華中師範大學
CENTRAL CHINA NORMAL UNIVERSITY

NRQCD因子化定理对重夸克偶素高阶修正的研究

报告人：张余栋
合作者：冯锋 桑文龙 杨德山 周明震 白晓卫
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第一届武汉高校“突破边界，前方高能”研究生论坛

- Two-Loop QCD Corrections to C-even Bottomonium Exclusive Decays to Double J/ψ

Y.D.Zhang*, X.W.Bai, F.Feng, W.L.Sang and M.Z.Zhou

arXiv: 2310.07453

- Z-boson radiative decays to an S-wave quarkonium at NNLO and NLL accuracy

W.L.Sang, D.S.Yang and **Y.D.Zhang***

Phys. Rev. D 108, no.1, 014021 (2023)

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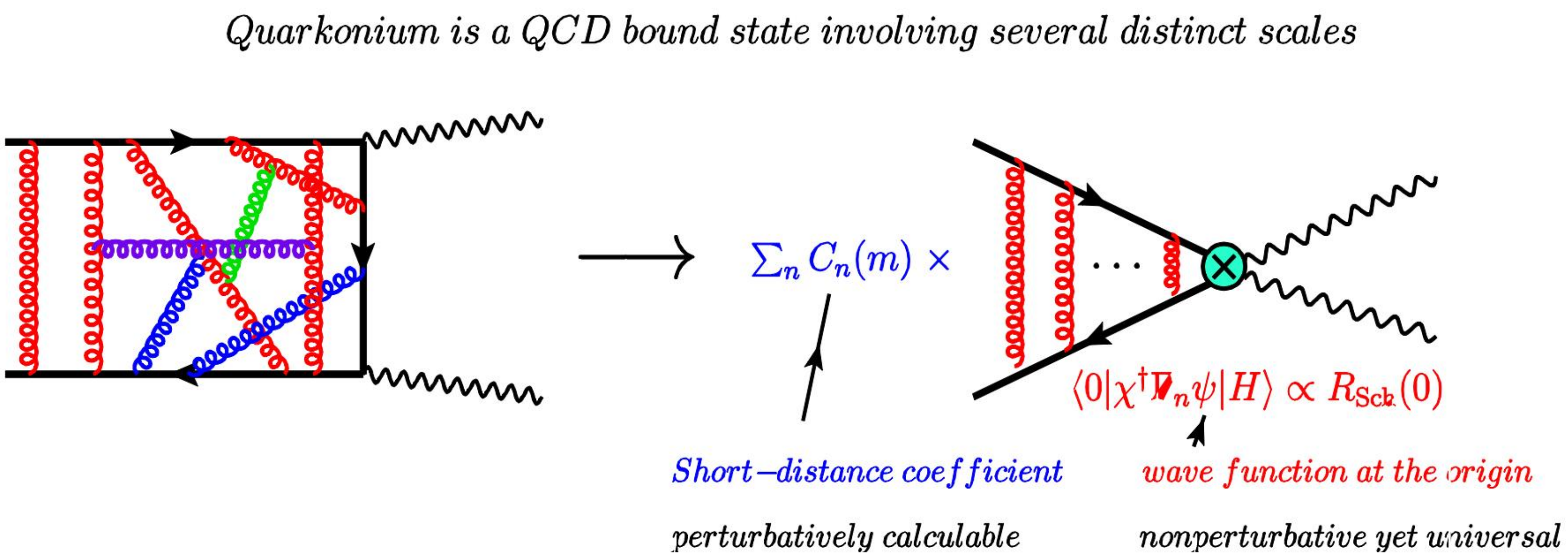
引言

- 类似于QED中的正负电子偶素，重夸克偶素是由一对正反重夸克构成的非相对论性的束缚态。

- 重夸克偶素是研究QCD的理想探针，有助于我们更好地认识QCD的非微扰效应。
- NRQCD是描述重夸克偶素的QCD有效场论。



NRQCD因子化定理

G. T. Bodwin, E. Braaten and G. P. Lepage Phys. Rev. D 51, 1125-1171 (1995)

NRQCD是专门为描述重夸克偶素而发展的QCD有效场论。

NRQCD可以系统地分离短程(微扰效应)和长程(非微扰效应)。

NRQCD微扰部分可以关于速度和耦合常数双重展开。

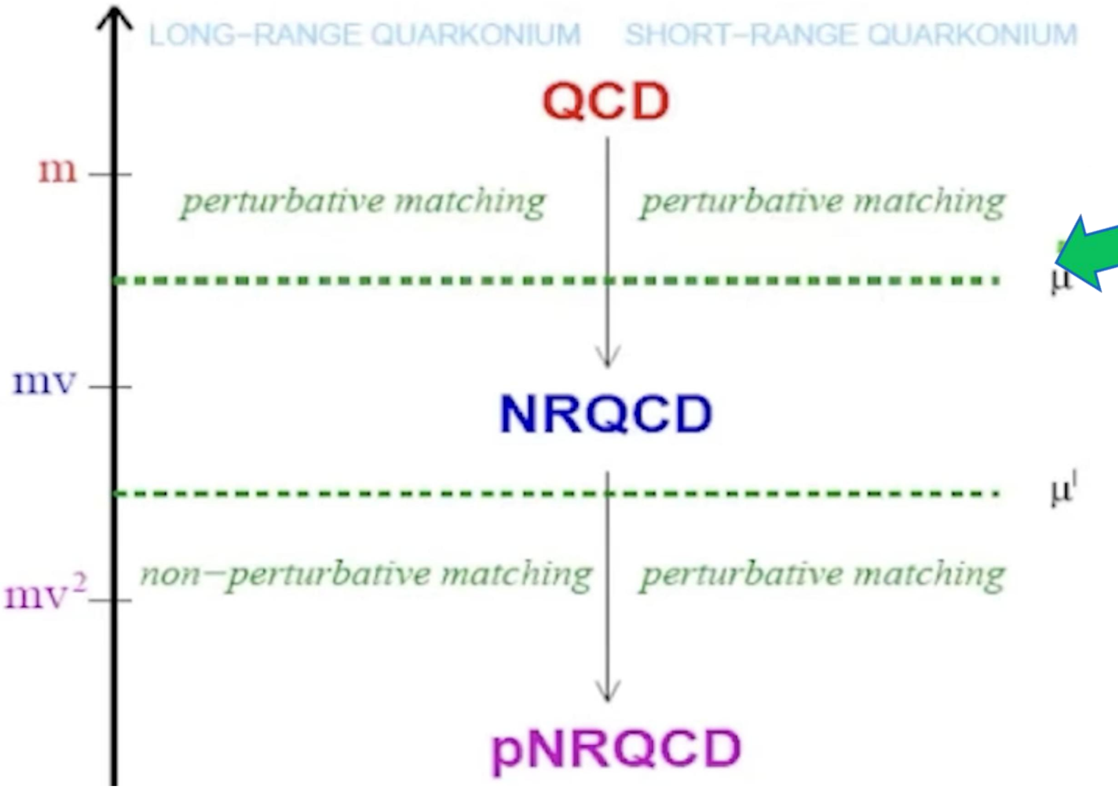
NRQCD被认为是模型无关理论，它的输入参量很少。

底夸克偶素特征速度：$v^2 \approx 0.1$

粲夸克偶素特征速度：$v^2 \approx 0.3$

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
M	1.5 GeV	4.7 GeV	180 GeV
Mv	0.9 GeV	1.5 GeV	16 GeV
Mv^2	0.5 GeV	0.5 GeV	1.5 GeV

$m \gg mv \gg mv^2$



微扰匹配：短程系数不敏感于长程物理

微扰的短程系数

$$\Gamma(H) = \sum_n \frac{\boxed{f_n(\Lambda)}}{m^{d_n-4}} \boxed{\langle 0|\mathcal{O}_n(\Lambda)|0\rangle}$$

非微扰长程矩阵元

研究现状

近年来，在NRQCD因子化框架下，对双粲偶素产生的研究取得了重要的理论进展。

1. $e^+ e^- \to J/\psi\ \eta_c$过程两圈微扰修正被计算，微扰展开收敛性较好，与实验也比较吻合；

2. $e^+ e^- \to J/\psi\ \chi_{cJ}$过程的两圈修正被计算，截面预言与实验较为符合。

3. $e^+ e^- \to J/\psi J/\psi$过程的高阶修正被计算，通过重新组合微扰展开，得到了精确的收敛的理论预言。

4. $\Upsilon \to J/\psi\ \eta_c(\chi_{cJ})$过程的两圈微扰修正被研究，重整化标度依赖性得到改善。

Feng, Jia, Mo, Sang, Zhang, arXiv: 1901.08447
Huang, Gong and Wang, JHEP 02, 049 (2023)
Sang, Feng, Jia, Mo, Zhang, PLB 843, 138057(2023)
Sang, Feng, Jia, Mo, Pan, Zhang PRL (2023)
Zhang, Sang and Zhang, PRL (2022)
……

$\eta_b(\chi_{bJ}) \to J/\psi\ J/\psi$过程的高阶微扰修正如何?

Two-Loop QCD Corrections to C-even Bottomonium Exclusive Decays to Double J/ψ

Y.D.Zhang*, X.W.Bai, F.Feng, W.L.Sang and M.Z.Zhou

arXiv: 2310.07453

$$\boxed{\eta_b(\chi_{bJ}) \to J/\psi + J/\psi}$$

研究动机

2008年， Yu Jia. 基于NRQCD计算了$\eta_b \to J/\psi + J/\psi$的LO相对论修正。
Phys. Rev. D 78, 054003 (2008)

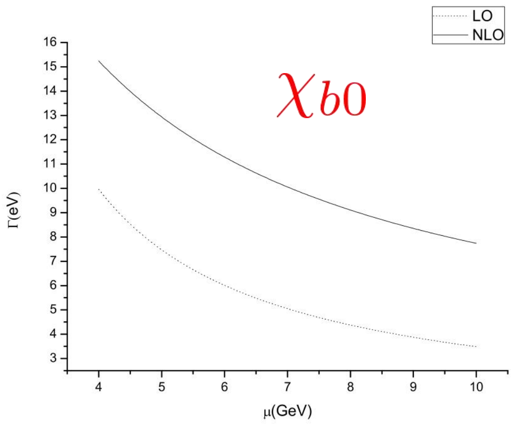
2009年，Yu Jia and Jian Xiong Wang，*et al.* 基于NRQCD计算了$\eta_b \to J/\psi + J/\psi$的NLO辐射修正。
Phys. Lett. B 670 (2009), 350-355

2010年，Braguta，*et al.* 基于Light Cone 计算了$\eta_b \to J/\psi + J/\psi$和 $\chi_{bJ} \to J/\psi + J/\psi$ 过程。
Phys. Rev. D 81 (2010), 014012
Phys. Atom. Nucl. 73, 1054-1068 (2010)

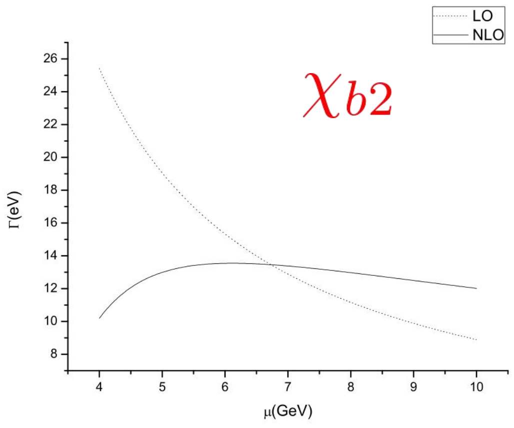
2011年，Cong Feng Qiao, *et al.* 基于NRQCD计算了$\eta_b \to J/\psi + J/\psi$的NLO辐射修正。
Phys. Lett. B 702 (2011), 49-54

2011年，Feng Feng，*et al.* 基于NRQCD计算了$\chi_{bJ} \to J/\psi + J/\psi$ 的NLO相对论修正。
Phys. Rev. D 84 (2011), 094031

2014年，Cong Feng Qiao，*et al.* 基于NRQCD计算了$\chi_{bJ} \to J/\psi + J/\psi$的NLO辐射修正。
Phys. Rev. D 89 (2014) no.7, 074004



(a) $\chi_{b0} \to J/\psi J/\psi$



(b) $\chi_{b2} \to J/\psi J/\psi$

研究动机

Belle, Phys. Rev. D 85 (2012), 071102

Channel	n^{up}	$\varepsilon(\%)$	$\sigma_{\mathrm{sys}}(\%)$	$\mathcal{B}_R$
$\chi_{b0} \to J/\psi J/\psi$	21	5.8	16	7.1×10^{-5}
$\chi_{b1} \to J/\psi J/\psi$	13	6.3	30	2.7×10^{-5}
$\chi_{b2} \to J/\psi J/\psi$	22	5.9	27	4.5×10^{-5}
$\chi_{b0} \to J/\psi \psi'$	20	3.4	17	1.2×10^{-4}
$\chi_{b1} \to J/\psi \psi'$	5.8	3.8	15	1.7×10^{-5}
$\chi_{b2} \to J/\psi \psi'$	17	3.5	16	4.9×10^{-5}
$\chi_{b0} \to \psi' \psi'$	3.0	2.1	20	3.1×10^{-5}
$\chi_{b1} \to \psi' \psi'$	12	2.2	17	6.2×10^{-5}
$\chi_{b2} \to \psi' \psi'$	3.3	2.1	12	1.6×10^{-5}

L.B.Chen and C.F.Qiao, Phys. Rev. D 89 (2014) no.7, 074004

	$\chi_{b0} \to J/\psi J/\psi$	$\chi_{b1} \to J/\psi J/\psi$	$\chi_{b2} \to J/\psi J/\psi$
Γ^{NLO}(eV)	$13.13^{+2.32+1.24+2.10}_{-1.93-1.18-5.39}$	$0.58^{+0.09+0.01+0.28}_{-0.12-0.01-0.40}$	$12.85^{+2.47+2.11+0.70}_{-2.03-1.90-2.66}$
$\mathrm{Br}^{\mathrm{NLO}}(10^{-5})$	$1.80^{+0.32+0.17+0.29}_{-0.26-0.16-0.74}$	$0.63^{+0.10+0.01+0.30}_{-0.13-0.01-0.43}$	$5.85^{+1.12+0.96+0.32}_{-0.92-0.86-1.21}$
Γ^{LO}(eV) [16]	5.54	9.04×10^{-7}	10.6
$\mathrm{Br}^{\mathrm{EXP}}(10^{-5})$ [27]	<7.1	<2.7	<4.5

研究动机

$$\Upsilon \to J/\psi + \eta_c(\chi_{cJ})$$

- Y. Jia.
  Phys. Rev. D 76 (2007), 074007
- J. Xu, *et al.*
  Phys. Rev. D 87 (2013) no.9, 094004
- W. L. Sang, *et al.*
  Phys. Rev. D 92 (2015) no.1, 014025
- Y. D. Zhang, W. L. Sang* and H. F. Zhang.
  Phys. Rev. Lett. 129 (2022) no.11, 112002

S.D.Yang et al.[Belle], Phys. Rev. D 90 (2014) no.11,112008

Channels	$\mathcal{B}_R(\times 10^{-6})$
$\Upsilon(1S) \to J/\psi + \eta_c$	< 2.2
$\Upsilon(1S) \to J/\psi + \chi_{c0}$	< 3.4
$\Upsilon(1S) \to J/\psi + \chi_{c1}$	$3.90 \pm 1.21 \pm 0.23$
$\Upsilon(1S) \to J/\psi + \chi_{c2}$	< 1.4
$\Upsilon(1S) \to J/\psi + \eta_c(2S)$	< 2.2

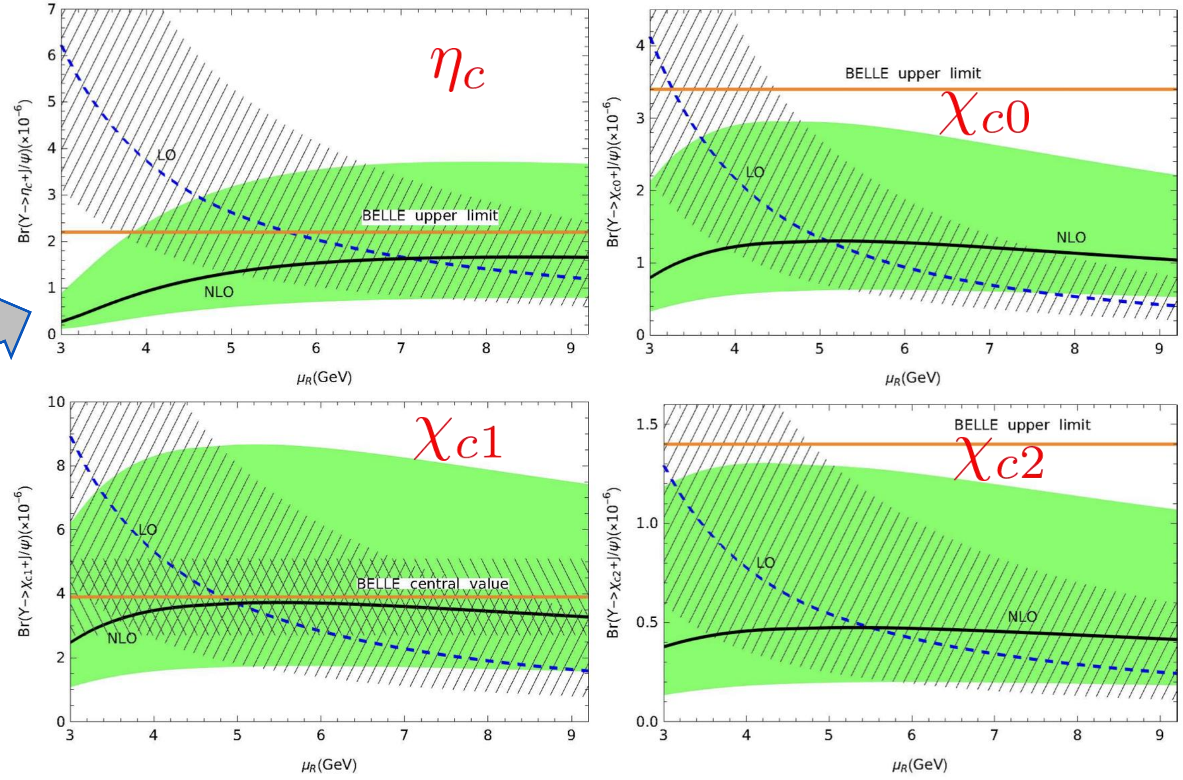


FIG. 2: NRQCD results for $\mathrm{Br}(\Upsilon \to J/\psi + \eta_c(\chi_{cJ}))$ as functions of μ_R. The uncertainty bands for the theoretical results correspond to the choices of the charm and bottom quark mass. In addition, the uncertainty from the Belle measurement for $J/\psi + \chi_{c1}$ is also illustrated.

衰变宽度

Two-Loop QCD Corrections to C-even Bottomonium Exclusive Decays to Double J/ψ

Y.D.Zhang*, X.W.Bai, F.Feng, W.L.Sang and M.Z.Zhou. **arXiv: 2310.07453**

衰变宽度
$$\Gamma\left[H \to J/\psi(\lambda_1)J/\psi(\lambda_2)\right] = \frac{|\mathbf{P}|}{16\pi m_H^2}\left|A^H_{\lambda_1,\lambda_2}\right|^2 \int_0^1 \frac{1}{2J+1}\sum_{S_z}\left|d^J_{S_z,\lambda_1-\lambda_2}(\theta)\right|^2 d\cos\theta$$
$$= \frac{|\mathbf{P}|}{16\pi(2J+1)m_H^2}\left|A^H_{\lambda_1,\lambda_2}\right|^2.$$

$$\Gamma(\eta_b \to J/\psi J/\psi) = \frac{|\mathbf{P}|}{16\pi m_{\eta_b}^2}\left(2|A^{\eta_b}_{1,1}|^2\right),$$
$$\Gamma(\chi_{b0} \to J/\psi J/\psi) = \frac{|\mathbf{P}|}{16\pi m_{\chi_{b0}}^2}\left(2|A^{\chi_{b0}}_{1,1}|^2 + |A^{\chi_{b0}}_{0,0}|^2\right),$$
$$\Gamma(\chi_{b1} \to J/\psi J/\psi) = \frac{|\mathbf{P}|}{48\pi m_{\chi_{b1}}^2}\left(4|A^{\chi_{b1}}_{1,0}|^2\right),$$
$$\Gamma(\chi_{b2} \to J/\psi J/\psi) = \frac{|\mathbf{P}|}{80\pi m_{\chi_{b2}}^2}\left(2|A^{\chi_{b2}}_{1,-1}|^2 + 2|A^{\chi_{b2}}_{1,1}|^2 + 4|A^{\chi_{b2}}_{1,0}|^2 + |A^{\chi_{b2}}_{0,0}|^2\right).$$

H: η_b 或 χ_{bJ}

$\mathbf{P}$: 末态两粒子的三动量

λ_1和λ_2: 分别表示末态两个J/ψ 粒子的螺旋度

根据NRQCD因子化理论

微扰的短程系数

$$A^H_{\lambda_1,\lambda_2} = \sqrt{2m_H 2m_{J/\psi}}\,\boxed{f^H_{\lambda_1,\lambda_2}}\,\boxed{\frac{\sqrt{\langle\mathcal{O}\rangle_H}}{m_b^n}\frac{\langle\mathcal{O}\rangle_{J/\psi}}{m_c^3}};$$

非微扰长程矩阵元

势模型

格点QCD

实验

计算过程

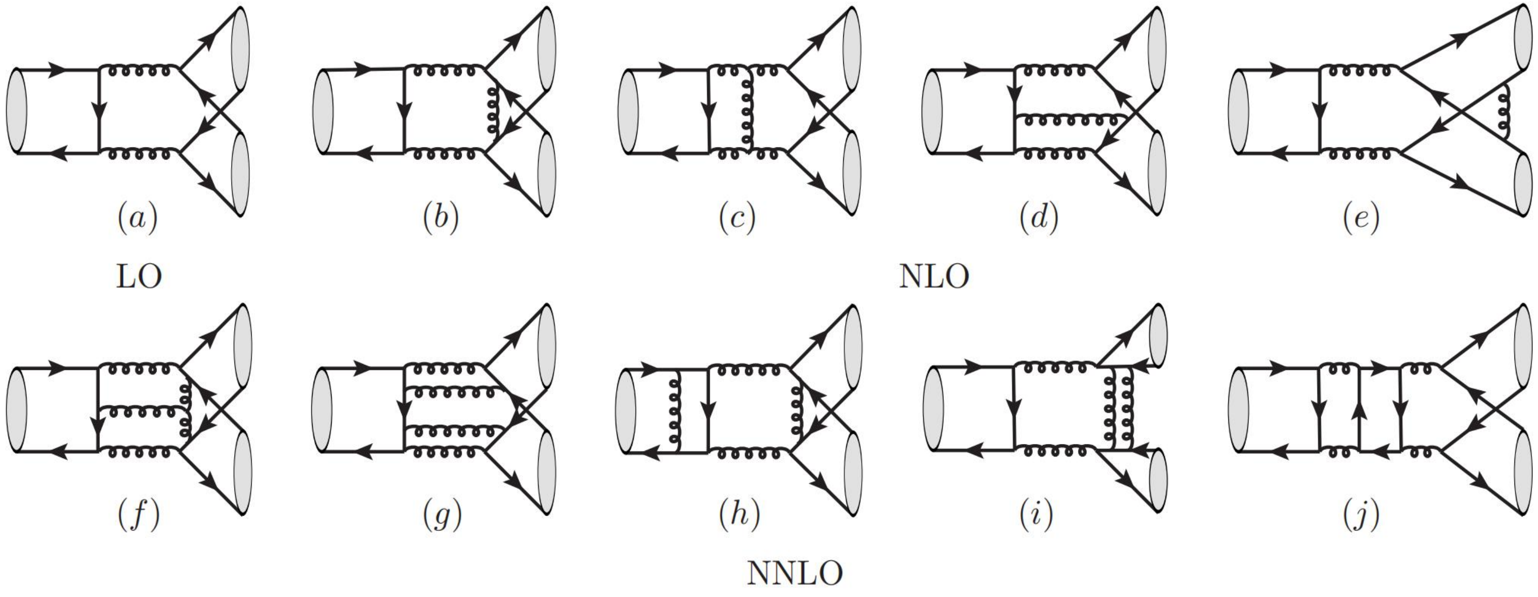


FIG. 1: Some representative Feynman diagrams for the process $\eta_b(\chi_{bJ}) \to J/\psi J/\psi$ up to $\mathcal{O}(\alpha_s^2)$.

- Sector Decomposition(*FIEASTA*)
- Differential equations
- Mellin-Barnes
- ...

AMFlow

X. Liu, Y. Q. Ma, C. Y. Wang, arXiv:1711.09572
X. Liu, Y. Q. Ma, arXiv:2107.01864
Z. F. Liu, Y. Q. Ma, arXiv:2201.11637(PRL)
X. Liu, Y. Q. Ma, arXiv:2201.11669(Package)

CalcLoop

Y.Q.Ma, et al.
https://gitlab.com/multiloop-pku/calcloop

方法1: FeynArts → FeynCalc → FIRE → AMFlow

两圈图：3781个

两圈MIs：约1400个

交叉检验

方法2: FeynArts → CalcLoop → AMFlow

计算结果

短程系数

$$f^{H}_{\lambda_1,\lambda_2} = \alpha_s^2\Big[f^{H,(0)}_{\lambda_1,\lambda_2} + \frac{\alpha_s}{\pi}\left(\frac{\beta_0}{2}\ln\frac{\mu_R^2}{m_b^2} f^{H,(0)}_{\lambda_1,\lambda_2} + f^{H,(1)}_{\lambda_1,\lambda_2}\right) + \frac{\alpha_s^2}{\pi^2}\Big(\frac{3\beta_0^2}{16}\ln^2\frac{\mu_R^2}{m_b^2} f^{H,(0)}_{\lambda_1,\lambda_2} + \Big(\frac{\beta_1}{8} f^{H,(0)}_{\lambda_1,\lambda_2} + \frac{3\beta_0}{4} f^{H,(1)}_{\lambda_1,\lambda_2}\Big)\ln\frac{\mu_R^2}{m_b^2} + \left(2\gamma_{J/\psi} + \gamma_H\right)\ln\frac{\mu_\Lambda^2}{m_c^2} f^{H,(0)}_{\lambda_1,\lambda_2} + f^{H,(2)}_{\lambda_1,\lambda_2}\Big)\Big] + \mathcal{O}\left(\alpha_s^5\right),$$

TABLE I: Numerical values for various SDCs with $m_b = 4.7$ GeV and $m_c = 1.5$ GeV.

$m_c = 1.50\,\mathrm{GeV}, m_b = 4.70\,\mathrm{GeV}$				
H	(λ_1, λ_2)	$f^{(0)}_{\lambda_1,\lambda_2}$	$f^{(1)}_{\lambda_1,\lambda_2}$	$f^{(2)}_{\lambda_1,\lambda_2}$
η_b	(1,1)	–	$0.551 - 1.410i$	$21.624 - 10.293i$
χ_{b0}	(1,1)	0.439	$-0.266 + 2.726i$	$-86.938 + 29.858i$
	(0,0)	-1.716	$-2.830 - 6.283i$	$184.697 - 75.742i$
χ_{b1}	(1,0)	–	$-0.281 + 1.188i$	$8.812 + 7.876i$
χ_{b2}	(1,1)	-0.621	$3.801 + 0.233i$	$84.482 - 2.120i$
	(1,0)	1.685	$-9.666 - 1.178i$	$-221.561 - 1.457i$
	(1,-1)	-3.733	$21.499 + 2.214i$	$465.006 + 10.900i$
	(0,0)	-2.145	$11.326 + 2.567i$	$278.731 + 3.346i$

$f^{H,(0)}_{\lambda_1,\lambda_2}$，$f^{H,(0)}_{\lambda_1,\lambda_2}$和 $f^{H,(0)}_{\lambda_1,\lambda_2}$分别表示领头阶，次领头阶和次次领头阶的短程系数。

总衰变宽度

TABLE II: Total decay widths of χ_{bJ}.

H	$\Gamma[\chi_{bJ}\to\gamma\Upsilon]$(keV)[47]	$\mathrm{Br}[\chi_{bJ}\to\gamma\Upsilon]$[44]	Γ_{tot}(MeV)
χ_{b0}	22.2	(1.94±0.27)%	$1.144^{+0.185}_{-0.140}$
χ_{b1}	27.8	(35.2±2.0)%	$0.079^{+0.005}_{-0.004}$
χ_{b2}	31.6	(18.0±1.0)%	$0.176^{+0.010}_{-0.009}$

$$\Gamma_{\eta_b} = 10^{+5}_{-4}\ \mathrm{MeV}.$$

$$\Gamma_{tot}(\chi_{bJ}) = \frac{\Gamma[\chi_{bJ}\to\gamma\Upsilon]}{\mathrm{Br}[\chi_{bJ}\to\gamma\Upsilon]}$$

计算结果和现象分析

TABLE III: Theoretical predictions on various (un)polarized decay widths (in units of eV) and branching fractions ($\times 10^{-5}$).

H	Order	$\Gamma_{0,0}$	$\Gamma_{1,0}$	$\Gamma_{1,1}$	$\Gamma_{1,-1}$	$\Gamma_{\rm Unpol}$	$\rm Br_{th}$	$\rm Br_{exp}$[3]
η_b	LO	–	–	–	–	–	–	
	NLO	–	–	$1.080^{+1.663}_{-0.714}$	–	$2.160^{+3.325}_{-1.428}$	$0.022^{+0.033+0.014}_{-0.014-0.007}$	–
	NNLO	–	–	$4.084^{+3.987}_{-2.232}$	–	$8.168^{+7.973}_{-4.463}$	$0.082^{+0.080+0.054}_{-0.045-0.027}$	
χ_{b0}	LO	$8.542^{+7.358}_{-4.393}$	–	$0.559^{+0.482}_{-0.288}$	–	$9.660^{+8.321}_{-4.968}$	$0.844^{+0.727+0.117}_{-0.434-0.118}$	
	NLO	$11.140^{+1.233}_{-2.500}$	–	$0.616^{+0.084}_{-0.124}$	–	$12.372^{+1.400}_{-2.748}$	$1.081^{+0.122+0.150}_{-0.240-0.151}$	<7.1
	NNLO	$6.449^{+1.710}_{-1.955}$	–	$0.329^{+0.371}_{-0.012}$	–	$7.107^{+1.741}_{-1.212}$	$0.621^{+0.152+0.086}_{-0.106-0.086}$	
χ_{b1}	LO	–	–	–	–	–	–	
	NLO	–	$0.007^{+0.011}_{-0.005}$	–	–	$0.027^{+0.042}_{-0.018}$	$0.035^{+0.053+0.002}_{-0.023-0.002}$	<2.7
	NNLO	–	$0.014^{+0.009}_{-0.006}$	–	–	$0.057^{+0.035}_{-0.026}$	$0.072^{+0.044+0.004}_{-0.033-0.004}$	
χ_{b2}	LO	$2.663^{+2.294}_{-1.370}$	$1.643^{+1.416}_{-0.845}$	$0.223^{+0.192}_{-0.115}$	$8.067^{+6.949}_{-4.149}$	$25.818^{+22.239}_{-13.279}$	$14.669^{+12.636+0.884}_{-7.545-0.789}$	
	NLO	$1.094^{+0.281}_{-0.679}$	$0.604^{+0.199}_{-0.423}$	$0.075^{+0.030}_{-0.057}$	$2.943^{+0.987}_{-2.084}$	$9.545^{+3.111}_{-6.655}$	$5.424^{+1.768+0.327}_{-3.781-0.292}$	<4.5
	NNLO	$0.071^{+0.476}_{-0.048}$	$0.020^{+0.267}_{-0.015}$	$0.001^{+0.032}_{-0.001}$	$0.157^{+1.351}_{-0.130}$	$0.467^{+4.311}_{-0.360}$	$0.265^{+2.450+0.016}_{-0.205-0.014}$	

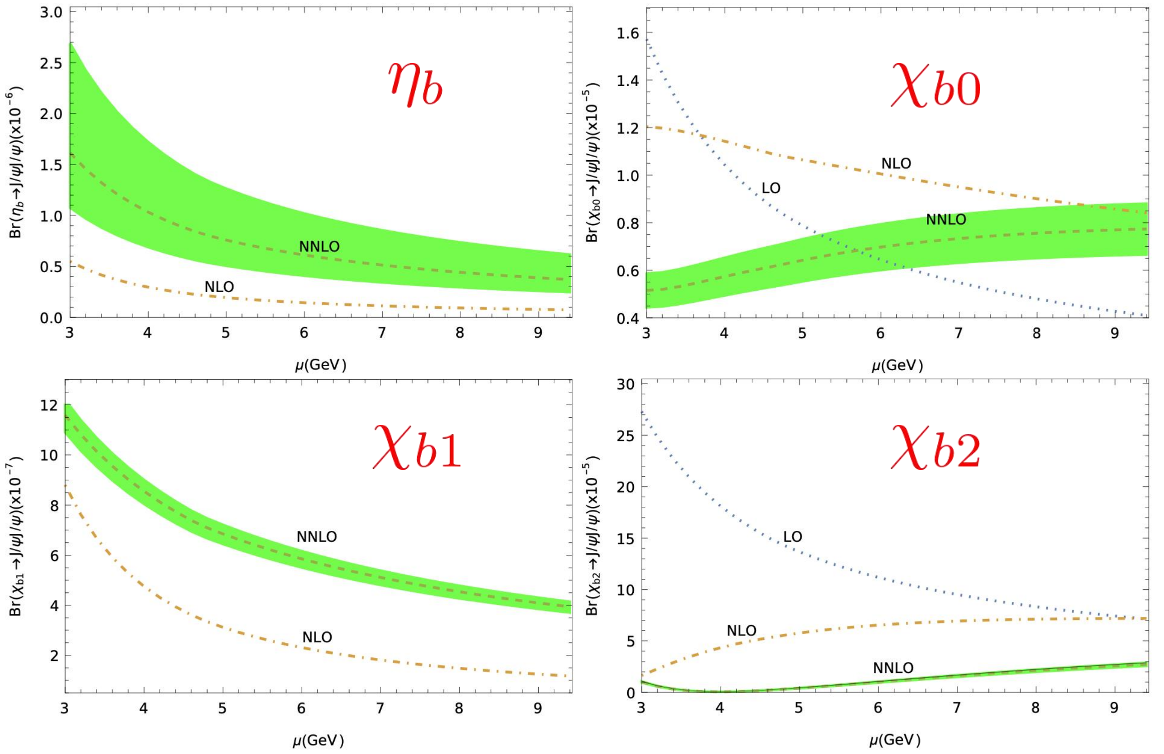


FIG. 2: Theoretical predictions for $\mathrm{Br}[\eta_b(\chi_{bJ}) \to J/\psi J/\psi]$ as a function of μ_R at various levels of accuracy in α_s.

$$\sigma(pp \to \chi_{b0} + X) = 1.5\,\mu\mathrm{b}$$
$$\sigma(pp \to \chi_{b2} + X) = 2.0\,\mu\mathrm{b}$$
$$\sigma(pp \to \eta_b + X) = 15\,\mu\mathrm{b}$$

Braguta, *et al* Phys. Rev. D 72, 094018 (2005)

$$\mathcal{L} = 100\,\mathrm{fb}^{-1}, \qquad \mathrm{Br}[J/\psi \to \ell\bar{\ell}] = 12\%,$$

$$(5-10)\times 10^3\ \eta_b(\chi_{bJ}) \to J/\psi J/\psi \to \ell\bar{\ell}\ell\bar{\ell}$$

计算结果和现象分析

$r = \frac{m_c}{m_b}$

TABLE IV: Theoretical predictions on various unpolarized decay widths (in units of eV) and branching fractions ($\times 10^{-5}$).

H	r	0.267		0.286		0.308		0.333		0.364		0.400	
	Order	Γ	Br	Γ	Br	Γ	Br	Γ	Br	Γ	Br	Γ	Br
η_b	LO	–	–	–	–	–	–	–	–	–	–	–	–
	NLO	2.407	0.024	2.341	0.023	2.231	0.022	2.059	0.021	1.796	0.018	1.401	0.014
	NNLO	9.245	0.092	8.944	0.089	8.468	0.085	7.749	0.077	6.688	0.067	5.151	0.052
χ_{b0}	LO	14.978	1.309	12.664	1.107	10.563	0.923	8.690	0.760	7.065	0.618	5.722	0.500
	NLO	21.144	1.848	17.267	1.509	13.826	1.209	10.831	0.947	8.299	0.725	6.250	0.546
	NNLO	12.177	1.064	9.951	0.870	7.956	0.695	6.202	0.542	4.708	0.412	3.502	0.306
χ_{b1}	LO	–	–	–	–	–	–	–	–	–	–	–	–
	NLO	0.044	0.055	0.037	0.047	0.031	0.039	0.024	0.030	0.016	0.021	0.009	0.012
	NNLO	0.099	0.125	0.082	0.104	0.065	0.082	0.048	0.061	0.032	0.040	0.017	0.022
χ_{b2}	LO	33.173	18.848	30.015	17.054	27.096	15.396	24.423	13.877	22.002	12.501	19.848	11.277
	NLO	13.545	7.696	11.845	6.730	10.253	5.825	8.764	4.979	7.370	4.187	6.058	3.442
	NNLO	0.410	0.233	0.438	0.249	0.459	0.261	0.472	0.268	0.473	0.269	0.58	0.260

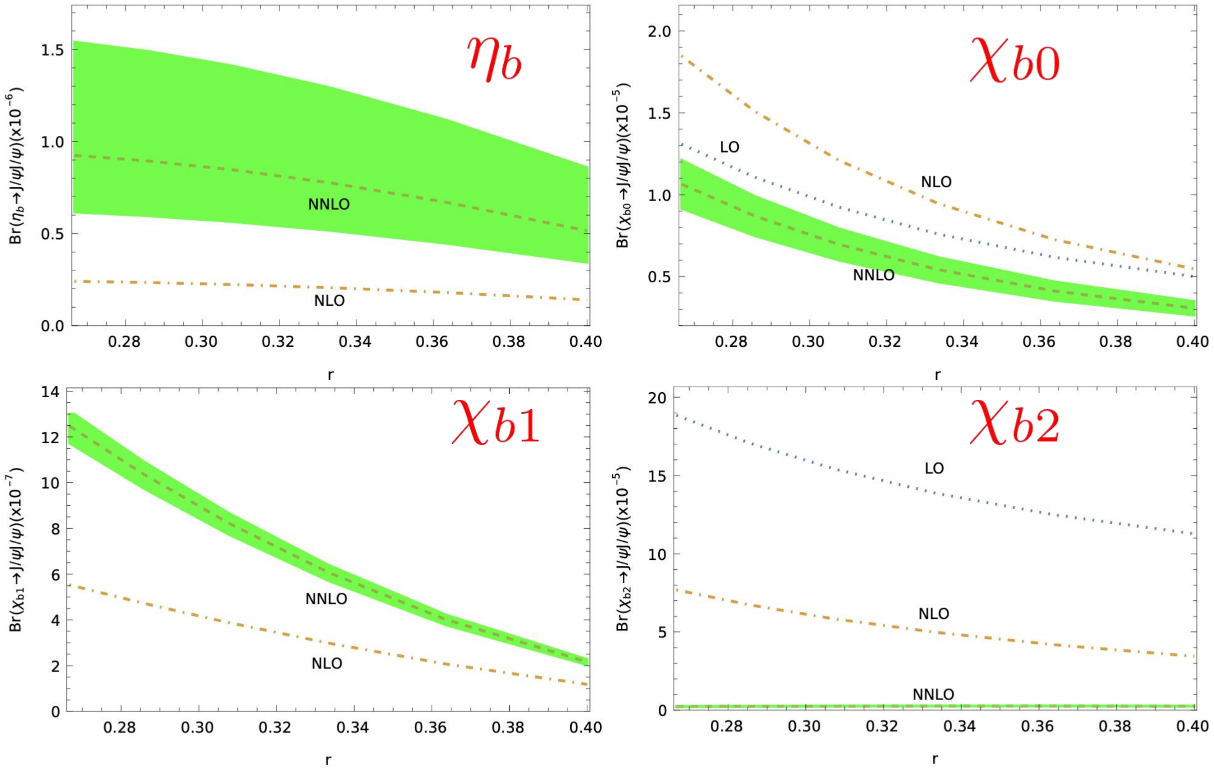


FIG. 3: Branching fractions of $\eta_b(\chi_{bJ}) \to J/\psi J/\psi$ as a function of r.

η_b和$\chi_{b0,1}$的分支比随着质量增加而减小，但χ_{b2}的NNLO结果与粲夸克质量几乎无关。

Z-boson radiative decays to an S-wave quarkonium at NNLO and NLL accuracy

W.L.Sang, D.S.Yang and Y.D.Zhang*

Phys. Rev. D 108, no.1, 014021 (2023)

$$Z \to \eta_Q/J/\psi/\Upsilon + \gamma$$

研究动机

- G.T.Bodwin, H.S.Chung, J.H.Ee and J.Lee Phys. Rev. D 97 (2018) no.1, 016009

$Z \to J/\psi + \gamma$ NLO+NLL(+18%)

$Z \to \Upsilon(1S) + \gamma$ NLO+NLL(+11%)

由于过程中涉及m_Z 和$m_Q(Q=c,b)$两个标度，NRQCD因子化的短程系数中包含 m_Z^2/m_Q^2 的大对数项，因此需要根据Light Cone做重求和计算。

- W.L.Sang*, D.S.Yang and Y.D.Zhang Phys. Rev. D 106, no.9, 094023 (2022)

TABLE III: Squared leading-twist SDCs $|\mathcal{C}_{0,1}^H|^2$ at various levels of accuracy. We take $\mu_R = m_Z$ and $\mu_\Lambda = 1$ GeV.

H	$\mathcal{K}_H^{\rm LL}$	LO	LO+LL	NLO	NLO+LL	NNLO	NNLO+LL
χ_{c0}	0.859	7.96	5.87	8.01	6.47	8.85	7.90
χ_{c1}	1.222	48.13	71.88	51.15	57.38	47.07	50.63
χ_{c2}	0.859	16.04	11.86	10.89	8.37	8.87	7.54
h_c	0.859	24.00	17.73	17.31	13.41	13.51	11.50
χ_{b0}	0.930	7.65	6.59	7.85	7.21	8.87	8.58
χ_{b1}	1.145	49.08	64.25	47.72	50.31	45.52	46.78
χ_{b2}	0.930	16.36	14.18	11.30	10.23	9.90	9.47
h_b	0.930	24.00	20.77	17.68	16.02	15.26	14.61

$Z \to \chi_{QJ}(h_Q) + \gamma$

对于χ_{c0}, χ_{c2}和 h_c衰变道，LL重求和对LO结果近-25%的压低。
对于χ_{c1}衰变道，LL重求和对LO结果近+50%的增高。

计算过程

衰变宽度

$$\Gamma(Z \to \eta_Q + \gamma) = \frac{1}{3}\frac{1}{2m_Z}\frac{1}{8\pi}\frac{2|\mathbf{P}|}{m_Z}\left(2|A_{0,1}^{\eta_Q}|^2\right),$$

$$\Gamma(Z \to J/\psi(\Upsilon) + \gamma) = \frac{1}{3}\frac{1}{2m_Z}\frac{1}{8\pi}\frac{2|\mathbf{P}|}{m_Z}\left(2|A_{1,1}^{J/\psi(\Upsilon)}|^2 + 2|A_{0,1}^{J/\psi(\Upsilon)}|^2\right)$$

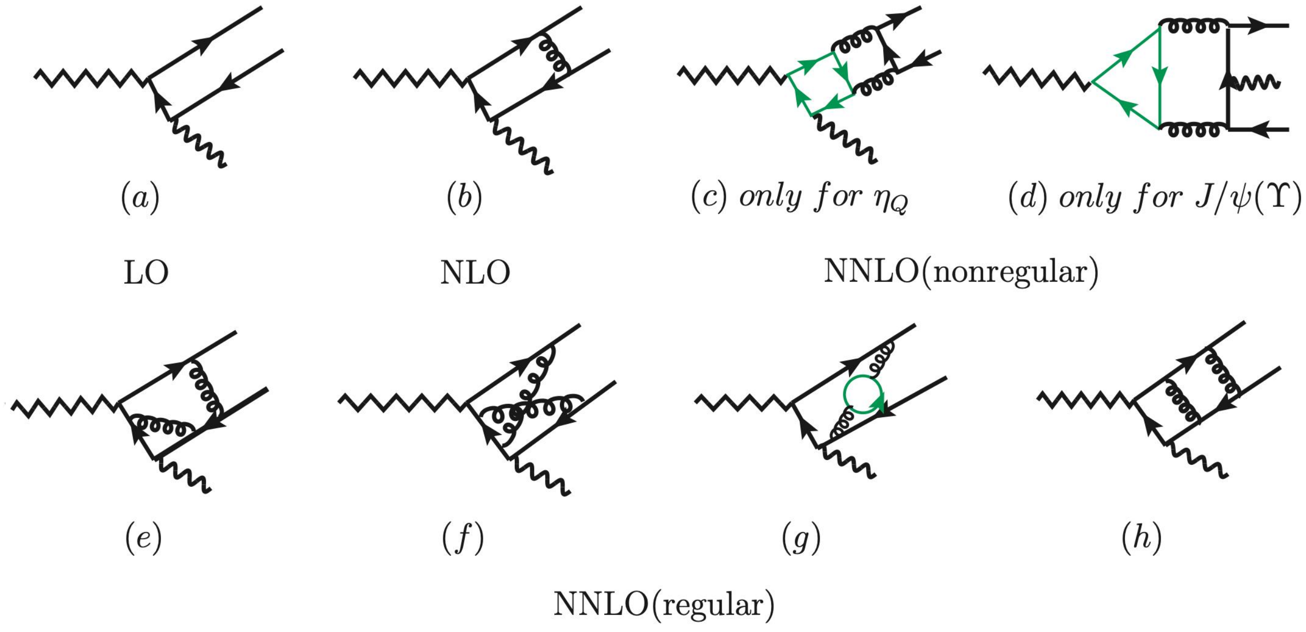


FIG. 1: Some representative Feynman diagrams for the process $Z \to H + \gamma$ up to $\mathcal{O}(\alpha_s^2)$.

短程系数

$$\mathcal{C}^H_{\lambda_1,\lambda_2} = \mathcal{C}^{H,(0)}_{\lambda_1,\lambda_2}\left[1 + \frac{\alpha_s}{\pi}\mathcal{C}^{H,(1)}_{\lambda_1,\lambda_2} + \frac{\alpha_s^2}{\pi^2}\left(\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_Q^2}\mathcal{C}^{H,(1)}_{\lambda_1,\lambda_2} + \gamma_H \ln\frac{\mu_\Lambda^2}{m_Q^2} + \mathcal{C}^{H,(2)}_{\mathrm{reg},\lambda_1,\lambda_2} + \mathcal{C}^{H,(2)}_{\mathrm{nonreg},\lambda_1,\lambda_2}\right)\right] + \mathcal{O}(\alpha_s^3),$$

TABLE I: NRQCD predictions to the various helicity SDCs. For simplicity, we define the symbols $f_1 \equiv \frac{g_V^d}{g_V^u} = -\frac{3-4S_W^2}{3-8S_W^2}$, $f_2 \equiv \frac{g_V^u - g_V^d}{g_V^u} = \frac{6-12S_W^2}{3-8S_W^2}$, $\bar{f}_1 \equiv \frac{g_V^u}{g_V^d} = -\frac{3-8S_W^2}{3-4S_W^2}$ and $\bar{f}_2 \equiv \frac{2g_V^d - 2g_V^u}{g_V^d} = \frac{12-24S_W^2}{3-4S_W^2}$, where g_V^u and g_V^d correspond to the values of g_V for up-type quark and down-type quark respectively.

H	(λ_1,λ_2)	$\mathcal{C}^{(1)}_{\lambda_1,\lambda_2}$	$\mathcal{C}^{(2)}_{\mathrm{reg},\lambda_1,\lambda_2}$	$\mathcal{C}^{(2)}_{\mathrm{nonreg},\lambda_1,\lambda_2}$
η_c	(0,1)	$1.035 - 1.679i$	$-60.56 + 27.66i - (0.88 + 0.74i)n_L$ $+(0.02 - 0.75i)n_c - (0.05 + 0.77i)n_b$	$-(2.52 - 4.28i)n_c + (0.70 - 1.26i)f_1 n_b$ $-(5.06 - 0.99i)f_2$
J/ψ	(1,1)	$0.929 - 2.088i$	$-59.12 + 40.50i - (0.36 + 1.12i)n_L$ $+(0.17 - 1.12i)n_c - (0.07 + 1.15i)n_b$	$1.51 + 1.49i$
	(0,1)	$0.122 - 1.684i$	$-50.17 + 28.13i - (0.54 + 0.74i)n_L$ $-(0.06 + 0.75i)n_c - (0.40 + 0.77i)n_b$	$1.42 + 1.56i$
η_b	(0,1)	$-0.127 - 1.628i$	$-46.21 + 13.95i - (1.17 + 0.13i)n_L$ $+(0.72 - 0.14i)n_c - (0.25 + 0.16i)n_b$	$(7.07 - 2.89i)\bar{f}_1 n_c - (0.74 - 4.07i)n_b$ $-(3.37 - 0.87i)\bar{f}_2$
Υ	(1,1)	$-0.454 - 2.065i$	$-37.76 + 22.93i - (0.87 + 0.37i)n_L$ $+(0.81 - 0.38i)n_c - (0.30 + 0.40i)n_b$	$-1.47 - 1.50i$
	(0,1)	$-0.985 - 1.665i$	$-35.71 + 15.10i - (0.85 + 0.14i)n_L$ $+(0.83 - 0.15i)n_c - (0.33 + 0.17i)n_b$	$-1.43 - 1.56i$

重求和计算

有限阶截断

短程系数用Light Cone表示为

$$C_{0,1}^{H} \equiv C_{0,1}^{H,(0)}\mathcal{K}^{H} = C_{0,1}^{H,(0)}\int_0^1 dx T_H(x,m_Z,\mu)\hat{\phi}_H(x,m_Q,\mu) + \mathcal{O}(m_Q^2/m_Z^2)$$

LCDA遵循ERBL演化方程

$$\mu^2\frac{d}{d\mu^2}\hat{\phi}_H(x,m_Q,\mu) = \int_0^1 dy V_H\left(x,y;\alpha_s(\mu)\right)\hat{\phi}_H(y,m_Q,\mu)$$

求解得到两个K因子

$$\begin{aligned}
\mathcal{K}^{P,\mathrm{NLL}} &= 1 + \frac{\alpha_s(m_Z)}{4\pi}C_F\left[(3-2\ln 2)\left(\ln\frac{m_Z^2}{m_Q^2} - i\pi\right) + \ln^2 2 + 3\ln 2 - \frac{\pi^2}{3} - 9\right] \\
&+ \left(\frac{\alpha_s(m_Z)}{4\pi}\right)^2 C_F\Bigg\{\left[C_F\left(\ln^2 2 - 8\ln 2 - \frac{\pi^2}{6} + \frac{9}{2}\right) + \frac{\beta_0}{2}(3-2\ln 2)\right]\ln^2\frac{m_Z^2}{m_Q^2} \\
&+ \Bigg[C_F\Bigg(\frac{7}{2}\zeta(3) - \frac{4}{3}\ln^3 2 + \frac{5}{3}\pi^2\ln 2 + 6\ln^2 2 + 21\ln 2 - \frac{5}{3}\pi^2 - \frac{51}{2} \\
&- 2i\pi\left(\ln^2 2 - 8\ln 2 - \frac{\pi^2}{6} + \frac{9}{2}\right)\Bigg) - C_A\left(\frac{3}{2}\zeta(3) - \frac{4}{3}\ln 2 - 1\right) \\
&- \beta_0\left(\ln^2 2 - \frac{2}{3}\ln 2 + \frac{\pi^2}{6} - \frac{1}{2}\right)\Bigg]\ln\frac{m_Z^2}{m_Q^2}\Bigg\} + \ldots,
\end{aligned}$$

$$\begin{aligned}
\mathcal{K}^{V,\mathrm{NLL}} &= \mathcal{K}^{P,\mathrm{NLL}} + \frac{\alpha_s(m_Z)}{4\pi}C_F\left[-4\ln 2\right] \\
&+ \left(\frac{\alpha_s(m_Z)}{4\pi}\right)^2\ln\frac{m_Z^2}{m_Q^2}\left\{C_F^2\left[4\ln^2 2 - 4\ln 2\right] - 4C_F\beta_0\ln 2\right\} + \ldots;
\end{aligned}$$

$$\boxed{\begin{aligned}
\widetilde{C}_{0,1}^{H,\mathrm{LL}} &\equiv C_{0,1}^{H,(0)}\mathcal{K}^{H,\mathrm{LL}}, \\
\widetilde{C}_{0,1}^{H,\mathrm{NLL}} &\equiv C_{0,1}^{H,(0)}\mathcal{K}^{H,\mathrm{NLL}},
\end{aligned}}$$

上面的$\mathcal{K}^{P,\mathrm{NLL}}$ 和 $\mathcal{K}^{V,\mathrm{NLL}}$ 分别对应$H=\eta_Q$和$H=J/\Psi(\Upsilon)$两种情形。

所有阶求和

LCDA和hard-kernel分别做Gegenbauer展开

$$\hat{\phi}_H(x,m_Q,\mu) \equiv \sum_{n=0}^{\infty}\hat{\phi}_{H,n}(\mu)x(1-x)C_n^{(3/2)}(2x-1)$$

$$T_H(x,m_Z,\mu) = \sum_{n=0}^{\infty}\frac{4(2n+3)}{(n+1)(n+2)}T_{H,n}(\mu)C_n^{(3/2)}(2x-1)$$

求解得到

$$\mathcal{K}^H = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}T_{H,n}(m_Z)U_{n,k}^H(m_Z,m_Q)\hat{\phi}_{H,k}(m_Q)$$

$$\mathcal{K}^H = \mathcal{K}^{H(0,0)} + \frac{\alpha_s(m_Z)}{4\pi}\mathcal{K}^{H(1,0)} + \frac{\alpha_s(m_Q)}{4\pi}\mathcal{K}^{H(0,1)}$$

$$\boxed{\begin{aligned}
C_{0,1}^{H,\mathrm{LL}} &\equiv C_{0,1}^{H,(0)}\mathcal{K}^{H(0,0)}, \\
C_{0,1}^{H,\mathrm{NLL}} &\equiv C_{0,1}^{H,(0)}\mathcal{K}^{H},
\end{aligned}}$$

固定阶短程系数

NRQCD短程系数

有限阶截断短程系数

所有阶求和短程系数

$$\begin{aligned}
C_{0,1}^{H,\mathrm{LO+(N)LL}} &= C_{0,1}^{H,\mathrm{LO}} - \widetilde{C}_{0,1}^{H,\mathrm{(N)LL}}\Big|_{\alpha_s^0} + C_{0,1}^{H,\mathrm{(N)LL}}, \\
C_{0,1}^{H,\mathrm{NLO+(N)LL}} &= C_{0,1}^{H,\mathrm{NLO}} - \widetilde{C}_{0,1}^{H,\mathrm{(N)LL}}\Big|_{\alpha_s^1} + C_{0,1}^{H,\mathrm{(N)LL}}, \\
C_{0,1}^{H,\mathrm{NNLO+(N)LL}} &= C_{0,1}^{H,\mathrm{NNLO}} - \widetilde{C}_{0,1}^{H,\mathrm{(N)LL}}\Big|_{\alpha_s^2} + C_{0,1}^{H,\mathrm{(N)LL}},
\end{aligned}$$

计算结果和现象分析

[8] Luchinsky arXiv:1706.04091
[14] Y.Grossman, M.Konig and M.Neubert, *et al.* JHEP 04, 101 (2015)

Channel	Order	$\Gamma_{\text{total}}(\text{eV})$	$\text{Br}(\times 10^{-9})$	$\text{Br}(\times 10^{-9})$ [8]	$\text{Br}(\times 10^{-9})$ [14
$Z \to \eta_c + \gamma$	LO	29.1	11.7	$9.4 \pm 1 \pm 0.1$	–
	NLO	31.7	12.7		
	NNLO	$27.3^{+0.4}_{-0.5}$	$10.9^{+0.2}_{-0.2}$		
	NNLO+NLL	$23.8^{+0.4}_{-0.5}$	$9.5^{+0.2}_{-0.2}$		
$Z \to J/\psi + \gamma$	LO	197.7	79.2	$88 \pm 9 \pm 0.9$	$80.2^{+1.4+2.0+3.9}_{-1.5-2.0-3.6}$
	NLO	200.6	80.4		
	NNLO	$178.8^{+2.1}_{-2.5}$	$71.6^{+0.9}_{-1.0}$		
	NNLO+NLL	$143.5^{+1.9}_{-2.3}$	$57.5^{+0.8}_{-0.9}$		
$Z \to \eta_b + \gamma$	LO	65.9	26.4	–	–
	NLO	65.5	26.3		
	NNLO	$64.0^{+0.2}_{-0.2}$	$25.6^{+0.1}_{-0.1}$		
	NNLO+NLL	$60.7^{+0.2}_{-0.2}$	$24.3^{+0.1}_{-0.1}$		
$Z \to \Upsilon + \gamma$	LO	139.2	55.8	–	$53.9^{+1.0+0.8+1.1}_{-1.0-0.8-0.8}$
	NLO	129.2	51.8		
	NNLO	$125.6^{+0.4}_{-0.4}$	$50.3^{+0.2}_{-0.2}$		
	NNLO+NLL	$115.4^{+0.4}_{-0.4}$	$46.3^{+0.2}_{-0.2}$		

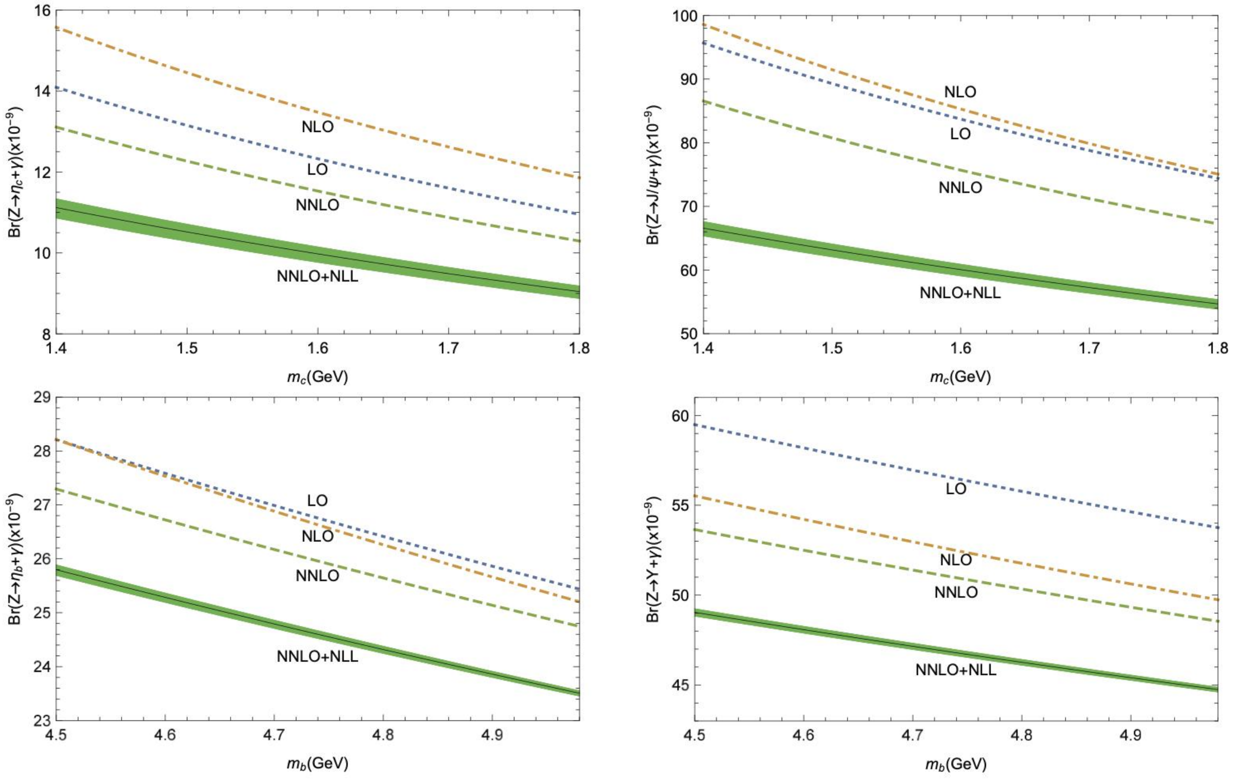


FIG. 2: Branching fraction for $Z \to H + \gamma$ as a function of m_Q. The green band denotes the uncertainty from μ_R.

- 对于η_b和 Υ 的产生，高阶修正可以忽略不计，而对于其他两个衰变道，修正可以达到10% 左右。
- NLL的重求和可以显著改变NRQCD的预言结果，特别是对于$Z \to J/\psi + \gamma$过程。
- 我们画出 $Z \to \eta_Q/J/\psi/\Upsilon + \gamma$ 的分支比关于夸克质量的函数，发现分支比随着夸克质量的增加而单调减小。
- 在未来的超级Z工厂中，Z玻色子产率可以达到 7×10^{11}。预计将产生Υ 或J/ψ的事例数可以达到5×10^4，产生$\eta_{c,b}$ 的事例数大约10^4，期待未来可以在超级Z工厂测量到这些衰变道。

恳请各位老师和同学批评指正

谢谢

THANKS