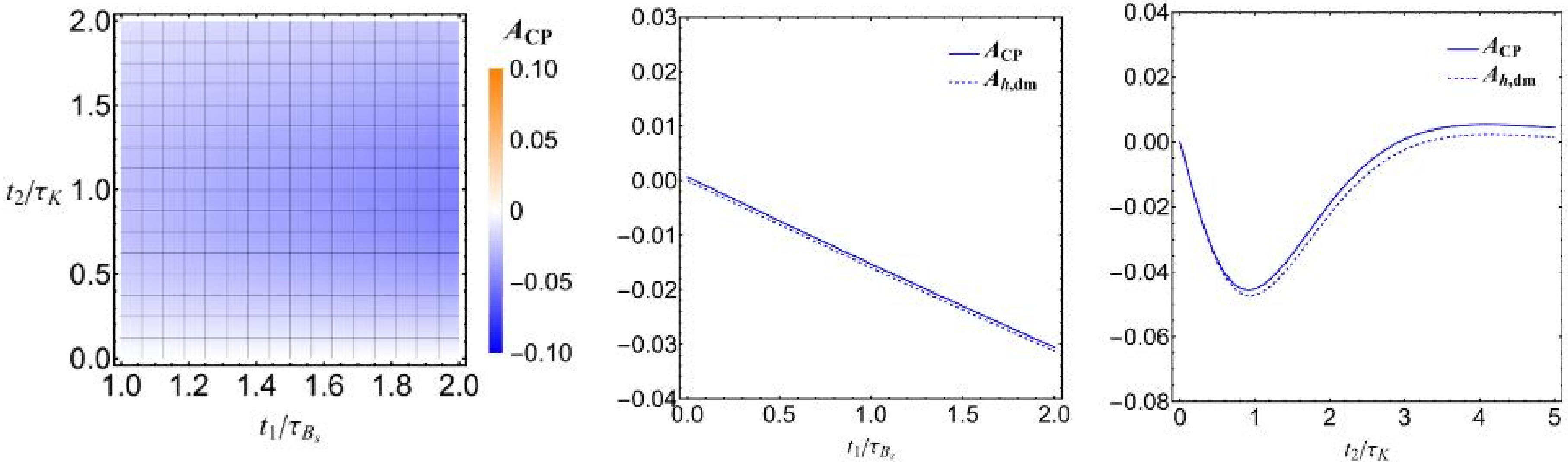
- Integrate out the t_1 from τ_{B_s} to $5\tau_{B_s}$

First experimental attempt to measure the double-mixing CP violation:

$$\frac{\mathcal{B}[B_s^0/\overline{B_s^0}\,(t_1) \to \rho^0 K(t_2) \to \rho^0(\pi^-\ell^+\nu_\ell)] - \mathcal{B}[B_s^0/\overline{B_s^0}\,(t_1) \to \rho^0 K(t_2) \to \rho^0(\pi^+\ell^-\overline{\nu_\ell})]}{\mathcal{B}[B_s^0/\overline{B_s^0}\,(t_1) \to \rho^0 K(t_2) \to \rho^0(\pi^-\ell^+\nu_\ell)] + \mathcal{B}[B_s^0/\overline{B_s^0}\,(t_1) \to \rho^0 K(t_2) \to \rho^0(\pi^+\ell^-\overline{\nu_\ell})]}$$

Combine the two decay processes: $B_s^0\,(t_1) \to \rho^0 K(t_2) \to \rho^0(\pi^-\ell^+\nu_\ell)$ **and** $B_s^0\,(t_1) \to \rho^0 K(t_2) \to \rho^0(\pi^+\ell^-\overline{\nu_\ell})$

- Initial tagging for B mesons in experiments is not necessary, thus the corresponding efficiency loss is prevented.



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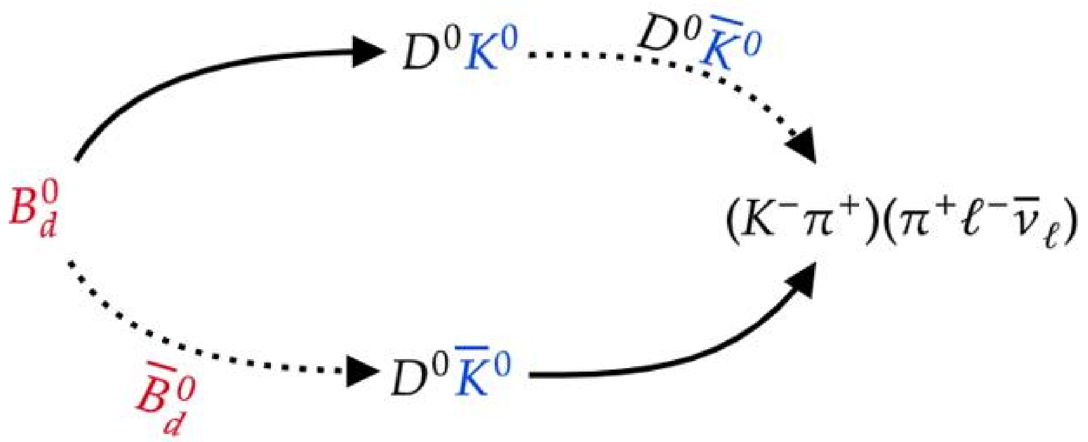
CKM phase extraction

✓ Take $B_d^0(t_1) \to D^0 K(t_2) \to (K^-\pi^+)(\pi^+\ell^-\overline{\nu_\ell})$ as an example (penguin = 0)

$$\langle D^0\overline{K}^0|\overline{B}_d^0\rangle = \langle\overline{D}^0K^0|B_d^0\rangle e^{i\theta_1}$$

$$\langle D^0K^0|B_d^0\rangle = \langle\overline{D}^0K^0|B_d^0\rangle\, r_B e^{i(\delta+\theta_2)}$$



- $A_{CP}(t_1, t_2) \propto \sin\omega'$
- **Parameters:** r_B、δ、$\underbrace{\phi_1 - \phi_2 + \theta_2 - \theta_1}_{\omega'} \approx 2\beta + \gamma$
- **Related phases:** $\theta_2 \to \gamma$, $\phi_1 \to 2\beta$

$$q_B/p_B = |q_B/p_B|\, e^{-i\phi_1}$$

$$q_K/p_K = |q_K/p_K|\, e^{-i\phi_2}$$

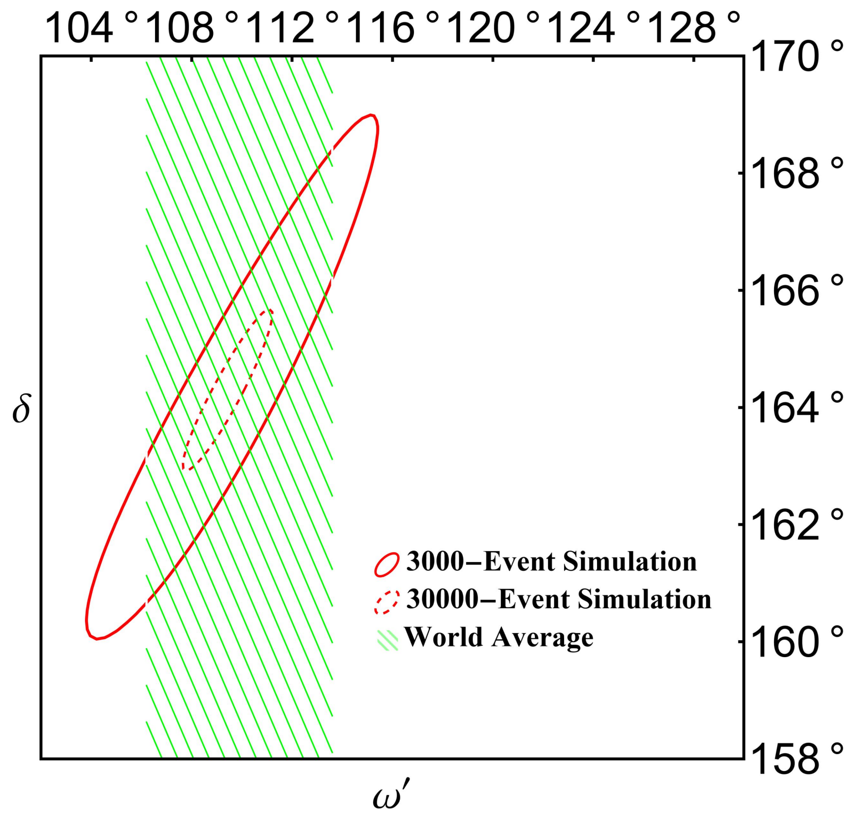
CKM phase extraction

✓ Take $B_d^0(t_1) \to D^0 K(t_2) \to (K^-\pi^+)(\pi^+\ell^-\overline{\nu_\ell})$ as an example (penguin = 0)

Number of Events	Parameter	Fitted Value	Input Value
3000	r_B	0.370 ± 0.015	0.366
	ω'	$(109.1 \pm 5.7)^\circ$	$(110 \pm 4)^\circ$
	δ	$(163.9 \pm 4.4)^\circ$	164°
30000	r_B	0.365 ± 0.004	0.366
	ω'	$(109.4 \pm 1.8)^\circ$	$(110 \pm 4)^\circ$
	δ	$(164.2 \pm 1.4)^\circ$	164°

$\omega' = \phi_1 - \phi_2 + \theta_2 - \theta_1$

$109^\circ \pm 3.7^\circ$ (Experiment)



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Summary

- A novel type of CP violation effect, the **double-mixing** CP asymmetry, is proposed.
- It does not require nonzero strong phases.
- The double-mixing CP asymmetry can be very **significant** in some decay modes. $B_s^0 \to \rho^0 \overline{K}^0 \to \rho^0(\pi^- \ell^+ \nu_\ell)$
- Strong phases can be **extracted from experimental data** without theoretical input. $B_d^0 \to D^0 K^0 \to (K^- \pi^+)(\pi^+ \ell^- \overline{\nu_\ell})$

Thanks !

Appendix

Case 1: $B_s^0(t_1) \to \rho^0 K(t_2) \to \rho^0(\pi^- \ell^+ \nu_\ell)$

$$
\begin{aligned}
S_h(t_2) &= \frac{e^{-\Gamma_2 t_2}}{2}\Big[-2\sin(\Delta m_2 t_2)\sin(\phi_1+\phi_2+2\delta) \\
&+ \sinh\frac{\Delta\Gamma_2 t_2}{2}\left(\left|\frac{q_1}{p_1}\right|\left|\frac{p_2}{q_2}\right| - \left|\frac{p_1}{q_1}\right|\left|\frac{q_2}{p_2}\right|\right) \\
&\times \cos(\phi_1+\phi_2+2\delta)\Big] , \\
S_n(t_2) &= \frac{e^{-\Gamma_2 t_2}}{2}\Big[2\sinh\frac{\Delta\Gamma_2 t_2}{2}\sin(\phi_1+\phi_2+2\delta) \\
&+ \sin(\Delta m_2 t_2)\left(\left|\frac{q_1}{p_1}\right|\left|\frac{p_2}{q_2}\right| - \left|\frac{p_1}{q_1}\right|\left|\frac{q_2}{p_2}\right|\right) \\
&\times \cos(\phi_1+\phi_2+2\delta)\Big] ,
\end{aligned}
$$

$$
\begin{aligned}
C'_+(t_2) &= 2|g_{2,-}(t_2)|^2 , \\
C'_-(t_2) &= 2|g_{2,+}(t_2)|^2 , \\
S'_h(t_2) &= e^{-\Gamma_2 t_2}\sinh\frac{\Delta\Gamma_2 t_2}{2}\cos(\phi_1+\phi_2+2\delta) , \\
S'_n(t_2) &= e^{-\Gamma_2 t_2}\sin(\Delta m_2 t_2)\cos(\phi_1+\phi_2+2\delta) ,
\end{aligned}
$$

Appendix

$$e^{2i\delta} = -\frac{V_{ub}^{*}V_{ud}}{V_{ub}V_{ud}^{*}} \qquad CP|\rho^0 K^0\rangle = -|\rho^0 \overline{K}^0\rangle$$

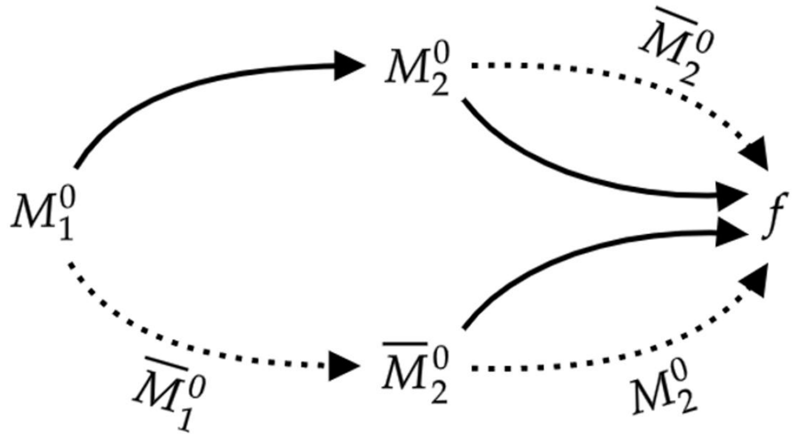
$$e^{-i\phi_1} = \frac{V_{tb}^{*}V_{ts}}{V_{tb}V_{ts}^{*}}, \qquad e^{-i\phi_2} = \frac{V_{cd}^{*}V_{cs}}{V_{cb}V_{cs}^{*}}.$$

TABLE I: The input parameters and their values.

Parameter	Value
$\lvert q_1/p_1\rvert$	1.0003 ± 0.0014 [21]
ϕ_1	$(-2.106 \pm 0.135)^\circ$ [22]
$x_1 = 2\Delta m_1/(\Gamma_{B_s,L} + \Gamma_{B_s,S})$	27.01 ± 0.10 [22]
$y_1 = \Delta\Gamma_1/(\Gamma_{B_s,L} + \Gamma_{B_s,S})$	-0.064 ± 0.003 [22]
$\lvert q_2/p_2\rvert$	0.996774 ± 0.000019 [22]
ϕ_2	$(0.176 \pm 0.001)^\circ$ [22]
$x_2 = 2\Delta m_2/(\Gamma_{K,L} + \Gamma_{K,S})$	0.946 ± 0.002 [22]
$y_2 = \Delta\Gamma_2/(\Gamma_{K,L} + \Gamma_{K,S})$	-0.996506 ± 0.000016 [22]
2δ	$(-48.907 \pm 3.094)^\circ$ [22]

$$|M_{H,L}\rangle = p\,|M^0\rangle \mp q\,|\bar{M}^0\rangle$$

$$g_\pm(t) = \frac{1}{2}\left[e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right]$$



Appendix

Case 2 : $B_d^0(t_1) \to D^0 K(t_2) \to (K^-\pi^+)(\pi^+\ell^-\overline{\nu_\ell})$

$$A_{\rm CP}(t_1,t_2) = \frac{e^{-\Gamma_1 t_1}\sinh\frac{\Delta\Gamma_1 t_1}{2}\,S_h(t_2) + e^{-\Gamma_1 t_1}\sin\left(\Delta m_1 t_1\right)\,S_n(t_2)}{|g_{1,+}(t_1)|^2 r^2 C'_+(t_2) + |g_{1,-}(t_1)|^2 C'_-(t_2) + e^{-\Gamma_1 t_1}\sinh\frac{\Delta\Gamma_1 t_1}{2}\,S'_h(t_2) + e^{-\Gamma_1 t_1}\sin\left(\Delta m_1 t_1\right)\,S'_n(t_2)}\,,$$

$$S_h(t_2) = -e^{-\Gamma_2 t_2}\, r\sin\omega'[\sin\delta\sinh\frac{\Delta\Gamma_K}{2}t_2 + \cos\delta\sin\Delta m_K t_2]\,,$$

$$S_n(t_2) = e^{-\Gamma_2 t_2}\, r\sin\omega'[\cos\delta\sinh\frac{\Delta\Gamma_K}{2}t_2 - \sin\delta\sin\Delta m_K t_2]\,,$$

$$S'_h(t_2) = e^{-\Gamma_2 t_2}\, r\cos\omega'[\cos\delta\sinh\frac{\Delta\Gamma_K}{2}t_2 - \sin\delta\sin\Delta m_K t_2]\,,$$

$$S'_n(t_2) = e^{-\Gamma_2 t_2}\, r\cos\omega'[\sin\delta\sinh\frac{\Delta\Gamma_K}{2}t_2 + \cos\delta\sin\Delta m_K t_2]\,.$$