

# Hyperon weak radiative decays in the light-front approach

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- Introduction
- Light-front quark model in the hyperon
- Hyperon weak radiative decays

## Conclusion



#### Hyperon in the particle physice

- A large number of hyperon are producted in various colliders.
- Rich dynamics effect. CP violation, EDM and so on.
   2105.11155, 2307.04364
- A good platform for texting CKM matrix element.
- Precise texting the SM and QCD.



Hyperon weak radiative decays

#### Puzzle of WRHD

• The parity-violating amplitudes were predicted to be zero for  $\Sigma \to p\gamma$  and  $\Xi \to \Sigma\gamma$  processes in SU(3) symmetry.

$$\begin{split} \langle B'\gamma | \mathcal{H}(0) | B \rangle &= \frac{iG_F}{\sqrt{2}} \bar{u}_{B'} (a+b\gamma_5) \sigma_{\mu\nu} u_B \frac{q^{\nu} \epsilon^{*\mu}}{m_B}, \\ Br(B \to B'\gamma) &= \frac{G_F^2}{16\pi\Gamma_B} m_B \left( 1 - \frac{m_{B'}^2}{m_B^2} \right)^3 (|a|^2 + |b|^2), \quad \alpha = \frac{\Re(a \cdot b)}{|a|^2 + |b|^2} \,. \end{split}$$

• Confict with the experiment data.

$$\alpha(\Sigma^+ \to p\gamma) = -0.652 \pm 0.056 \pm 0.020.$$

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#### Hyperon weak radiative decays

The BESIII experiment published its precise measurement of the decay  $\Sigma \rightarrow p\gamma$ .  $Br(\Sigma^+ \rightarrow p\gamma) = (0.996 \pm 0.021 \pm 0.018) \times 10^{-3}$ 

$$\alpha(\Sigma^+ \to p\gamma) = -0.652 \pm 0.056 \pm 0.020.$$

This measurement is lower than its world average value by 4.2 standard deviations.  $Br \qquad \alpha$ 

$\Sigma^+ \to p \gamma$	$0.996 \pm 0.027$	$-0.652 \pm 0.060$
$\Lambda \to n\gamma$	$0.832 \pm 0.066$	$-0.16\pm0.11$
$\Xi^0\to\Lambda\gamma$	$1.17\pm0.07$	$-0.704 \pm 0.067$
$\Xi^0\to\Sigma^0\gamma$	$3.33\pm0.10$	$-0.69\pm0.06$
$\Xi^-\to \Sigma^- \gamma$	$0.127 \pm 0.023$	$1.0 \pm 1.3$



#### Light-front quark model with three quark picture

# Recent years, the LFQM has been explored to the three quark picture. 2212.00300, 2303.02946, 2304.07698

$$\begin{aligned} |\mathcal{B}(P,S,S_{z})\rangle &= \int \{d^{3}\tilde{p}_{1}\}\{d^{3}\tilde{p}_{2}\}\{d^{3}\tilde{p}_{3}\}2(2\pi)^{3}\delta^{3}(\tilde{P}-\tilde{p}_{1}-\tilde{p}_{2}-\tilde{p}_{3})\frac{1}{\sqrt{P^{+}}} \\ &\times \sum_{\lambda_{1},\lambda_{2},\lambda_{3}}\Psi^{SS_{z}}(\tilde{p}_{1},\tilde{p}_{2},\tilde{p}_{3},\lambda_{1},\lambda_{2},\lambda_{3})C^{ijk}|q_{1}^{i}(p_{1},\lambda_{1})q_{2}^{j}(p_{2},\lambda_{2})q_{3}^{k}(p_{3},\lambda_{3})\rangle, \end{aligned}$$

$$\begin{split} \Psi_{0}^{S=\frac{1}{2},S_{z}}(\tilde{p}_{i},\lambda_{i}) & \Psi_{1}^{S=\frac{1}{2},S_{z}}(\tilde{p}_{i},\lambda_{i}) \\ =& A_{0}\bar{u}(p_{3},\lambda_{3})(\bar{\not\!\!P}+M_{0})(-\gamma_{5})C\bar{u}^{T}(p_{2},\lambda_{2}) & =& A_{1}\bar{u}(p_{3},\lambda_{3})(\bar{\not\!P}+M_{0})(\gamma^{\mu}-v^{\mu})C\bar{u}^{T}(p_{2},\lambda_{2}) \\ & \times \bar{u}(p_{1},\lambda_{1})u(\bar{P},S_{z})\Phi(x_{i},k_{i\perp}), & \times \bar{u}(p_{1},\lambda_{1})(\frac{1}{\sqrt{3}}\gamma_{\mu}\gamma_{5})u(\bar{P},S_{z})\Phi(x_{i},k_{i\perp}), \end{split}$$



Wavefunction of heavy baryon

$$\begin{split} \Psi_0^{S=\frac{1}{2},S_z}(\tilde{p}_i,\lambda_i) \\ =& A_0 \bar{u}(p_3,\lambda_3)(\bar{I} + M_0)(-\gamma_5)C\bar{u}^T(p_2,\lambda_2) \\ &\times \bar{u}(p_1,\lambda_1)u(\bar{P},S_z)\Phi(x_i,k_{i\perp}), \end{split}$$

$$\begin{split} &\left\langle \frac{1}{2} \frac{1}{2}; s_3 s_2 \left| 0 s_{23} \right\rangle \left\langle \frac{1}{2} 0; s_1 0 \left| \frac{1}{2} S_z \right\rangle \right. \\ &= A_0 \bar{u}(p_3, s_3) (\bar{I} + M_0) (-\gamma_5) C \bar{u}^T(p_2, s_2) \\ &\times \bar{u}(p_1, s_1) u(\bar{P}, S_z) \end{split}$$

$$\begin{split} \Psi_{1}^{S=\frac{1}{2},S_{z}}(\tilde{p}_{i},\lambda_{i}) \\ =& A_{1}\bar{u}(p_{3},\lambda_{3})(\bar{I}^{p}+M_{0})(\gamma^{\mu}-v^{\mu})C\bar{u}^{T}(p_{2},\lambda_{2}) \\ &\times \bar{u}(p_{1},\lambda_{1})(\frac{1}{\sqrt{3}}\gamma_{\mu}\gamma_{5})u(\bar{P},S_{z})\Phi(x_{i},k_{i\perp}), \\ &\left\langle \frac{1}{2}\frac{1}{2};s_{3}s_{2} \middle| 1s_{23} \right\rangle \left\langle \frac{1}{2}1;s_{1}s_{23} \middle| \frac{1}{2}S_{z} \right\rangle \\ &= A_{1}\bar{u}(p_{3},s_{3})(\bar{I}^{p}+M_{0})(\gamma^{\mu}-v^{\mu})C\bar{u}^{T}(p_{2},s_{2}) \\ &\times \bar{u}(p_{1},s_{1})\left(\frac{1}{\sqrt{3}}\gamma_{\mu}\gamma_{5}\right)u(\bar{P},S_{z}) \\ & \mathbf{Phys. \, Rev. \, D \, 107, \, no.11, \, 116025} \, (2023) \end{split}$$



#### Wavefunction of light baryon

 $8_{M_A}$ 

 $3 \otimes 3 \otimes 3 = (6_S \oplus \overline{3}_A) \otimes 3 = (10_S \oplus 8_{M_S}) \oplus (8_{M_A} \oplus 1_A)$ 

$$\begin{split} 8_{M_S}: & p: \frac{1}{\sqrt{6}} (2uud - udu - duu) \\ & n: \frac{1}{\sqrt{6}} (-2ddu + dud + udd) \\ & \Sigma^+: \frac{1}{\sqrt{6}} (2uus - usu - suu) \\ & \Sigma^o: \frac{1}{\sqrt{12}} (2uds - usd - dsu + 2dus - sud - sa) \\ & \Sigma^-: \frac{1}{\sqrt{6}} (2dds - dsd - dsd) \\ & \Lambda^o: \frac{1}{2} (usd + sud - sdu - dsu) \\ & \Xi^o: \frac{1}{\sqrt{6}} (sus + uss - 2ssu) \\ & \Xi^-: \frac{1}{\sqrt{6}} (sds + dss - 2ssd) \end{split}$$

$$\begin{array}{ll} : & p: \frac{1}{\sqrt{2}}(udu - duu) \\ & n: \frac{1}{\sqrt{2}}(udd + dud) \\ & \Sigma^+: \frac{1}{\sqrt{2}}(usu - suu) \\ & \Sigma^o: \frac{1}{2}(usd + dsu - sdu - sud) \\ & \Sigma^-: \frac{1}{\sqrt{2}}(dsd - sdd) \\ & \Lambda^o: \frac{1}{\sqrt{12}}(2uds - dsu - sud - 2dus - sdu + usd) \\ & \Xi^o: \frac{1}{\sqrt{2}}(uss - sus) \\ & \Xi^-: \frac{1}{\sqrt{2}}(dss - sds) \end{array}$$



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#### Wavefunction of light baryon

 $\begin{array}{rcl} 2 \otimes 2 \otimes 2 &=& (2 \otimes 2) \otimes 2 = (1 \oplus 3) \otimes 2 = (1 \otimes 2) \oplus (3 \otimes 2) = 2 \oplus 2 \oplus 4. \\ \\ 2_{M_S}: & |1/2, 1/2\rangle: & \frac{1}{\sqrt{6}}(2 \uparrow \uparrow \downarrow - \uparrow \downarrow \uparrow - \downarrow \uparrow \uparrow) & 2_{M_A}: & |1/2, 1/2\rangle: & \frac{1}{\sqrt{2}}(\uparrow \downarrow \uparrow - \downarrow \uparrow \uparrow) \\ & |1/2, -1/2\rangle: & \frac{1}{\sqrt{6}}(\uparrow \downarrow \downarrow + \downarrow \uparrow \downarrow - 2 \downarrow \downarrow \uparrow) & |1/2, -1/2\rangle: & \frac{1}{\sqrt{2}}(\uparrow \downarrow \downarrow - \downarrow \uparrow \downarrow) \end{array}$ 

**Flavor symmetry wavefunction.** 

$$\begin{aligned} \frac{1}{\sqrt{2}} \left( \psi_{M_S}^f \eta_{M_S}^s + \psi_{M_A}^f \eta_{M_A}^s \right) \\ |\mathcal{B}(P, S, S_z)\rangle &= \int \{d^3 \tilde{p}_1\} \{d^3 \tilde{p}_2\} \{d^3 \tilde{p}_3\} 2(2\pi)^3 \delta^3 (\tilde{P} - \tilde{p}_1 - \tilde{p}_2 - \tilde{p}_3) \frac{1}{\sqrt{P^+}} \\ &\times \sum_{\lambda_1, \lambda_2, \lambda_3} \Psi^{S, S_z} (\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \lambda_1, \lambda_2, \lambda_3) C^{ijk} |q_1^i(p_1, \lambda_1) q_2^j(p_2, \lambda_2) q_3^k(p_3, \lambda_3)\rangle. \end{aligned}$$

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#### **Spin Wavefunction**

$$\begin{split} \Psi_{S/A}^{S,S_z} &= \bar{u}(p_3,\lambda_3)_{\alpha} \bar{u}^T(p_2,\lambda_2)_{\beta} \bar{u}(p_1,\lambda_1)_{\gamma} \Gamma_{S/A}^{\alpha\beta\gamma} A_1 \Phi(x_i,k_{i\perp}), \\ \\ Spin information \end{split} \begin{array}{l} \Gamma_S^{\alpha\beta\gamma} &= -[(\bar{I}\!\!\!/\, + M_0)(\gamma^\mu - v^\mu)C]^{\alpha\beta} \left[\frac{1}{\sqrt{3}}\gamma_\mu\gamma_5 u(\bar{P},S_z)\right]^{\gamma}, & 2MS \\ \Gamma_A^{\alpha\beta\gamma} &= [(\bar{I}\!\!\!/\, + M_0)(-\gamma_5)C]^{\alpha\beta}[u(\bar{P},S_z)]^{\gamma}, & 2MA \\ \end{split} \end{split}$$

$$\Psi^{S,S_z} = \frac{1}{\sqrt{2}} A_1 \Phi(x_i, k_{i\perp}) \left( M^S_{\alpha\beta\gamma} \Gamma^{\alpha\beta\gamma}_S + M^A_{\alpha\beta\gamma} \Gamma^{\alpha\beta\gamma}_A \right),$$
  

$$M^S_{\alpha\beta\gamma} = \frac{1}{\sqrt{6}} \left( 2\bar{u}(p_3, \lambda_3)_{\alpha} \bar{u}(p_2, \lambda_2)_{\beta} \bar{d}(p_1, \lambda_1)_{\gamma} - \bar{u}(p_3, \lambda_3)_{\alpha} \bar{d}(p_2, \lambda_2)_{\beta} \bar{u}(p_1, \lambda_1)_{\gamma} - \bar{d}(p_3, \lambda_3)_{\alpha} \bar{u}(p_2, \lambda_2)_{\beta} \bar{u}(p_1, \lambda_1)_{\gamma} \right),$$
  

$$M^A_{\alpha\beta\gamma} = \frac{1}{\sqrt{2}} \left( \bar{u}(p_3, \lambda_3)_{\alpha} \bar{d}(p_2, \lambda_2)_{\beta} \bar{u}(p_1, \lambda_1)_{\gamma} - \bar{d}(p_3, \lambda_3)_{\alpha} \bar{u}(p_2, \lambda_2)_{\beta} \bar{u}(p_1, \lambda_1)_{\gamma} \right),$$



### **Flavor Wavefunction**

One can define  $p_i$  as the momentum of a quark with a specified flavor  $p_3 \longrightarrow u$ 

$$\begin{array}{l} p_2 \longrightarrow \mathsf{u} \\ p_1 \longrightarrow \mathsf{d} \\ |\mathcal{B}(P,S,S_z)\rangle \ = \ \int \{d^3 \tilde{p}_1\} \{d^3 \tilde{p}_2\} \{d^3 \tilde{p}_3\} \times 2(2\pi)^3 \delta^3 (\tilde{P} - \tilde{p}_1 - \tilde{p}_2 - \tilde{p}_3) \frac{1}{\sqrt{P^+}} \\ \times \ \sum_{\lambda_1,\lambda_2,\lambda_3} \Psi^{S,S_z}(\tilde{p}_1,\tilde{p}_2,\tilde{p}_3,\lambda_1,\lambda_2,\lambda_3) C^{ijk} | u^i(p_3,\lambda_1) u^j(p_2,\lambda_2) d^k(p_1,\lambda_3) \rangle. \end{array}$$

$$\Psi^{S,S_z} = \frac{1}{\sqrt{2}} A_1 \Phi(x_i, k_{i\perp}) \bar{u}(p_3, \lambda_3)_{\alpha} \bar{u}(p_2, \lambda_2)_{\beta} \bar{u}(p_1, \lambda_1)_{\gamma} \times \left(\frac{1}{\sqrt{6}} (2\Gamma_S^{\alpha\beta\gamma} - \Gamma_S^{\beta\gamma\alpha}) - \Gamma_S^{\beta\gamma\alpha}) + \frac{1}{\sqrt{2}} (\Gamma_A^{\alpha\gamma\beta} - \Gamma_A^{\gamma\beta\alpha}) \right).$$



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#### Simplify the wavefunction

$$\Psi^{S,S_z} = \frac{1}{\sqrt{2}} A_1 \Phi(x_i, k_{i\perp}) \bar{u}(p_3, \lambda_3)_{\alpha} \bar{u}(p_2, \lambda_2)_{\beta} \bar{u}(p_1, \lambda_1)_{\gamma} \times \left(\frac{1}{\sqrt{6}} (2\Gamma_S^{\alpha\beta\gamma} - \Gamma_S^{\beta\gamma\alpha}) + \frac{1}{\sqrt{2}} (\Gamma_A^{\alpha\gamma\beta} - \Gamma_A^{\gamma\beta\alpha})\right).$$

#### **Fierz transformation**

$$\begin{split} \Gamma_{S}^{\alpha\gamma\beta} &= -\frac{1}{4\sqrt{3}} \bigg( \frac{1}{2} [(\bar{I}\!\!\!/ + M_{0})\sigma^{\mu\nu}\gamma_{5}C]^{\alpha\beta} [\sigma_{\mu\nu}u(\bar{P},S_{z})]^{\gamma} - [(\bar{I}\!\!\!/ + M_{0})(\gamma^{\mu} - v^{\mu})C]^{\alpha\beta} [\gamma_{\mu}\gamma_{5}u(\bar{P},S_{z})]^{\gamma} \\ &\quad +7[(\bar{I}\!\!\!/ + M_{0})\gamma_{5}C]^{\alpha\beta} [u(\bar{P},S_{z})]^{\gamma} - [(\bar{I}\!\!\!/ + M_{0})\gamma^{\mu}\gamma_{5}C]^{\alpha\beta} [\gamma_{\mu}u(\bar{P},S_{z})]^{\gamma} \bigg) \\ \Gamma_{S}^{\beta\gamma\alpha} &= -\frac{1}{4\sqrt{3}} \bigg( \frac{1}{2} [(\bar{I}\!\!\!/ + M_{0})\sigma^{\mu\nu}\gamma_{5}C]^{\alpha\beta} [\sigma_{\mu\nu}u(\bar{P},S_{z})]^{\gamma} - [(\bar{I}\!\!\!/ + M_{0})(\gamma^{\mu} - v^{\mu})C]^{\alpha\beta} [\gamma_{\mu}\gamma_{5}u(\bar{P},S_{z})]^{\gamma} \\ &\quad -[(\bar{I}\!\!\!/ + M_{0})\gamma^{\mu}\gamma_{5}C]^{\alpha\beta} [\gamma_{\mu}u(\bar{P},S_{z})]^{\gamma} - [5(\bar{I}\!\!\!/ + M_{0})(\gamma^{\mu} - v^{\mu})C]^{\alpha\beta} [\gamma_{\mu}\gamma_{5}u(\bar{P},S_{z})]^{\gamma} \bigg) \\ \Gamma_{A}^{\alpha\gamma\beta} &= \frac{-1}{4} \bigg( -\frac{1}{2} [(\bar{I}\!\!\!/ + M_{0})\sigma^{\mu\nu}\gamma_{5}C]^{\alpha\beta} [\sigma_{\mu\nu}u(\bar{P},S_{z})]^{\gamma} + [(\bar{I}\!\!/ + M_{0})(\gamma^{\mu} + v^{\mu})\gamma_{5}C]^{\alpha\beta} [\gamma_{\mu}u(\bar{P},S_{z})]^{\gamma} \\ &\quad +[(\bar{I}\!\!/ + M_{0})(\gamma^{\mu} - v^{\mu})C]^{\alpha\beta} [\gamma_{\mu}\gamma_{5}u(\bar{P},S_{z})]^{\gamma} \bigg) \\ \Gamma_{A}^{\beta\gamma\alpha} &= \frac{1}{4} \bigg( \frac{1}{2} [(\bar{I}\!\!/ + M_{0})\sigma^{\mu\nu}\gamma_{5}C]^{\alpha\beta} [\sigma_{\mu\nu}u(\bar{P},S_{z})]^{\gamma} - [(\bar{I}\!\!/ + M_{0})(\gamma^{\mu} - v^{\mu})C]^{\alpha\beta} [\gamma_{\mu}\gamma_{5}u(\bar{P},S_{z})]^{\gamma} \\ &\quad -[(\bar{I}\!\!/ + M_{0})\sigma^{\mu\nu}\gamma_{5}C]^{\alpha\beta} [\sigma_{\mu\nu}u(\bar{P},S_{z})]^{\gamma} + [3(\bar{I}\!\!/ + M_{0})\gamma_{5}C]^{\alpha\beta} [u(\bar{P},S_{z})]^{\gamma} \bigg) \bigg) \end{split}$$



#### The wavefunction of octet baryon

$$\begin{split} \Psi_{p}^{S,S_{z}} &= \bar{u}(p_{3},\lambda_{3})_{\alpha}\bar{u}(p_{2},\lambda_{2})_{\beta}\bar{u}(p_{1},\lambda_{1})_{\gamma}\Gamma_{p}^{\alpha\beta\gamma}A_{p}\Phi(x_{i},k_{i\perp}) \\ \Gamma_{p}^{\alpha\beta\gamma} &= \left( [(\bar{I}\!\!\!/P + M_{0})\sigma^{\mu\nu}\gamma_{5}C]^{\alpha\beta}[\sigma_{\mu\nu}u(\bar{P},S_{z})]^{\gamma} - 4[(\bar{I}\!\!\!/P + M_{0})(\gamma^{\mu} - v^{\mu})C]^{\alpha\beta}[\gamma_{\mu}\gamma_{5}u(\bar{P},S_{z})]^{\gamma} \\ &- 2[(\bar{I}\!\!\!/P + M_{0})(\gamma^{\mu} - v^{\mu})\gamma_{5}C]^{\alpha\beta}[\gamma_{\mu}u(\bar{P},S_{z})]^{\gamma} \right). \end{split}$$

 $\begin{array}{l} \text{the lowering operators} \\ T_{-}u = d, \quad U_{-}d = s, \quad V_{-}u = s. \\ T_{-}|p\rangle = |n\rangle, \ -U_{-}|p\rangle = |\Sigma^{+}\rangle, \ \frac{1}{\sqrt{2}}T_{-}U_{-}|p\rangle = |\Sigma^{0}\rangle\frac{1}{2}T_{-}T_{-}U_{-}|p\rangle = |\Sigma^{-}\rangle, \\ -V_{-}U_{-}|p\rangle = |\Xi^{0}\rangle, \\ T_{-}V_{-}U_{-}|p\rangle = |\Xi^{0}\rangle, \\ T_{-}V_{-}U_{-}|p\rangle = |\Xi^{-}\rangle, \ \frac{-1}{\sqrt{6}}(V_{-} + U_{-}T_{-})|p\rangle = |\Lambda\rangle. \end{array}$ 



The wavefunction of octet baryon

$$\begin{aligned} |\mathcal{B}(P,S,S_{z})\rangle &= \int \{d^{3}\tilde{p}_{1}\}\{d^{3}\tilde{p}_{2}\}\{d^{3}\tilde{p}_{3}\}2(2\pi)^{3}\delta^{3}(\tilde{P}-\tilde{p}_{1}-\tilde{p}_{2}-\tilde{p}_{3})\frac{1}{\sqrt{P^{+}}}\\ &\times \sum_{\lambda_{1},\lambda_{2},\lambda_{3}}\Psi^{SS_{z}}(\tilde{p}_{1},\tilde{p}_{2},\tilde{p}_{3},\lambda_{1},\lambda_{2},\lambda_{3})C^{ijk}|q_{1}^{i}(p_{1},\lambda_{1})q_{2}^{j}(p_{2},\lambda_{2})q_{3}^{k}(p_{3},\lambda_{3})\rangle, \end{aligned}$$

Order of q1 q2 q3  

$$p: uud, n: ddu, \Sigma^{+}: uus, \Sigma^{0}: uds, \Sigma^{-}: dds, \Xi^{0}: ssu, \Xi^{-}: ssd, \Lambda: uds. V_{-} = \Gamma_{-}^{\alpha\beta\gamma}$$

$$\Gamma_{N}^{\alpha\beta\gamma} \equiv \Gamma_{p}^{\alpha\beta\gamma} = -\Gamma_{n}^{\alpha\beta\gamma}, \Gamma_{\Sigma}^{\alpha\beta\gamma} \equiv \Gamma_{\Sigma^{-}}^{\alpha\beta\gamma} = \sqrt{2}\Gamma_{\Sigma^{0}}^{\alpha\beta\gamma} = -\Gamma_{\Sigma^{+}}^{\alpha\beta\gamma}, \Gamma_{\Xi}^{\alpha\beta\gamma} \equiv \Gamma_{\Xi^{0}}^{\alpha\beta\gamma} = -\Gamma_{\Xi^{-}}^{\alpha\beta\gamma}$$

$$\Gamma_{N}^{\alpha\beta\gamma} = \Gamma_{\Xi}^{\alpha\beta\gamma} = \Gamma_{\Xi}^{\alpha\beta\gamma}.$$
The wavefunction of Lambda  

$$\bar{u}(p_{3}, \lambda_{3})_{\alpha}\bar{u}(p_{2}, \lambda_{2})_{\beta}\bar{u}(p_{1}, \lambda_{1})_{\gamma}\Gamma_{\Lambda}^{\alpha\beta\gamma}A_{\Lambda}\Phi(x_{i}, k_{i\perp}),$$

$$\Gamma_{\Lambda}^{\alpha\beta\gamma} = [(\bar{P} + M_{0})\gamma_{5}C]^{\alpha\beta}[u(\bar{P}, S_{z})]^{\gamma}.$$
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#### The symmetry of three quark

$$\begin{split} \Psi_{p}^{S,S_{z}} &= \bar{u}(p_{3},\lambda_{3})_{\alpha}\bar{u}(p_{2},\lambda_{2})_{\beta}\bar{u}(p_{1},\lambda_{1})_{\gamma}\Gamma_{p}^{\alpha\beta\gamma}A_{p}\Phi(x_{i},k_{i\perp}) \\ \Gamma_{p}^{\alpha\beta\gamma} &= \left( [(\vec{P}+M_{0})\sigma^{\mu\nu}\gamma_{5}C]^{\alpha\beta}[\sigma_{\mu\nu}u(\vec{P},S_{z})]^{\gamma} - 4[(\vec{P}+M_{0})(\gamma^{\mu}-v^{\mu})C]^{\alpha\beta}[\gamma_{\mu}\gamma_{5}u(\vec{P},S_{z})]^{\gamma} \\ -2[(\vec{P}+M_{0})(\gamma^{\mu}-v^{\mu})\gamma_{5}C]^{\alpha\beta}[\gamma_{\mu}u(\vec{P},S_{z})]^{\gamma} \right). \\ \mathbf{Fierz transformation} \\ [(\vec{P}+M_{0})(\gamma^{\mu}-v^{\mu})C]^{\beta\alpha}[\gamma_{\mu}\gamma_{5}u(\vec{P},S_{z})]^{\gamma} &= [(\vec{P}+M_{0})(\gamma^{\mu}-v^{\mu})C]^{\alpha\beta}[\gamma_{\mu}\gamma_{5}u(\vec{P},S_{z})]^{\gamma}, \\ [(\vec{P}+M_{0})(\gamma^{\mu}-v^{\mu})\gamma_{5}C]^{\beta\alpha}[\gamma_{\mu}u(\vec{P},S_{z})]^{\gamma} &= [(\vec{P}+M_{0})\gamma_{5}C]^{\alpha\beta}[u(\vec{P},S_{z})]^{\gamma} + [(\vec{P}-M_{0})\gamma^{\mu}\gamma_{5}C]^{\alpha\beta}[\gamma_{\mu}u(\vec{P},S_{z})]^{\gamma}, \\ [(\vec{P}+M_{0})\sigma^{\mu\nu}\gamma_{5}C]^{\beta\alpha}[\sigma_{\mu\nu}u(\vec{P},S_{z})]^{\gamma} &= [(\vec{P}+M_{0})\sigma^{\mu\nu}\gamma_{5}C]^{\alpha\beta}[\sigma_{\mu\nu}u(\vec{P},S_{z})]^{\gamma} - 4[M_{0}\gamma^{\mu}\gamma_{5}C]^{\alpha\beta}[\gamma_{\mu}u(\vec{P},S_{z})]^{\gamma} \\ +4[\vec{P}\gamma_{5}C]^{\alpha\beta}[u(\vec{P},S_{z})]^{\gamma}. \end{split}$$

$$(A3)$$

This symmetry can only be constructed based on the fact that the spin wave function in our work is symmetric under the exchange of  $\lambda 1$  and  $\lambda 2$ 



$$G_A(p_i,\lambda_i) = A(\bar{u}(p_3,\lambda_3),\bar{u}(p_2,\lambda_2),\bar{u}(p_1,\lambda_1))\left\langle\frac{1}{2}\frac{1}{2};\lambda_3\lambda_2|0\lambda_{23}\right\rangle\left\langle0\frac{1}{2};0\lambda_1|\frac{1}{2}S_z\right\rangle,$$
  

$$G_S(p_i,\lambda_i) = A(\bar{u}(p_3,\lambda_3),\bar{u}(p_2,\lambda_2),\bar{u}(p_1,\lambda_1))\left\langle\frac{1}{2}\frac{1}{2};\lambda_3\lambda_2|1\lambda_{23}\right\rangle\left\langle1\frac{1}{2};1\lambda_1|\frac{1}{2}S_z\right\rangle,$$

summing up all the helicities

$$\begin{split} \sum_{\lambda_1,\lambda_2,\lambda_3} G_A(p_i,\lambda_i) &= \frac{1}{\sqrt{2}} \bigg[ A\left(\bar{u}(p_3,\frac{1}{2}),\bar{u}(p_2,-\frac{1}{2}),\bar{u}(p_1,\frac{1}{2})\right) - A\left(\bar{u}(p_3,-\frac{1}{2}),\bar{u}(p_2,\frac{1}{2}),\bar{u}(p_1,\frac{1}{2})\right) \bigg] \\ \sum_{\lambda_1,\lambda_2,\lambda_3} G_S(p_i,\lambda_i) &= \frac{1}{\sqrt{6}} \bigg[ 2A\left(\bar{u}(p_3,\frac{1}{2}),\bar{u}(p_2,\frac{1}{2}),\bar{u}(p_1,-\frac{1}{2})\right) - A\left(\bar{u}(p_3,\frac{1}{2}),\bar{u}(p_2,-\frac{1}{2}),\bar{u}(p_1,\frac{1}{2})\right) \\ &- A(\bar{u}(p_3,-\frac{1}{2}),\bar{u}(p_2,\frac{1}{2}),\bar{u}(p_1,\frac{1}{2})) \bigg]. \end{split}$$

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$$\begin{split} \sum_{\lambda_{1},\lambda_{2},\lambda_{3}} \Psi^{S,S_{z}} &= \overline{A_{1}\Phi(x_{i},k_{i\perp})\frac{1}{\sqrt{18}} \bigg[ 2A\left(\bar{u}(p_{3},\frac{1}{2}),\bar{u}(p_{2},\frac{1}{2}),\bar{u}(p_{1},-\frac{1}{2})\right) - A\left(\bar{u}(p_{3},\frac{1}{2}),\bar{u}(p_{2},-\frac{1}{2}),\bar{u}(p_{1},\frac{1}{2})\right) \\ &- A\left(\bar{u}(p_{3},-\frac{1}{2}),\bar{u}(p_{2},\frac{1}{2}),\bar{u}(p_{1},\frac{1}{2})\right) - A\left(\bar{u}(p_{3},\frac{1}{2}),\bar{u}(p_{2},\frac{1}{2}),\bar{u}(p_{1},-\frac{1}{2})\right) \\ &+ 2A\left(\bar{u}(p_{3},\frac{1}{2}),\bar{u}(p_{2},-\frac{1}{2}),\bar{u}(p_{1},\frac{1}{2})\right) - A\left(\bar{u}(p_{3},-\frac{1}{2}),\bar{u}(p_{2},\frac{1}{2}),\bar{u}(p_{1},\frac{1}{2})\right) \\ &- A\left(\bar{u}(p_{3},\frac{1}{2}),\bar{u}(p_{2},\frac{1}{2}),\bar{u}(p_{1},-\frac{1}{2})\right) + 2A\left(\bar{u}(p_{3},\frac{1}{2}),\bar{u}(p_{2},-\frac{1}{2}),\bar{u}(p_{1},\frac{1}{2})\right) \\ &- A\left(\bar{u}(p_{3},-\frac{1}{2}),\bar{u}(p_{2},\frac{1}{2}),\bar{u}(p_{1},-\frac{1}{2})\right) \\ &= \frac{1}{\sqrt{18}}A_{1}\Phi(x_{i},k_{i\perp})\bigg[ 2A\left(\bar{u}(p_{3},\frac{1}{2}),\bar{u}(p_{2},\frac{1}{2}),\bar{u}(p_{1},\frac{1}{2})\right) - A\left(\bar{u}(p_{3},\frac{1}{2}),\bar{u}(p_{2},-\frac{1}{2}),\bar{u}(p_{1},\frac{1}{2})\right) \\ &- A\left(\bar{u}(p_{3},-\frac{1}{2}),\bar{u}(p_{2},\frac{1}{2}),\bar{u}(p_{1},\frac{1}{2})\right) - A\left(\bar{u}(p_{3},\frac{1}{2}),\bar{u}(p_{2},-\frac{1}{2}),\bar{u}(p_{1},\frac{1}{2})\right) \\ &- A\left(\bar{u}(p_{3},-\frac{1}{2}),\bar{u}(p_{2},\frac{1}{2}),\bar{u}(p_{1},\frac{1}{2})\right) + \text{permutations}\bigg]. \end{split}$$



$$\sum_{\lambda_1,\lambda_2,\lambda_3} \Psi^{S,S_z} = \frac{1}{\sqrt{18}} A_1 \Phi(x_i, k_{i\perp}) \left[ 2A\left(\bar{u}(p_3, \frac{1}{2}), \bar{u}(p_2, \frac{1}{2}), \bar{u}(p_1, -\frac{1}{2})\right) - A\left(\bar{u}(p_3, \frac{1}{2}), \bar{u}(p_2, -\frac{1}{2}), \bar{u}(p_1, \frac{1}{2})\right) - A\left(\bar{u}(p_3, -\frac{1}{2}), \bar{u}(p_2, \frac{1}{2}), \bar{u}(p_1, \frac{1}{2})\right) - A\left(\bar{u}(p_3, -\frac{1}{2}), \bar{u}(p_3, -\frac{1}{2}), \bar{u}(p_3, \frac{1}{2}), \bar{u}(p_3, \frac{1}{2})\right) - A\left(\bar{u}(p_3, -\frac{1}{2}), \bar{u}(p_3, \frac{1}{2}), \bar{u}(p_3, \frac{1}{2})\right) - A\left(\bar{u}(p_3, -\frac{1}{2}), \bar{u}(p_3, \frac{1}{2}), \bar{u}(p_3, \frac{1}{2})\right) - A\left(\bar{u}(p_3, \frac{1}{2}), \frac{1}{2}), \bar{u}(p_3, \frac{1}{2})\right)$$

$$\begin{split} \Phi(x_i, k_{i\perp}) &= \sqrt{\frac{e_1 e_2 e_3}{x_1 x_2 x_3 M_0}} 16 \left(\frac{\pi}{\beta^2}\right)^{3/2} \pi^{-3/16} \times \exp\left[-\sum_{i \le j}^3 \frac{\vec{k}_{i\perp} \cdot \vec{k}_{j\perp} + k_{iz} k_{jz}}{2\beta^2}\right] \\ &\int \left(\prod_{i=1}^3 \frac{dx_i d^2 k_{i\perp}}{2(2\pi)^3}\right) 2(2\pi)^3 |\Phi(x_i, k_{i\perp})|^2 \times \delta\left(1 - \sum x_i\right) \delta^2\left(\sum k_{i\perp}\right) = 1. \end{split}$$

The shape parameter can be determined by the decay constent which estimated by Lattice QCD

$$\begin{split} \lambda_{1}^{B\neq\Lambda} &= \frac{1}{4m_{B}^{2}} \int \frac{dx_{2}d^{2}\vec{k}_{2\perp}}{2(2\pi)^{3}} \frac{dx_{3}d^{2}\vec{k}_{3\perp}}{2(2\pi)^{3}} \frac{1}{\sqrt{x_{1}x_{2}x_{3}}} A_{B\neq\Lambda} \Phi(x_{i},k_{i\perp}) \\ &\times \left( -2\mathrm{Tr}[(\vec{I}\!\!\!/ + M_{0})(\gamma^{\alpha} - v^{\alpha})\gamma_{5}(\not\!\!/ _{2} - m_{2})(1 - \gamma_{5})\gamma^{\mu}(\not\!\!/ _{3} + m_{3})]\mathrm{Tr}[\gamma_{\mu}(1 + \gamma_{5})(\not\!\!/ _{1} + m_{1})\gamma_{\alpha}(\vec{I}\!\!\!/ + M_{0})] \\ &-4\mathrm{Tr}[(\vec{I}\!\!\!/ + M_{0})(\gamma^{\alpha} - v^{\alpha})(\not\!\!/ _{2} - m_{2})(1 - \gamma_{5})\gamma^{\mu}(\not\!\!/ _{3} + m_{3})]\mathrm{Tr}[\gamma_{\mu}(1 + \gamma_{5})(\not\!\!/ _{1} + m_{1})\gamma_{\alpha}\gamma_{5}(\vec{I}\!\!\!/ + M_{0})] \\ &+\mathrm{Tr}[(\vec{I}\!\!\!/ + M_{0})\sigma^{\alpha\beta}\gamma_{5}(\not\!\!/ _{2} - m_{2})(1 - \gamma_{5})\gamma^{\mu}(\not\!\!/ _{3} + m_{3})]\mathrm{Tr}[\gamma_{\mu}(1 + \gamma_{5})(\not\!\!/ _{1} + m_{1})\sigma_{\alpha\beta}(\vec{I}\!\!\!/ + M_{0})] \Big), \end{split}$$

$$\begin{split} \lambda_{1}^{\Lambda} &= -\frac{1}{4m_{\Lambda}^{2}} \int \frac{dx_{2}d^{2}\vec{k}_{2\perp}}{2(2\pi)^{3}} \frac{dx_{3}d^{2}\vec{k}_{3\perp}}{2(2\pi)^{3}} \frac{\sqrt{6}}{\sqrt{x_{1}x_{2}x_{3}}} A_{\Lambda} \Phi(x_{i},k_{i\perp}) \\ &\times \left( \mathrm{Tr}[(\vec{I} + M_{0})\gamma_{5}(\not\!\!\!p_{2} - m_{d})(1 - \gamma_{5})\gamma^{\mu}(\not\!\!\!p_{3} + m_{u})] \mathrm{Tr}[\gamma_{\mu}(1 + \gamma_{5})(\not\!\!\!p_{1} + m_{s})(\vec{I} + M_{0})] \right). \end{split}$$



$$\sum_{\lambda_1,\lambda_2,\lambda_3} \Psi^{S,S_z} = \frac{1}{\sqrt{18}} A_1 \Phi(x_i, k_{i\perp}) \left[ 2A\left(\bar{u}(p_3, \frac{1}{2}), \bar{u}(p_2, \frac{1}{2}), \bar{u}(p_1, -\frac{1}{2})\right) - A\left(\bar{u}(p_3, \frac{1}{2}), \bar{u}(p_2, -\frac{1}{2}), \bar{u}(p_1, \frac{1}{2})\right) - A\left(\bar{u}(p_3, -\frac{1}{2}), \bar{u}(p_2, -\frac{1}{2}), \bar{u}(p_1, \frac{1}{2})\right) - A\left(\bar{u}(p_3, -\frac{1}{2}), \bar{u}(p_2, -\frac{1}{2}), \bar{u}(p_1, \frac{1}{2})\right) + \text{permutations} \right].$$

$$\begin{split} \Phi(x_i, k_{i\perp}) &= \sqrt{\frac{e_1 e_2 e_3}{x_1 x_2 x_3 M_0}} 16 \left(\frac{\pi}{\beta^2}\right)^{3/2} \pi^{-3/16} \times \exp\left[-\sum_{i \le j}^3 \frac{\vec{k}_{i\perp} \cdot \vec{k}_{j\perp} + k_{iz} k_{jz}}{2\beta^2}\right] \\ &\int \left(\prod_{i=1}^3 \frac{dx_i d^2 k_{i\perp}}{2(2\pi)^3}\right) 2(2\pi)^3 |\Phi(x_i, k_{i\perp})|^2 \times \delta\left(1 - \sum x_i\right) \delta^2\left(\sum k_{i\perp}\right) = 1. \end{split}$$

The shape parameter can be determined by the decay constent which estimated by Lattice QCD

baryons	Ν	Σ	Ξ	Λ		
$\lambda_1^B(10^{-3}{ m GeV}^2)$	-44.9	-46.1	-49.8	-42.2		
$\beta({ m GeV})$	0.361	0.345	0.434	0.378		



#### Hamiltonians

$$\begin{aligned} \mathcal{H}(s \to d\gamma) &= \frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* \sum_{i}^{1-6,12} [(z_i(\mu) + \tau y_i(\mu))Q_i(\mu)], \\ Q_1 &= [\bar{u}_{\alpha}\gamma_{\mu}(1-\gamma_5)s_{\beta}][\bar{d}_{\beta}\gamma^{\mu}(1-\gamma_5)u_{\alpha}], \ Q_2 &= [\bar{u}_{\alpha}\gamma_{\mu}(1-\gamma_5)s_{\alpha}][\bar{d}_{\beta}\gamma^{\mu}(1-\gamma_5)u_{\beta}], \\ Q_3 &= [\bar{d}_{\alpha}\gamma_{\mu}(1-\gamma_5)s_{\alpha}] \sum_{q} [\bar{q}_{\beta}\gamma^{\mu}(1-\gamma_5)q_{\beta}], \ Q_4 &= [\bar{d}_{\alpha}\gamma_{\mu}(1-\gamma_5)s_{\beta}] \sum_{q} [\bar{q}_{\beta}\gamma^{\mu}(1-\gamma_5)q_{\alpha}], \\ Q_5 &= [\bar{d}_{\alpha}\gamma_{\mu}(1-\gamma_5)s_{\alpha}] \sum_{q} [\bar{q}_{\beta}\gamma^{\mu}(1+\gamma_5)q_{\beta}], \ Q_6 &= [\bar{d}_{\alpha}\gamma_{\mu}(1-\gamma_5)s_{\beta}] \sum_{q} [\bar{q}_{\beta}\gamma^{\mu}(1+\gamma_5)q_{\alpha}], \\ Q_{12} &= \frac{e}{16\pi^2} m_s \bar{d}\sigma^{\mu\nu}(1+\gamma_5)sF_{\mu\nu}, \quad \tau &= -\frac{V_{ts}V_{td}^*}{V_{us}V_{ud}^*}, \end{aligned}$$

 $z_1 = -0.606, \ z_2 = 1.346, \ y_1 = y_2 = 0, \ z_{12} = -0.081, \ y_{12} = -0.383.$ 





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#### Matrix element



#### **Matrix element**

 $\mathcal{M} \equiv \langle B'\gamma | \mathcal{H}(0) | B \rangle = \frac{G_F}{\sqrt{2}} (\mathcal{M}_P + \mathcal{M}_T),$  $\mathcal{M}_{T} = V_{us}V_{ud}^{*}\sum(z_{i}+\tau y_{i})\langle B'\gamma|Q_{i}|B\rangle, \quad \mathcal{M}_{P} = V_{us}V_{ud}^{*}(z_{12}+\tau y_{12})\frac{e}{8\pi^{2}}m_{s}\times\langle B'\gamma|\bar{d}\sigma^{\mu\nu}(1+\gamma_{5})sF_{\mu\nu}|B\rangle,$  $\mathcal{M}_{P} = -V_{us}V_{ud}^{*}(z_{12} + \tau y_{12})\frac{e}{8\pi^{2}}m_{s}\Gamma_{B}^{abc}\overline{\Gamma}_{B'}^{\alpha\beta\gamma} \int \frac{dx_{1}d^{2}\vec{k}_{1\perp}}{2(2\pi)^{3}}\frac{dx_{2}d^{2}\vec{k}_{2\perp}}{2(2\pi)^{3}}\frac{\Phi(x_{i},k_{i\perp})\Phi^{*}(x_{i}',k_{i\perp}')}{\sqrt{x_{1}(1-x_{2}'-x_{3}')}}$  $\times \ [(p_1' + m_1')\sigma^{\mu\nu}(1 + \gamma_5)(p_1 + m_1)]_{\gamma c} \times [p_3 + m_3]_{\alpha a}[p_2 + m_2]_{\beta b}A_BA_{B'}(2iq_{\nu}\epsilon_{\mu}^*(q)),$  $\mathcal{M}_{T}^{b} = C \int dx_{1} dx_{2} dx_{1}^{\prime} \frac{d^{2}k_{1\perp}}{2(2\pi)^{3}} \frac{d^{2}k_{2\perp}}{2(2\pi)^{3}} \frac{d^{2}k_{1\perp}^{\prime}}{2(2\pi)^{3}} \frac{A_{B}A_{B^{\prime}}\epsilon_{\mu}^{*}(q)\Gamma_{B}^{def}\overline{\Gamma}_{B^{\prime}}^{abc}\Phi(x_{i},k_{i\perp})\Phi^{*}(x_{i}^{\prime},k_{i\perp}^{\prime})}{\sqrt{x_{1}(1-x_{1}-x_{2})x_{1}^{\prime}(1-x_{1}^{\prime}-x_{2})}(k^{2}-m_{2}^{\prime 2})}$  $\times \ [(p_{2}' + m_{3}')\gamma^{\mu}(\not k + m_{3}')\gamma_{\nu}(1 - \gamma_{5})(\not p_{1} + m_{1})]_{af} \times [(\not p_{1}' + m_{1}')\gamma^{\nu}(1 - \gamma_{5})(\not p_{2} + m_{3})]_{cd}[\not p_{2} + m_{2}]_{be},$  $\mathcal{M}_{T}^{c} = C \int dx_{1} dx_{2} dx_{1}^{\prime} \frac{d^{2} k_{1\perp}}{2(2\pi)^{3}} \frac{d^{2} k_{2\perp}}{2(2\pi)^{3}} \frac{d^{2} k_{1\perp}^{\prime}}{2(2\pi)^{3}} \frac{A_{B} A_{B^{\prime}} \epsilon_{\mu}^{*}(q) \Gamma_{B}^{def} \overline{\Gamma}_{B^{\prime}}^{abc} \Phi(x_{i}, k_{i\perp}) \Phi^{*}(x_{i}^{\prime}, k_{i\perp}^{\prime})}{\sqrt{x_{1}(1 - x_{1} - x_{2})x_{1}^{\prime}(1 - x_{1}^{\prime} - x_{2})} (k^{2} - m_{1}^{2})}$  $\times \ [(p_3' + m_3')\gamma_{\nu}(1 - \gamma_5)(k + m_1)\gamma^{\mu}(p_1 + m_1)]_{af} \times [(p_1' + m_1')\gamma^{\nu}(1 - \gamma_5)(p_3 + m_3)]_{cd}[p_2 + m_2]_{be},$  $\mathcal{M}_{T}^{f} = C \int dx_{1} dx_{1}' \frac{d^{2}k_{1\perp}}{2(2\pi)^{3}} \frac{d^{2}k_{1\perp}'}{2(2\pi)^{3}} A_{B} A_{B'} \epsilon_{\mu}^{*}(q) \Gamma_{B}^{def} \overline{\Gamma}_{B'}^{abc} \frac{1}{\sqrt{x_{1}(1-x_{1}-x_{2})x_{1}'(1-x_{1}'-x_{2}')x_{2}P^{+}q^{-}}}$  $\times \left( -\int dx_2 \frac{d^2 k_{2\perp}}{2(2\pi)^3} [(\not\!\!p_2 - \not\!\!q + m_2) \gamma^{\mu} (\not\!\!p_2 + m_2)]_{be} + \int dx_2' \frac{d^2 k_{2\perp}'}{2(2\pi)^3} [(\not\!\!p_2' + m_2) \gamma^{\mu} (\not\!\!p_2' + \not\!\!q + m_2)]_{be} \right)$  $C = eV_{us}V_{ud}^*[z_1 - z_2 + \frac{22}{7}(y_1 - y_2))],$  $\times [(p_3' + m_3')\gamma_{\nu}(1 - \gamma_5)(p_1 + m_1)]_{af}\Phi(x_i, k_{i\perp}) \times [(p_1' + m_1')\gamma^{\nu}(1 - \gamma_5)(p_3 + m_3)]_{cd}\Phi^*(x_i', k_{i\perp}'),$ 



#### **Symmetry factor**

For every specific radiative decay process, we need to consider an additional symmetry factor  $SF_{B\to B'}^{T/P}$  arising from the flavor and color symmetries of each baryon in the penguin or tree diagram,

$$\begin{split} SF^P_{\Sigma^+ \to p\gamma} &= 2, \qquad SF^T_{\Sigma^+ \to p\gamma} = 4, \qquad SF^P_{\Lambda/\Sigma^0 \to n\gamma} = 2, \qquad SF^T_{\Lambda/\Sigma^0 \to n\gamma} = 2\\ SF^P_{\Xi^0 \to \Lambda/\Sigma^0 \gamma} &= 2, \qquad SF^T_{\Xi^0 \to \Lambda/\Sigma^0 \gamma} = 2, \qquad SF^P_{\Xi^- \to \Sigma^- \gamma} = 4, \qquad SF^T_{\Xi^- \to \Sigma^- \gamma} = 0. \end{split}$$



#### **Numerical results**

TABLE III. The corresponding form factor for each Feynman diagram in unit  $(10^{-3} \text{GeV}^2)$ . Here we show the numerical results for penguin diagram. Actually,  $a^P$ ,  $b^P$  have complex phase from CKM matrix as  $e^{i\delta}$  with  $\delta = -3.14$ . The total form factor can be expressed as  $a = a^P e^{i\delta} + a^W$  and  $b = b^P e^{i\delta} + b^W$ .

form factors	$a^P$	$a^b$	$a^c$	$a^d$	$a^e$	$a^f$	$b^P$	$b^b$	$b^c$	$b^d$	$b^e$	$b^f$	$a^W$	$b^W$
$\Sigma^+ \to p\gamma$	0.73	-5.96	-4.77	6.75	-4.86	1.44	0.21	0.83	-0.43	-0.43	0.49	0.54	-7.40	1.00
$\Lambda \to n\gamma$	-0.31	-0.79	-1.58	3.97	-6.30	11.67	0.06	0.07	-0.75	-0.62	0.34	-0.92	6.97	-1.88
$\Sigma^0 \to n\gamma$	1.06	1.44	1.97	-2.09	-1.16	-1.68	0.30	-0.42	-0.10	-0.29	-0.64	2.36	-1.52	0.91
$\Xi^0\to\Lambda\gamma$	-0.07	0.17	9.56	4.05	-2.05	-20.52	0.006	0.32	0.42	-0.20	-1.07	-1.62	-8.79	-2.15
$\Xi^0\to\Sigma^0\gamma$	0.02	11.12	-27.56	10.49	-1.68	29.03	0.01	0.97	0.85	0.10	-0.10	-4.98	21.40	-3.16
$\Xi^-\to \Sigma^- \gamma$	-0.04	-	-	-	-	-	-0.01	-	-	-	-	-	-	-

#### The W exchange diagram give the mainly contribution.

#### We are working hard to recalculate the results



#### **Phenomenological analysis**

$$Br(B \to B'\gamma) = \frac{G_F^2}{16\pi\Gamma_B} m_B \left(1 - \frac{m_{B'}^2}{m_B^2}\right)^3 (|a|^2 + |b|^2), \quad \alpha = \frac{\Re(a \cdot b)}{|a|^2 + |b|^2}.$$

#### polarization

$$\begin{split} \frac{dBr^{\pm}(B \to B'\gamma)}{d\cos(\theta_{\gamma})} &= \frac{G_F^2}{16\pi\Gamma_B} m_B \left(1 - \frac{m_{B'}^2}{m_B^2}\right)^3 \times \left(|a|^2 + |b|^2 \mp 2Re(a \cdot b^*)\cos(\theta_{\gamma})\right) \\ A_{FB}^{\pm} &= \frac{\left(\int_0^1 d\cos\theta_{\gamma} \frac{dBr^{\pm}}{d\cos\theta_{\gamma}} - \int_{-1}^0 d\cos\theta_{\gamma} \frac{dBr^{\pm}}{d\cos\theta_{\gamma}}\right)}{Br^{\pm}}. \end{split}$$



#### Phenomenological analysis

The disparity in  $\alpha(\Sigma + \rightarrow p\gamma)$  indicates that these processes may entail a substantial strong phase

$$a = |a^{P}|e^{i\delta}e^{i\delta_{a}^{P}} + |a^{W}|e^{i\delta_{a}^{W}}, \ b = |b^{P}|e^{i\delta}e^{i\delta_{b}^{P}} + |b^{W}|e^{i\delta_{b}^{W}}$$

The weak phase is determined by the CKM matrix element. One can try to determine the strong phase by fit the  $\alpha$ .



#### **Conclusion and outlook**

- We explore the LFQM in a three-quark picture for the light baryon octetRich dynamics effect. CP violation, EDM and so on.
- Our calculations indicate that the main contribution to hyperon weak radiative decays induced by s → d comes from the W exchange Feynman diagram.
- We are working hard to recalculate the results
- We are trying decrease are error from the vegas integration and give an error analysis.
- We are trying to estimate the contibution of QCD penguin diagram.