# Fully-Heavy Hadronic Molecules $B_c^{(*)+} B_c^{(*)-}$ Bound by Fully-Heavy Mesons

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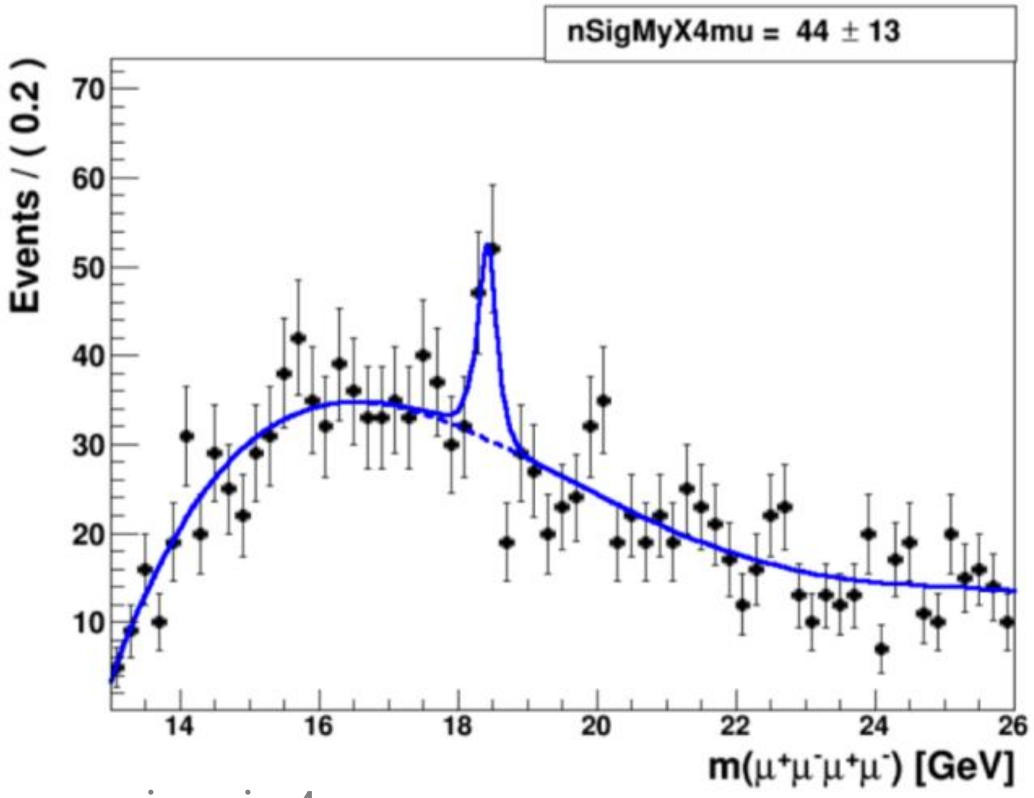
# Outline

- Motivation
- Coupled-channel Bethe-Salpeter equation
- Possible fully-heavy molecular states
- Summary and Outlook

# Signs of all heavy quark states




signs in $4\mu$

- An exotic structure in $\mu^+ \mu^- \mu^+ \mu^-$ channel at invariant mass,

  $18.4 \pm 0.1(stat) \pm 0.2(syst)$ GeV

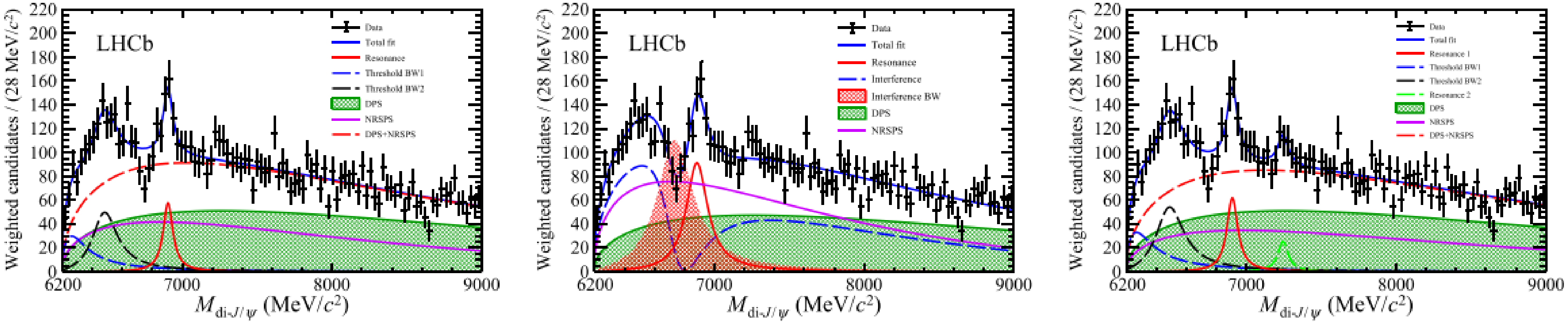
  with a global significance of $3.6\sigma$ was reported by CMS collaboration.

- Later, the LHCb collaboration searched for $X_{\bar{b}b\bar{b}b}$ in the $\Upsilon(1s)\mu^+ \mu^-$ final state, but no significant excess in the range $17.5 - 20.0$ GeV has been found.

[CMS, https://meetings.aps.org/Meeting/APR18/Session/U09.6]
[LHCb, J. High Energy Phys. JHEP10(2018)086]

# Signs of all heavy quark states

[LHCb, Sci.Bull. 65 (2020) 23, 1983-1993]




(a) (b) (c)

di-J/ψ Invariant mass spectra

(a) without interferences, (b) with interferences, (c) with an additional BW function.

- In 2020, the LHCb collaboration reported the observation of two exotic structures in the di-$J/\psi$ invariant mass spectrum:

$$X(6900): M = 6905 \pm 11 \pm 7 \text{ MeV}, \quad \Gamma = 80 \pm 19 \pm 33 \text{ MeV}.$$

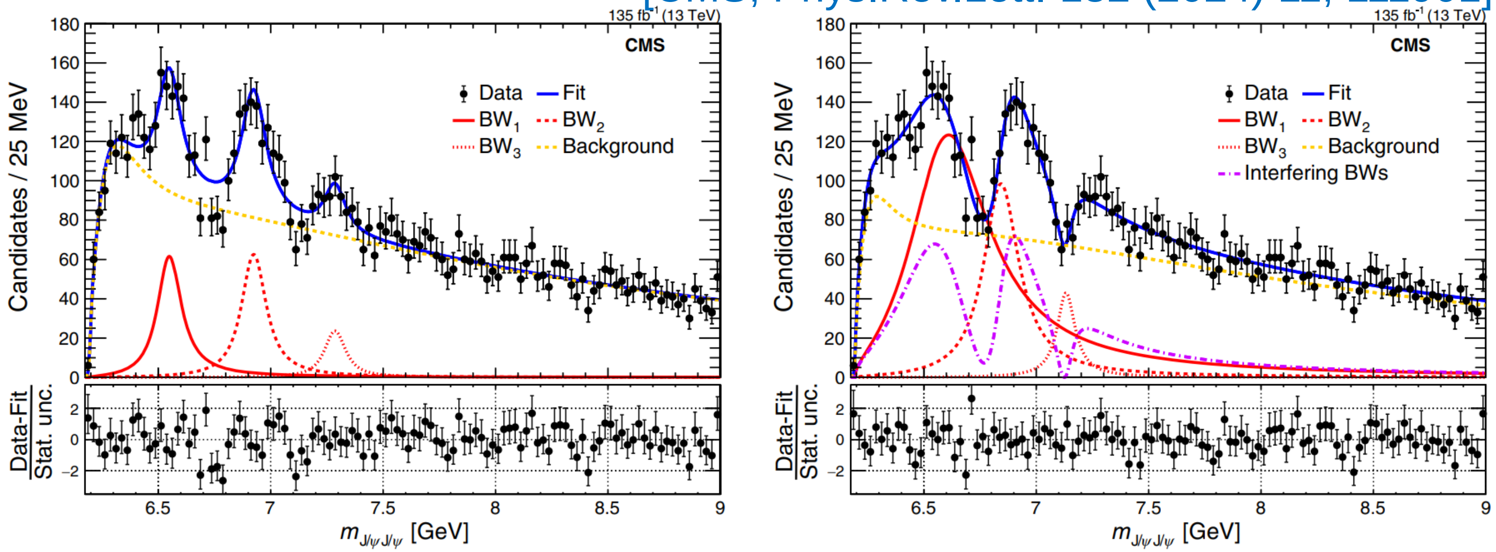
Fit(a)

$$X(6900): M = 6886 \pm 11 \pm 11 \text{ MeV}, \quad \Gamma = 168 \pm 33 \pm 69 \text{ MeV}.$$

Fit(b)

# Signs of all heavy quark states

[CMS, Phys.Rev.Lett. 132 (2024) 11, 111901]




Invariant mass spectrum of di-J/ψ. Left: without interferences. Right: with interferences.

- The $X(6900)$ was examined at the di-$J/\psi$ mass spectrum by CMS collaboration, with two new structures:

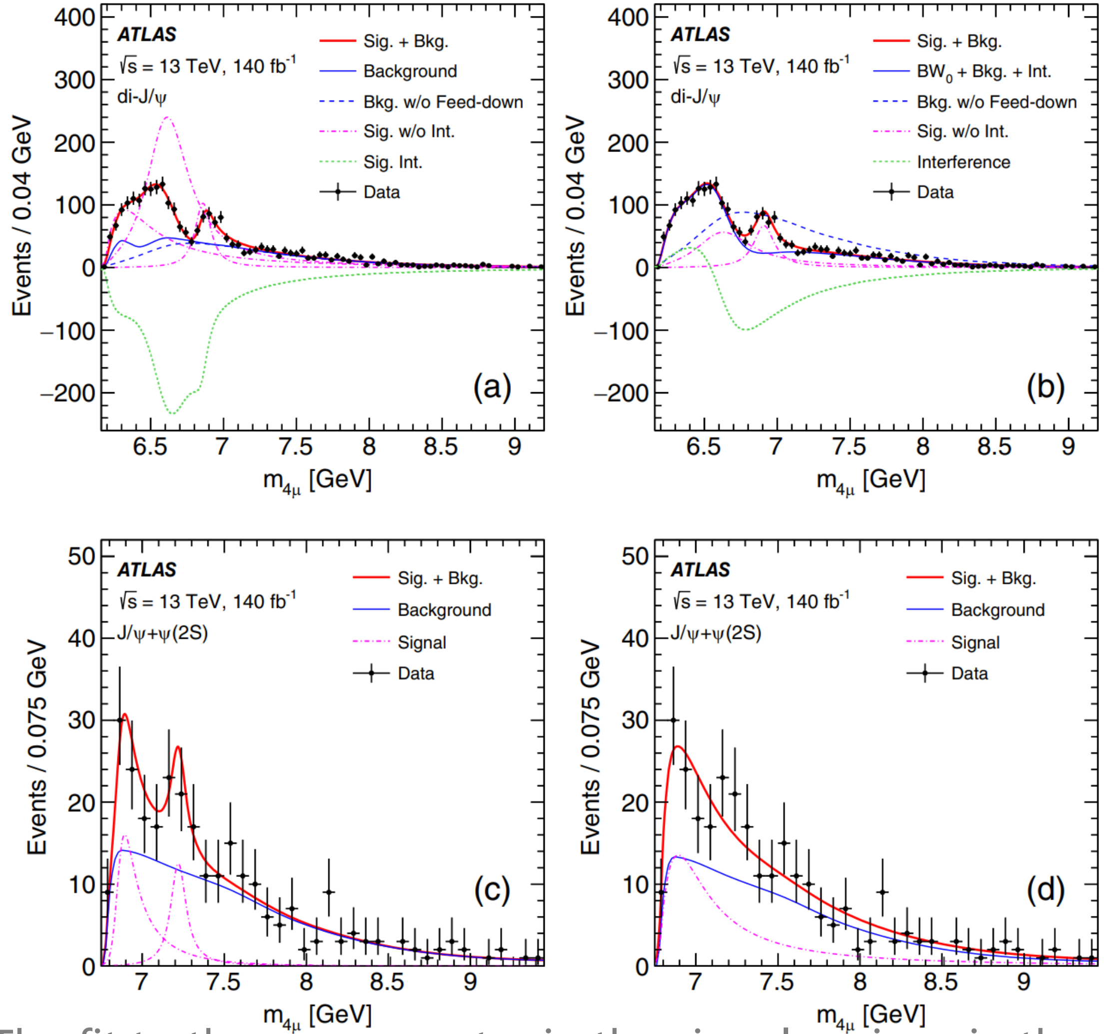
$$X(6600): M = 6552 \pm 10 \pm 12 \text{ MeV}, \quad \Gamma = 124 \pm 29 \pm 34 \text{ MeV};$$

$$X(6900): M = 6927 \pm 9 \pm 5 \text{ MeV}, \quad \Gamma = 122 \pm 22 \pm 19 \text{ MeV};$$

$$X(7200): M = 7287 \pm 19 \pm 5 \text{ MeV}, \quad \Gamma = 95 \pm 46 \pm 20 \text{ MeV}.$$

# Signs of all heavy quark states

[ATLAS, Phys.Rev.Lett. 131 (2023) 15, 151902]



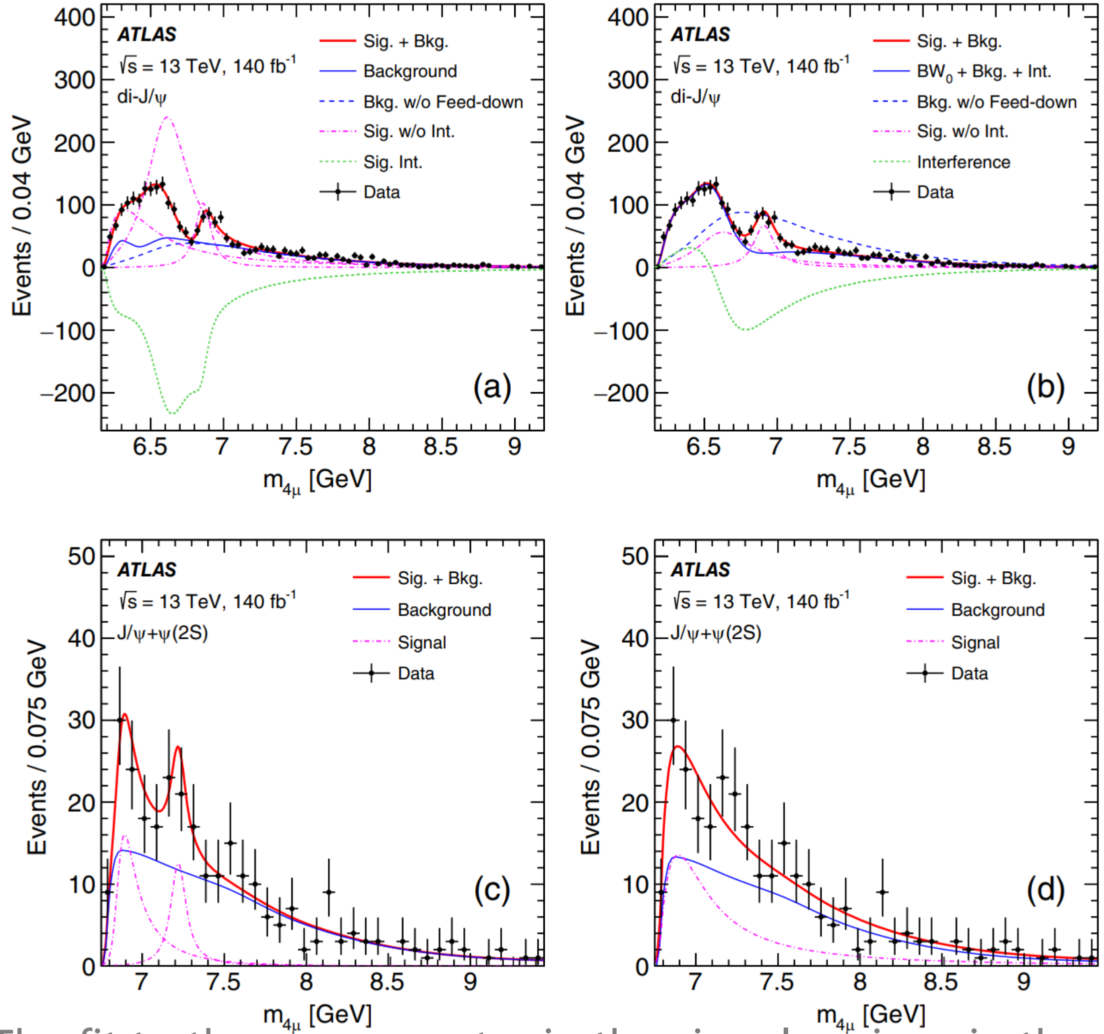

The fit to the mass spectra in the signal regions in the di-$J/\psi$ (a,b) and $J/\psi$ + $\psi(2s)$ (c,d) channels.

- The ATLAS collaboration examined the di-$J/\psi$ mass spectra, and their best fit was performed with three interfering resonances:

| Di-$J/\psi$ | Model A | Model B |
|---|---|---|
| $m_0$ | $6.41 \pm 0.08^{+0.08}_{-0.03}$ | $6.65 \pm 0.02^{+0.03}_{-0.02}$ |
| $\Gamma_0$ | $0.59 \pm 0.35^{+0.12}_{-0.20}$ | $0.44 \pm 0.05^{+0.06}_{-0.05}$ |
| $m_1$ | $6.63 \pm 0.05^{+0.08}_{-0.01}$ | ... |
| $\Gamma_1$ | $0.35 \pm 0.11^{+0.11}_{-0.04}$ | |
| $m_2$ | $6.86 \pm 0.03^{+0.01}_{-0.02}$ | $6.91 \pm 0.01 \pm 0.01$ |
| $\Gamma_2$ | $0.11 \pm 0.05^{+0.02}_{-0.01}$ | $0.15 \pm 0.03 \pm 0.01$ |
| $\Delta s/s$ | $\pm 5.1\%^{+8.1\%}_{-8.9\%}$ | ... |

# Signs of all heavy quark states

[ATLAS, Phys.Rev.Lett. 131 (2023) 15, 151902]




The fit to the mass spectra in the signal regions in the di-$J/\psi$ (a,b) and $J/\psi$ + $\psi(2s)$ (c,d) channels.

- The ATLAS collaboration also examined the $J/\psi\psi(2s)$ mass spectra, and the evidence for an enhancement at 6.9 GeV and a resonance at 7.2 GeV was reported :

| $J/\psi + \psi(2S)$ | Model $\alpha$ | Model $\beta$ |
|---|---|---|
| $m_3$ | $7.22 \pm 0.03^{+0.01}_{-0.04}$ | $6.96 \pm 0.05 \pm 0.03$ |
| $\Gamma_3$ | $0.09 \pm 0.06^{+0.06}_{-0.05}$ | $0.51 \pm 0.17^{+0.11}_{-0.10}$ |
| $\Delta s/s$ | $\pm 21\%^{+25\%}_{-15\%}$ | $\pm 20\% \pm 12\%$ |

# Theoretical work on fully-heavy states $X(6900)$

- Compact tetraquark

  - $P$-wave $cc\bar{c}\bar{c}$ tetraquark in QCD sum rule

    [Chen W, Chen H-X, Liu X, Steele T G and Zhu S-L 2017, Phys. Lett. B 773 247–51]

  - $2S$-wave $cc\bar{c}\bar{c}$ tetraquark in NR quark model

    [Liu M-S, Liu F-X, Zhong X-H and Zhao Q, arXiv:2006.11952]

  - $2S$-wave $cc\bar{c}\bar{c}$ tetraquark in string junction picture

    [Karliner M and Rosner J L, Phys. Rev. D 102 114039]

  ……

- Non-resonant

  - Rescattering mechanism

    [Wang J-Z, Chen D-Y, Liu X and Matsuki T, Phys. Rev. D 103 071503]

    [Dong X-K, Baru V, Guo F-K, Hanhart C and Nefediev A, Phys. Rev. Lett. 126 132001]

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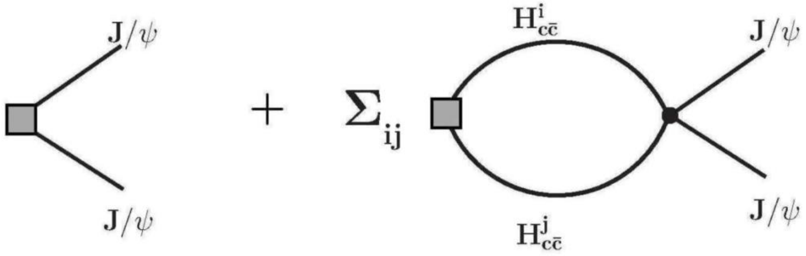
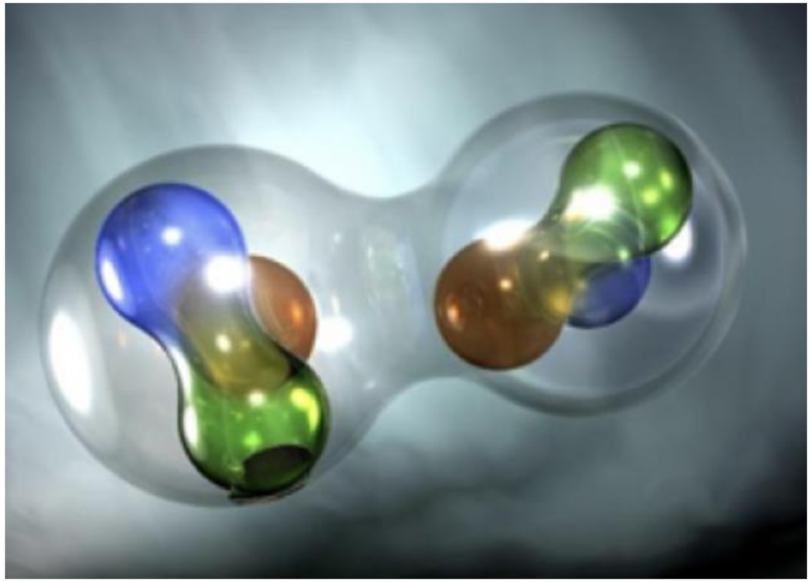


# hadronic molecules

Hadronic molecular states seem to be universal:

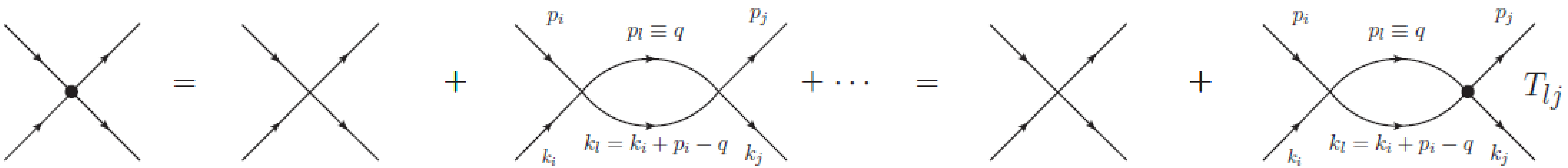
- Deuteron can be regarded as a molecular state composed of $p$ and $n$.
- $f_0(980)$, $a_0(980)$ could be considered a $\bar{K}K$ molecular state.
- $D_{s0}^*(2317)$ could be considered a $DK$ molecular state.
- $X(3872)$ could be considered a $D^*\bar{D}$ molecular state.
- $Z_b(10610)$ could be considered a $B^*\bar{B}$ molecular state.
- $\Lambda(1405)$ could be considered a $N\bar{K}$ molecular state.
- $\Lambda_c(2940)$ could be considered a $ND^*$ molecular state.
- $P_{cs}(4459)$ could be considered a $\bar{D}^*\Xi_c^0$ molecular state.



[Guo F-K, Hanhart C, Meißner U-G, Wang Q, Zhao Q and Zou B-S 2018 Hadronic molecules Rev. Mod. Phys. 90 015004]

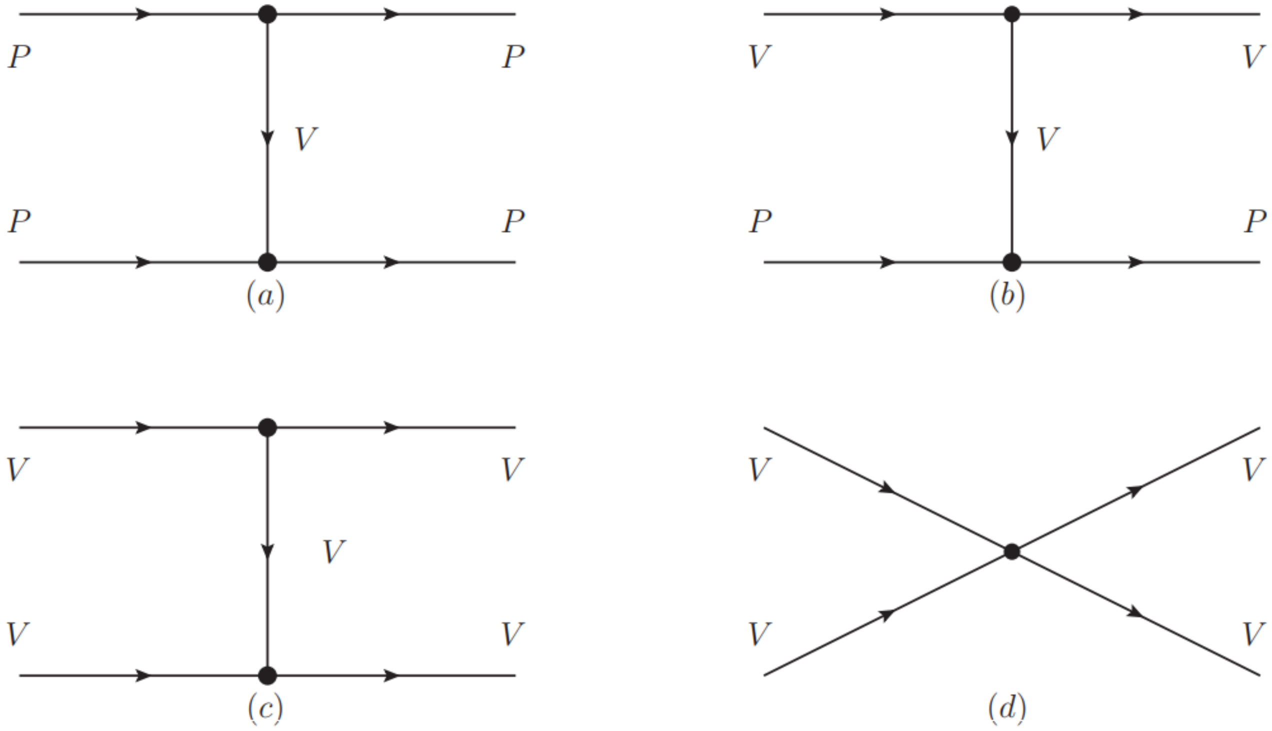
# BS-eq within LHG

Scattering matrix solved through the Bethe-Salpeter equation in coupled channels




$$T_{ij} = V_{ij} + \sum_l V_{il} G_{ll} V_{lj} + \cdots = V_{ij} + V_{il} G_{ll} T_{lj}$$

# BS-eq within LHG




$$\mathcal{L}_{VPP} = -ig\,\langle [P, \partial_\mu P] V^\mu \rangle\,,$$
$$\mathcal{L}_{VVV} = ig\,\langle (V^\mu \partial_\nu V_\mu - \partial_\nu V^\mu V_\mu) V^\nu \rangle\,,$$
$$\mathcal{L}_{VVVV} = \frac{g^2}{2} \langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle\,.$$

$$P = \begin{pmatrix} \eta_c & B_c^+ \\ B_c^- & \eta_b \end{pmatrix}$$

$$V = \begin{pmatrix} J/\psi & B_c^{*+} \\ B_c^{*-} & \Upsilon \end{pmatrix}$$

P: pseudoscalar meson, V: vector meson

# coupling constant g

$$\mathcal{L}_{VPP} = -ig\langle[P,\partial_\mu P]V^\mu\rangle\,,$$
$$\mathcal{L}_{VVV} = ig\langle(V^\mu\partial_\nu V_\mu - \partial_\nu V^\mu V_\mu)V^\nu\rangle\,,$$
$$\mathcal{L}_{VVVV} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu\rangle\,.$$

$$P = \begin{pmatrix} \eta_c & B_c^+ \\ B_c^- & \eta_b \end{pmatrix}$$

$$V = \begin{pmatrix} J/\psi & B_c^{*+} \\ B_c^{*-} & \Upsilon \end{pmatrix}$$

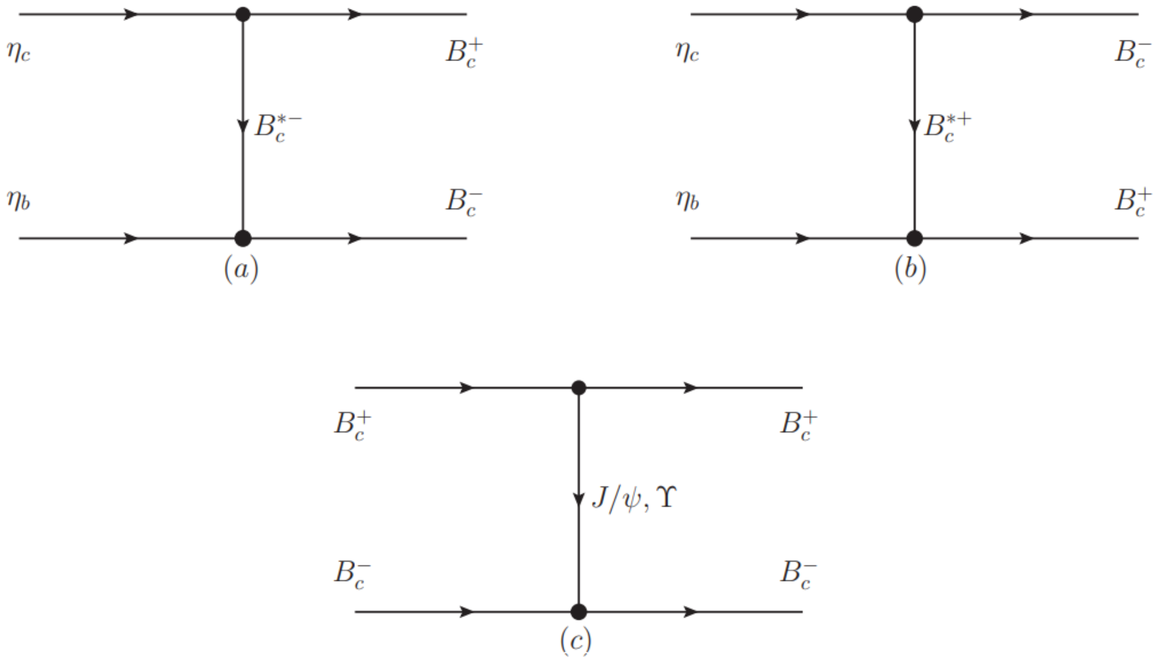
- Within the framework of $SU(5)$ flavor symmetry, a uniform coupling constant is required.
- A coupling constant $g = \frac{M_V}{2f}$ are often used to calculate light vector meson exchanges.
- In order to consider the broken of symmetry, the coupling constant is generalized using the physical mass and decay constant.

$$M_{J/\psi} = 3096.9 \text{ MeV [35]},\ f_{\eta_c} = 387/\sqrt{2} \text{ MeV [36]},$$
$$M_{B_c^*} = 6331 \text{ MeV [37]},\ f_{B_c} = 427/\sqrt{2} \text{ MeV [38]},$$
$$M_\Upsilon = 9460.4 \text{ MeV [35]},\ f_{\eta_b} = 667/\sqrt{2} \text{ MeV [38]},$$

# P-P interaction in $bc\bar{b}\bar{c}$ system

- Two coupled channels in $bc\bar{b}\bar{c}$ system,

$$\eta_c\eta_b\,, \quad B_c^+B_c^-$$




$$\begin{aligned} V_{PP}(s) &= C^t_{PP}\times g^2(p_1+p_3)(p_2+p_4) \\ &+ C^u_{PP}\times g^2(p_1+p_4)(p_2+p_3)\,, \end{aligned}$$

$$C^t_{PP} = \left(\begin{array}{c|cc} J=0 & \eta_c\eta_b & B_c^+B_c^- \\ \hline \eta_c\eta_b & 0 & \lambda\frac{1}{m^2_{B_c^*}} \\ B_c^+B_c^- & \lambda\frac{1}{m^2_{B_c^*}} & -\left(\frac{1}{m^2_{J/\psi}}+\frac{1}{m^2_{\Upsilon}}\right) \end{array}\right)$$

$$C^u_{PP} = \left(\begin{array}{c|cc} J=0 & \eta_c\eta_b & B_c^+B_c^- \\ \hline \eta_c\eta_b & 0 & \lambda\frac{1}{m^2_{B_c^*}} \\ B_c^+B_c^- & \lambda\frac{1}{m^2_{B_c^*}} & 0 \end{array}\right)$$

# V-P interaction in $bc\bar{b}\bar{c}$ system

- Four coupled channels in $bc\bar{b}\bar{c}$ system,

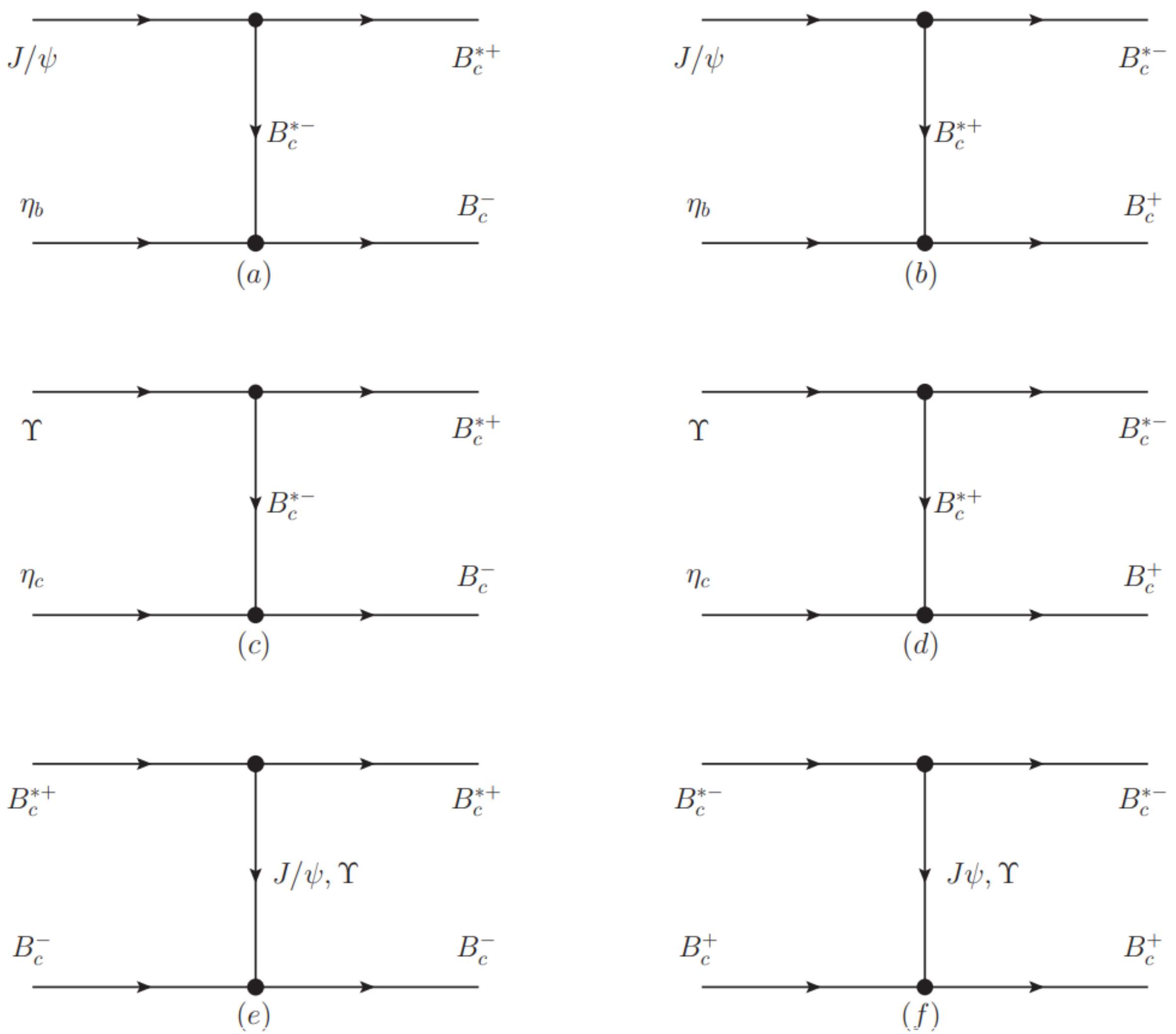
$$J/\psi\eta_b\,,\quad \Upsilon\eta_c\,,\quad B_c^{*+}B_c^-\,,\quad B_c^{*-}B_c^+$$

- The single channel with positive $C$-parity:

$$B_c^*\bar{B}_c^{(C=+)} \equiv \left(B_c^{*+}B_c^- + c.c.\right)/\sqrt{2}\,,$$

- The three coupled channel with negative $C$-parity:

$$J/\psi\eta_b\,,\ \Upsilon\eta_c\,,\ B_c^*\bar{B}_c^{(C=-)} \equiv \left(B_c^{*+}B_c^- - c.c.\right)/\sqrt{2}$$



# V-P interaction in $bc\bar{b}\bar{c}$ system

$$
\begin{aligned}
V_{VP(\pm)}(s) &= C^t_{VP(\pm)} \times g^2 (p_1+p_3)(p_2+p_4)\ \epsilon_1\cdot\epsilon_3 \\
&+ C^u_{VP(\pm)} \times g^2 (p_1+p_4)(p_2+p_3)\ \epsilon_1\cdot\epsilon_3
\end{aligned}
$$

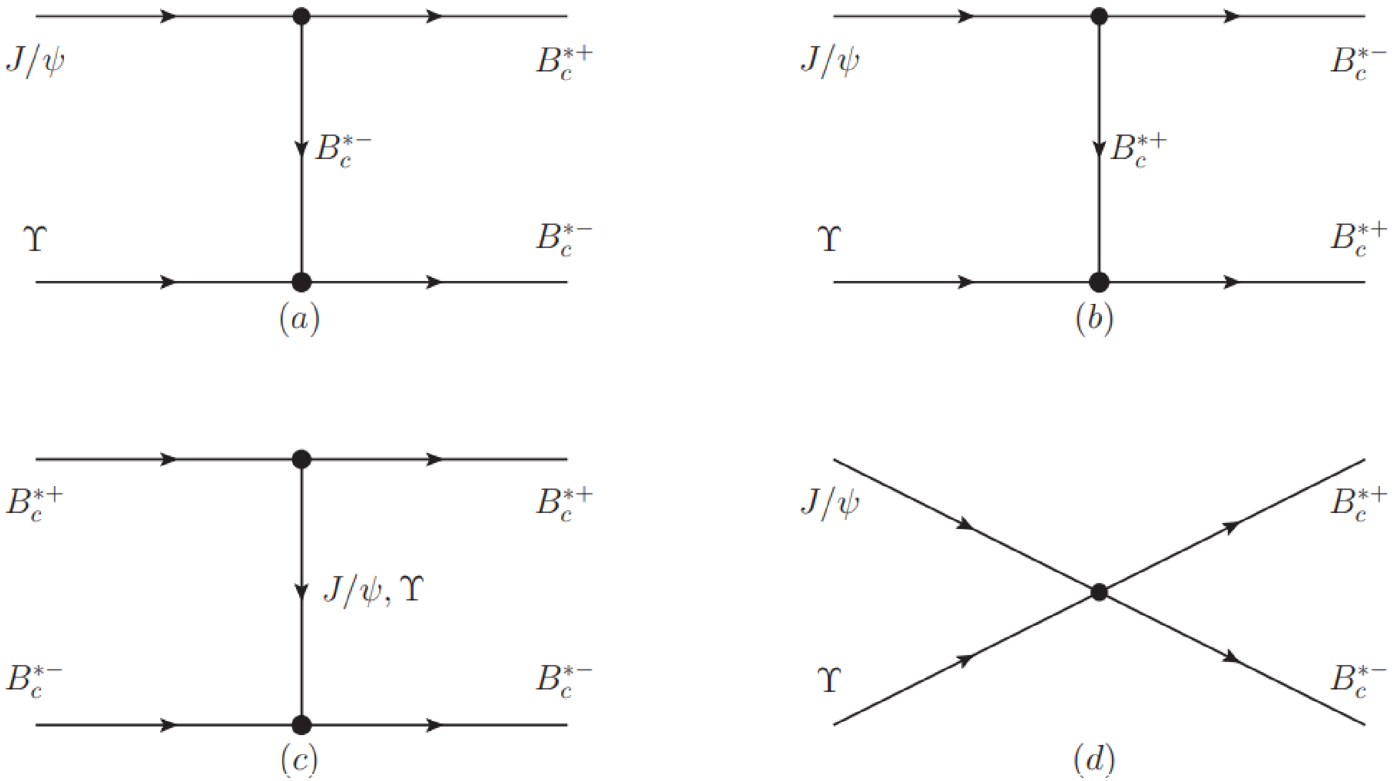
$$
\begin{aligned}
C^t_{VP(+)} &= -\left(\frac{1}{m^2_{J/\psi}}+\frac{1}{m^2_{\Upsilon}}\right), \\
C^u_{VP(+)} &= 0.
\end{aligned}
$$

$C^t_{VP(-)} =$

| $J=1$ | $J/\psi\eta_b$ | $\Upsilon\eta_c$ | $B_c^*\bar{B}_c^{(C=-)}$ |
|---|---|---|---|
| $J/\psi\eta_b$ | $0$ | $0$ | $\frac{\sqrt{2}\lambda}{m^2_{B_c^*}}$ |
| $\Upsilon\eta_c$ | $0$ | $0$ | $\frac{\sqrt{2}\lambda}{m^2_{B_c^*}}$ |
| $B_c^*\bar{B}_c^{(C=-)}$ | $\frac{\sqrt{2}\lambda}{m^2_{B_c^*}}$ | $\frac{\sqrt{2}\lambda}{m^2_{B_c^*}}$ | $-\left(\frac{1}{m^2_{J/\psi}}+\frac{1}{m^2_{\Upsilon}}\right)$ |

- $VVP$ vertices are not considered here, so there is no $u$-channel vector exchange contribution in the $VP$ sector.

# V-V interaction in $bc\bar{b}\bar{c}$ system




- Additional four-vector contact terms are taken into account from :

$$\mathcal{L}_{VVVV} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle$$

$$g^2 = \sqrt{g_{V_1} g_{V_2} g_{V_3} g_{V_4}}$$

# Loop function

- Bethe-Salpeter equation:

$$T_{PP/VP/VV}(s) = \frac{V_{PP/VP/VV}(s)}{1 - V_{PP/VP/VV}(s)G(s)}$$

- Loop function:

$$G_{ii}(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - m_1^2 + i\epsilon}\frac{1}{(p-q)^2 - m_2^2 + i\epsilon}.$$

Cutoff regularization

$$G_{ii}(s) = \int_0^{q_{\max}} \frac{d^3q}{(2\pi)^3}\frac{\omega_1+\omega_2}{2\omega_1\omega_2}\frac{1}{s-(\omega_1+\omega_2)^2 + i\epsilon}$$

# Loop function

- Looking for poles on complex plane,

$$G_{ii}^{II}(s) = G_{ii}(s) + i\frac{k}{4\pi\sqrt{s}}, \qquad k(s) = \sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}/(2\sqrt{s})$$

- Coupling constants are defined as the residue of the amplitude at the poles:

$$T_{ij}(s) = \frac{g_i g_j}{s - s_p^2} \qquad g_i^2 = \lim_{\sqrt{s}\to s_p}(s - s_p^2)T_{ii}(s)$$

# Results of $bc\bar{b}\bar{c}$ system

TABLE I. Pole positions $E_p$ with respect to the cutoff momentum $\Lambda$, in units of MeV. We only list the poles that can qualify as hadronic molecules.

| Pole | $\Lambda = 400$ | $\Lambda = 600$ | $\Lambda = 800$ | $\Lambda = 1000$ | $\Lambda = 1200$ | $\Lambda = 1400$ |
|---|---|---|---|---|---|---|
| $\lvert B_c^+ B_c^-; J^{PC} = 0^{++}\rangle$ | $--$ | $12503.3 - i126.8$ | $12383.1 - i115.2$ | $12379.0 - i0$ | $12324.5 - i0$ | $12227.8 - i0$ |
| $\lvert B_c^{*+} B_c^-; J^{PC} = 1^{++}\rangle$ | $--$ | $--$ | $12604.8 - i0$ | $12598.1 - i0$ | $12581.5 - i0$ | $12552.5 - i0$ |
| $\lvert B_c^{*+} B_c^-; J^{PC} = 1^{+-}\rangle$ | $--$ | $12559.9 - i111.6$ | $12495.8 - i76.8$ | $12444.4 - i42.2$ | $12394.8 - i0$ | $12296.4 - i0$ |
| $\lvert B_c^{*+} B_c^{*-}; J^{PC} = 0^{++}\rangle$ | $--$ | $--$ | $--$ | $--$ | $12549.4 - i0$ | $12508.2 - i0$ |
| $\lvert B_c^{*+} B_c^{*-}; J^{PC} = 1^{+-}\rangle$ | $--$ | $--$ | $--$ | $--$ | $--$ | $--$ |
| $\lvert B_c^{*+} B_c^{*-}; J^{PC} = 2^{++}\rangle$ | $12660.7 - i45.7$ | $12572.0 - i96.7$ | $12556.3 - i0$ | $12510.2 - i0$ | $12418.8 - i0$ | $12287.7 - i0$ |

- Four bound state poles in $b\bar{c}c\bar{b}$ system with cutoff parameter $\Lambda = 400 \sim 1400$ MeV.
- The $0^{++}$ and $1^{+-}$ poles in VV interaction are more difficult to form a bound state due to the additional repulsive contact terms.

# Results of $bc\bar{b}\bar{c}$ system

TABLE II. Pole positions $E_p$ and their couplings $g_i$ to various coupled channels, with the cutoff momentum $\Lambda = 500$ MeV. We only list the poles that can qualify as hadronic molecules.

| State | $E_p$ (MeV) | Channel | $\lvert g_i \rvert$ (GeV) |
|---|---|---|---|
| $\lvert B_c^+ B_c^-; J^{PC} = 0^{++}\rangle$ | $12541 - i83$ | $\eta_c \eta_b$ | 20 |
| | | $B_c^+ B_c^-$ | 123 |
| $\lvert B_c^{*+} B_c^-; J^{PC} = 1^{+-}\rangle$ | $12597 - i74$ | $J/\psi \eta_b$ | 14 |
| | | $\Upsilon \eta_c$ | 14 |
| | | $B_c^* \bar{B}_c^{(C=-)}$ | 114 |
| $\lvert B_c^{*+} B_c^{*-}; J^{PC} = 2^{++}\rangle$ | $12630 - i80$ | $J/\psi \Upsilon$ | 23 |
| | | $B_c^{*+} B_c^{*-}$ | 135 |

- The $0^{++}$, $1^{+-}$ and $2^{++}$ states are more likely to exist.
- All 3 states are strongly coupled to $B_c^{(*)} \bar{B}_c^{(*)}$.

# Results of $bc\bar{b}\bar{c}$ system

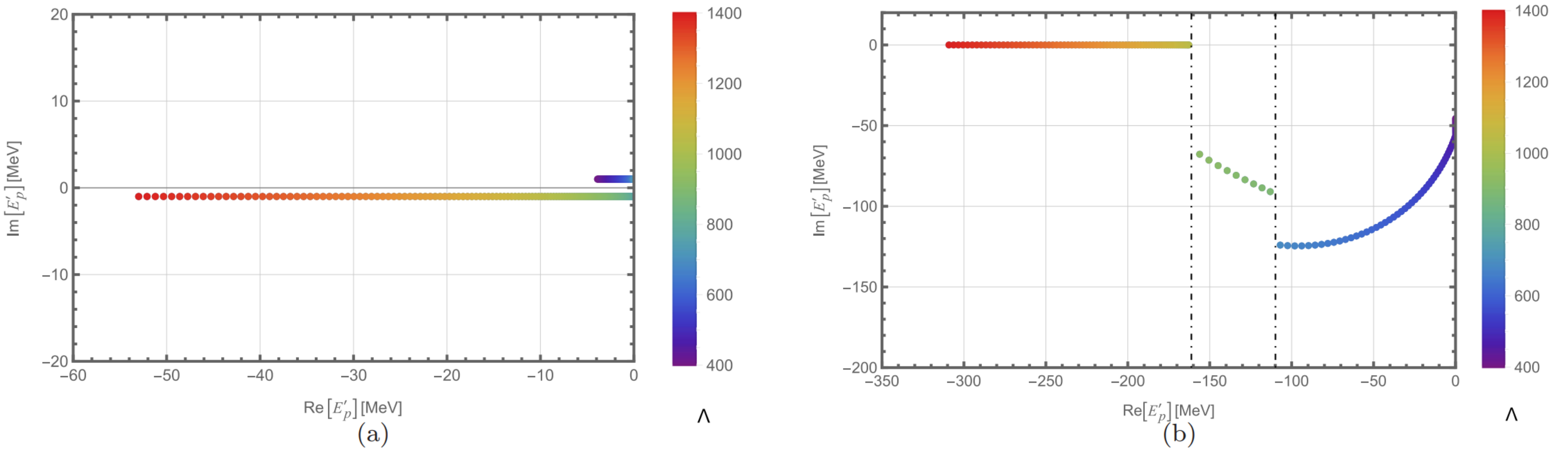



FIG. 2. Pole positions $E'_p = E_p - M_{B_c^*} - M_{B_c}$ of the hadronic molecules (a) $|B_c^{*+}B_c^-; J^{PC} = 1^{++}\rangle$ and (b) $|B_c^{*+}B_c^-; J^{PC} = 1^{+-}\rangle$ with respect to the cutoff momentum $\Lambda = 400 \sim 1400$ MeV. A small imaginary part is added to the poles of the subfigure (a). The $J/\psi\eta_b$ and $\Upsilon\eta_c$ thresholds are indicated by dotted lines in the subfigure (b).

- The $1^{++}$ pole involves single channel $B_c^*\bar{B}_c^{(C=+)}$ corresponds to a virtual state when $\Lambda < 710$ MeV, and a bound state when $\Lambda > 710$ MeV.
- The $1^{++}$ pole can still be a bound state for $\Lambda > 940$ MeV, when only the contribution of $J/\psi$ exchange is retained.

# Results of $bc\bar{b}\bar{c}$ system

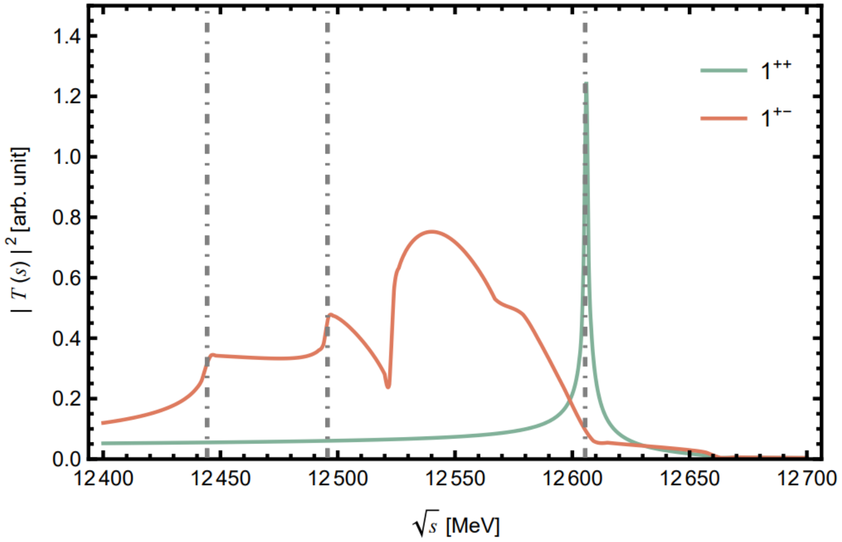



FIG. 3. The scattering amplitudes $|T_{VP}^{B_c^*\bar{B}_c^{(C=+)}\to B_c^*\bar{B}_c^{(C=+)}}(s)|^2$ and $|T_{VP}^{B_c^*\bar{B}_c^{(C=-)}\to B_c^*\bar{B}_c^{(C=-)}}(s)|^2$ for the cutoff momentum $\Lambda = 600$ MeV.

- The pole of $J^{PC} = 1^{++}$ in VP interaction is identified as a virtual state, so it enhances the near-threshold cusp effect at $B_c^{*+}B_c^-$ threshold.
- The pole of $J^{PC} = 1^{+-}$ in VP interaction is identified as a bound/resonance state, so it appears as a normal peak under $B_c^{*+}B_c^-$ threshold, with the threshold effects of the lower coupled channels.

# Summary and Outlook

- Within the extensional LHG formalism, fully-heavy hadronic molecules $B_c^{(*)}\bar{B}_c^{(*)}$ could arise from the exchange of fully-heavy vector mesons $J/\psi$, $B_c^*$ and $\Upsilon$.
- 3 states: $|B_c^+B_c^-; J^{PC}=0^{++}\rangle$, $|B_c^{*+}B_c^-; J^{PC}=1^{+-}\rangle$ and $|B_c^{*+}B_c^{*-}; J^{PC}=2^{++}\rangle$ are more likely to exist, and $|B_c^{*+}B_c^-; J^{PC}=1^{++}\rangle$ may exist or behave as threshold cusp.
- A similar approach is used for the fully-heavy systems $bc\bar{c}\bar{c}$, $bb\bar{c}\bar{c}$ and $bb\bar{b}\bar{c}$. Our results do not support the existence of deeply-bound hadronic molecules in these systems.
- The tetraquark state may have a larger effect, as well as the direct gluon exchange.

Thanks for your attention

# Back-up

# BS-eq within LHG

$$P = \begin{pmatrix} \frac{\eta}{\sqrt{3}}+\frac{\eta'}{\sqrt{6}}+\frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ & \bar{D}^0 & B^+ \\ \pi^- & \frac{\eta}{\sqrt{3}}+\frac{\eta'}{\sqrt{6}}-\frac{\pi^0}{\sqrt{2}} & K^0 & D^- & B^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}}+\sqrt{\frac{2}{3}}\eta' & D_s^- & B_s^0 \\ D^0 & D^+ & D_s^+ & \eta_c & B_c^+ \\ B^- & \bar{B}^0 & \bar{B}_s^0 & B_c^- & \eta_b \end{pmatrix}, \quad V = \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} & B^{*+} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} & D^{*-} & B^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} & B_s^{*0} \\ D^{*0} & D^{*+} & D_s^{*+} & J/\psi & B_c^{*+} \\ B^{*-} & \bar{B}^{*0} & \bar{B}_s^{*0} & B_c^{*-} & \Upsilon \end{pmatrix}.$$

A flavor SU(5) symmetry is assumed

# BS-eq within LHG

$VV$ **interaction**

$J/\psi\Upsilon\,, \quad B_c^{*+}B_c^{*-}\,.$

$$V_{VV}(s) = V_{VV}^{ex}(s) + V_{VV}^{co}(s)\,.$$

$$\begin{aligned} V_{VV}^{ex}(s) &= C_{VV}^{t} \times g^2(p_1+p_3)(p_2+p_4)\epsilon_1\cdot\epsilon_3\epsilon_2\cdot\epsilon_4 \\ &+ C_{VV}^{u} \times g^2(p_1+p_4)(p_2+p_3)\epsilon_1\cdot\epsilon_4\epsilon_2\cdot\epsilon_3\,, \end{aligned}$$

$$C_{VV}^{t} = \left(\begin{array}{c|cc} J=0,1,2 & J/\psi\Upsilon & B_c^{*+}B_c^{*-} \\ \hline J/\psi\Upsilon & 0 & \lambda\frac{1}{m_{B_c^*}^2} \\ B_c^{*+}B_c^{*-} & \lambda\frac{1}{m_{B_c^*}^2} & -(\frac{1}{m_{J/\psi}^2}+\frac{1}{m_{\Upsilon}^2}) \end{array}\right)$$

$$C_{VV}^{u} = \left(\begin{array}{c|cc} J=0,1,2 & J/\psi\Upsilon & B_c^{*+}B_c^{*-} \\ \hline J/\psi\Upsilon & 0 & \lambda\frac{1}{m_{B_c^*}^2} \\ B_c^{*+}B_c^{*-} & \lambda\frac{1}{m_{B_c^*}^2} & 0 \end{array}\right)\,.$$

$$V_{J/\psi\Upsilon\to B_c^{*+}B_c^{*-}}^{co}(s) = \begin{cases} -4g^2 & \text{for } J=0, \\ 0 & \text{for } J=1, \\ 2g^2 & \text{for } J=2, \end{cases}$$

$$V_{B_c^{*+}B_c^{*-}\to B_c^{*+}B_c^{*-}}^{co}(s) = \begin{cases} 4g^2 & \text{for } J=0, \\ 6g^2 & \text{for } J=1, \\ -2g^2 & \text{for } J=2. \end{cases}$$

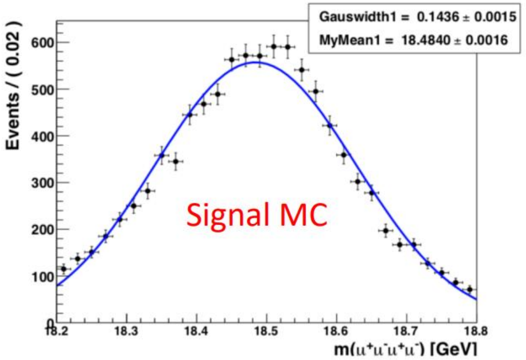
Loop function:

$$G_{ii}(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - m_1^2 + i\epsilon}\frac{1}{(p-q)^2 - m_2^2 + i\epsilon},$$

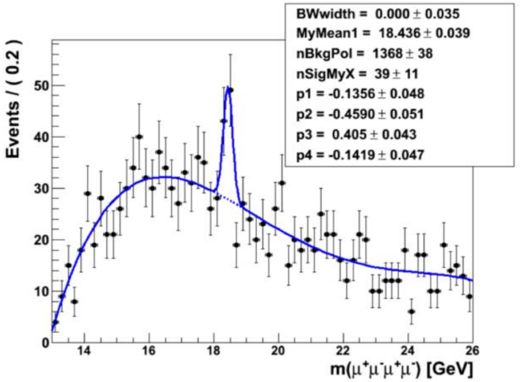
dimensional regularization

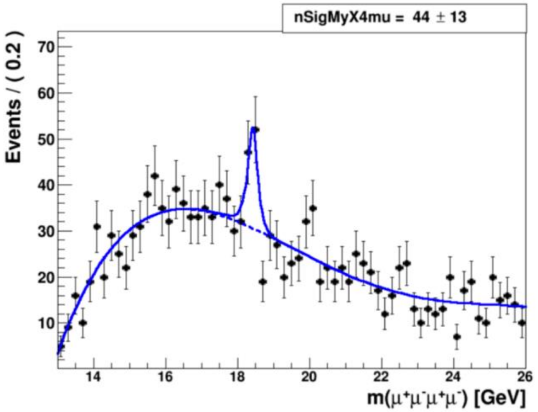
$$\begin{aligned}G_{ii}(s) =& \frac{1}{16\pi^2}\Big\{a_{ii}(\mu) + \ln\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}\ln\frac{m_2^2}{m_1^2} \\ &+ \frac{q_{cmi}(s)}{\sqrt{s}}\big[\ln\left(s - (m_2^2 - m_1^2) + 2q_{cmi}(s)\sqrt{s}\right) \\ &+ \ln\left(s + (m_2^2 - m_1^2) + 2q_{cmi}(s)\sqrt{s}\right) \\ &- \ln\left(-s - (m_2^2 - m_1^2) + 2q_{cmi}(s)\sqrt{s}\right) \\ &- \ln\left(-s + (m_2^2 - m_1^2) + 2q_{cmi}(s)\sqrt{s}\right)\big]\Big\},\end{aligned}$$

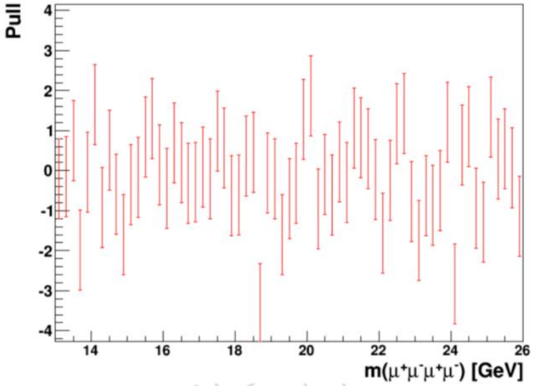
# Intrinsic Width and significance in 4μ




- Fit the signal to a single Gaussian, width = 144 MeV.
- Fit the signal shape to a Breit-Wigner convolved with a Gaussian with a fixed width.
- Returned intrinsic width $0 \pm 35$ MeV.



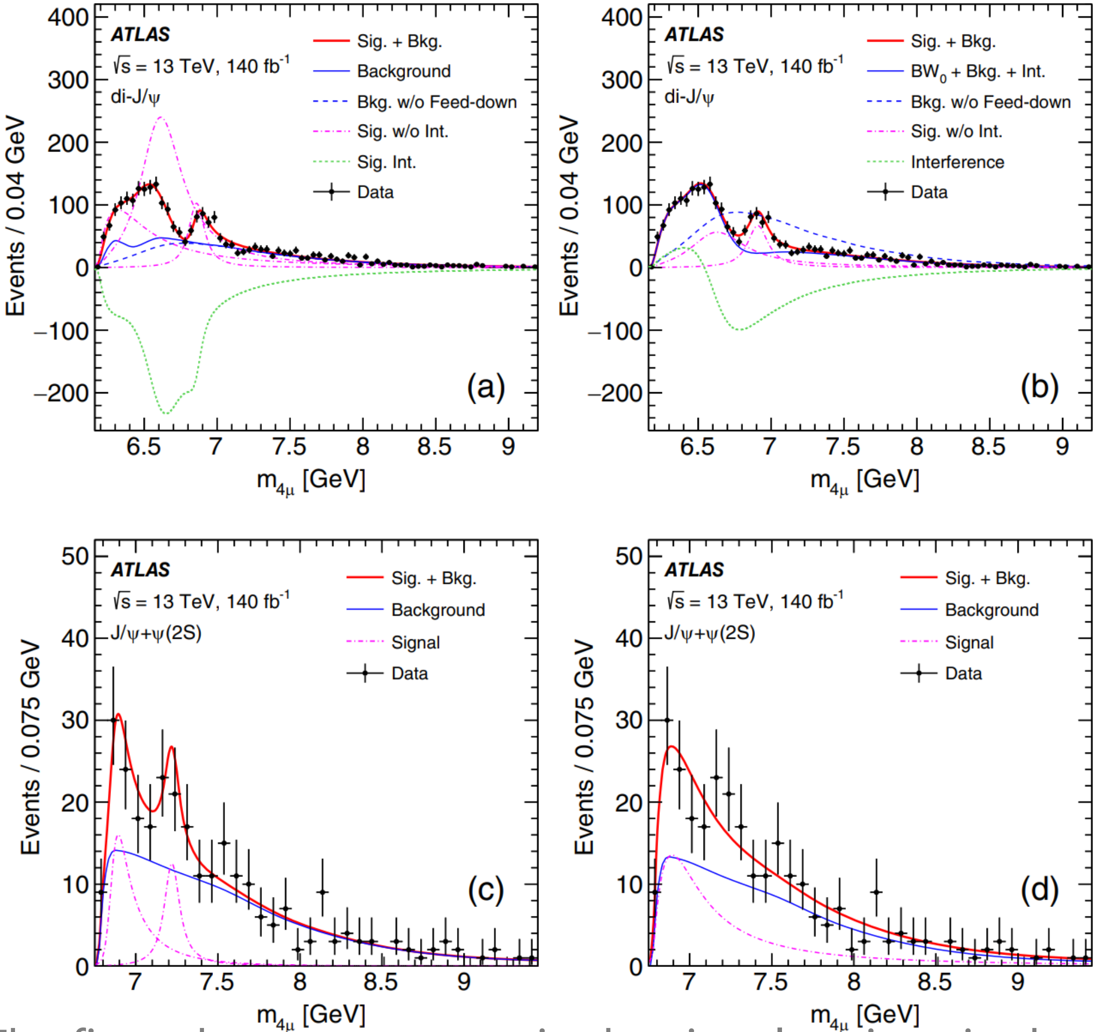




- Evaluate the significance with $\sqrt{-2\ln {}^{L_0}/_{L_s}}$, $L_0$: null hypothesis, $L_s$: signal hypothesis
- 44±13 signal yield with 3.86**σ** local significance.
- Mass : 18.4 ± 0.1 (stat.) ± 0.2 (syst.) GeV

# Signs of all heavy quark states

ATLAS, Phys.Rev.Lett. 131 (2023) 15, 151902.




The fit to the mass spectra in the signal regions in the di-$J/\psi$ (a,b) and $J/\psi + \psi(2s)$ (c,d) channels.

TABLE II. The fitted masses and natural widths (in GeV), and relative uncertainties of signal yields ($\Delta s/s$) in the di-$J/\psi$ and $J/\psi + \psi(2S)$ channels. The results of both the models are given in each channel. The first uncertainties are statistical while the second ones are systematic.

| Di-$J/\psi$ | Model A | Model B |
|---|---|---|
| $m_0$ | $6.41 \pm 0.08^{+0.08}_{-0.03}$ | $6.65 \pm 0.02^{+0.03}_{-0.02}$ |
| $\Gamma_0$ | $0.59 \pm 0.35^{+0.12}_{-0.20}$ | $0.44 \pm 0.05^{+0.06}_{-0.05}$ |
| $m_1$ | $6.63 \pm 0.05^{+0.08}_{-0.01}$ | ... |
| $\Gamma_1$ | $0.35 \pm 0.11^{+0.11}_{-0.04}$ | |
| $m_2$ | $6.86 \pm 0.03^{+0.01}_{-0.02}$ | $6.91 \pm 0.01 \pm 0.01$ |
| $\Gamma_2$ | $0.11 \pm 0.05^{+0.02}_{-0.01}$ | $0.15 \pm 0.03 \pm 0.01$ |
| $\Delta s/s$ | $\pm 5.1\%^{+8.1\%}_{-8.9\%}$ | ... |

| $J/\psi + \psi(2S)$ | Model $\alpha$ | Model $\beta$ |
|---|---|---|
| $m_3$ | $7.22 \pm 0.03^{+0.01}_{-0.04}$ | $6.96 \pm 0.05 \pm 0.03$ |
| $\Gamma_3$ | $0.09 \pm 0.06^{+0.06}_{-0.05}$ | $0.51 \pm 0.17^{+0.11}_{-0.10}$ |
| $\Delta s/s$ | $\pm 21\%^{+25\%}_{-15\%}$ | $\pm 20\% \pm 12\%$ |