Modular Flavor Symmetry in Heterotic E_6 GUT

Hajime Otsuka (Kyushu University)

References :

- Modular flavor and CP symmetries in Calabi-Yau compactifications : 2010.10782, 2107.00487, 2402.13563
- Phenomenology : 2112.00493, 2204.12325, 2207.14014

with K. Ishiguro (KEK), T. Kai, T. Kobayashi (Hokkaido U.), H. Okada,
S. Nishimura, M. Tanimoto (Niigata U.), K. Yamamoto (Hiroshima Inst. Tech.)

2024/4/8 @ Workshop on Grand Unified Theories: Phenomenology and Cosmology, HIAS, UCAS

String theory

⇩ Compactifications

Modular symmetry

⇩ Strong constraints on EFTs

- Flavor symmetry
- CP
- Phenomenology

See Gui-Jun Ding and Arsenii Titov talk

Outline

1. Why modular symmetries ?

2. Modular flavor symmetry in Heterotic E_6 GUT

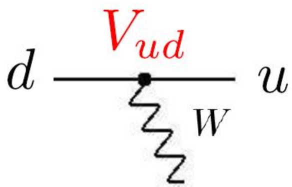
3. Hierarchical structure of physical Yukawa couplings

4. Conclusion

Flavor puzzle

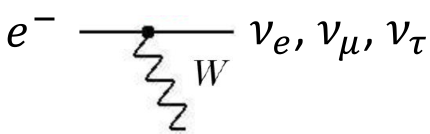
- Origin of flavor and CP : important issue in the SM

PDG ('20)



$$V_{\rm CKM} = \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

NuFIT 5.0 (2020)



$$|U|^{\rm w/o\ SK\text{-}atm}_{3\sigma} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.156 \\ 0.233 \to 0.507 & 0.461 \to 0.694 & 0.631 \to 0.778 \\ 0.261 \to 0.526 & 0.471 \to 0.701 & 0.611 \to 0.761 \end{pmatrix}$$

- Hierarchical structure of quarks/lepton masses

Mass [MeV]	2	5	100	1270	4180	172000
	u	*d*	*s*	*c*	*b*	*t*

→ Non-trivial structure of Yukawa couplings

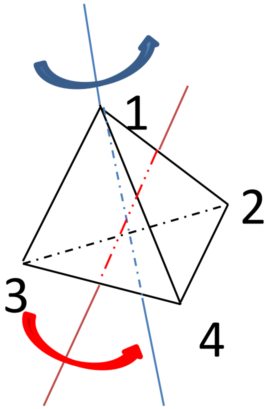
“Traditional” flavor symmetry

- Field transformations:

$$\phi_i \xrightarrow{g} \rho_i(g)\phi_i \qquad g \in G_{\rm flavor}$$

- Non-Abelian discrete symmetries
  well explain the flavor structure in the lepton sector

E.g., $\Gamma_3 \simeq A_4$: Tetrahedral sym.



- Flavor symmetries should be broken.
  → Many free parameters in symmetry breaking sector

$$m_{ij}(\tau) = m_{ij}^0 + f_{ij}(\tau)$$

Vacum alignment determined by flavon fields τ

"Modular" flavor symmetry

F. Feruglio, 1706.08749

- Modular transformations:

- Moduli fields
  (symmetry breaking fields)

$$\tau \to R(\tau) = \frac{p\tau + q}{s\tau + t}$$

$p, q, s, t \in \mathbb{Z}$
$pt - qs = 1$

- Matter fields

$$\phi_i \to (s\tau + t)^{-k_i} \rho_i(g) \phi_i$$

Automorphy factor

Weight $k_i \in \mathbb{Z}$

- Yukawa couplings

$$Y(\tau) \to (s\tau + t)^{k_Y} \rho_Y(g) Y(\tau)$$

- Non-Abelian discrete symmetries ⊂ modular symmetry
- <u>Small parameters</u>

“Modular” flavor symmetry

F. Feruglio, 1706.08749

$SL(2,\mathbb{Z})$ modular sym. = geometrical sym. of T^2 torus

$T^{\mathbf{2}} = \mathbb{C}/\Lambda$

$\neq$

τ τ'

“Modular” flavor symmetry

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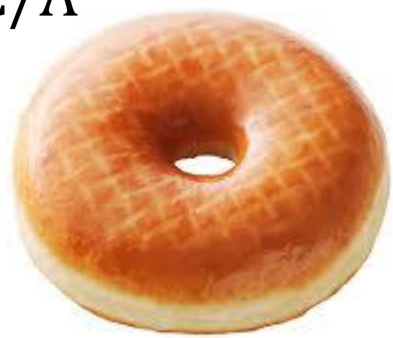

$=$

τ

$$\tau' = \frac{p\tau + q}{s\tau + t}$$

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F. Feruglio, 1706.08749

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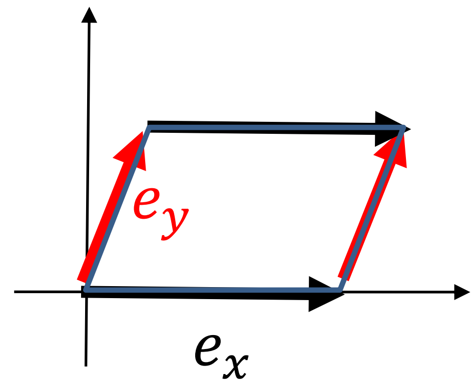
=



$$\tau = \frac{e_y}{e_x}$$

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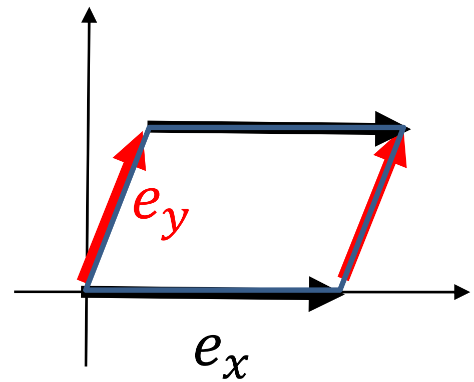


$\tau = \dfrac{e_y}{e_x}$

- Lattice vectors are related under $SL(2,\mathbb{Z})$ modular transformation:

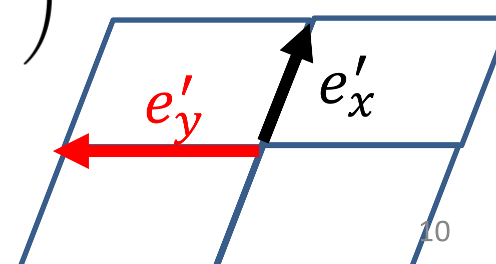
$$\begin{pmatrix} e'_y \\ e'_x \end{pmatrix} = \begin{pmatrix} p & q \\ s & t \end{pmatrix} \begin{pmatrix} e_y \\ e_x \end{pmatrix}$$

$p, q, s, t \in \mathbb{Z}$ satisfying $pt - qs = 1$

$$\tau \to \tau' = \frac{p\tau + q}{s\tau + t}$$

Two generators : S and T

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$



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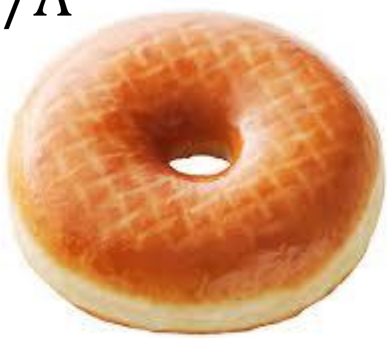

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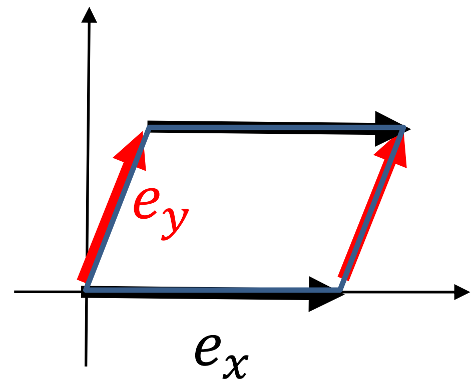

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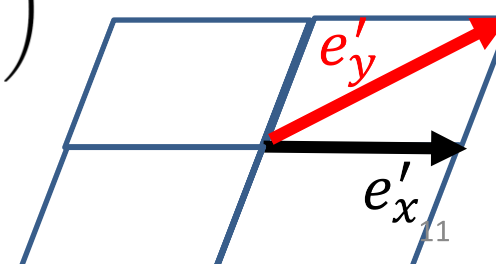
$$\tau \to \tau' = \frac{p\tau + q}{s\tau + t}$$

Two generators : S and T

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$



Finite subgroups of modular group

Modular group

$$\Gamma \simeq \{S, T \mid S^2 = 1, (ST)^3 = 1\}$$

Finite subgroups

$$\Gamma_N = \Gamma/\Gamma(N)$$

$$\Gamma_N \simeq \{S, T \mid S^2 = 1, (ST)^3 = 1, T^N = 1\}$$

Non-abelian discrete groups :

$$\Gamma_2 \simeq S_3, \quad \Gamma_3 \simeq A_4, \quad \Gamma_4 \simeq S_4, \quad \Gamma_5 \simeq A_5,$$

Flavor symmetries of quarks/leptons

F. Feruglio, 1706.08749

Finite subgroups of modular group

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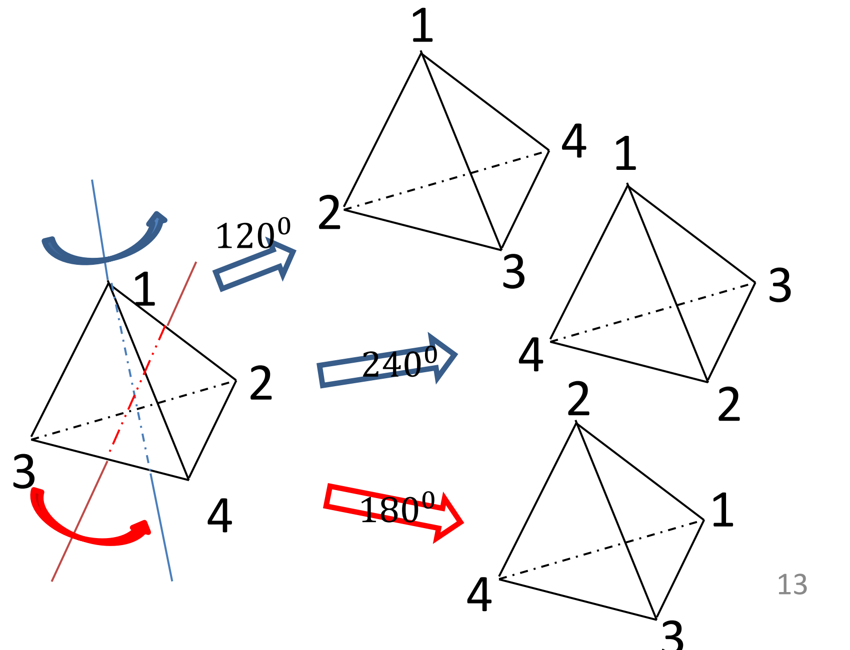
E.g., $\Gamma_3 \simeq A_4$: Tetrahedral sym.

Generators : S and T

$$S^2 = T^3 = (ST)^3 = 1$$



Modular invariant 4D supersymmetric EFT

$$K = -\ln\big(i(\bar{\tau} - \tau)\big) + \sum_i \frac{|\phi_i|^2}{\big(i(\bar{\tau} - \tau)\big)^{k_i}}$$

$$W = \sum_n Y_{i_1 \dots i_n}(\tau)\, \phi_{i_1} \cdots \phi_{i_n}$$

ϕ_i : chiral superfields with modular weight k_i

$Y_{i_1 \dots i_n}(\tau)$: couplings

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Modular transformations:

$\gamma \in \Gamma_N \subset SL(2,Z)$

$$\tau \to R(\tau) = \frac{p\tau+q}{s\tau+t}$$

$$\phi_i \to (s\tau+t)^{-k_i}\rho_i(\gamma)\phi_i$$

$$Y(\tau) \to (s\tau+t)^{k_Y}\rho_Y(\gamma)Y(\tau) = Y(R(\tau))$$

Representation matrix of Γ_N

Modular invariant 4D supersymmetric EFT

$$K = -\ln\big(i(\bar{\tau}-\tau)\big) + \sum_i \frac{|\phi_i|^2}{\big(i(\bar{\tau}-\tau)\big)^{k_i}}$$

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Representation matrix of Γ_N

- Modular invariant W requires $k_Y = \sum_i k_i$ and $\rho_Y \bigotimes_i \rho_i \ni 1$
- **Couplings are described by the modular function**
- Flavor structure/CP violation are determined by the value of τ

- Couplings are described by the modular function (modular forms)

$$Y_i(R(\tau)) = (s\tau + t)^{k_Y} \rho_Y(\gamma)_{ij} Y_j(\tau)$$

$$\tau \to R(\tau) = \frac{p\tau + q}{s\tau + t}$$

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$$Y_i(R(\tau)) = (s\tau + t)^{k_Y} \rho_Y(\gamma)_{ij} Y_j(\tau)$$

$$\tau \to R(\tau) = \frac{p\tau + q}{s\tau + t}$$

- Modular form $Y(\tau)$ is a holomorphic function @ $\mathrm{Im}\tau > 0$ and $\mathrm{Im}\tau \to \infty$

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad \tau \to \tau + 1 \qquad Y(\tau + 1) = Y(\tau)$$

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \tau \to -1/\tau \qquad Y\left(-\frac{1}{\tau}\right) = (-\tau)^{k_Y} Y(\tau)$$

$$I = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \tau \to \tau \qquad Y\left(\frac{-\tau}{-1}\right) = (-1)^{k_Y} Y(\tau)$$

k_Y : even

- Finite number of modular forms depending on k_Y

- Couplings are described by the modular function (modular forms)

$$Y_i(R(\tau)) = (s\tau + t)^{k_Y} \rho_Y(\gamma)_{ij} Y_j(\tau)$$

$$\tau \to R(\tau) = \frac{p\tau + q}{s\tau + t}$$

Ex. A_4 triplet of modular function with $k = 2$

η : Dedekind eta-function

$$\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right) \\
Y_2(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right), \\
Y_3(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right),
\end{aligned}$$

F. Feruglio, 1706.08749

- Couplings are described by the modular function (modular forms)

$$Y_i(R(\tau)) = (s\tau + t)^{k_Y} \rho_Y(\gamma)_{ij} Y_j(\tau)$$

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Ex. $\underline{A_4 \text{ triplet of modular function with } k = 2}$

η : Dedekind eta-function

$$\begin{aligned} Y_1(\tau) &= 1 + 12q + 36q^2 + 12q^3 + \cdots, \\ Y_2(\tau) &= -6q^{1/3}(1 + 7q + 8q^2 + \cdots), \\ Y_3(\tau) &= -18q^{2/3}(1 + 2q + 5q^2 + \cdots). \end{aligned}$$

$$q = e^{2\pi i\tau}$$

$\mathrm{Im}\tau \gg 1$

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$$q = e^{2\pi i\tau}$$

$\mathrm{Im}\tau \gg 1$

- Modulus-dependent Yukawa couplings would lead to

(1) Mass hierarchy of charged lepton masses

(2) Differences of neutrino masses squared and mixing angles

Outline

1. Why modular symmetries ?

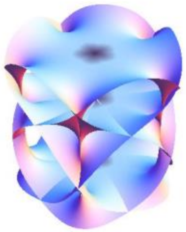
2. Modular flavor symmetry in Heterotic E_6 GUT

3. Hierarchical structure of physical Yukawa couplings

4. Conclusion

4D SUSY E_6 GUT from Heterotic string on 6D Calabi-Yau

Candelas-Horowitz-Strominger-Witten ('85)



- 4D gauge symmetry :

$$E_8 \times E_8^{(\mathrm{hidden})} \rightarrow E_6 \times SU(3) \times E_8^{(\mathrm{hidden})}$$

- Matters (E_6 : 27 or $\overline{27}$) ≈ Moduli

$$27^i \approx \text{Kahler Moduli } t^i$$

(2-cycle volume)

$(i = 1,2, \dots , h^{1,1})$

- Yukawa couplings ($27^{\mathbf{3}}$)

$$W = F_{ijk} 27^i 27^j 27^k$$

$F_{ijk} = \partial_{t^i}\partial_{t^j}\partial_{t^k}F$ （$F(t)$: prepotential）

Symplectic Modular Symmetric in CY moduli space

A. Strominger ('90),
P. Candelas, X. de la Ossa ('91)

- Symplectic transformations :

Moduli: $t^i \to \tilde{t}^i \simeq \frac{\partial \tilde{X}^i}{\partial X^j} t^j$

Matters: $27^i \to \widetilde{27}^i \simeq \frac{\partial \tilde{X}^i}{\partial X^j} 27^j$

Yukawa couplings: $F_{ijk} \to \tilde{F}_{ijk} = \frac{\partial X^l}{\partial \tilde{X}^i} \frac{\partial X^m}{\partial \tilde{X}^j} \frac{\partial X^n}{\partial \tilde{X}^k} F_{lmn}$

(X^0, X^i):
projective coordinates
with the gauge $X^0 = 1$
$(\, i = 1,2, \dots, h^{1,1})$

$F_{ijk} = \partial_{X^i} \partial_{X^j} \partial_{X^k} F$
$(F(t)$: prepotential)

Yukawa couplings : tensor rep. under the modular symmetry ($t \to \gamma t$)

Symplectic modular symmetry of 6D CY
⊃ Flavor symmetry

$G^{(27)}_{\text{flavor}} \subset Sp(2h^{1,1} + 2, \mathbb{Z})$

Ishiguro-Kobayashi-Otsuka, 2107.00487

- Yukawa couplings

Classical level

Prepotential : $F = \frac{\kappa_{ijk}}{6} t^i t^j t^k$

→ Constant Yukawa coupling : $\partial_i \partial_j \partial_k F = \kappa_{ijk}$

$$W = \kappa_{ijk} 27^i 27^j \, 27^k$$

Quantum level

Prepotential : $F = \frac{\kappa_{ijk}}{6} t^i t^j t^k + O(e^{2\pi i t})$ Instanton effects

→ Yukawa coupling : $\partial_i \partial_j \partial_k F = \kappa_{jik} + O(e^{2\pi i t})$

- Instanton effects will lead to non-trivial flavor structure

K. Ishiguro, T. Kobayashi, S. Nishimura, H.O. , arXiv:2402.13563

Instanton-corrected Yukawa couplings on 6D CY

$$y_{ijk} = \kappa_{ijk} + \sum_{d_1, d_2, \ldots, d_n = 0}^{\infty} \frac{(d_i d_j d_k) n_{d_1, d_2, \ldots, d_m}}{1 - \prod_{l=1}^{m} q_l^{d_l}} \prod_{l=1}^{m} q_l^{d_l}$$

$$q_l \equiv e^{2\pi i t_l}$$

Gromov-Witten invariants

We discuss two examples, where $SL(2,\mathbb{Z})$ modular symmetry emerges in asymptotic regions of the CY moduli space

Ex.1 Instanton-corrected Yukawa couplings

$P^{1,1,1,6,9}[18]$ with two Kahler moduli（$h^{1,1}=2$）

- Prepotential :

$$F = -\frac{1}{6}(9t_1^3 + 9t_1^2 t_2 + 3t_1 t_2^2) + \cdots$$

- Yukawa couplings :

Candelas-Font-Katz-Morrison, 9403187

$$y_{ijk} = \kappa_{ijk} + \sum_{d_1,d_2=0}^{\infty} c_{ijk}(d_1,d_2) n_{d_1,d_2} \frac{q_1^{d_1} q_2^{d_2}}{1 - q_1^{d_1} q_2^{d_2}}$$

$d_1 \setminus d_2$	0	1	2	3
0		3	-6	27
1	540	-1080	2700	-17280
2	540	143370	-574560	5051970
3	540	204071184	74810520	-913383000

Table 1: Instanton numbers up to $d_1, d_2 \leq 3$.

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Ex.1 Instanton-corrected Yukawa couplings

$P^{1,1,1,6,9}[18]$ with two Kahler moduli ($h^{1,1}=2$)

- Prepotential :

$$F=-\frac{1}{6}\left(\frac{9}{4}t^3+3ts^2\right)+\cdots \qquad t=t_1, \qquad s=\frac{3}{2}t_1+t_2$$

- Yukawa couplings :

Candelas-Font-Katz-Morrison, 9403187

If we take $q_s \to 0$ ($\mathrm{Im}s \to \infty$), d_2=0 is relevant

$d_1 \setminus d_2$	0	1	2	3
0		3	-6	27
1	540	-1080	2700	-17280
2	540	143370	-574560	5051970
3	540	204071184	74810520	-913383000

Table 1: Instanton numbers up to $d_1, d_2 \le 3$.

$$y_{ttt}=\frac{9}{4}E_4(t) \text{ (weight 4)}$$
$$y_{tss}=1$$
$$y_{tts}=y_{sss}=0$$

$$E_4(t)=1+240\sum_{k=0}^{\infty}\frac{k^3q^k}{1-q^k}$$

Ex.1 Instanton-corrected Yukawa couplings

$P^{1,1,1,6,9}[18]$ with two Kahler moduli（ $h^{1,1}=2$ ）

- Moduli Kahler potential :

$$K=-\ln[i(\frac{3}{8}(t-\bar{t})^3+\frac{1}{2}(t-\bar{t})(s-\bar{s})^2)]$$

- Matter Kahler metric :

Dixon-Kaplunovsky-Louis（'90)

$$K_{t\bar{t}}^{(27)}\sim e^{-\frac{K}{3}}K_{t\bar{t}}\simeq\frac{1}{(t-\bar{t})^{5/3}}$$

$$K_{s\bar{s}}^{(27)}\sim e^{-\frac{K}{3}}K_{s\bar{s}}\simeq(t-\bar{t})^{1/3}$$

- Matter modular weight :

$-5/3$ for $A_t^{(27)}$

$1/3$ for $A_s^{(27)}$

The action is invariant under $SL(2,\mathbb{Z})_t$: $t\to\frac{at+b}{ct+d}$

Ex.2 Instanton-corrected Yukawa couplings

Two Kahler moduli ($h^{1,1}=2$) : $\begin{matrix}\mathbb{CP}^2 \\ \mathbb{CP}^2\end{matrix}\begin{bmatrix}3\\3\end{bmatrix}$

- Prepotential :

$$F = -\frac{1}{6}(9t_1^2 t_2 + 9t_1 t_2^2) + \cdots$$

- Yukawa couplings :

$$y_{ijk} = \kappa_{ijk} + \sum_{d_1,d_2=0}^{\infty} c_{ijk}(d_1,d_2) n_{d_1,d_2} \frac{q_1^{d_1} q_2^{d_2}}{1 - q_1^{d_1} q_2^{d_2}}$$

$d_1 \setminus d_2$	0	1	2	3	4	5	6
0		189	189	162	189	189	162
1	189	8262	142884	1492290	11375073	69962130	
2	189	142884	13108392	516953097	12289326723		
3	162	1492290	516953097	55962304650			
4	189	11375073	12289326723				
5	189	69962130					
6	162						

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- Prepotential :

$$F = -\frac{1}{6}(9t_1^2 t_2 + 9 t_1 t_2^2) + \cdots$$

- Yukawa couplings :

$$y_{ijk} = \kappa_{ijk} + \sum_{d_1,d_2=0}^{\infty} c_{ijk}(d_1,d_2) n_{d_1,d_2} \frac{q_1^{d_1} q_2^{d_2}}{1 - q_1^{d_1} q_2^{d_2}}$$

$d_1 \setminus d_2$	0	1	2	3	4	5	6
0		189	189	162	189	189	162
1	189	8262	142884	1492290	11375073	69962130	
2	189	142884	13108392	516953097	12289326723		
3	162	1492290	516953097	55962304650			
4	189	11375073	12289326723				
5	189	69962130					
6	162						

Ex.2 Instanton-corrected Yukawa couplings

Two Kahler moduli ($h^{1,1} = 2$) : $\begin{matrix} \mathbb{CP}^2 \\ \mathbb{CP}^2 \end{matrix} \left[\begin{matrix} 3 \\ 3 \end{matrix} \right]$

- Prepotential :

$$F = -\frac{1}{6}\left(-\frac{9}{4}t^3 + 9ts^2\right) + \cdots$$

$t = t_1, \quad s = \frac{1}{2}t_1 + t_2$

- Yukawa couplings :

If we take $q_s \to 0$ $(\mathrm{Im}s \to \infty)$, d_2=0 is relevant

$d_1 \setminus d_2$	0	1	2	3	4
0		189	189	162	189
1	189	8262	142884	1492290	11375073
2	189	142884	13108392	516953097	12289326723
3	162	1492290	516953097	55962304650	
4	189	11375073	12289326723		
5	189	69962130			
6	162				

Table 3: Instanton numbers up to bidegree $d_1 + d_2 \leq 6$. Note that there symmetry $n_{d_1,d_2} = n_{d_2,d_1}$.

$$y_{ttt} = \frac{63}{80}E_4(t) - \frac{243}{80}E_4(3t) \quad (\Gamma_0(N) \text{ modular form of weight 4})$$

$$y_{tss} = 9$$

$t \to \frac{pt+q}{st+t} \qquad s \equiv 0 \pmod 3$

Note that $E_4(nt)$ is a $\Gamma_0(N)$ modular form if $n|N$

Ex.2 Instanton-corrected Yukawa couplings

Two Kahler moduli ($h^{1,1} = 2$) : $\begin{matrix} \mathbb{CP}^2 \\ \mathbb{CP}^2 \end{matrix} \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

- Moduli Kahler potential :

$$K = -\ln\left(i\left(\frac{1}{2}(t-\bar{t})(s-\bar{s})^2\right)\right)$$

- Matter Kahler metric :

Dixon-Kaplunovsky—Louis ('90)

$$K^{(27)}_{t\bar{t}} \sim e^{-\frac{K}{3}} K_{t\bar{t}} \simeq \frac{1}{(t-\bar{t})^{5/3}}$$

$$K^{(27)}_{s\bar{s}} \sim e^{-\frac{K}{3}} K_{s\bar{s}} \simeq (t-\bar{t})^{1/3}$$

- Matter modular weight :

$-5/3$ for $A^{(27)}_t$

$1/3$ for $A^{(27)}_s$

The action is invariant under $\Gamma_0(3)_t$: $t \to \frac{pt+q}{st+t}$ $\quad s \equiv 0 \pmod 3$

Outline

1. Why modular symmetries ?
2. Modular flavor symmetry in Heterotic E_6 GUT
3. Hierarchical structure of physical Yukawa couplings
4. Conclusion

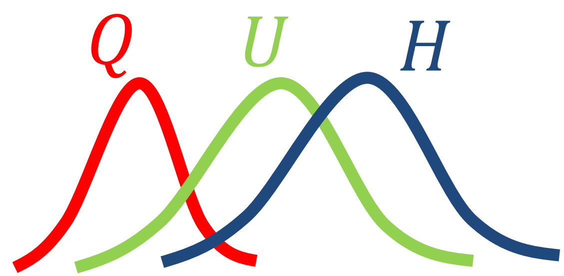
Non-trivial structure of 4D Yukawa couplings

- Mechanisms

1. Charge assignments of quarks/leptons
   under continuous or discrete flavor symmetries *U(1) : Froggatt-Nielsen ('79),…*

2. Localization of matter wavefunctions in extra-dimensional spaces
   Arkani-Hamed and Schmaltz ('99), Kaplan-Tait ('00) ,…

$$Y = \int$$



Extra-dimensional space

They can be engineered in the UV completion of the SM,
such as string theory

Yukawa couplings in 4D N=1 SUSY

Kinetic term of matters A^i : $K = K_{i\bar{j}} A^i \bar{A}^{\bar{j}}$

Holomorphic Yukawa couplings : $W = y_{ijk} A^i A^j A^k$

Physical Yukawa couplings (after canonically normalizing fields A^i)

$$Y_{abc} = e^{\frac{K}{2}} L_a^i L_b^j L_c^k y_{ijk}$$

L_a^i: diagonalizing the kinetic terms $K_{i\bar{j}}$

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Hierarchical structure of Yukawa couplings :

1. Flavor structure of holomorphic Yukawa couplings y_{ijk}
(controlled by modular symmetries (modular forms))

$y_{ttt} \propto E_4(t)$ (weight 4)
$y_{tss} =$ consant

K. Ishiguro, T. Kobayashi, HO, arXiv: 2103.10240

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1. Flavor structure of holomorphic Yukawa couplings y_{ijk}
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$y_{ttt} \propto E_4(t)$ (weight 4)
$y_{tss} =$ consant

2. Kinetic mixing of matter field Kahler metric $K_{i\bar{J}}$
(positive and negative modular weights → large kinetic mixing)

$$K_{t\bar{t}}^{(27)} \sim e^{-\frac{K}{3}} K_{t\bar{t}} \simeq \frac{1}{(t-\bar{t})^{5/3}}$$

$$K_{s\bar{s}}^{(27)} \sim e^{-\frac{K}{3}} K_{s\bar{s}} \simeq (t-\bar{t})^{1/3}$$

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Conclusion

String theory

⇩

Compactifications
- Geometric symmtries
- $SL(2,\mathbb{Z})$ for toroidal backgrounds
- $Sp(2g,\mathbb{Z})$ for multi-moduli

Modular symmetry

⇩

Strong constraints on the EFT

- Flavor symmetry $\subset$ Modular symmetry

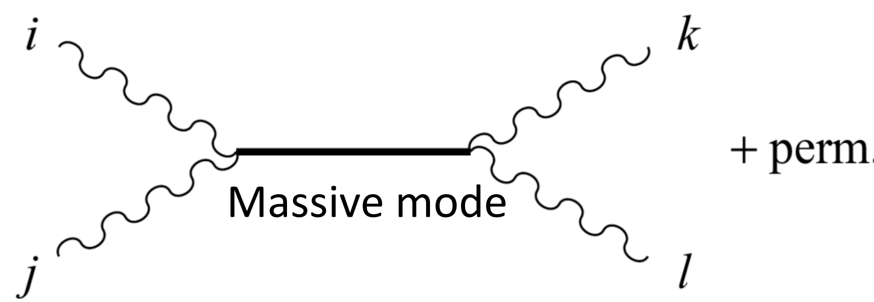
- Holormorphic Yukawa couplings $\sim$ modular forms
- Large kinetic mixings induced by modular weights

Discussion (1/2) Higher-order couplings in SUSY E_6 GUT

Bershadsky-Cecotti-Ooguri-Vafa ('93)

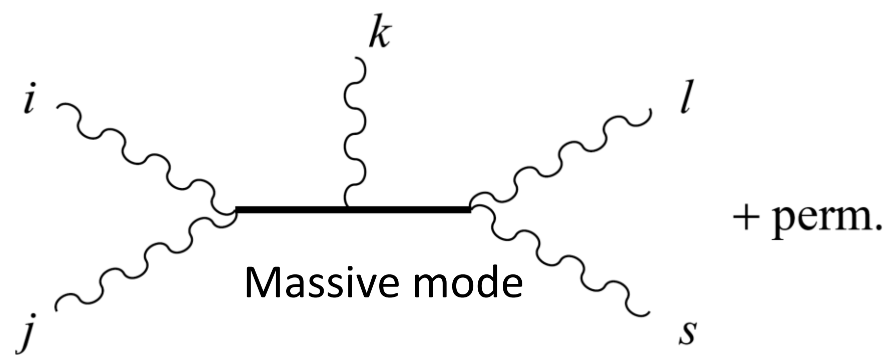
- <u>Dimension-5</u>

$$\frac{F_{ijkl}}{\Lambda} 27_i 27_j 27_k 27_l$$



- <u>Dimension-6</u>

$$\frac{F_{ijkls}}{\Lambda^2} 27_i 27_j 27_k 27_l 27_s$$



From hep-th/9309140

- *n*-point couplings : $F_{ij\dots n} = \partial_i \partial_j \dots \partial_n F$ F : prepotential

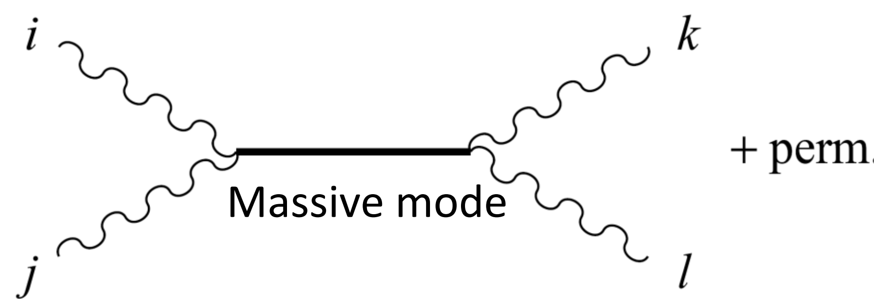
— Non-trivial representations under $Sp(2h+2, \mathbb{Z})$

Discussion (2/2) Higher-order couplings in SUSY E_6 GUT

Bershadsky-Cecotti-Ooguri-Vafa ('93)

- Dimension-5

$$\frac{F_{ijkl}}{\Lambda} 27_i 27_j 27_k 27_l$$



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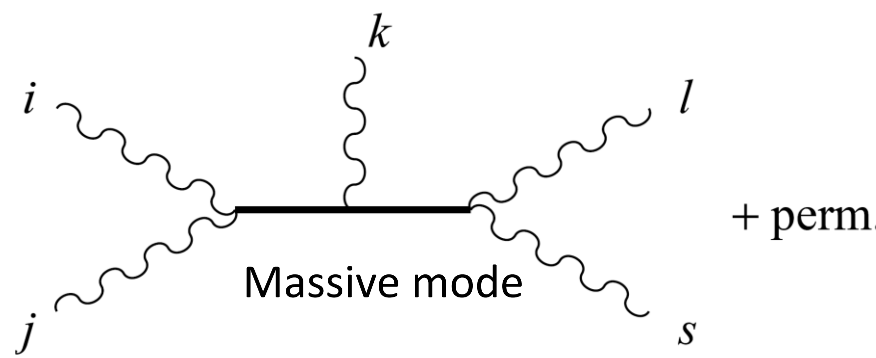

From hep-th/9309140

- Prepotential : $F = F_{\text{cubic polynomial}} + F_{\text{instanton}}$

  E.g., $F_{ijkl} = \partial_i \partial_j \partial_k \partial_l \, F_{\text{instanton}}$ are exponentially suppressed

  -> no dangerous flavor/CP-violating processes under $\text{Im}t^i \gg 1$

Thank you!

Appendix

4D CP and modular symmetry

- 4D CP ⊂ 10D proper Lorentz transformation

Consider simultaneous transformations of

Strominger-Witten ('85)
Dine-Leigh-MacIntire ('92)
Choi-Kaplan-Nelson ('92)

— 4D parity

— 6D orientation reversing : $z_i \to -\bar{z}_i \ \ (i = 1,2,3)$

(z_i: local coordinates of 6D space)

(Volume form : $dV \to -dV$)

$dV \propto dz_1 \wedge dz_2 \wedge dz_3 \wedge d\bar{z}_1 \wedge d\bar{z}_2 \wedge d\bar{z}_3$

10D Majorana-Weyl spinor under $SO(1,9) = SO(1,3) \times SO(6)$:

$$16 = (2, 4_+) \oplus (2', \bar{4}_-)$$

$2,\ 2'$: left- and right-handed spinors of $SL(2,\mathbb{C})$

$4_+, \bar{4}_-$: $+$ and $-$ chirality spinors of $SU(4)$

4D CP and modular symmetry

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$(2, 4_+) \to (2', \bar{4}_-)$ E.g., in heterotic string, E_6 : $\overline{27} \to \overline{27}^*$

- <u>Such transformations correspond to 4D CP</u>

4D CP and modular symmetry

- 4D CP $\subset$ 10D proper Lorentz transformation

Consider simultaneous transformations of

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$\tau^i \to (\tau^i)^*$

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CP as an outer automorphism of symplectic modular group

- Under CP and symplectic modular transf. $\gamma \in Sp(2h^{2,1}+2,\mathbb{Z})$

$$\Pi \xrightarrow{\mathrm{CP}} \mathcal{CP}\bar{\Pi} \xrightarrow{\gamma} \mathcal{CP}\cdot\gamma\bar{\Pi} \xrightarrow{\mathrm{CP}^{-1}} \mathcal{CP}\cdot\gamma\cdot\mathcal{CP}^{-1}\Pi$$

$$\gamma = \begin{pmatrix} d & c \\ b & a \end{pmatrix} \to \mathcal{Q}(\gamma) \equiv \mathcal{CP}\cdot\gamma\cdot\mathcal{CP}^{-1} = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$$

- <u>Outer automorphism</u> $\mathcal{Q}$:

(i) $\mathcal{Q}(\gamma_1)\mathcal{Q}(\gamma_2) = \mathcal{CP}\cdot\gamma_1\cdot\mathcal{CP}^{-1}\mathcal{CP}\cdot\gamma_2\cdot\mathcal{CP}^{-1} = \mathcal{Q}(\gamma_1\gamma_2)$

(ii) No group element $\hat{\gamma} \in Sp(2h^{2,1}+2,\mathbb{Z})$ exists s.t. $\mathcal{Q}(\gamma) = \hat{\gamma}^{-1}\gamma\hat{\gamma}$

Enlarging the symplectic modular group

- Symplectic modular group $Sp(2h^{2,1}+2,\mathbb{Z})$ is enlarged to

$$Sp(2h^{2,1}+2,\mathbb{Z}) \rtimes \mathcal{CP}$$

- Natural extension of T^2 toroidal case (Modular group : $SL(2,\mathbb{Z})$)

$$GL(2,\mathbb{Z}) \simeq SL(2,\mathbb{Z}) \rtimes \mathcal{CP}$$

$$SL(2,\mathbb{Z}) \simeq Sp(2,\mathbb{Z})$$

H. P. Nilles, M. Ratz, A. Trautner, P. K. S. Vaudrevange ('18),
P .P. Novichkov, J. T. Penedo, S. T. Petcov, A. V. Titov ('19)