武汉高校“突破边界，前方高能”研究生论坛
— 类光超曲面上的玻色场论

刘文斌 D202280089 导师：龙江

华中科技大学 物理学院 基础理论物理研究所

2023 年 12 月 24 日

Related works

This presentation is based on the following works:

[1] W.-B. Liu and J. Long, "Symmetry group at future null infinity: Scalar theory," *Phys. Rev. D* **107** (2023), no. 12, 126002, 2210.00516.

[2] W.-B. Liu and J. Long, "Symmetry group at future null infinity II: Vector theory," *JHEP* **07** (2023) 152, 2304.08347.

[3] W.-B. Liu and J. Long, "Symmetry group at future null infinity III: Gravitational theory," *JHEP* **10** (2023) 117, 2307.01068.

[4] A. Li, W.-B. Liu, J. Long, and R.-Z. Yu, "Quantum flux operators for Carrollian diffeomorphism in general dimensions," *JHEP* **11** (2023) 140, 2309.16572.

[5] W.-B. Liu, J. Long, and X.-H. Zhou, "Quantum flux operators in higher spin theories," 2311.11361.

[6] W.-B. Liu and J. Long, "Holographic dictionary from bulk reduction," under review.

Related methods versus ours

There are mainly three ways to study the physics of $\mathscr{I}^+$:

- Asymptotic symmetry analysis leads to BMS group, extending the Poincaré group. This method imposes specified gauge conditions and fall-offs, and then analyzes the transformations preserving them [Bondi, 1962].
- Amplitude approach leads to equivalence among BMS asymptotic symmetries, soft theorems, and memory effects [Strominger,2017; Weinberg,1965; Zeldovich,1974].
- Carrollian approach. Carrollian physics is ultra-relativistic limit ($c \to 0$) of the Lorentz one. BMS group is conformal Carrollian group of level 2 [Lévy-Leblond,1965; Duval,2014].

Our methods: We study theories at future null infinity $\mathscr{I}^+$ using bulk reduction. By imposing the appropriate fall-offs, we reduce bulk theory to $\mathscr{I}^+$, quantize the theory, and realize boundary symmetry through commutators among quantum flux operators.

Overview of our methods

Using the above methods, we get a new central charge and regularize it. Moreover, we include a new operator to close the algebra. This operator concerns (EM) duality transformations. We summarize various correspondences between the bulk and boundary theories in the right table.

Bulk	Boundary
AFS	Carrollian manifold
Leading radiative modes	Fundamental fields $F_{A(s)}(u,\Omega)$
Other modes	Descendant fields
EOMs	Constraints $C(F, F^{(k)}) = 0$
Symplectic form $\mathbf{\Omega}^{\mathcal{H}}(\delta\mathtt{f};\delta\mathtt{f})$	Symplectic form $\mathbf{\Omega}(\delta F;\delta F)$
Leaky fluxes	Hamiltonians
# of Propagating DOFs	Proportion of central charges
Scattering amplitudes	Correlation functions

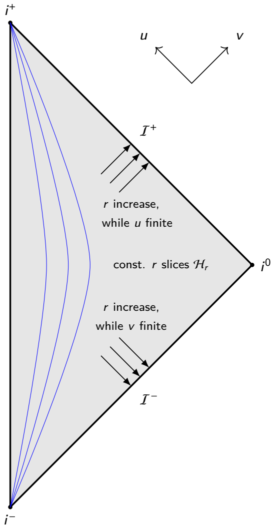
Scri plus

- $\mathcal{I}^+ = \mathbb{R} \times S^2$ has a degenerate metric

$$ds^2_{\mathcal{I}^+} = \gamma = d\theta^2 + \sin^2\theta d\phi^2,$$

and a vector $\chi = \partial_u$ to generate time direction.

- In amplitude approach, one sees $\mathcal{I}^-$ as the place of in states, and $\mathcal{I}^+$ as the place of out states, which leads to 2d celestial CFT.
- In Carrollian approach, $\mathcal{I}^+$ is seen as a boundary along which there is a Carrollian time evolution. This leads to 3d Carrollian CFT [Donnay, 2022].



Symmetry and fields

- Carrollian diffeomorphism is generated by

$$\xi = f(u,\Omega)\partial_u + Y^A(\Omega)\partial_A \ \in \ \mathrm{Diff}(S^2) \ltimes C^\infty(\mathcal{I}^+). \tag{1}$$

- We first consider Carrollian diffeomorphism as a symmetry of $\mathcal{I}^+$

$$f = a^\mu n_\mu,\ \ Y^A = \omega^{\mu\nu} Y^A_{\mu\nu} \quad \Longrightarrow \quad \dot{f} = \tfrac{1}{2}\nabla\cdot Y,\ \ Y^A = \omega^{\mu\nu} Y^A_{\mu\nu}$$

Poincaré (1900s) $\Longrightarrow$ original BMS (1960s)

$$\Longrightarrow \quad \dot{f} = \tfrac{1}{2}\nabla\cdot Y,\ \ \dot{Y} = 0 \quad \Longrightarrow \quad f(u,\Omega),\ \ \dot{Y} = 0$$

extended BMS (2010s) $\Longrightarrow$ Carrollian diffeomorphism (2020s)

- We expand the fields $f_{A(s)}(t,\boldsymbol{x}) = r^{s-1}F_{A(s)}(u,\Omega) + \mathcal{O}(r^{s-2})$, e.g.,

$$s=0: \quad \Phi(t,\boldsymbol{x}) = r^{-1}\Sigma(u,\Omega) + \mathcal{O}(r^{-2}), \tag{2a}$$

$$s=1: \quad a_A(t,\boldsymbol{x}) = A_A(u,\Omega) + \mathcal{O}(r^{-1}), \tag{2b}$$

$$s=2: \quad g_{AB}(t,\boldsymbol{x}) = r^2\gamma_{AB} + rC_{AB}(u,\Omega) + \mathcal{O}(r^0). \tag{2c}$$

Symplectic form

The boundary symplectic forms read (with $32\pi G = 1$ for gravitational theory)

$$\mathbf{\Omega}_s(\delta F;\delta F) = \int du d\Omega \delta F^{A(s)} \wedge \delta \dot{F}_{A(s)}, \tag{3}$$

More explicitly, we have

$$s=0: \quad \mathbf{\Omega}(\delta\Sigma;\delta\Sigma) = \int du d\Omega \delta\Sigma \wedge \delta\dot{\Sigma}, \tag{4a}$$

$$s=1: \quad \mathbf{\Omega}(\delta A;\delta A) = \int du d\Omega \delta A_A \wedge \delta \dot{A}^A, \tag{4b}$$

$$s=2: \quad \mathbf{\Omega}(\delta C;\delta C) = \int du d\Omega \delta C_{AB} \wedge \delta \dot{C}^{AB}. \tag{4c}$$

Fundamental commutators and correlators

- From the boundary symplectic form, we could work out the commutators

$$[\Sigma(u,\Omega),\dot{\Sigma}(u',\Omega')] = \frac{i}{2}\delta(u-u')\delta(\Omega-\Omega'), \tag{5a}$$

$$[A_A(u,\Omega),\dot{A}_B(u',\Omega')] = \frac{i}{2}\gamma_{AB}\delta(u-u')\delta(\Omega-\Omega'), \tag{5b}$$

$$[C_{AB}(u,\Omega),\dot{C}_{CD}(u',\Omega')] = \frac{i}{2}X_{ABCD}\delta(u-u')\delta(\Omega-\Omega'). \tag{5c}$$

- These fundamental commutators are of a unified form

$$[F_{A(s)}(u,\Omega),\dot{F}_{B(s)}(u',\Omega')] = \frac{i}{2}X_{A(s)B(s)}\delta(u-u')\delta(\Omega-\Omega'). \tag{6}$$

- We also have fundamental correlator

$$\langle 0|\, F_{A(s)}(u,\Omega)\dot{F}_{B(s)}(u',\Omega')\,|0\rangle = X_{A(s)B(s)}\frac{\delta(\Omega-\Omega')}{4\pi(u-u'-i\epsilon)}. \tag{7}$$

Hamiltonians

- We could use

$$i_{\xi}\Omega(\delta F, \delta F) = \delta H_{\xi} \tag{8}$$

to find the corresponding Hamiltonians

$$\begin{aligned}\mathcal{T}_f^s &= \int du d\Omega \ f(u,\Omega)\dot{F}_{A(s)}\dot{F}^{A(s)}, \\ \mathcal{M}_Y^s &= \frac{1}{2}\int du d\Omega \ Y_A(\Omega)(\dot{F}_{B(s)}\nabla_C F_{D(s)} - F_{B(s)}\nabla_C \dot{F}_{D(s)})P^{AB(s)CD(s)}.\end{aligned} \tag{9}$$

- One could also find these from Fourier transforming Poincaré flux densities which come from stress tensor $T_{\mu\nu}$.
- Normal order will be imposed to obtain quantized operators.

Fluxes as generators

- All the physical operators have the same form

$$\int dud\Omega : \dot{F}^{A(s)} \oint F_{A(s)} : \ . \tag{10}$$

- For supertranslations, we obtain

$$\oint_f F_{A(s)} = \delta_f F_{A(s)} \equiv i[\mathcal{T}_f^s, F_{A(s)}] = f \dot{F}_{A(s)}. \tag{11}$$

- For superrotations, we have $\Delta_Y = \oint_Y - \oint_{f=\frac{1}{2}u\nabla\cdot Y}$, acting on fields as

$$\Delta_Y^s F_{A(s)} = \Delta_{A(s)}(Y;F) \equiv i[\mathcal{M}_Y^s, F_{A(s)}]. \tag{12}$$

- More explicitly, they act on fields as

$$\begin{aligned}\Delta_{A(s)}(Y;F;u,\Omega) =& Y^D \nabla^C F^{B(s)} \rho_{DB(s)CA(s)} + \frac{1}{2}\nabla^C Y^D F^{B(s)} P_{DB(s)CA(s)} \\ =& \frac{1}{2}\nabla_B Y^B F_{A(s)} + Y^B \nabla_B F_{A(s)} - s\nabla_{[A} Y_{B]} F^B_{A(s-1)}.\end{aligned} \tag{13}$$

Commutation relations and central charges

- The commutators are

$$[\mathcal{T}^s_{f_1},\mathcal{T}^s_{f_2}] = \mathrm{C}^s_T(f_1,f_2) + i\mathcal{T}^s_{f_1\dot{f}_2-f_2\dot{f}_1}, \tag{14a}$$
$$[\mathcal{T}^s_f,\mathcal{M}^s_Y] = -i\mathcal{T}^s_{Y^A\nabla_A f}, \tag{14b}$$
$$[\mathcal{M}^s_Y,\mathcal{M}^s_Z] = i\mathcal{M}^s_{[Y,Z]} + is\mathcal{O}^s_{o(Y,Z)}. \tag{14c}$$

- Central charge for $s=0$ is

$$\mathrm{C}^{(s=0)}_T(f_1,f_2) = -\frac{i\delta^{(2)}(0)}{48\pi}\int du d\Omega\left(f_1\dddot{f}_2 - f_2\dddot{f}_1\right). \tag{15}$$

- It is interesting to find $\mathrm{C}^{(s=1)}_T = \mathrm{C}^{(s=2)}_T = 2\mathrm{C}^{(s=0)}_T$.
- (14a) is a Virasoro algebra.

Duality transformations I

- We need a new operator concerning duality transformation

$$\mathcal{O}_g^s = \int du d\Omega : \dot{F}^{A(s)} \eth_g F_{A(s)} : \quad . \tag{16}$$

- This is helicity flux, evaluating the difference between the numbers of particles with right-hand and left-hand helicity.
- Duality transformations rotate the field strength tensors and their duals.
- The infinitesimal transformations at $\mathcal{I}^+$ are

$$s = 1: \quad \delta_\epsilon A_A = \epsilon \widetilde{A}_A, \qquad \delta_\epsilon \widetilde{A}_A = -\epsilon A_A, \tag{17a}$$

$$s = 2: \quad \delta_\epsilon C_{AB} = \epsilon \widetilde{C}_{AB}, \qquad \delta_\epsilon \widetilde{C}_{AB} = -\epsilon C_{AB}. \tag{17b}$$

- $\widetilde{A}_A$ and $\widetilde{C}_{AB}$ are dual vector field and shear tensor

$$\widetilde{A}_A = \epsilon_{BA} A^B, \qquad \widetilde{C}_{AB} = \epsilon_{CA} C_B^C = Q_{ABCD} C^{CD}. \tag{18}$$

Duality transformations II

- Original EM duality transformations correspond to $g = \mathrm{const.}$, belonging to $SO(2)$ [Oliver, 1892; Dirac, 1931].
- We first compute the helicity flux $\mathcal{O}_g^{(s=2)}$ for gravity, which is a potential observable about gravitational radiation.
- $g \in C^\infty(S^2)$ lifts $so(2)$ to a infinite-dimensional algebra, also generalizes global transformation to be local.
- It is first time to find the appearance of internal symmetry in the algebra regarding spacetime symmetry.
- At last, we need to complete the algebra

$$[\mathcal{T}_f, \mathcal{O}_g] = 0, \tag{19a}$$
$$[\mathcal{M}_Y^s, \mathcal{O}_g] = i\mathcal{O}_{Y^A \nabla_A g}, \tag{19b}$$
$$[\mathcal{O}_{g_1}, \mathcal{O}_{g_2}] = 0. \tag{19c}$$

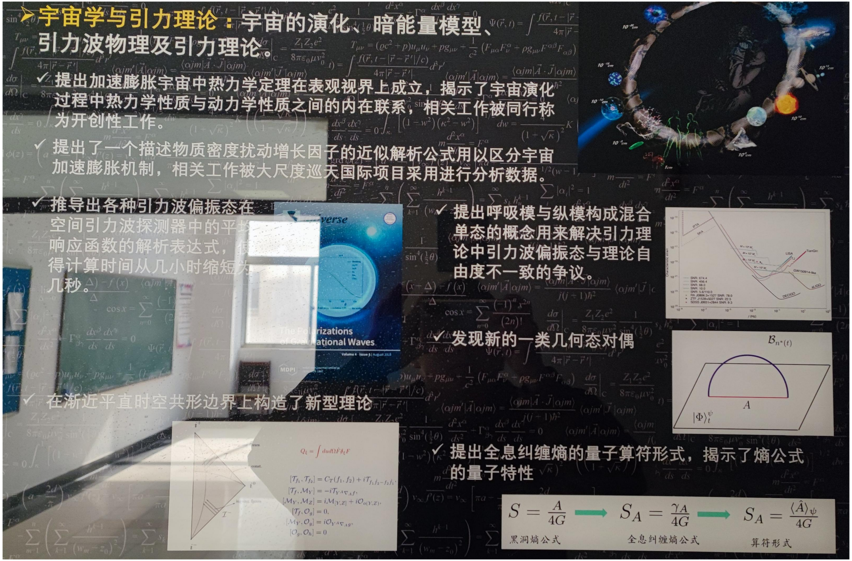
Future interests

We have mainly analyzed the symmetry of bosonic field theories. There are various topics derserving further study:

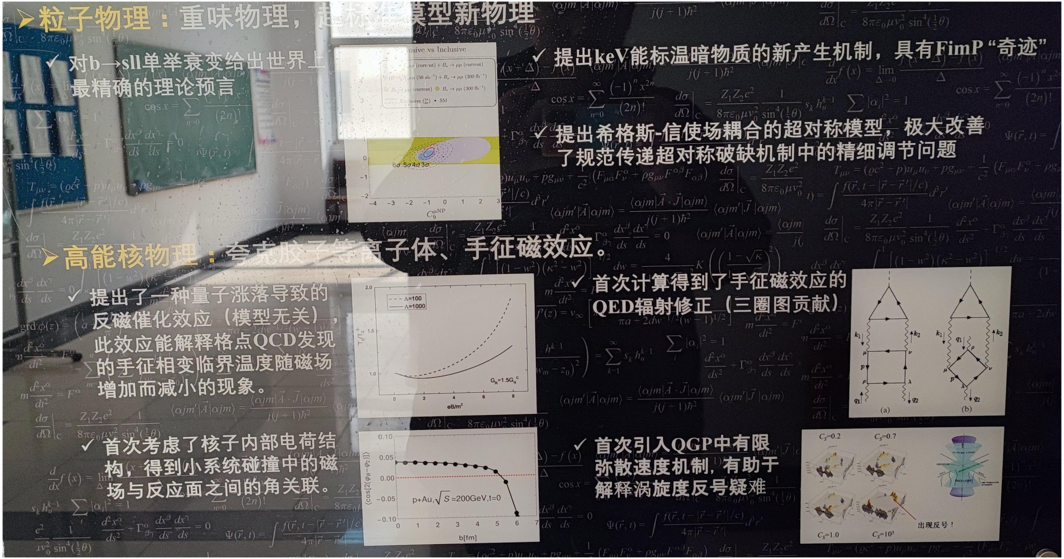
- Reduce Dirac field, and even higher spin fermionic field to boundary. This will finally lead to the consideration of supersymmetry.
- Generalize to general dimensions and general null hypersurfaces. One of main difficulties lies at the analogy of helicity in higher dimensions.
- Find why duality operator $\mathcal{O}_g$ appears. Moreover, study the algebra including gauge transformation and its EM dual counterpart.
- Calculate scattering amplitudes which are observables in QFT.
- Construct flat holography: find how to intrisically define boundary field theory from bulk reduction, construct a detailed dictionary, and contribute to bulk quantum gravity.

About our institute

About our institute



About our institute



Thanks for your attention!