# Precision Study of Gluon Saturation

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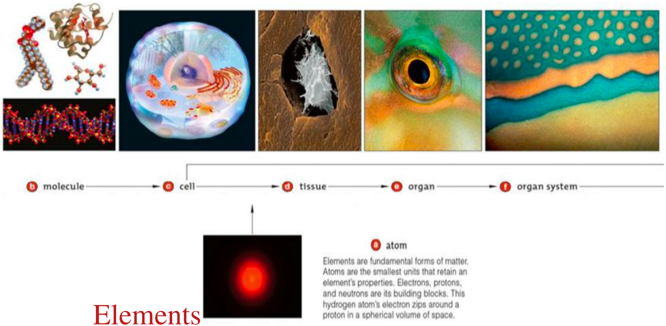
G. A. Chirilli, Bo-Wen Xiao, Feng Yuan, Phys. Rev. Lett. **108**, 122301 (2012).
Y. Shi, L. Wang, S.Y. Wei, Bo-Wen Xiao, Phys. Rev. Lett. **128**, 202302 (2022).
Y. Hagiwara, X.B. Tong, B.W. Xiao, In Preparation.

# Life is an Emergent Property

The philosophical concept of “emergent properties": Understanding the individual parts alone is insufficient to understand or predict critical properties of systems. Thus, emergent properties necessarily come from the interactions of the parts of the larger system.



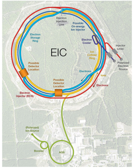
Elements are fundamental forms of matter. Atoms are the smallest units that retain an element's properties. Electrons, protons, and neutrons are its building blocks. This hydrogen atom's electron zips around a proton in a spherical volume of space.

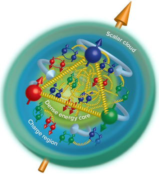
Systems

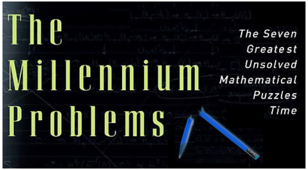
Elements

# Emergent Phenomena in QCD

To understand our physical world, we have to understand QCD!







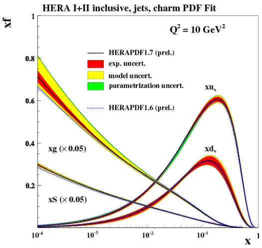
Three pillars of EIC Physics:

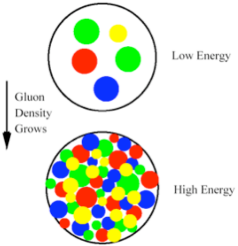
- How does the spin of proton arise? (Spin puzzle)
- What are the emergent properties of dense gluon system?
- How does proton mass arise? Mass gap: million dollar question.

EICs: keys to unlocking these mysteries!

# Saturation Physics, Color Glass Condensate

Describe the emergent property of high density gluons inside proton and nuclei.

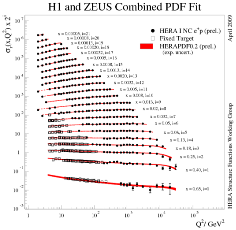


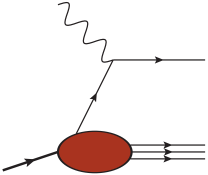


- Gluon density grows rapidly as $x$ gets small.
- Many gluons with fixed size packed in a confined hadron, gluons overlap and recombine ⇒ Non-linear QCD dynamics ⇒ ultra-dense gluonic matter with collective property.

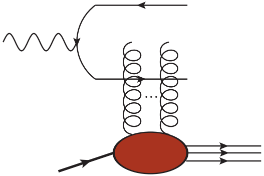
# Dual Descriptions of Deep Inelastic Scattering

深度非弹性散射的双重描述:





Bjorken frame



Dipole frame

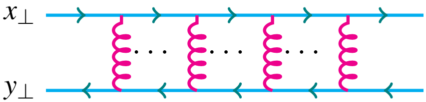
- Bjorken: partonic picture is manifest. Saturation shows up as limit of number density.
- Dipole: the partonic picture is no longer manifest. Saturation appears as the unitarity limit for scattering. Convenient to resum the multiple gluon interactions.

$$F_2(x,Q^2) = \sum_f e_f^2 \frac{Q^2}{4\pi^2\alpha_{\text{em}}} S_\perp \int_0^1 \mathrm{d}z \int \mathrm{d}^2 r_\perp \left|\psi\left(z, r_\perp, Q\right)\right|^2 \left[1 - S^{(2)}\left(Q_s r_\perp\right)\right]$$

# Wilson Lines in Color Glass Condensate Formalism

The Wilson loop (color singlet dipole) in McLerran-Venugopalan (MV) model

$$\frac{1}{N_c}\left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp)\right\rangle = e^{-\frac{Q_s^2(x_\perp - y_\perp)^2}{4}}$$

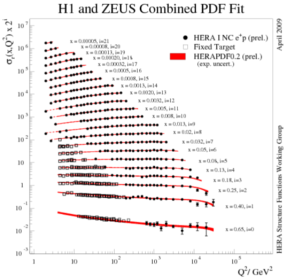


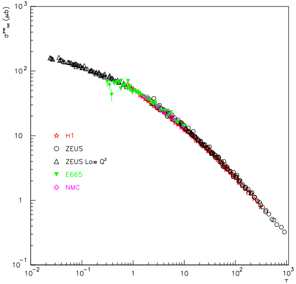
- Dipole amplitude $S^{(2)}$ then produces the quark $k_T$ spectrum via Fourier transform

$$\mathcal{F}(k_\perp) \equiv \frac{dN}{d^2k_\perp} = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} \frac{1}{N_c}\left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp)\right\rangle.$$

# Geometrical Scaling in DIS

深度非弹性散射总截面[Golec-Biernat, Stasto, Kwiecinski; 01, Munier, Peschanski, 03]



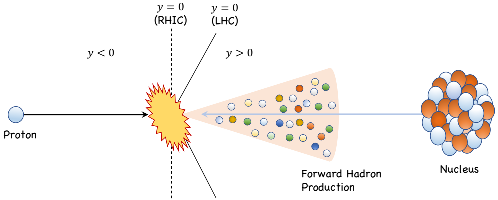


- All data ($x \leq 0.01$, $Q^2 \leq 450 GeV^2$) is function of a single variable $\tau = Q^2/Q_s^2$.
- Define $Q_s^2(x) = (x_0/x)^\lambda \mathrm{GeV}^2$ with $x_0 = 3.04 \times 10^{-3}$ and $\lambda = 0.288$.

# Forward hadron production in *pA* collisions

[Dumitru, Jalilian-Marian, 02] Dilute-dense factorization at forward rapidity

$$\frac{d\sigma_{\rm LO}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2}\left[x_1 q_f(x_1,\mu)\mathcal{F}_{x_2}(k_\perp)D_{h/q}(z,\mu) + x_1 g(x_1,\mu)\tilde{\mathcal{F}}_{x_2}(k_\perp)D_{h/g}(z,\mu)\right].$$

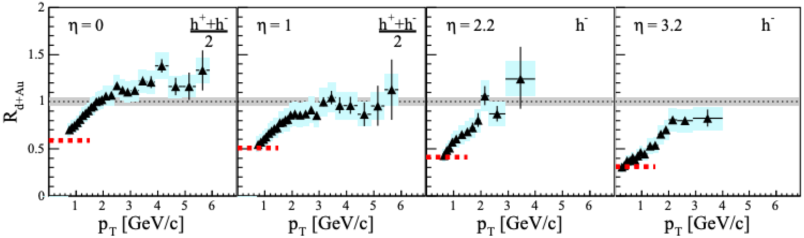


- $\mathcal{F}(k_\perp)$ (dipole gluon distribution) encodes dense gluon info.
- Early attempts: [Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11] [Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 14]
- Full NLO: [Chirilli, BX and Yuan, 12]

# d+Au collisions at RHIC

相对论重离子对撞机上的单强子产生: 氘+金核碰撞/质子+质子碰撞

$$R_{d+Au} = \frac{1}{\langle N_{\rm coll} \rangle} \frac{d^2N_{d+Au}/d^2p_T d\eta}{d^2N_{pp}/d^2p_T d\eta}$$



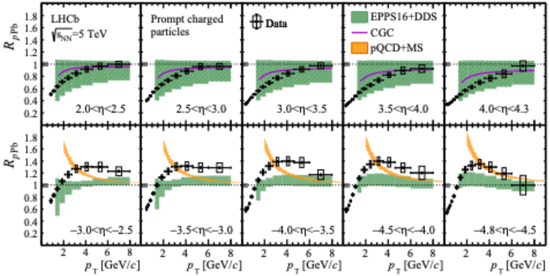
BRAHMS

- Cronin effect at middle rapidity
- Rapidity evolution of the nuclear modification factors $R_{d+Au}$
- Promising evidence for gluon saturation effects

# New LHCb Results

[R. Aaet al. (LHCb Collaboration), Phys. Rev. Lett. 128 (2022) 142004]

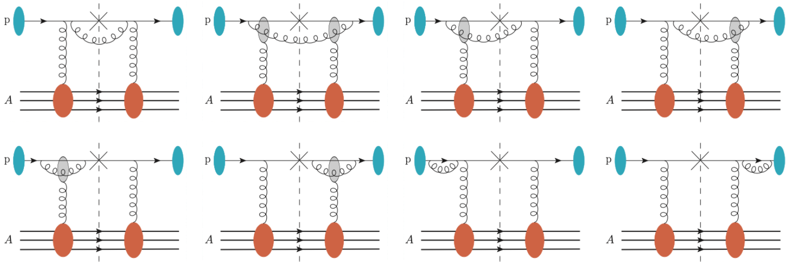
$$R_{pPb} = \frac{1}{\langle N_{\text{coll}} \rangle} \frac{d^2N_{p+Pb}/d^2p_T d\eta}{d^2N_{pp}/d^2p_T d\eta}$$



- Rapidity evolution of the nuclear modification factors $R_{pPb}$ similar to RHIC

# NLO diagrams in the $q \to q$ channel

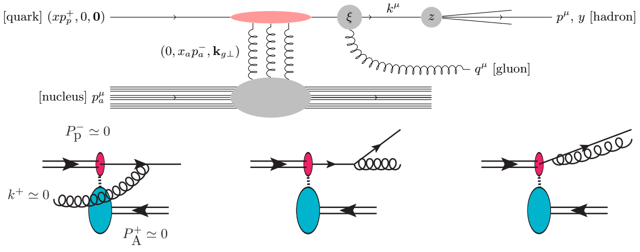
G. A. Chirilli, Bo-Wen Xiao, Feng Yuan, ▸ Phys. Rev. Lett. **108**, 122301 (2012).



- Take into account real (top) and virtual (bottom) diagrams together!
- Non-linear multiple interactions inside the grey blobs!
- Integrate over gluon phase space ⇒Divergences!.

# Factorization for single inclusive hadron productions

Factorization for the $p + A \to H + X$ process [Chirilli, BX and Yuan, 12]



Rapidity Divergence Collinear Divergence (P) Collinear Divergence (F)

- Include all real and virtual graphs in all channels $q \to q$, $q \to g$, $g \to q(\bar{q})$ and $g \to g$.
- 1. collinear to target nucleus; rapidity divergence $\Rightarrow$ BK evolution for UGD $\mathcal{F}(k_\perp)$.
- 2. collinear to the initial quark; $\Rightarrow$ DGLAP evolution for PDFs
- 3. collinear to the final quark. $\Rightarrow$ DGLAP evolution for FFs.

# Factorization and NLO Calculation

- Factorization is about separation of short distant physics (perturbatively calculable hard factor) from large distant physics (Non perturbative).

$$\sigma \sim xf(x) \otimes \mathcal{H} \otimes D_h(z) \otimes \mathcal{F}(k_\perp)$$

- NLO (1-loop) calculation always contains various kinds of divergences.
  - Some divergences can be absorbed into the corresponding evolution equations.
  - Renormalization: cutting off infinities and hiding the ignorance.
  - The rest of divergences should be canceled.
- Hard factor

$$\mathcal{H} = \mathcal{H}^{(0)}_{\rm LO} + \frac{\alpha_s}{2\pi}\mathcal{H}^{(1)}_{\rm NLO} + \cdots$$

should always be finite and free of divergence of any kind.

# Numerical implementation of the NLO result

Single inclusive hadron production up to NLO

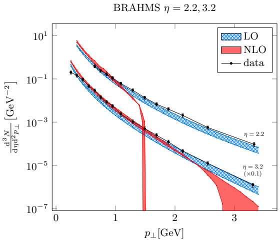
$$d\sigma = \int xf_a(x) \otimes D_a(z) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} + \frac{\alpha_s}{2\pi} \int xf_a(x) \otimes D_b(z) \otimes \mathcal{F}_{(N)ab}^{x_g} \otimes \mathcal{H}_{ab}^{(1)}.$$

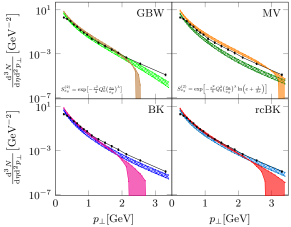
Consistent implementation should include all the NLO $\alpha_s$ corrections.

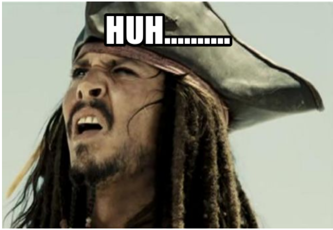
- NLO parton distributions. (MSTW or CTEQ)
- NLO fragmentation function. (DSS or others.)
- Use NLO hard factors. [Chirilli, BX and Yuan, 12]
- Use the one-loop approximation for the running coupling
- rcBK evolution equation for the dipole gluon distribution [Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]. Full NLO BK evolution not available.
- Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]

# Numerical implementation of the NLO result

Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]

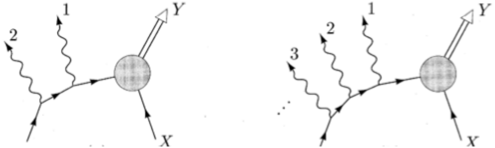






- Reduced factorization scale dependence!
- Catastrophe: Negative NLO cross-sections at high $p_T$.
- Fixed order calculation in field theories is not guaranteed to be positive.

# Large Logarithms



- NLO vs NLL      Naive $\alpha_s$ expansion sometimes is not sufficient!

| | LO | NLO | NNLO | $\cdots$ |
|---|---|---|---|---|
| LL | 1 | $\alpha_s L$ | $(\alpha_s L)^2$ | $\cdots$ |
| NLL | | $\alpha_s$ | $\alpha_s(\alpha_s L)$ | $\cdots$ |
| $\cdots$ | | | $\cdots$ | $\cdots$ |

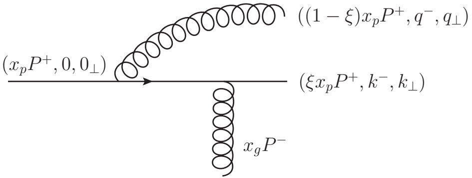
- Evolution → Resummation of large logs.
  LO evolution resums LL; NLO ⇒ NLL.

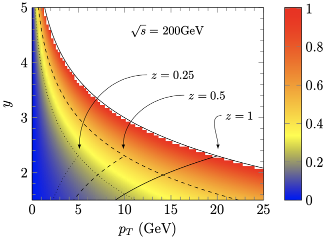
# Extending the applicability of CGC calculation

- Goal: find a solution within our current factorization (exactly resum $\alpha_s \ln 1/x_g$) to extend the applicability of CGC. Other scheme choices certainly is possible.
- A lot of logs arise in pQCD loop-calculations: DGLAP, small-$x$, threshold, Sudakov.
- Breakdown of $\alpha_s$ expansion occurs due to the appearance of logs in certain PS.
- Demonstrate onset of saturation and visualize smooth transition to dilute regime.
- Add'l consideration: numerically challenging due to limited computing resources.
- Towards a more complete framework. [Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 14; Kang, Vitev, Xing, 14; Ducloue, Lappi and Zhu, 16, 17; Iancu, Mueller, Triantafyllopoulos, 16; Liu, Ma, Chao, 19; Kang, Liu, 19; Kang, Liu, Liu, 20;]

# Gluon Radiation at the Threshold

Near threshold: radiated gluon has to be soft! $\tau = \frac{p_\perp e^y}{\sqrt{s}}$ density ($\tau = x_p \xi z \le 1$)





- Gluon momentum: $q^+ = (1-\xi)p_q^+ \to 0$
- Introduce an additional semi-hard scale $\Lambda^2$.

# Threshold Logarithms

Y. Shi, L. Wang, S.Y. Wei, Bo-Wen Xiao, ▸ Phys. Rev. Lett. **128**, 202302 (2022).

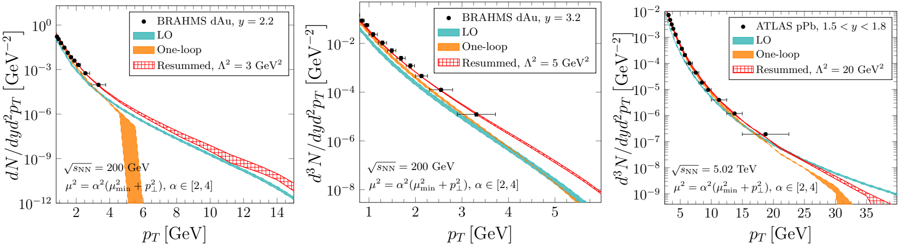
- Threshold enhancement for $\sigma$: $e^{-x} = 1 - x + \frac{x^2}{2} + \cdots$
- In the coordinate space, we can identify two types of logarithms

$$\text{single log: } \ln\frac{k_\perp^2}{\mu_r^2} \to \ln\frac{k_\perp^2}{\Lambda^2}, \quad \ln\frac{\mu^2}{\mu_r^2} \to \ln\frac{\mu^2}{\Lambda^2}; \quad \text{double log: } \ln^2\frac{k_\perp^2}{\mu_r^2} \to \ln^2\frac{k_\perp^2}{\Lambda^2},$$

with $\mu_r \equiv c_0/r_\perp$ with $c_0 = 2e^{-\gamma_E}$.

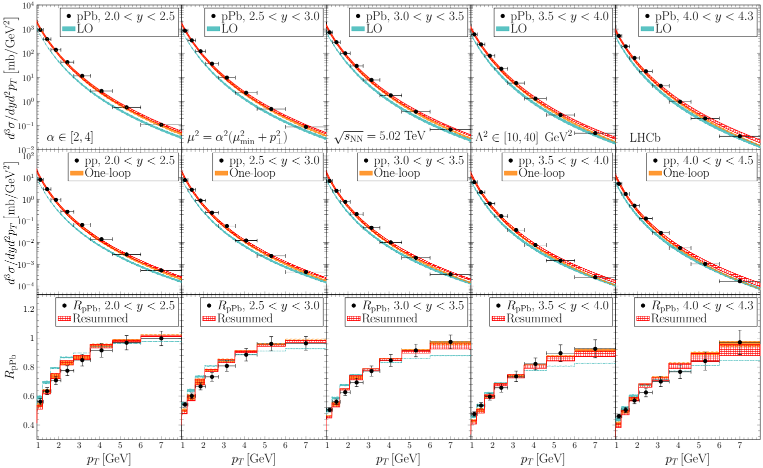
- Introduce a semi-hard auxiliary scale $\Lambda^2 \sim \mu_r^2 \gg \Lambda_{QCD}^2$. Identify dominant $r_\perp$!
- Dependence on $\mu^2$, $\Lambda^2$ cancel order by order. Choose “natural" values at fixed order.
- For running coupling, $\Lambda^2 = \Lambda_{QCD}^2 \left[\frac{(1-\xi)k_\perp^2}{\Lambda_{QCD}^2}\right]^{C_R/[C_R+\beta_1]}$. Akin to CSS & Catani *et al*.

# Numerical Results for *pA* spectra



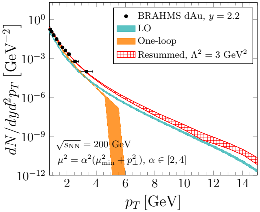
Nice agreement with data across many orders of magnitudes for different energies and $p_T$ ranges!

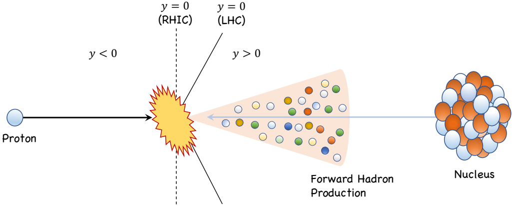
# Comparison with the new LHCb data

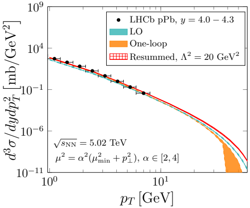


- LHCb data: 2108.13115
- ▸ Data Link ▸ DIS2021
- Threshold effect is not important at low $p_T$ for LHCb data. Saturation effects are still dominant.
- Predictions are improved from LO to NLO.
- Solve the negativity problem at both RHIC and LHC.

# Summary





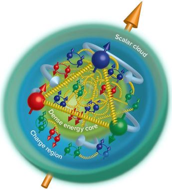


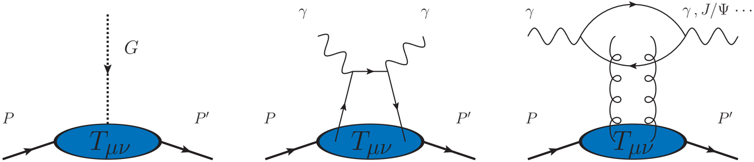
- Ten-Year Odyssey in NLO hadron productions in *pA* collisions in CGC.
- Towards the precision test of saturation physics (CGC) at RHIC and LHC.
- Next Goal:Global analysis for CGC combining data from pA and DIS.
- Exciting time of NLO CGC phenomenology with the upcoming EIC.

# Outlook: Energy-Momentum Tensor and Gravitational Form Factors

Determining the gluonic gravitational form factors of the proton B. Duran, *et al.*,

▸ Nature **615** (2023) no.7954, 813-816.





- Impossible to use graviton (spin 2) to probe proton mass distribution (GFF).
- [Ji, 97]; [Kharzeev, 96] Use two photons/gluons (spin 1) to study GPDs and GFFs, and probe quark and gluon parts, respectively.
- [Hagiwara, Tong, Xiao, in preparation] Theoretical understanding the gluon core.

# Understanding GFFs with the WW Method

Compute Momentum GFF from photon/gluon GTMD and WW method

$$\begin{aligned} A_g(t) &= \int_0^1 dx \int d^2k_\perp x\mathcal{G}_x(k_\perp, \Delta_\perp), \\ &= \frac{N_c}{\alpha_s} \int_0^1 dx \int \frac{d^2b_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} \vec{\nabla}^2_{r_\perp} S_x(b_\perp, r_\perp)\Big|_{r_\perp=0}, \\ &= \frac{N_c}{\alpha_s} \int_0^1 dx \int \frac{d^2b_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} Q_s^2(x, b_\perp). \end{aligned}$$

- New Relation between GFF and Dipole Scattering Amplitude.
- Gaussian Ansatz. $S_x(b_\perp, r_\perp) = \exp[-\frac{r_\perp^2}{4} Q_s^2(x, b_\perp)] \Rightarrow$ Probe $Q_s$.
- Gluon Radius in the proton $\sqrt{\langle b_\perp^2 \rangle_g} \approx 0.55\text{fm}$ and $\sqrt{\langle r^2 \rangle_g} \approx 0.61\text{fm}$. Gluon core!
- Assume uniform gluon in p and n. $\Rightarrow$ Neutron Radius in Nuclei at the EIC and EicC!

# Threshold resummation in the CGC formalism

Threshold logarithms: Sudakov soft gluon part and Collinear (plus-distribution) part.

- Soft single and double logs ($\ln k_\perp^2/\Lambda^2$, $\ln^2 k_\perp^2/\Lambda^2$ ) are resummed via Sudakov factor. Performing Fouier transformations

$$\int \frac{d^2 r_\perp}{(2\pi)^2} S(r_\perp) \ln\frac{\mu^2}{\mu_r^2} e^{-ik_\perp\cdot r_\perp} = -\int \frac{d^2 l_\perp}{\pi l_\perp^2}\left[F(k_\perp+l_\perp) - J_0(\frac{c_0}{\mu}l_\perp)F(k_\perp)\right]$$

$$= \quad -\frac{1}{\pi}\int \frac{d^2 l_\perp}{(l_\perp-k_\perp)^2}\left[F(l_\perp) - \frac{\Lambda^2}{\Lambda^2+(l_\perp-k_\perp)^2}F(k_\perp)\right] + F(k_\perp)\ln\frac{\mu^2}{\Lambda^2}.$$

- Two equivalent methods to resum the collinear part ($P_{ab}(\xi)\ln\Lambda^2/\mu^2$):
  1. Reverse DGLAP evolution; 2. RGE method (threshold limit $\xi \to 1$).
- Introduce forward threshold quark jet function $\Delta^q(\Lambda^2, \mu^2, \omega)$, which satisfies

$$\frac{\mathrm{d}\Delta^q(\omega)}{\mathrm{d}\ln\mu^2} = -\frac{\mathrm{d}\Delta^q(\omega)}{\mathrm{d}\ln\Lambda^2} = -\frac{\alpha_s C_F}{\pi}\left[\ln\omega + \frac{3}{4}\right]\Delta^q(\omega) + \frac{\alpha_s C_F}{\pi}\int_0^\omega \mathrm{d}\omega' \frac{\Delta^q(\omega) - \Delta^q(\omega')}{\omega - \omega'}$$

- Consistent with the threshold resummation in SCET[Becher, Neubert, 06]!