Conserved charge fluctuations and correlations in finite-density QCD

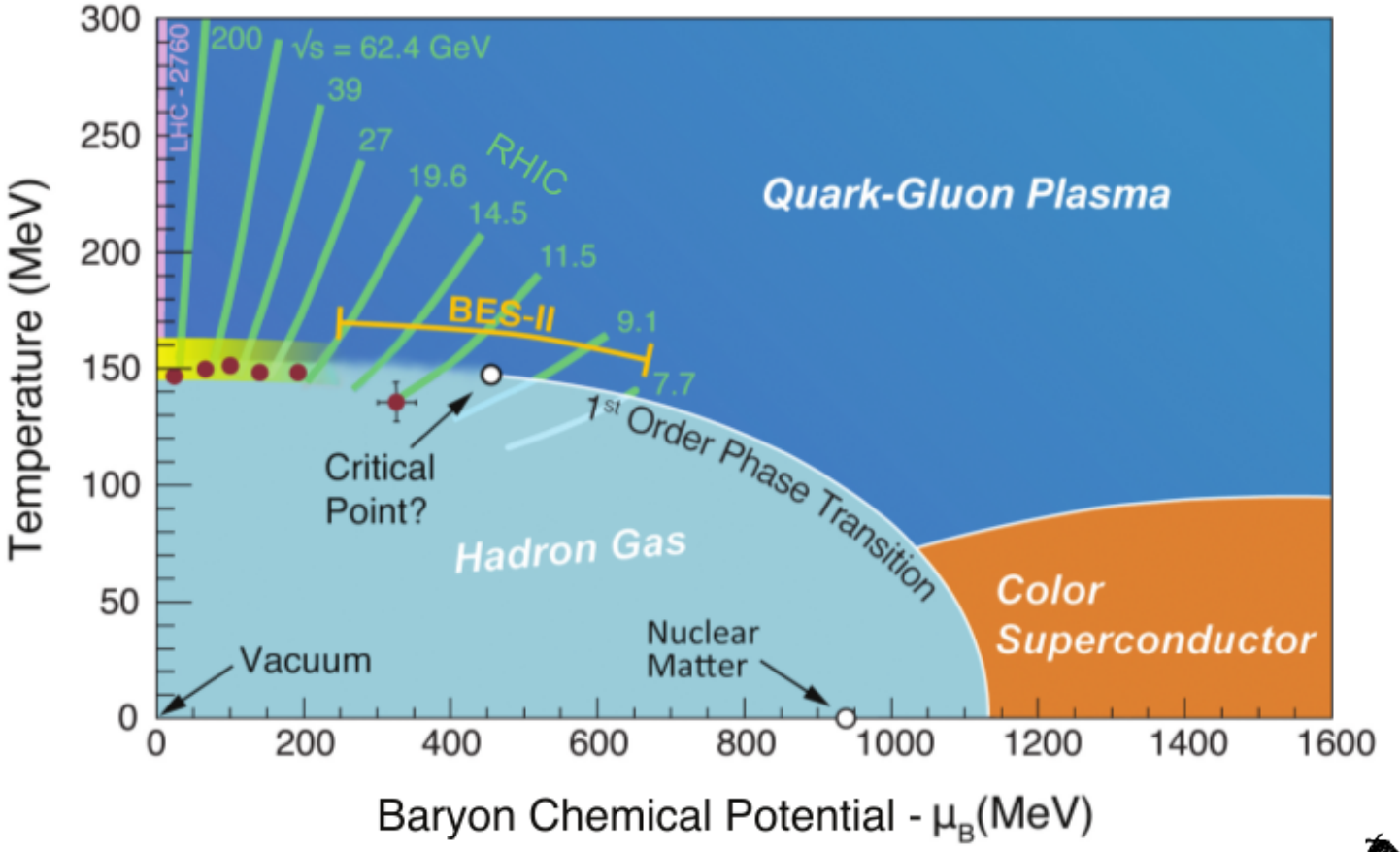
Frithjof Karsch
Bielefeld University

UNIVERSITÄT BIELEFELD
Faculty of Physics

- QCD phase diagram – constraining the location of a critical endpoint
- Net baryon-number – strangeness correlations on the pseudo-critical line
- Higher order cumulants of net baryon-number fluctuations on the pseudo-critical line

CRC-TR 211 Strong-interaction matter under extreme conditions
DFG Deutsche Forschungsgemeinschaft

Exploring the phase diagram of strong-interaction matter



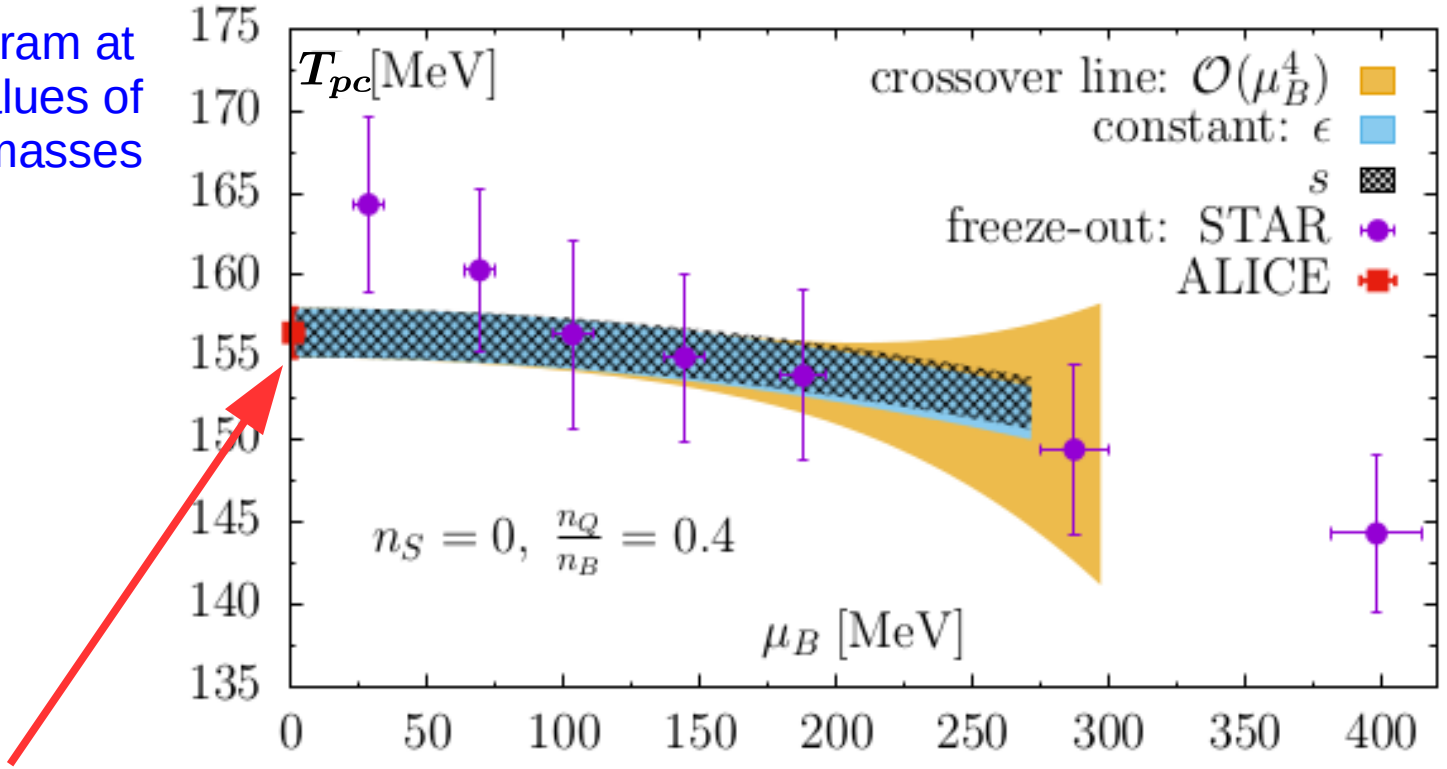
Where is the critical point?



Pseudo-critical line for physical quark mass values

$$T_{pc}(\mu_B) = T_{pc}(0)\left(1 - \kappa_2\left(\frac{\mu_B}{T_{pc}(\mu_B)}\right)^2 - \kappa_4\left(\frac{\mu_B}{T_{pc}(\mu_B)}\right)^4 + \ldots\right)$$

phase diagram at physical values of the quark masses



STAR: arXiv:1701.07065

A. Andronic et al., Nature 561 (2018) 321

$T_{pc} = (156.5 \pm 1.5)$ MeV

$\kappa_2 = 0.012(4)$

$\kappa_4 = 0.000(4)$

A. Bazavov et al. [HotQCD], Phys. Lett. B795 (2019), arXiv:1812.08235

$T_{pc} = (158.0 \pm 0.6)$ MeV

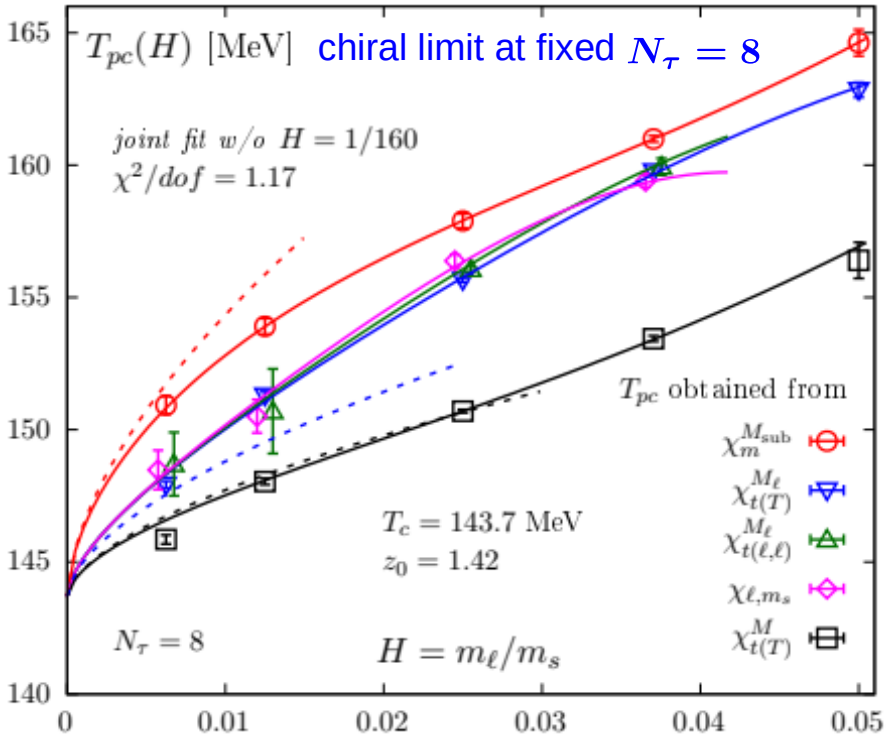
$\kappa_2 = 0.0153(18)$

$\kappa_4 = 0.00032(67)$

S. Borsanyi, et al, PRL 125 (2020) arXiv:2002.02821

Pseudo-critical and critical temperatures

$$T_{pc}(H) = T_c + z_x T_c H^{1/\beta\delta} \quad , \quad x = \text{peak}, \delta, ...$$



H.T.Ding, O. Kaczmarek, FK, P. Petreczky, Mugdha Sarkar, C. Schmidt, Sipaz Sharma, arXiv:2403.09390

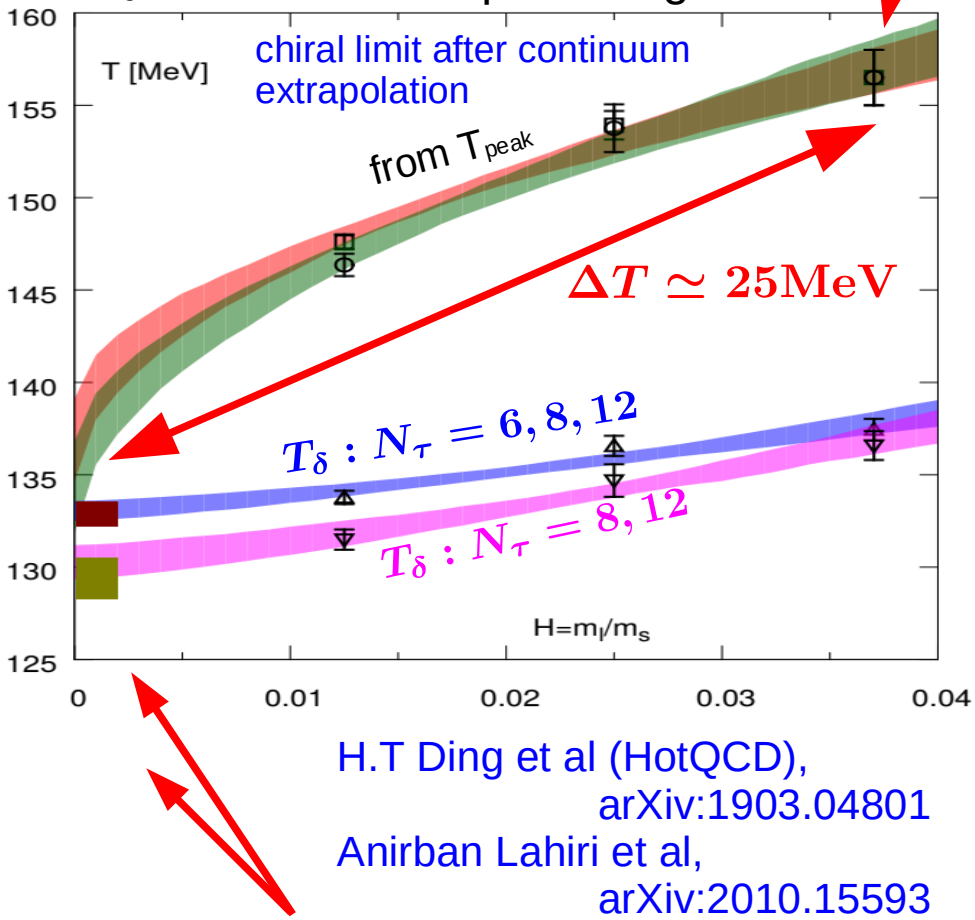
physical masses

$$T_{pc}^{phys} = (156.5 \pm 1.5)\ \text{MeV}$$

A. Bazavov et al (HotQCD), arXiv:1812.08235

physical masses

T_δ : T at ~60% of peak height



H.T Ding et al (HotQCD), arXiv:1903.04801

Anirban Lahiri et al, arXiv:2010.15593

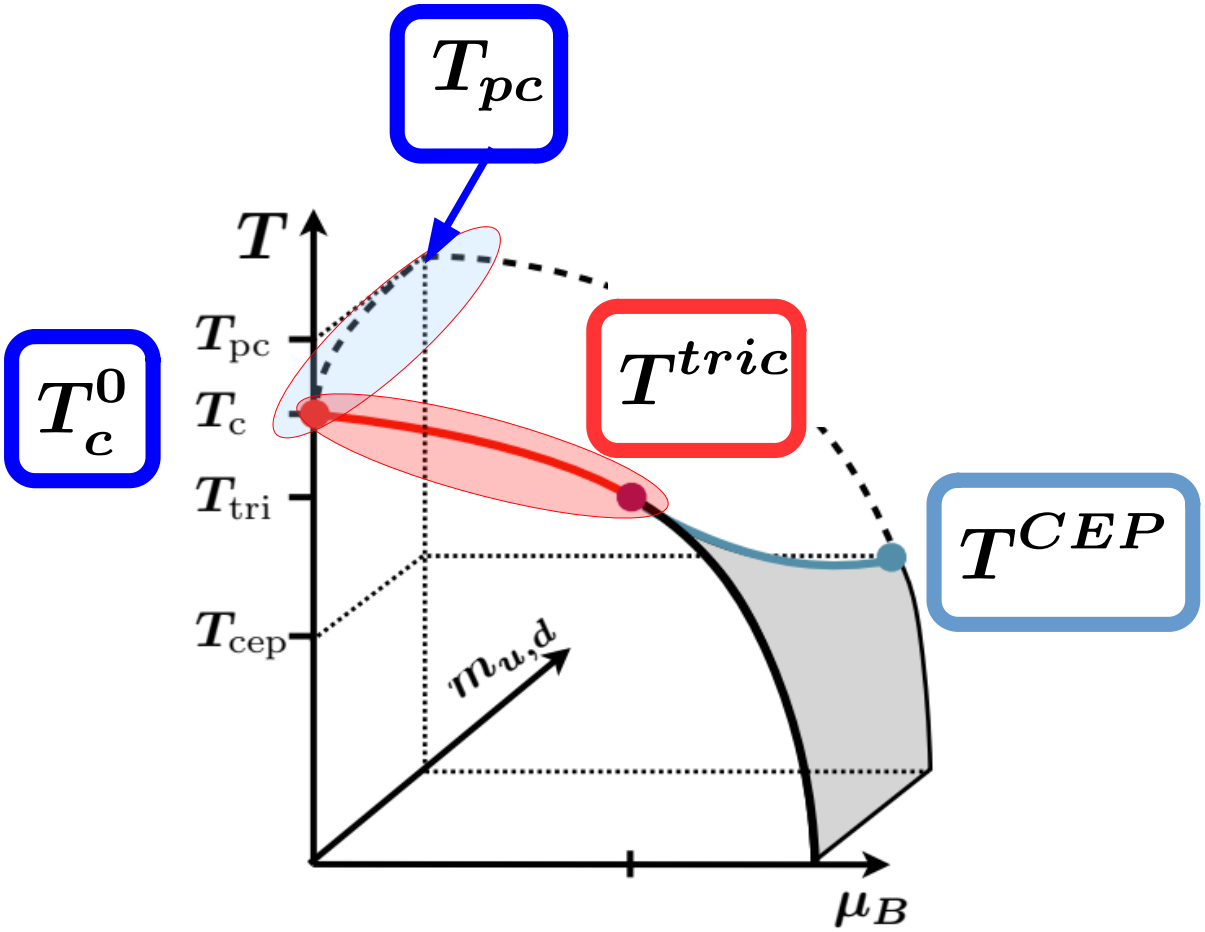
chiral limit extrapolations

$$T_c^0 = 132^{+3}_{-6}\ \text{MeV}$$

also: A. Y. Kotov et al., arXiv: 2105.09842

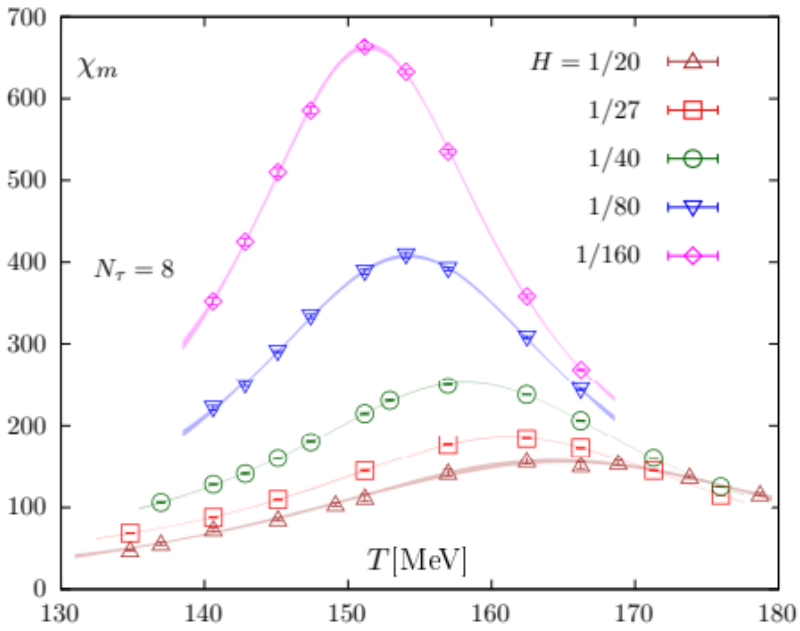
Phases of strong-interaction matter

determination of T_c^0 puts an upper limit on T^{CEP}



pseudo-critical temperatures from maxima of "appropriate" susceptibilities

– no unique definition –



$H = m_\ell/m_s$

$m_\pi = (160 - 55)\text{MeV}$

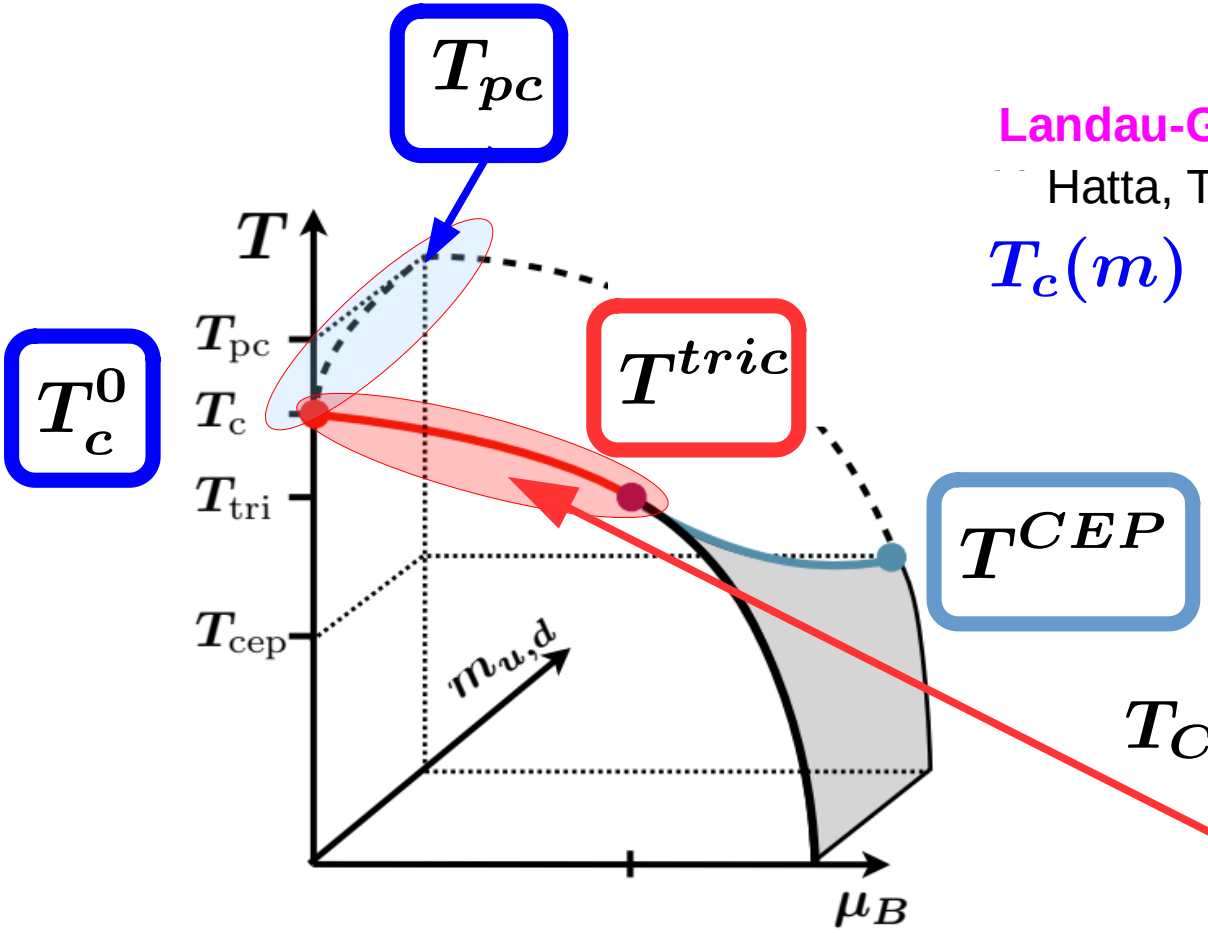
Random Matrix Model — A. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov, J.J.M. Verbaarschot, Phys. Rev. D58 (1998) 096007

QCD motivated — M. Stephanov, Phys. Rev. D73 (2006) 094508

NJL — M. Buballa, S. Carignano, Phys. Lett. B791 (2019) 361

Phases of strong-interaction matter

determination of T_c^0 puts an upper limit on T^{CEP}



Landau-Ginzburg potential:

Hatta, T. Ikeda, Phys. Rev. D67 (2003) 014028

$$T_c(m) = T^{tric} - \mathrm{c}\, m^{2/5}$$

$$\mathrm{non-universal,\ but\ c} > 0$$

$$T_{CEP} < T^{tric} < T_c^0 < T_{pc}^0$$

curvature in the chiral limit;
weak quark mass dependence:

H.-T. Ding et al. PRD 109 (2024) 114516

Random Matrix Model	A. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov, J.J.M. Verbaarschot, Phys. Rev. D58 (1998) 096007
QCD motivated	M. Stephanov, Phys. Rev. D73 (2006) 094508
NJL	M. Buballa, S. Carignano, Phys. Lett. B791 (2019) 361

determination of T_c^0 puts an upper limit on T^{CEP}

T_{pc}

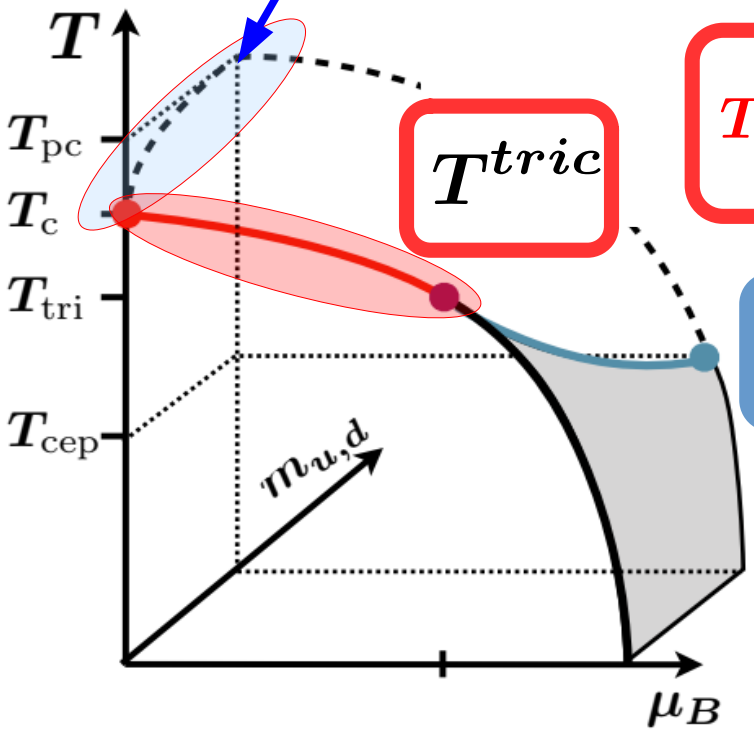
$T_{pc} = 156.5(1.5)\mathrm{MeV}$

$T_c^0 = 132^{+3}_{-6}\mathrm{MeV}$

A. Bazavov et al (HotQCD), arXiv:1812.08235

H.T. Ding et al (HotQCD), arXiv:1903.04801

T_c^0



T^{tric}

T^{CEP}

$$T^{tric} \simeq T_c^0 - 1.9(6)\ \mathrm{MeV} \left(\frac{\mu_B}{T_c}\right)^2_{tric}$$

Taylor series suggests

$$(\mu_B/T)_{tric} \geq 2$$

$$T^{CEP} < T^{tric} < T_c^0$$

$$T^{CEP} < 125\ \mathrm{MeV}$$

the temperature range below T_c^0 is most important for getting information on a possible CEP

upper limit on T^{CEP} puts constraint on HIC searches for the CEP

– pseudo-critical temperatures at physical quark mass values

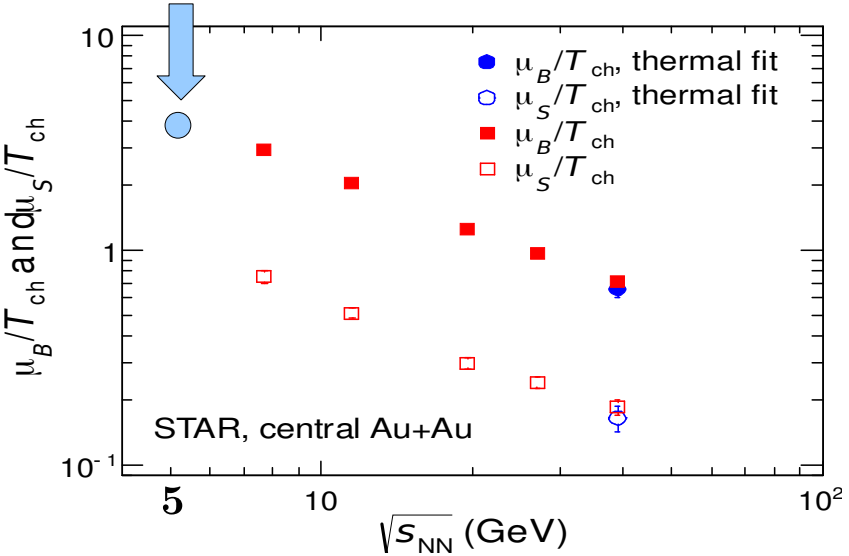
$$T_{pc}(\mu_B) = 156.5(1.5)\text{MeV}\left(1 - 0.012(4)\left(\frac{\mu_B}{T}\right)^2 - / + \ldots\right)$$

to reach $T < 125\text{MeV}$ $\quad$ $T < 110\text{MeV}$

need $\mu_B/T \simeq 4$ $\quad$ $\mu_B/T \simeq 5$

$\mu_B \geq 500\text{MeV}$ $\quad$ $\mu_B \gtrsim 550\text{MeV}$

my extrap.



Search for the Critical End Point requires beam energies

$$\sqrt{s_{NN}} \leq 5 \text{ GeV}$$

STAR freeze-out parameter, arXiv:1906.03732
also: HADES collaboration, arXiv:1512.07070

QCD thermodynamics at non-zero net baryon-density
– Taylor expansion –

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu_{B,Q,S}=0} , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

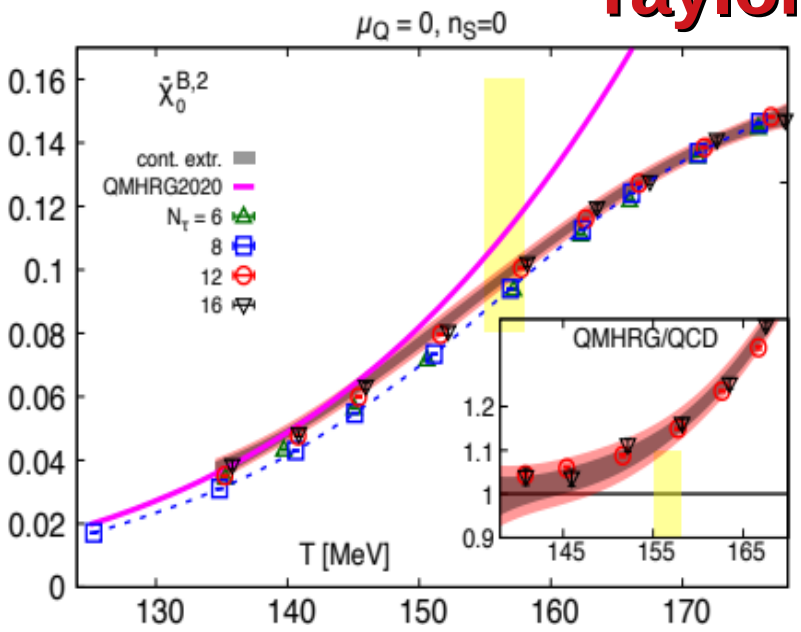
constrained Taylor series, demanding strangeness neutrality: $n_S = 0$

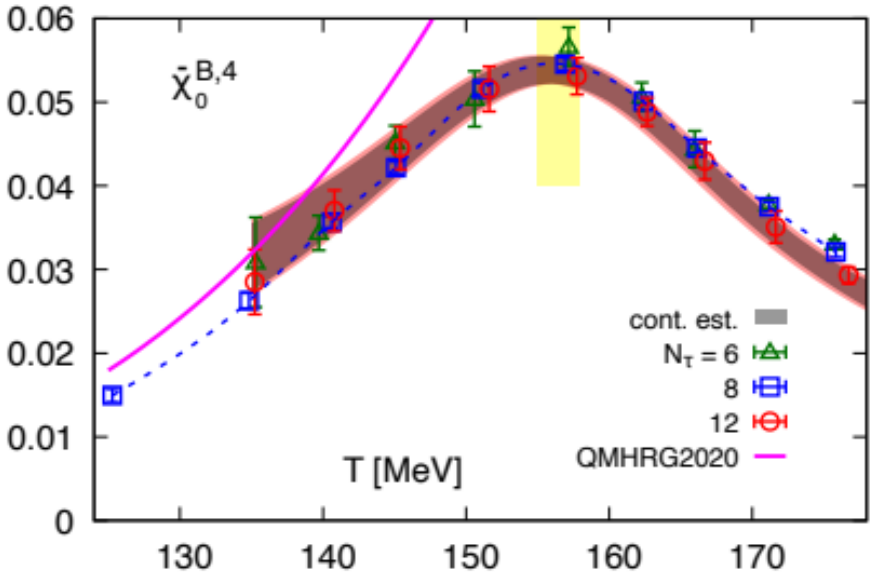
and: $n_Q/n_B = 0.4$

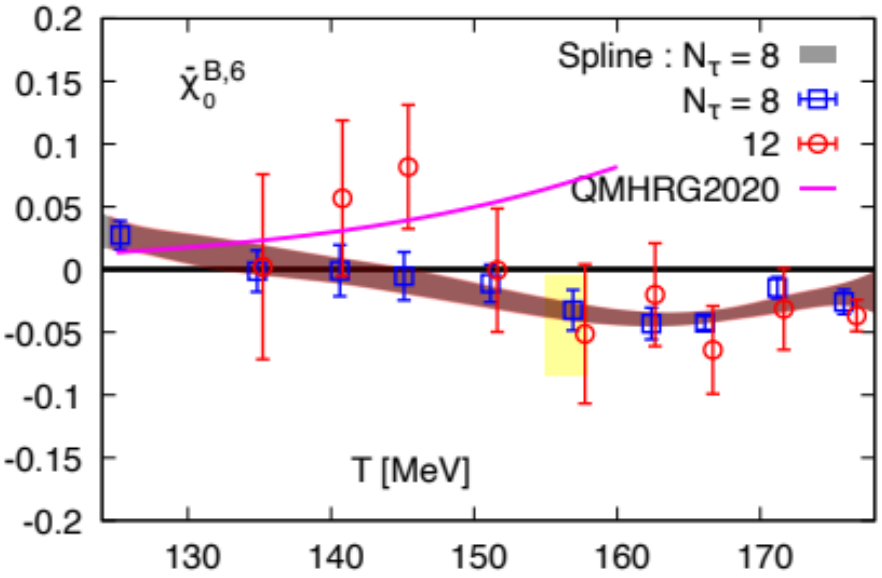
$$\begin{aligned} \hat{\mu}_Q(T, \mu_B) &= q_1(T)\ \hat{\mu}_B + q_3(T)\ \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5) \\ \hat{\mu}_S(T, \mu_B) &= s_1(T)\ \hat{\mu}_B + s_3(T)\ \hat{\mu}_B^3 + \mathcal{O}(\hat{\mu}_B^5) \end{aligned}$$

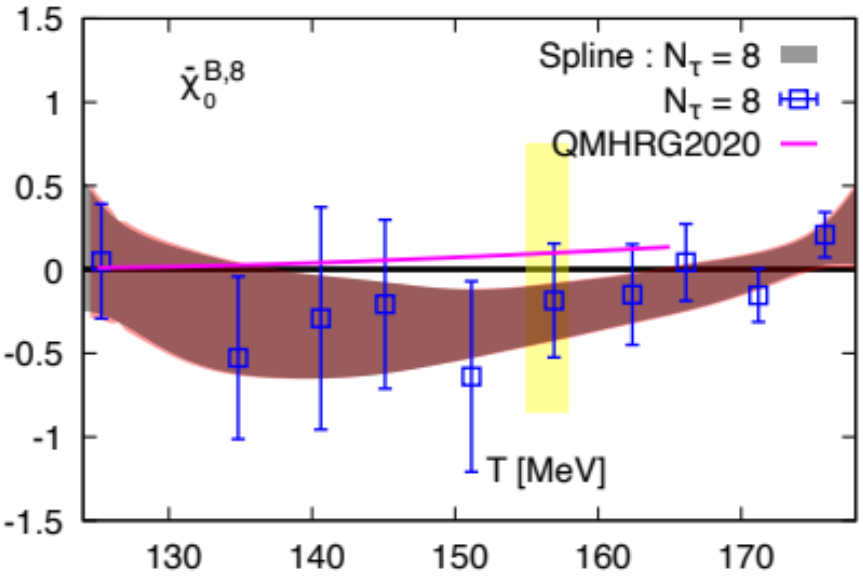
$$\frac{P}{T^4} = \sum_{k=0}^{\infty} \frac{1}{k!} \tilde{\chi}_k^B(T) \left(\frac{\mu_B}{T}\right)^k$$

Up to 8th order cumulants are used "frequently" – Taylor expansion –







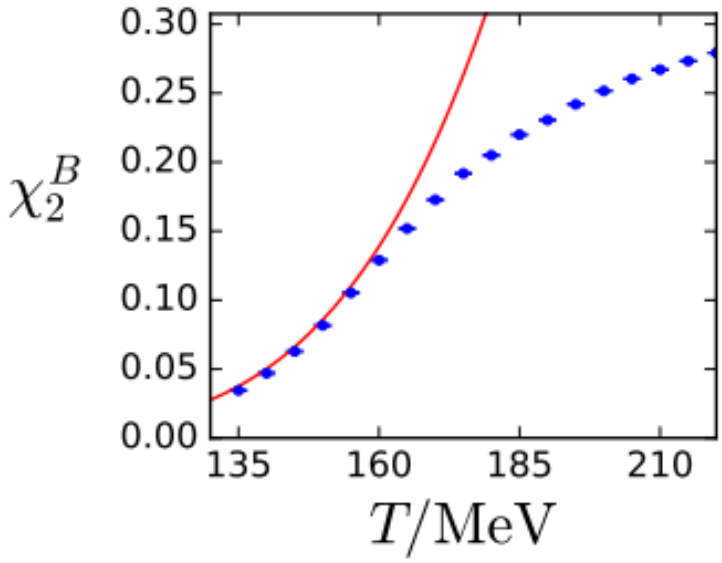


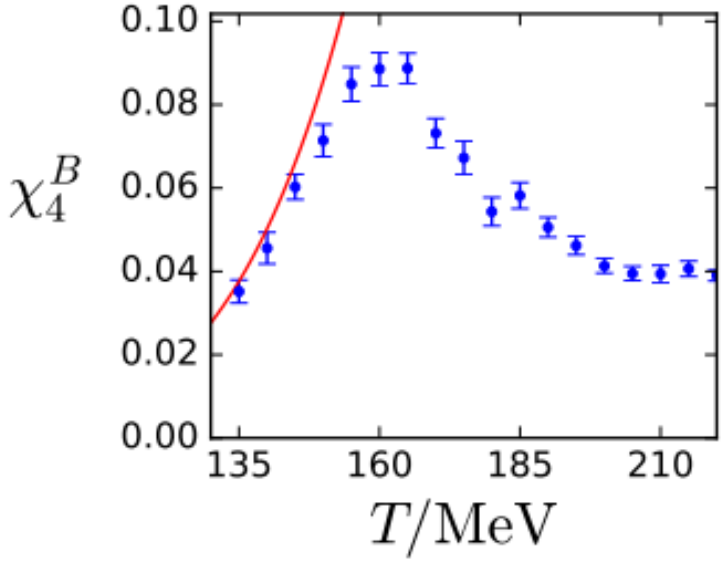
control basic features of skewness up to hyper-kurtosis ratios

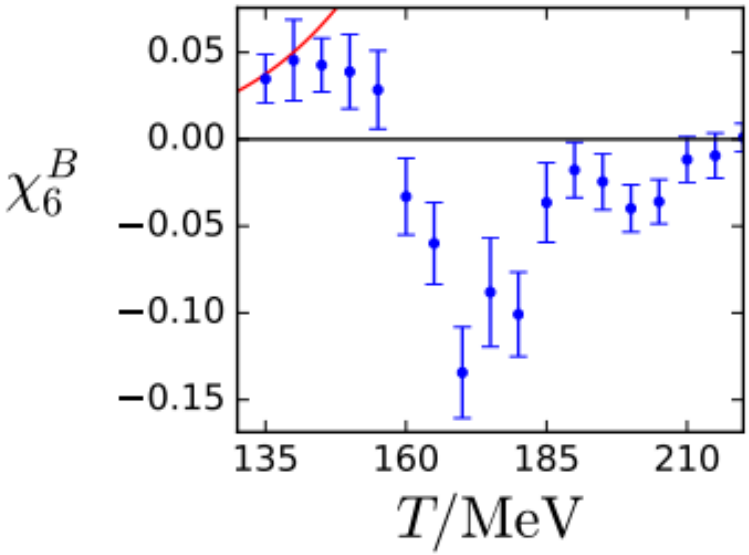
A. Bazavov et al. (HotQCD), Phys. Rev. D 105 (2022) 074511 arXiv:2202.09184

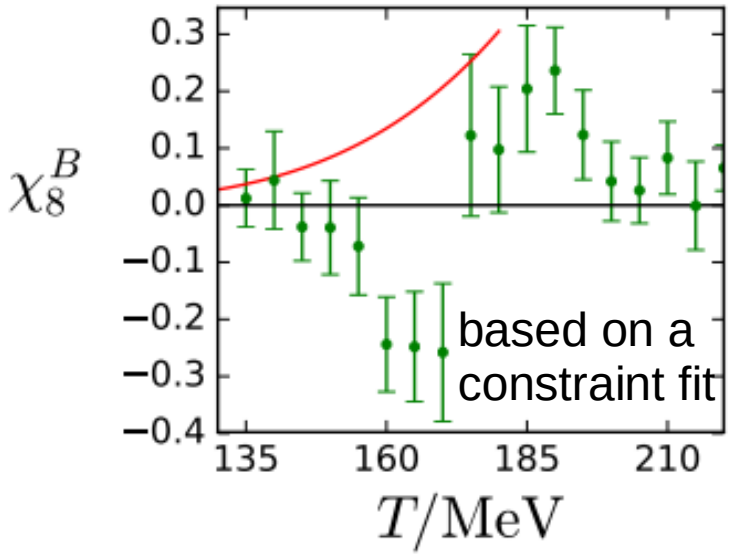
Up to 8th order cumulants are used frequently – imag. chem. pot. extrapolations –

$48^3 \times 12$









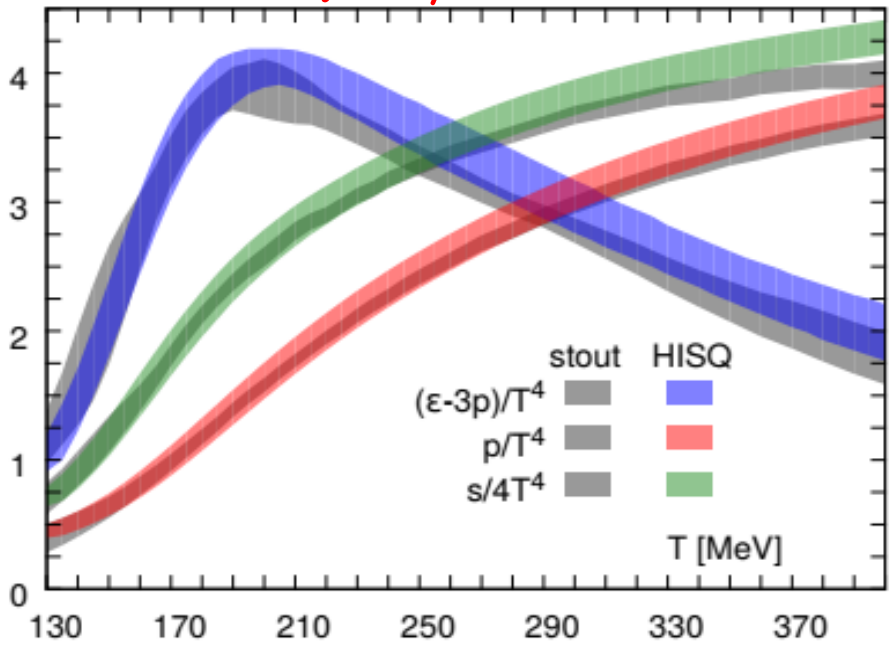
S. Borsanyi et al. , JHEP 10 (2018) 205, arXiv:1805.04445

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{\Delta p(T,\mu_B)}{T^4} = \frac{p(T,\mu_B) - p(T,0)}{T^4} = P_2(T)\left(\frac{\mu_B}{T}\right)^2 + P_4(T)\left(\frac{\mu_B}{T}\right)^4 + P_6(T)\left(\frac{\mu_B}{T}\right)^6 + \ldots$$

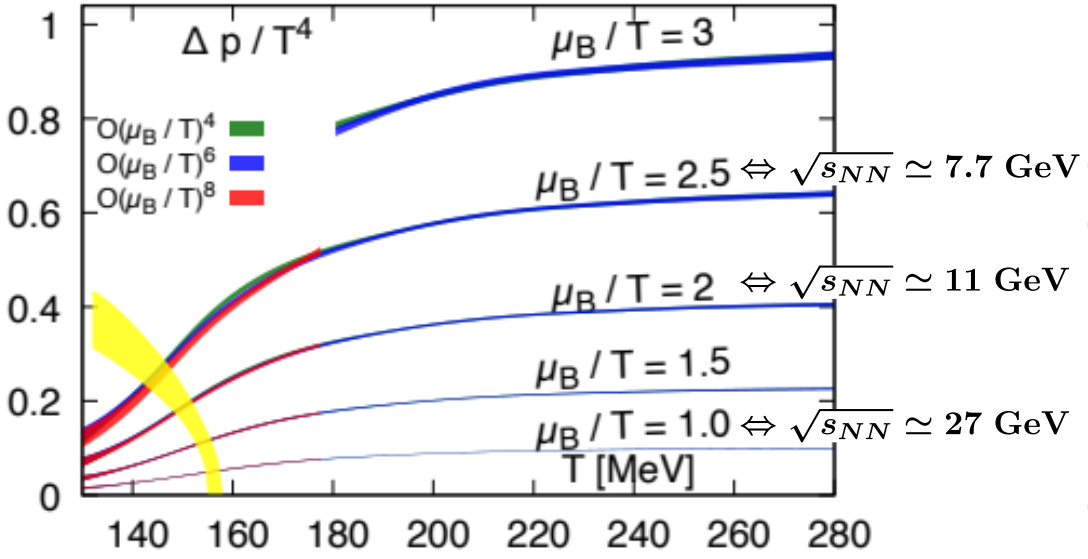
EoS 2017: $\mu_B = 0$

$\mu_B/T = 0$



EoS 2022: non-zero μ_B contribution

$\mu_B/T = 2$: (10-30)% contribution to total pressure at



BES-II range (collider mode)

A. Bazavov et al (HotQCD), arXiv:1701.04325

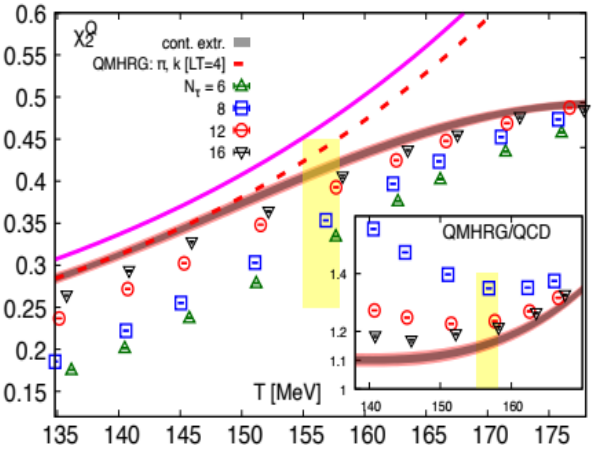
D. Bollweg et al (HotQCD), arXiv:2212.09043

2nd order cumulants: fluctuations and correlations

two constraints: only 4 out of 6
2nd order observables are independent

$T_c = (156.5 \pm 1.5)\mathrm{MeV}$

$\chi_2^B = 2\chi_{11}^{BQ} - \chi_{11}^{BS}$ $\chi_2^S = 2\chi_{11}^{QS} - \chi_{11}^{BS}$

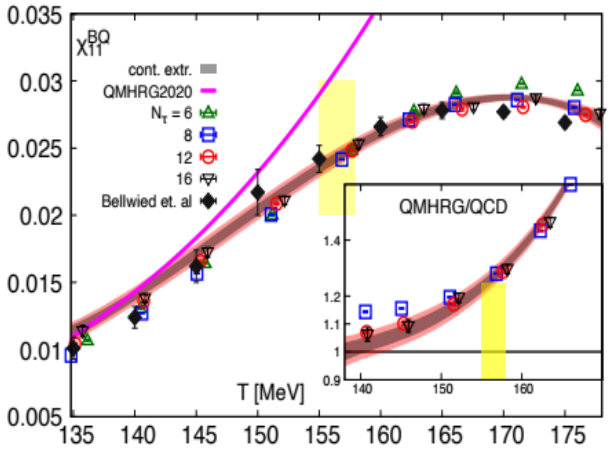


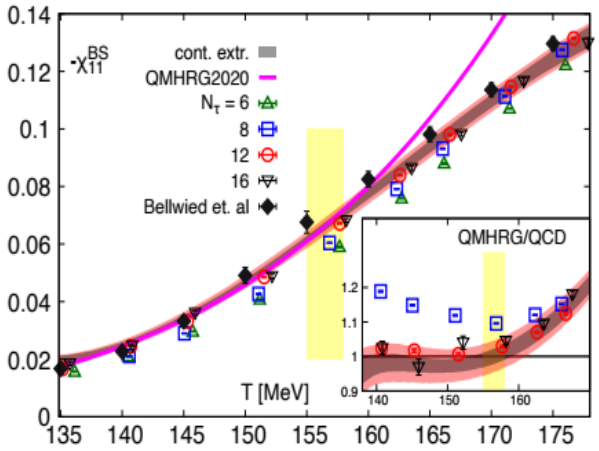
– exact relations in QCD for
– satisfied in HRG models to better than 1%

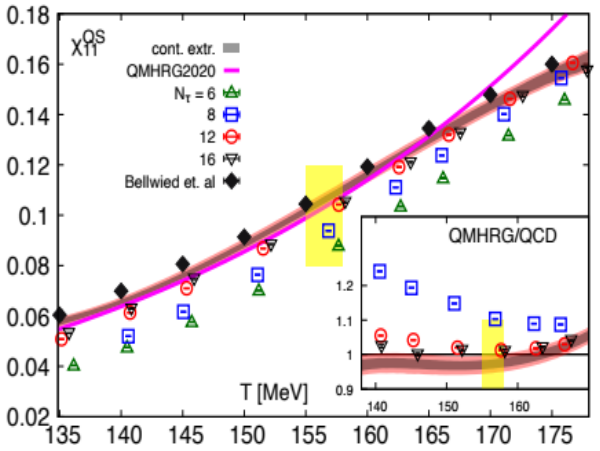
– **basic features:**

cumulants involving net electric charge differ earlier from HRG model calculations

cumulants involving strangeness agree with appropriately chosen HRG model up to T_{pc}





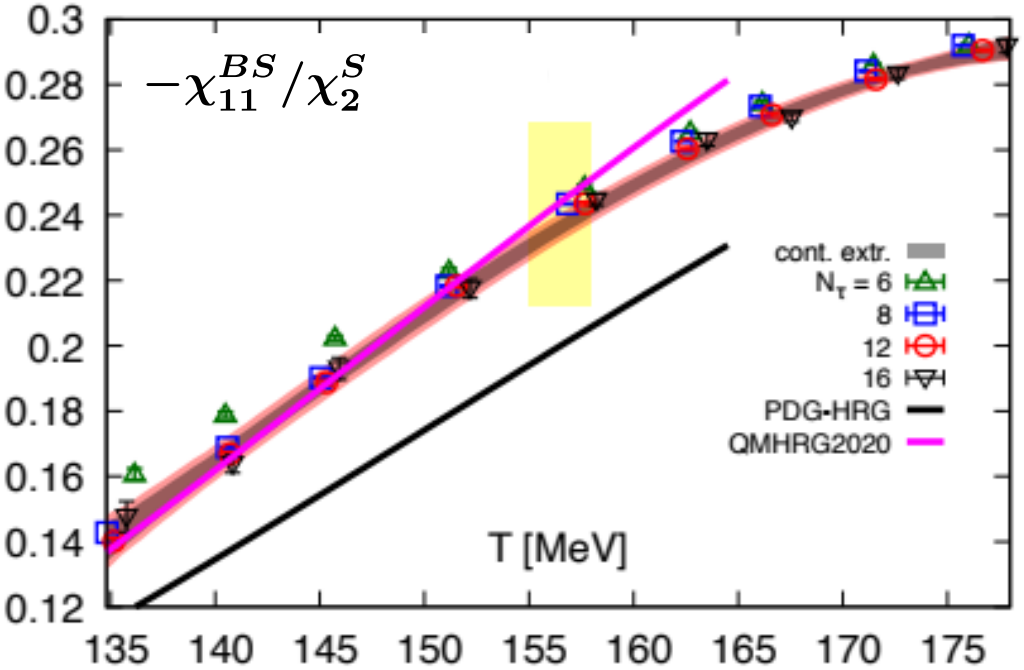


– continuum extrapolated results for all 2nd order cumulants in (2+1)-flavor QCD

D. Bollweg et al (HotQCD), arXiv:2107.10011

Ratio of baryon number – strangeness correlation and net strangeness fluctuations

$$\frac{\mu_S}{\mu_B} \equiv -\frac{\chi_{11}^{BS}}{\chi_2^S} - q_1 \frac{\chi_{11}^{QS}}{\chi_2^S} + \mathcal{O}(\mu_B^2)$$



- evidence for experimentally not yet observed strange baryons?
- evidence for strong flavor correlations

$$-\frac{\chi_{11}^{BS}}{\chi_2^S} = \frac{1}{3} + \frac{2}{3}\frac{\chi_{11}^{us}}{\chi_2^s}$$

D. Bollweg et al. (HotQCD), arXiv:2107.10011

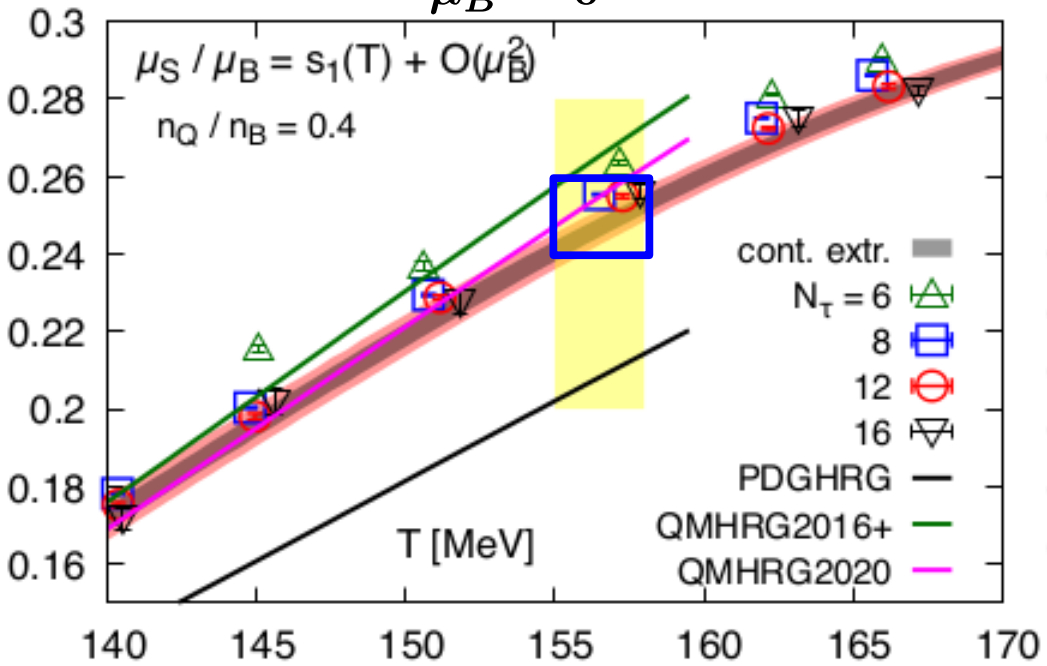
PDG-HRG: uses experimentally known hadron spectrum listed by the Particle Data Group
QM-HRG: uses additional hadrons predicted to exist in Quark Model calculations

Ratio of baryon number – strangeness correlation and net strangeness fluctuations

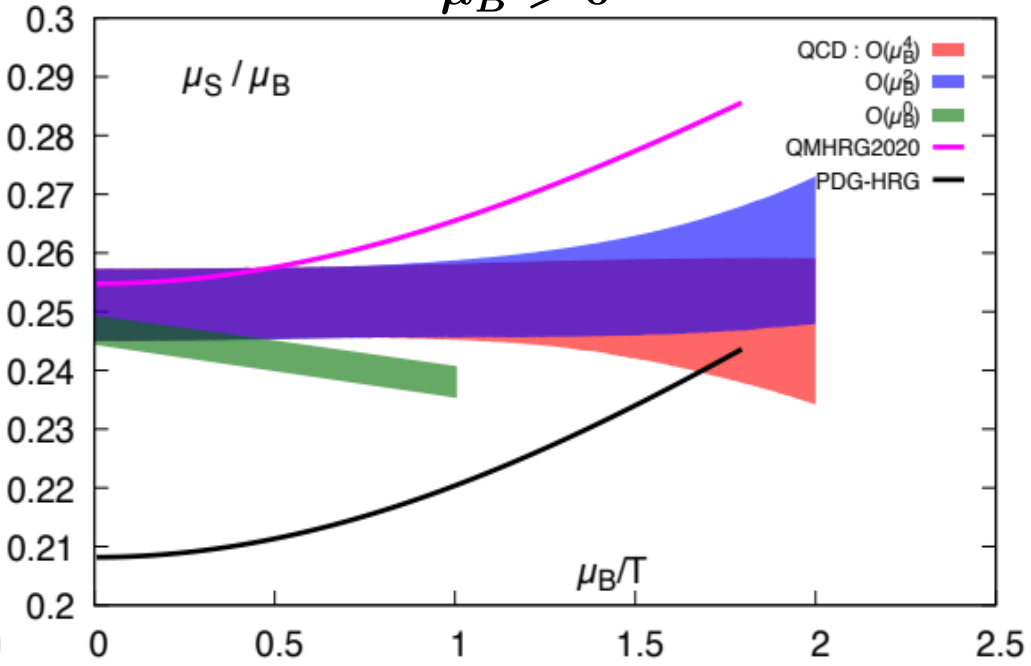
$$\frac{\mu_S}{\mu_B} \equiv -\frac{\chi_{11}^{BS}}{\chi_2^S} - q_1 \frac{\chi_{11}^{QS}}{\chi_2^S} + \mathcal{O}(\mu_B^2)$$

$$\left.\frac{\mu_S}{\mu_B}\right|_{T_{pc}(\mu_B)}$$ on the pseudo-critical line

$\mu_B = 0$



$\mu_B > 0$



HotQCD, arXiv:2107.10011

at T_{pc} QCD: $-\chi_{11}^{BS}/\chi_2^S = 0.236(11)$

$-q_1\chi_{11}^{QS}/\chi_2^S = 0.009(1)$

for $n_Q/n_B = 0.4 \Rightarrow \dfrac{\mu_S}{\mu_B} = 0.245(11)$

on $T_{pc}(\mu_B)$**:** Taylor series converges well up to $\mu_B/T \simeq 2$

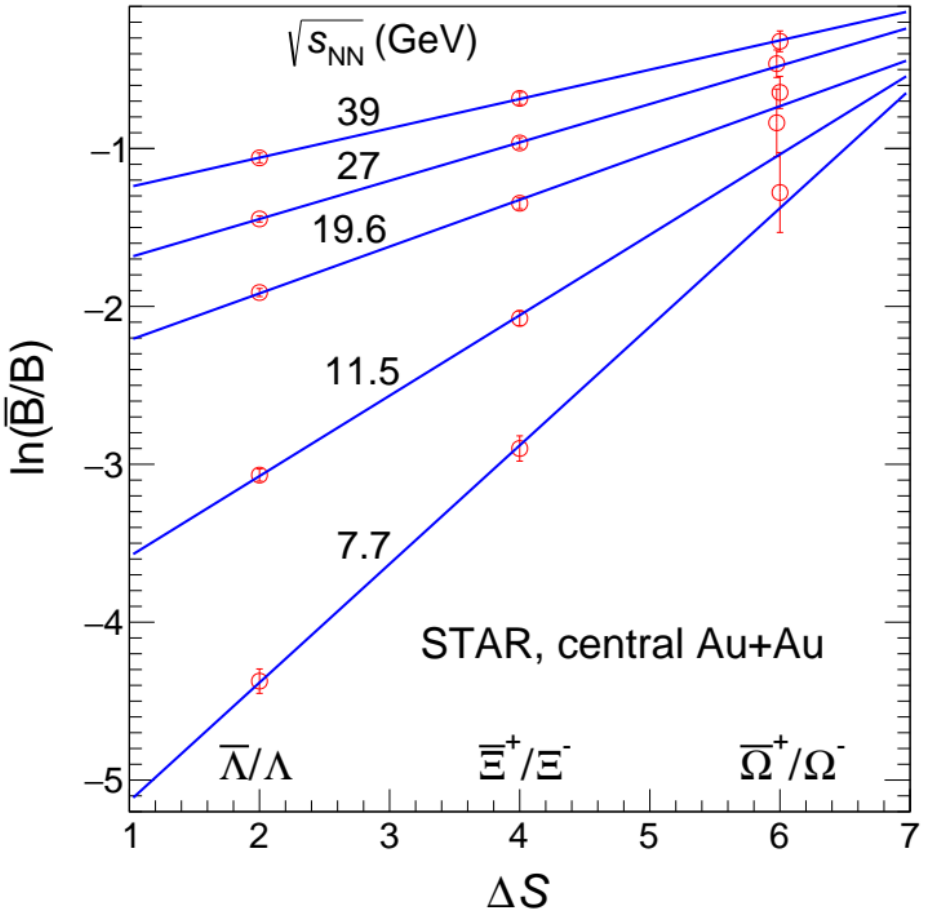
deviations from QMHRG increase with increasing μ_B/T: O(10%) at $\mu_B/T \simeq 2$

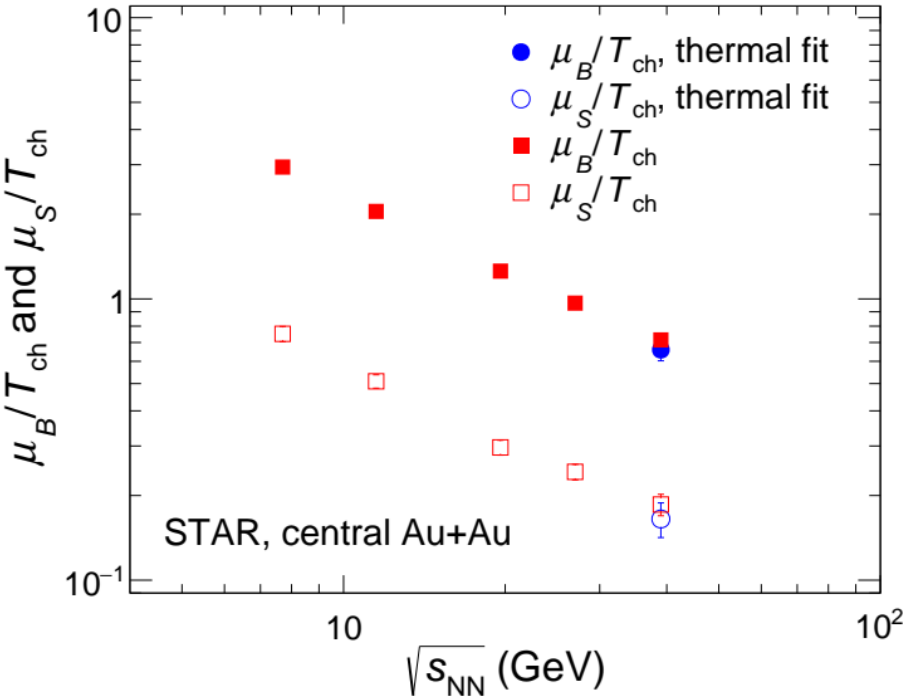
Freeze-out parameter from strange baryon yields

STAR2020: Phys. Rev. C 102 (2020) 034909

data are consistent with HRG-model ansatz: $Y_H \sim e^{(B\mu_B + Q\mu_Q + S\mu_S)/T}$

Determination of freeze-out parameters by comparing ratios of multi-strange baryon yields with HRG-motivated Boltzmann ansatz $\ln(\overline{B}/B) = -2\mu_B/T_{ch} + \mu_S/T_{ch} \cdot \Delta S$





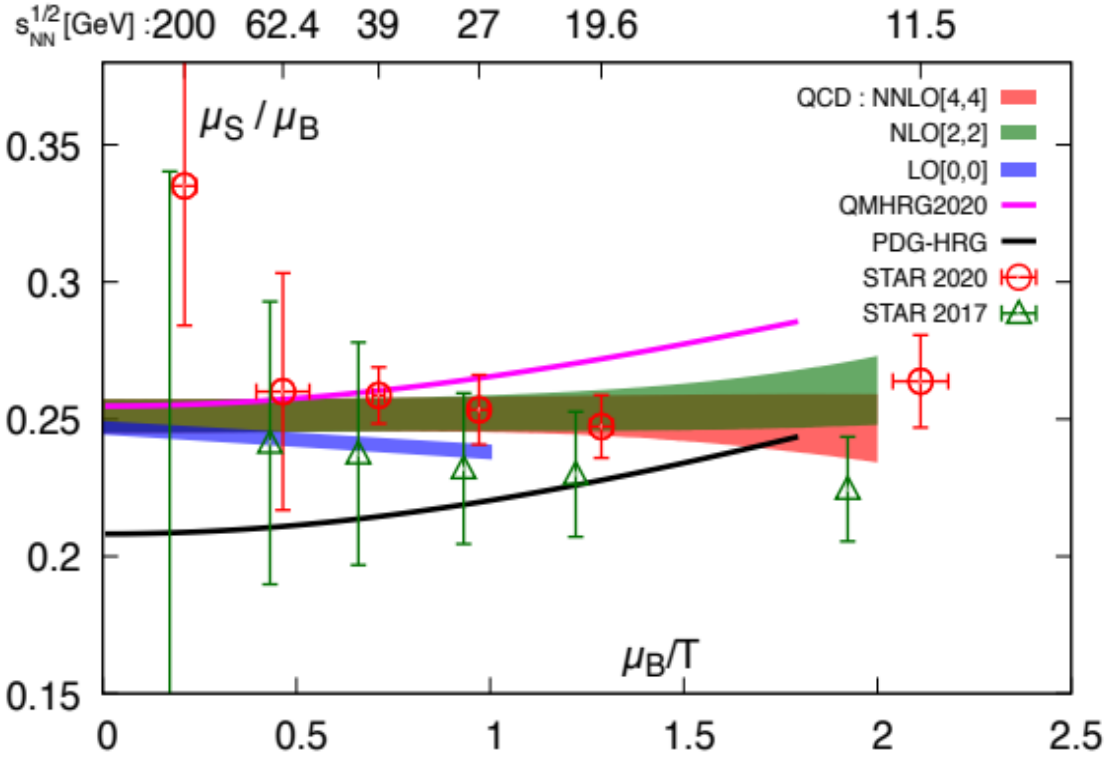
Baryon number – strangeness chemical potentials at freeze-out from strange baryon yields vs. QCD

QCD:

$$\frac{\mu_S}{\mu_B} \equiv -\frac{\chi_{11}^{BS}}{\chi_2^S} - q_1 \frac{\chi_{11}^{QS}}{\chi_2^S} + \mathcal{O}(\mu_B^2)$$

STAR:

$$\ln(\overline{\mathrm{B}}/\mathrm{B}) = -2\mu_B/T_{\mathrm{ch}} + \mu_S/T_{\mathrm{ch}} \cdot \Delta S$$

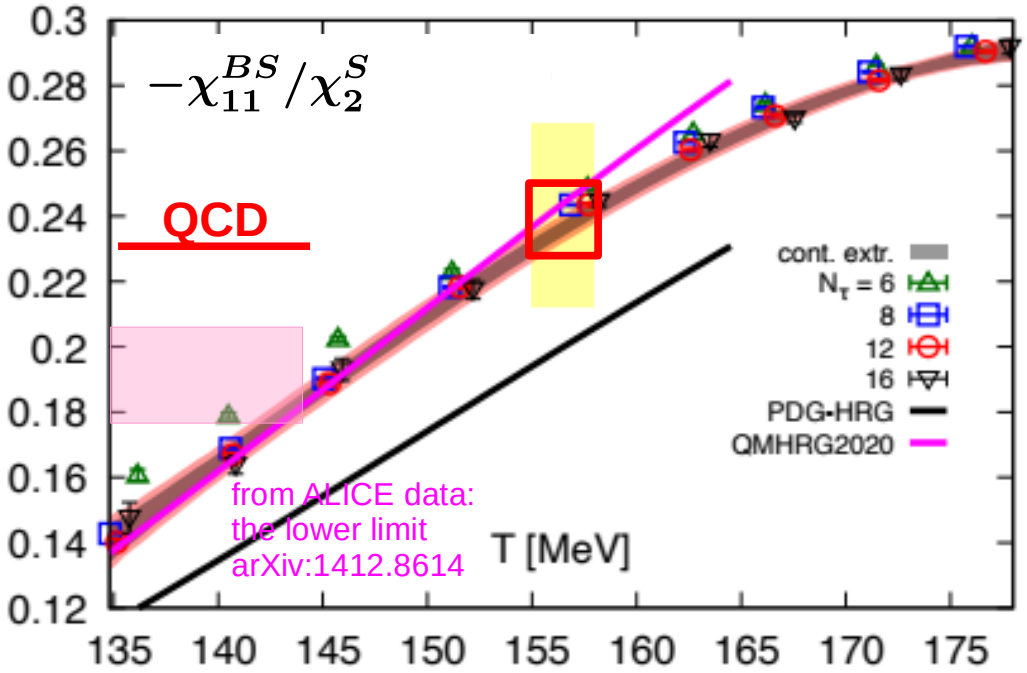


HotQCD, Phys. Rev. D 104 (2021) 074512
arXiv:2107.10011

STAR2017: Phys. Rev. C 96 (2017) 044904
STAR2020: Phys. Rev. C 102 (2020) 034909

STAR multi-strange baryon yields are consistent with freeze-out at T_{pc} and a μ_S/μ_B that reflects contributions from additional strange baryons

Baryon number – strangeness correlations
HIC at freeze-out vs. QCD on the pseudo-critical line



testing assumption of Skellam distribution:

for Skellam distributions mean values and second order cumulants as well as correlations are related:

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(\langle N_q\rangle + \langle N_{-q}\rangle)$$

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}\sum_{n=1}^{|q|} n^2(\langle N_n\rangle + \langle N_{-n}\rangle)$$

$$\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3}\sum_{n=-q_N}^{q_N}\sum_{m=-q_M}^{q_M} nm\langle N_{n,m}\rangle$$

$$|q| = 1\ (B),\ 2\ (Q),\ 3\ (S)$$

$$-\frac{\chi_{BS}}{T^2} > \frac{1}{VT^3}[2\langle\Lambda+\Sigma^0\rangle + 4\langle\Sigma^+\rangle + 8\langle\Xi\rangle + 6\langle\Omega^-\rangle] = 97.4 \pm 5.8.$$

unknown feed-down corrections, eg $\Sigma^* \to N\bar{K}$??

$$\frac{\chi_S}{T^2} \simeq \frac{1}{VT^3}[(\langle K^+\rangle + \langle K^0\rangle + \langle\Lambda+\Sigma^0\rangle + \langle\Sigma^+\rangle + \langle\Sigma^-\rangle + 4\langle\Xi^-\rangle + 4\langle\Xi^0\rangle + 9\langle\Omega^-\rangle + \text{antiparticles}) - (\Gamma_{\phi\to K^+} + \Gamma_{\phi\to K^-} + \Gamma_{\phi\to K^0} + \Gamma_{\phi\to\bar{K}^0})\langle\phi\rangle] = (504 \pm 24).$$

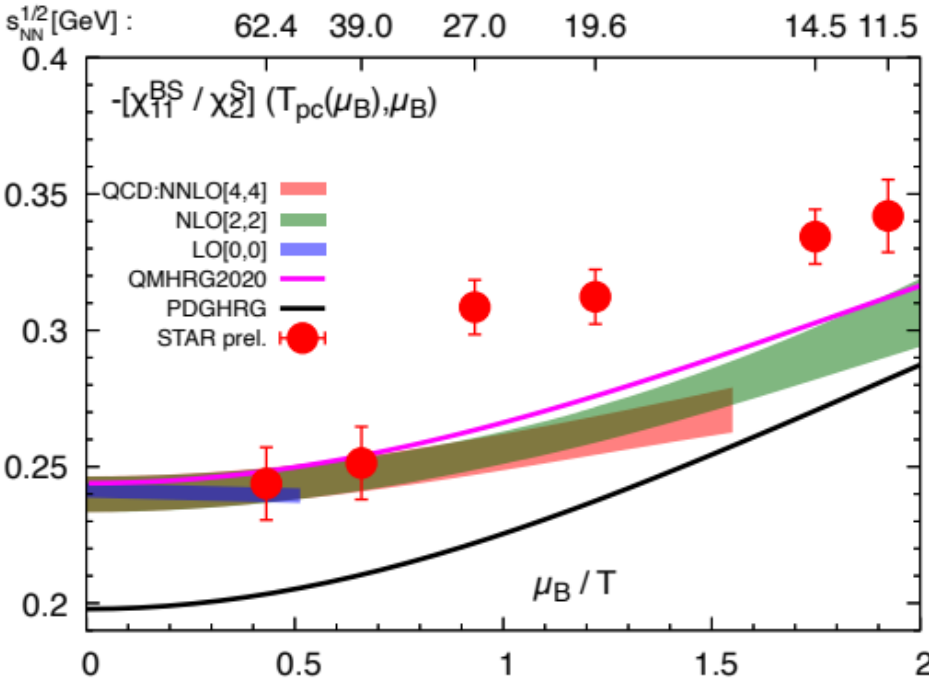
getting control over feed-down corrections, in particular to χ_{11}^{BS} is important!!!

$$\frac{\chi_{BS}}{\chi_S} \geq 0.193(22) + ??$$

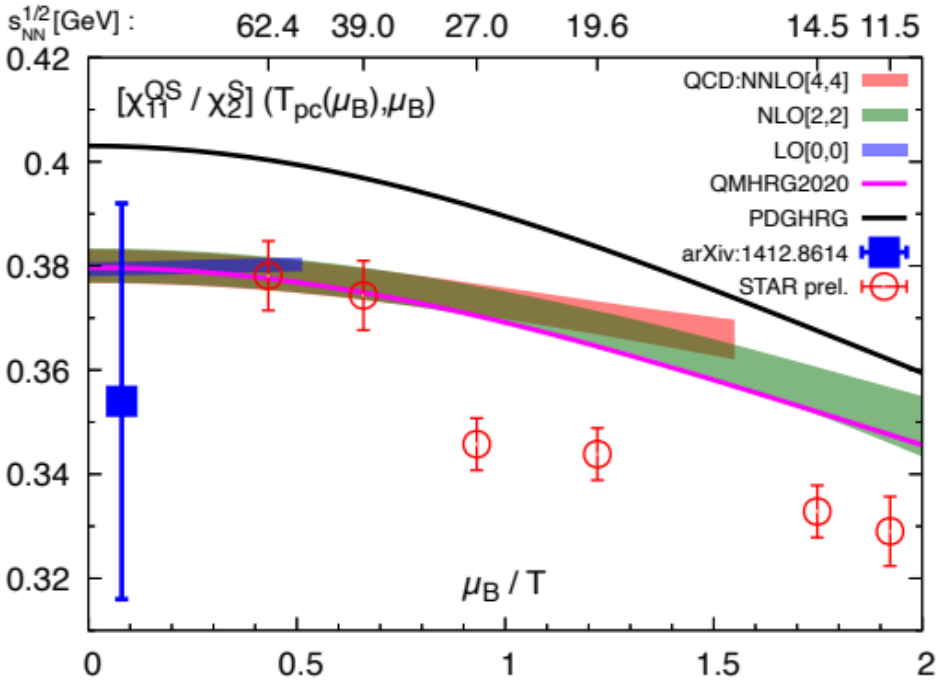
P. Braun-Munzinger, A. Kalweit, K. Redlich, J. Stachel, Phys. Lett. B747 (2015) 292, arXiv:1412.8614

Baryon number – strangeness correlations
HIC at freeze-out vs. QCD on the pseudo-critical line

QCD Taylor series up to $\mathcal{O}((\mu_B/T)^4)$ compared with preliminary STAR data



strangeness-electric charge correlations



H.W. Feng (STAR), CPOD 2024

D. Bollweg, H.-T. Ding, J.Goswami, FK, Swagato Mugherjee, P. Petreczky, C. Schmidt, to appear on arXiv on Monday

using that isospin violations are small in QCD:

$$2\frac{\chi_{11}^{QS}}{\chi_2^S} + \frac{\chi_{11}^{BS}}{\chi_2^S} \simeq 1$$

holds to better than 1%

Higher order net baryon-number cumulant ratios on the pseudo-critical line

A. Bazavov et al. (HotQCD), PRD 101 (2020) 074502
PRD 96 (2017) 074510

constrained Taylor series, demanding strangeness neutrality:

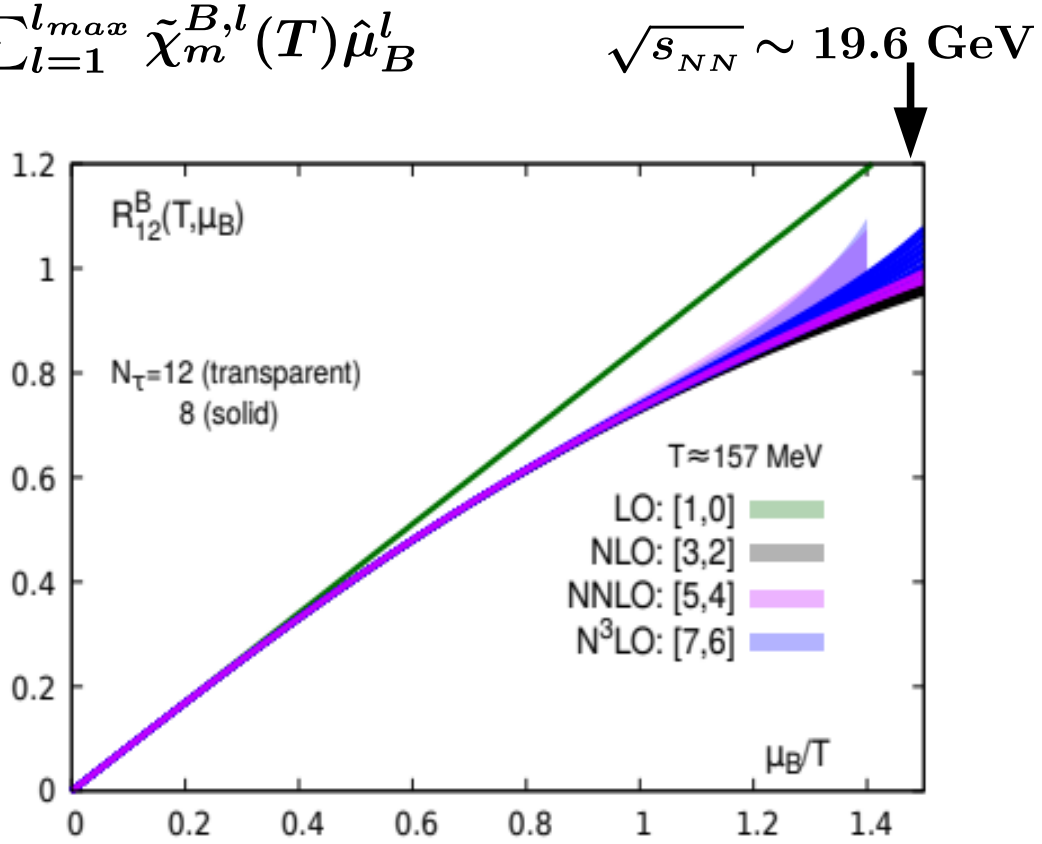
$n_S = 0$ and $n_Q/n_B = 0.4$

$$\Rightarrow \quad R^B_{nm} = \frac{\chi^B_n(T,\mu_B)}{\chi^B_m(T,\mu_B)} = \frac{\sum_{k=1}^{k_{max}} \tilde{\chi}^{B,k}_n(T)\hat{\mu}^k_B}{\sum_{l=1}^{l_{max}} \tilde{\chi}^{B,l}_m(T)\hat{\mu}^l_B}$$

mean over variance

$$R^B_{12} = (M/\sigma^2)_B = \frac{\chi^B_1}{\chi^B_2}$$

– good QCD observable to relate "physical observable" to model dependent chemical potential



Higher order net baryon-number cumulant ratios on the pseudo-critical line

A. Bazavov et al. (HotQCD), PRD 101 (2020) 074502
PRD 96 (2017) 074510

constrained Taylor series, demanding strangeness neutrality:

$n_S = 0$ and $n_Q/n_B = 0.4$

$$\Rightarrow \quad R^B_{nm} = \frac{\chi^B_n(T,\mu_B)}{\chi^B_m(T,\mu_B)} = \frac{\sum_{k=1}^{k_{max}} \tilde{\chi}^{B,k}_n(T)\hat{\mu}^k_B}{\sum_{l=1}^{l_{max}} \tilde{\chi}^{B,l}_m(T)\hat{\mu}^l_B}$$

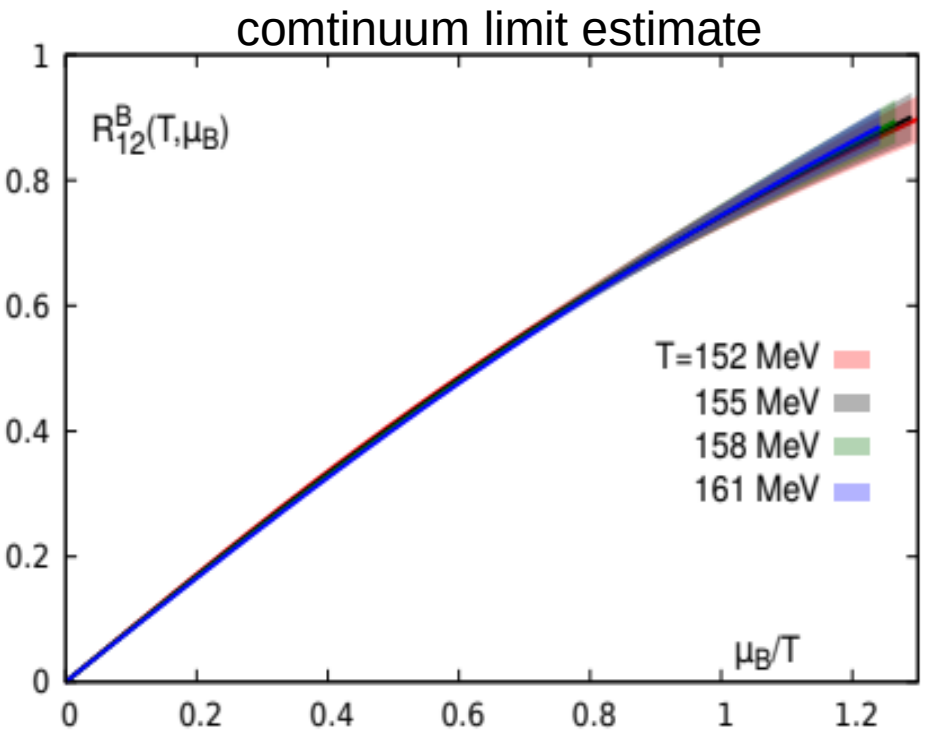
$\sqrt{s_{NN}} \sim 19.6$ GeV

mean over variance

$$R^B_{12} = (M/\sigma^2)_B = \frac{\chi^B_1}{\chi^B_2}$$

– good QCD observable to relate "physical observable" to model dependent chemical potential

– weak temperature dependence



Higher order net baryon-number cumulant ratios on the pseudo-critical line

A. Bazavov et al. (HotQCD), PRD 101 (2020) 074502
PRD 96 (2017) 074510

constrained Taylor series, demanding strangeness neutrality:

$n_S = 0$ and $n_Q/n_B = 0.4$

$$R^B_{nm} = \frac{\chi^B_n(T,\mu_B)}{\chi^B_m(T,\mu_B)} = \frac{\sum_{k=1}^{k_{max}} \tilde{\chi}^{B,k}_n(T)\hat{\mu}^k_B}{\sum_{l=1}^{l_{max}} \tilde{\chi}^{B,l}_m(T)\hat{\mu}^l_B}$$

new STAR data on proton-number ratio $R^p_{12}(\sqrt{s_{NN}})$ presented at CPOD 2024 by A. Pandav

mean over variance

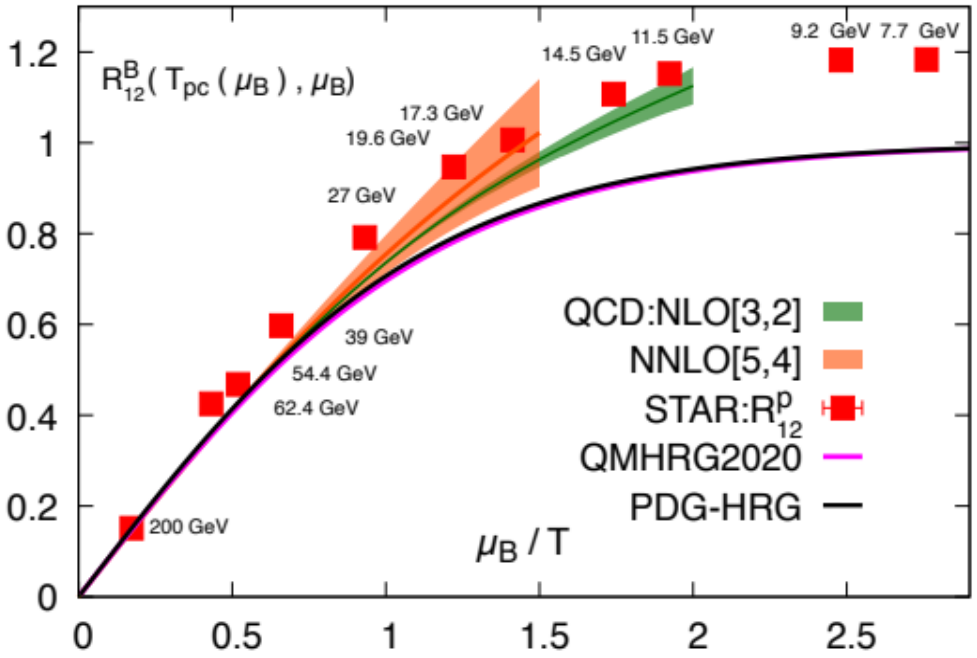
$$R^X_{12} = (M/\sigma^2)_X = \frac{\chi^X_1}{\chi^X_2}$$

X=B (QCD)
P (STAR)

– used STAR results on hadron yields to convert $\sqrt{s_{-NN}} \Leftrightarrow \mu_B$

L. Adamczyk (STAR), PRC 96 (2017) 044904

– **thermal conditions determined from STAR proton cumulants at freeze-out are consistent with those determined in QCD from baryon cumulants on the pseudo-critical line**



D. Bollweg et al., to appear on arXiv on Monday

Higher order cumulant ratios on the pseudo-critical line

A. Bazavov et al. (HotQCD), PRD 101 (2020) 074502
PRD 96 (2017) 074510

skewness

$$R_{31}^B = \left(\frac{S\sigma^3}{M}\right)_B = \frac{\chi_3^B}{\chi_1^B}$$

kurtosis

$$R_{42}^B = (\kappa\sigma^2)_B = \frac{\chi_4^B}{\chi_2^B}$$

$\mathcal{O}(\mu_B^4)$ Taylor series $\Longleftrightarrow$ $\mathcal{O}(\mu_B^8)$ Taylor series for pressure

$$R_{nm}^B = \frac{\chi_n^B(T,\mu_B)}{\chi_m^B(T,\mu_B)} = \frac{\sum_{k=1}^{k_{max}} \tilde{\chi}_n^{B,k}(T)\hat{\mu}_B^k}{\sum_{l=1}^{l_{max}} \tilde{\chi}_m^{B,l}(T)\hat{\mu}_B^l} \qquad n_S = 0,\ n_Q/n_B = 0.4$$

$$R_{31}^B(T,\mu_B) = r_{31}^{B,0}(T) + r_{31}^{B,2}(T)\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

$$R_{42}^B(T,\mu_B) = r_{42}^{B,0}(T) + r_{42}^{B,2}(T)\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

$\mu_S = \mu_Q = 0$

$$r_{42}^{B,0} = r_{31}^{B,0}$$

$$r_{42}^{B,2} = 3 r_{31}^{B,2}$$

$n_S = 0,\ n_Q/n_B = 0.4$

$$\frac{r_{42}^{B,0}}{r_{31}^{B,0}} = \frac{1 + s_1\frac{\chi_{11}^{BS}}{\chi_2^B} + q_1\frac{\chi_{11}^{BQ}}{\chi_2^B}}{1 + s_1\frac{\chi_{31}^{BS}}{\chi_4^B} + q_1\frac{\chi_{31}^{BQ}}{\chi_4^B}}$$

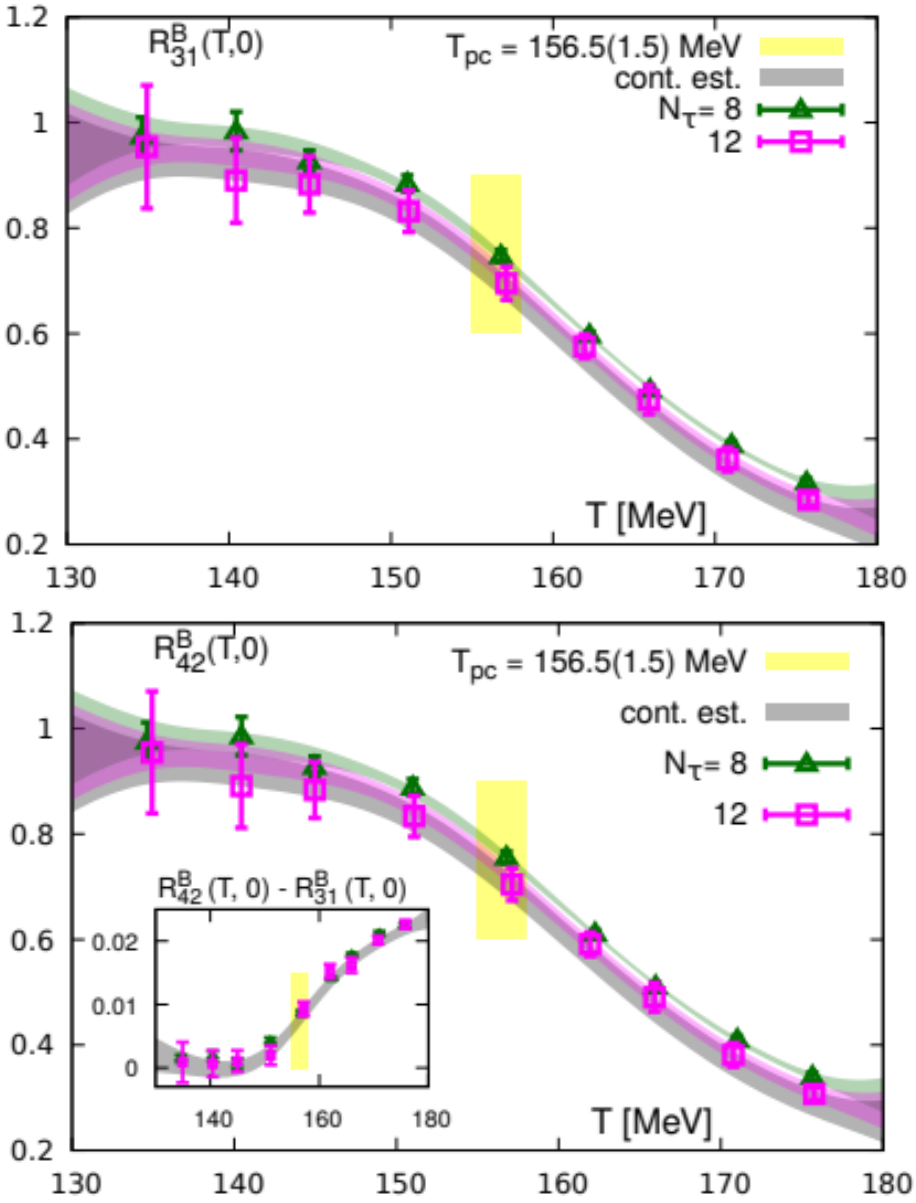
$T_{pc}(\mu_B) = T_0(1 - \kappa_2^f(\mu_B/T)^2)$

$$r_{nm}^{B,k} \rightarrow r_{nm,f}^{B,k} \equiv r_{nm}^{B,k}(T_0) - \kappa_2^f T_0 \left.\frac{\mathrm{d}r_{nm}^{B,k-2}}{\mathrm{d}T}\right|_{T=T_0}$$

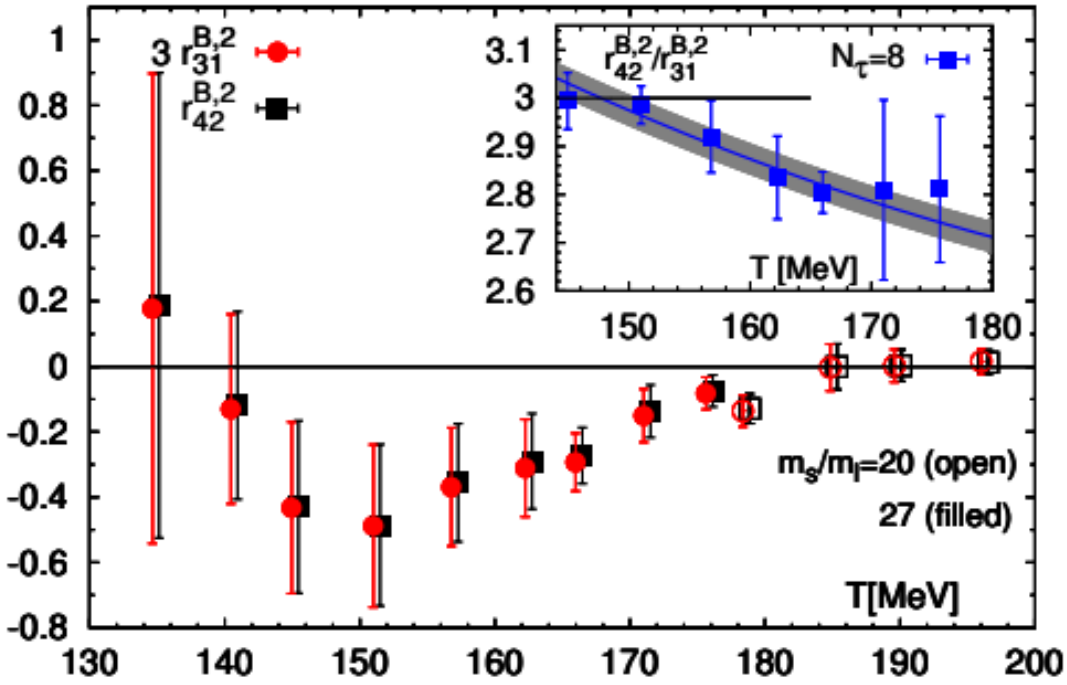
HotQCD, Phys.Rev.D 96 (2017) 074510, arXiv:1708.04897

Higher order cumulant ratios on the pseudo-critical line

A. Bazavov et al. (HotQCD), PRD 101 (2020) 074502
PRD 96 (2017) 074510



$$n_S = 0,\ n_Q/n_B = 0.4$$



$$\mu_S = \mu_Q = 0$$

$$r_{42}^{B,0} = r_{31}^{B,0}$$

$$r_{42}^{B,2} = 3 r_{31}^{B,2}$$

still well fulfilled also for

$$n_S = 0$$

$$n_Q/n_B = 0.4$$

Higher order cumulant ratios on the pseudo-critical line

A. Bazavov et al. (HotQCD), PRD 101 (2020) 074502
PRD 96 (2017) 074510

skewness

$$R^B_{31} = \left(\frac{S\sigma^3}{M}\right)_B = \frac{\chi^B_3}{\chi^B_1}$$

kurtosis

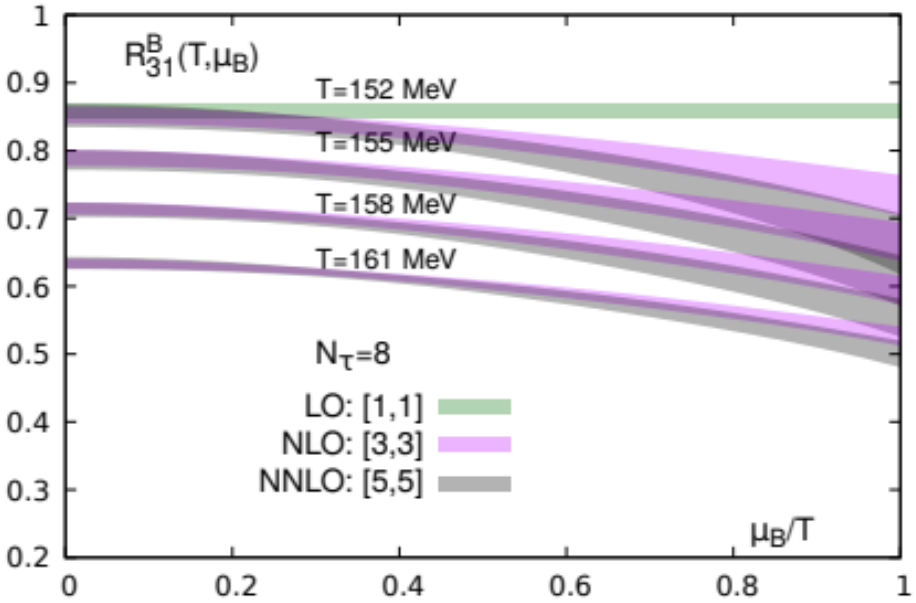
$$R^B_{42} = (\kappa\sigma^2)_B = \frac{\chi^B_4}{\chi^B_2}$$

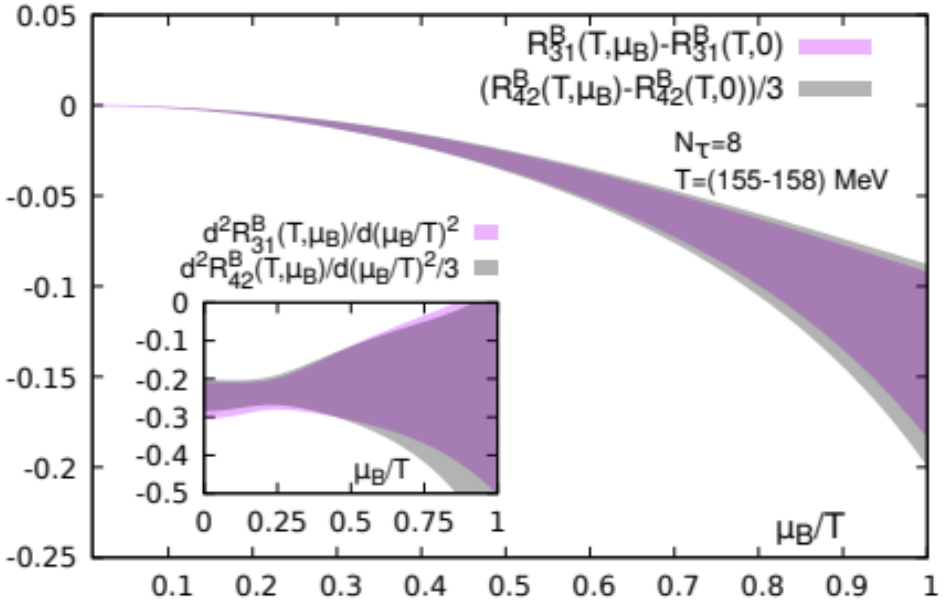
$$n_S = 0,\ n_Q/n_B = 0.4$$

$\mathcal{O}(\mu_B^4)$ Taylor series $\Leftrightarrow$ $\mathcal{O}(\mu_B^8)$ Taylor series for pressure

$$R^B_{nm} = \frac{\chi^B_n(T,\mu_B)}{\chi^B_m(T,\mu_B)} = \frac{\sum_{k=1}^{k_{max}} \tilde{\chi}^{B,k}_n(T)\hat{\mu}^k_B}{\sum_{l=1}^{l_{max}} \tilde{\chi}^{B,l}_m(T)\hat{\mu}^l_B}$$

also at NNLO curvature of R^B_{42} is about a factor 3 larger than that of R^B_{31}





Skewness and kurtosis ratios

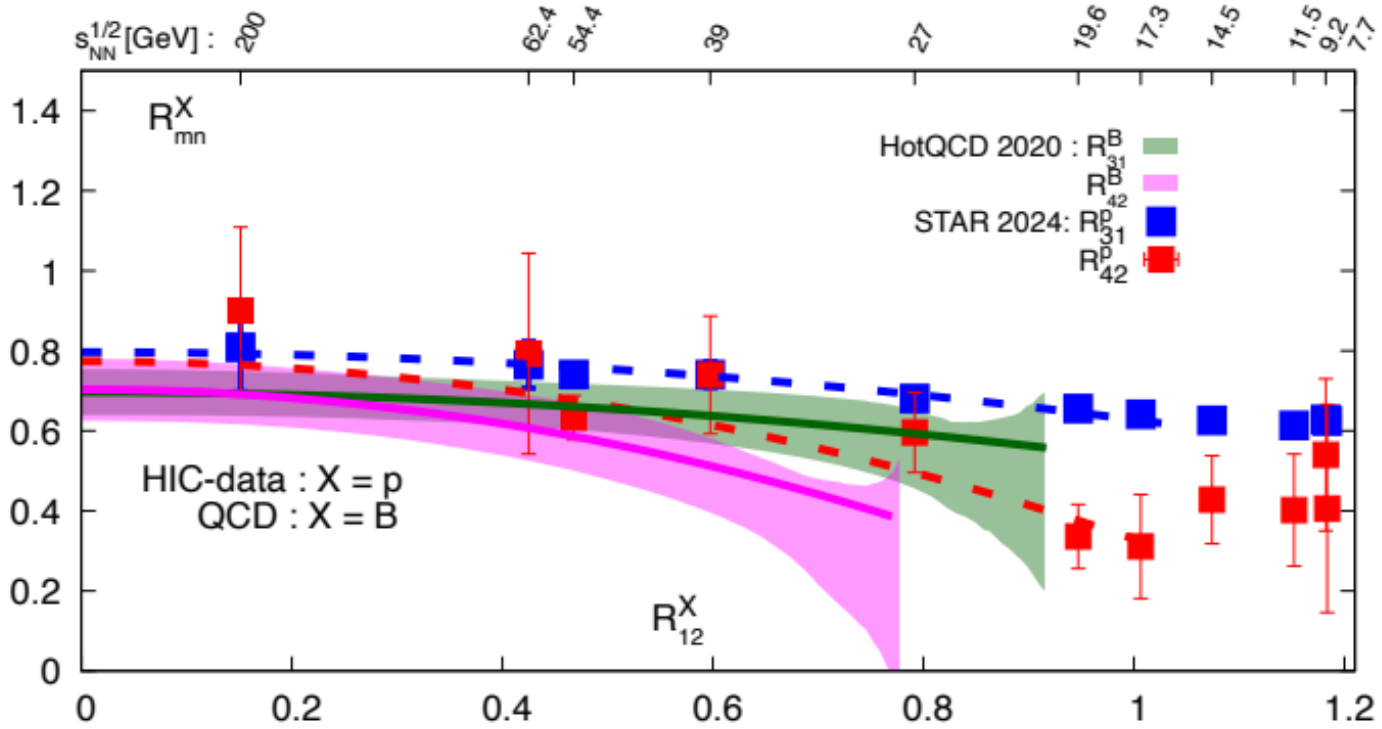
QCD on the pseudo-critical line vs. STAR data at freeze-out

$\mathcal{O}(\mu_B^4)$ QCD for $n_S = 0,\ n_Q/n_B = 0.4$
on the pseudo-critical line

new STAR data:
presented at CPOD 2024
by A. Pandav

$\underline{\mu_B = 0}$

$$\left.\frac{\chi_4^B}{\chi_2^B}\right|_{T_{pc}(0)} = 0.69(3)$$



STAR, CPOD 2024

$$R^p_{31} = 0.80(1) - 0.17(2)\,(R^p_{12})^2$$

$$R^p_{42} = 0.77(6) - 0.44(12)\,(R^p_{12})^2$$

fit for $0 \le R^p_{12} \le 1$

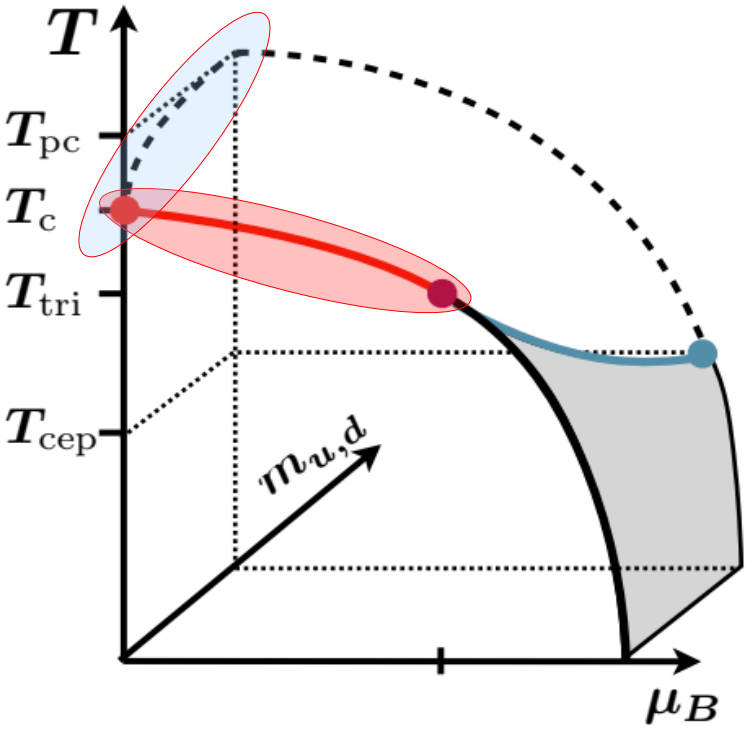
HotQCD 2017, 2020:

$$R^B_{31} = 0.70(x) - 0.169(x)\,(R^B_{12})^2$$

$$R^B_{42} = 0.70(x) - 0.56(x)\,(R^B_{12})^2$$

fit for $0 \le R^B_{12} \le 0.5$

Conclusions

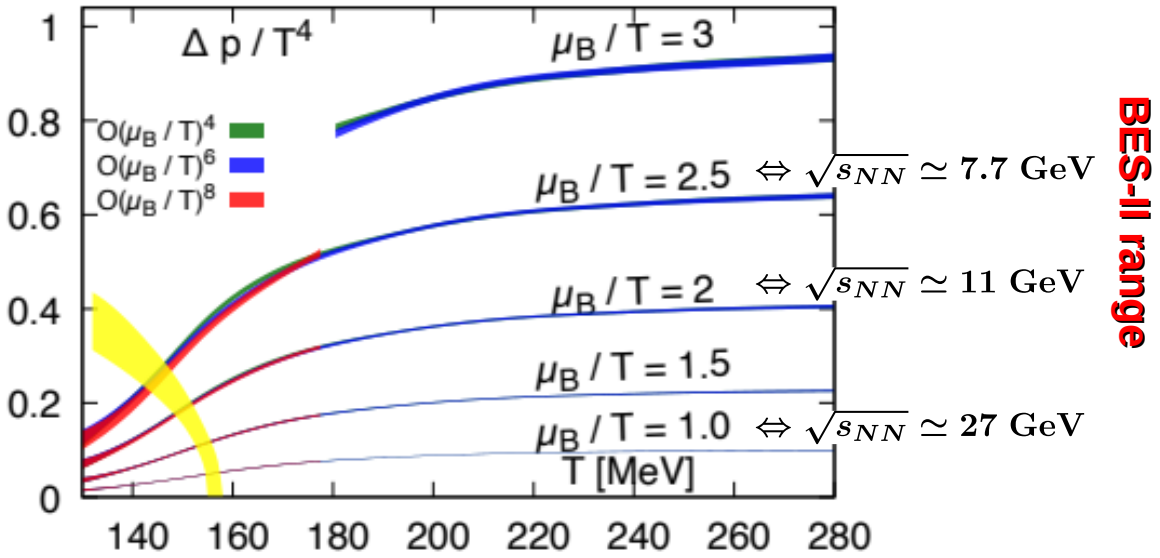


What we learned so far about the CEP in QCD from lattice QCD calculations:

I) the critical temperature is below $T_c = 132^{+3}_{-6}$ MeV

II) the corresponding critical chemical potential is likely to be above 500 MeV

→ Taylor expansions need to be resummed in order to reach higher μ_B/T



BES-II range

– no CEP for $\mu_B/T \leq 2.5$

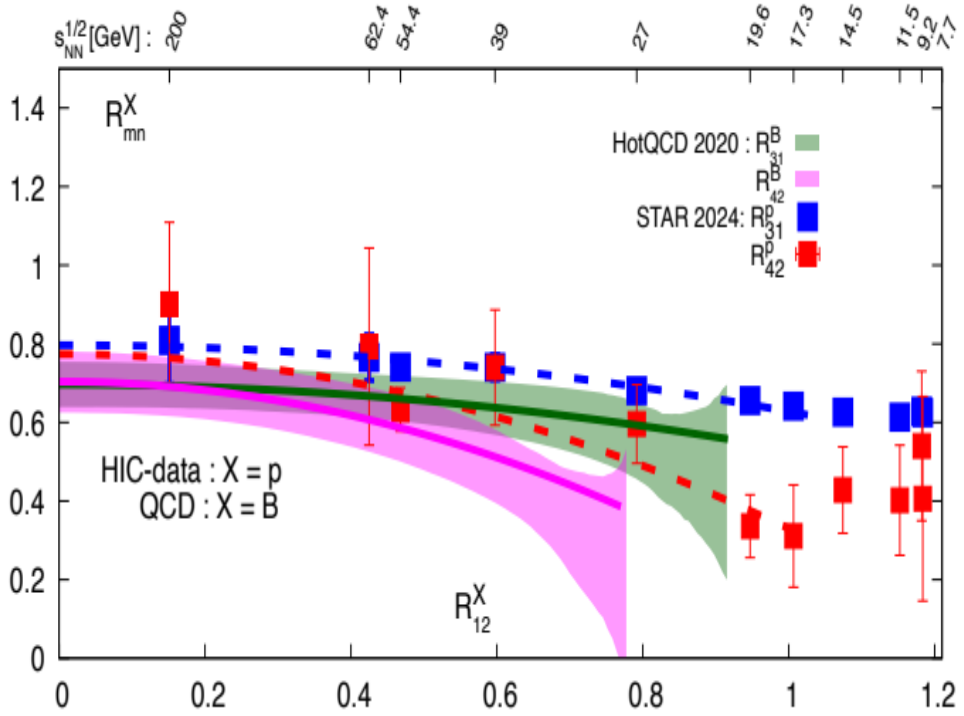
– CEP not in the BES-II range (in collider mode)

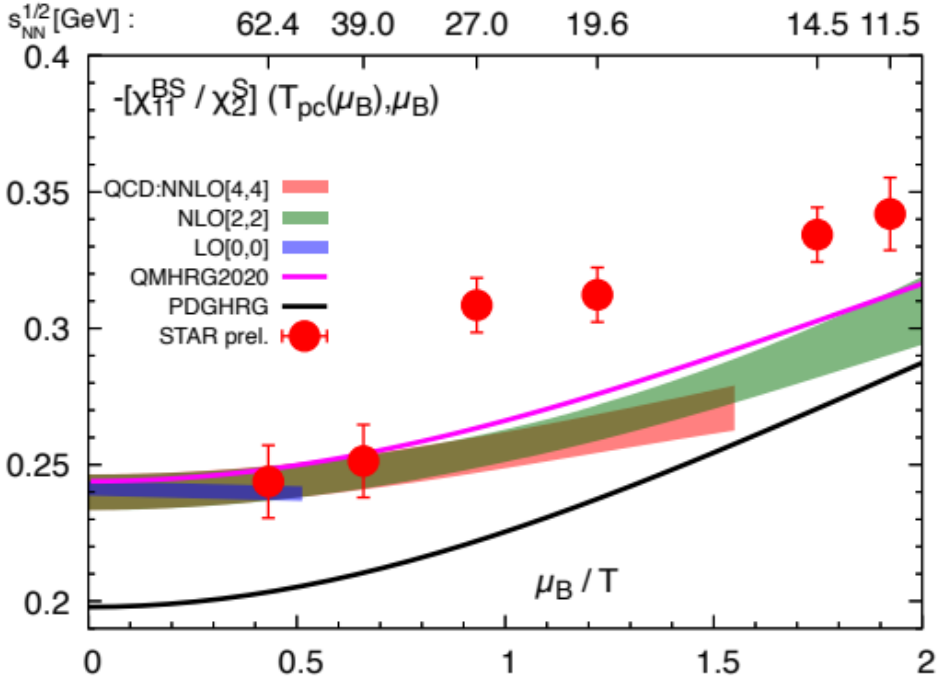
– EoS (pressure & number density) well controlled for

$\mu_B/T \leq 2.0 \; \forall T > 135$ MeV

(larger range for higher T)

– reliable μ_B - range is smaller for higher order cumulants, given only an 8th order Taylor series for the pressure





Skewness and kurtosis ratios

- QCD and STAR results are in good agreement for $\sqrt{s_{NN}} \geq$ **19.6** GeV
- $\mathcal{O}((R^B_{12})^2)$ QCD results and quadratic fit to STAR results for R^p_{31} and R^p_{42} agree well on curvature coefficient.
- difference in LO coefficients may suggest that freeze-out temperature is slightly below T_{pc}.

Baryon number – Strangeness correlations

- QCD and STAR results are in good agreement for $\sqrt{s_{NN}} \geq$ **39** GeV
- Significant differences between QCD and STAR results for $\sqrt{s_{NN}} \leq$ **27** GeV
- Good agreement between QCD, QMHRG2020 and STAR data for large $\sqrt{s_{NN}}$ suggest sensitivity of data to missing baryon resonances, which requires careful handling of feed-down corrections

Critical behavior in QCD

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

singular regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

$$t \sim \frac{T - T_c}{T_c} + \kappa_2 \left(\frac{\mu_l}{T}\right)^2$$

$$h \sim \frac{m_l}{T_c}$$

Pseudo-critical temperatures

response functions
2nd order cumulants

- magnetic

$$\frac{\partial^2 \ln Z}{\partial h^2} \sim \left(\frac{m_l}{T_c}\right)^{1/\delta - 1} \sim -0.79$$

mixed

$$\frac{\partial^2 \ln Z}{\partial h\, \partial t} \sim \left(\frac{m_l}{T_c}\right)^{(\beta-1)/\beta\delta} \sim -0.34$$

thermal

$$\frac{\partial^2 \ln Z}{\partial t^2} \sim \left(\frac{m_l}{T_c}\right)^{-\alpha/\beta\delta} \sim +0.11$$

O(4) critical exponents

$\alpha = -0.21$

$\beta = 0.38$

$\delta = 4.82$

divergence: strong moderate none

Chiral observables in QCD

– chiral condensate: $\langle\bar{\psi}\psi\rangle_l = \dfrac{\partial P/T}{\partial m_l/T}$, $\langle\bar{\psi}\psi\rangle_l = (\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d)/2$

– chiral order parameter: $M = \dfrac{2}{f_K^4}\left[m_s\langle\bar{\psi}\psi\rangle_l - m_l\langle\bar{\psi}\psi\rangle_s\right]$

$m_l = (m_u + m_d)/2$

– chiral susceptibility: $\chi_M = m_s\left(\dfrac{\partial M}{\partial m_u} + \dfrac{\partial M}{\partial m_d}\right)$ **magnetic**

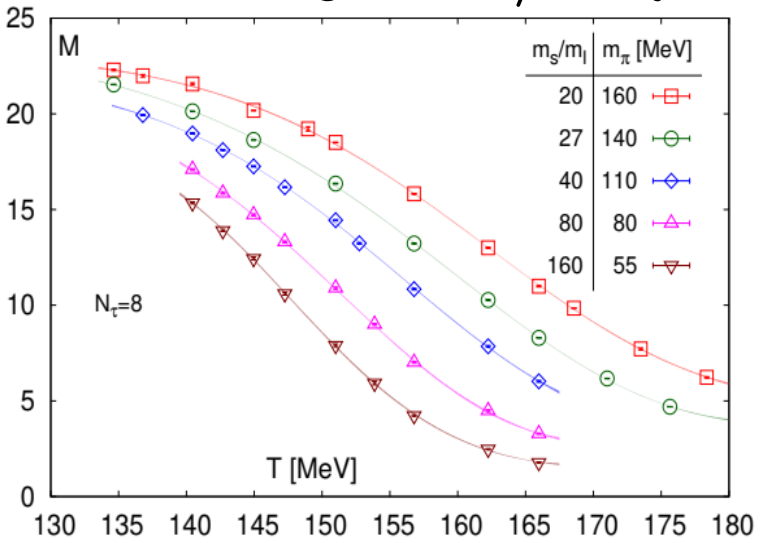
– mixed chiral susceptibility: $\chi_t = T\dfrac{\partial M}{\partial T}$ **mixed**

– conserved charge fluctuations: $\chi_x = T^2\dfrac{\partial^2 P/T^4}{\partial\mu_X^2}\Big|_{\mu_X=0}$ **thermal**

$X = B,\ S, \ldots$

The Chiral PHASE TRANSITION in (2+1)-flavor QCD

$$M \sim m_s \partial \ln Z / \partial m_l$$



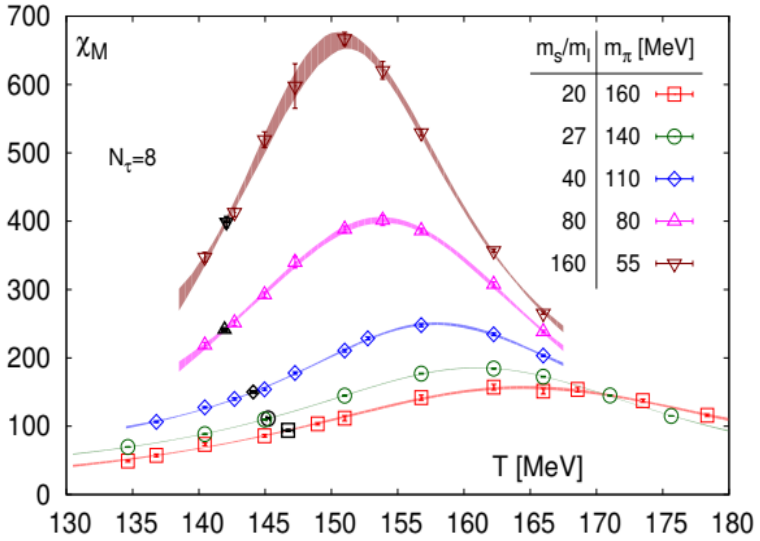
m_s fixed, physical

$m_l \Rightarrow 0$

"magnetic" susceptibility

$$\chi_M \sim \frac{\partial^2 \ln Z}{\partial m_l^2} \sim (m_s/m_l)^{0.79}$$

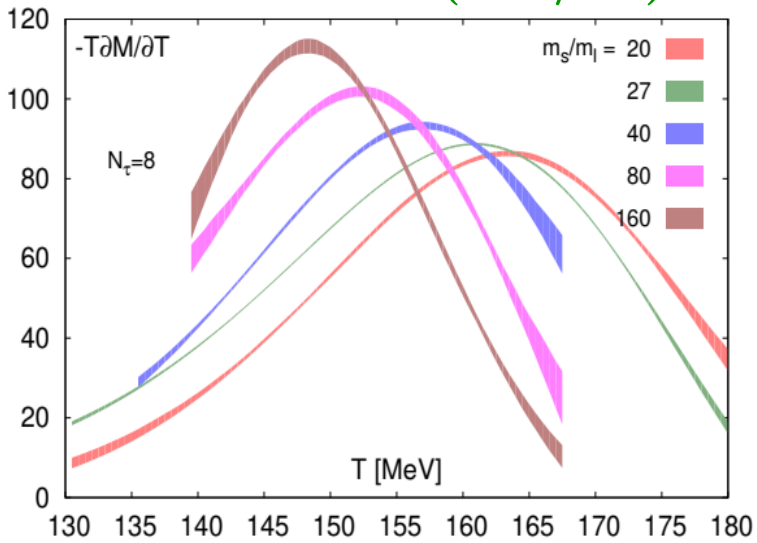
$(160/27)^{0.79} \sim 4$



"mixed" susceptibility

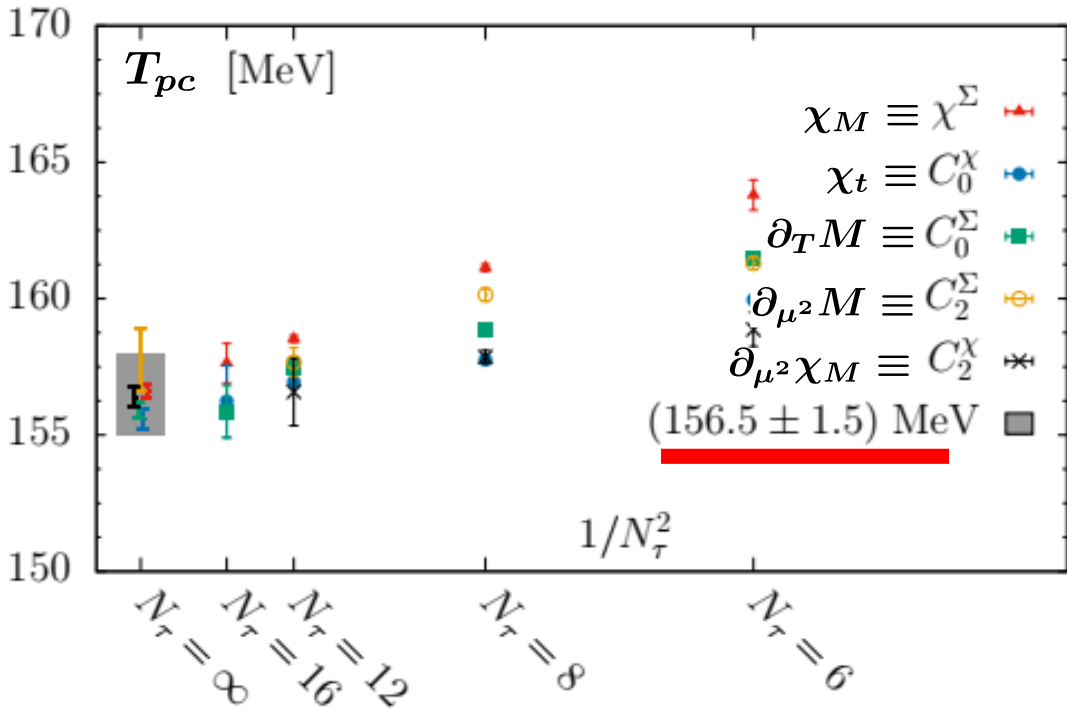
$$\chi_t \sim \frac{\partial^2 \ln Z}{\partial T \partial m_l} \sim (m_s/m_l)^{0.34}$$

$(160/27)^{0.34} \sim 1.8$



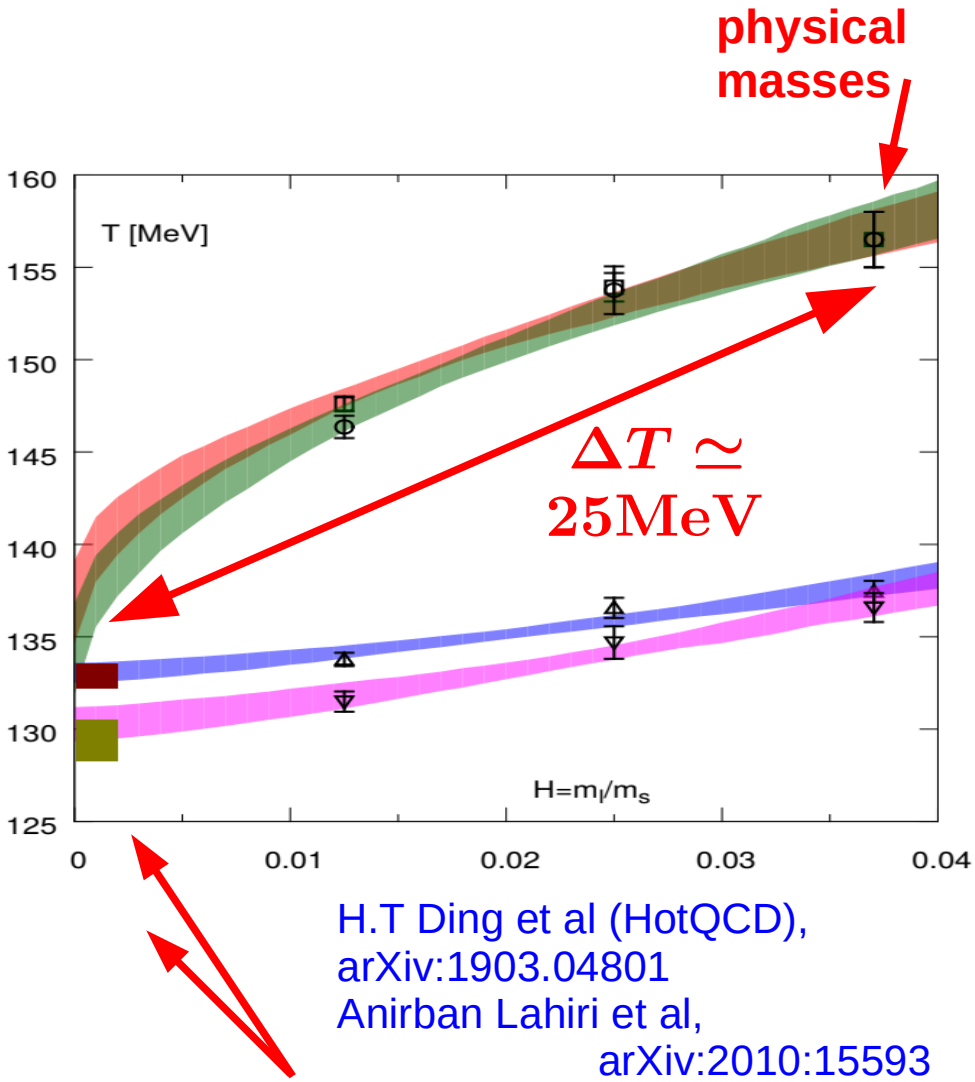
H.T. Ding et al. (HotQCD), PRL 123 (2019) 15, arXiv:1903.04801

The Chiral PHASE TRANSITION in (2+1)-flavor QCD



A. Bazavov et al (HotQCD), arXiv:1812.08235

physical masses



H.T Ding et al (HotQCD), arXiv:1903.04801
Anirban Lahiri et al, arXiv:2010:15593

physical masses

$$T_{pc}^{phys} = (156.5 \pm 1.5)\ \mathrm{MeV}$$

chiral limit extrapolations

$$T_c^0 = 132^{+3}_{-6}\ \mathrm{MeV}$$