奇特重味四夸克态的质量和衰变

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武汉

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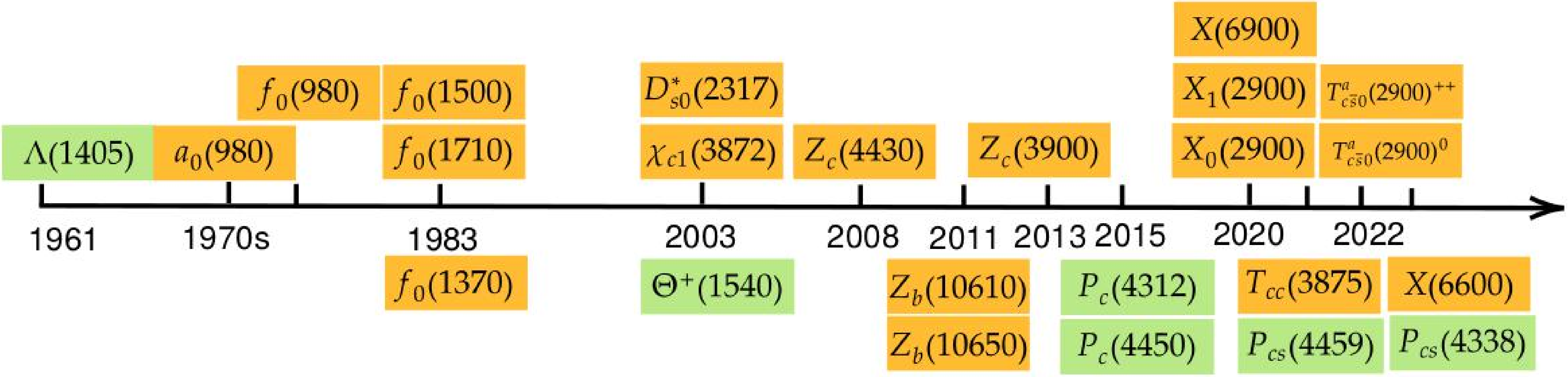
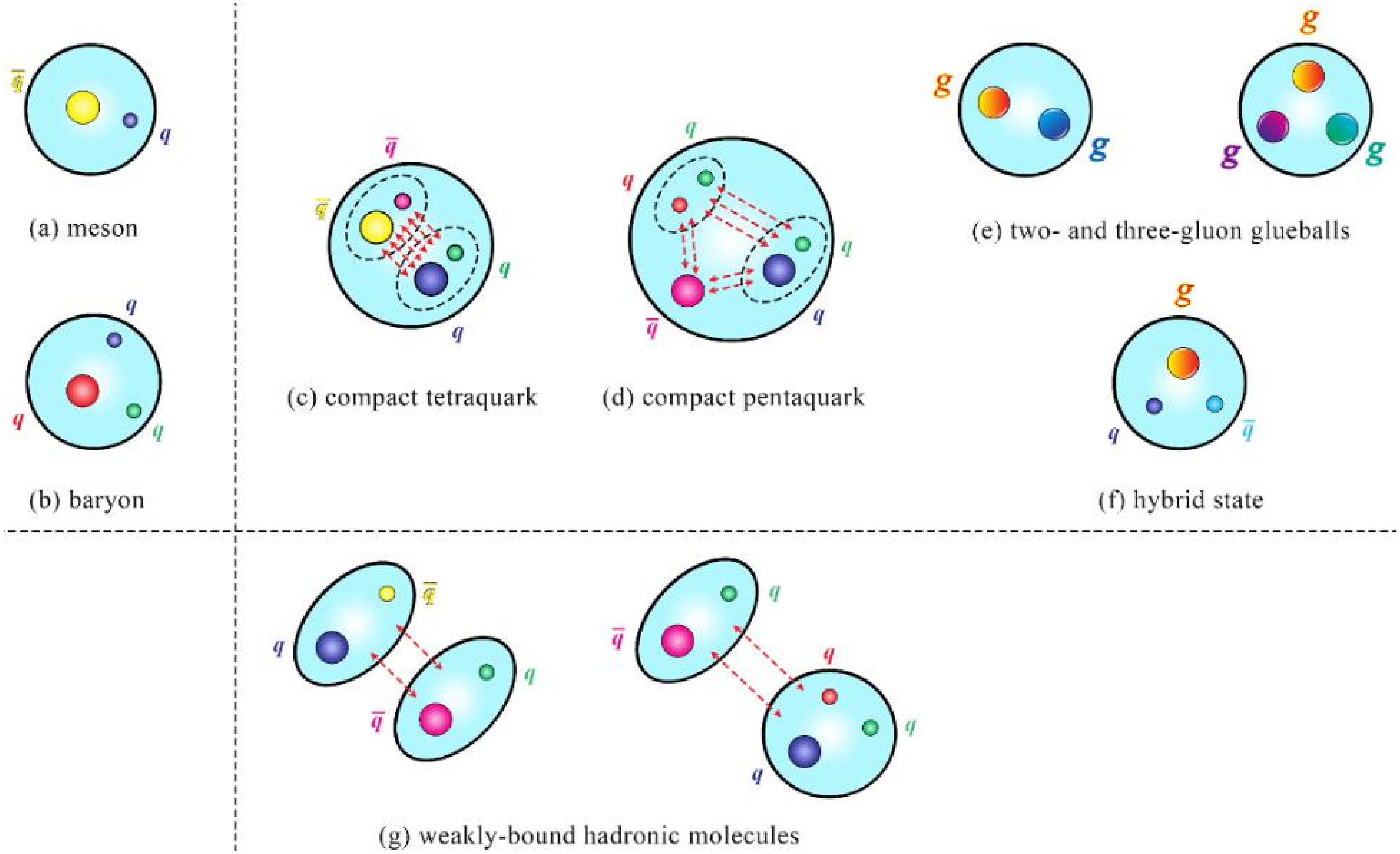
Content

Y.R. Liu, H.X. Chen, W. Chen, X. Liu, S.L. Zhu, Prog. Part. Nucl. Phys. 107, 237 (2019);
H.X. Chen, W. Chen, X. Liu, Y.R. Liu, S.L. Zhu, Rep. Prog. Phys. 86, 026201 (2023).



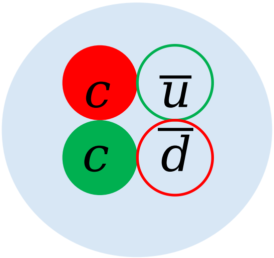
Some of the observed exotic states

Chen et al., Rep. Prog. Phys. 86, 026201 (2023).



$cs\bar{c}\bar{s}$

State	Mass(MeV)	Γ (MeV)	Observed channels
$\chi_{c1}(4140)$[CDF:2009jgo]	$4143.0 \pm 2.9 \pm 1.2$	$11.7^{+8.3}_{-5.0} \pm 3.7$	$B^+ \to J/\psi\phi K^+$
$X(4350)$[Belle:2009rkh]	$4350^{+4.6}_{-5.1} \pm 0.7$	$13^{+18}_{-9} \pm 4$	$\gamma\gamma \to J/\psi\phi$
$\chi_{c1}(4274)$[CDF:2011pep]	$4274^{+8.4}_{-6.7} \pm 1.9$	$32.3^{+21.9}_{-15.3} \pm 7.6$	$B^+ \to J/\psi\phi K^+$
$\chi_{c0}(4500)$[LHCb:2016axx]	$4506 \pm 11^{+12}_{-15}$	$92 \pm 21^{+21}_{-20}$	$B^+ \to J/\psi\phi K^+$
$\chi_{c0}(4700)$[LHCb:2016axx]	$4704 \pm 10^{+14}_{-24}$	$120 \pm 31^{+42}_{-33}$	$B^+ \to J/\psi\phi K^+$
$X(4630)$[LHCb:2021uow]	$4626 \pm 16^{+18}_{-110}$	$174 \pm 27^{+134}_{-73}$	$B^+ \to J/\psi\phi K^+$
$X(4685)$[LHCb:2021uow]	$4684 \pm 7^{+13}_{-16}$	$126 \pm 15^{+37}_{-41}$	$B^+ \to J/\psi\phi K^+$
$X(3960)$[LHCb:2022aki]	$3956 \pm 5 \pm 10$	$43 \pm 13 \pm 8$	$B^+ \to D_s^+ D_s^- K^+$
$X_0(4140)$[LHCb:2022aki]	$4133 \pm 6 \pm 6$	$67 \pm 17 \pm 7$	$B^+ \to D_s^+ D_s^- K^+$



T_{cc}

$cc\bar{c}\bar{c}$

State	Mass (MeV)	Γ(MeV)	Observed channels
$X(6900)$[LHCb:2020bwg]	$6905 \pm 11 \pm 7$ MeV	$80 \pm 19 \pm 33$ MeV	di-J/ψ
$X(6600)$[CMS:2023owd]	$6552 \pm 10 \pm 12$ MeV	$124^{+32}_{-26} \pm 33$ MeV	diJ/ψ
$X(7200)$[CMS:2023owd]	$7287^{+20}_{-18} \pm 5$MeV	$95^{+59}_{-40} \pm 19$MeV	di-J/ψ
$X(6400)$[ATLAS:2023bft]	$6.41 \pm 0.08^{+0.08}_{-0.03}$ GeV	$0.59 \pm 0.35^{+0.12}_{-0.2}$GeV	di-J/ψ
$X(6600)$[ATLAS:2023bft]	$6.63 \pm 0.05^{+0.08}_{-0.01}$ GeV	$0.35 \pm 0.11^{+0.11}_{-0.04}$GeV	di-J/ψ
$X(7200)$[ATLAS:2023bft]	$7.22 \pm 0.03^{+0.01}_{-0.04}$GeV	$0.095 \pm 0.6^{+0.06}_{-0.05}$ GeV	$J/\psi + \psi(2S)$

$cs\bar{n}\bar{n}$

$cn\bar{s}\bar{n}$

State	Mass(MeV)	Γ (MeV)	Observed channels
$T_{cs0}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+ D^- K^+$
$T_{cs1}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$B^+ \to D^+ D^- K^+$
$T^a_{c\bar{s}0}(2900)^0$[LHCb:2022sfr,LHCb:2022lzp]	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to \bar{D}^0 D_s^+ \pi^-$
$T^a_{c\bar{s}0}(2900)^{++}$[LHCb:2022sfr,LHCb:2022lzp]	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^- D_s^+ \pi^+$

Introduction

Related papers:

Y.R. Liu, X. Liu, S.L. Zhu, Phys. Rev. D 93, 074023 (2016)
J. Wu, Y.R. Liu, K.Chen, X.Liu, S.L. Zhu, Phys. Rev. D 94, 094031 (2016)
S.Q. Luo, K. Chen, X. Liu, Y.R. Liu, S.L. Zhu, Eur. Phys. J. C 77, 709 (2017)
J. Wu, Y.R. Liu, K. Chen, X. Liu, S.L. Zhu, Phys. Rev. D 95, 034002 (2017)
J. Wu, Y.R. Liu, K.Chen, X.Liu, S.L. Zhu, Phys. Rev. D 97, 094015 (2018)
J. Wu, X. Liu, Y.R. Liu, S.L. Zhu, Phys. Rev. D 99, 014037 (2019)
J.B. Cheng, Y.R. Liu, Phys. Rev. D 100, 054002 (2019)
J.B. Cheng, S.Y. Li, Y.R. Liu, Y.N. Liu, Z.G. Si, T. Yao, Phys. Phys. D 101, 114017 (2020)
J.B. Cheng, S.Y. Li, Y.R. Liu, Z.G. Si, T. Yao, Chin. Phys. C 45, 043102 (2021)
S.Y. Li, Y.R. Liu, Z.L. Man, Z.G. Si, J. Wu, Phys. Rev. D 108, 056015 (2023)
S.Y. Li, Y.R. Liu, Z.L. Man, Z.G. Si, J. Wu, arXiv: 2308.06768 [hep-ph]
S.Y. Li, Y.R. Liu, Z.L. Man, Z.G. Si, J. Wu, arXiv: 2401.00115 [hep-ph]

Formalism: color-magnetic interaction (CMI) model [symmetry analysis]

$$H = \sum_i m_i^{eff} + H_{eff},$$
$$H_{eff} = -\sum_{i<j} C_{ij} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j.$$

$$M = \sum_i m_i + E_{\rm CMI}$$

- Although problems:
  - Dynamics (no);
  - Effective quark masses (system to system);
  - Effective coupling constants (conventional → multiquark);
  - Estimated masses (uncertainty).
- Simple for estimation of rough positions of multiquark states
- CMI model for mass splittings can catch basic features of spectra
- Research methods in this model:
  - (1) Construct flavor-color-spin wave function bases;
  - (2) Mixing between different color-spin structures
    - → Base independent results

Hadron	CMI	Hadron	CMI	Parameter(MeV)
N	$-8C_{nn}$	Δ	$8C_{nn}$	$C_{nn}=18.4$
Σ	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{ns}$	Σ^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{ns}$	$C_{ns}=12.4$
Ξ^0	$\frac{8}{3}(C_{ss}-4C_{ns})$	Ξ^{*0}	$\frac{8}{3}(C_{ss}+C_{ns})$	
Ω	$8C_{ss}$			$C_{ss}=6.5$
Λ	$-8C_{nn}$			
D	$-16C_{c\bar{n}}$	D^*	$\frac{16}{3}C_{c\bar{n}}$	$C_{c\bar{n}}=6.7$
D_s	$-16C_{c\bar{s}}$	D_s^*	$\frac{16}{3}C_{c\bar{s}}$	$C_{c\bar{s}}=6.7$
B	$-16C_{b\bar{n}}$	B^*	$\frac{16}{3}C_{b\bar{n}}$	$C_{b\bar{n}}=2.1$
B_s	$-16C_{b\bar{s}}$	B^*	$\frac{16}{3}C_{b\bar{s}}$	$C_{b\bar{s}}=2.3$
η_c	$-16C_{c\bar{c}}$	J/ψ	$\frac{16}{3}C_{c\bar{c}}$	$C_{c\bar{c}}=5.3$
η_b	$-16C_{b\bar{b}}$	Υ	$\frac{16}{3}C_{b\bar{b}}$	$C_{b\bar{b}}=2.9$
Σ_c	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{cn}$	Σ_c^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{cn}$	$C_{cn}=4.0$
Ξ_c'	$\frac{8}{3}C_{ns}-\frac{16}{3}C_{cn}-\frac{16}{3}C_{cs}$	Ξ_c^*	$\frac{8}{3}C_{ns}+\frac{8}{3}C_{cn}+\frac{8}{3}C_{cs}$	$C_{cs}=4.8$
Σ_b	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{bn}$	Σ_b^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{bn}$	$C_{bn}=1.3$
Ξ_b'	$\frac{8}{3}C_{ns}-\frac{16}{3}C_{bn}-\frac{16}{3}C_{bs}$	Ξ_b^*	$\frac{8}{3}C_{ns}+\frac{8}{3}C_{bn}+\frac{8}{3}C_{bs}$	$C_{bs}=1.2$

$$H=\sum_i m_i^{eff}-\sum_{i<j}C_{ij}\lambda_i\cdot\lambda_j\sigma_i\cdot\sigma_j$$

$$\begin{aligned} m_n &= 361.8\text{ MeV}, \\ m_s &= 540.4\text{ MeV}, \\ m_c &= 1724.8\text{ MeV}, \\ m_b &= 5052.9\text{ MeV}. \end{aligned}$$

$(n=u,d)$

TABLE III. Comparison for hadron masses between experimental data and theoretical estimation. All the values are in units of MeV.

Hadron	Theory	Experiment	Deviation	Hadron	Theory	Experiment	Deviation
D	1975.9	1864.8	111.1	D^*	2121.0	2007.0	114.0
D_s	2154.5	1968.3	186.2	D_s^*	2299.5	2112.1	187.4
η_c	3361.0	2983.6	377.4	J/ψ	3474.1	3096.9	377.2
Σ_c	2452.9	2454.0	1.1	Σ_c^*	2516.9	2518.4	−1.5
Ω_c	2796.2	2695.2	101.0	Ω_c^*	2845.3	2765.9	79.4
Ξ_c	2525.9	2471.0	54.9	Ξ_c'	2612.3	2577.9	34.4
Ξ_c^*	2680.6	2645.9	34.7				

Formalism: CMI model

- Our scheme to study multiquark spectrum:

$$M = \sum_i m_i + E_{\text{CMI}} \quad \longrightarrow \quad \boxed{M = [M_{ref} - (E_{\text{CMI}})_{ref}] + E_{\text{CMI}}}$$

(1) Important color-mixing (base independent results)

(2) $M_{upper\ limit} = \sum_i m_i + E_{CMI}$

(3) Reference scale → hadron-hadron threshold

M= $[M_{ref=threshold} - (E_{CMI})_{ref}] + E_{CMI}$,

more reasonable masses

(4) But, from studies for

$cs\overline{cs}$, $QQ\overline{QQ}$, $qq\overline{QQ}$, $Qq\overline{Q}\overline{q}$, $QQ\overline{Q}\overline{q}$;

$c\bar{c}qqq$, $QQqq\bar{q}$, $QQQq\bar{q}$ 偏低 → $M_{lower\ limit}$

(5) Reference scale → mass of X(4140)

Formalism: CMI model→mass splitting model

Li et al., PRD 108, 056015 (2023)

Assumption: $X(4140)$ observed in $J/\psi\phi$ as the lowest 1^{++} $cs\bar{c}\bar{s}$ tetraquark

$$M = M_{X(4140)} - (E_{CMI})_{X(4140)} + \sum_{ij} n_{ij}\Delta_{ij} + E_{CMI}$$

where $\Delta_{ij} = m_i - m_j$ denotes the effective quark mass gap between i quark and j quark

C_{ij}	n	s	c	b	$C_{i\bar{j}}$	$\bar{n}$	$\bar{s}$	$\bar{c}$	$\bar{b}$
n	18.3	12.1	4.0	1.3	n	29.8	18.7	6.6	2.1
s		6.5	4.3	1.3	s		9.8	6.7	2.3
c			3.5	2.0	c			5.3	3.3
b				1.9	b				2.9

$$\frac{C_{cc}}{C_{c\bar{c}}} = \frac{C_{bb}}{C_{b\bar{b}}} = \frac{C_{bc}}{C_{b\bar{c}}} = \frac{C_{nn}}{C_{n\bar{n}}} \approx \frac{2}{3}$$

Godfrey-Isgur model: $m_{B_c^*} - m_{B_c} = 70$ MeV

Wu et al., PRD 99, 014037 (2019);
Cheng et al., PRD 101, 114017 (2020)

$$\Delta_{bc}=3340.2\text{MeV},$$
$$\Delta_{cn}=1280.7\text{MeV},$$
$$\Delta_{sn}=90.6\text{MeV},$$
$$\Delta_{cs}=1180.6\text{MeV},$$
$$\Delta_{bs}=4520.2\text{MeV}.$$

Approximate relations:

$$\Delta_{cn}\approx\Delta_{cs}+\Delta_{sn},$$
$$\Delta_{bs}\approx\Delta_{bc}+\Delta_{cs}.$$

Formalism: rearrangement decay

- Combine information from spectrum and decay to analyze multiquark properties
- A simple decay scheme:

1. decay Hamiltonian is a constant: $\boldsymbol{H_{decay}} = \mathcal{C}$
2. measured width ≈sum of two-body rearrangement decay widths: $\boldsymbol{\Gamma_{exp}} \approx \boldsymbol{\Gamma_{sum}}$

$$\mathcal{M} = \langle initial | H_{decay} | final \rangle = \mathcal{C} \sum_{ij} x_i y_j$$

$$\Psi_{initial} = \sum_i x_i (q_1 q_2 \bar{q}_3 \bar{q}_4),$$

$$\Psi_{final} = \sum_i y_i (q_1 q_2 \bar{q}_3 \bar{q}_4).$$

$$\Gamma = |\mathcal{M}|^2 \frac{|\mathbf{P}|}{8\pi M_{initial}^2}$$

system-dependent $\mathcal{C}$

Analyzed masses and widths of the Pc states in:

PRD100, 054002(2019); 108, 056015 (2023)

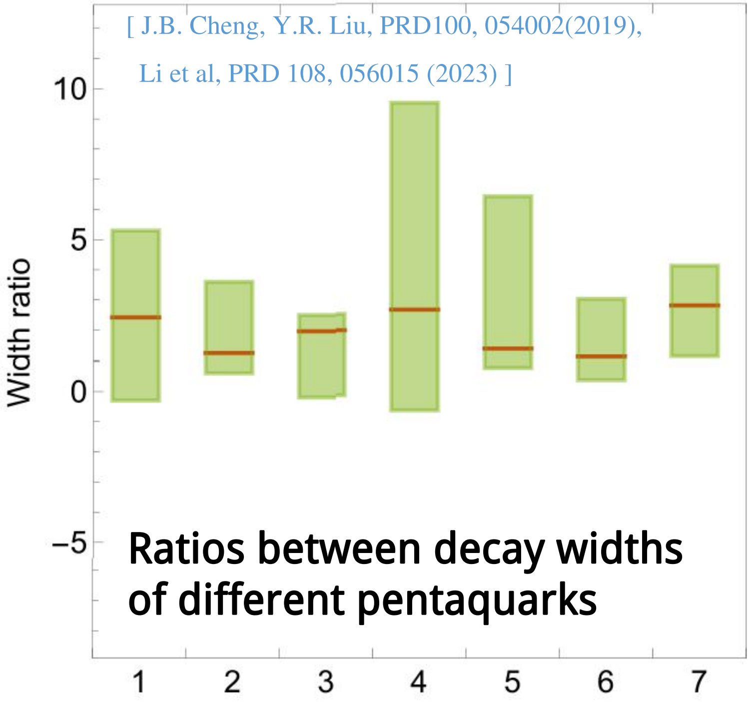
State	Mass(MeV)	Γ (MeV)	Observed channels
$P_\psi^N(4380)^+$[22]	$4380 \pm 8 \pm 29$	$215 \pm 18 \pm 86$	$\Lambda_b^0 \to J/\psi p K^-$
$P_\psi^N(4312)^+$[23]	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	$\Lambda_b^0 \to J/\psi p K^-$
$P_\psi^N(4440)^+$[23]	$4440 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.2}$	$\Lambda_b^0 \to J/\psi p K^-$
$P_\psi^N(4457)^+$[23]	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	$\Lambda_b^0 \to J/\psi p K^-$
$P_{\psi s}^\Lambda 4459)^0$[54]	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$	$\Xi_b^- \to J/\psi \Lambda K^-$
$P_\psi^N(4337)^+$[53]	$4337^{+7}_{-4}{}^{+2}_{-2}$	$29^{+26}_{-12}{}^{+14}_{-14}$	$B_s^0 \to J/\psi p \bar{p}$
$P_{\psi s}^\Lambda(4338)^0$ [55]	$4338.2 \pm 0.7 \pm 0.4$	$7.0 \pm 1.2 \pm 1.3$	$B^- \to J/\psi \Lambda \bar{p}$

Example of formalism: Pc states

$(nnn)_{8_c}(c\bar{c})_{8_c}$-$(nnn)_{1_c}(c\bar{c})_{1_c}$

Assume Pc(4312) as the second lowest

$I(J^P) = \frac{1}{2}(\frac{3}{2}^-)\ nnnc\bar{c}$



Ratios between decay widths of different pentaquarks

Theoretical states

$$\Gamma(\tilde{P}_c(4421)^+) : \Gamma(\tilde{P}_c(4461)^+) = 2.42,$$
$$\Gamma(\tilde{P}_c(4421)^+) : \Gamma(\tilde{P}_c(4312)^+) = 1.24,$$
$$\Gamma(\tilde{P}_c(4312)^+) : \Gamma(\tilde{P}_c(4461)^+) = 1.96,$$
$$\Gamma(\tilde{P}_c(4324)^+) : \Gamma(\tilde{P}_c(4461)^+) = 2.64,$$
$$\Gamma(\tilde{P}_c(4324)^+) : \Gamma(\tilde{P}_c(4312)^+) = 1.35,$$
$$\Gamma(\tilde{P}_c(4324)^+) : \Gamma(\tilde{P}_c(4421)^+) = 1.09.$$

Experimental states

$$\Gamma(P_c(4440)^+) : \Gamma(P_c(4457)^+) = 3.2^{+2.1}_{-3.5},$$
$$\Gamma(P_c(4440)^+) : \Gamma(P_c(4312)^+) = 2.1^{+1.5}_{-1.5},$$
$$\Gamma(P_c(4312)^+) : \Gamma(P_c(4457)^+) = 1.5^{+1.0}_{-1.7},$$
$$\Gamma(P_c(4337)^+) : \Gamma(P_c(4457)^+) = 4.5^{+5.0}_{-5.2},$$
$$\Gamma(P_c(4337)^+) : \Gamma(P_c(4312)^+) = 3.0^{+3.4}_{-2.3},$$
$$\Gamma(P_c(4337)^+) : \Gamma(P_c(4440)^+) = 1.4^{+1.6}_{-1.1}.$$

$P_c(4457)^+$, $P_c(4440)^+$, $P_c(4337)^+$ can be regarded as the J=3/2, J=1/2, and J=1/2 pentaquark states, respectively.

For $P_c(4457)^+$ $\Gamma(\Sigma_c^*\bar{D}) : \Gamma(\Lambda_c\bar{D}) : \Gamma(NJ/\psi) = 2.3 : 4.0 : 1.0$

For $P_c(4440)^+$ $\Gamma(\Lambda_c\bar{D}^*) : \Gamma(\Sigma_c\bar{D}) : \Gamma(\Lambda_c\bar{D}) : \Gamma(NJ/\psi) : \Gamma(N\eta_c) = 45.5 : 3.0 : 3.0 : 7.5 : 1.0$

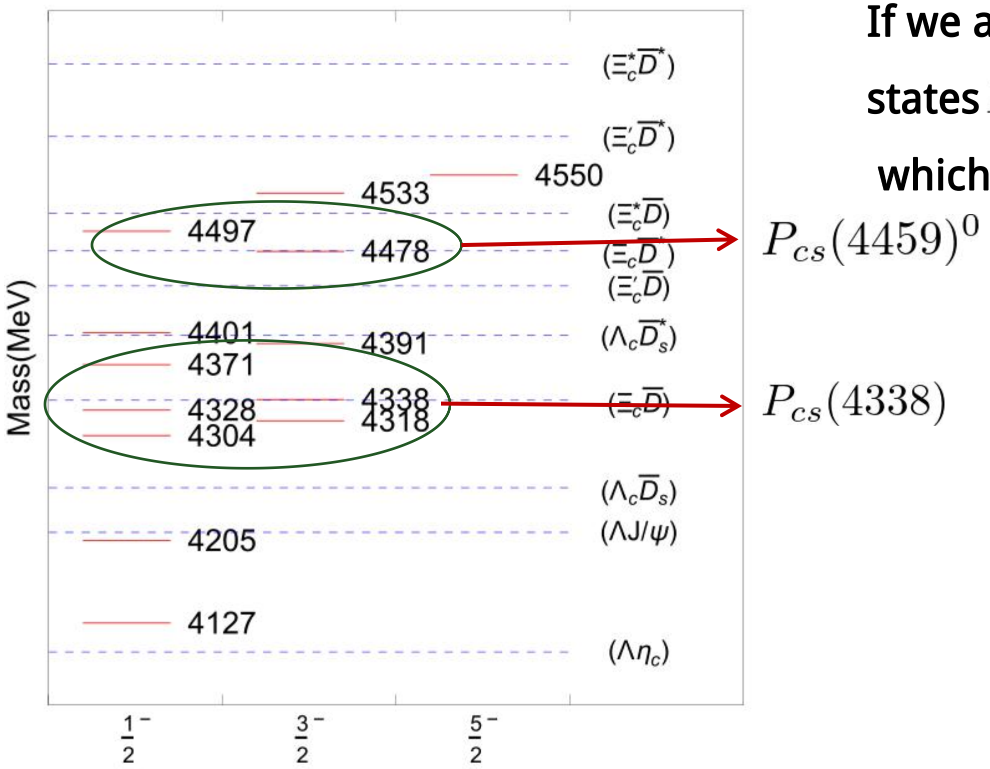
For $P_c(4312)^+$ $\Gamma(NJ/\psi) : \Gamma(\Lambda_c\bar{D}^*) = 1.1$

For $P_c(4337)^+$ $\Gamma(\Lambda_c\bar{D}) : \Gamma(NJ/\psi) = 1.3$

Predictions

Example of formalism: Pcs states

$(nns)_{8_c}(c\bar{c})_{8_c}$-$(nns)_{1_c}(c\bar{c})_{1_c}$



$I=0$ $nnsc\bar{c}$ states

If we assign the $P_{cs}(4459)^0$, $P_{cs}(4338)^0$ to be J=3/2 pentaquark states $\tilde{P}_{cs}(4478)$, $\tilde{P}_{cs}(4338)$, respectively, $\Gamma(\tilde{P}_{cs}(4478)):\Gamma(\tilde{P}_{cs}(4338)) \sim 0.12$ which is contradicted with the experimantal value.

Other possible assignments:

$$\begin{aligned}
\Gamma(\tilde{P}_{cs}(4478)^0):\Gamma(\tilde{P}_{cs}(4371)^0) &= 0.15,\\
\Gamma(\tilde{P}_{cs}(4478)^0):\Gamma(\tilde{P}_{cs}(4328)^0) &= 0.56,\\
\Gamma(\tilde{P}_{cs}(4478)^0):\Gamma(\tilde{P}_{cs}(4318)^0) &= 2.57,\\
\Gamma(\tilde{P}_{cs}(4478)^0):\Gamma(\tilde{P}_{cs}(4304)^0) &= 0.17,\\
\Gamma(\tilde{P}_{cs}(4497)^0):\Gamma(\tilde{P}_{cs}(4371)^0) &= 0.72,\\
\Gamma(\tilde{P}_{cs}(4497)^0):\Gamma(\tilde{P}_{cs}(4338)^0) &= 0.61,\\
\Gamma(\tilde{P}_{cs}(4497)^0):\Gamma(\tilde{P}_{cs}(4328)^0) &= 2.78,\\
\Gamma(\tilde{P}_{cs}(4497)^0):\Gamma(\tilde{P}_{cs}(4318)^0) &= 12.71,\\
\Gamma(\tilde{P}_{cs}(4497)^0):\Gamma(\tilde{P}_{cs}(4304)^0) &= 0.83.
\end{aligned}$$

Theoretical widths are much smaller than the measured results.

$J^P = \frac{1}{2}^-$

Both $\boldsymbol{P_{cs}(4338)^0}$ and $\boldsymbol{P_{cs}(4459)^0}$ can be regarded as $\boldsymbol{\frac{1}{2}^-}$ pentaquark states, respectively.

For $P_{cs}(4338)$, $\Gamma(\Lambda J/\Psi):\Gamma(\Lambda_c\bar{D}_s)=3.0$

For $P_{cs}(4459)^0$, $\Gamma(\Lambda_c\bar{D}_s^*):\Gamma(\Xi_c\bar{D}^*):\Gamma(\Lambda J/\Psi)=2.3:1.1:1.0$

Predictions

The J=5/2 state，the lighest J=3/2 state, and the lighest J=1/2 state are narrow.

Exp: $\Gamma(P_{cs}(4459)^0):\Gamma(P_{cs}(4338)^0) = 2.5^{+1.6}_{-1.4}$

$cs\bar{c}\bar{s}$ states

$$Q_1 q_2 \bar{Q}_3 \bar{q}_4 \to (Q_1 \bar{Q}_3)_{1c} + (q_2 \bar{q}_4)_{1c},$$

$$Q_1 q_2 \bar{Q}_3 \bar{q}_4 \to (Q_1 \bar{q}_4)_{1c} + (q_2 \bar{Q}_3)_{1c}.$$

$$M = [M_{ref} - (E_{\mathrm{CMI}})_{ref}] + E_{\mathrm{CMI}}$$

With $M_{X(4140)} = 4146.5\ \mathrm{MeV}$

$$\Psi_{tetra} = \sum_i x_i (Q_1 q_2 \bar{Q}_3 \bar{q}_4),$$

$$\Psi_{final} = \sum_i y_i (Q_1 q_2 \bar{Q}_3 \bar{q}_4).$$

$\chi_{c1}(4140)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
$19\ {}^{+7}_{-5}$ OUR AVERAGE					
$162 \pm 21\ {}^{+24}_{-49}$	24k	[1] AAIJ	21E	LHCB	$B^+ \to J/\psi \phi K^+$
$15.3^{+10.4}_{-6.1} \pm 2.5$	19	[2] AALTONEN	17	CDF	$B^+ \to J/\psi \phi K^+$
$16.3 \pm 5.6 \pm 11.4$	616	[3] ABAZOV	15M	D0	$p\bar{p} \to J/\psi \phi$ + anything
$20 \pm 13\ {}^{+3}_{-8}$	52	[4] ABAZOV	14A	D0	$B^+ \to J/\psi \phi K^+$
$28\ {}^{+15}_{-11} \pm 19$	0.3k	[5] CHATRCHYAN	14M	CMS	$B^+ \to J/\psi \phi K^+$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$83 \pm 21\ {}^{+21}_{-14}$	4289	[6,7] AAIJ	17C	LHCB	$B^+ \to J/\psi \phi K^+$
$11.7^{+8.3}_{-5.0} \pm 3.7$	14	[8,9] AALTONEN	09AH	CDF	$B^+ \to J/\psi \phi K^+$

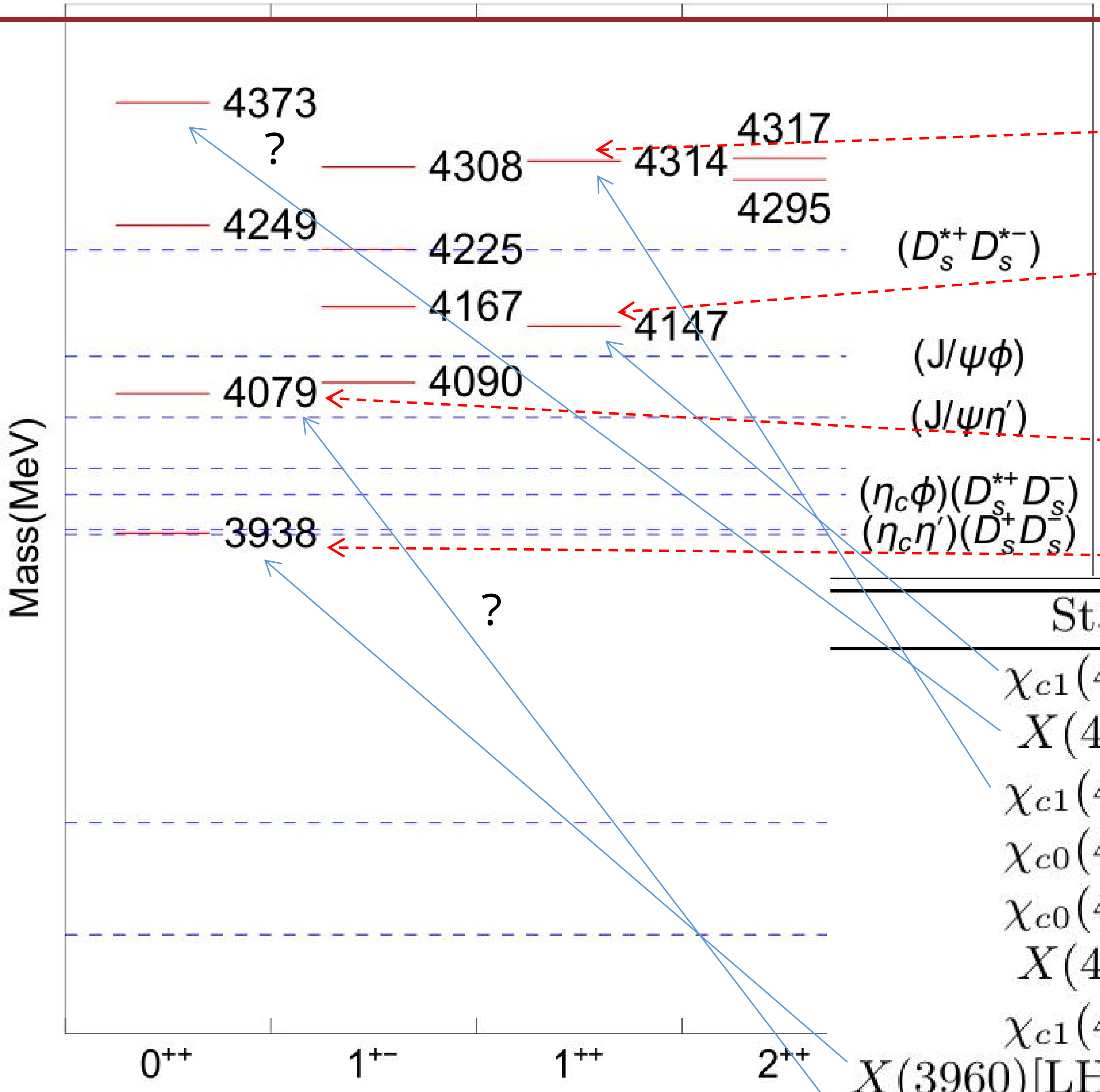
$$|\mathcal{M}|^2 = \mathcal{C}^2 |\sum_{ij} x_i y_j|^2,$$

$$\Gamma = |\mathcal{M}|^2 \frac{|\mathbf{P}|}{8\pi M^2_{Qq\bar{q}\bar{q}}}$$

$$\mathcal{C} = 7282.15\ \mathrm{MeV\ from}\ X(4140)$$

Li et al., arXiv: 2308.06768 [hep-ph]

$cs\overline{cs}$ states



$$\Gamma(J/\psi\phi) : \Gamma(D_s^*\bar{D}_s) \simeq 4.9,$$

$$\Gamma(J/\psi\phi) : \Gamma(D_s^*\bar{D}_s) \simeq 0.0.$$

$$\Gamma(\eta_c\eta') : \Gamma(\eta_c\eta) : \Gamma(D_s^+D_s^-) \simeq 1 : 1.6 : 3.2,$$

$$\Gamma(\eta_c\eta) : \Gamma(D_s^+D_s^-) \simeq 7.8.$$

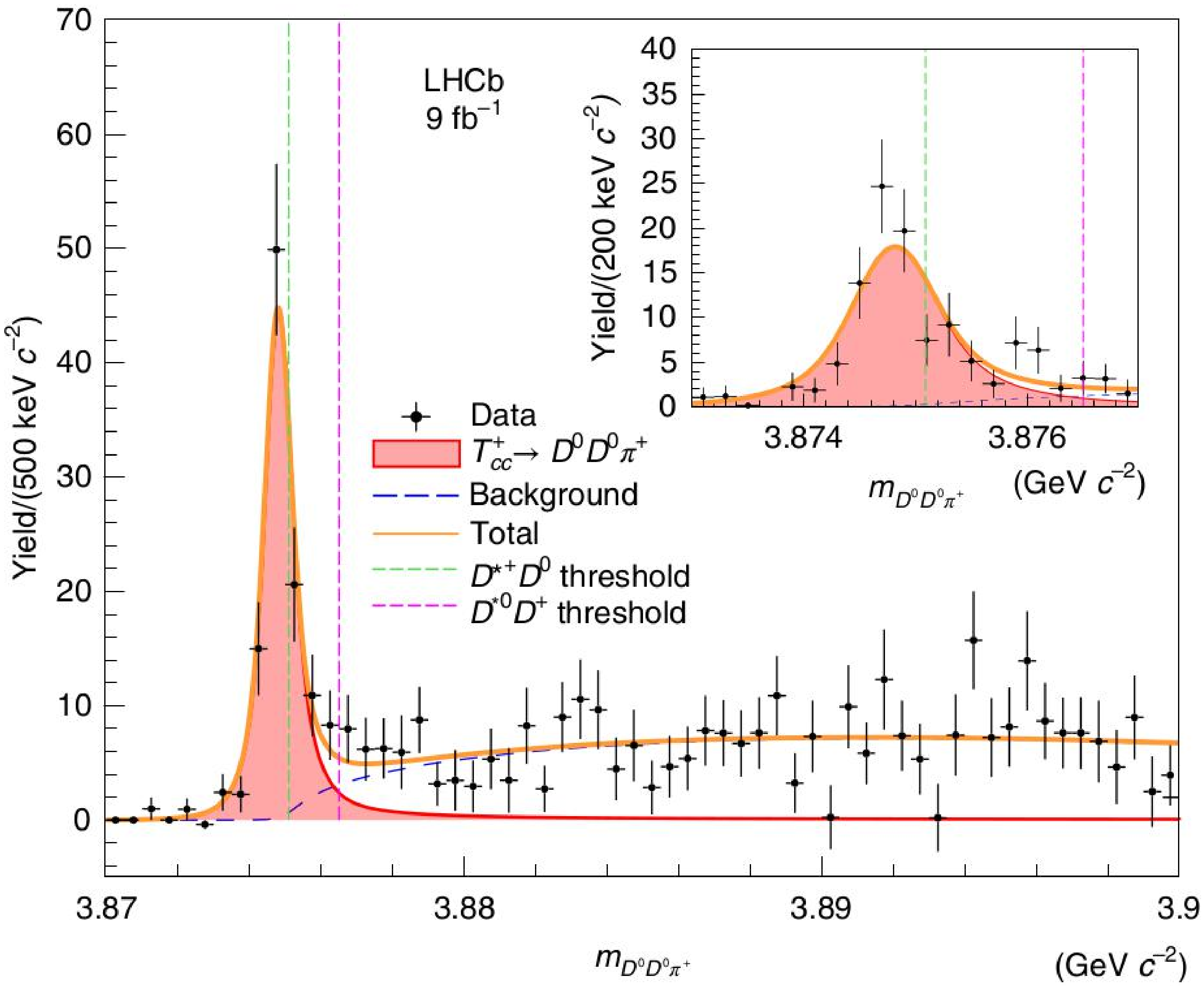
State	Mass_{PDG}(MeV)	Γ_{PDG} (MeV)	Γ_{sum}
$\chi_{c1}(4140)$	4146.5 ± 3.0	19^{+7}_{-5}	83.0 MeV
$X(4350)$	$4350^{+4.6}_{-5.1} \pm 0.7$	$13^{+18}_{-9} \pm 4$	74.0 MeV
$\chi_{c1}(4274)$	4286^{+8}_{-9}	51 ± 7	76.9 MeV
$\chi_{c0}(4500)$	$4474 \pm 3 \pm 3$	$77 \pm 6^{+10}_{-8}$	
$\chi_{c0}(4700)$	$4694 \pm 4^{+16}_{-3}$	$87 \pm 8^{+16}_{-6}$	
$X(4630)$	$4626 \pm 16^{+18}_{-110}$	$174 \pm 27^{+134}_{-73}$	
$\chi_{c1}(4685)$	$4684 \pm 7^{+13}_{-16}$	$126 \pm 15^{+37}_{-41}$	
$X(3960)$[LHCb:2022aki]	$3956 \pm 5 \pm 10$	$43 \pm 13 \pm 8$	31.8 MeV
$X_0(4140)$[LHCb:2022aki]	$4133 \pm 6 \pm 6$	$67 \pm 17 \pm 7$	60.2 MeV

Li et al., arXiv: 2308.06768 [hep-ph]

J^{PC}	Mass	Channels					Γ_{sum}
2^{++}		$J/\psi\phi$	$D_s^{*+}D_s^{*-}$				
	4316.9	(83.4, 53.8)	(47.5, 23.9)				77.7
	4294.6	(16.6, 10.2)	(52.5, 23.2)				33.4
1^{++}		$J/\psi\phi$	$(D_s^{*+}D_s^{-} - D_s^{+}D_s^{*-})/\sqrt{2}$				
	4313.6	(99.8, 63.9)	(8.2, 13.0)				76.9
	4146.5	(0.2, 0.1)	(91.8, 82.9)				83.0
0^{++}		$J/\psi\phi$	$\eta_c\eta'$	$\eta_c\eta$	$D_s^{*+}D_s^{*-}$	$D_s^{+}D_s^{-}$	
	4372.6	(57.1, 40.9)	(0.0, 0.0)	(0.0, 0.0)	(52.8, 32.9)	(0.1, 0.2)	74.0
	4249.0	(39.5, 21.2)	(0.8, 0.7)	(0.7, 0.8)	(42.7, 11.4)	(2.3, 2.2)	36.3
	4078.5	(3.1, –)	(18.0, 10.4)	(16.0, 16.6)	(3.8, –)	(49.2, 33.2)	60.2
	3938.2	(0.3, –)	(34.0, –)	(30.3, 28.2)	(0.8, –)	(48.4, 3.6)	31.8
1^{+-}		$J/\psi\eta'$	$J/\psi\eta$	$\eta_c\phi$	$D_s^{*+}D_s^{*-}$	$(D_s^{*+}D_s^{-} + D_s^{+}D_s^{*-})/\sqrt{2}$	
	4308.0	(0.9, 0.6)	(0.8, 0.8)	(8.5, 6.9)	(97.7, 46.9)	(0.2, 0.4)	55.5
	4225.1	(3.0, 1.8)	(2.7, 2.7)	(36.8, 26.0)	(1.6, 0.1)	(23.4, 30.3)	60.9
	4166.9	(2.2, 1.1)	(1.9, 1.9)	(54.5, 33.8)	(0.1, –)	(49.6, 50.8)	87.5
	4089.8	(46.9, 13.5)	(41.7, 38.2)	(0.2, 0.1)	(0.6, –)	(26.8, 9.3)	61.0

Li et al., arXiv: 2308.06768 [hep-ph]

$QQ\overline{q}\overline{q}$ states



LHCb, Nature Phys. 18, 751 (2022):

$$m_{D^{*+}} + m_{D^0} = 3875.1 \text{ MeV}$$

$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{BW} = 273 \pm 61 \pm 5^{+11}_{-14} \text{ keV}$$

$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}$$

LHCb, Nature Commun. 13,3351 (2022):

$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0} \text{ keV}$$

$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14} \text{ keV}$$

Minimal quark content: $cc\overline{u}\overline{d}$

$QQ\overline{q}\overline{q}$ states

$$\tilde{m} = M_{X(4140)} - \langle H_{CMI} \rangle_{X(4140)},$$
$$M_{cc\bar{n}\bar{n}} = \tilde{m} + E_{CMI} - 2\Delta_{sn},$$
$$M_{cc\bar{n}\bar{s}} = \tilde{m} + E_{CMI} - \Delta_{sn},$$
$$M_{cc\bar{s}\bar{s}} = \tilde{m} + E_{CMI},$$
$$M_{bc\bar{n}\bar{n}} = \tilde{m} + E_{CMI} - 2\Delta_{sn} + \Delta_{bc},$$
$$M_{bc\bar{n}\bar{s}} = \tilde{m} + E_{CMI} - \Delta_{sn} + \Delta_{bc},$$
$$M_{bc\bar{s}\bar{s}} = \tilde{m} + E_{CMI} + \Delta_{bc},$$
$$M_{bb\bar{n}\bar{n}} = \tilde{m} + E_{CMI} - 2\Delta_{sn} + 2\Delta_{bc},$$
$$M_{bb\bar{n}\bar{s}} = \tilde{m} + E_{CMI} - \Delta_{sn} + 2\Delta_{bc},$$
$$M_{bb\bar{s}\bar{s}} = \tilde{m} + E_{CMI} + 2\Delta_{bc}.$$

Li et al., arXiv: 2401.00115 [hep-ph]

$$(Q_1Q_2)(\bar{q}_3\bar{q}_4) \to (Q_1\bar{q}_3)_{1c} + (Q_2\bar{q}_4)_{1c},$$
$$(Q_1Q_2)(\bar{q}_3\bar{q}_4) \to (Q_1\bar{q}_4)_{1c} + (Q_2\bar{q}_3)_{1c}.$$

$$\Psi_{tetra} = \sum_i x_i (Q_1Q_2\bar{q}_3\bar{q}_4),$$
$$\Psi_{final} = \sum_i y_i (Q_1Q_2\bar{q}_3\bar{q}_4).$$

$$|\mathcal{M}|^2 = \mathcal{C}^2 |\sum_{ij} x_i y_j|^2,$$

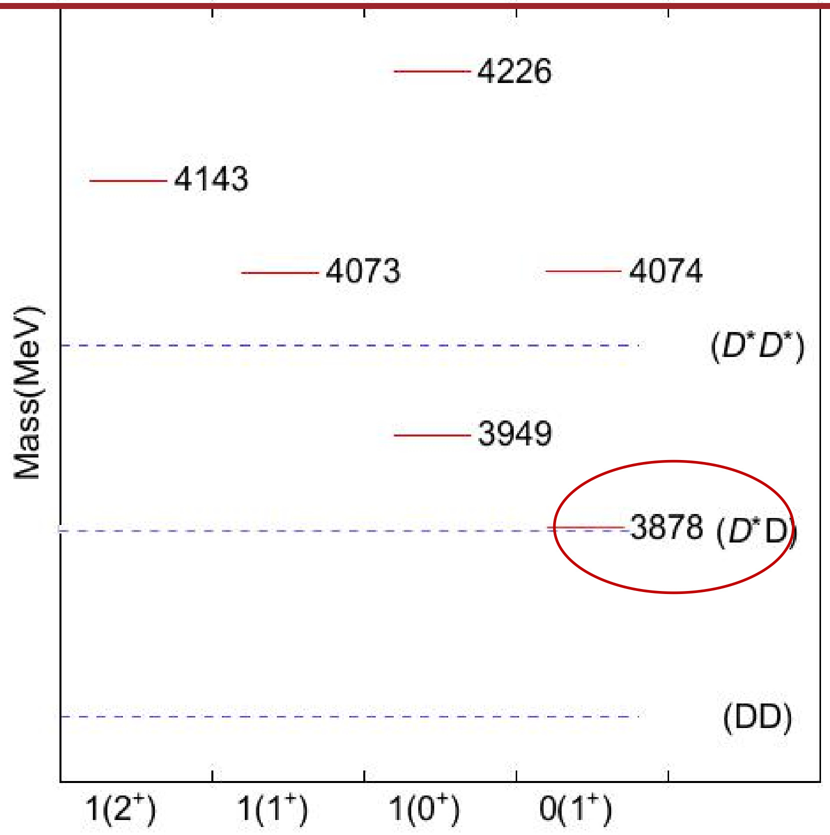
$$\Gamma = |\mathcal{M}|^2 \frac{|\mathbf{P}|}{8\pi M^2_{QQ\bar{q}\bar{q}}}$$

$\mathcal{C}$=7282.15 MeV from X(4140)

Lowest $I(J^P) = 0(1^+)\ cc\overline{n}\overline{n}$ tetraquark state: $T_{cc} = cc\bar{u}\bar{d}$

$$\Gamma_{BW} = 410 \pm 165 \pm 43^{+18}_{-38}\ \text{keV}$$

$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14}\ \text{keV}$$



If M → 3876 MeV, Γ=3.0 MeV;
If M → 3880 MeV, Γ=9.7 MeV.

Width sensitive to mass for near-threshold states.

With measured mass $M_{Tcc} = M_{D^{*+}} + M_D - 273$ keV, quasi-two-body decay width [Capstick, Roberts, PRD 49, 4570 (1994)]:

$$\Gamma = \int_0^{k_{max}} dk \frac{\Gamma_{D^{*+}\to D^0\pi^+}}{(M_{T_{cc}^+} - E_{D^{*+}}(k) - E_{D^0}(k))^2 + \frac{1}{4}\Gamma_{D^{*+}}} \frac{k^2|\mathcal{M}|^2}{(2\pi)^2 M_{T_{cc}^+} E_{D^{*+}}(k) E_{D^0}(k)}$$

~105 keV

$cc\bar{n}\bar{n}$				
$I(J^P)$	Mass	Channels		Γ
		D^*D^*		
$1(2^+)$	[4143.2]	[(33.3, 20.8)]		[20.8]
		D^*D		
$1(1^+)$	[4072.8]	[(16.7, 53.0)]		[53.0]
		D^*D^*	DD	
$1(0^+)$	4225.9 3948.8	(55.7, 43.2) (2.6, –)	(0.3, 0.3) (41.4, 35.9)	43.5 35.9
		D^*D^*	D^*D	
$0(1^+)$	4074.0 3878.2	(48.4, 20.9) (1.6, –)	(6.2, 19.8) (18.8, 7.2)	40.7 7.2

$$k_{max} = \frac{\sqrt{M^2_{T^+_{cc}} - (2M_{D^0} + M_\pi)^2}\sqrt{M^2_{T^+_{cc}} - M^2_\pi}}{2M_{T^+_{cc}}}$$

Lowest $I(J^P) = 0(1^+)$ $cc\bar{n}\bar{n}$ tetraquark state: $T_{cc} = cc\bar{u}\bar{d}$

PHYSICAL REVIEW D **104,** 114009 (2021)

Color and baryon number fluctuation of preconfinement system in production process and T_{cc} structure

Yi Jin,[1] Shi-Yuan Li,[2] Yan-Rui Liu,[2] Qin Qin,[3] Zong-Guo Si,[2] and Fu-Sheng Yu[4,5,6]

IV. CONCLUSION

The consistency between the theoretical analysis on the T_{cc} production by Qin, Shen and Yu [37] and the data [8,9] strongly favors that the newly discovered resonance T_{cc} is produced as a real four-quark state. We in this paper clarify

lation. The cross section $pp \to T_{DD^*} + X$ is around $3 \times 10^2 pb$, which is one order lower than that of the production rate of the four-quark state [37].

Chinese Physics C Vol. 45, No. 10 (2021) 103106

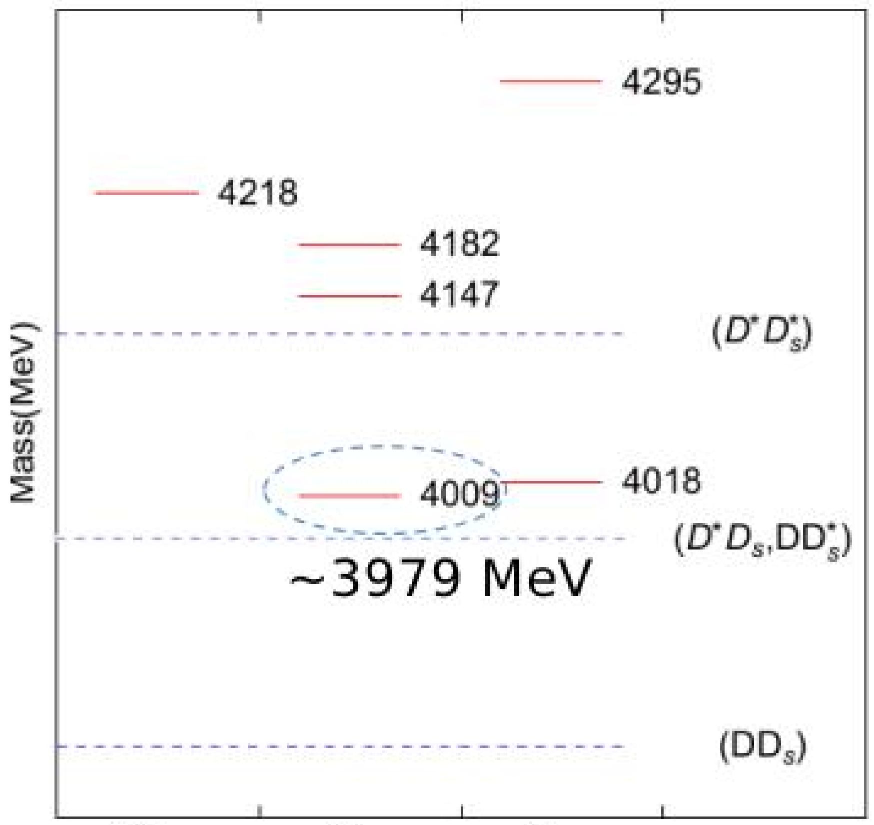
Discovery potentials of double-charm tetraquarks*

Qin Qin(秦溱)[1†] Yin-Fa Shen(沈胤发)[1] Fu-Sheng Yu(于福升)[2,3,4‡]

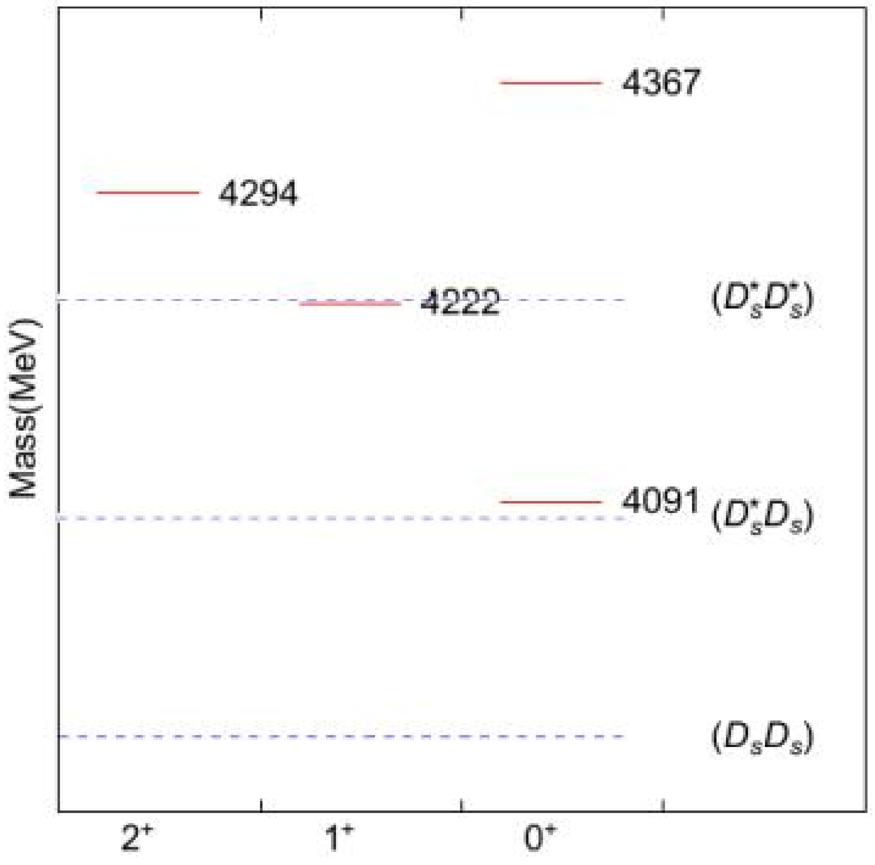
From mass, width, and production properties, it is possible to assign the LHCb T_{cc} as the lowest $I(J^P) = 0(1^+)$ $cc\bar{u}\bar{d}$ tetraquark state.

Mass difference between lowest J=1 tetraquarks: $cn\bar{c}\bar{n} - cc\bar{n}\bar{n}$~75 MeV

$c\,c\,\overline{n}\,\overline{s}$ and $c\,c\,\overline{s}\,\overline{s}$ states: spectrum



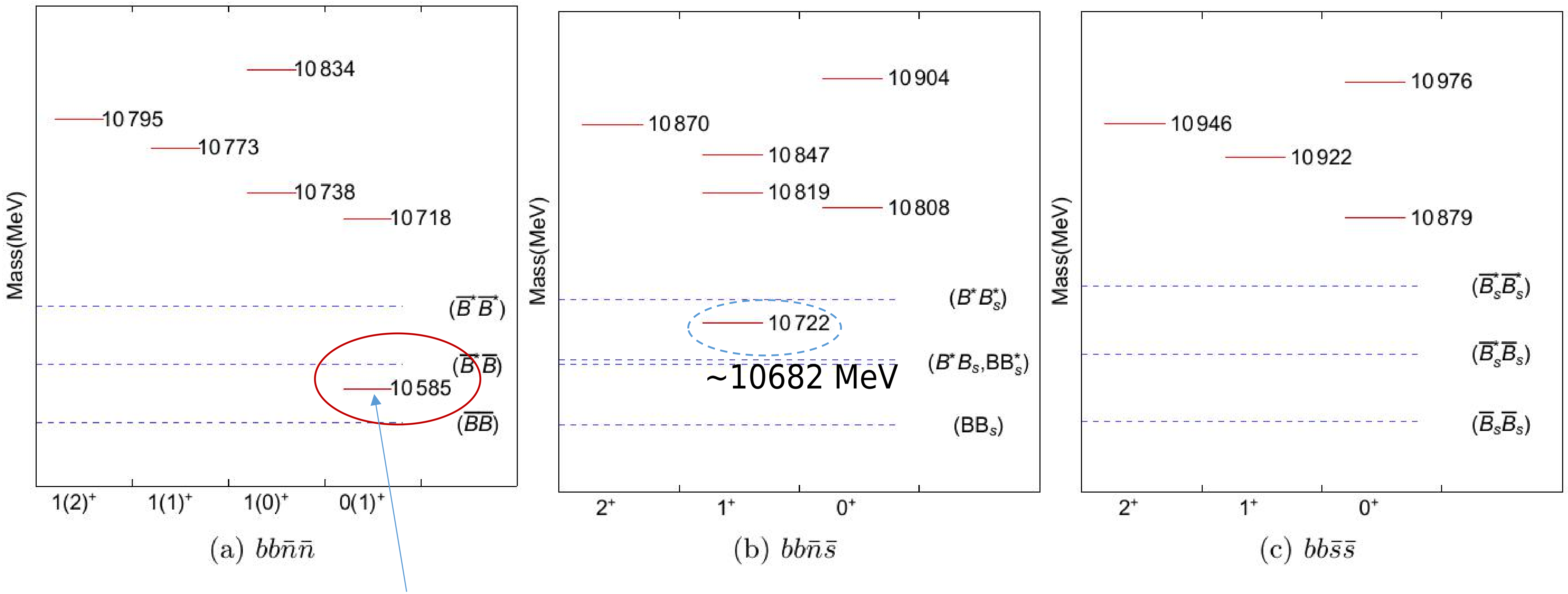
(b) $cc\bar{n}\bar{s}$



(c) $cc\bar{s}\bar{s}$

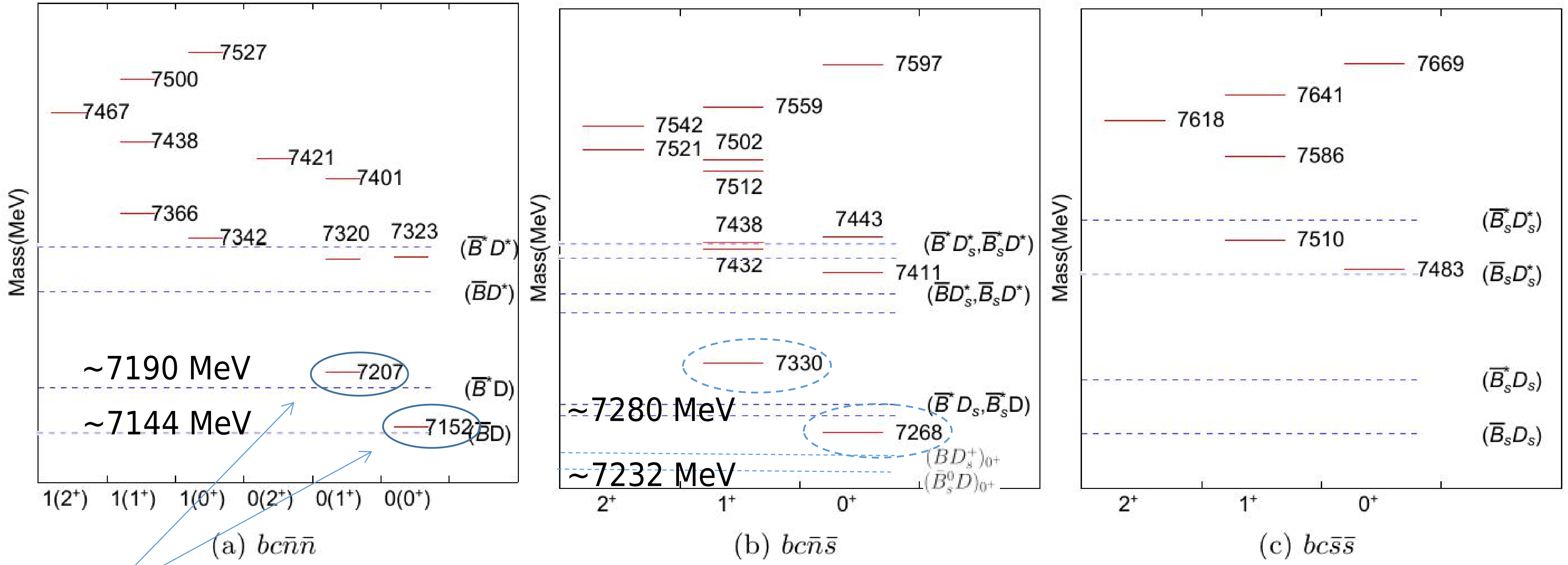
Belle: PRD 105, 032002 (2022): No $X_{cc\overline{ss}}$ is observed

$b\,b\,\overline{q}\,\overline{q}$ states: spectrum



Almost all theoretical studies support this bound $bb\bar{u}\bar{d}$.

$b\,c\,\bar{q}\,\bar{q}$ states: spectrum



Situation similar to Tcc before LHCb's observation!

J.B. Cheng et al, CPC 45, 043102 (2021): 7167 MeV & 7223 MeV;
Karliner, Rosner, PRL 119, 202001 (2017): 11 MeV below BD;
Alexandrou et al, 2312.02925: shallow bound $bc\bar{u}\bar{d}$ with J=0 and 1.

Table 10. Stability of the double-heavy tetraquarks in various studies. The meanings of "S," "US," and "ND" are "stable," "unstable," and "not determined," respectively.

Reference	$(cc\bar{n}\bar{n})$	$(cc\bar{n}\bar{s})$	$(cc\bar{s}\bar{s})$	$(bb\bar{n}\bar{n})$	$(bb\bar{n}\bar{s})$	$(bb\bar{s}\bar{s})$	$(bc\bar{n}\bar{n})$	$(bc\bar{n}\bar{s})$	$(bc\bar{s}\bar{s})$
This work	US	US	US	S	S	US	ND	US	US
[8]	S	S		S	S		S	US	
[11]	S	S	US	S	S	US	S	S	US
[16]	S			S					
[18]	S			S			S		
[19]	US			S			S		
[20]	US			S	S		US	US	
[24]	S			S			S		
[28]	S	US	US	S	S	US	S	US	US
[29]	S			S			S		
[30]	US	US	US	S	US	US	US	US	US
[31]	US	US	US	S	US	US	US	US	US
[32]			US			US			US
[33]	US	US	US	S	S	S			
[34]								S	S
[39]	US			S					
[44, 45]	US	US		S	S		S	US	
[47]							S		
[48]				S	S		US	US	
[63]	US			S			ND		
[69]							ND	US	
[83]	US	US	US	S	S	US	US	US	US
[84]	US	US	US	S	S	US	US	US	US

J.B. Cheng et al, CPC 45, 043102 (2021)

$$T_{cc} < 3965 \text{ MeV}$$

$$T_{bb} < 10627 \text{ MeV}$$

$$T_{bc} < 7199 \text{ MeV}$$

$QQ\bar{q}\bar{q}$ states: rearrangement decay

$cc\bar{n}\bar{n}$

$I(J^P)$	Mass	Channels			Γ
$1(2^+)$	4143.2	D^*D^*: (33.3, 20.8)			20.8
$1(1^+)$	4072.8	D^*D: (16.7, 53.0)			53.0
$1(0^+)$	4225.9	D^*D^*: (55.7, 43.2)	DD: (0.3, 0.3)		43.5
	3948.8	(2.6, –)	(41.4, 35.9)		35.9
$0(1^+)$	4074.0	D^*D^*: (48.4, 20.9)	D^*D: (6.2, 19.8)		40.7
	3878.2	(1.6, –)	(18.8, 7.2)		7.2

$cc\bar{n}\bar{s}$

J^P	Mass	Channels			Γ
2^+	4217.5	$D^*D_s^*$: (33.3, 35.5)			35.5
1^+	4182.2	$D^*D_s^*$: (49.6, 42.7)	D^*D_s: (4.1, 6.4)	DD_s^*: (4.5, 7.0)	56.1
	4146.6	(0.0, 0.0)	(16.7, 24.1)	(16.6, 23.7)	47.8
	4009.2	(0.4, –)	(20.9, 13.9)	(20.5, 13.1)	27.0
0^+	4295.1	$D^*D_s^*$: (55.3, 76.7)	DD_s: (0.2, 0.4)		77.1
	4018.8	(3.0, –)	(41.5, 65.0)		65.0

$cc\bar{s}\bar{s}$

J^P	Mass	Channels			Γ
2^+	4293.5	$D_s^*D_s^*$: (33.3, 14.6)			14.6
1^+	4222.0	$D_s^*D_s$: (16.7, 42.7)			42.7
0^+	4366.6	$D_s^*D_s^*$: (55.0, 33.6)	D_sD_s: (0.1, 0.1)		33.8
	4090.7	(3.3, –)	(41.5, 29.1)		29.1

$bb\bar{n}\bar{n}$

$I(J^P)$	Mass	Channels			Γ
$1(2^+)$	10795.3	B^*B^*: (33.3, 5.3)			5.3
$1(1^+)$	10772.9	$\bar{B}^*\bar{B}$: (16.7, 11.5)			11.5
$1(0^+)$	10834.4	$\bar{B}^*\bar{B}^*$: (57.4, 10.3)	$\bar{B}\bar{B}$: (1.2, 0.3)		10.5
	10738.4	(0.9, 0.1)	(40.5, 7.2)		7.4
$0(1^+)$	10717.8	$\bar{B}^*\bar{B}^*$: (41.2, 4.6)	$\bar{B}^*\bar{B}$: (12.2, 7.0)		11.6
	10584.5	(8.8, –)	(12.8, –)		0

$bb\bar{n}\bar{s}$

J^P	Mass	Channels			Γ
2^+	10869.9	$B^*B_s^*$: (33.3, 10.0)			10.0
1^+	10846.5	$\bar{B}^*\bar{B}_s^*$: (0.1, 0.0)	$\bar{B}^*\bar{B}_s$: (15.1, 4.9)	$\bar{B}\bar{B}_s^*$: (18.2, 5.9)	10.9
	10819.1	(44.2, 10.4)	(11.3, 3.4)	(8.8, 2.6)	16.3
	10722.2	(5.7, –)	(15.3, 2.3)	(14.7, 2.1)	4.3
0^+	10903.8	$\bar{B}^*\bar{B}_s^*$: (56.7, 18.9)	$\bar{B}\bar{B}_s$: (0.7, 0.3)		19.2
	10807.8	(1.7, 0.4)	(41.0, 13.8)		14.2

$bb\bar{s}\bar{s}$

J^P	Mass	Channels			Γ
2^+	10946.1	$B_s^*B_s^*$: (33.3, 4.7)			4.7
1^+	10921.6	$\bar{B}_s^*\bar{B}_s$: (16.7, 10.3)			10.3
0^+	10975.7	$\bar{B}_s^*\bar{B}_s^*$: (55.8, 8.7)	$\bar{B}_s\bar{B}_s$: (0.3, 0.1)		8.8
	10878.7	(2.5, 0.2)	(41.3, 6.5)		6.8

Ratios between partial widths as predictions for tetraquark states having two or three rearrangement decay channels.

$bc\bar{s}\bar{s}$

J^P	Mass	Channels			Γ
2^+	7617.9	$\bar{B}_s^*D_s^*$: (33.3, 12.8)			12.8
1^+	7640.5	$\bar{B}_s^*D_s^*$: (46.2, 19.7)	$\bar{B}_s^*D_s$: (0.4, 0.2)	$\bar{B}_sD_s^*$: (8.1, 4.1)	24.1
	7586.4	(3.0, 0.9)	(1.4, 0.8)	(29.3, 12.3)	14.1
	7510.2	(0.8, –)	(39.8, 18.2)	(4.3, 1.0)	19.2
0^+	7668.6	$\bar{B}_s^*D_s^*$: (55.2, 26.1)	$\bar{B}_sD_s$: (0.2, 0.1)		26.3
	7482.6	(3.2, –)	(41.5, 20.6)		20.6

$cc\bar{c}\bar{c}$ states

TABLE I. Summary of the fit results for the $J/\psi J/\psi$ invariant mass distribution. The mass m and natural width Γ for both the no-interference model and the interference model, and the signal yields N for the no-interference model, are given for the three signal structures. The dual uncertainties are the statistical followed by the systematic components, and single uncertainties are only statistical.

CMS		BW_1	BW_2	BW_3
No interference	m (MeV)	$6552 \pm 10 \pm 12$	$6927 \pm 9 \pm 4$	$7287^{+20}_{-18} \pm 5$
	Γ (MeV)	$124^{+32}_{-26} \pm 33$	$122^{+24}_{-21} \pm 18$	$95^{+59}_{-40} \pm 19$
	N	470^{+120}_{-110}	492^{+78}_{-73}	156^{+64}_{-51}
Interference	m (MeV)	6638^{+43+16}_{-38-31}	6847^{+44+48}_{-28-20}	7134^{+48+41}_{-25-15}
	Γ (MeV)	$440^{+230+110}_{-200-240}$	191^{+66+25}_{-49-17}	97^{+40+29}_{-29-26}

ATLAS

Di-J/ψ	Model A	Model B
m_0	$6.41 \pm 0.08^{+0.08}_{-0.03}$	$6.65 \pm 0.02^{+0.03}_{-0.02}$
Γ_0	$0.59 \pm 0.35^{+0.12}_{-0.20}$	$0.44 \pm 0.05^{+0.06}_{-0.05}$
m_1	$6.63 \pm 0.05^{+0.08}_{-0.01}$	...
Γ_1	$0.35 \pm 0.11^{+0.11}_{-0.04}$	
m_2	$6.86 \pm 0.03^{+0.01}_{-0.02}$	$6.91 \pm 0.01 \pm 0.01$
Γ_2	$0.11 \pm 0.05^{+0.02}_{-0.01}$	$0.15 \pm 0.03 \pm 0.01$
$\Delta s/s$	$\pm 5.1\%^{+8.1\%}_{-8.9\%}$	...

$J/\psi + \psi(2S)$	Model α	Model β
m_3	$7.22 \pm 0.03^{+0.01}_{-0.04}$	$6.96 \pm 0.05 \pm 0.03$
Γ_3	$0.09 \pm 0.06^{+0.06}_{-0.05}$	$0.51 \pm 0.17^{+0.11}_{-0.10}$
$\Delta s/s$	$\pm 21\%^{+25\%}_{-15\%}$	$\pm 20\% \pm 12\%$

$$m[X(6900)] = 6905 \pm 11 \pm 7\ \text{MeV}/c^2,$$

and LHCb w/o interference

$$\Gamma[X(6900)] = 80 \pm 19 \pm 33\ \text{MeV},$$

$$m[X(6900)] = 6886 \pm 11 \pm 11\ \text{MeV}/c^2$$

and LHCb w/ interference

$$\Gamma[X(6900)] = 168 \pm 33 \pm 69\ \text{MeV}.$$

$cc\bar{c}\bar{c}$ states

Assume X(6600)
is a scalar tetraquark

State	Mass (MeV)	Γ(MeV)	Observed channels
$X(6900)$[LHCb:2020bwg]	$6905 \pm 11 \pm 7$ MeV	$80 \pm 19 \pm 33$ MeV	di-J/ψ
$X(6600)$[CMS:2023owd]	$6552 \pm 10 \pm 12$ MeV	$124^{+32}_{-26} \pm 33$ MeV	diJ/ψ
$X(7200)$[CMS:2023owd]	$7287^{+20}_{-18} \pm 5$MeV	$95^{+59}_{-40} \pm 19$MeV	di-J/ψ
$X(6400)$[ATLAS:2023bft]	$6.41 \pm 0.08^{+0.08}_{-0.03}$ GeV	$0.59 \pm 0.35^{+0.12}_{-0.2}$GeV	di-J/ψ
$X(6600)$[ATLAS:2023bft]	$6.63 \pm 0.05^{+0.08}_{-0.01}$ GeV	$0.35 \pm 0.11^{+0.11}_{-0.04}$GeV	di-J/ψ
$X(7200)$[ATLAS:2023bft]	$7.22 \pm 0.03^{+0.01}_{-0.04}$GeV	$0.095 \pm 0.6^{+0.06}_{-0.05}$ GeV	$J/\psi + \psi(2S)$

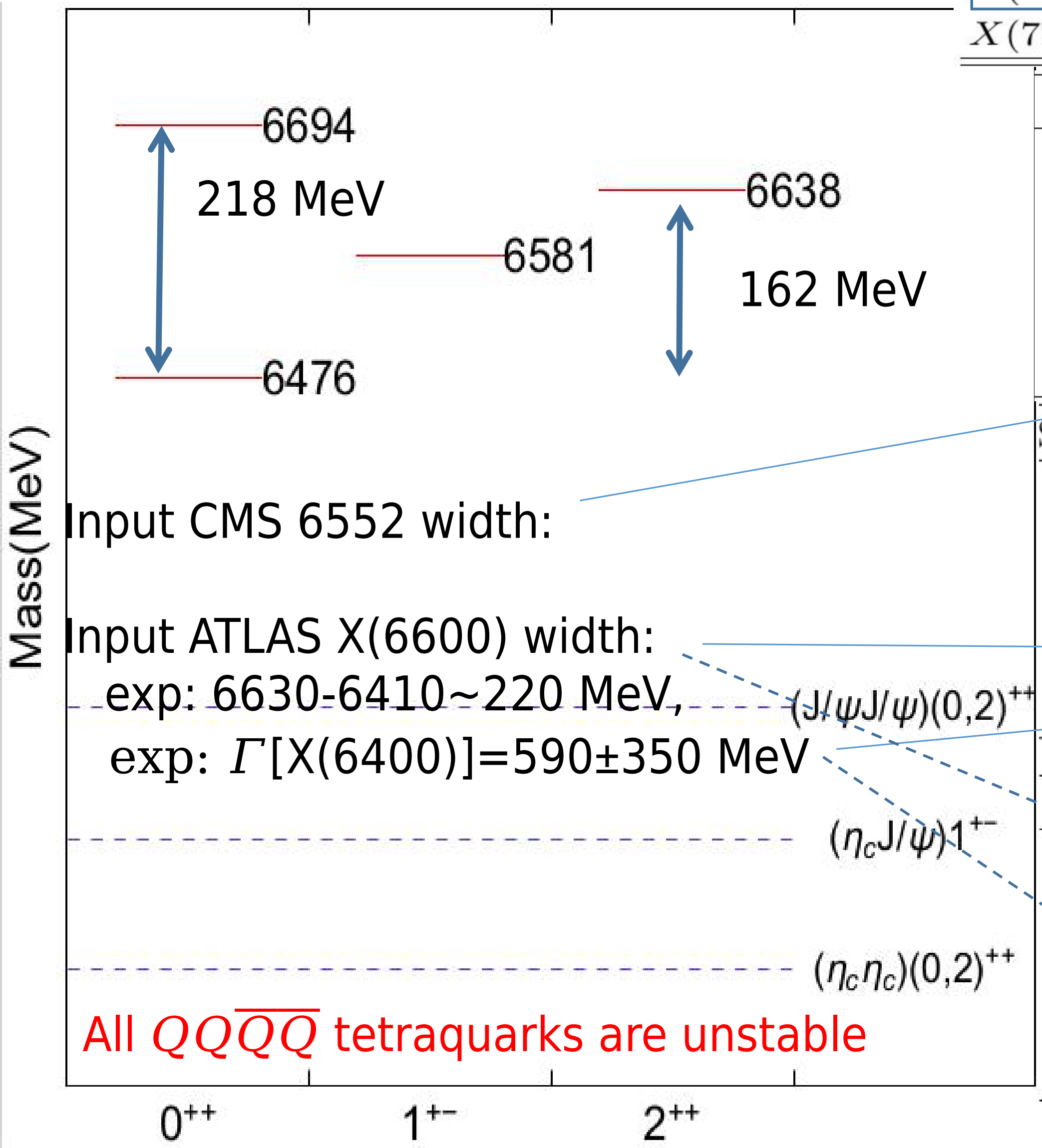
$$\tilde{m} = M_{X(4140)} - \langle H_{CMI} \rangle_{X(4140)},$$
$$M_{cc\bar{c}\bar{c}} = \tilde{m} + \langle H_{CMI} \rangle + 2\Delta_{cs},$$
$$M_{bb\bar{b}\bar{b}} = \tilde{m} + \langle H_{CMI} \rangle + 2\Delta_{bs} + 2\Delta_{bc},$$
$$M_{bb\bar{c}\bar{c}} = \tilde{m} + \langle H_{CMI} \rangle + \Delta_{bc} + \Delta_{bs} + \Delta_{cs},$$
$$M_{bb\bar{b}\bar{c}} = \tilde{m} + \langle H_{CMI} \rangle + 2\Delta_{bc} + \Delta_{bs} + \Delta_{cs},$$
$$M_{cc\bar{c}\bar{b}} = \tilde{m} + \langle H_{CMI} \rangle + \Delta_{bs} + \Delta_{cs},$$
$$M_{bc\bar{b}\bar{c}} = \tilde{m} + \langle H_{CMI} \rangle + 2\Delta_{bc} + 2\Delta_{cs}.$$

$$Q_1 Q_2 \bar{Q}_3 \bar{Q}_4 \to (Q_1 \bar{Q}_3)_{1c} + (Q_2 \bar{Q}_4)_{1c},$$
$$Q_1 Q_2 \bar{Q}_3 \bar{Q}_4 \to (Q_1 \bar{Q}_4)_{1c} + (Q_2 \bar{Q}_3)_{1c}.$$

From X(6600),
$\mathcal{C}$=14.95 GeV (CMS, lower 0^{++});
$\mathcal{C}$=24.38 GeV (ATLAS, higher 0^{++});
$\mathcal{C}$=31.32 GeV (ATLAS, higher 2^{++})

$cc\overline{c}\overline{c}$ states

State	Mass (MeV)	Γ(MeV)	Observed channels
$X(6900)$[LHCb:2020bwg]	$6905 \pm 11 \pm 7$ MeV	$80 \pm 19 \pm 33$ MeV	di-J/ψ
$X(6600)$[CMS:2023owd]	$6552 \pm 10 \pm 12$ MeV	$124^{+32}_{-26} \pm 33$ MeV	diJ/ψ
$X(7200)$[CMS:2023owd]	$7287^{+20}_{-18} \pm 5$MeV	$95^{+59}_{-40} \pm 19$MeV	di-J/ψ
$X(6400)$[ATLAS:2023bft]	$6.41 \pm 0.08^{+0.08}_{-0.03}$ GeV	$0.59 \pm 0.35^{+0.12}_{-0.2}$GeV	di-J/ψ
$X(6600)$[ATLAS:2023bft]	$6.63 \pm 0.05^{+0.08}_{-0.01}$ GeV	$0.35 \pm 0.11^{+0.11}_{-0.04}$GeV	di-J/ψ
$X(7200)$[ATLAS:2023bft]	$7.22 \pm 0.03^{+0.01}_{-0.04}$GeV	$0.095 \pm 0.6^{+0.06}_{-0.05}$ GeV	$J/\psi + \psi(2S)$



System	$J^{P(C)}$	Mass	Channels $J/\psi J/\psi$	$J/\psi\eta_c$	$\eta_c\eta_c$	Γ
$cc\bar{c}\bar{c}$	2^{++}	6637.5	(33.3, 80.2)			80.2
	1^{+-}	6581.0		(16.7, 172.1)		172.1
	0^{++}	6694.3	(54.9, 138.2)		(0.1, 0.3)	138.5
		6476.4	(3.5, 6.9)		(41.6, 110.7)	117.7

Input CMS 6552 width:

System	$J^{P(C)}$	Mass	Channels $J/\psi J/\psi$	$J/\psi\eta_c$	$\eta_c\eta_c$	Γ
$cc\bar{c}\bar{c}$	2^{++}	6637.5	(33.3, 213.5)			213.5
	1^{+-}	6581.0		(16.7, 458.0)		458.0
	0^{++}	6694.1	(54.9, 367.7)		(0.1, 0.9)	368.6
		6476.4	(3.5, 18.5)		(41.6, 294.8)	313.2

Input ATLAS X(6600) width:

exp: 6630-6410~220 MeV,

exp: Γ[X(6400)]=590±350 MeV

System	$J^{P(C)}$	Mass	Channels $J/\psi J/\psi$	$J/\psi\eta_c$	$\eta_c\eta_c$	Γ
$cc\bar{c}\bar{c}$	2^{++}	6637.5	(33.3, 352.3)			352.3
	1^{+-}	6581.0		(16.7, 755.8)		755.8
	0^{++}	6694.1	(54.9, 606.8)		(0.1, 1.4)	608.2
		6476.4	(3.5, 30.5)		(41.6, 486.4)	516.9

All $QQ\overline{Q}\overline{Q}$ tetraquarks are unstable

$Qq\bar{q}\bar{q}$ states

State	Mass(MeV)	Γ (MeV)	Observed channels
$T_{cs0}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+ D^- K^+$
$T_{cs1}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$B^+ \to D^+ D^- K^+$
$T^a_{c\bar{s}0}(2900)^0$[LHCb:2022sfr,LHCb:2022lzp]	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to D^0 D_s^+ \pi^-$
$T^a_{c\bar{s}0}(2900)^{++}$[LHCb:2022sfr,LHCb:2022lzp]	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^- D_s^+ \pi^+$

$$\tilde{m}=M_{X(4140)} - \langle H_{CMI}\rangle_{X(4140)},$$
$$M_{cn\bar{n}\bar{n}}=\tilde{m} + E_{CMI} - 2\Delta_{sn} - \Delta_{cn},$$
$$M_{cn\bar{s}\bar{n}}=\tilde{m} + E_{CMI} - \Delta_{sn} - \Delta_{cn},$$
$$M_{cn\bar{s}\bar{s}}=\tilde{m} + E_{CMI} - \Delta_{cn},$$
$$M_{cs\bar{n}\bar{n}}=\tilde{m} + E_{CMI} - \Delta_{sn} - \Delta_{cn},$$
$$M_{cs\bar{s}\bar{n}}=\tilde{m} + E_{CMI} - \Delta_{cn},$$
$$M_{cs\bar{s}\bar{s}}=\tilde{m} + E_{CMI} - \Delta_{cs},$$
$$M_{bn\bar{n}\bar{n}}=\tilde{m} + E_{CMI} - 2\Delta_{sn} - \Delta_{cn} + \Delta_{bc},$$
$$M_{bn\bar{s}\bar{n}}=\tilde{m} + E_{CMI} - \Delta_{sn} - \Delta_{cn} + \Delta_{bc},$$
$$M_{bn\bar{s}\bar{s}}=\tilde{m} + E_{CMI} - \Delta_{cn} + \Delta_{bc},$$
$$M_{bs\bar{n}\bar{n}}=\tilde{m} + E_{CMI} - \Delta_{sn} - \Delta_{cn} + \Delta_{bc},$$
$$M_{bs\bar{s}\bar{n}}=\tilde{m} + E_{CMI} - \Delta_{cn} + \Delta_{bc},$$
$$M_{bs\bar{s}\bar{s}}=\tilde{m} + E_{CMI} - \Delta_{cs} + \Delta_{bc}.$$

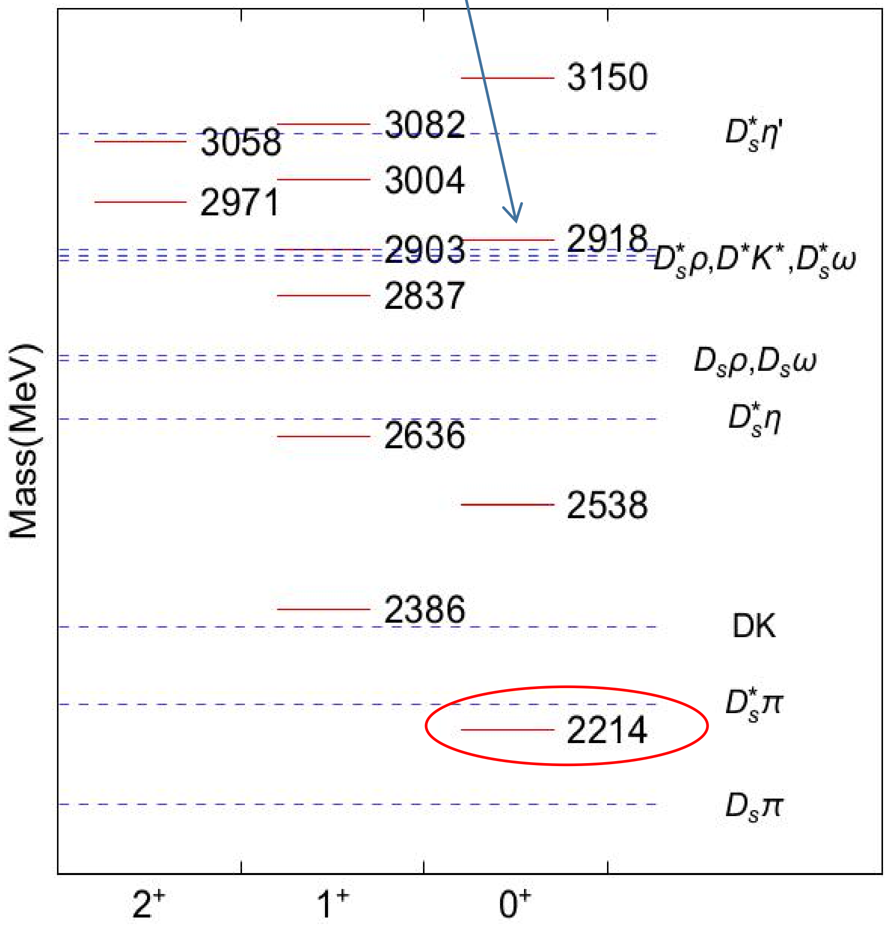
$$Q_1 q_2 \bar{q}_3 \bar{q}_4 \to (Q_1 \bar{q}_3)_{1c} + (q_2 \bar{q}_4)_{1c},$$
$$Q_1 q_2 \bar{q}_3 \bar{q}_4 \to (Q_1 \bar{q}_4)_{1c} + (q_2 \bar{q}_3)_{1c}.$$

Assume $T^a_{c\bar{s}0}(2900)$ to be the second highest 0^+ $cn\bar{s}\bar{n}$
→ $\mathcal{C}$=13.577 GeV

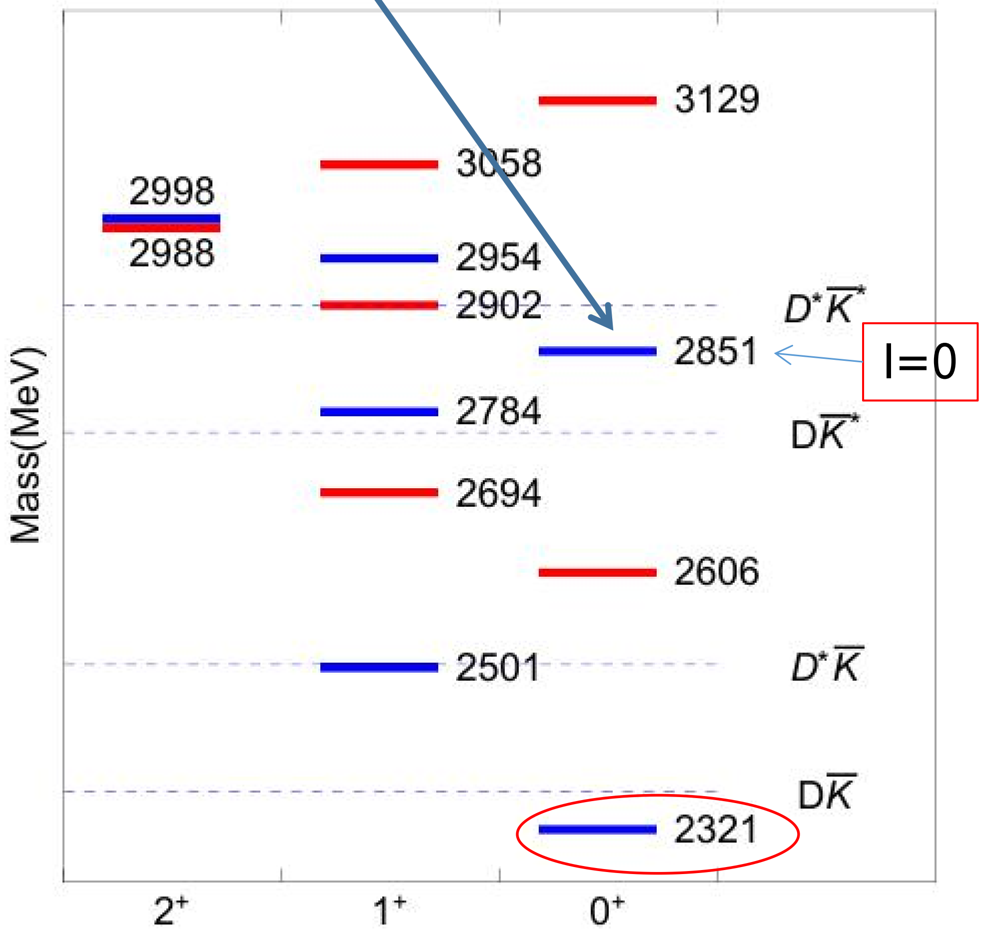
$Qq\overline{qq}$ states

State	Mass(MeV)	Γ (MeV)	Observed channels
$T_{cs0}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+ D^- K^+$
$T_{cs1}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$B^+ \to D^+ D^- K^+$
$T^a_{c\bar{s}0}(2900)^0$[LHCb:2022sfr,LHCb:2022lzp]	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to \bar{D}^0 D_s^+ \pi^-$
$T^a_{c\bar{s}0}(2900)^{++}$[LHCb:2022sfr,LHCb:2022lzp]	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^- D_s^+ \pi^+$

Width $\to \mathcal{C}$



$cn\overline{sn}$: I=0 & I=1 degenerate

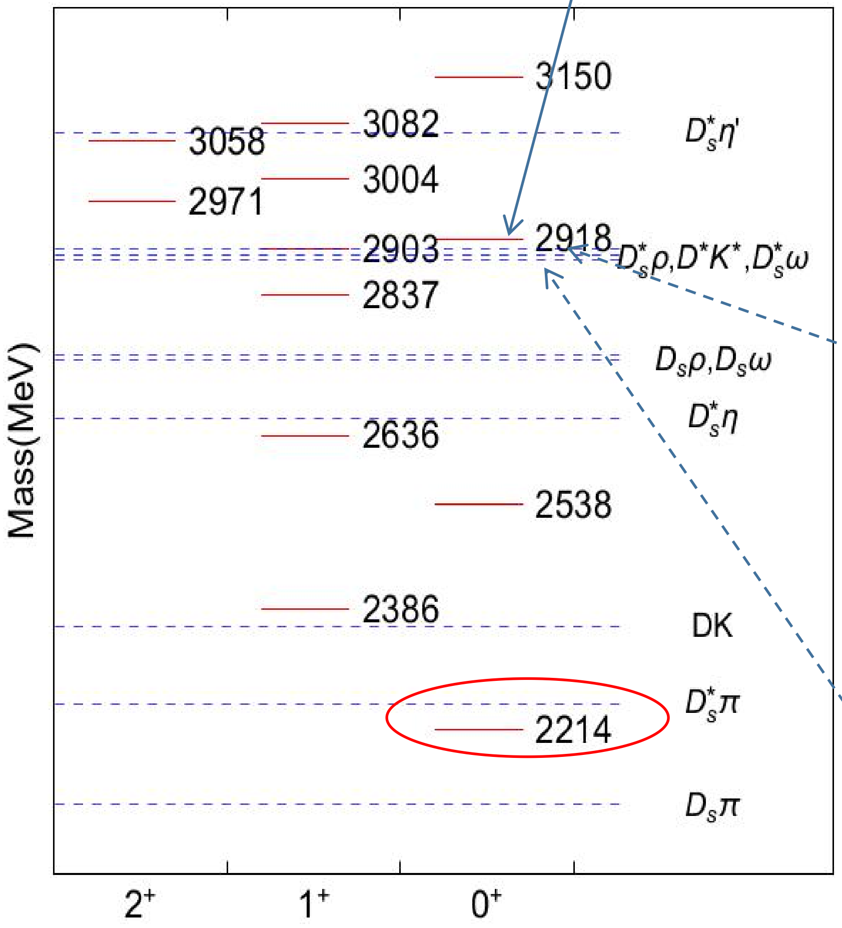


(e) $I = 0/1\ cs\bar{n}\bar{n}$ states

$Qq\overline{q}\overline{q}$ states

State	Mass(MeV)	Γ (MeV)	Observed channels
$T_{cs0}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+ D^- K^+$
$T_{cs1}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$B^+ \to D^+ D^- K^+$
$T^a_{c\bar{s}0}(2900)^0$[LHCb:2022sfr,LHCb:2022lzp]	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to \bar{D}^0 D_s^+ \pi^-$
$T^a_{c\bar{s}0}(2900)^{++}$[LHCb:2022sfr,LHCb:2022lzp]	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^- D_s^+ \pi^+$

Width $\to \mathcal{C}$



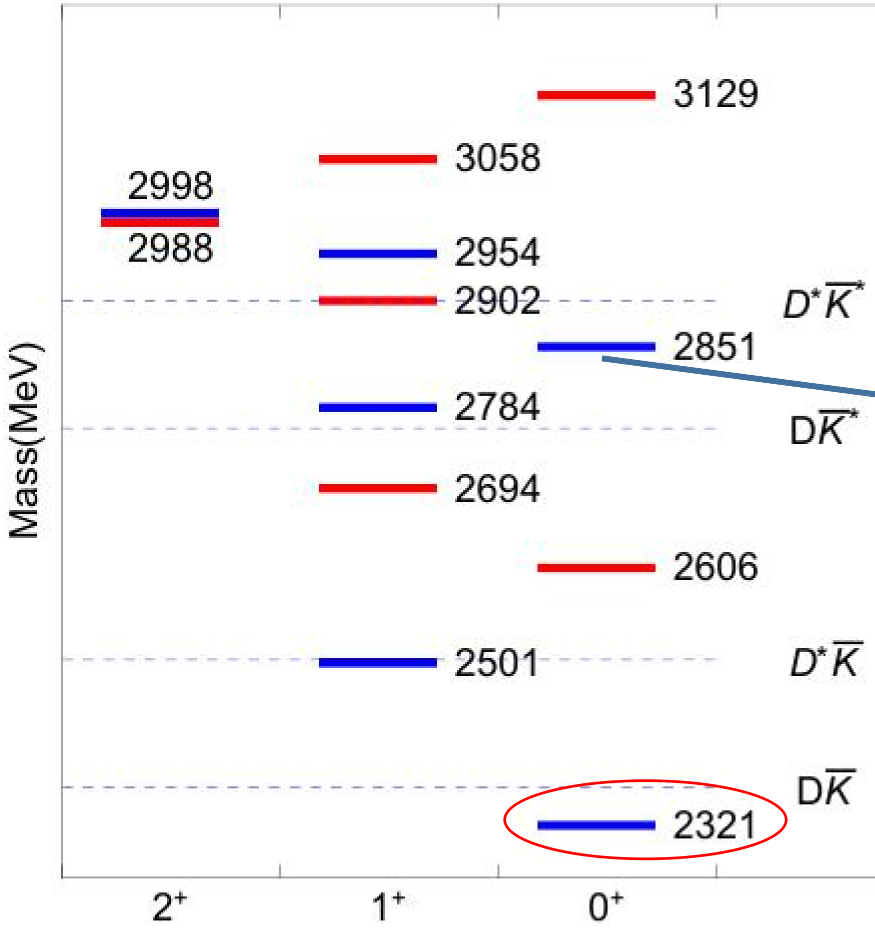
$I(J^P)$	Mass	Channels						Γ
$1(2^+)$		$D_s^*\rho$	D^*K^*					
	3058.0	(95.8, 340.4)	(26.9, 94.8)					435.2
	2971.3	(4.2, 10.9)	(73.1, 179.3)					190.1
$1(1^+)$		$D_s^*\rho$	$D_s^*\pi$	$D_s\rho$	D^*K^*	D^*K	DK^*	
	3082.1	(71.1, 266.6)	(0.0, 0.2)	(4.5, 22.7)	(22.5, 84.1)	(0.8, 4.6)	(9.3, 46.7)	424.9
	3004.4	(26.8, 80.9)	(0.3, 2.0)	(22.7, 103.6)	(61.8, 181.3)	(1.3, 7.5)	(1.3, 6.1)	381.4
	2902.7	(0.0, 0.0)	(0.7, 4.2)	(44.1, 166.0)	(12.6, 2.7)	(0.0, 0.2)	(12.4, 45.5)	218.5
	2837.3	(1.3, –)	(0.0, 0.0)	(26.8, 80.2)	(0.1, –)	(3.2, 16.5)	(70.9, 198.4)	295.1
	2636.4	(0.8, –)	(10.7, 51.5)	(1.8, –)	(1.5, –)	(72.7, 258.8)	(4.4, –)	310.3
	2385.9	(0.0, –)	(88.2, 253.0)	(0.0, –)	(1.4, –)	(22.1, –)	(1.7, –)	253.0
$1(0^+)$		$D_s^*\rho$	$D_s\pi$	D^*K^*	DK			
	3149.7	(63.3, 266.6)	(0.0, 0.3)	(46.3, 196.1)	(0.2, 1.7)			464.6
	2917.9	(34.1, 54.9)	(0.7, 5.1)	(47.4, 56.9)	(3.1, 20.1)			137.0
	2537.5	(2.5, –)	(17.3, 96.4)	(4.1, –)	(67.0, 297.1)			393.5
	2214.3	(0.1, –)	(81.9, 235.6)	(2.1, –)	(29.6, –)			235.6

$I(J^P)$	Mass	Channels							Γ
$0(2^+)$		$D_s^*\omega$	D^*K^*						
	3058.0	(95.8, 333.6)	(26.9, 94.8)						428.4
	2971.3	(4.2, 10.4)	(73.1, 179.3)						189.7
$0(1^+)$		$D_s^*\omega$	$D_s^*\eta$	$D_s^*\eta'$	$D_s\omega$	D^*K^*	D^*K	DK^*	
	3082.1	(71.1, 262.0)	(0.0, 0.1)	(0.0, 0.0)	(4.5, 22.5)	(22.5, 84.1)	(0.8, 4.6)	(9.3, 46.7)	420.0
	3004.4	(26.8, 78.5)	(0.2, 0.8)	(0.2, –)	(22.7, 102.3)	(61.8, 181.3)	(1.3, 7.5)	(1.3, 6.1)	376.4
	2902.7	(0.0, 0.0)	(0.3, 1.4)	(0.4, –)	(44.1, 162.4)	(12.6, 2.7)	(0.0, 0.2)	(12.4, 45.5)	212.1
	2837.3	(1.3, –)	(0.3, 0.0)	(0.0, –)	(26.8, 77.1)	(0.1, –)	(3.2, 16.5)	(70.9, 198.4)	292.0
	2636.4	(0.8, –)	(5.0, –)	(5.7, –)	(1.8, –)	(1.5, –)	(72.7, 258.8)	(4.4, –)	258.8
	2385.9	(0.0, –)	(41.5, –)	(46.7, –)	(0.0, –)	(1.4, –)	(22.1, –)	(1.7, –)	–
$0(0^+)$		$D_s^*\omega$	$D_s\eta$	$D_s\eta'$	D^*K^*	DK			
	3149.7	(63.3, 263.3)	(0.0, 0.1)	(0.0, 0.1)	(46.3, 196.1)	(0.2, 1.7)			461.3
	2917.9	(34.1, 47.8)	(0.4, 2.0)	(0.4, 0.1)	(47.4, 56.9)	(3.1, 20.1)			126.9
	2537.5	(2.5, –)	(8.1, 12.6)	(9.1, –)	(4.1, –)	(67.0, 297.1)			309.7
	2214.3	(0.1, –)	(38.6, –)	(43.3, –)	(2.1, –)	(29.6, –)			–

$cn\overline{sn}$: I=0 & I=1 degenerate

$Qq\bar{q}\bar{q}$ states

State	Mass(MeV)	Γ (MeV)	Observed channels
$T_{cs0}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$B^+ \to D^+ D^- K^+$
$T_{cs1}(2900)^0$[LHCb:2020pxc,LHCb:2020bls]	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$B^+ \to D^+ D^- K^+$
$T^a_{c\bar{s}0}(2900)^0$[LHCb:2022sfr,LHCb:2022lzp]	$2892 \pm 21 \pm 2$	119 ± 29	$B^0 \to \bar{D}^0 D_s^+ \pi^-$
$T^a_{c\bar{s}0}(2900)^{++}$[LHCb:2022sfr,LHCb:2022lzp]	$2921 \pm 23 \pm 2$	137 ± 35	$B^+ \to D^- D_s^+ \pi^+$



(e) $I = 0/1\ cs\bar{n}\bar{n}$ states

$I(J^P)$	Mass	Channels			Γ
$0(2^+)$	[2998.3]	D^*K^* [(66.7, 380.2)]			[380.2]
$0(1^+)$	[2953.5 2784.3 2500.6]	$D^*\bar{K}^*$ [(47.6, 203.0) (2.4, −) (0.1, −)]	$D^*\bar{K}$ [(1.3, 14.5) (1.8, 17.8) (55.2, −)]	$D\bar{K}^*$ [(10.6, 88.5) (47.6, 151.9) (0.1, −)]	[306.1 169.7 0]
$0(0^+)$	[2850.5 2320.7]	$D^*\bar{K}^*$ [(41.4, −) (0.3, −)]	$D\bar{K}$ [(2.7, 34.4) (55.6, −)]		[34.4 0]

$I(J^P)$	Mass	Channels			Γ
$1(2^+)$	[2987.5]	D^*K^* [(33.3, 180.1)]			[180.1]
$1(1^+)$	[3058.0 2902.3 2693.7]	$D^*\bar{K}^*$ [(46.9, 329.8) (2.2, 0.3) (0.9, −)]	$D^*\bar{K}$ [(0.4, 4.5) (1.1, 12.1) (40.2, 333.5)]	$D\bar{K}^*$ [(7.1, 69.0) (30.7, 225.3) (3.9, −)]	[403.3 237.7 333.5]
$1(0^+)$	[3128.9 2606.4]	$D^*\bar{K}^*$ [(55.1, 451.1) (3.2, −)]	$D\bar{K}$ [(0.1, 2.1) (41.5, 419.3)]		[453.2 419.3]

Summary

With one mass formulae and a simple decay scheme:

- X(3960) is a good candidate of the lowest $\mathbf{0^{++}}$ $\boldsymbol{cs\bar{c}\bar{s}}$ tetraquark state.

- The lowest $\mathbf{0(1^+)}$ $\boldsymbol{cc\bar{u}\bar{d}}$ tetraquark state can be used to understand the LHCb $\boldsymbol{T_{cc}}$ state. [mass, width, production]

- X(6660)/X(6400) consistent with $\mathbf{0^{++}}$ $\boldsymbol{cc\bar{c}\bar{c}}$ tetraquark states [$\mathbf{2^{++}}$?].

- $\boldsymbol{T^a_{c\bar{s}0}(2900)}$ as the second highest I=1 $\boldsymbol{cn\bar{s}\bar{n}}$ tetraquark state;

  $\boldsymbol{T_{cs0}(2900)}$ as the higher I=0 $\boldsymbol{cs\bar{u}\bar{d}}$ tetraquark state.

Thanks for your attention!