Ripples of the QCD critical point

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Dalian University of Technology

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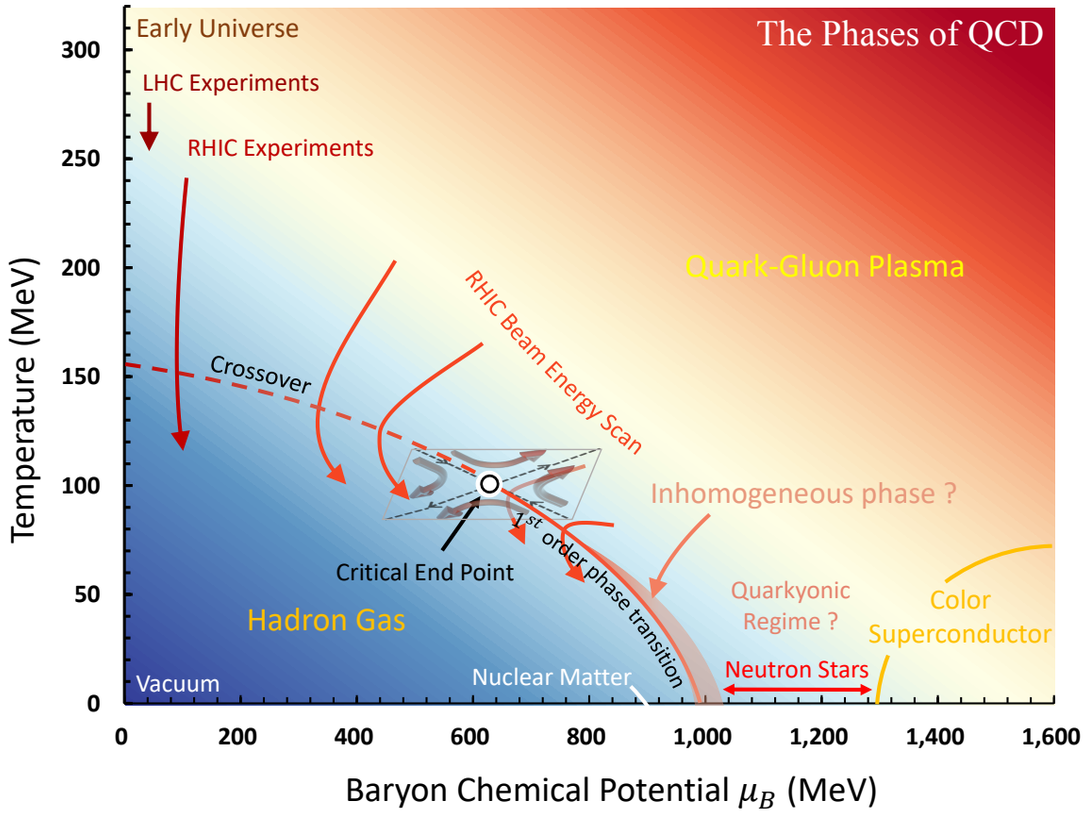
Based on:
WF, Xiaofeng Luo, Jan M. Pawlowski, Fabian Rennecke, Shi Yin, *Ripples of the QCD Critical Point*, arXiv: 2308.15508;
WF, Xiaofeng Luo, Jan M. Pawlowski, Fabian Rennecke, Rui Wen, Shi Yin, *Hyper-order baryon number fluctuations at finite temperature and density*, PRD 104 (2021) 094047, arXiv: 2101.06035.

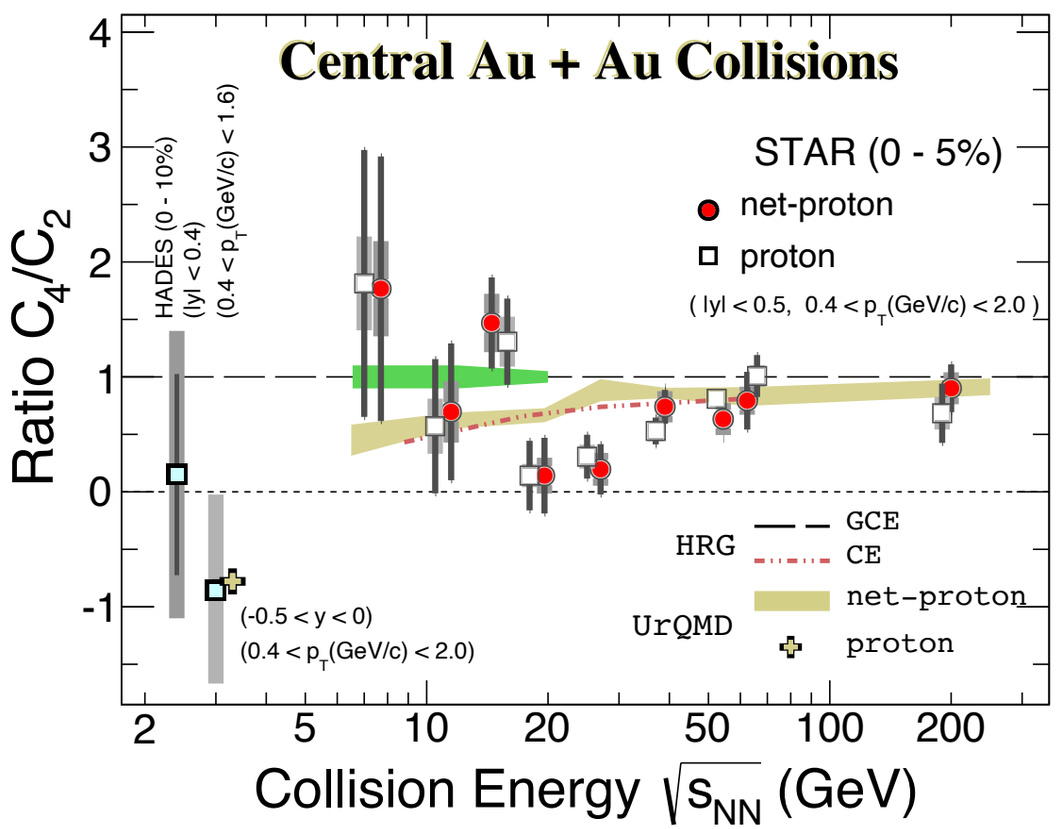
fQCD collaboration:
Braun, Chen, Fu, Gao, Geissel, Huang, Lu, Ihssen, Pawlowski, Rennecke, Sattler, Schallmo, Stoll, Tan, Töpfel, Turnwald, Wessely, Wen, Wink, Yin, Zheng, Zorbach

CEP in QCD phase diagram

QCD phase diagram



Fluctuations measured in BES-I

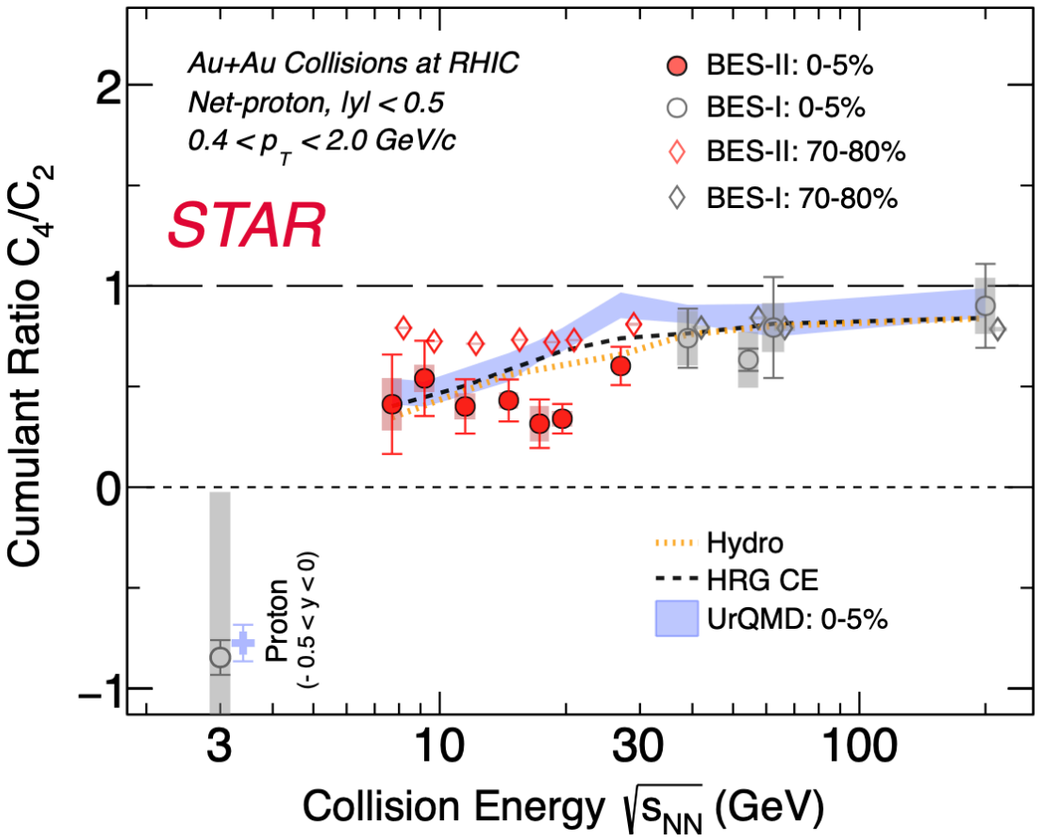


J. Adam *et al.* (STAR), *PRL* 126 (2021), 092301;
M. Abdallah *et al.* (STAR), *PRC* 104 (2021), 024902;
M. Abdallah *et al.* (STAR), *PRL* 128 (2022) 20, 202303

- The non-monotonicity of the kurtosis is observed with 3.1σ significance in the phase I of BES program (BES-I) at STAR

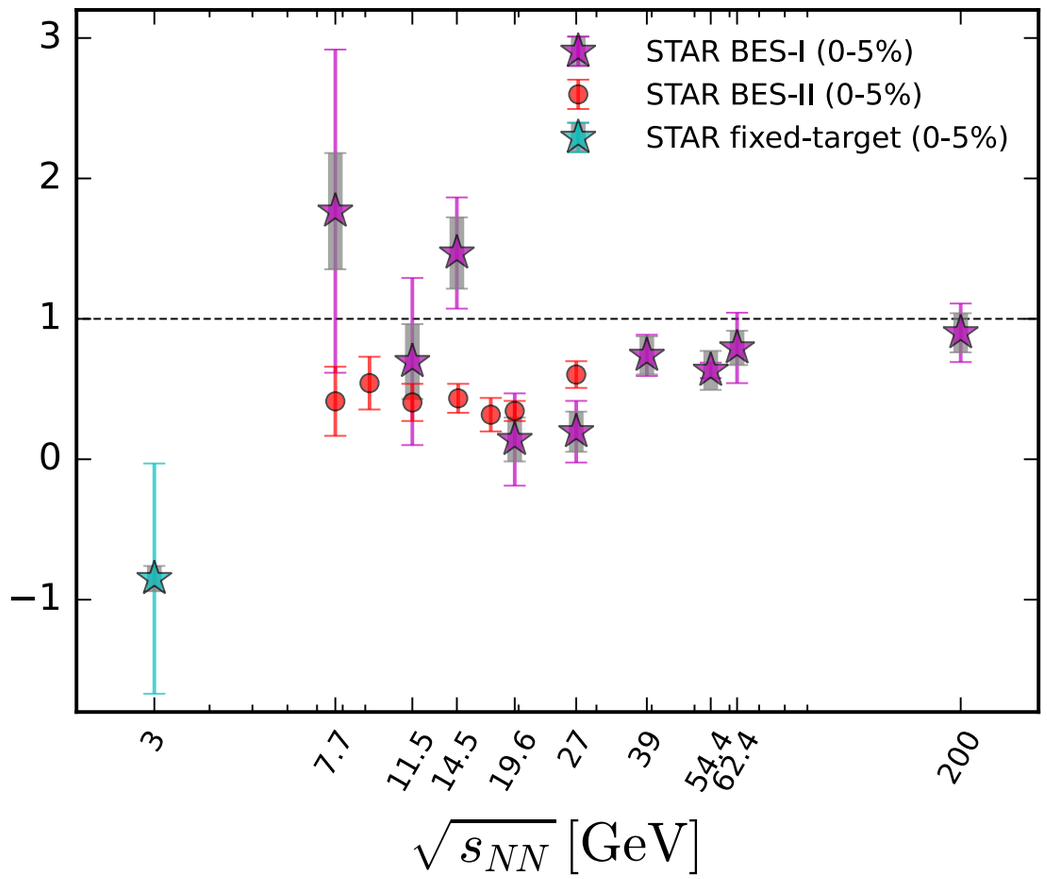
Recent results in BES-II

Net proton kurtosis



Ashish Pandav for STAR Collaboration in CPOD2024

Results in BES-I and BES-II

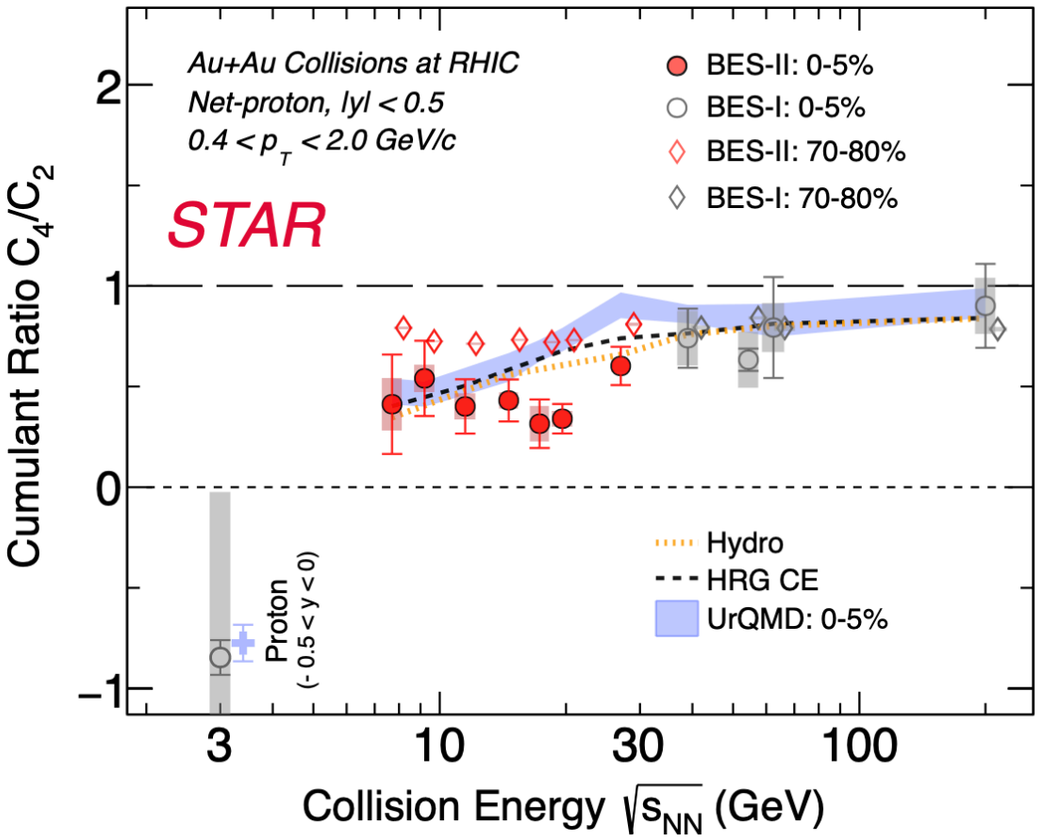


also cf. talk by Xiaofeng Luo

- The kurtosis in the energy regime of fixed-target experiments, i.e. 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV, then become very pivotal.
- Is there a "peak" structure?

Recent results in BES-II

Net proton kurtosis



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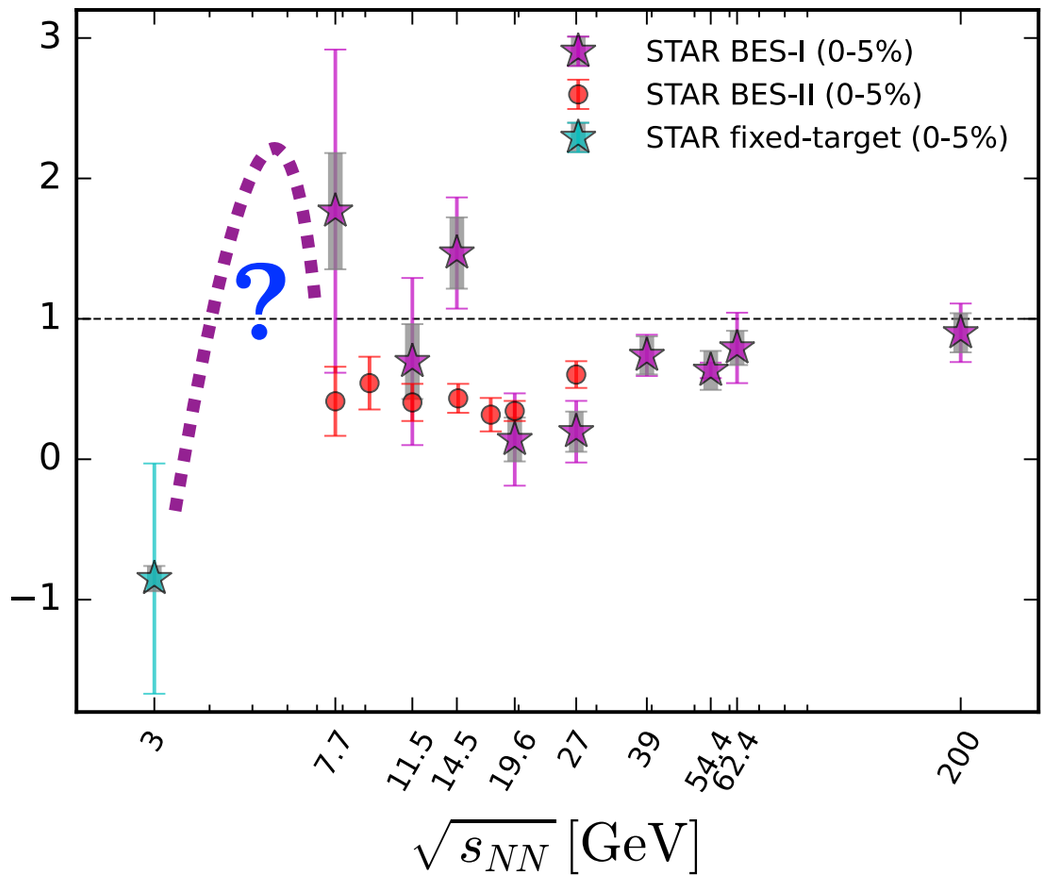
Results in BES-I and BES-II



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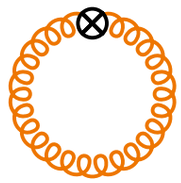
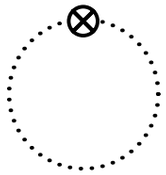
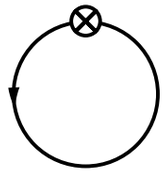
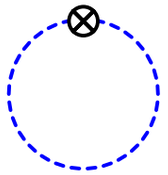
- The kurtosis in the energy regime of fixed-target experiments, i.e. 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV, then become very pivotal.
- Is there a "peak" structure?

Outline

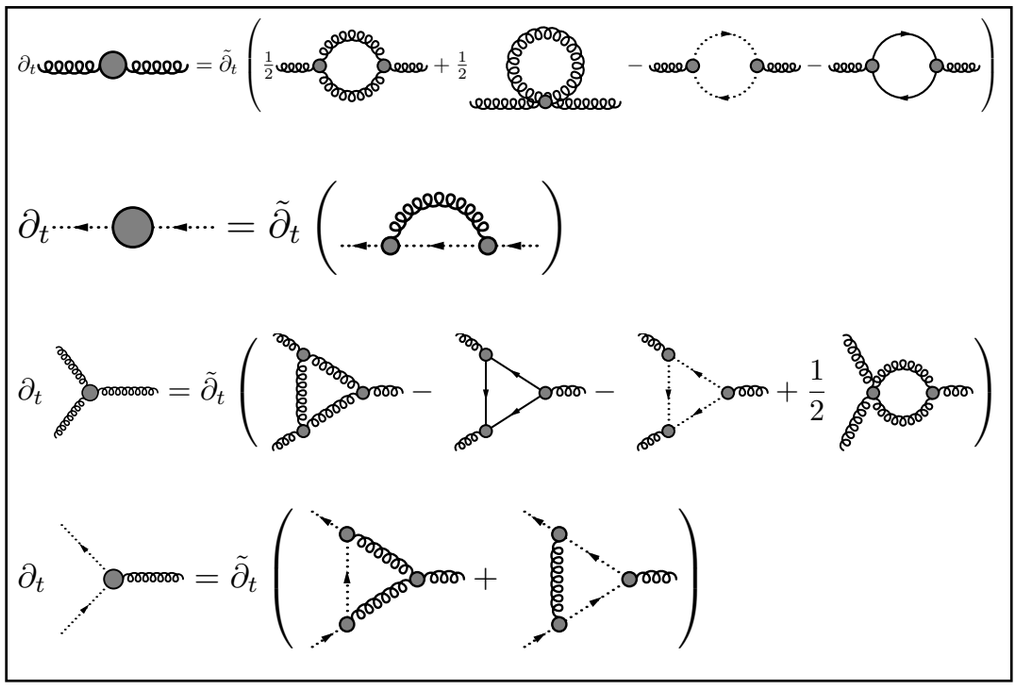
- Introduction
- Recent advance of QCD phase structure from functional QCD
- Baryon number fluctuations at high density
- Ripples of the QCD critical point
- Summary

First-principles QCD within fRG

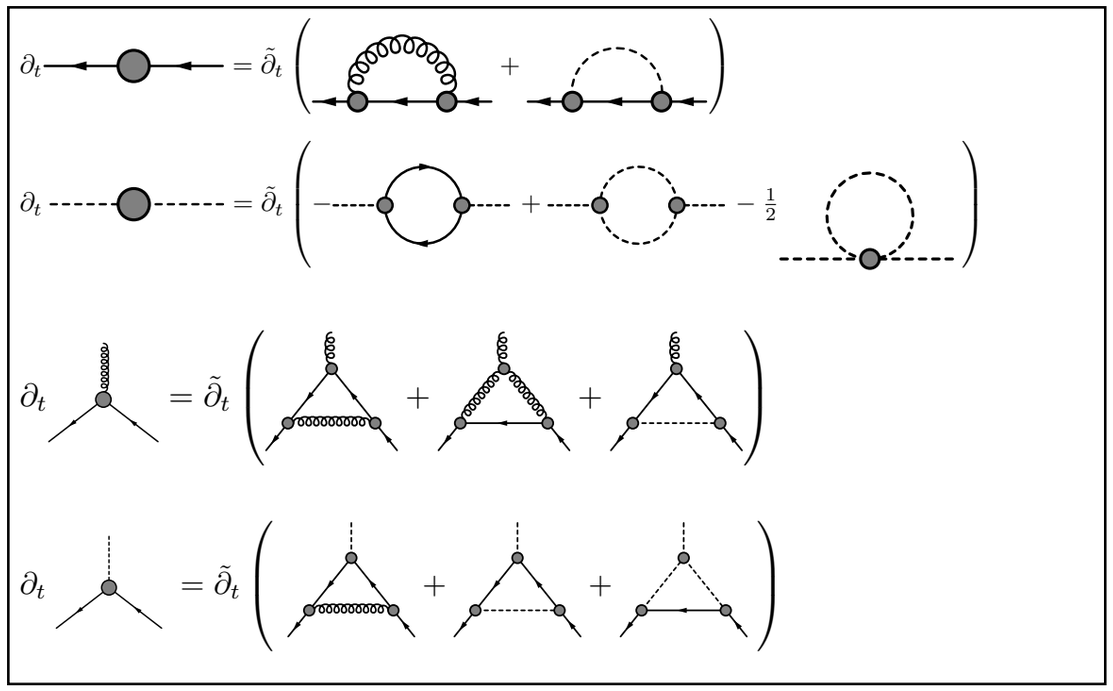
QCD flow equation:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}$$  $$-$$  $$-$$  $$+\frac{1}{2}$$ 

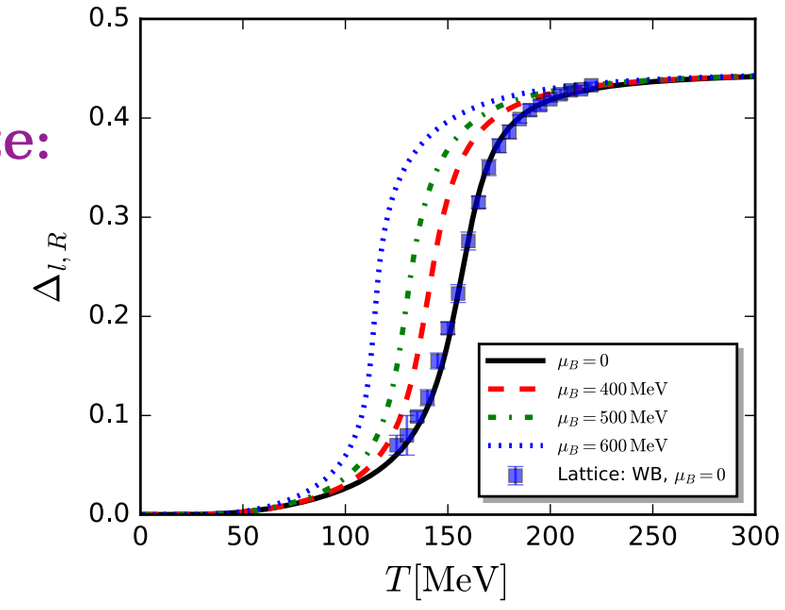
Glue sector:

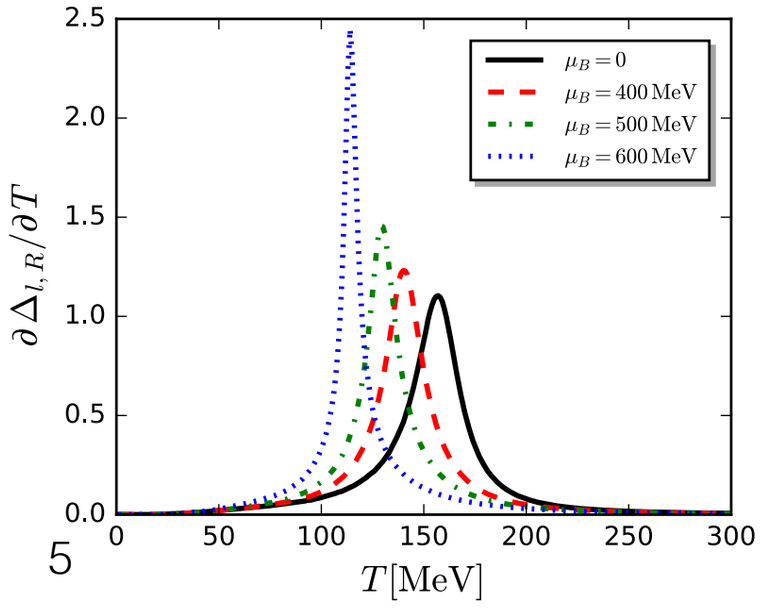


Matter sector:



quark condensate:

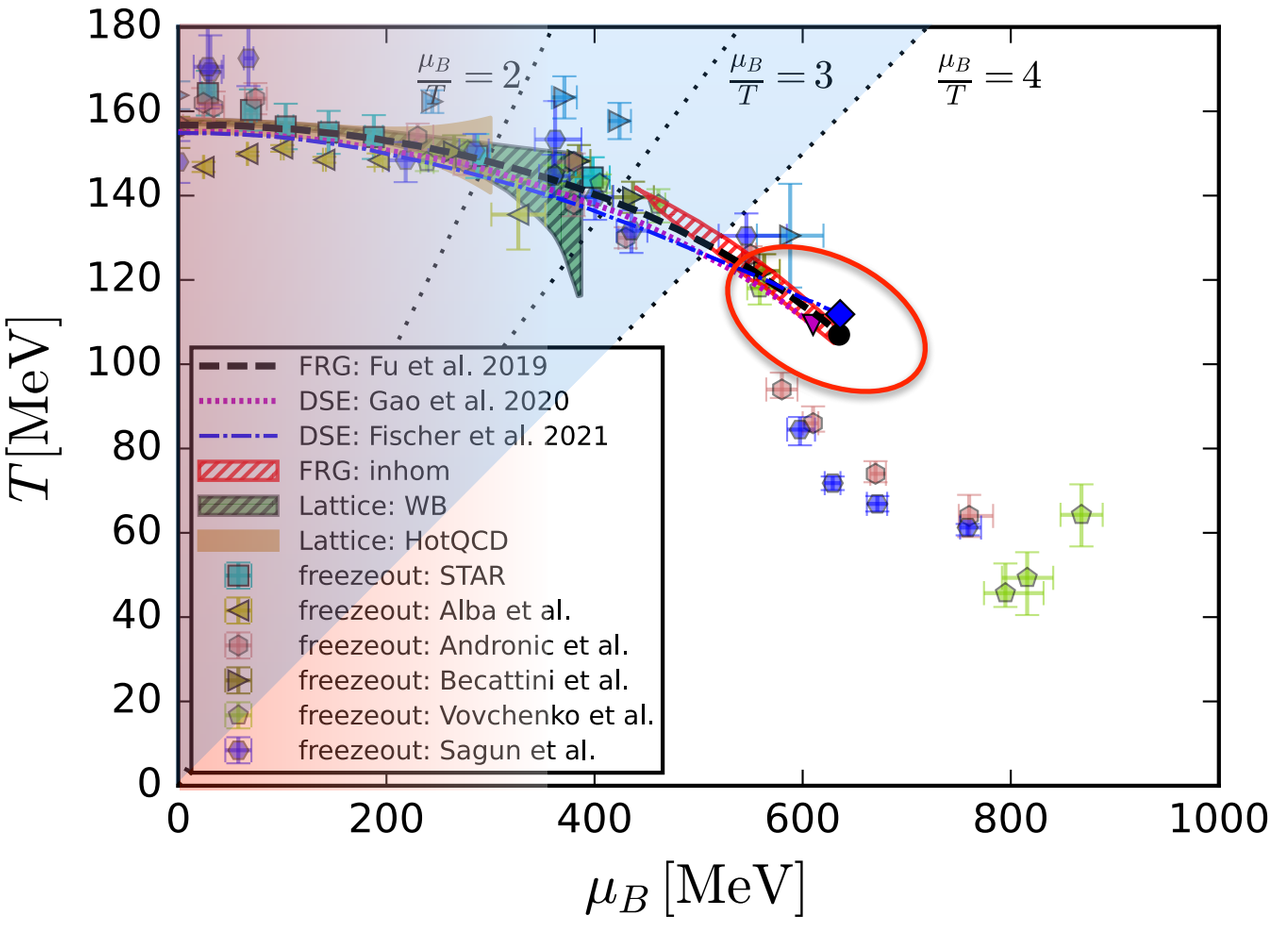




fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

Lattice: Borsanyi *et al.* (WB), *JHEP* 09 (2010) 073

CEP from first-principles functional QCD



Passing through strict benchmark tests in comparison to lattice QCD at vanishing and small μ_B.

Regime of quantitative reliability of functional QCD with $\mu_B/T \lesssim 4$.

also cf. talk by Jan M. Pawlowski

Estimates of the location of CEP from first-principles functional QCD:

fRG:

- $(T, \mu_B)_{\text{CEP}} = (107, 635)\textbf{MeV}$

fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

DSE:

▼ $(T, \mu_B)_{\text{CEP}} = (109, 610)\textbf{MeV}$

DSE (fRG): Gao, Pawlowski, *PLB* 820 (2021) 136584

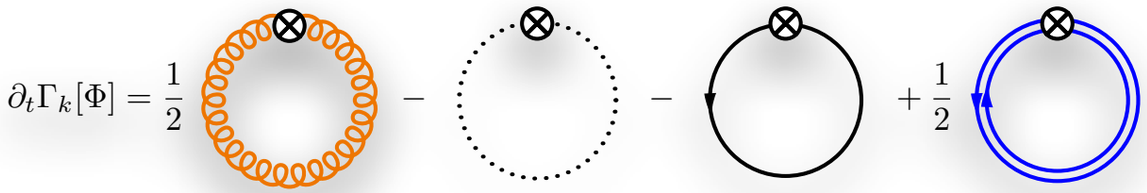
◆ $(T, \mu_B)_{\text{CEP}} = (112, 636)\textbf{MeV}$

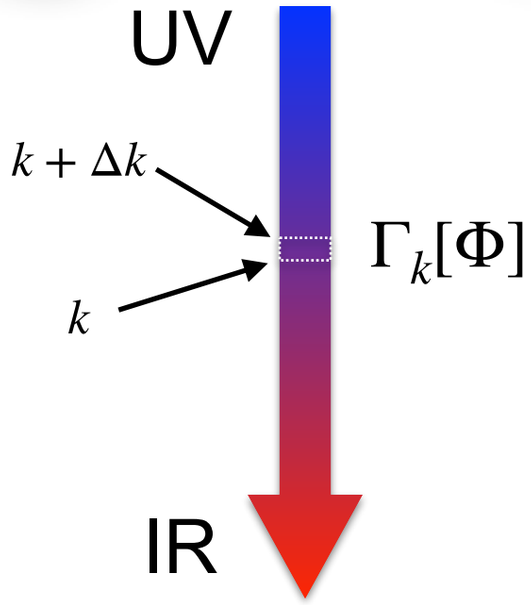
DSE: Gunkel, Fischer, *PRD* 104 (2021) 5, 054022

- No CEP observed in $\mu_B/T \lesssim 2 \sim 3$ from lattice QCD. Karsch, *PoS* CORFU2018 (2019)163
- Recent studies of QCD phase structure from both fRG and DSE have shown convergent estimate for the location of CEP: 600 MeV $\lesssim \mu_{B_{\text{CEP}}} \lesssim$ 650 MeV.

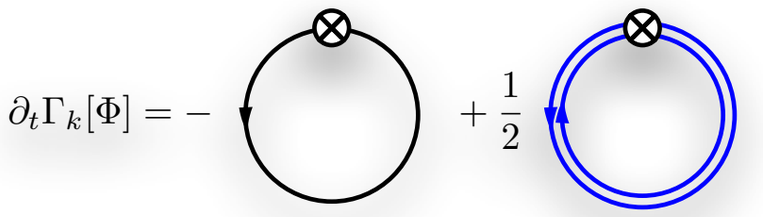
QCD-assisted LEFT

QCD flow equation:



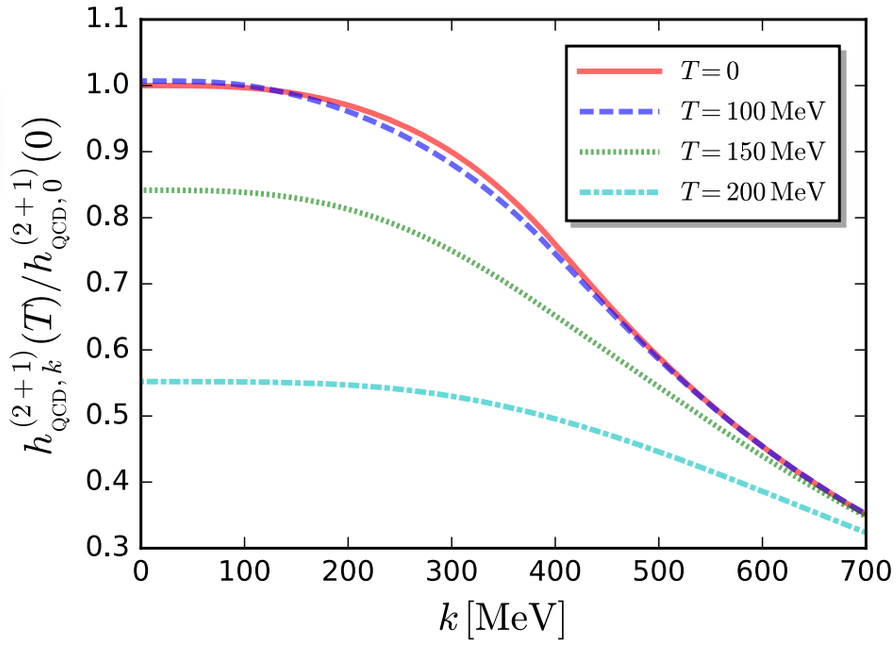


LEFT flow equation:



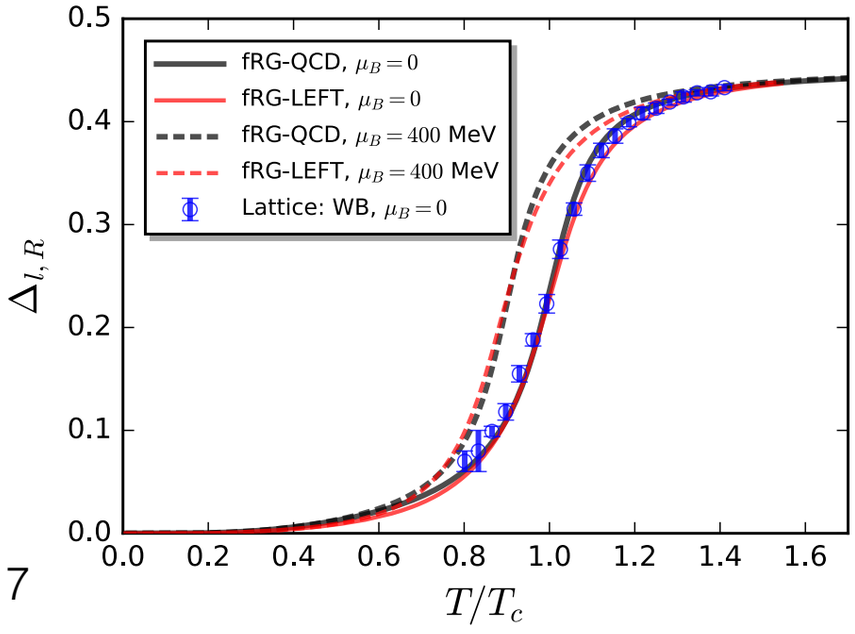
quark meson

- **Yukawa couplings obtained in QCD inputted in QCD-assisted LEFT**



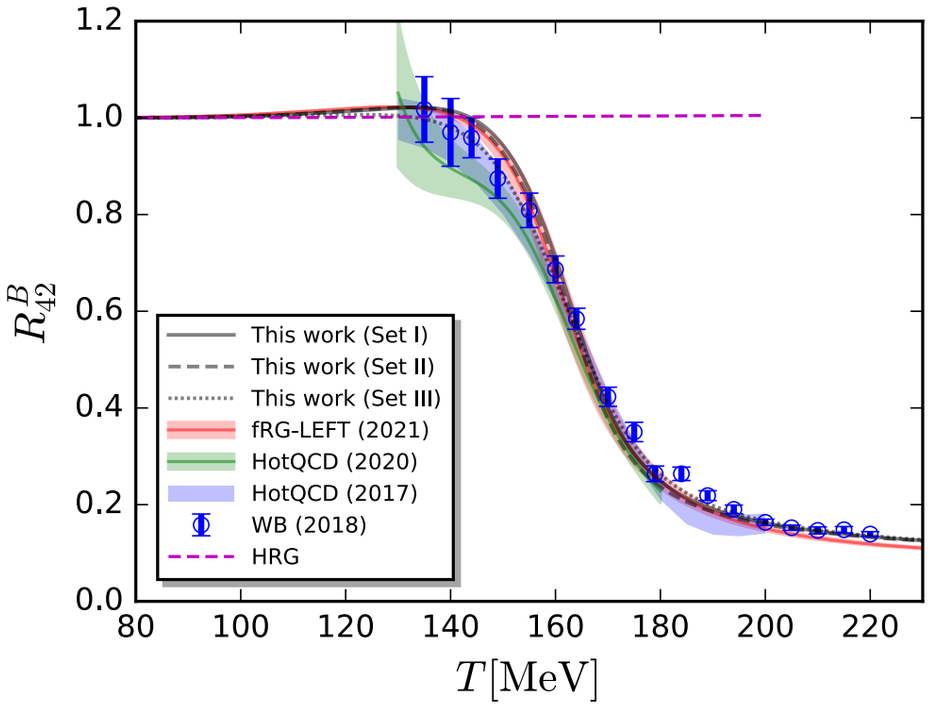
WF, Pawlowski, Rennecke, *PRD* 101 (2020) 054032

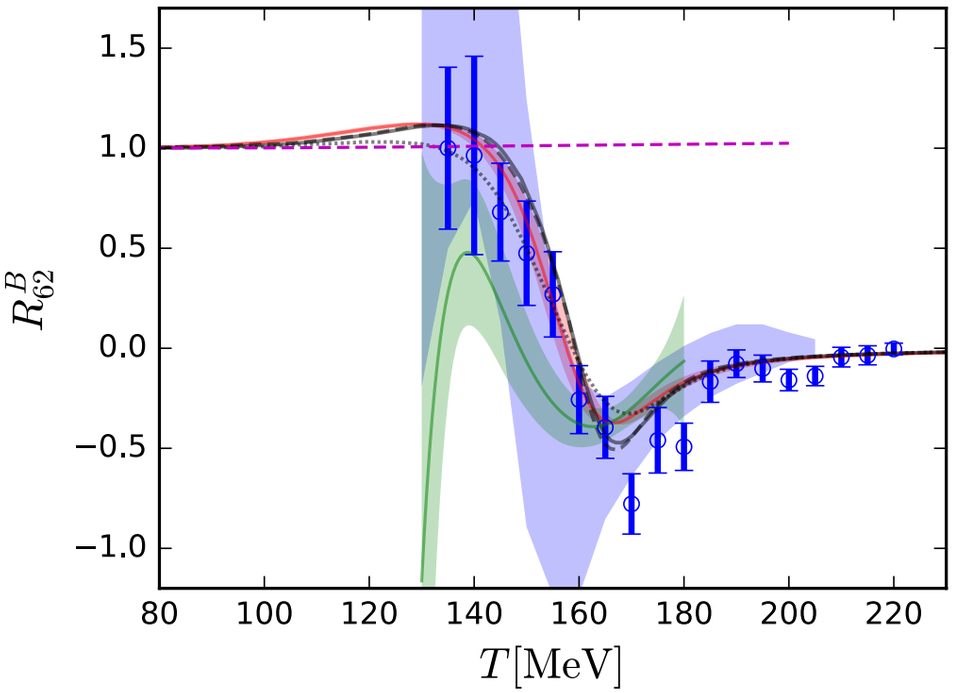
- **Chiral condensates in QCD and QCD-assisted LEFT in agreement**



WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

Baryon number fluctuations





$\mu_B = 0$

fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508;
WF, Luo, Pawlowski, Rennecke, Wen, Yin, *PRD* 104 (2021) 094047

HotQCD: A. Bazavov *et al.*, arXiv: *PRD* 95 (2017), 054504; *PRD* 101 (2020), 074502

WB: S. Borsanyi *et al.*, arXiv: *JHEP* 10 (2018) 205

baryon number fluctuations

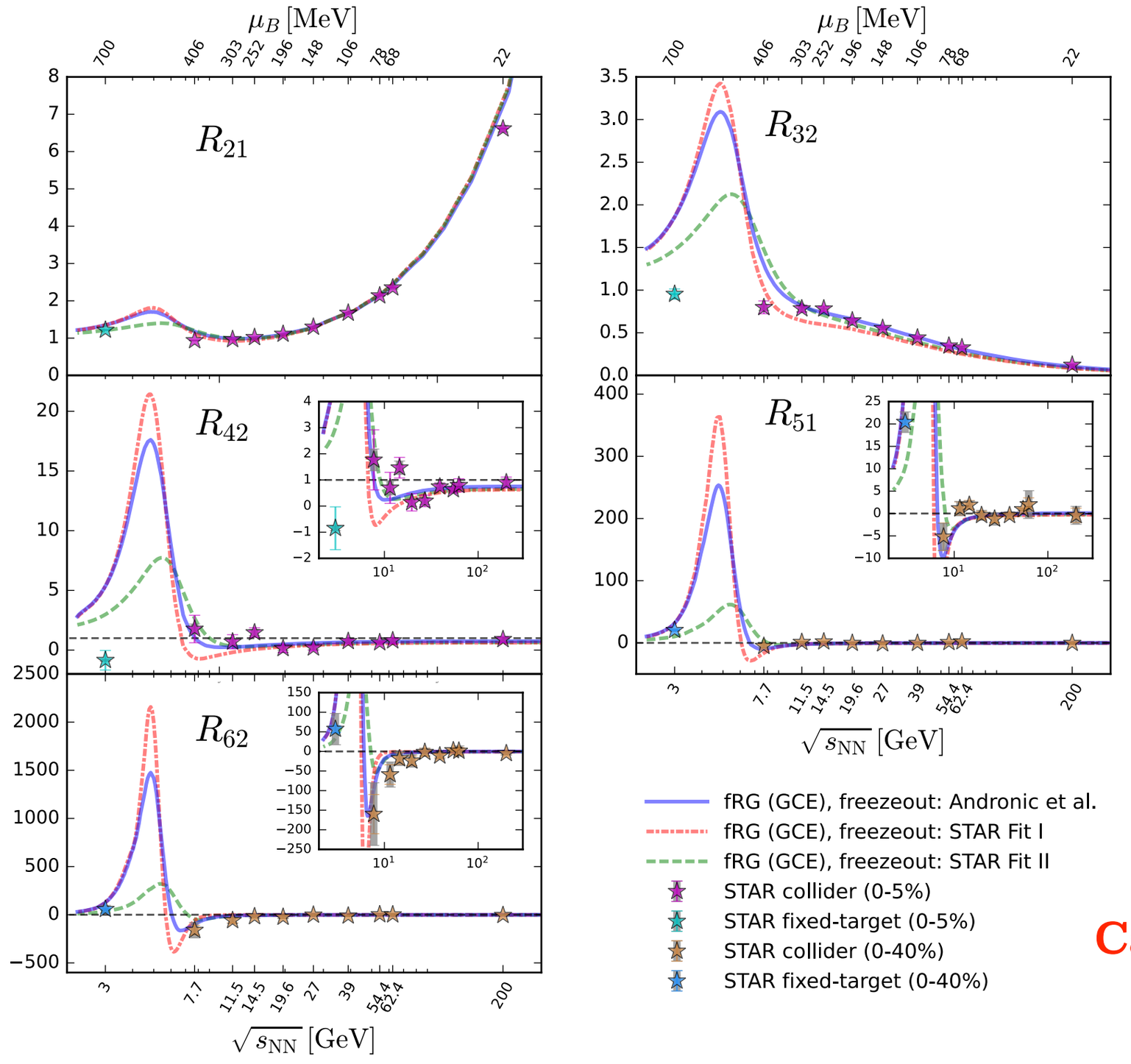
$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4} \qquad R^B_{nm} = \frac{\chi_n^B}{\chi_m^B}$$

relation to the cumulants

$$\frac{M}{VT^3} = \chi_1^B\,,\ \frac{\sigma^2}{VT^3} = \chi_2^B\,,\ S = \frac{\chi_3^B}{\chi_2^B\sigma}\,,\ \kappa = \frac{\chi_4^B}{\chi_2^B\sigma^2}\,,$$

- **In comparison to lattice results and our former results, the improved results of baryon number fluctuations at vanishing chemical potential in the QCD-assisted LEFT are convergent and consistent.**

Grand canonical fluctuations at the freeze-out



STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301; Abdallah *et al.* (STAR), *PRL* 128 (2022) 202303; Aboona *et al.* (STAR), *PRL* 130 (2023) 082301

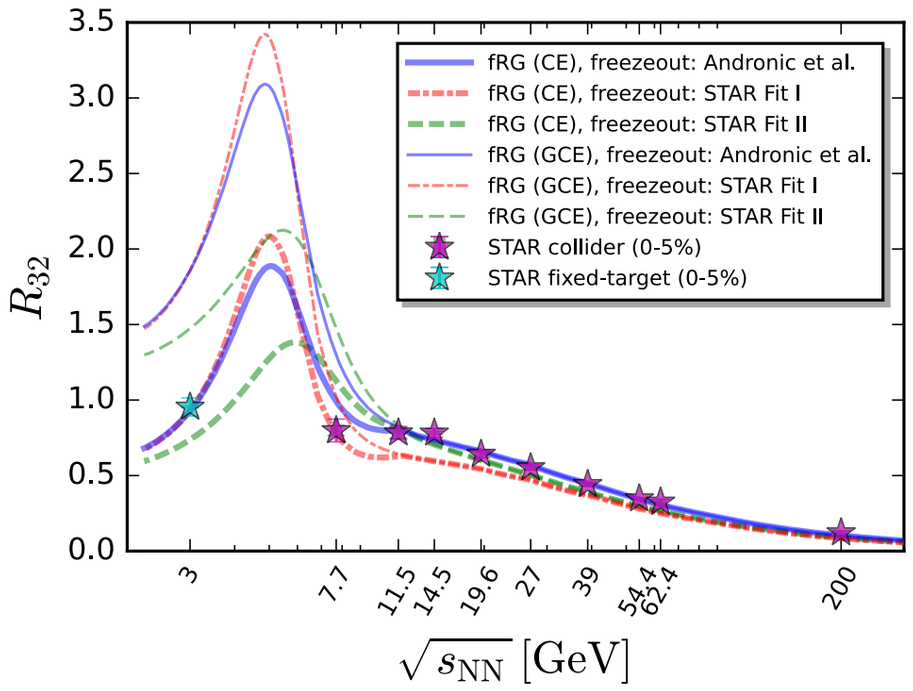
fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

- **Results in fRG are obtained in the QCD-assisted LEFT with a CEP at $(T_{\rm CEP}, \mu_{B_{\rm CEP}}) = (98{,}643)$ MeV.**
- **Peak structure is found in 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV.**
- **Agreement between the theory and experiment is worsening with $\sqrt{s_{\rm NN}} \lesssim$ 11.5 GeV.**
- **Effects of global baryon number conservation in the regime of low collision energy should be taken into account.**

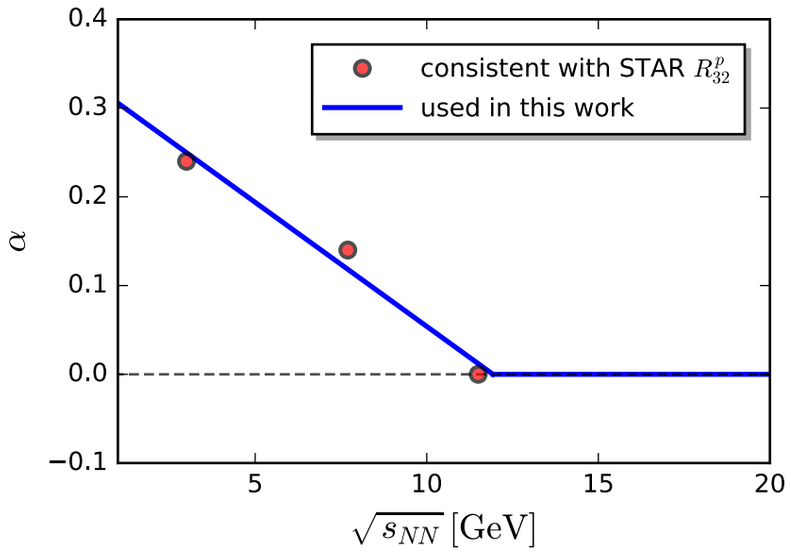
Caveat:

Fluctuations of baryon number in theory are compared with those of proton number in experiments.

Canonical corrections with SAM



- **Experimental data R_{32} is used to constrain the parameter α in the range $\sqrt{s_{\rm NN}} \lesssim 11.5$ GeV.**
- **We choose the simplest linear dependence**



$$\alpha(\bar{s}) = a\left(1-\sqrt{\bar{s}}\right)\theta\left(1-\bar{s}\right)$$

$$a = 0.33\,, \qquad \sqrt{\bar{s}} = \frac{\sqrt{s_{\rm NN}}}{11.9\,\text{GeV}}$$

SAM:

- We adopt the subensemble acceptance method (SAM) to take into account the effects of global baryon number conservation:

$$\alpha = \frac{V_1}{V}$$

V_1: the subensemble volume measured in the acceptance window, V: the volume of the whole system.

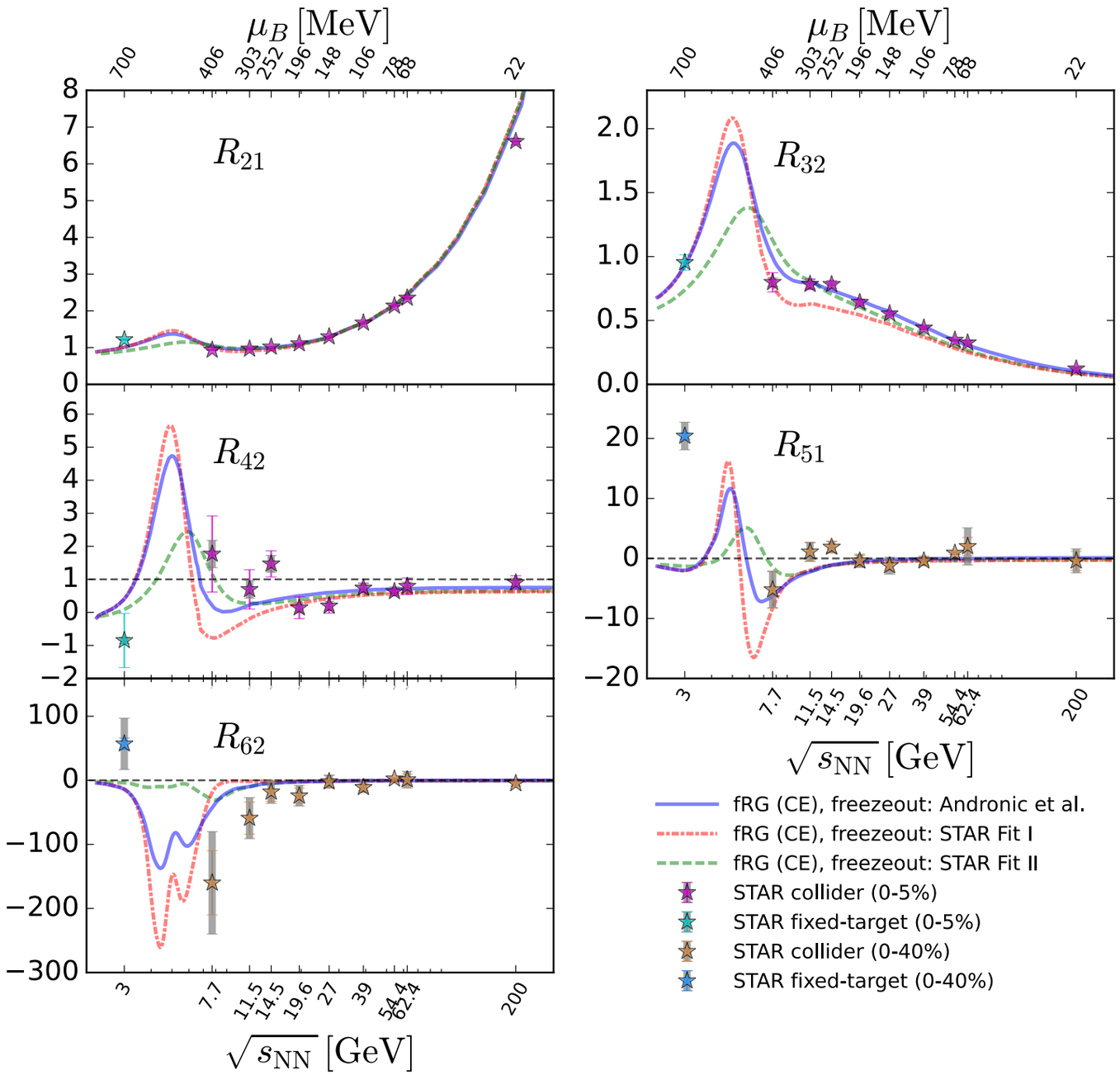
- fluctuations with canonical corrections are related to grand canonical fluctuations as follows:

$$\bar{R}^B_{21} = \beta R^B_{21}, \qquad \bar{R}^B_{32} = (1-2\alpha)R^B_{32},$$

$$\bar{R}^B_{42} = (1-3\alpha\beta)R^B_{42} - 3\alpha\beta(R^B_{32})^2$$

SAM: Vovchenko, Savchuk, Poberezhnyuk, Gorenstein, Koch , *PLB* 811 (2020) 135868

Canonical fluctuations at the freeze-out



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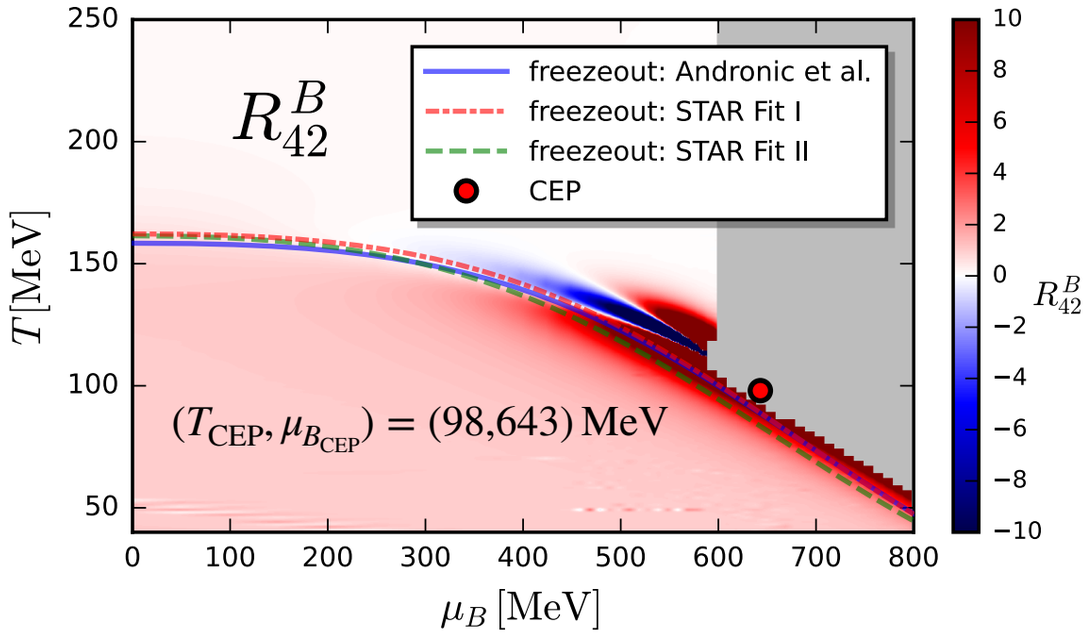
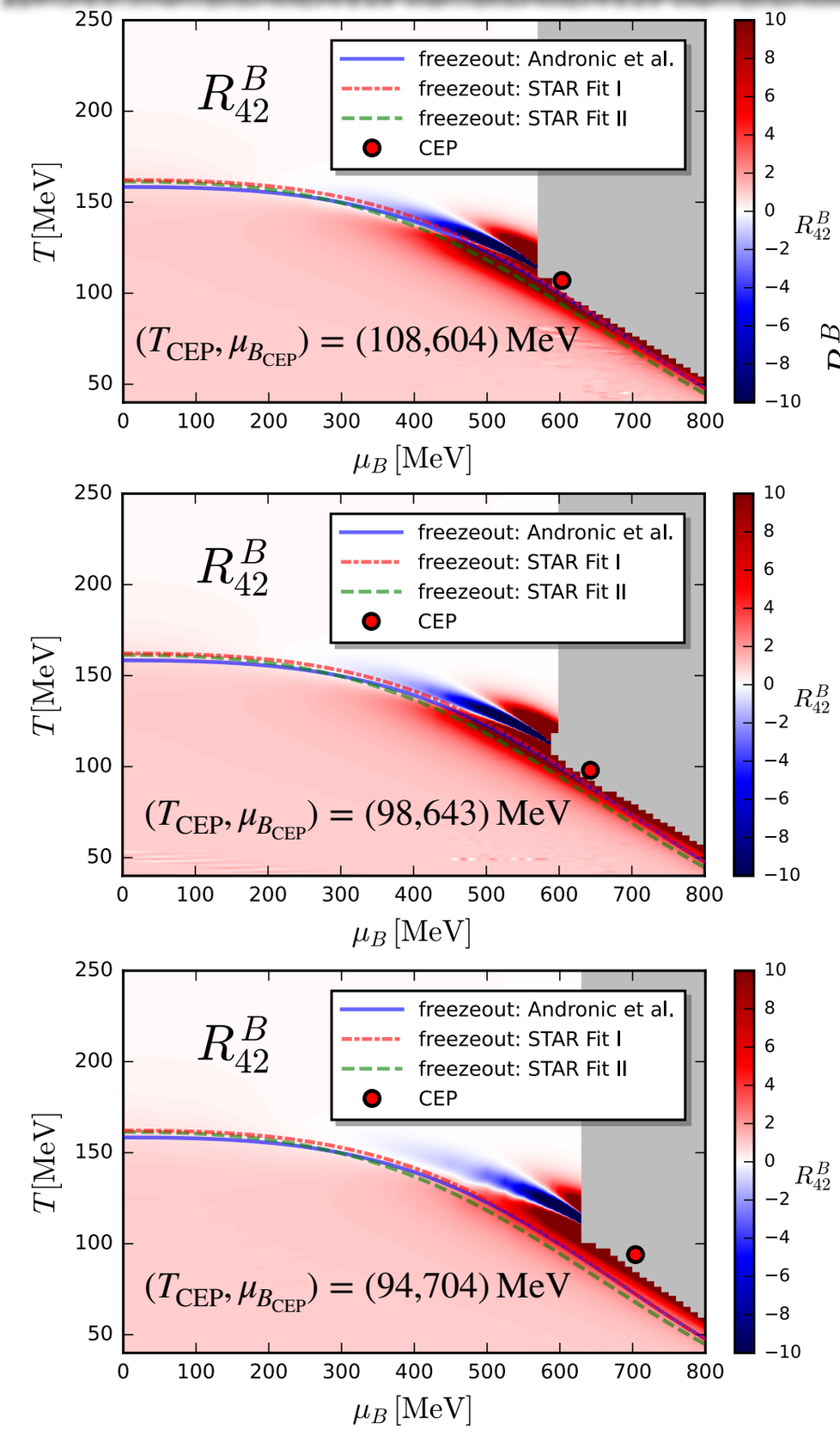
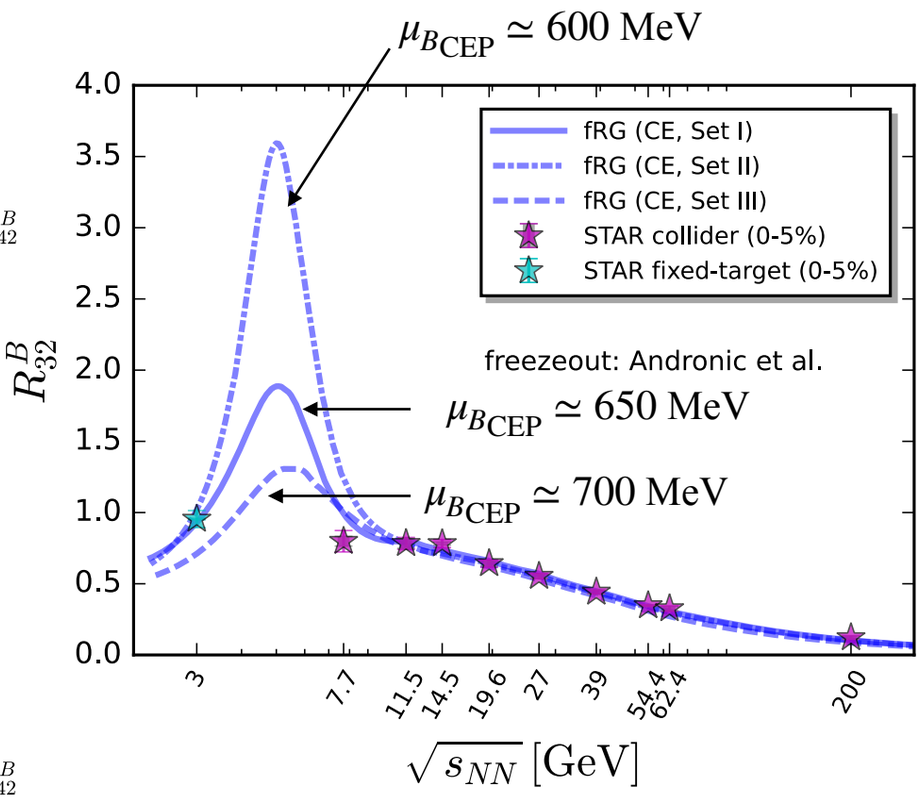
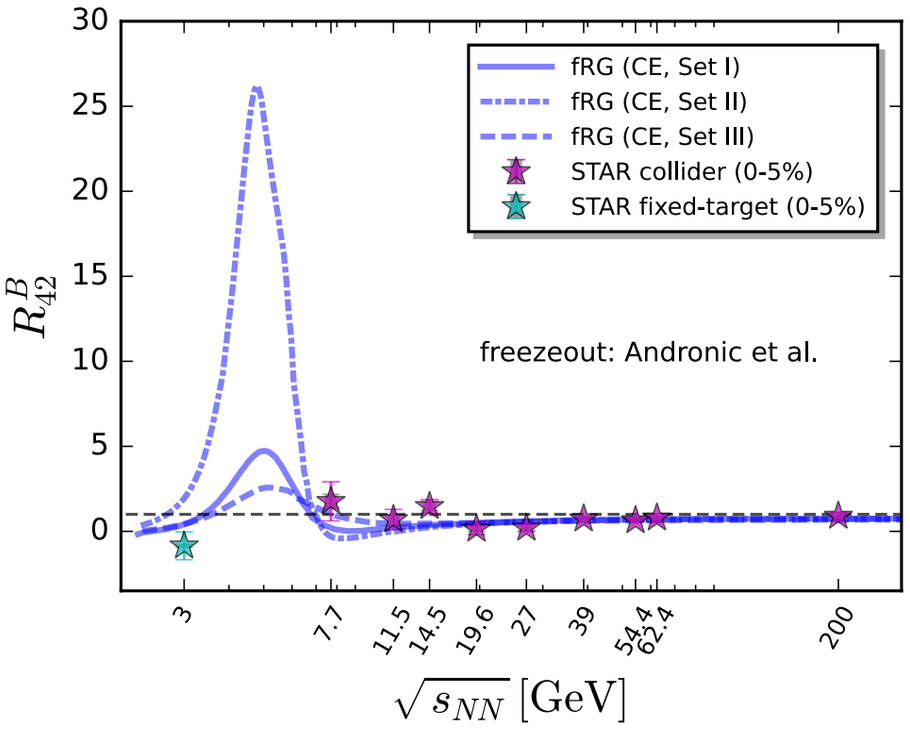


- **Peak structure is found in 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV.**
- **Position of peak in R_{42} is $\mu_{B_{\rm peak}} =$ 536, 541 and 486 MeV for the three freeze-out curves, significantly smaller than $\mu_{B_{\rm CEP}} = 643$ MeV.**

Dependence on the location of the CEP







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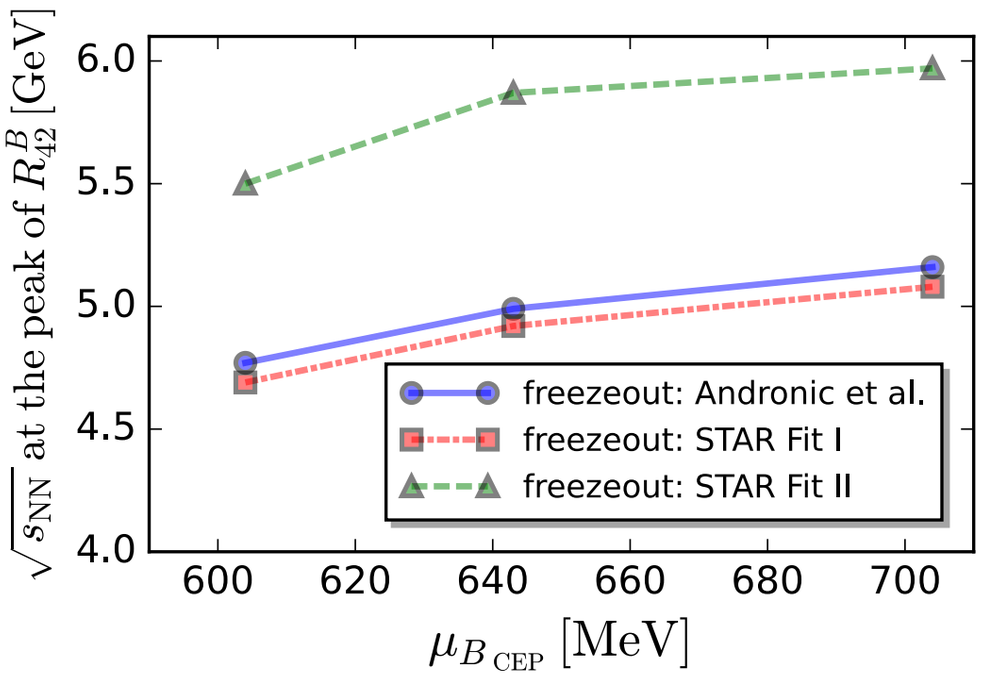
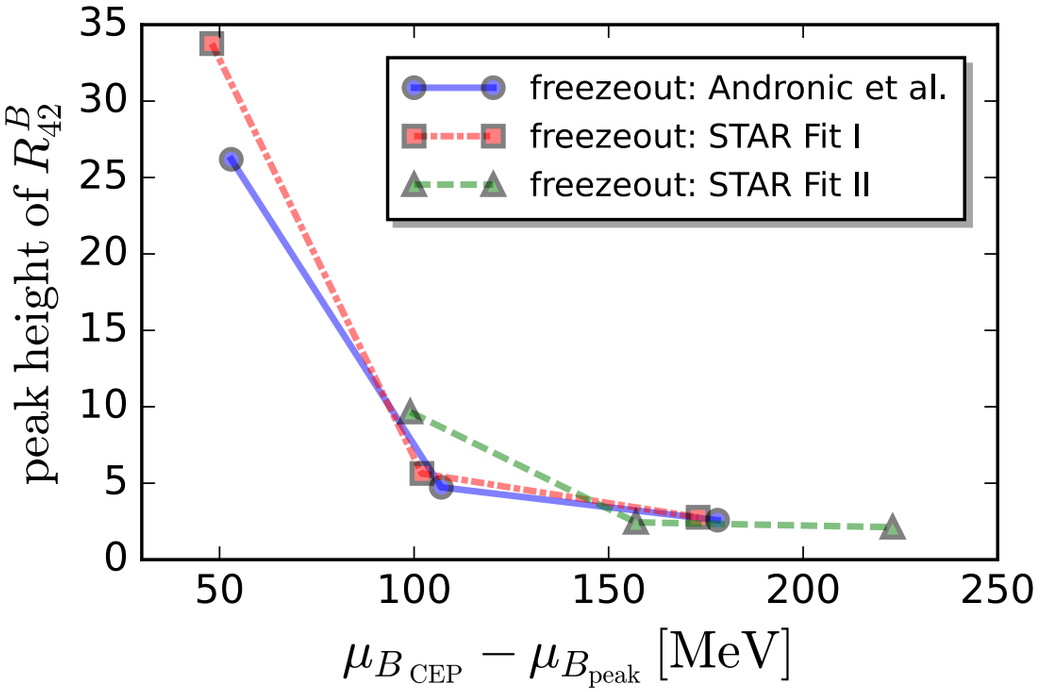
- **Position of the peak is insensitive to the location of CEP.**
- **Height of peak decreases as CEP moves towards larger μ_B.**

Ripples of the QCD critical point

Position of peak:



Height of peak:



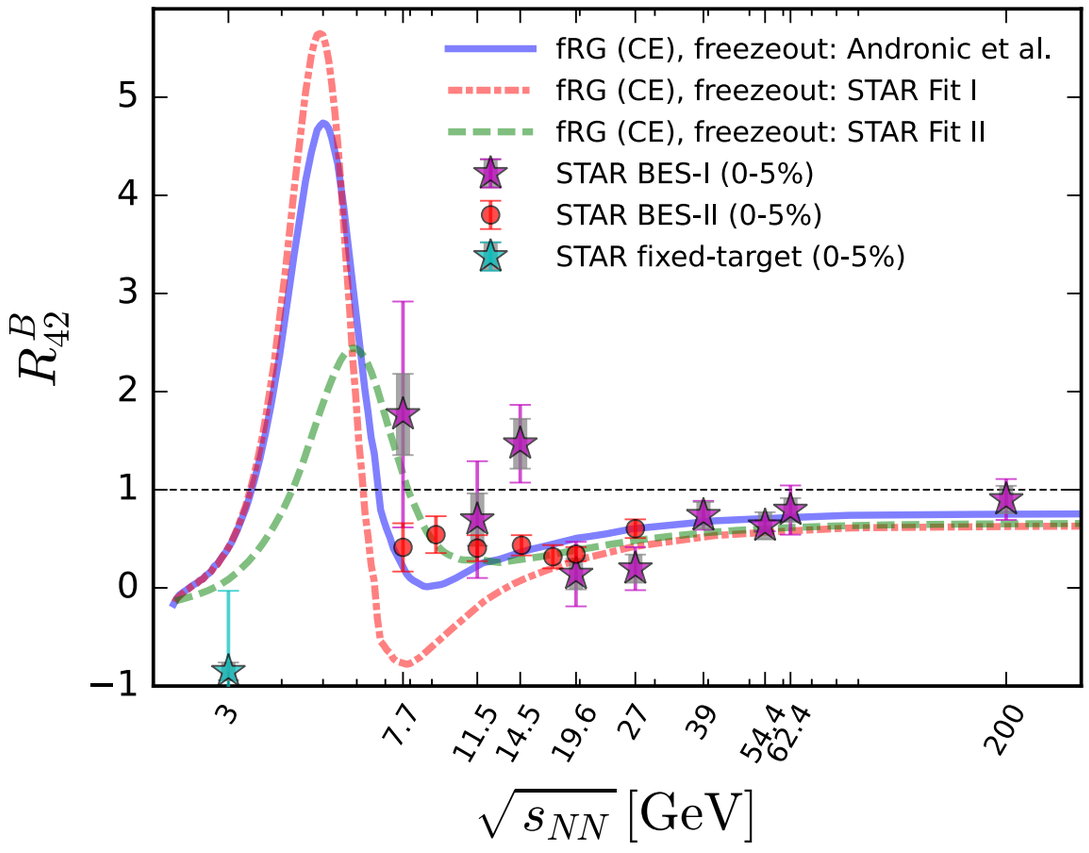


fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508

- **Note that the ripples of CEP are far away from the critical region characterized by the universal scaling properties, e.g., the critical slowing down.**
- **But, the information of CEP, such as its location and properties, etc., is still encoded in the ripples.**

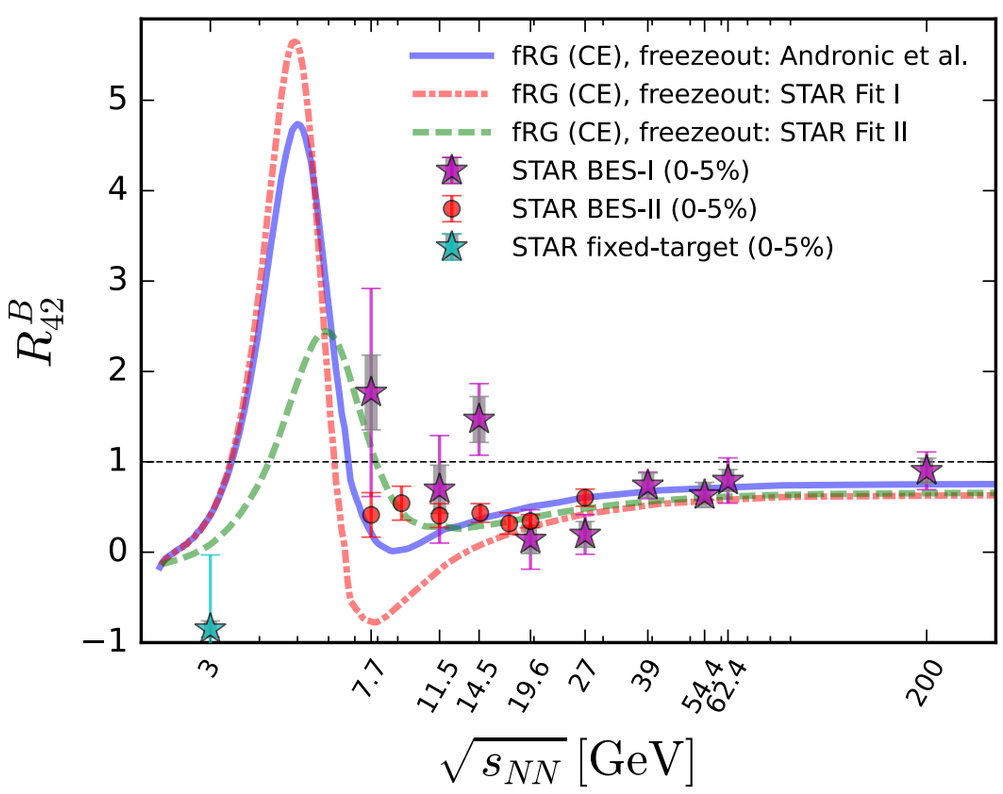
Comparison to BES-II

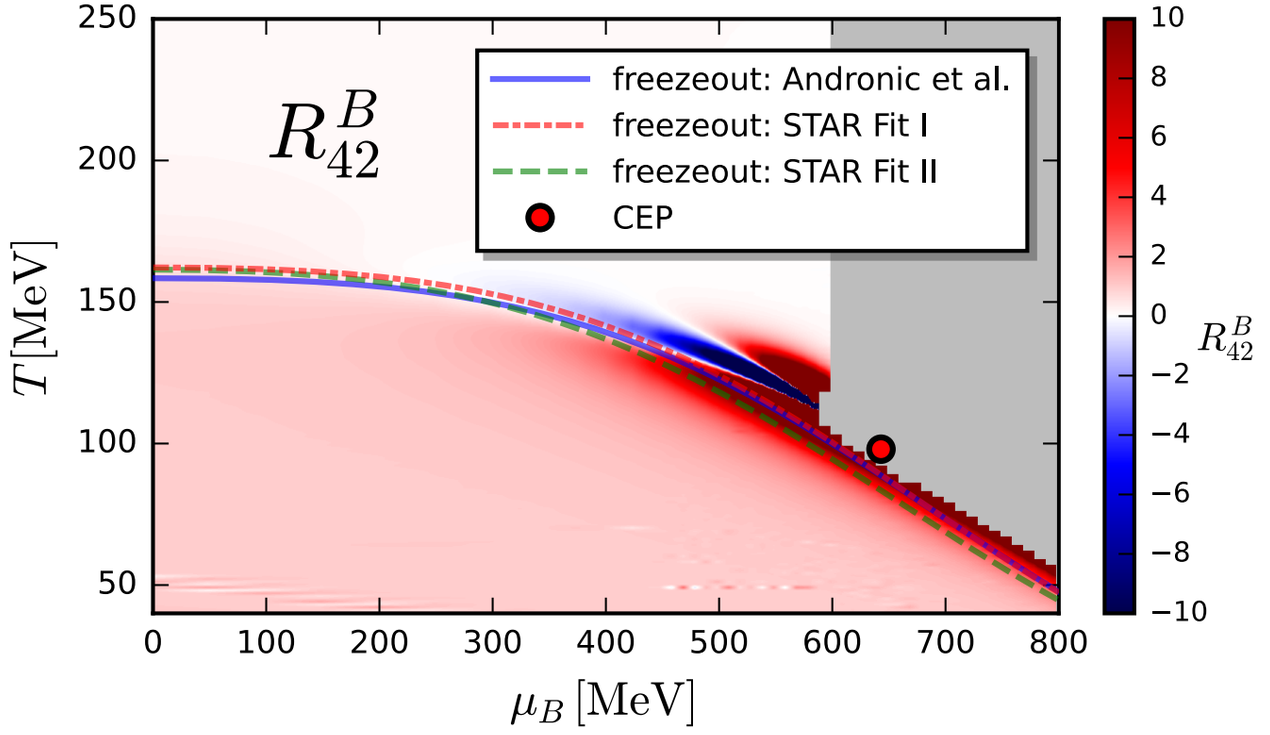
Net baryon (proton) number Kurtosis:



- **In comparison to BES-I, BES-II results are better consistent with the theoretical prediction.**
- **Experimental results in the energy regime of fixed-target experiments, i.e. 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV, are now very important!! It will finally tell us whether there is a CEP.**

Summary

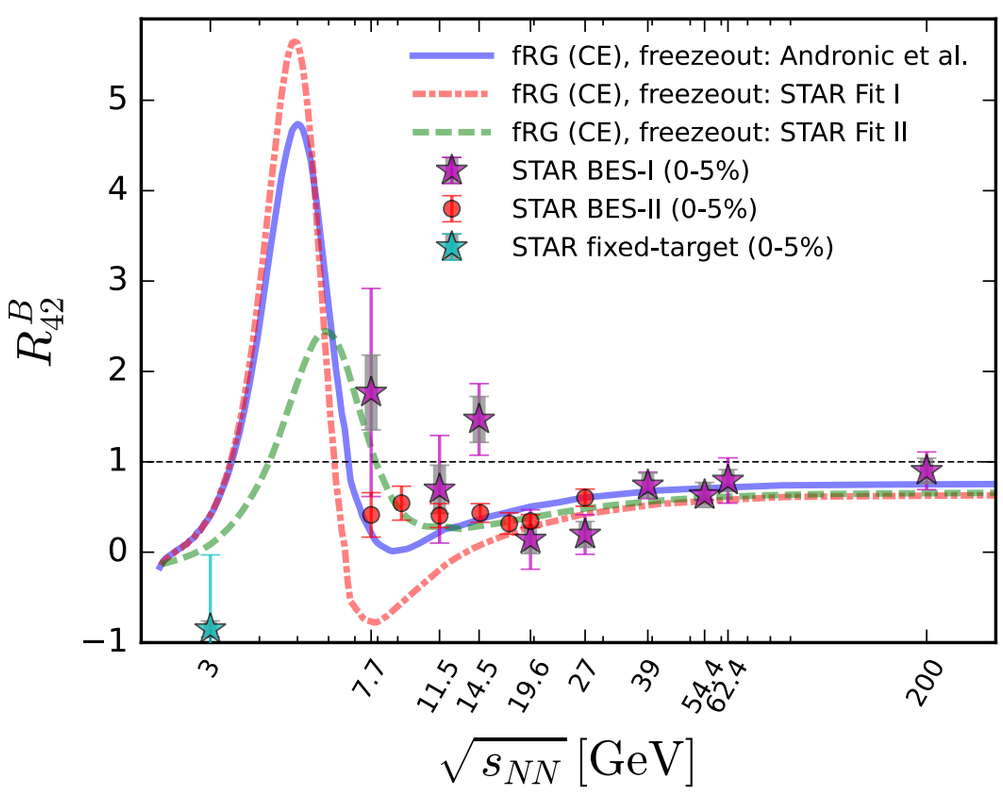


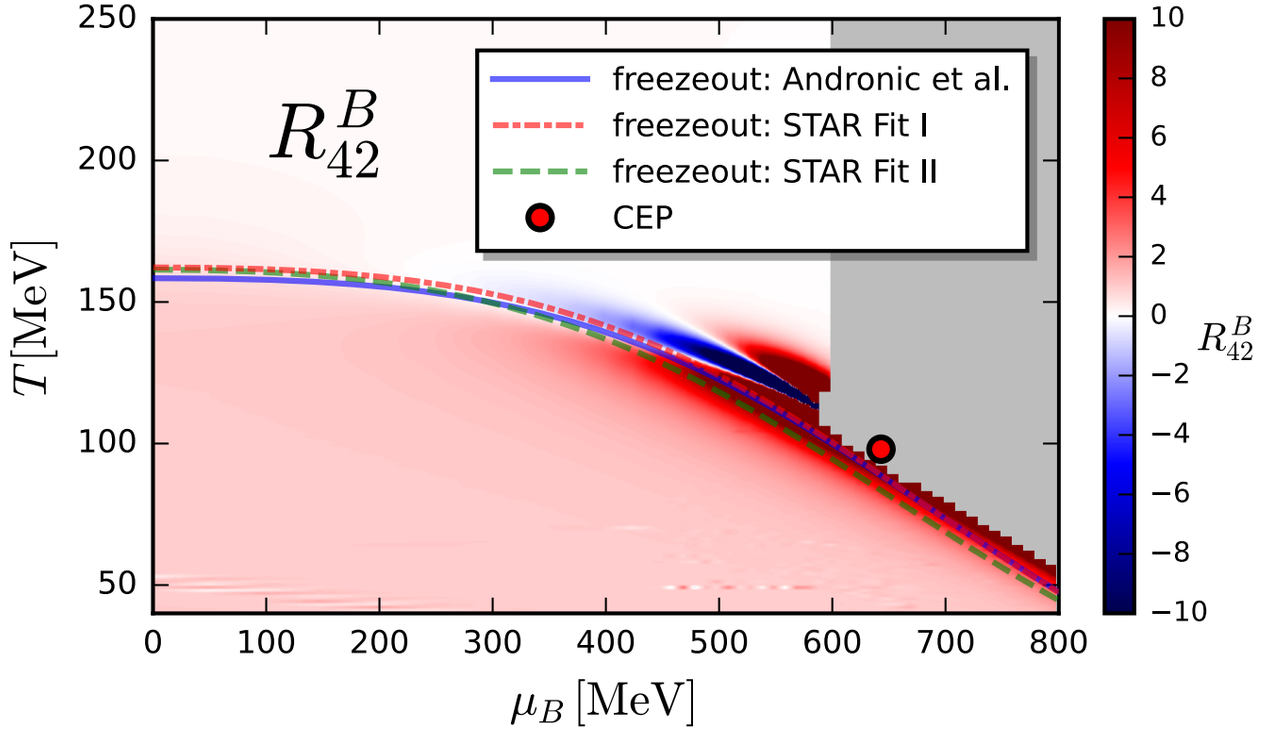


★ Recent BES-II fluctuation data indicate that, very possibly, there is no CEP with $\mu_B \lesssim 400$ MeV.

★ A prominent peak structure is predicted in baryon number fluctuations in the collision energy range of 3 GeV $\lesssim \sqrt{s_{\rm NN}} \lesssim$ 7.7 GeV, which need to be confirmed in experiments in the near future.

Summary





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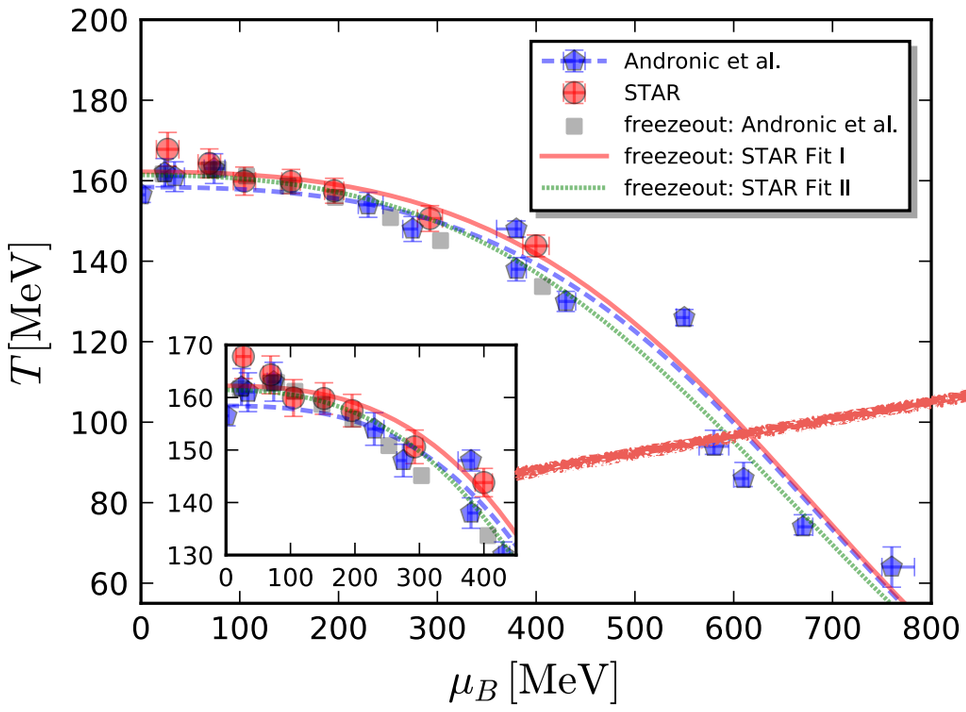
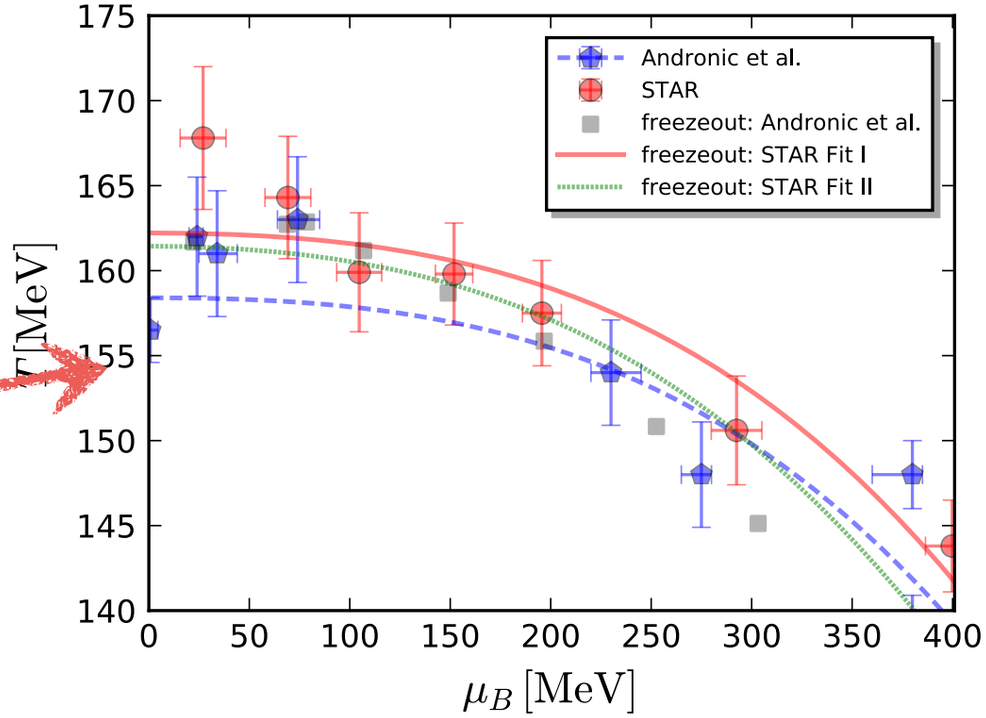
Thank you very much for your attentions!

Backup

Determination of the freeze-out curve





three freeze-out curves

1. freeze-out: Andronic *et al.*

Andronic, Braun-Munzinger, Redlich, *Nature* 561 (2018) 7723, 321

2. freeze-out: STAR Fit I

L. Adamczyk *et al.* (STAR), *PRC* 96 (2017), 044904

3. freeze-out: STAR Fit II

$$\mu_{B\,\mathrm{CF}} = \frac{a}{1 + 0.288\sqrt{s_{\mathrm{NN}}}}\,,$$

$$T_{\mathrm{CF}} = \frac{T^{(0)}_{\mathrm{CF}}}{1 + \exp\left(2.60 - \ln(\sqrt{s_{\mathrm{NN}}})/0.45\right)}$$

neglecting first two at low μ_B and the last one

all data points

- freeze-out curve should not rise with μ_B
- convexity of the freeze-out curve

Functional renormalization group

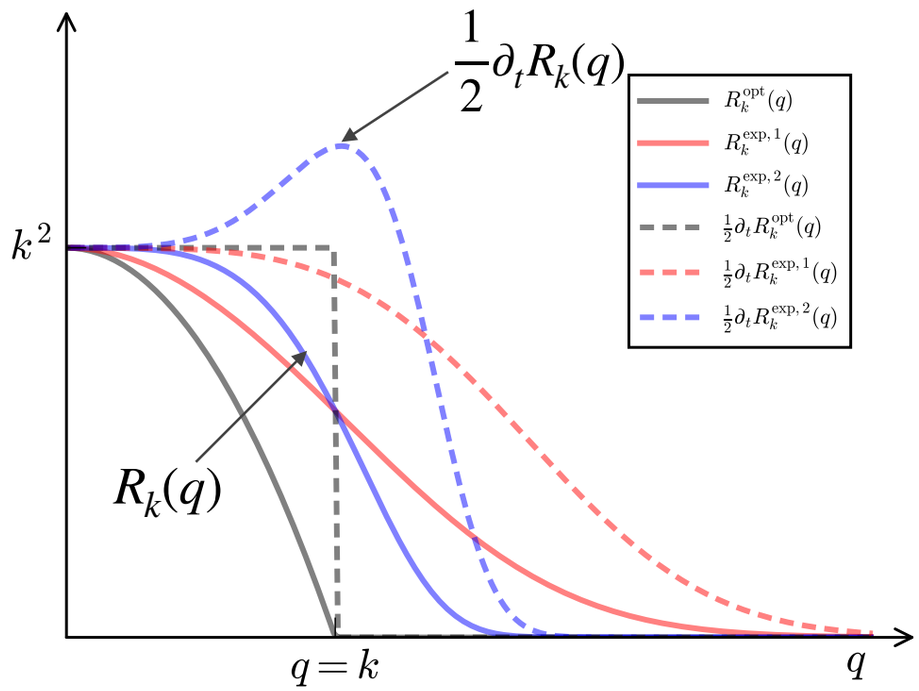
Functional integral with an IR regulator

$$Z_k[J] = \int (\mathscr{D}\hat{\Phi}) \exp\Big\{ - S[\hat{\Phi}] - \Delta S_k[\hat{\Phi}] + J^a \hat{\Phi}_a \Big\}$$

$$W_k[J] = \ln Z_k[J]$$

regulator:

$$\Delta S_k[\varphi] = \frac{1}{2} \int \frac{d^4q}{(2\pi)^4} \varphi(-q) R_k(q) \varphi(q)$$



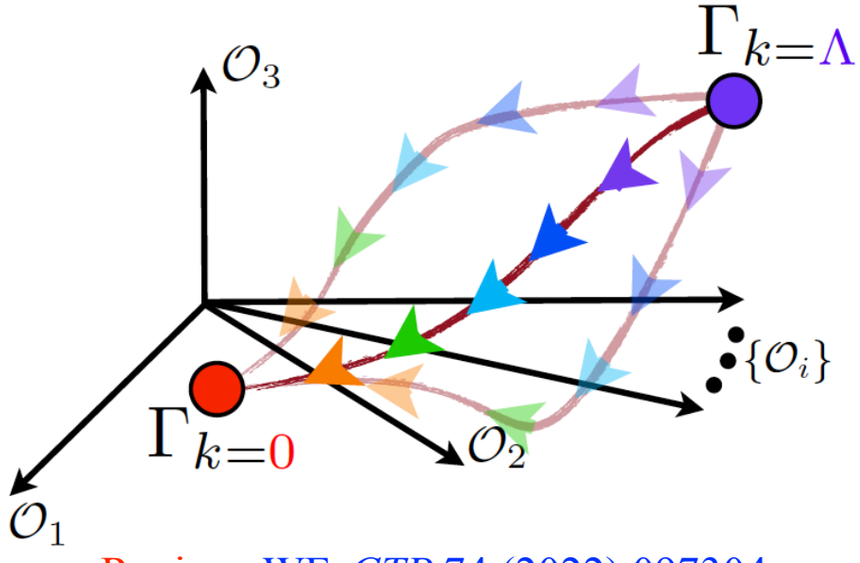
flow of the Schwinger function:

$$\partial_t W_k[J] = -\frac{1}{2} \mathrm{STr}\Big[\big(\partial_t R_k\big) G_k \Big] - \frac{1}{2} \Phi_a \partial_t R_k^{ab} \Phi_b$$

$$G_{k,ab} = \gamma^c{}_a \Big(\Gamma_k^{(2)}[\Phi] + \Delta S_k^{(2)}[\Phi] \Big)^{-1}_{cb},$$

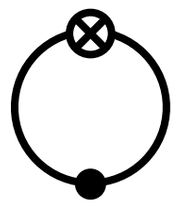
Legendre transformation:

$$\Gamma_k[\Phi] = - W_k[J] + J^a \Phi_a - \Delta S_k[\Phi]$$



flow of the effective action:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{STr}\Big[\big(\partial_t R_k\big) G_k \Big] = \frac{1}{2}$$

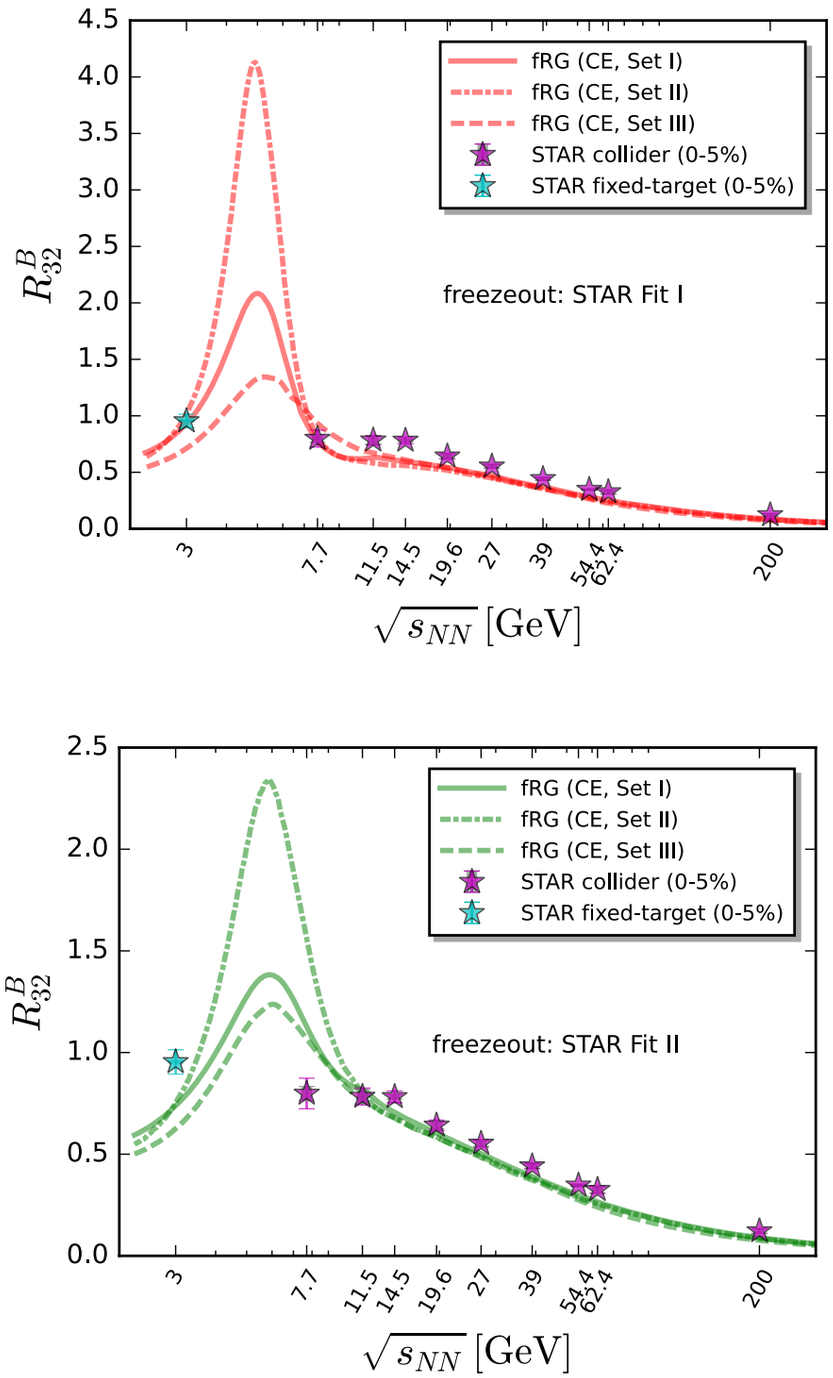


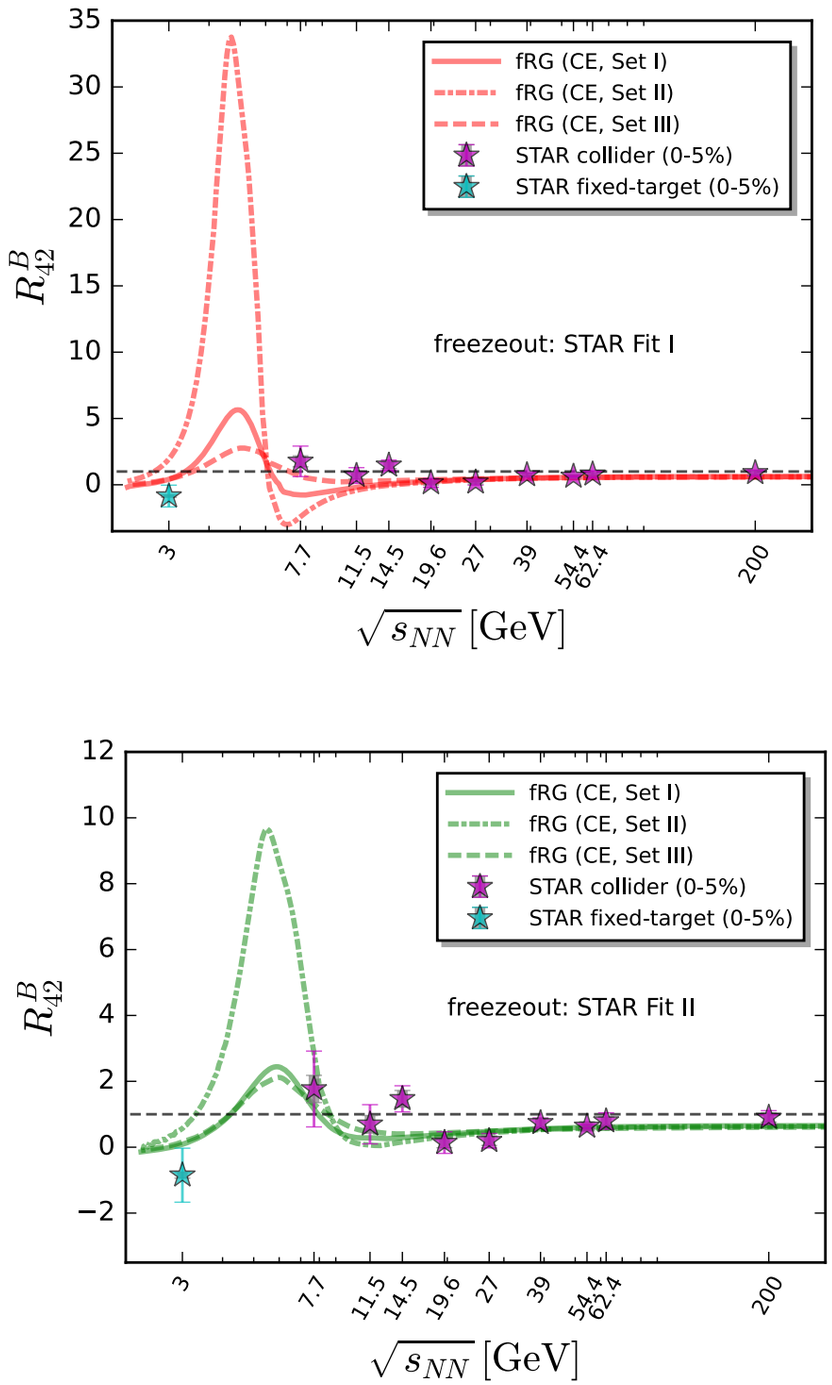
Wetterich formula

C. Wetterich, *PLB*, 301 (1993) 90

Review: WF, *CTP* 74 (2022) 097304, arXiv: 2205.00468 [hep-ph]

Dependence of the location of CEP





STAR: Adam *et al.* (STAR), *PRL* 126 (2021) 092301

fRG: WF, Luo, Pawlowski, Rennecke, Yin, arXiv: 2308.15508