Susy breaking in a UV finite GUT

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BB, Del Piano, Sannino 2308.13311 , work in progress

Introduction

Renormalisable SO(10) GUTs need large representations

Neutrino mass: $16\,126_H\,16$

(alternatively $16\,16_H\,16_H\,16$ - nonrenormalisable)

In supersymmetry this means the appearance of Landau pole below Planck scale

Example: $3 \times 16 + 126_H + \overline{126}_H + 210_H + 10_H + 54_H$

Clark, Kuo, Nakagawa, '82

Aulakh, Mohapatra, '83

Aulakh, BB, Melfo, Senjanović, Vissani, hep-ph/0306242

Babu, BB, Saad, 1805.10631

For some time it the theory with $126_H + \overline{126}_H + 210_H + 10_H$ was the minimal renormalisable supersymmetric SO(10)

until it was found that neutrino masses are too small

Aulakh, hep-ph/0506291

BB, Melfo, Senjanović, Vissani, hep-ph/0511352

Aulakh, Garg, hep-ph/0512224

Bertolini, Schwetz, Malinsky, hep-ph/0605006

This can be saved easily in a nonminimal model, for example adding a new field (54_H)

Babu, BB, Saad, 1805.10631

We will consider this model as a prototype, toy model of a consistent asymptotically UV interacting (= SAFE) susy GUT

$$b_{1-loop} = -3 \times 8 + (3 \times 2 + 56 + 35 + 35 + 1 + 12) = 121$$

The Landau pole is obtained from the 1-loop RGE

$$\frac{dg_{10}}{d\log\mu} = \frac{b_1}{(4\pi)^2} g_{10}^3$$

$$\rightarrow \qquad g_{10}^2(\mu) = \frac{g_{10}^2(\mu_0)}{1 - \frac{g_{10}^2(\mu_0)}{8\pi^2} b_1 \log\left(\frac{\mu}{\mu_0}\right)}$$

The solution diverges when

$$g_{10}^2(\mu_{\text{Landau pole}}) = \infty \qquad \rightarrow$$

$$\mu_{\text{Landau pole}} = M_{GUT} \exp\left(\frac{8\pi^2}{b_1 g_{10}^2(M_{GUT})}\right) \approx 3 \times M_{GUT} \ll M_{Planck}$$

What happens there?

Higher loops could save the situation and make the theory UV safe

Litim,Sannino, 1406.2337

I.e. higher loops change the 1-loop infinite result, making all couplings finite, although nonzero (the theory is not UV free!)

This is the UV analogue of the Banks Zaks IR fixed point

Banks, Zaks, '82

But perturbation theory is not applicable here, 1-loop large, 2-loops even larger, etc. We cannot follow the RG flow, the theory becomes nonperturbative in the UV

All we can do in a supersymmetric theory is to look for possible fixed points and check if various non-perturbative constraints (positivity bounds) are satisfied.

The main one is on the a-central charge:

$$a_{UV} \geq a_{IR}$$

There is a prescription how to calculate this central charge in susy:

$$a = \sum_i a_1(R_i)$$

with

$$a_1(R) = 3(R-1)^3 - (R-1)$$

and R_i the R-charge of the superfield i

If all known constraints are satisfied, the fixed point is allowed.

A candidate for such UV fixed point has been found, assuming first generation of matter superfields has zero yukawas.

All fields except 16_1 have $R = 2/3$

$$R_{16_1} = \frac{125}{6}$$

Then

$$a_{UV} - a_{IR} = 3.74 \times 10^5 > 0$$

and the fixed point is a consistent candidate for a UV safe theory

Bajc, Sannino, 1610.09681

The massless first generation is clearly a problem, but we will not dwell on it further here. We assume this can be somehow corrected.

What we are interested here is in the supersymmetry breaking.

In fact the above analysis was done in a supersymmetric theory, but in the IR one needs the SM, which is not supersymmetric. So susy has to be broken somehow. The above picture did not consider it. The purpose here is to show how to break it without destroying the existence of the UV safe fixed point.

We will use here an implementation, employed for SU(5),

Bajc, Melfo, 0801.4349

of an earlier idea for dynamical supersymmetry breaking

Witten, '81

Dimopoulos, Dvali, Rattazzi, Giudice, hep-ph/9705307

The idea is the following: take two gauge non-singlets $\phi_{1,2}$:

$$W_{sb} = \mu\phi_1\phi_2 + \lambda\phi_1^2\phi_2 + \ldots$$

The potential is

$$V = \left|\frac{\partial W_{sb}}{\partial\phi_1}\right|^2 + \left|\frac{\partial W_{sb}}{\partial\phi_2}\right|^2$$

Extremisation of the potential

$$\begin{aligned}
\frac{\partial V}{\partial\phi_1} &= \frac{\partial^2 W_{sb}}{\partial\phi_1^2}\left(\frac{\partial W_{sb}}{\partial\phi_1}\right)^* + \frac{\partial^2 W_{sb}}{\partial\phi_1\partial\phi_2}\left(\frac{\partial W_{sb}}{\partial\phi_2}\right)^* = 0 \\
\frac{\partial V}{\partial\phi_2} &= \frac{\partial^2 W_{sb}}{\partial\phi_1\partial\phi_2}\left(\frac{\partial W_{sb}}{\partial\phi_1}\right)^* + \frac{\partial^2 W_{sb}}{\partial\phi_2^2}\left(\frac{\partial W_{sb}}{\partial\phi_2}\right)^* = 0
\end{aligned}$$

The potential is extremised in different ways

1) Supersymmetric minimum:

$$\frac{\partial V}{\partial \phi_1} = \frac{\partial^2 W_{sb}}{\partial \phi_1^2} \underbrace{\left(\frac{\partial W_{sb}}{\partial \phi_1}\right)^*}_{=0} + \frac{\partial^2 W_{sb}}{\partial \phi_1 \partial \phi_2} \underbrace{\left(\frac{\partial W_{sb}}{\partial \phi_2}\right)^*}_{=0} = 0$$

$$\frac{\partial V}{\partial \phi_2} = \frac{\partial^2 W_{sb}}{\partial \phi_1 \partial \phi_2} \underbrace{\left(\frac{\partial W_{sb}}{\partial \phi_1}\right)^*}_{=0} + \frac{\partial^2 W_{sb}}{\partial \phi_2^2} \underbrace{\left(\frac{\partial W_{sb}}{\partial \phi_2}\right)^*}_{=0} = 0$$

This means

$$\frac{\partial W_{sb}}{\partial \phi_1} = 0 \quad \to \quad \phi_2 = 0$$

$$\frac{\partial W_{sb}}{\partial \phi_2} = 0 \quad \to \quad \phi_1 = -\frac{\mu}{\lambda}$$

But it can be also a

2) Non-supersymmetric extremum:

$$\frac{\partial V}{\partial \phi_1} = \frac{\partial^2 W_{sb}}{\partial \phi_1^2} \underbrace{\left(\frac{\partial W_{sb}}{\partial \phi_1} \right)^*}_{=0} + \underbrace{\frac{\partial^2 W_{sb}}{\partial \phi_1 \partial \phi_2}}_{=0} \left(\frac{\partial W_{sb}}{\partial \phi_2} \right)^* = 0$$

$$\frac{\partial V}{\partial \phi_2} = \underbrace{\frac{\partial^2 W_{sb}}{\partial \phi_1 \partial \phi_2}}_{=0} \left(\frac{\partial W_{sb}}{\partial \phi_1} \right)^* + \underbrace{\frac{\partial^2 W_{sb}}{\partial \phi_2^2}}_{=0} \left(\frac{\partial W_{sb}}{\partial \phi_2} \right)^* = 0$$

This is obtained by

$$\frac{\partial W_{sb}}{\partial \phi_1} = \phi_2 \frac{\partial^2 W_{sb}}{\partial \phi_1 \partial \phi_2} = 0 \quad \rightarrow \quad \phi_1 = -\frac{\mu}{2\lambda}$$

$$\frac{\partial^2 W_{sb}}{\partial \phi_2^2} = 0 \quad \text{automatically} \qquad \rightarrow \quad \phi_2 \ \text{ undetermined}$$

Supersymmetry is broken because

$$F_2 = \frac{\partial W_{sb}}{\partial \phi_2} = -\frac{\mu^2}{4\lambda} \neq 0$$

At this point not clear yet if a minimum or a maximum

(ϕ_2 classical flat direction)

But susy is broken so radiative corrections will lift the potential

$$V = \frac{|F_2|^2}{Z_2(\phi_2)}$$

$Z_2(\phi_2)\ldots$ wave function renormalisation of ϕ_2

Imagine we add in our SO(10) model two 54 (2-index symmetric):

$$W_{sb} = \mu\, Tr\left(\phi_1\phi_2\right) + \lambda\, Tr\left(\phi_1^2\phi_2\right) + \ldots$$

$\phi_{1,2} = 54_{1,2}$

What is $Z_2(\phi_2)$?

It can be computed as an RGE:

$$\frac{d(\log Z_2)}{d\tau} = 20 g_{10}^2 - \frac{28}{5}\lambda^2$$

$$\tau = \frac{\log(\phi_2)}{8\pi^2}$$

We need now to add two more RGEs:

$$\begin{aligned} \frac{d g_{10}^2}{d\tau} &= 133 g_{10}^4 \\ \frac{d(\log \lambda^2)}{d\tau} &= -60 g_{10}^2 + 28\lambda^2 \end{aligned}$$

Closed system of RGEs for $g_{10}(\phi_2)$, $\lambda(\phi_2)$, $Z_2(\phi_2)$

The extremum of the potential

$$\frac{\partial V}{\partial \phi_2} = -\frac{|F_2|^2}{Z_2^2}\frac{\partial Z_2}{\partial \phi_2}$$

vanishes when

$$\frac{\partial Z_2}{\partial \phi_2} = 0 \to \lambda^2 = \frac{20}{\frac{28}{5}} g_{10}^2$$

The second derivative of the potential at the extremum

$$\frac{\partial^2 V}{\partial \phi_2^2} = -\frac{|F_2|^2}{Z_2^2}\frac{\partial^2 Z_2}{\partial \phi_2^2}$$

is positive if $\frac{\partial^2 Z_2}{\partial \phi_2^2} < 0$

However this is not the case for our situation:

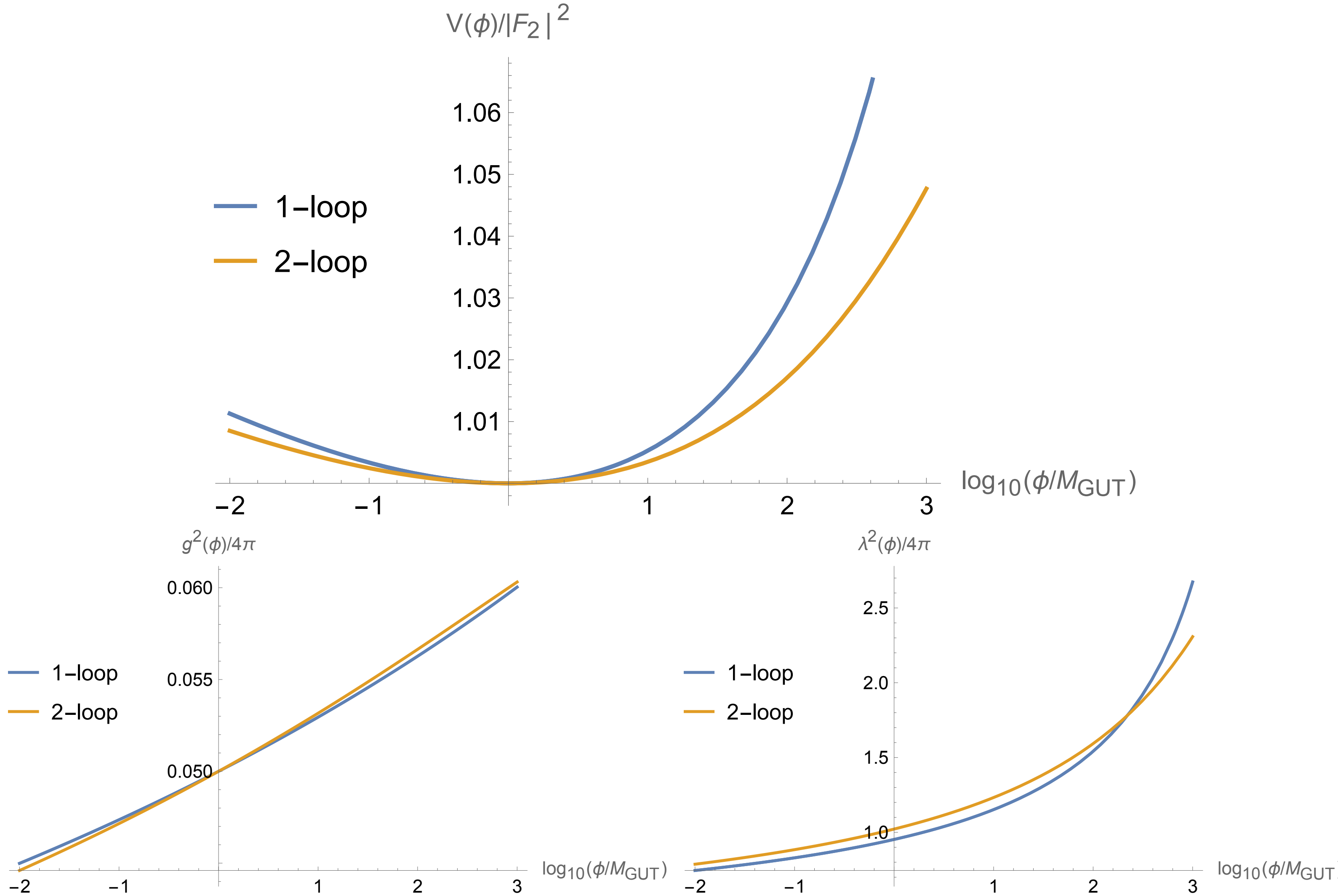
$$\frac{\partial^2 Z_2}{\partial \phi_2^2} > 0$$

and the potential has a maximum.

Can we use something else instead of 54? The two fields $\phi_{1,2}$ needs to have both quadratic and cubic gauge invariants. Another possibility is for example 210. It turns out that it is even worse, i.e. bigger the representation more positive the second derivative of Z_2 in the extremum

Our SO(10) model cannot be used to break supersymmetry this way.

However in SU(5) two 24 can break supersymmetry



SU(5) superpotential for susy breaking

$$W = \mu Tr\left(\phi_1\phi_2\right) + \lambda Tr\left(\phi_1^2\phi_2\right) + \ldots$$

$\phi_{1,2} = 24_{1,2}$

develops a susy breaking minimum of the potential

Bajc, Melfo, 0801.4349

What we need is thus to spontaneously break SO(10)→SU(5) first

The superpotential with 210_H, 126_H, $\overline{126}_H$, 10_H, 54_H has a minimum in the SU(5) direction

BB, Melfo, Senjanović, Vissani, hep-ph/0402122

Fukuyama, Ilakovac, Kikuchi, Meljanac, Okada, hep-ph/0405300

One problem is left:

$$54_2 = 24_2 + 15_2 + \overline{15}_2$$

and this extra $15_2 + \overline{15}_2$ pair stays light and makes the theory blow up before reaching even the SU(5) scale. To avoid it, we add a 45 representation and the term

$$\Delta W = Tr\left(45\, 54_1 54_2\right)$$

Being 45 two index antisymmetric, the 45 vev gives mass $M_{SO(10)}$ to both $15 + \overline{15}$ pairs leaving the two $24_{1,2}$ light

Coupling this new 45 with the other Higgs representations (210_H, 126_H, $\overline{126}_H$, 10_H, 54_H)

$$\delta W_{45} = 45^2\, 210_H + 45\, 210_H^2 + 45\, 126_H\, \overline{126}_H + 45^2\, 54_H$$

does not spoil the SU(5) minimum.

$$\langle 45 \rangle \neq 0$$

Fukuyama, Ilakovac, Kikuchi, Meljanac, Okada, hep-ph/0405300

Last comment:

The superpotential

$$W_{sb} = \mu \, Tr \left(54_1 54_2 \right) + \lambda \, Tr \left(54_1^2 54_2 \right)$$

gives some light states (color octet and weak triplet chiral supermultiplet) - mainly components of the linear 54_2

This does not destroy unification but increase the scales

$M_{susy} \sim 10^6$ GeV

$M_{SU(5)} \sim 10^{19}$ GeV

To get more "usual" scales one needs two other heavy $54_{3,4}$:

$$W_{csb} = Tr\left(M 54_3 54_4 + 54_3\left(\mu_1 54_1 + \lambda_1 54_1^2\right) + 54_4\left(\mu_2 54_2 + \lambda_2 54_1 54_2\right)\right)$$

Bajc, Melfo, 0801.4349

By integrating out $54_{3,4}$ one gets the previous W_{sb} plus terms proportional to $1/M$ giving

$M_{susy} \sim 10^{4-5}$ GeV

$M_{SU(5)} \sim 10^{16}$ GeV

We have still to check if the new model, i.e. with

4 extra 54 dimensional SO(10) representations

1 extra 45 dimensional SO(10) representations

on top of the original

$$3 \times 16 + 210_H + 126_H + \overline{126}_H + 10_H + 54_H$$

still has an allowed UV fixed point

It is easy to determine the R-charges of the new fields from the new superpotential terms yielding

$$R(54_{1,2,3,4}) = R(45) = \frac{2}{3}$$

This by itself would not change the value of the a-central charge.

However these new superfields also contribute to the NSVZ relation (R-charge non-anomalous - triangle $RGG = 0$):

$$\sum_i T_i(R_i - 1) = 0$$

the new fields change the value of

$$R(16_1) = \frac{181}{6}$$

with then

$$a_{UV} - a_{IR} = 1.19 \times 10^6 > 0$$

still compatible with all positivity constraints.

Thus this SO(10) model is able to break supersymmetry and have a UV fixed point

Same result but different R_{16_1} and $\Delta a > 0$ in the model with only two 54

Massless representation

Can 16_1 get enough mass from loops? The answer is no because

$$\delta K = \frac{1}{M_{SU(5)}^4} \frac{\alpha}{4\pi} 16_1\, 16_1\, 10 \, \langle 16_1^*\, 16_1^*\, 54_2^* \rangle$$

$$\langle 54_2 \rangle \sim \theta\theta F \quad , \quad \langle 16_1 \rangle \to M_{SU(5)} \quad , \quad m_{susy} \sim \frac{\alpha}{4\pi} \frac{F}{M_{SU(5)}}$$

$$\to y_1 \sim \frac{m_{susy}}{M_{SU(5)}} \sim 10^{-11}$$

far too small

Conclusions

- supersymmetric renormalisable SO(10) have a Landau pole much before the Planck scale
- consistent candidates (susy SO(10)) for a UV fixed point are known from the literature (modulo first generation mass)
- supersymmetry breaking cannot be obtained directly in SO(10) but the model must first be broken into SU(5)
- although details slightly change, the UV fixed point of such model is still consistent