Analysis of three-body charmed B meson decays

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Based on:

$B \to (D^* \to)DP_1 P_2$ *Phys.Rev.D* 104 (2021) 11, 116012

in collaboration with Run-Hui Li, Zheng-Yi Wei, Cai-Dian Lu

$B \to D(V \to)P_1P_2$ *ArXiv:2404.XXXX*

Outline

- Motivation for three-body B meson decays
- Experimental and theoretical investigation of $B \to DP_1 P_2$
- Factorization Assisted Topological Amplitude approach
- Numerical results for $B \to (D^* \to)DP_1 P_2, \; B \to D(V \to)P_1 P_2$
- Summary

Rich physics in three-body B decay

- Broaden the study of B decay mechanisms

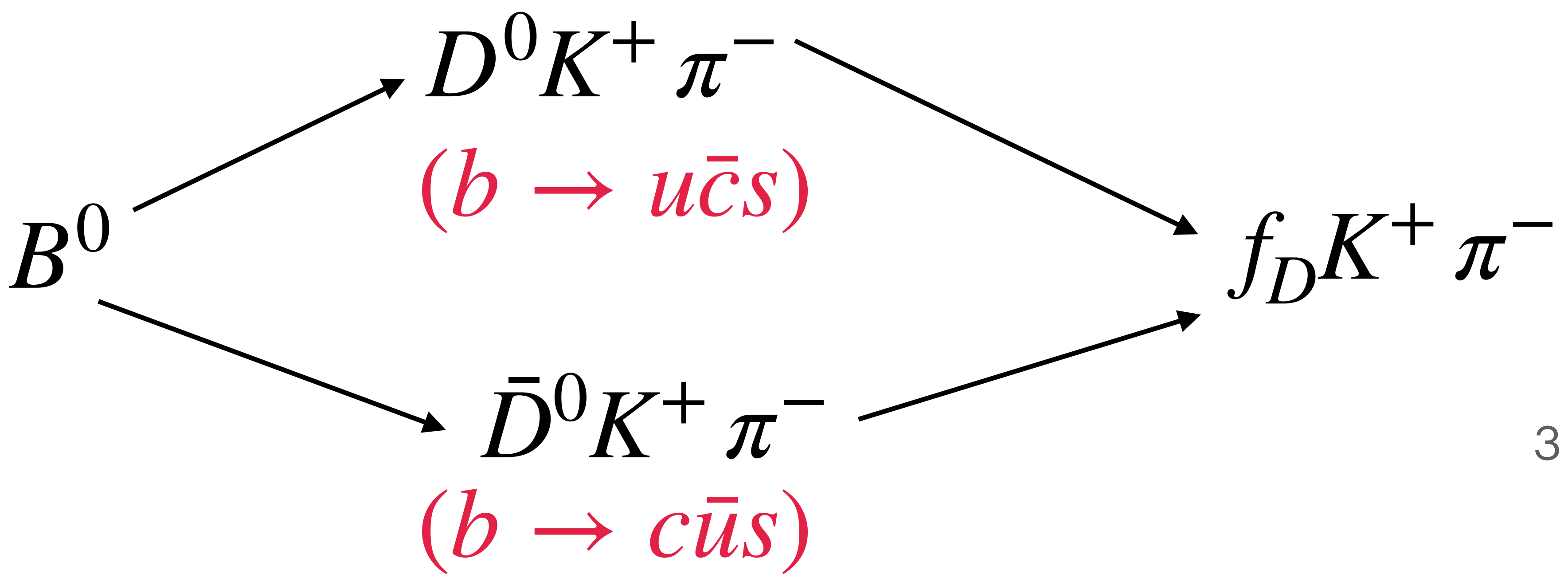
  such as testing the standard model,

  studying the emergence of quantum chromodynamics

- Provide additional possibilities for CP violation searches.

  - besides tree and penguin amplitudes interference as in two-body B decays,
  - the interference between different resonant states in three-body B decays.

  $B^0 \to DK^+\pi^-$ with D representing $D^0, \bar{D}^0$, measure the unitarity triangle angle γ



$$\gamma \equiv \arg[-V_{ud}V^*_{ub}/(V_{cd}V^*_{cb})]$$

comparable in magnitude and potentially enhancing CP violation effects

Provide opportunities for the analysis on the hadron spectroscopy

$$B \to D(p_D)\, P_1(p_1)\, P_2(p_2)$$

$$m_{12}^2 = (p_1 + p_2)^2,\ m_{13}^2 = (p_1 + p_D)^2$$

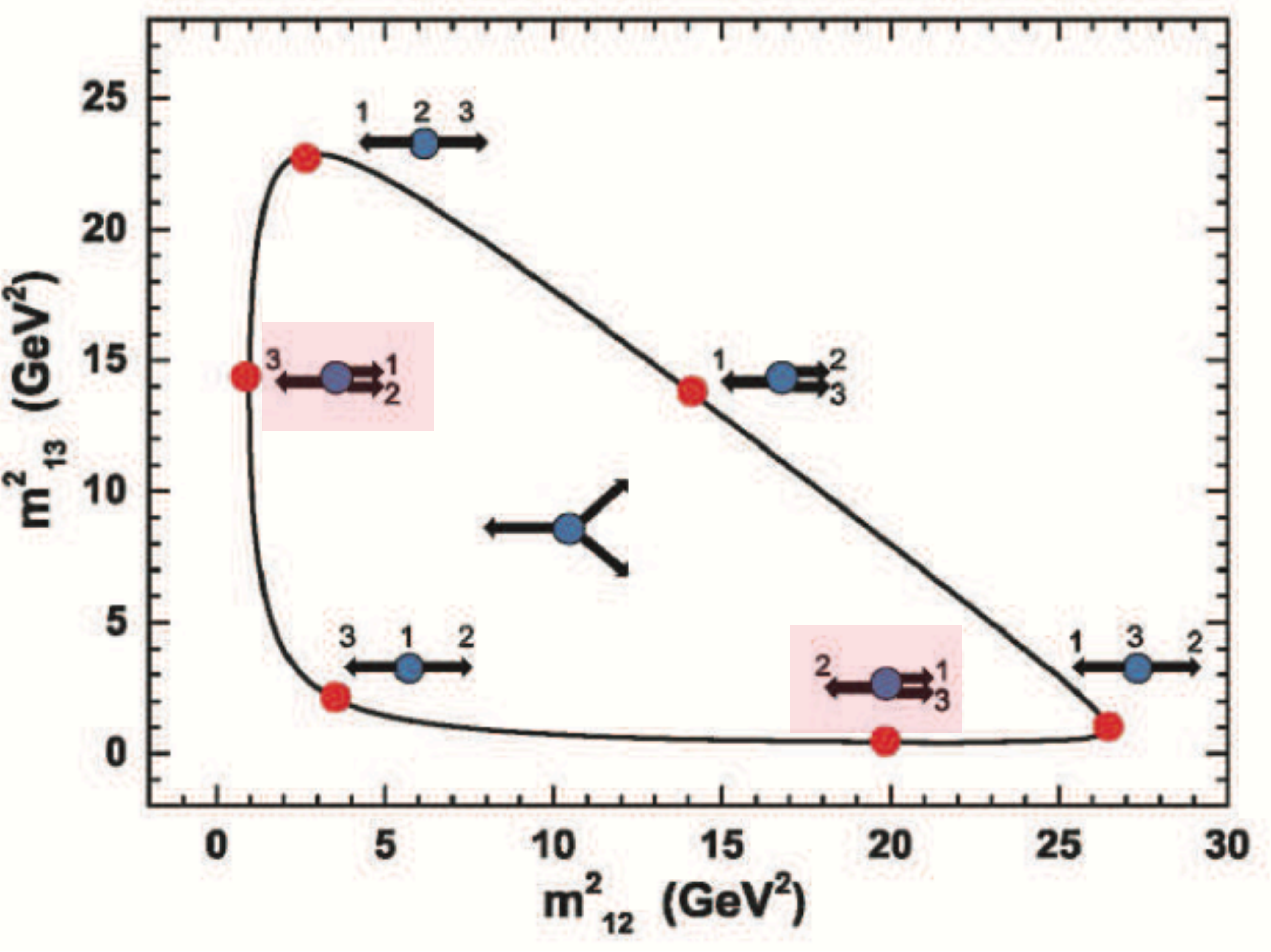


Fig from arXiv:2112.00315 Ying Li etc.

Dalitz plot regions:

the center, the corners and the edges regions

- the two energetic particles are collinear and form a moving-fast meson-pair, called quasi-two-body decay
- $m^2(P_1P_2), m^2(P_1P_D)$ generally peak as resonances, intermediate resonances in three-body B meson decays show up

$B \to D\,(V, S\ldots \to)\,P_1\,P_2$ with vector, scaler… resonances

$B \to (D^* \to)\,DP_1\,P_2$ with charmed resonances D^*

Experimental and theoretical investigation

Applying Dalitz plot analysis,

- LHCb have investigated $B_{(s)} \to D(s)\,K\,\pi$

  [Phys.Rev.D 90(2014), Phys.Rev.D 92(2015)…]

- LHCb, Belle and Babar for $B_{(s)} \to D(s)\,\pi\,\pi$

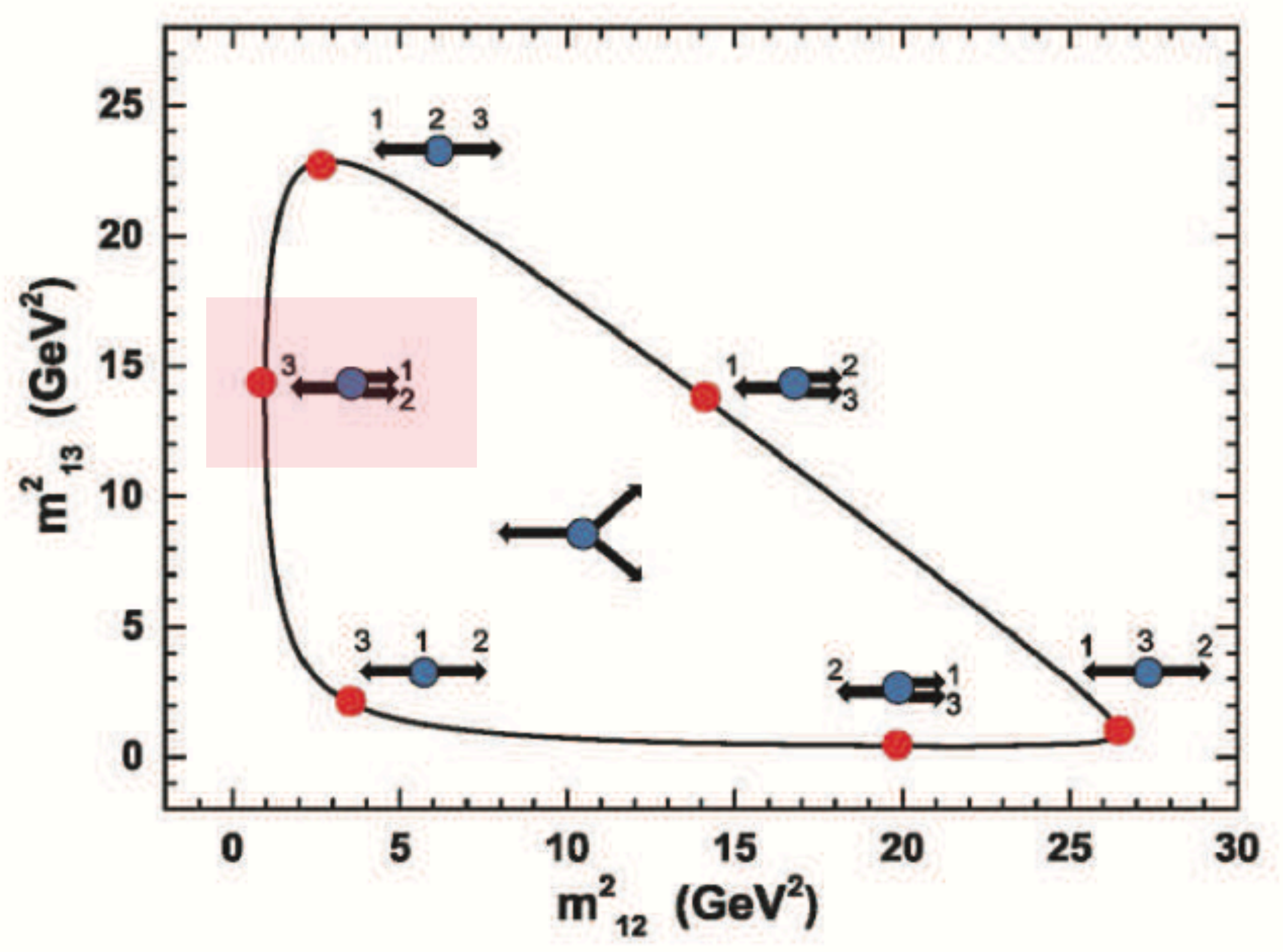
  [Phys.Rev.D 92(2015), Phys.Rev.D 76(2007),Phys.Rev.D 79(2009)…]

  - structures of ground and excited states of D^*, K^* and ρ
  - their corresponding fit fractions in isobar model

- Recently, ρ-like resonances in $B_{(s)} \to D(s)\,K^-\,K_S$ by Belle II

  [arXiv:2305.01321]

Actually, virtual effect from $\rho(770) \to KK$,
when $m(\rho) < m(K) + m(\bar{K})$



isobar model

$$A = \sum_{i=1}^{N} c^i A^i$$

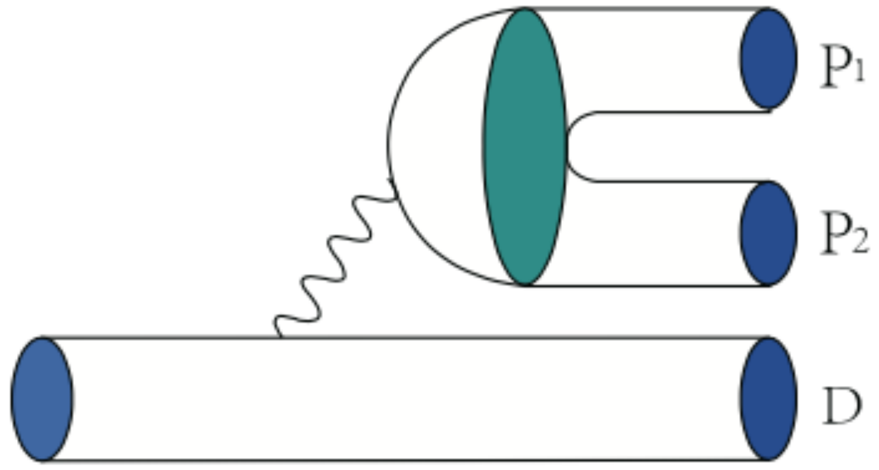
resonances generally described by
relativistic Breit-Wigner model

- The theoretical approaches based on the factorization hypothesis have been proposed for quasi-two-body decays.

  - PQCD approach for charmed B decays
    $B_{(s)} \to D_{(s)}\pi\pi,\ B_{(s)} \to D_{(s)}K\pi,\ B_{(s)} \to D_{(s)}KK$

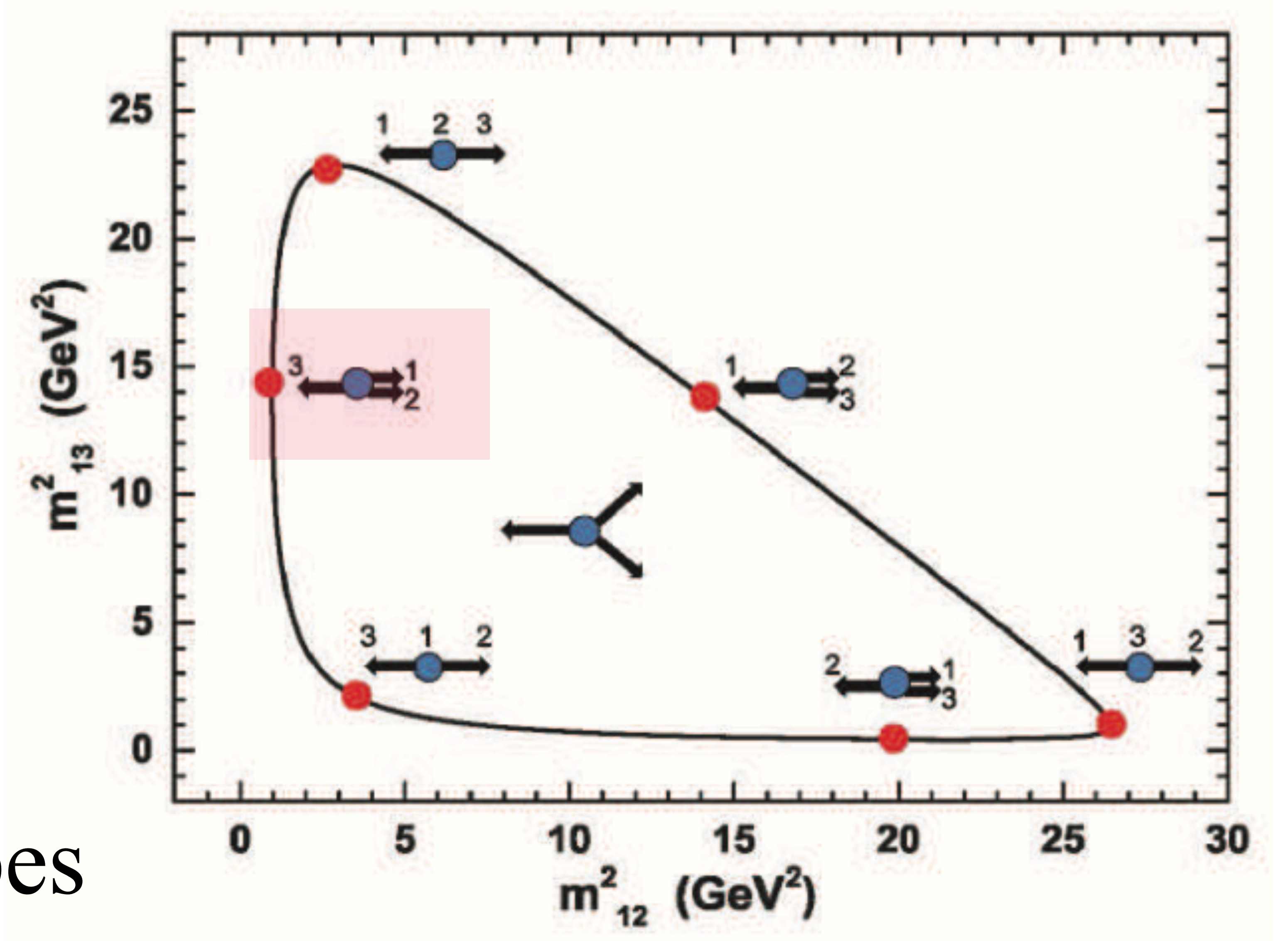
    [Phys.Rev.D 108(2023), Phys.Rev.D 103(2021)…]

the interactions between the meson-pair and the bachelor particle are power suppressed naturally.



$$A_i \sim H \otimes J \otimes \Phi_B \otimes \Phi_D \otimes \Phi_{P_1P_2,i}$$

two-meson wave function



parameterize as time-like form factor,

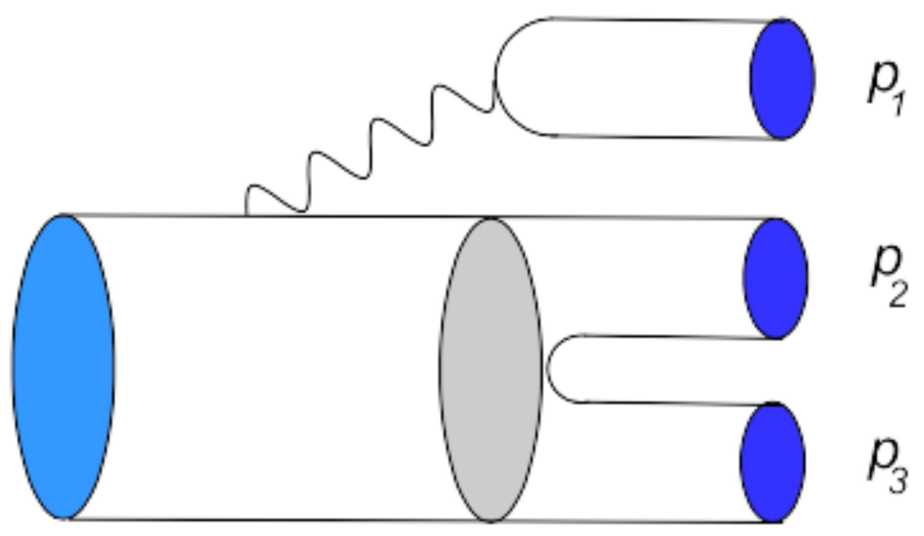
usually taken to be relativistic Breit-Wigner line shapes

$$L^{\mathrm{RBW}}(s) = \frac{m_R^2}{s - m_R^2 + i m_R \Gamma_R(s)}$$

- QCDF approach for charmless B decays

  $B_{(s)} \to \pi\pi\pi,\ B_{(s)} \to K\pi\pi,\ B_{(s)} \to KK\pi,\ B_{(s)} \to KKK$

  [Phys.Rev.D 88(2013), Nucl.Phys.B 899(2015), JHEP 06 (2020)073 …]



$$< P_1(p_2)P_2(p_3)\,|\,H_{eff}\,|\,B >$$

$$= \sum_i < P_2(p_2)P_3(p_3)\,|\,R_i > \frac{1}{s - m_{R_i}^2 + i m_{R_i}\Gamma_{R_i}} < R_i\,|\,H_{eff}\,|\,B >$$

- approaches based on the symmetry principles, such as SU(3) and isospin symmetry, for charmless B decays

  [Phys.Lett.B 726(2013), Phys.Rev.D91(2015)…]

factorization-assisted topological-amplitude approach

- factorization-assisted topological-amplitude approach (FAT)

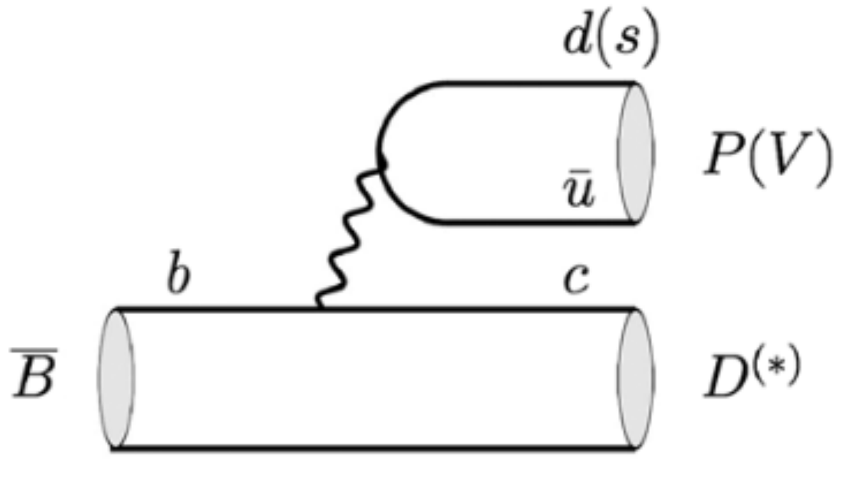
[H.Y.Cheng, etc. Phys.Rev.D 54 (1996)*]*

$$B \to DP, D^*P, DV$$

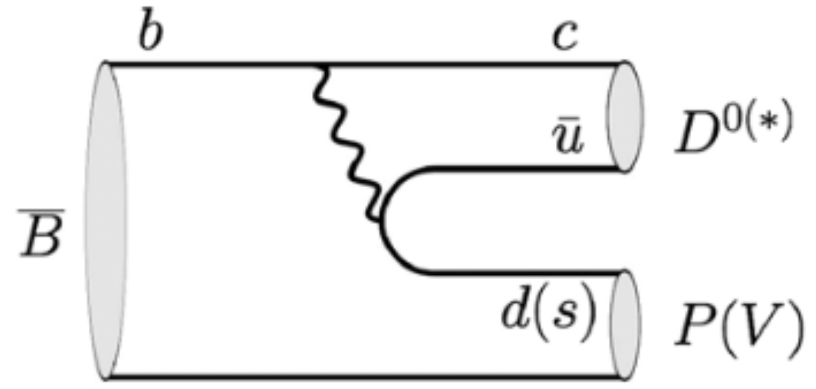
- Distinct by weak interaction and flavor flows with all strong interaction encoded, including non-perturbative ones.
- Amplitudes with strong phases extracted from data.
- Flavor SU(3) symmetry relate different amplitudes and strong phases of the same topological type.

$$T = |T|, \;\; C = |C| e^{i\delta_C}, \;\; E = |E| e^{i\delta_E},$$

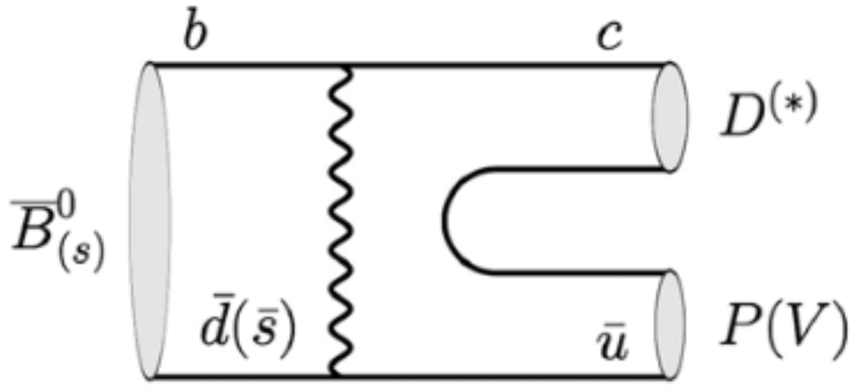
for $B_{(s)} \to D_{(s)}P, P = \pi, K, \eta, \eta'$



(a) T



(b) C



(c) E

factorization-assisted topological-amplitude approach

[H.Y.Cheng, etc. Phys.Rev.D 75 (2007) 074021]

	Scheme 1
$\|T\|$	$16.26^{+0.61}_{-0.68}$
$\|C\|$	$6.77^{+0.20}_{-0.21}$
$\|E\|$	$1.47^{+0.13}_{-0.15}$
δ_C (degrees)	$-69.0^{+9.2}_{-7.5}$
δ_E (degrees)	$-146.2^{+13.9}_{-12.0}$
ξ_T	1 (fixed)
ξ_C	1 (fixed)
$\chi^2_{\rm min}$	45.28
$\chi^2_{\rm min}/{\rm dof}$	11.32

- based on Flavor SU(3) symmetry

  $T = |T|,\ C = |C| e^{i\delta_C},\ E = |E| e^{i\delta_E},$

- Need to keep SU(3) breaking effects

factorization-assisted topological-amplitude approach

[Si-Hong Zhou , etc. Phys.Rev.D 92 (2015) 094016]

$$T_c^{DP} = i\frac{G_F}{\sqrt{2}}V_{cb}V_{uq}^* a_1(\mu) f_P (m_B^2 - m_D^2) F_0^{B\to D}(m_P^2),$$

$$C_c^{DP} = i\frac{G_F}{\sqrt{2}}V_{cb}V_{uq}^* f_D (m_B^2 - m_P^2) F_0^{B\to P}(m_D^2)\chi_c^C e^{i\phi_c^C},$$

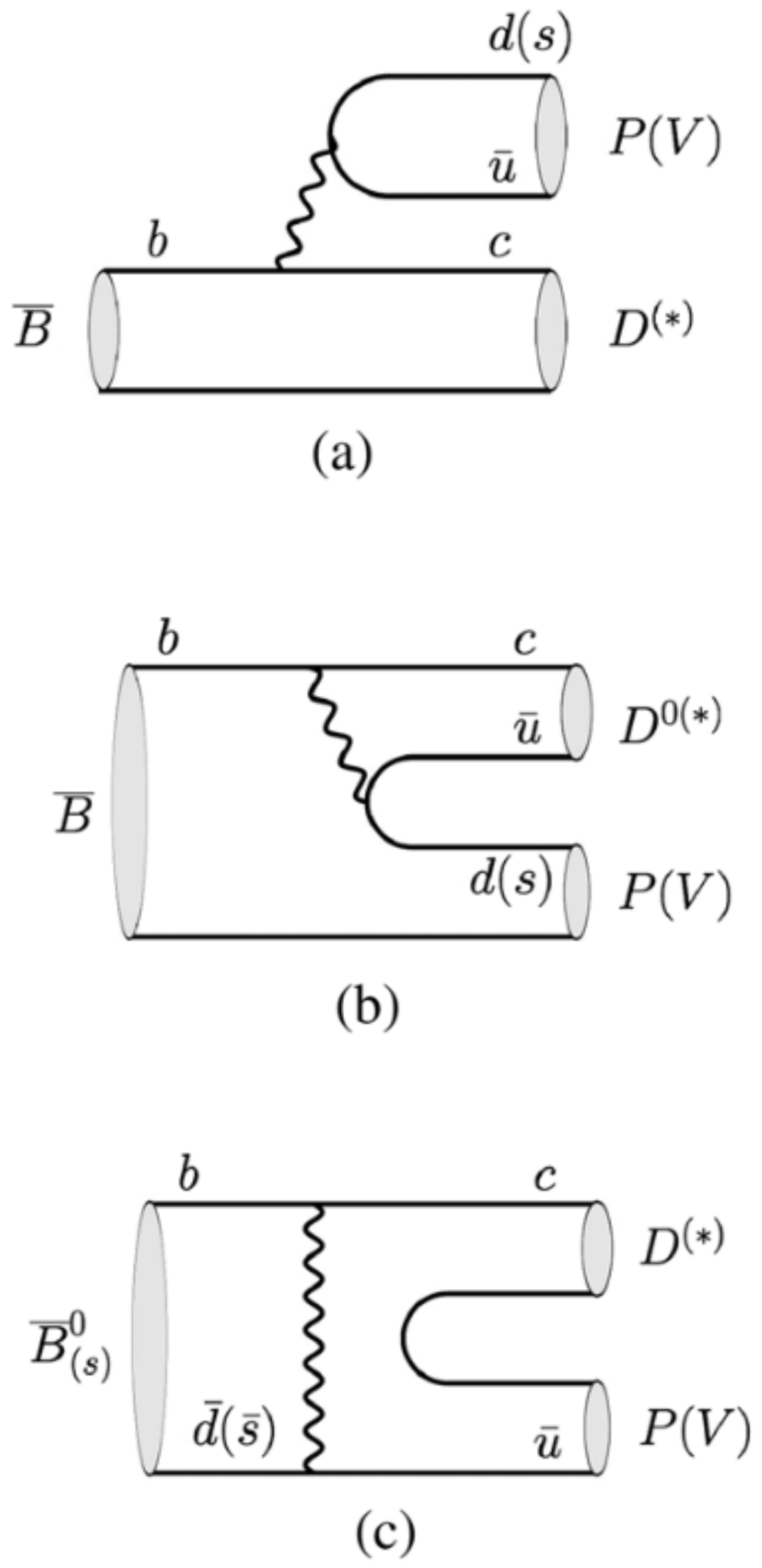
$$E_c^{DP} = i\frac{G_F}{\sqrt{2}}V_{cb}V_{uq}^* m_B^2 f_B \frac{f_{D_{(s)}} f_P}{f_D f_\pi}\chi_c^E e^{i\phi_c^E},$$

SU(3) breaking effect are kept in F, f,

$$\chi_c^C = 0.48 \pm 0.01, \qquad \phi_c^C = (56.6^{+3.2}_{-3.8})^\circ,$$

$$\chi_c^E = 0.024^{+0.002}_{-0.001}, \qquad \phi_c^E = (123.9^{+3.3}_{-2.2})^\circ,$$

with $\chi^2/\mathrm{d.o.f.} = 1.4$.



FAT firstly proposed in D meson decays

H. n. Li, C. D. Lu, F. S. Yu , Q. Qin

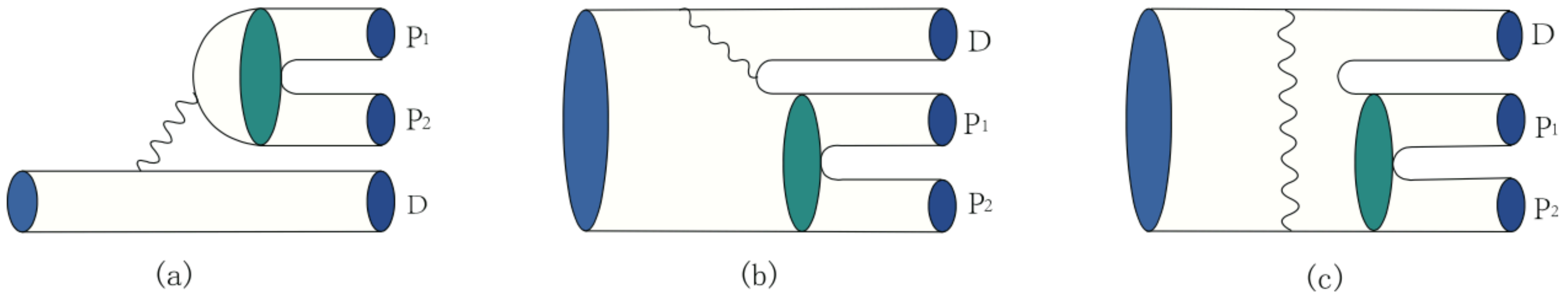
Applied successfully in B meson decays

S.H.Zhou, etc.

$B \to D P_1 P_2$ with resonances $D^*, \rho, K^*, \omega, \phi$ in FAT

- take $\bar{B}_{(s)} \to D_{(s)}(V \to)P_1P_2$ as example

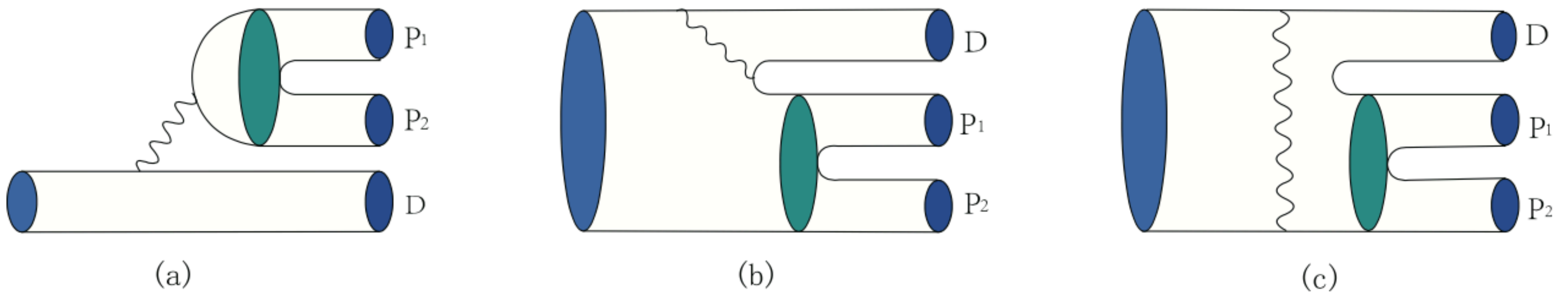
  - two subprocesses: $\bar{B}_{(s)} \to D_{(s)}V$ firstly, subsequently $V \to P_1P_2$



  - classify the topological diagrams into T, C, E by weak decays

    $b \to c\, q\, \bar{u}\ (q = d, s)$

two subprocesses: $\bar{B}_{(s)} \to D_{(s)} V$ firstly, subsequently $V \to P_1 P_2$



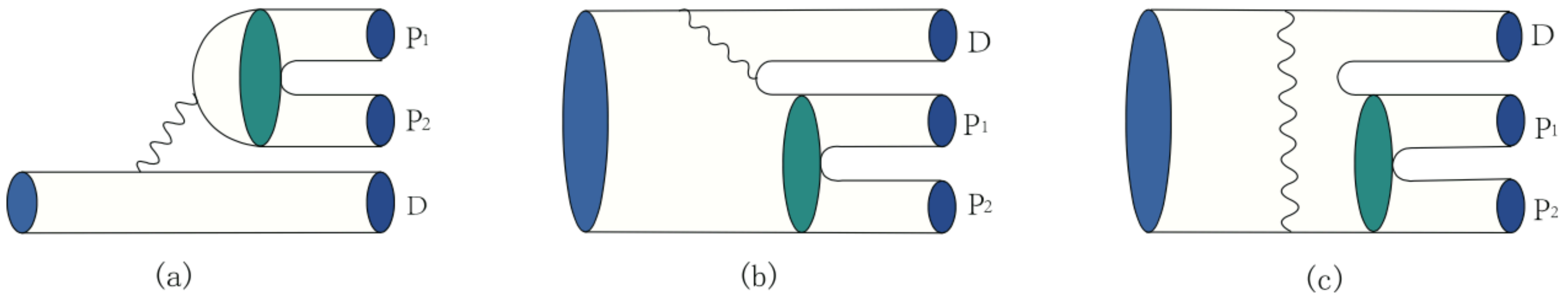
- amplitudes of $b \to c\, q\, \bar{u}$ $(q = d, s)$:

$$T^{DV} = \sqrt{2}\, G_F\, V_{cb} V_{uq}^{*}\, a_1(\mu) f_V\, m_V\, F_1^{B\to D}\left(m_V^2\right)\left(\varepsilon_V^{*} \cdot p_B\right),$$

$$C^{DV} = \sqrt{2}\, G_F\, V_{cb} V_{uq}^{*}\, f_{D_{(s)}}\, m_V\, A_0^{B\to V}\left(m_D^2\right)\left(\varepsilon_V^{*} \cdot p_B\right) \chi^C e^{i\phi^C},$$

$$E^{DV} = \sqrt{2}\, G_F\, V_{cb} V_{uq}^{*}\, m_V\, f_B \frac{f_{D_{(s)}} f_V}{f_D f_\pi}\left(\varepsilon_V^{*} \cdot p_B\right) \chi^E e^{i\phi^E}.$$

two subprocesses: $\bar{B}_{(s)} \to D_{(s)}V$ firstly, subsequently $V \to P_1P_2$



- RBW distribution for intermediate states ρ, K^*, ω, ϕ:

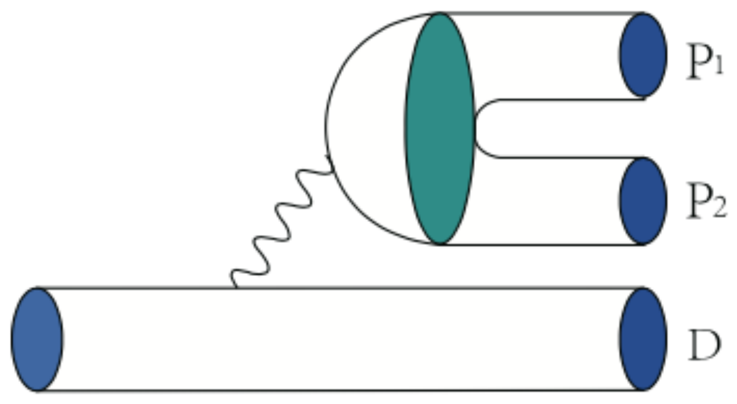
$$L^{\mathrm{RBW}}(s) = \frac{1}{s - m_V^2 + i m_V \Gamma_V(s)},$$

where $s = (p_1 + p_2)^2$, $\Gamma_V(s)$ is s-dependent width of vector resonances

- $\langle P_1(p_1)\, P_2(p_2) \,|\, V(p_V)\rangle$ can be parametrized as a strong coupling constant $g_{VP_1P_2}$

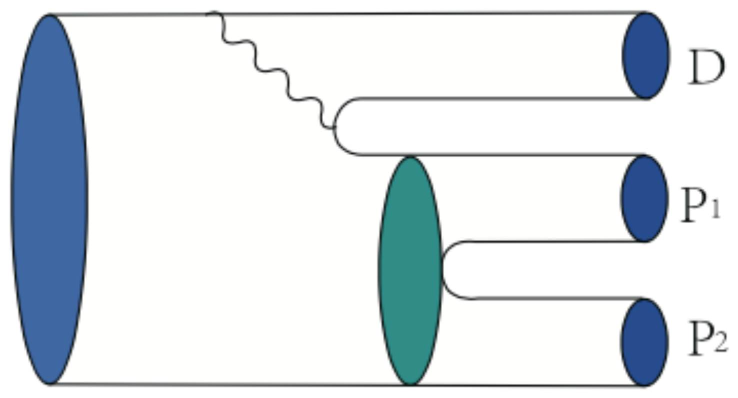
$$\Gamma_{V\to P_1P_2} = \frac{2}{3}\frac{p_c^3}{4\pi\, m_V^2}\, g_{VP_1P_2}^2,$$

Finally, combing the two subprocesses together, the decay amplitudes of each topological diagrams



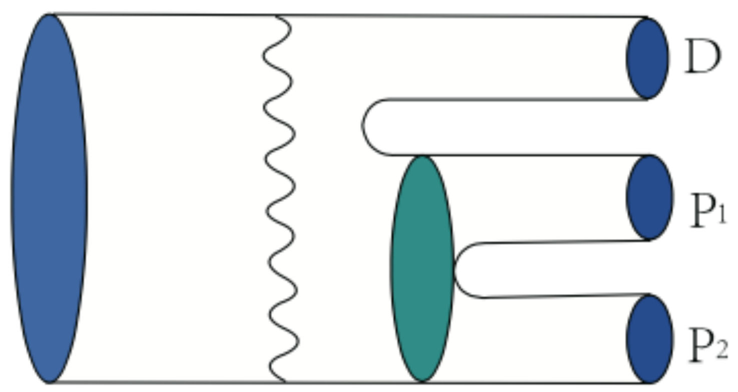
(a)

$$T = \langle P_1(p_1)\, P_2(p_2)\, |(\bar{q}u)_{V-A}|0\rangle \, \langle D(p_D)|(\bar{c}b)_{V-A}|B(p_B)\rangle$$
$$= \frac{\langle P_1(p_1)\, P_2(p_2)\, |V(p_V)\rangle}{s - m_V^2 + i m_V \Gamma_V(s)} \langle V(p_V)\, |(\bar{q}u)_{V-A}|\, 0\rangle \, \langle D\, (p_D)\, |(\bar{c}b)_{V-A}|\, B(p_B)\rangle$$
$$= p_D \cdot (p_1 - p_2)\, \sqrt{2}\, G_F\, V_{cb} V_{uq}^*\, a_1\, f_V m_V F_1^{B\to D}(s)\, \frac{g_{V P_1 P_2}}{s - m_V^2 + i m_V \Gamma_V(s)}\,,$$



(b)

$$C = \langle P_1(p_1)\, P_2(p_2)\, |(\bar{q}b)_{V-A}|\, B(p_B)\rangle \, \langle D\, (p_D)\, |(\bar{c}u)_{V-A}|\, 0\rangle$$
$$= \frac{\langle P_1(p_1)\, P_2(p_2)\, |V(p_V)\rangle}{s - m_V^2 + i m_V \Gamma_V(s)} \langle V(p_V)\, |(\bar{q}b)_{V-A}|\, B(p_B)\rangle \, \langle D\, (p_D)\, |(\bar{c}u)_{V-A}|\, 0\rangle$$
$$= p_D \cdot (p_1 - p_2)\, \sqrt{2}\, G_F\, V_{cb} V_{uq}^*\, f_D\, m_V A_0^{B\to V}(m_D^2)\, \chi^C e^{\phi^C}\, \frac{g_{V P_1 P_2}}{s - m_V^2 + i m_V \Gamma_V(s)}\,,$$



(c)

$$E = \langle D\, (p_D)\, P_1(p_1)\, P_2(p_2)\, |\mathcal{H}_{eff}|\, B(p_B)\rangle$$
$$= \frac{\langle P_1(p_1)\, P_2(p_2)\, |V(p_V)\rangle}{s - m_V^2 + i m_V \Gamma_V(s)} \langle D(p_D) V(p_V)\, |\mathcal{H}_{eff}|\, B(p_B)\rangle$$
$$= p_D \cdot (p_1 - p_2)\, \sqrt{2}\, G_F\, V_{cb} V_{uq}^*\, m_V\, f_B\, \frac{f_V f_{D_{(s)}}}{f_\pi f_D}\, \chi^E e^{i\phi^E}\, \frac{g_{V P_1 P_2}}{s - m_V^2 + i m_V \Gamma_V(s)}\,,$$

Numerical results and discussion

input parameters

- electroweak coefficients: CKM matrix elements and Wilson coefficients;
- nonperturbative QCD parameters: decay constants f, transition form factors F and nonfactorizable parameters $\chi^C, \phi^C, \chi^E, \phi^E$;

$$\chi^C = 0.48 \pm 0.01, \qquad \phi^C = \left(56.6^{+3.2}_{-3.8}\right)^\circ,$$
$$\chi^E = 0.024^{+0.002}_{-0.001}, \qquad \phi^E = \left(123.9^{+3.3}_{-2.2}\right)^\circ.$$

$C \gg E$

- Hadronic parameters: m_V, Γ_0 and $g_{VP_1P_2}$

Predict branching fractions for $\bar{B}_{(s)} \to D_{(s)} P_1 P_2,\ \bar{B}_{(s)} \to \bar{D}_{(s)} P_1 P_2$

integrate the the differential width over the kinematics region

$$d\Gamma = ds \frac{1}{(2\pi)^3} \frac{(|\mathbf{p_D}||\mathbf{p}_1|)^3}{6 m_B^3} |A(s)|^2,$$

- ground states of ρ, K^*, ω, ϕ

$B_{(s)} \to D_{(s)}(\rho \to)\pi\pi,$

$B_{(s)} \to D_{(s)}(K^* \to)K\pi,$

$B_{(s)} \to D_{(s)}(\rho, \omega, \phi \to)KK$

- ground states of D^*

$\bar{B}_{(s)} \to (D^*_{(s)} \to) D_{(s)} P_1 P_2$

Decay Modes	Amplitudes	Data	$\mathcal{B}_{\text{FAT}}$	$\mathcal{B}_{\text{PQCD}}$
$\bar{B} \to D(\rho \to)\pi\pi$	$V_{cb}V_{ud}^*$	10^{-4}	10^{-4}	10^{-4}
$B^- \to D^0(\rho^- \to)\pi^0\pi^-$	$T+C$	134 ± 18	$97.7^{+2.1+16.8+8.5}_{-2.3-15.8-8.1}$	115^{+59}_{-38}
$\bar{B}^0 \to D^+(\rho^- \to)\pi^0\pi^-$	$T+E$	76 ± 12	$60.0^{+0.5+13.0+6.4}_{-0.3-11.7-6.0}$	$82.3^{+49.2}_{-29.0}$
$\bar{B}^0 \to D^0(\rho^0 \to)\pi^+\pi^-$	$\frac{1}{\sqrt{2}}(E-C)$	3.21 ± 0.21	$2.50^{+0.14+0.24+0.03}_{-0.13-0.49-0.03}$	$1.39^{+1.24}_{-0.90}$
$\bar{B}^0_s \to D^+_s(\rho^- \to)\pi^-\pi^0$	T	95 ± 20	$74.5^{+0.0+15.6+7.9}_{-0.0-14.2-7.5}$	$77.2^{+40.2}_{-25.6}$
	$V_{cb}V_{us}^*$			
$\bar{B}^0_s \to D^+(\rho^- \to)\pi^-\pi^0$	E		$0.018^{+0.003+0+0.005}_{-0.001-0-0.004}$	$0.051^{+0.022}_{-0.014}$
$\bar{B}^0_s \to D^0(\rho^0 \to)\pi^+\pi^-$	$\frac{1}{\sqrt{2}}E$		$0.009^{+0.002+0+0.002}_{-0.001-0-0.001}$	$0.026^{+0.010}_{-0.006}$
$\bar{B} \to \bar{D}(\rho \to)\pi\pi$	$V_{ub}V_{cs}^*$	10^{-6}	10^{-6}	10^{-6}
$B^- \to D^-_s(\rho^0 \to)\pi^+\pi^-$	$\frac{1}{\sqrt{2}}T$		$16.7^{+0.0+3.5+1.7+1.5}_{-0.0-3.2-1.6-1.5}$	$15.2^{+11.1}_{-8.2}$
$\bar{B}^0 \to D^-_s(\rho^+ \to)\pi^+\pi^0$	T		$29.7^{+0.0+6.2+2.9+2.6}_{-0.0-5.6-2.8-2.6}$	$28.2^{+20.4}_{-15.3}$
$\bar{B}^0_s \to D^-(\rho^+ \to)\pi^+\pi^0$	E		$0.19^{+0.03+0+0.02+0.02}_{-0.02-0-0.02-0.02}$	$0.69^{+0.20}_{-0.16}$
$\bar{B}^0_s \to \bar{D}^0(\rho^0 \to)\pi^+\pi^-$	$\frac{1}{\sqrt{2}}E$		$0.09^{+0.02+0+0.01+0.01}_{-0.01-0-0.01-0.01}$	$0.34^{+0.10}_{-0.08}$
	$V_{ub}V_{cd}^*$			
$B^- \to D^-(\rho^0 \to)\pi^+\pi^-$	$\frac{1}{\sqrt{2}}(T-A)$		$0.35^{+0+0.10+0.01+0.03}_{-0-0.09-0.01-0.03}$	$0.53^{+0.36}_{-0.27}$
$B^- \to \bar{D}^0(\rho^- \to)\pi^+\pi^0$	$C+A$		$0.48^{+0.02+0.07+0.01+0.04}_{-0.02-0.06-0.01-0.04}$	$0.05^{+0.02}_{-0.01}$
$\bar{B}^0 \to D^-(\rho^+ \to)\pi^+\pi^0$	$T+E$		$1.03^{+0.01+0.23+0.01+0.09}_{-0.01-0.20-0.01-0.09}$	$0.76^{+0.59}_{-0.31}$
$\bar{B}^0 \to \bar{D}^0(\rho^0 \to)\pi^+\pi^-$	$\frac{1}{\sqrt{2}}(E-C)$		$0.11^{+0.01+0.02+0+0.01}_{-0.01-0.02-0-0.01}$	$0.013^{+0.009}_{-0.008}$

1. $B_{(s)} \to D_{(s)}\rho \to D_{(s)}\pi\pi$

Hierarchy of branching fraction

- CKM:

$\bar{B} \to D(\rho \to)\pi\pi$ induced via $b \to c\bar{u}q$

$\bar{B} \to \bar{D}(\rho \to)\pi\pi$ induced via $b \to u\bar{c}q$

Cabibbo favored decay modes are able to be measured firstly by experiments

- topological diagrams

FAT $|T| \sim 2|C| \sim 12|E|$

Decay Modes	Amplitudes	Data	$\mathcal{B}_{\rm FAT}$	$\mathcal{B}_{\rm PQCD}$
$\bar{B} \to D(K^* \to)K\pi$	$V_{cb}V_{ud}^*$	10^{-4}	10^{-4}	10^{-4}
$\bar{B}^0 \to D_s^+(K^{*-} \to)K^-\pi^0$	E		$0.11^{+0.02+0+0.02}_{-0.01-0-0.02}$	$0.52^{+0.14+0.05+0.05}_{-0.12-0.08-0.00}$
$\bar{B}_s^0 \to D^0(K^{*0} \to)K^+\pi^-$	C	2.86 ± 0.44	$3.74^{+0.16+0.79+0.04}_{-0.15-0.71-0.04}$	$2.86^{+1.67+0.43+0.05}_{-1.33-0.56-0.08}$
	$V_{cb}V_{us}^*$			
$B^- \to D^0(K^{*-} \to)K^-\pi^0$	$T + C$		$2.04^{+0.04+0.35+0.17}_{-0.05-0.33-0.16}$	$1.67^{+0.71+0.32+0.07}_{-0.53-0.34-0.07}$
$\bar{B}^0 \to D^+(K^{*-} \to)K^-\pi^0$	T		$1.31^{+0+0.28+0.13}_{-0-0.25-0.13}$	$1.24^{+0.55+0.15+0.06}_{-0.40-0.18-0.05}$
$\bar{B}^0 \to D^0(\bar{K}^{*0} \to)K^-\pi^+$	C	0.32 ± 0.05	$0.27^{+0.01+0.06+0.01}_{-0.01-0.05-0.01}$	$0.17^{+0.10+0.03+0.00}_{-0.08-0.03-0.01}$
$\bar{B}_s^0 \to D_s^+(K^{*-} \to)K^-\pi^0$	$T + E$		$1.42^{+0.01+0.31+0.15}_{-0.01-0.28-0.14}$	$1.11^{+0.45+0.20+0.05}_{-0.33-0.21-0.04}$
$\bar{B} \to \bar{D}(K^* \to)K\pi$	$V_{ub}V_{cs}^*$	10^{-6}	10^{-6}	10^{-6}
$B^- \to \bar{D}^0(K^{*-} \to)K^-\pi^0$	$C + A$		$4.46^{+0.18+0.74+0.06+0.39}_{-0.18-0.68-0.06-0.39}$	$1.00^{+0.43+0.20+0.00}_{-0.48-0.27-0.07}$
$B^- \to D^-(\bar{K}^{*0} \to)K^-\pi^+$	A		$0.72^{+0+0+0.03+0.06}_{-0-0-0.01-0.06}$	$0.21^{+0.10+0.03+0.04}_{-0.06-0.02-0.00}$
$\bar{B}^0 \to \bar{D}^0(\bar{K}^{*0} \to)K^-\pi^+$	C		$3.48^{+0.15+0.73+0.04+0.31}_{-0.14-0.66-0.04-0.31}$	$1.96^{+1.01+0.52+0.11}_{-0.87-0.41-0.12}$
$\bar{B}_s^0 \to D_s^-(K^{*+} \to)K^+\pi^0$	$T + E$		$8.59^{+0.14+1.92+0.86+0.76}_{-0.08-1.72-0.82-0.76}$	$13.3^{+6.84+0.76+0.80}_{-3.04-0.73-0.79}$
	$V_{ub}V_{cd}^*$			
$B^- \to D_s^-(K^{*0} \to)K^+\pi^-$	A		$0.037^{+0+0+0.002+0.003}_{-0-0-0.001-0.003}$	$0.014^{+0.008+0.004+0.002}_{-0.003-0.008-0.0002}$
$\bar{B}^0 \to D_s^-(K^{*+} \to)K^+\pi^0$	E		$0.005^{+0.0009+0+0.0007+0.0004}_{-0.0004-0-0.0007+0.0004}$	$0.005^{+0.003+0.001+0}_{-0.003-0.001-0}$
$\bar{B}_s^0 \to D^-(K^{*+} \to)K^+\pi^0$	T		$0.35^{+0+0.07+0.004+0.03}_{-0-0.07-0.004+0.03}$	$0.6^{+0.30+0.03+0.04}_{-0.15-0.04-0.04}$
$\bar{B}_s^0 \to \bar{D}^0(K^{*0} \to)K^+\pi^-$	C		$0.16^{+0.01+0.03+0.002+0.01}_{-0.01-0.03-0.002-0.01}$	$0.08^{+0.05+0.02+0.00}_{-0.03-0.02-0.00}$

2. $B_{(s)} \to D_{(s)}K^* \to D_{(s)}K\pi$

FAT $|T| \sim 2|C| \sim 12|E|$

comparable branching ratios or $10^{-6} - 10^{-4}$ are also measurable in experiments

Comparison with PQCD

FAT $|T| \sim 2|C| \sim 12|E|$

PQCD $|T| \gg |C| \sim |E|$

- all modes dominated only by C are larger than those in the PQCD
- more precise

3. $B_{(s)} \to D_{(s)}\rho \to D_{(s)}KK$

Decay Modes	$\mathcal{B}^v_{\rm FAT}$	$\mathcal{B}^v_{\rm PQCD}$
$\bar{B} \to D(\rho \to)KK$		
$B^- \to D^0(\rho^- \to)K^-K^0$	$7.01^{+0.13+1.26+0.63}_{-0.14-1.16-0.60} \times 10^{-5}$	$11.8^{+6.2+0.9+0.7}_{-4.0-1.2-0.9} \times 10^{-5}$
$\bar{B}^0 \to D^+(\rho^- \to)K^-K^0$	$4.64^{+0.03+1.00+0.49}_{-0.02-0.90-0.47} \times 10^{-5}$	$7.93^{+5.01+0.32+0.65}_{-2.93-0.30-0.63} \times 10^{-5}$
$\bar{B}^0 \to D^0(\rho^0 \to)K^+K^-$	$1.34^{+0.07+0.28+0.02}_{-0.07-0.26-0.02} \times 10^{-6}$	$1.07^{+0.46+0.80+0.01}_{-0.37-0.58-0.01} \times 10^{-6}$
$\bar{B}^0_s \to D^+_s(\rho^- \to)K^-K^0$	$5.63^{+0+1.18+0.60}_{-0-1.07-0.60} \times 10^{-5}$	$6.06^{+3.47+0.04+0.47}_{-2.06-0.04-0.45} \times 10^{-5}$
$\bar{B}^0_s \to D^+(\rho^- \to)K^-K^0$	$9.46^{+1.64+0+2.71}_{-0.77-0-1.89} \times 10^{-9}$	$4.22^{+0.58+0.90+0.40}_{-0.67-0.65-0.30} \times 10^{-8}$
$\bar{B}^0_s \to D^0(\rho^0 \to)K^+K^-$	$0.48^{+0.08+0+0.11}_{-0.04-0-0.07} \times 10^{-8}$	$1.05^{+0.15+0.23+0.10}_{-0.17-0.15-0.07} \times 10^{-8}$
$\bar{B} \to \bar{D}(\rho \to)KK$		
$B^- \to D^-(\rho^0 \to)K^+K^-$	$1.89^{+0+0.53+0.03+0.04}_{-0-0.46-0.03-0.04} \times 10^{-9}$	$3.22^{+0.52+0.86+0.01}_{-0.45-0.43-0.01} \times 10^{-9}$
$B^- \to \bar{D}^0(\rho^- \to)K^-K^0$	$2.51^{+0.10+0.37+0.04+0.17}_{-0.11-0.34-0.04-0.17} \times 10^{-9}$	$0.53^{+0.12+0.25+0.03}_{-0.06-0.17-0.01} \times 10^{-9}$
$B^- \to D^-_s(\rho^0 \to)K^+K^-$	$8.74^{+0+1.83+0.87+0.77}_{-0-1.66-0.83-0.77} \times 10^{-8}$	$6.26^{+1.69+2.69+0.03}_{-1.30-0.92-0.02} \times 10^{-8}$
$\bar{B}^0 \to D^-(\rho^+ \to)K^+K^0$	$5.37^{+0.07+1.18+0.06+0.05}_{-0.04-1.07-0.06-0.05} \times 10^{-9}$	$6.87^{+2.05+3.30+0.08}_{1.60-1.01-0.08} \times 10^{-9}$
$\bar{B}^0 \to \bar{D}^0(\rho^0 \to)K^+K^-$	$5.73^{+0.32+1.26+0.06+0.51}_{-0.31-1.13-0.06-0.51} \times 10^{-10}$	$0.78^{+0.20+0.46+0.08}_{-0.13-0.29-0.06} \times 10^{-10}$
$\bar{B}^0 \to D^-_s(\rho^+ \to)K^+K^0$	$1.51^{+0+0.32+0.15+0.13}_{-0-0.29-0.14-0.13} \times 10^{-7}$	$2.32^{+0.63+1.00+0.01}_{-0.48-0.34-0.01} \times 10^{-7}$
$\bar{B}^0_s \to D^-(\rho^+ \to)K^+K^0$	$0.99^{+0.17+0+0.11+0.09}_{-0.08-0-0.11-0.09} \times 10^{-9}$	$7.47^{+1.49+2.42+0.40}_{-0.32-1.83-0.37} \times 10^{-9}$
$\bar{B}^0_s \to \bar{D}^0(\rho^0 \to)K^+K^-$	$0.51^{+0.09+0+0.06+0.04}_{-0.04-0-0.05-0.04} \times 10^{-9}$	$1.85^{+0.36+0.61+0.09}_{-0.32-0.45-0.08} \times 10^{-9}$

$B_{(s)} \to D_{(s)}\phi \to D_{(s)}K\bar{K}$ pole mass dynamics

Belle II

$$\mathcal{B}(B^- \to D^0K^-K^0_S) = (1.89 \pm 0.16 \pm 0.10) \times 10^{-4},$$
$$\mathcal{B}(\overline{B}^0 \to D^+K^-K^0_S) = (0.85 \pm 0.11 \pm 0.05) \times 10^{-4},$$

a peaking structure in low invariant mass region of $K^{\pm}K^0$, with ρ-like resonances but ρ(770) is not favored

BWT (or off-shell) effects

$$B_{(s)} \to D_{(s)}\rho \to D_{(s)}K\bar{K}$$

- $m_\rho < m_K + m_{\bar{K}}$
- ρ(770) is about 20 % of the ρ-like resonance and nonresonance contribution
- are not very sensitive to the widths of resonances, $B_{(s)} \to D_{(s)}\rho, \omega \to D_{(s)}K\bar{K}$

4. $\bar{B}_{(s)} \to D^*_{(s)} P_2 \to D_{(s)} P_1 P_2$

Decay modes	$\mathcal{B}^{(\text{cut})}_{\text{FAT}}(10^{-5})$	$\mathcal{B}^{(\text{cut})}_{\text{PQCD}}(10^{-5})$ [14]	Data (10^{-5})
$B^+ \to \bar{D}^{*0}\pi^+ \to D^-\pi^+\pi^+$	$18.8^{+3.16}_{-2.96}$	$19.2^{+8.80}_{-6.20}$	22.3 ± 3.20 [63]
			10.9 ± 1.8 [59]
			10.9 ± 2.7 [30]
$B^+ \to \bar{D}^{*0}K^+ \to D^-\pi^+K^+$	$1.39^{+0.24}_{-0.21}$	$1.48^{+0.68}_{-0.47}$	0.56 ± 0.23 [47]
$B^0 \to D^{*-}\pi^+ \to \bar{D}^0\pi^-\pi^+$	$10.26^{+2.25}_{-2.02}$	$8.7^{+4.5}_{-2.9}$	8.8 ± 1.3 [26]
			7.8 [29]
$B^0 \to D^{*-}K^+ \to \bar{D}^0\pi^-K^+$	$0.83^{+0.17}_{-0.15}$	$0.72^{+0.36}_{-0.24}$	0.81 ± 0.38 [28]
$B^0_s \to D^{*-}_s\pi^+ \to \bar{D}^0K^-\pi^+$	$4.21^{+0.89}_{-0.81}$	$1.90^{+1.01}_{-0.68}$	4.70 ± 4.38 [32]

BWT (or off-shell) effects from D^*

$B^- \to (D^* \to)D^+\pi^- K^-$

by cut $s > 2.1GeV$

Resonance	Fit fraction
$D^*_0(2400)^0$	$8.3 \pm 2.6 \pm 0.6 \pm 1.9$
$D^*_2(2460)^0$	$31.8 \pm 1.5 \pm 0.9 \pm 1.4$
$D^*_1(2760)^0$	$4.9 \pm 1.2 \pm 0.3 \pm 0.9$
S-wave nonresonant	$38.0 \pm 7.4 \pm 1.5 \pm 10.8$
P-wave nonresonant	$23.8 \pm 5.6 \pm 2.1 \pm 3.7$
$D^*_v(2007)^0$	$7.6 \pm 2.3 \pm 1.3 \pm 1.5$
B^*_v	$3.6 \pm 1.9 \pm 0.9 \pm 1.6$

Summary

1. We systematically analyze three-body decays $B \to DP_1P_2$ with ρ, K^*, ω, ϕ, and D^* resonances.

——The intermediate subprocesses $B \to DV, B \to D^*P_2$ are calculated in FAT approach;

——The resonant states V and D^* are described by relativistic Breit-Wigner distribution successively strong decay to P_1P_2.

2. The branching fractions agree with data measured by Babar, LHCb and Belle, and BWT effects for $\rho^{\pm}(770)$ to $K^{\pm}K_S$ system reach a proportion of approximately 20% of all ρ-resonant and nonresonant components measured by Belle II recently.

3. Predictions for the other modes, especially with comparable branching ratio $10^{-6} - 10^{-4}$ are also measurable in LHCb and Belle II.

Thank you

Backup slides

$$
\begin{aligned}
T &= \langle P_1(p_1)\, P_2(p_2)\, |(\bar{u}b)_{V-A}| B(p_B)\rangle \left\langle \bar{D}(p_{\bar{D}}) |(\bar{c}q)_{V-A}|0\right\rangle \\
&= \frac{\langle P_1(p_1)\, P_2(p_2)\, |V(p_V)\rangle}{s-m_V^2+im_V\Gamma_V(s)} \langle V(p_V)\, |(\bar{u}b)_{V-A}|\, B(p_B)\rangle \left\langle \bar{D}(p_{\bar{D}})\, |(\bar{c}u)_{V-A}|\, 0\right\rangle \\
&= p_{\bar{D}} \cdot (p_1-p_2)\; \sqrt{2}\, G_F\, V_{ub}V_{cq}^*\, a_1\, f_D\, m_V\, A_0^{B\to V}(m_D^2)\, \frac{g_{VP_1P_2}}{s-m_V^2+im_V\Gamma_V(s)}\,,
\end{aligned}
$$

$$
\begin{aligned}
C &= \langle P_1(p_1)\, P_2(p_2)\, |(\bar{u}b)_{V-A}|\, B(p_B)\rangle \left\langle \bar{D}(p_{\bar{D}})\, |(\bar{c}q)_{V-A}|\, 0\right\rangle \\
&= \frac{\langle P_1(p_1)\, P_2(p_2)\, |V(p_V)\rangle}{s-m_V^2+im_V\Gamma_V(s)} \langle V(p_V)\, |(\bar{q}b)_{V-A}|\, B(p_B)\rangle \left\langle \bar{D}(p_{\bar{D}})\, |(\bar{c}u)_{V-A}|\, 0\right\rangle \\
&= p_{\bar{D}} \cdot (p_1-p_2)\; \sqrt{2}\, G_F\, V_{ub}V_{cq}^*\, f_D\, m_V\, A_0^{B\to V}(m_D^2)\, \chi^C e^{i\phi^C}\, \frac{g_{VP_1P_2}}{s-m_V^2+im_V\Gamma_V(s)}
\end{aligned}
$$

$$
\begin{aligned}
E &= \left\langle \bar{D}(p_{\bar{D}})\, P_1(p_1)\, P_2(p_2)\, |\mathcal{H}_{eff}|\, B(p_B)\right\rangle \\
&= \frac{\langle P_1(p_1)\, P_2(p_2)\, |V(p_V)\rangle}{s-m_V^2+im_V\Gamma_V(s)} \left\langle \bar{D}(p_{\bar{D}})V(p_V)\, |\mathcal{H}_{eff}|\, B(p_B)\right\rangle \\
&= p_{\bar{D}} \cdot (p_1-p_2)\; \sqrt{2}\, G_F\, V_{ub}V_{cq}^*\, m_V\, f_B\, \frac{f_V f_{D_{(s)}}}{f_\pi f_D}\, \chi^E e^{i\phi^E}\, \frac{g_{VP_1P_2}}{s-m_V^2+im_V\Gamma_V(s)}\,,
\end{aligned}
$$

$$
\begin{aligned}
A &= \left\langle \bar{D}(p_{\bar{D}})\, P_1(p_1)\, P_2(p_2)\, |\mathcal{H}_{eff}|\, B(p_B)\right\rangle \\
&= \frac{\langle P_1(p_1)\, P_2(p_2)\, |V(p_V)\rangle}{s-m_V^2+im_V\Gamma_V(s)} \left\langle \bar{D}(p_{\bar{D}})V(p_V)\, |\mathcal{H}_{eff}|\, B(p_B)\right\rangle \\
&= p_{\bar{D}} \cdot (p_1-p_2)\; \sqrt{2}\, G_F\, V_{ub}V_{cq}^*\, a_1\, f_B\, \frac{f_D g_{DDV} m_D^2}{m_B^2-m_D^2}\, \frac{g_{VP_1P_2}}{s-m_V^2+im_V\Gamma_V(s)}\,,
\end{aligned}
$$

$$
A^{\bar{D}V} = -\sqrt{2}\, G_F\, V_{ub}V_{cq}^*\, a_1(\mu)\, f_B\, \frac{f_{D_{(s)}} g_{DDV} m_D^2}{m_B^2-m_D^2} (\varepsilon_V^* \cdot p_B)\,.
$$

$b \to u$ transition, respectively, where $q = d, s$ and $p_V = p_1 + p_2 = \sqrt{s}$. T

$$\langle \pi^+(p_1)\pi^-(p_2)|(\bar{u}b)_{V-A}|B^-\rangle^R = \sum_i \langle \pi^+\pi^-|V_i\rangle \frac{1}{s - m_{V_i}^2 + i m_{V_i}\Gamma_{V_i}} \langle V_i|(\bar{u}b)_{V-A}|B^-\rangle + \sum_i \langle \pi^+\pi^-|S_i\rangle \frac{-1}{s - m_{S_i}^2 + i m_{S_i}\Gamma_{S_i}} \langle S_i|(\bar{u}b)_{V-A}|B^-\rangle,$$

$$\langle \pi^+(p_1)\pi^-(p_2)|(\bar{u}b)_{V-A}|B^-\rangle^R = \sum_i \frac{g^{V_i\to\pi^+\pi^-}}{s_{12} - m_{V_i}^2 + i m_{V_i}\Gamma_{V_i}} \sum_{\text{pol}} \varepsilon^* \cdot (p_1 - p_2)\langle V_i|(\bar{u}b)_{V-A}|B^-\rangle - \sum_i \frac{g^{S_i\to\pi^+\pi^-}}{s_{12} - m_{S_i}^2 + i m_{S_i}\Gamma_{S_i}} \langle S_i|(\bar{u}b)_{V-A}|B^-\rangle,$$

f_π	f_K	f_D	f_{D_s}	f_B	f_{B_s}	f_ρ	f_{K^*}	f_ω	f_ϕ
130.2 ± 1.7	155.6 ± 0.4	211.9 ± 1.1	258 ± 12.5	190.9 ± 4.1	225 ± 11.2	213 ± 11	220 ± 11	192 ± 10	225 ± 11

	$F_1^{B\to D}$	$F_1^{B_s\to D_s}$	$A_0^{B\to\rho}$	$A_0^{B\to K^*}$	$A_0^{B_s\to K^*}$	$A_0^{B\to\omega}$	$A_0^{B_s\to\phi}$
$F_i(0)$	0.54	0.58	0.30	0.33	0.27	0.26	0.30
α_1	2.44	2.44	1.73	1.51	1.74	1.60	1.73
α_2	1.49	1.70	0.17	0.14	0.47	0.22	0.41

TABLE I: Masses m_V and full widths Γ_0 of vector resonant states.

Resonance	Line shape Parameters	Resonance	Line shape Parameters
$\rho(770)$	$m_V = 775.26$ MeV	$\omega(782)$	$m_V = 782.65$ MeV
	$\Gamma_0 = 149.1$ MeV		$\Gamma_0 = 8.49$ MeV
$K^*(892)^+$	$m_V = 891.66$ MeV	$K^*(892)^0$	$m_V = 895.55$ MeV
	$\Gamma_0 = 50.8$ MeV		$\Gamma_0 = 47.3$ MeV
$\phi(1020)$	$m_V = 1019.46$ MeV		
	$\Gamma_0 = 4.25$ MeV		

$$g_{\rho\to\pi^+\pi^-} = 6.0\,, \quad g_{K^*\to K^+\pi^-} = 4.59\,, \quad g_{\phi\to K^+K^-} = -4.54\,.$$

$$g_{\rho\to K^+K^-} : g_{\omega\to K^+K^-} : g_{\phi\to K^+K^-} = 1 : 1 : -\sqrt{2},$$

$$g_{\rho^0\pi^+\pi^-} = g_{\rho^+\pi^0\pi^+}\,,\ g_{\rho^0\pi^0\pi^0} = g_{\omega\pi^+\pi^-} = 0\,,$$

$$g_{\rho^0K^+K^-} = -g_{\rho^0K^0\bar{K}^0} = g_{\omega K^+K^-} = g_{\omega K^0\bar{K}^0},\ g_{\phi K^+K^-} = g_{\phi K^0\bar{K}^0}$$

TABLE VI: The same as table IV, but for the quasi-two-body decays (top) $\bar{B}_{(s)} \to D(\phi \to)KK$ ($\times 10^{-4}$), and (bottom) $\bar{B}_{(s)} \to \bar{D}(\phi \to)KK$ ($\times 10^{-6}$).

Decay Modes	Amplitudes	$\mathcal{B}_{\rm FAT}$	$\mathcal{B}_{\rm PQCD}$
$\bar{B} \to D(\phi \to)KK$	$V_{cb}V_{us}^*$		
$\bar{B}_s^0 \to D^0(\phi \to)K^+K^-$	C	$0.193^{+0.008+0.041+0.002}_{-0.008-0.037-0.002}$	
$\to D^0(\phi \to)K^0\bar{K}^0$		$0.134^{+0.006+0.028+0.001}_{-0.006-0.026-0.001}$	
$\bar{B} \to D(\phi \to)KK$	$V_{ub}V_{cs}^*$		
$B^- \to D_s^-(\phi \to)K^+K^-$	A	$0.75^{+0+0+0.32+0.07}_{-0-0-0.32-0.07}$	$0.15^{+0.02+0.01+0.01}_{-0.02-0.01-0.01}$
$\to D_s^-(\phi \to) \to K^0K^0$		$0.52^{+0+0+0.32+0.05}_{-0-0-0.34-0.05}$	$0.10^{+0.01+0.01+0.01}_{-0.01-0.01-0.01}$
$\bar{B}_s^0 \to \bar{D}^0(\phi \to)K^+K^-$	C	$2.85^{+0.12+0.60+0.03+0.25}_{-0.12-0.54-0.03-0.25}$	
$\to \bar{D}^0(\phi \to)\bar{K}^0K^0$		$1.98^{+0.08+0.42+0.02+0.17}_{-0.08-0.38-0.02-0.17}$	

$\bar{B} \to D(\omega \to)KK$	
$\bar{B}^0 \to D^0(\omega \to)K^+K^-$	$1.83^{+0.09+0.15+0.03}_{-0.08-0.32-0.03} \times 10^{-6}$
$\bar{B}_s^0 \to D^0(\omega \to)K^+K^-$	$4.01^{+0.70+0+0.92}_{-0.33-0-0.57} \times 10^{-9}$
$\bar{B} \to \bar{D}(\omega \to)KK$	
$B^- \to D_s^-(\omega \to)K^+K^-$	$6.90^{+0+1.45+0.68+0.61}_{-0-1.31-0.65-0.61} \times 10^{-8}$
$B^- \to D^-(\omega \to)K^+K^-$	$4.15^{+0+0.68+0.06+0.37}_{-0-0.63-0.05-0.37} \times 10^{-9}$
$\bar{B}^0 \to \bar{D}^0(\omega \to)K^+K^-$	$6.14^{+0.30+1.16+0.09+0.54}_{-0.28-1.05-0.09-0.54} \times 10^{-10}$
$\bar{B}_s^0 \to \bar{D}^0(\omega \to)K^+K^-$	$4.21^{+0.73+0+0.49+0.37}_{-0.34-0-0.45-0.37} \times 10^{-10}$