The time evolution of elliptic flow in heavy-ion collisions at intermediate energies

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2nd Symposium on Intermediate-energy Heavy Ion Collisions (iHIC24) 12-16 July 2024



Huzhou city

East longitude
120°

North latitude
30°



World-famous
old town: Nanxun

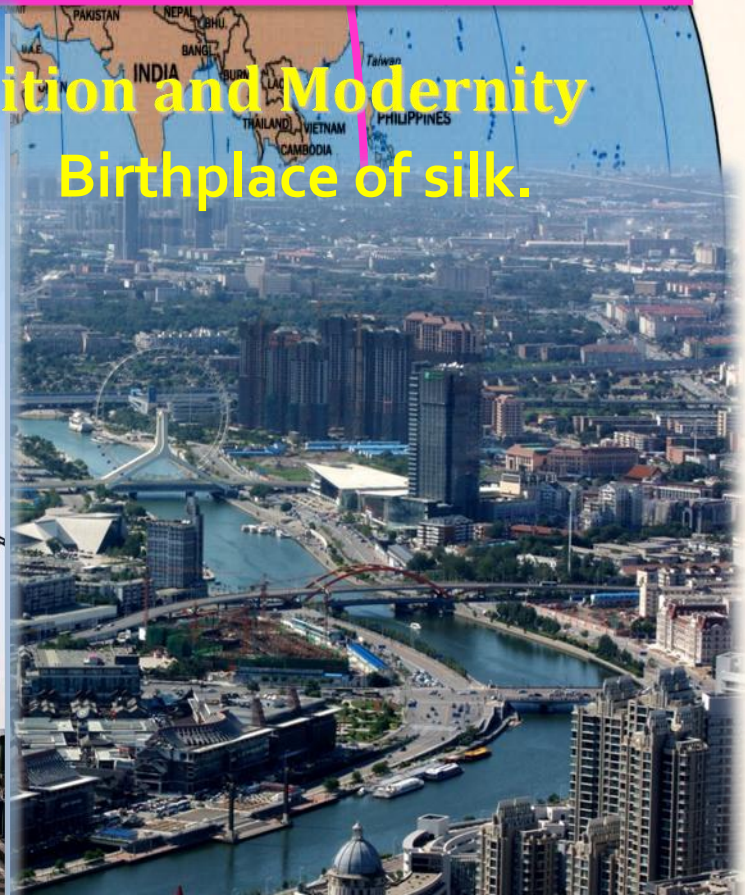


100 m tall



Twin towers
288 m tall

A city of Tradition and Modernity
Birthplace of silk.



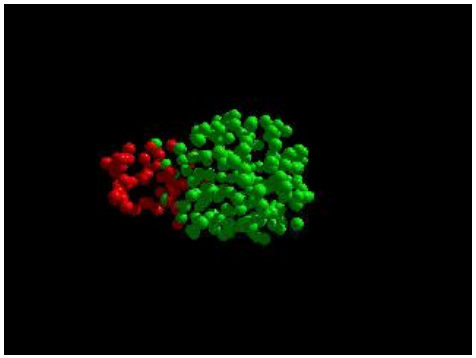
Outline

- **Introduction**
  - **Heavy-ion collisions (HIC) •Nuclear equation of state (EOS)**
- **Transport model for intermediate energy HIC**
- **Time evolution of elliptic flow**
- **The density at which the EOS is probed by elliptic flow**
- **Summary**

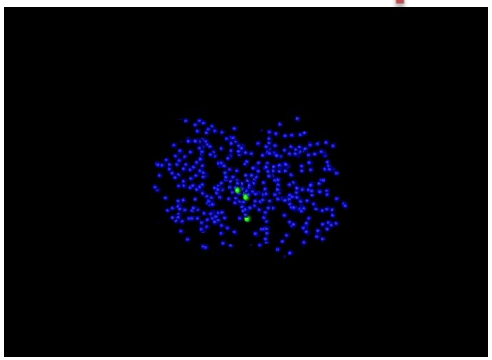
Overview of HIC

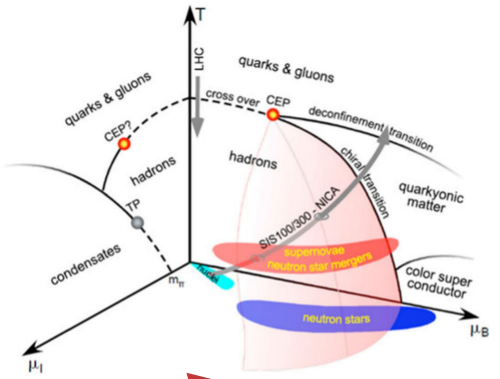
◆ intermediate energies HIC

QCD phase diagram, QGP properties



Nuclear equation of state





◆ Low energy fusion reaction

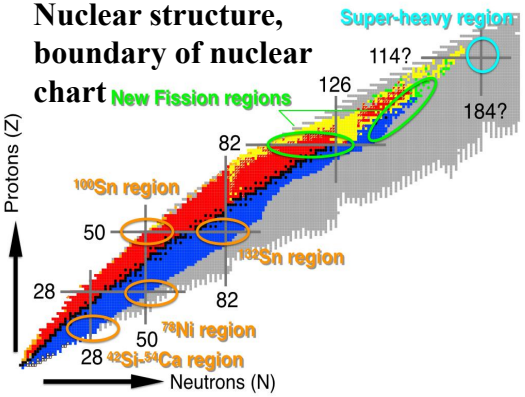
~100 MeV- a few GeV/nucleon

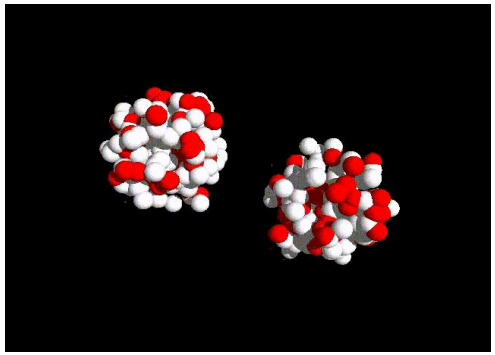
beam energy

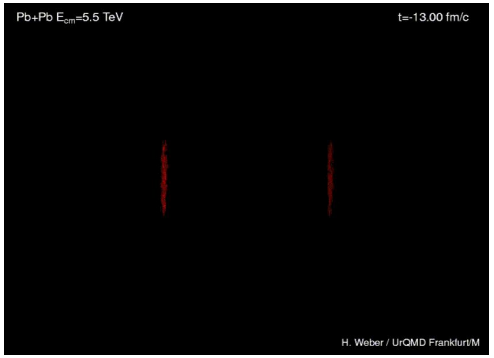
a few 100 MeV/nucleon

>tens of GeV/nucleon

Nuclear structure, boundary of nuclear chart







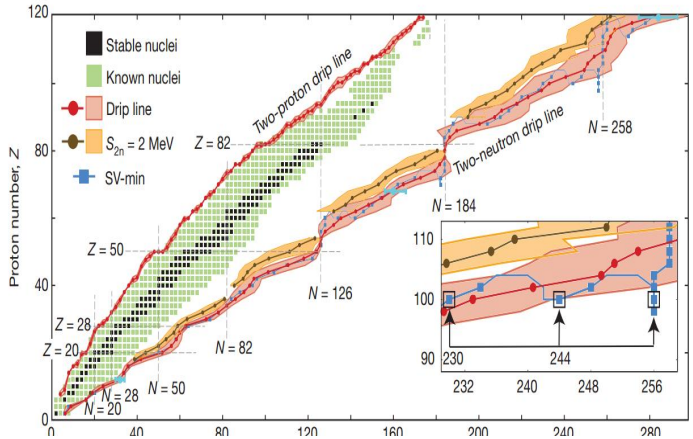
◆ Relativistic HIC

Nuclear equation of state (EOS)

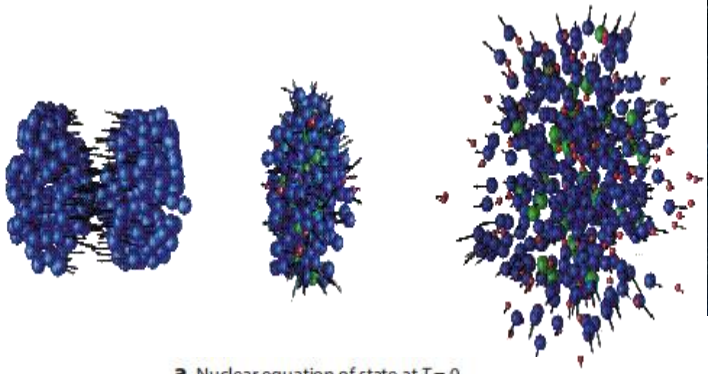
The thermodynamic relationship between the binding energy *E* (or pressure *P*) and density ρ, as well as the isospin asymmetry δ.

$$E(\rho,\delta)=E(\rho,0)+E_{sym}(\rho)\delta^2+\cdots, \qquad \delta=\frac{\rho_n-\rho_p}{\rho_n+\rho_p}$$

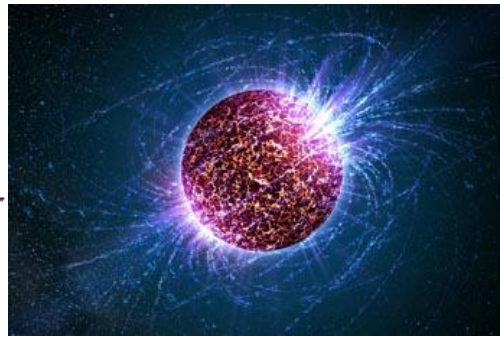
Nuclear landscape

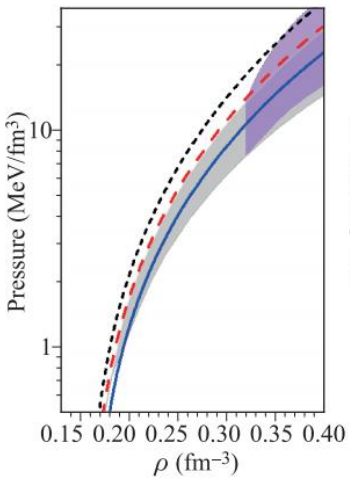


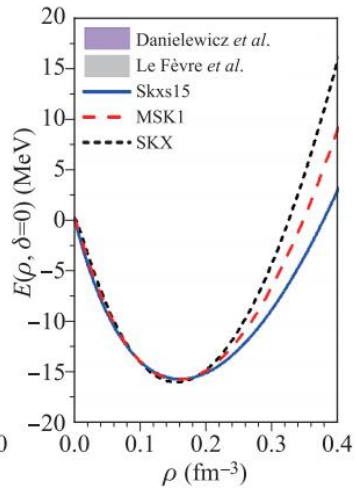
Heavy ion collision

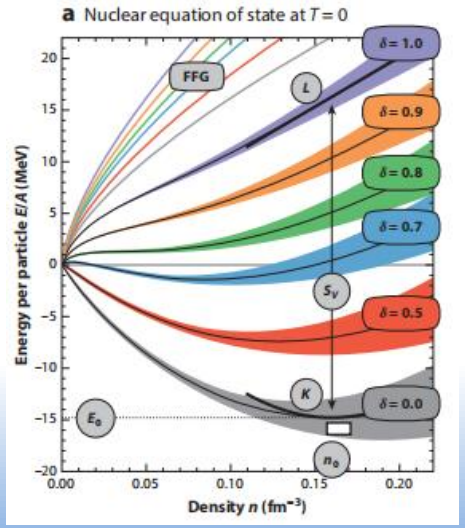


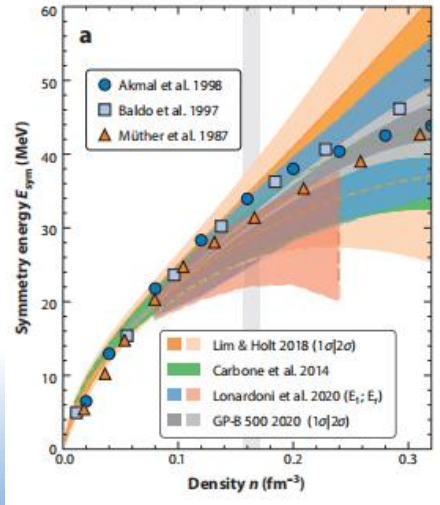
Neutron star











Nuclear equation of state (EOS)

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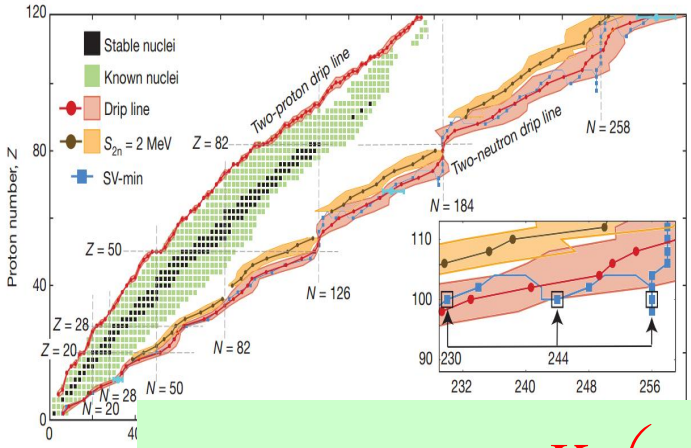
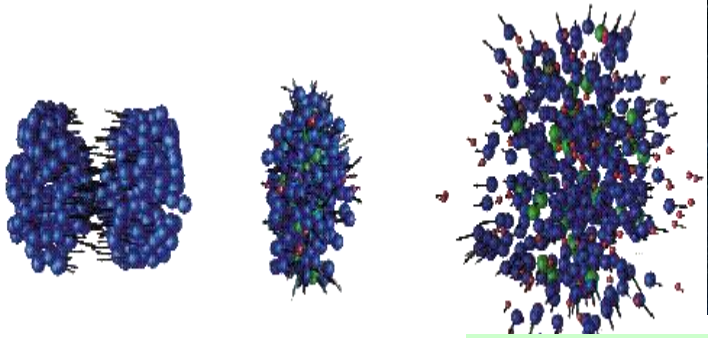
$$E(\rho,\delta)=E(\rho,0)+E_{sym}(\rho)\delta^2+\cdots,\qquad \delta=\frac{\rho_n-\rho_p}{\rho_n+\rho_p}$$

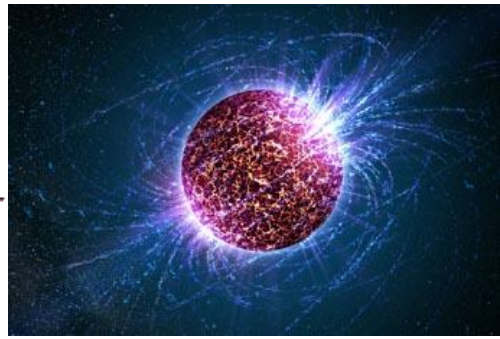
Nuclear landscape



Heavy ion collision



Neutron star



$$E(\rho,0)=E_0+\frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2+\ldots,$$

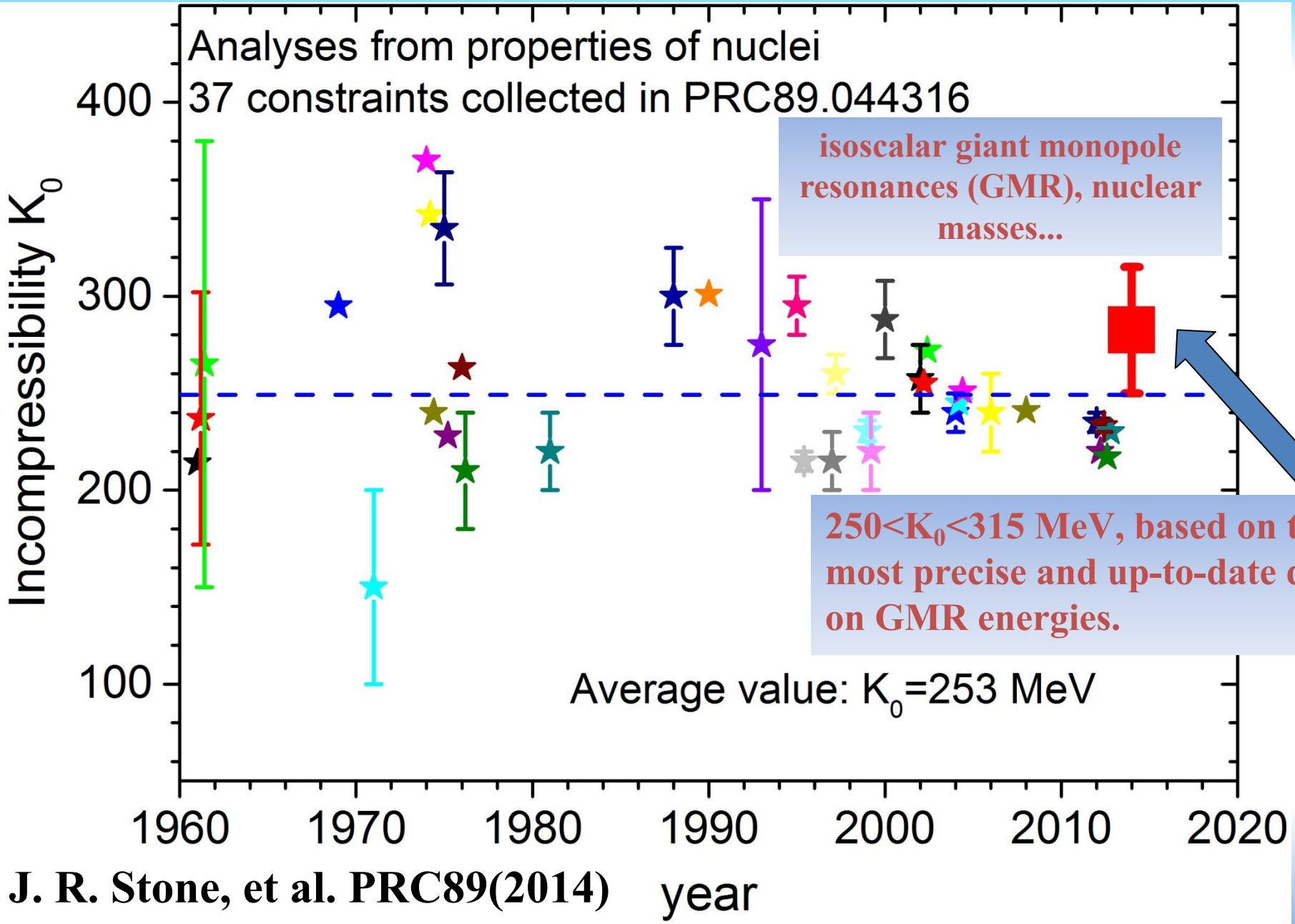
$$K_0=9\rho^2\left(\frac{\partial^2 E}{\partial\rho^2}\right)\Bigg|_{\rho=\rho_0}$$

$$E_{sym}(\rho)=S_0+L\left(\frac{\rho-\rho_0}{3\rho_0}\right)+\frac{K_{sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2+\ldots$$

$$L=3\rho\frac{dE_{sym}(\rho)}{d\rho}\Bigg|_{\rho=\rho_0}$$

K_0 and L determine the EOS in the vicinity of the saturation density.

The incompressibility K_0 from properties of nuclei

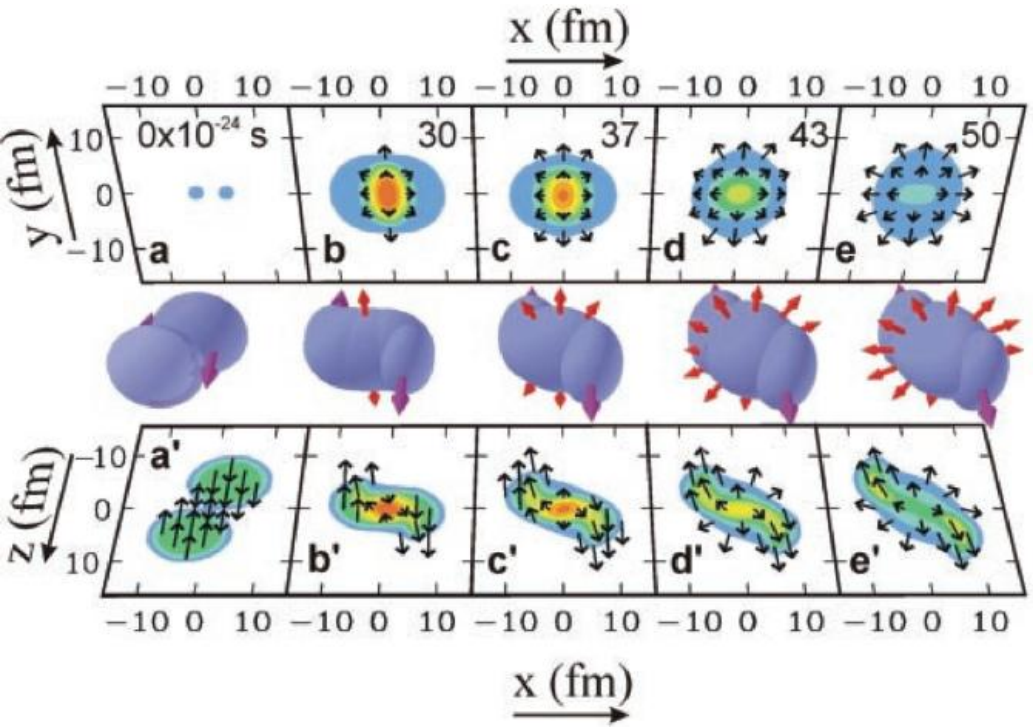


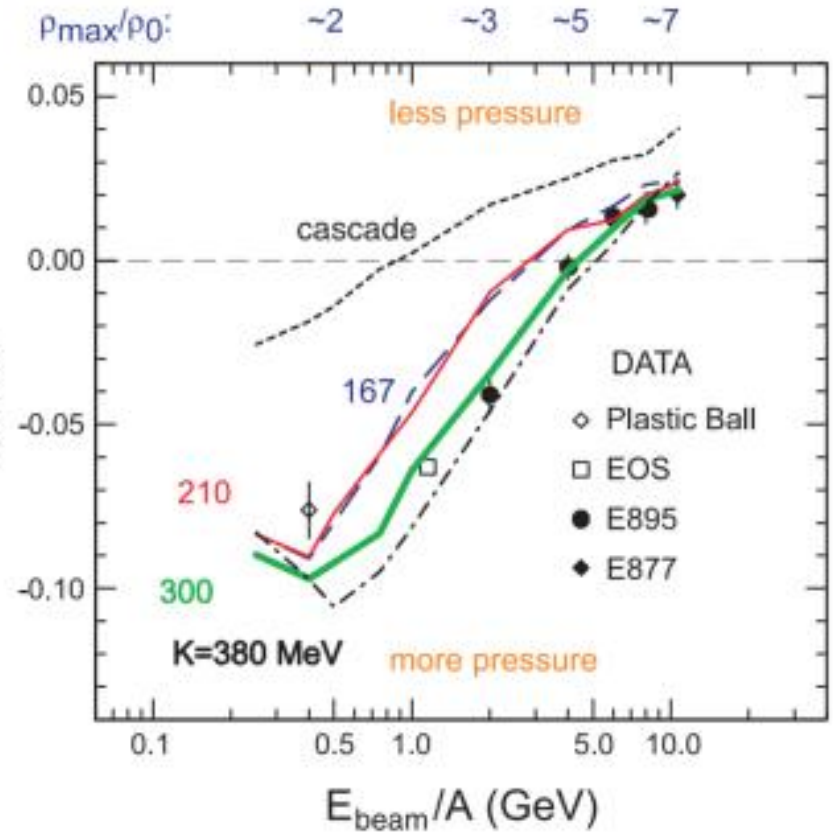
J. R. Stone, et al. PRC89(2014)

HIC offers a unique way to create nuclear matter with high density and isospin asymmetry in laboratory.

Science
AAAS

Determination of the Equation of State of Dense Matter
Pawel Danielewicz, *et al.*
Science **298**, 1592 (2002);
DOI: 10.1126/science.1078070





EOS can be deduced from the comparision bewteen experimental observables and transport model calculations.

Connections between small collisions and big collisions

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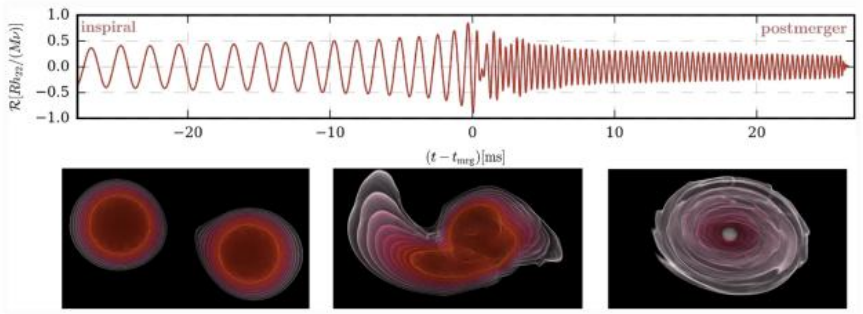
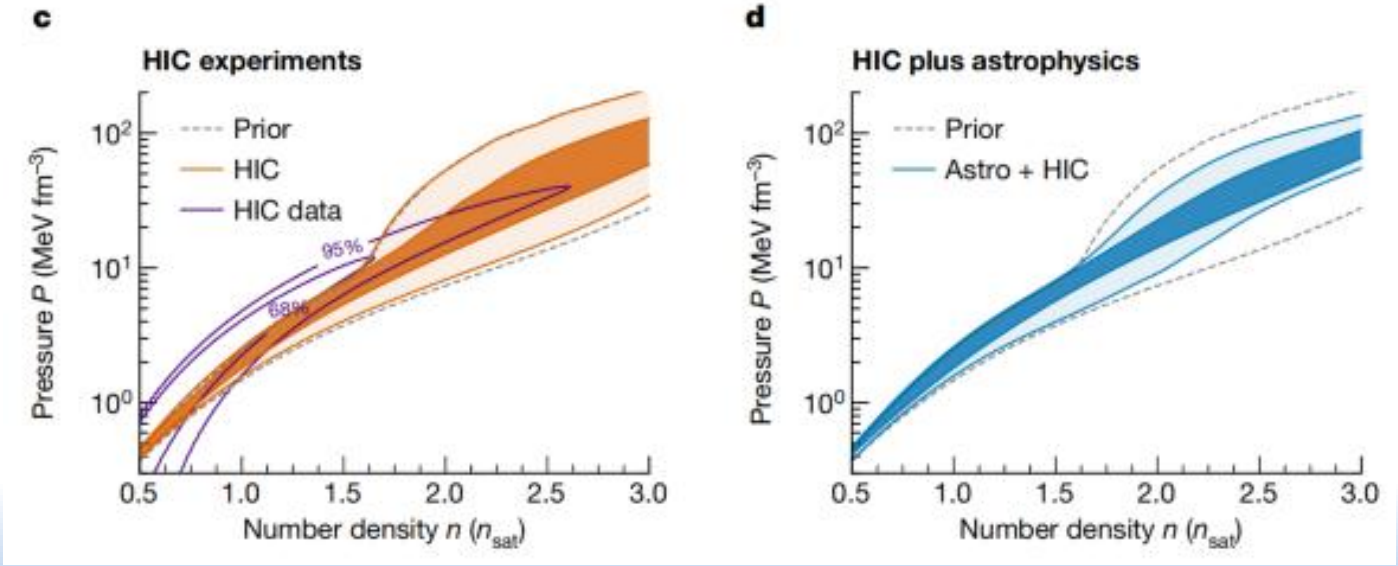
Constraining neutron-star matter with microscopic and macroscopic collisions

Sabrina Huth ✉, Peter T. H. Pang ✉, Ingo Tews, Tim Dietrich, Arnaud Le Fèvre, Achim Schwenk, Wolfgang Trautmann, Kshitij Agarwal, Mattia Bulla, Michael W. Coughlin & Chris Van Den Broeck

Nature **606**, 276–280 (2022) | Cite this article

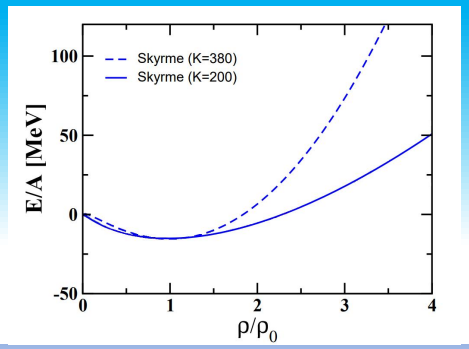
Neutron star properties

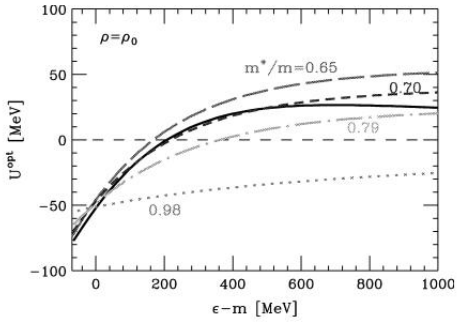




Elliptic flow (v_2)

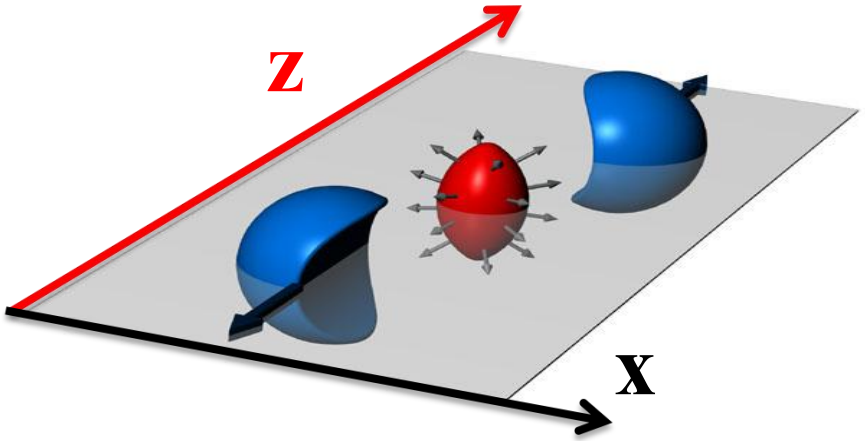
$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

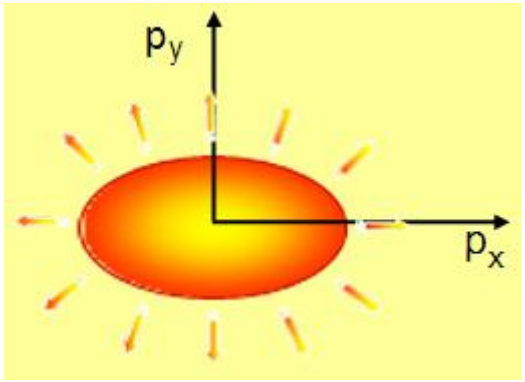




Momentum-space anisotropy

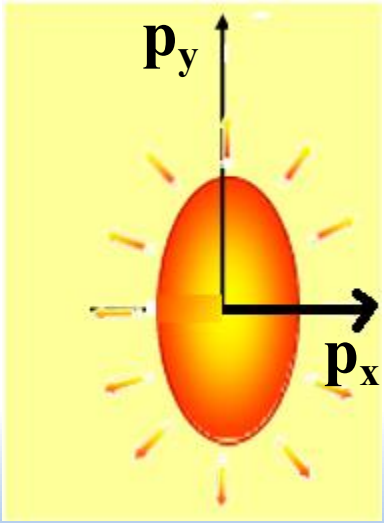
Coordinate-space anisotropy





In-plane
$v_2>0$

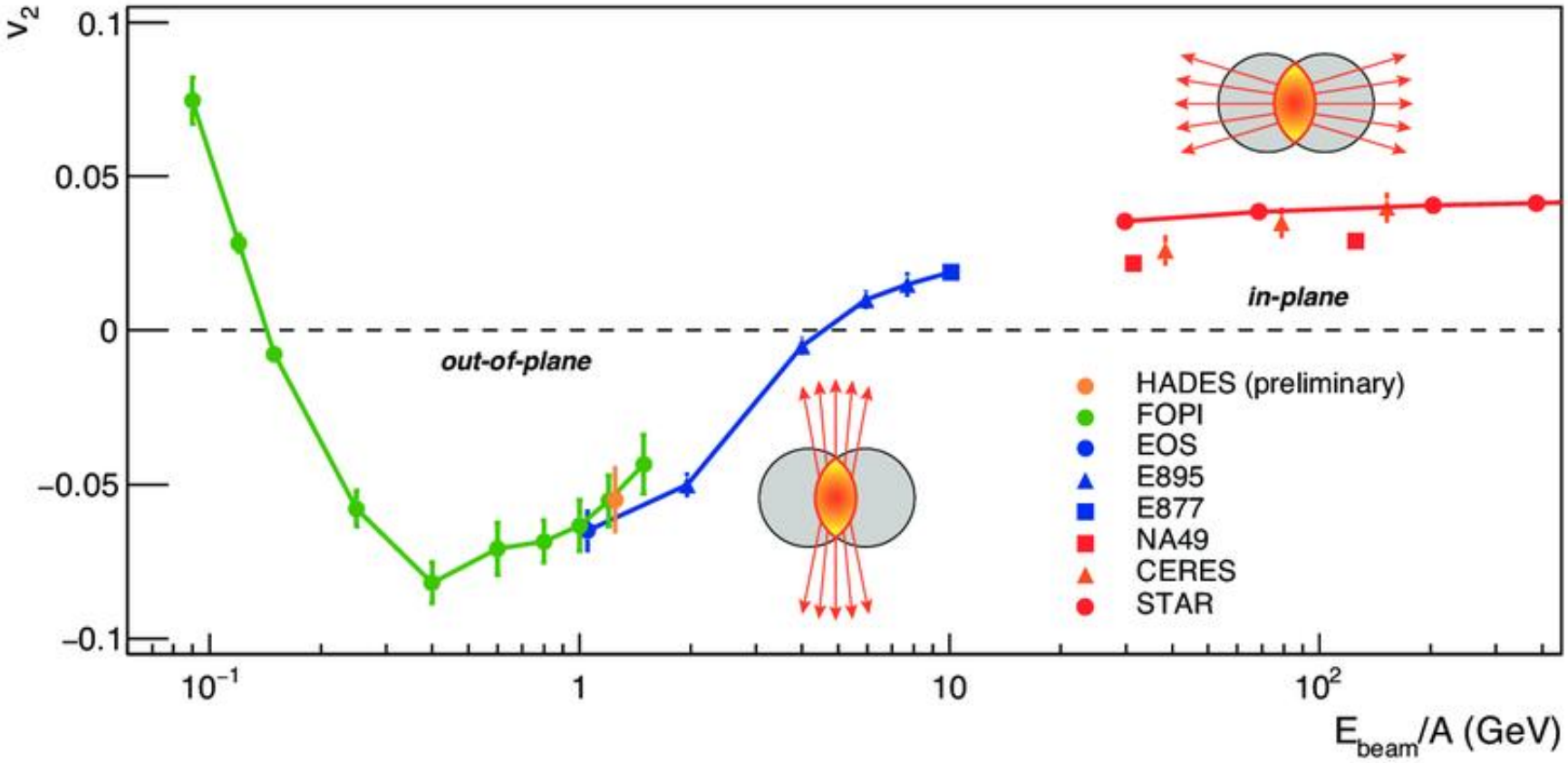
$$p_t = \sqrt{p_x^2 + p_y^2}\,; y = \frac{1}{2}\ln\left[\frac{E + p_z}{E - p_z}\right]$$



Out-of-plane
Squeeze out
$v_2<0$

v_2 has been widely used to probe the nuclear EOS, the nucleon effective mass,...

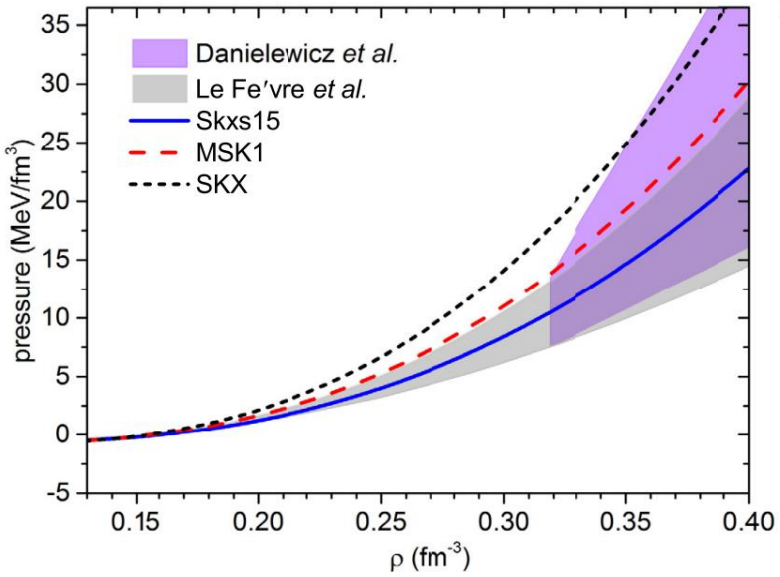
Elliptic flow (v_2)

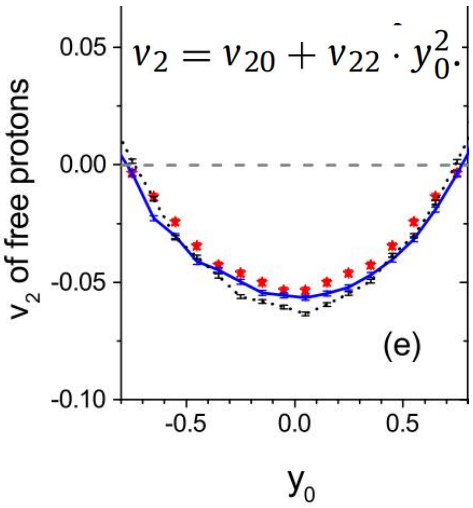


Elliptic flow is a gold mine.

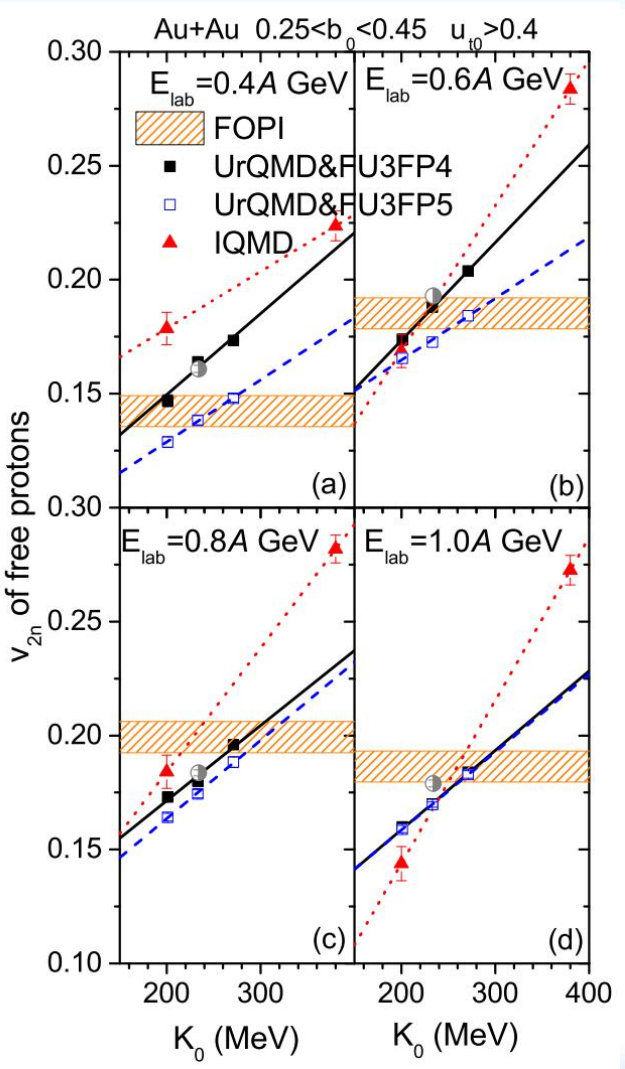
Deducing the nuclear EOS

	K_0 (MeV)	S_0 (MeV)	L (MeV)
Skxs15	201	31.88	34.79
MSK1	234	30.00	33.92
SKX	271	31.10	33.18
SV-sym34	234	34.00	80.95





$$v_{2n} = |v_{20}| + |v_{22}|$$

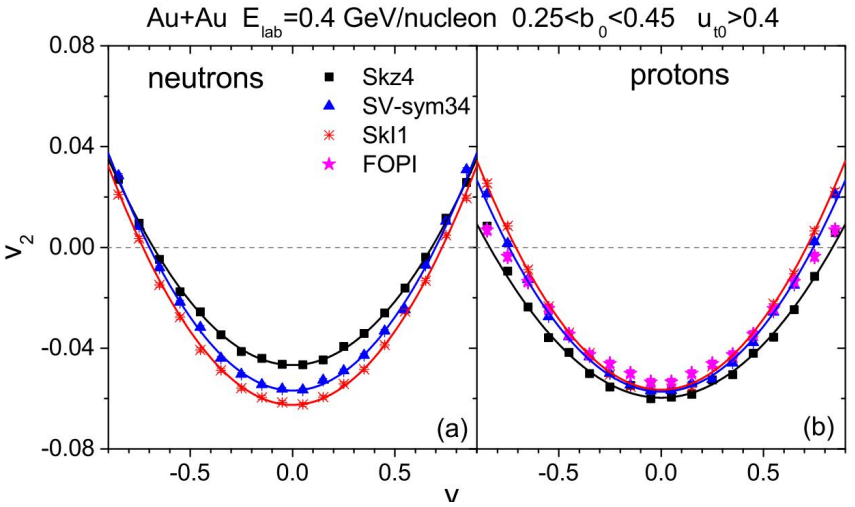


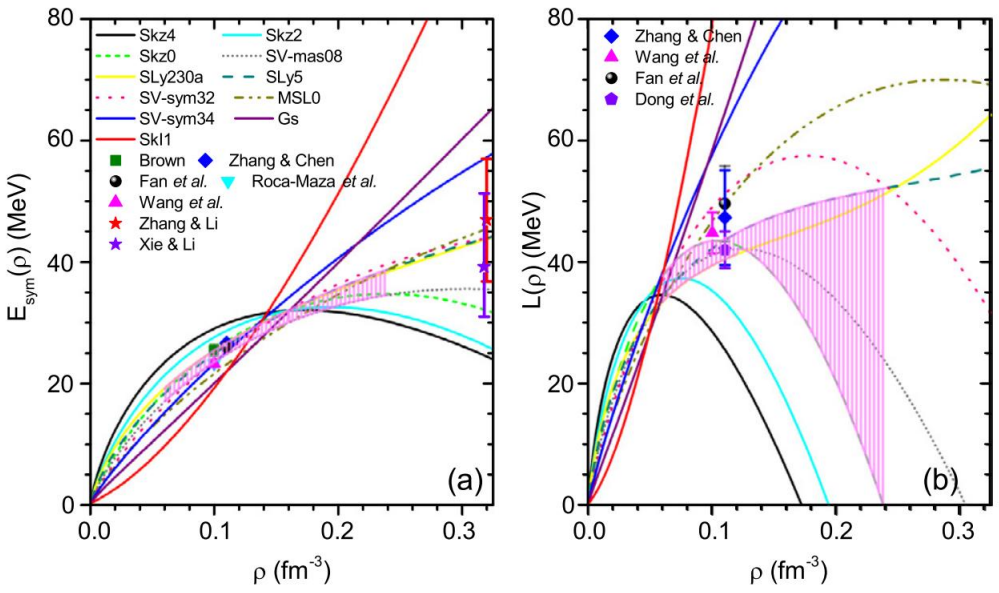
Determination of the nuclear incompressibility from the rapidity-dependent elliptic flow in heavy-ion collisions at beam energies 0.4A–1.0A GeV

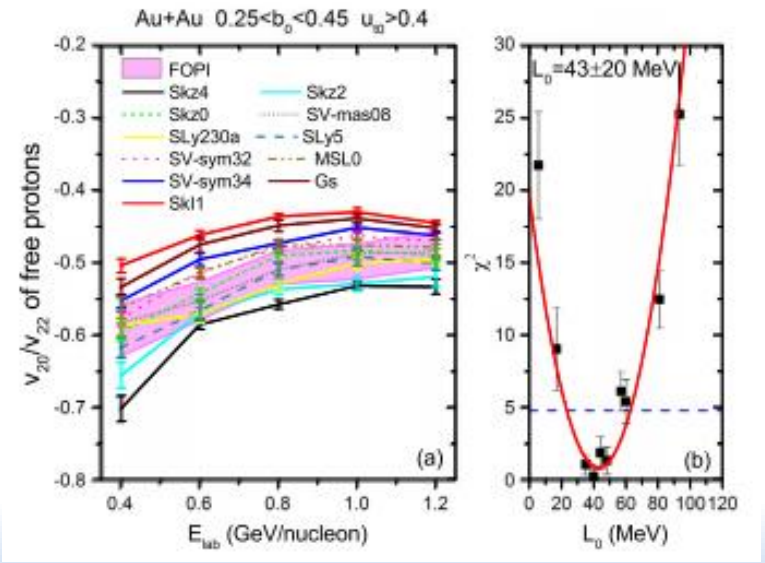
Yongjia Wang, et al., Physics Letters B 778 (2018) 207–212.

Deducing the nuclear symmetry energy

	ρ_0 (fm^{-3})	K_0 (MeV)	S_0 (MeV)	L_0 (MeV)	K_{sym} (MeV)
Skz4	0.16	230	32.0	5.8	-240.9
Skz2	0.16	230	32.0	16.8	-259.7
Skz0	0.16	230	32.0	35.1	-242.2
SV-mas08	0.16	233	30.0	40.2	-172.4
SLy230a	0.16	230	32.0	44.3	-98.2
SLy5	0.16	230	32.0	48.2	-112.7
SV-sym32	0.159	234	32.0	57.1	-148.8
MSL0	0.16	230	30.0	60.0	-99.3
SV-sym34	0.159	234	34.0	80.9	-79.1
Gs	0.158	237	31.1	93.3	14.1
SkI1	0.16	243	37.5	161.1	234.7







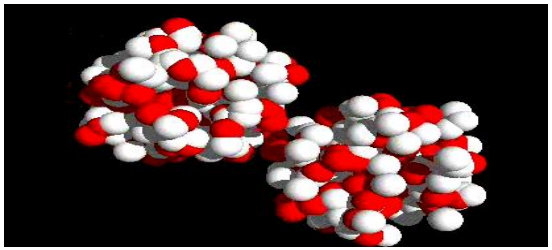
Yongjia Wang, et al., Physics Letters B 802 (2020) 135249.

Ultrarelativistic Quantum Molecular Dynamics Model (UrQMD)

1).Initialization

Get the coordinate **r**

and the momentum **p**



$$\phi_i(\vec{r}_i;t) = \frac{1}{(2\pi)^{3/4}(\Delta x)^{3/2}} \exp\left\{-\frac{[\vec{r}_i - \vec{R}_i(t)]^2}{(2\Delta x)^2} + i\vec{r}_i \cdot \vec{P}_i(t)\right\}.$$

2).Propagation

Nucleon moves in the mean-field.

Density, momentum, isospin-dependent.

$$\dot{\mathbf{p}}_i = -\frac{\partial H}{\partial \mathbf{r}_i}, \quad \dot{\mathbf{r}}_i = \frac{\partial H}{\partial \mathbf{p}_i}.$$

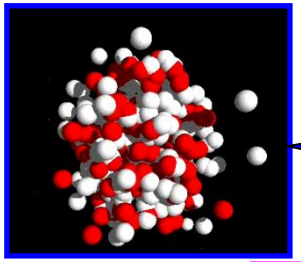
Input Skyrme forces

EOS, Symmetry energy

3).Collision term

Medium modified cross section.

Density, momentum, isospin-dependent. Pauli blocking.



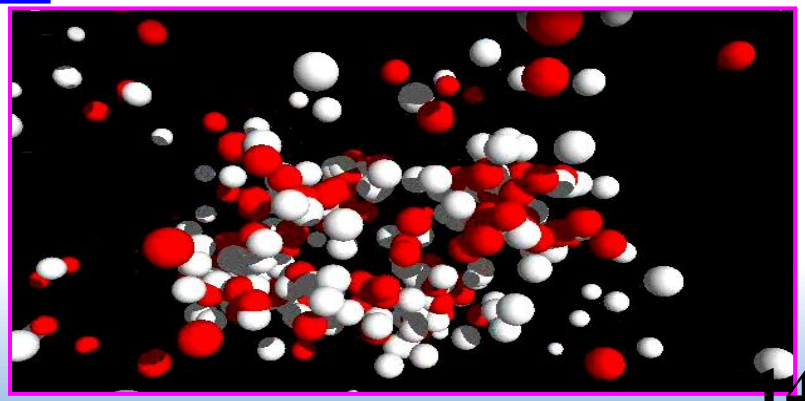
Input: $f\sigma_{pp}=f\sigma_{nn}$, and $f\sigma_{np}$

4). Cluster recognition

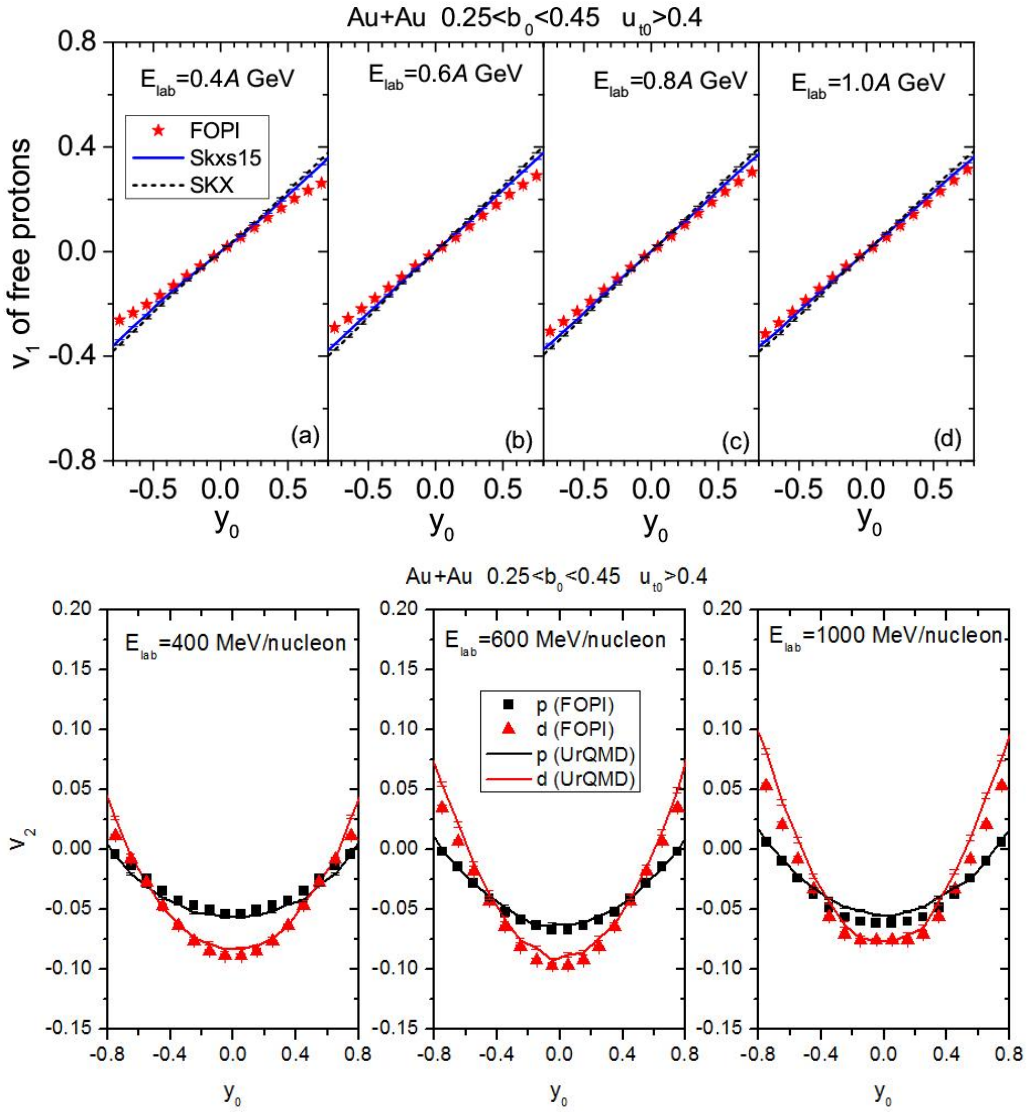
an isospin-dependent Minimum Spanning Tree

Then, compare the simulated results with experimental data, one can get the information of EoS and in-medium NN cross section.

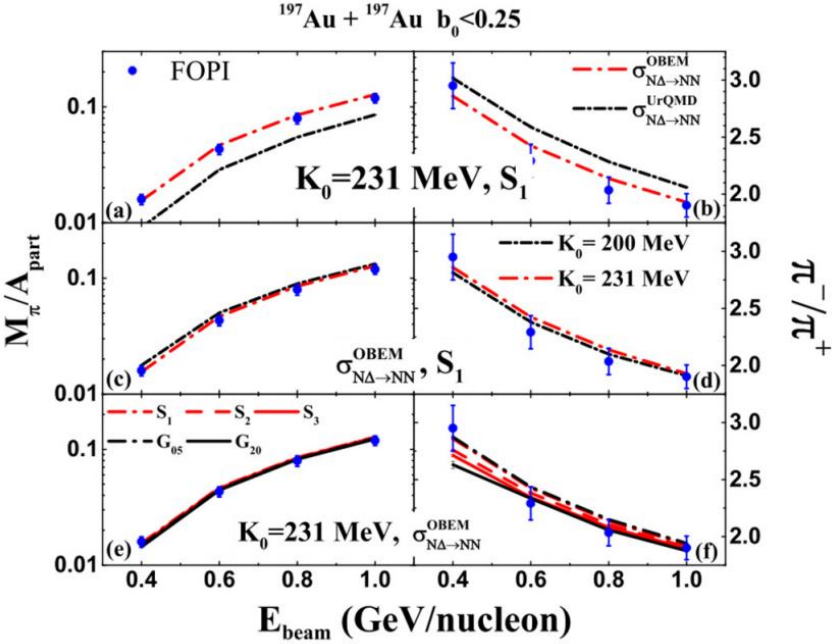
PRC 83.044617; 89.034606;



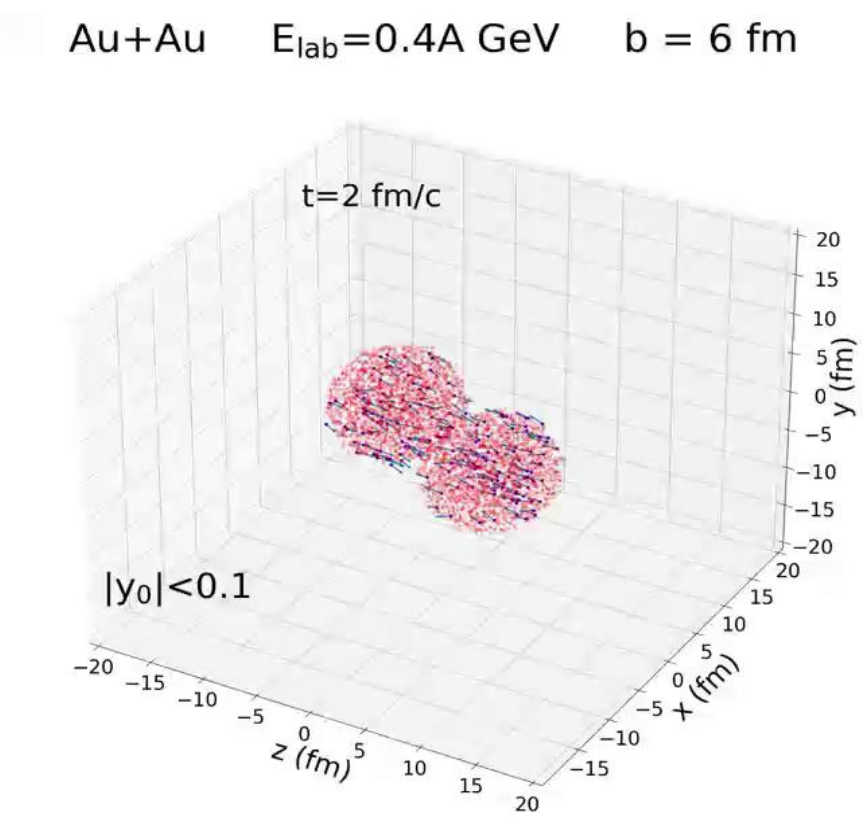
Ultrarelativistic Quantum Molecular Dynamics Model (UrQMD)

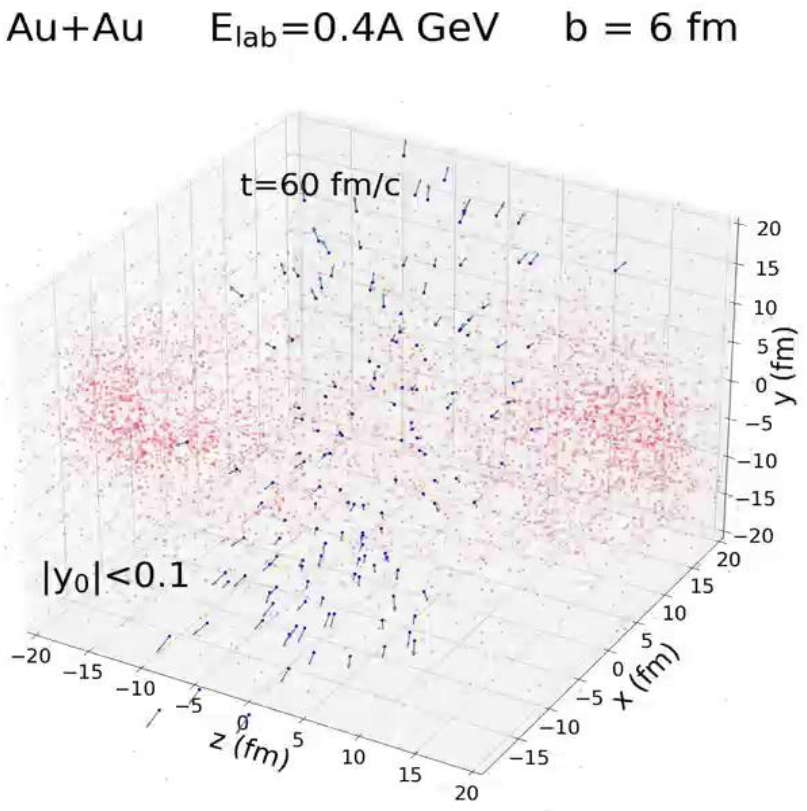


PRC83, 044617; PRC89, 034606; PRC97, 044620; PRC97, 034602; Frontiers of Physics, 15(4), 44302 (2020); PLB828, 137019; PRC103, 014616.....



In the framework of transport model, the momentum of each nucleon can be changed either by the force caused by the nuclear mean field (MF) potential or by nucleon-nucleon (NN) scattering.





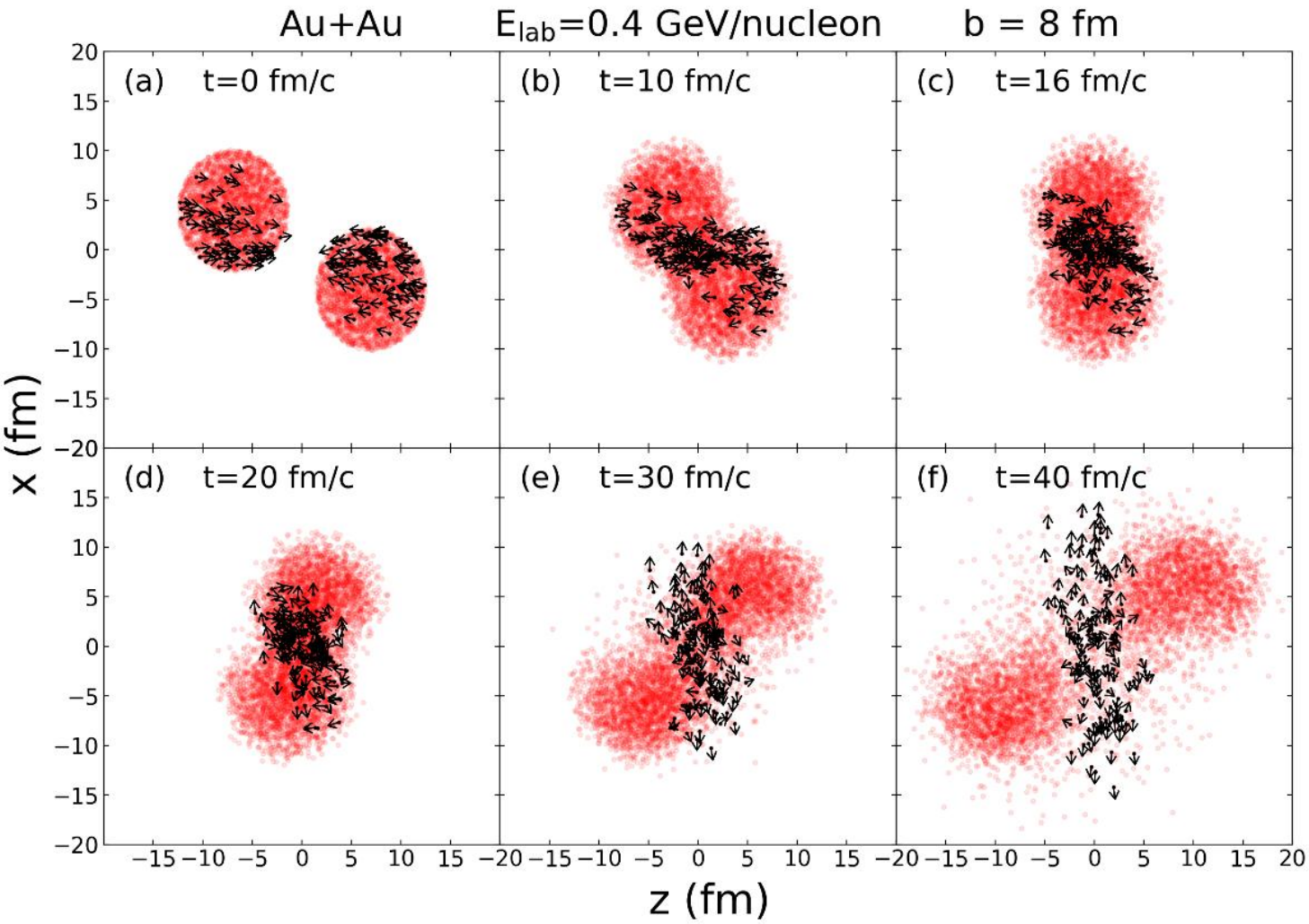
Reversely tracing nucleons that are finally emitted at mid-rapidity (-0.1<y_0<0.1). Recording the momentum changes casued by MF and NN collisions.

$$\Delta v_2^{\text{coll}}(t) = \left\langle V_2^{\text{aft.coll}}(t) - V_2^{\text{bef.coll}}(t) \right\rangle.$$

$$\Delta v_2^{\text{mf}}(t) = \left\langle V_2^{\text{aft.mf}}(t+\Delta t) - V_2^{\text{bef.mf}}(t) \right\rangle.$$

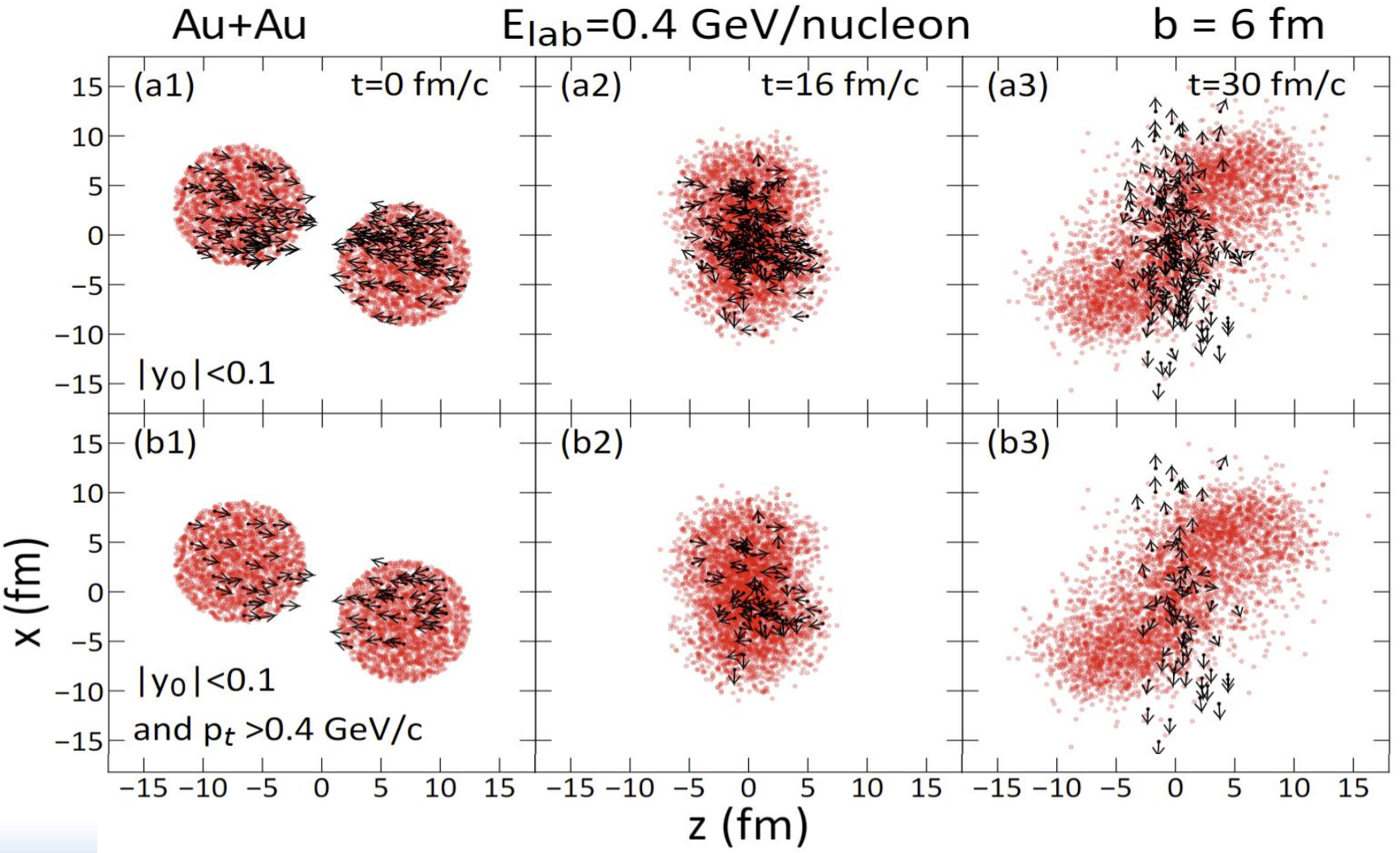
V_2 represents $\frac{p_x^2 - p_y^2}{p_t^2}$

Solid dots with arrows represent the traced nucleons (those will be appeared at mid-rapidity at the final stage), while circles denote other nucleons.

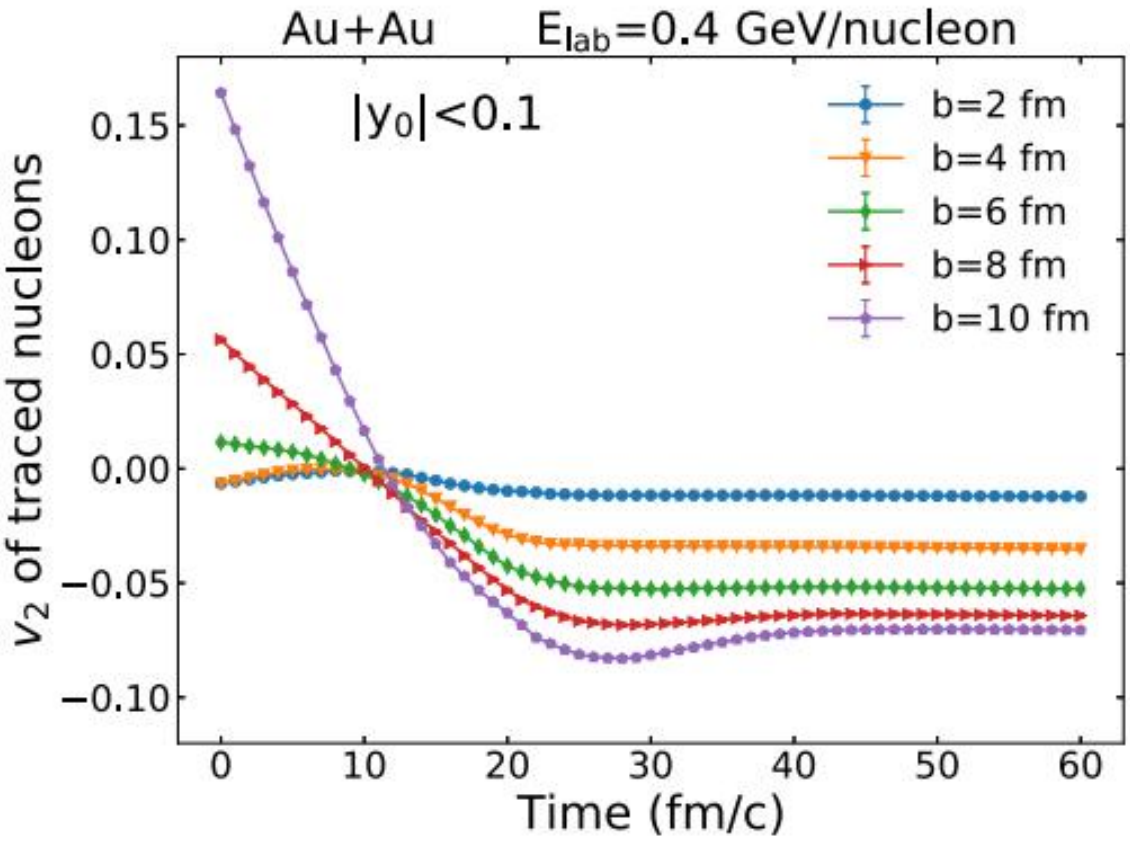


Gao B, **Wang Y**, Gao Z, et al. Phys. Lett. B, 838 (2023) 137685.

Time evolution of v_2 of the traced nucleons finally emitted at mid-rapidity with different transverse momentum.



Time evolution of v_2 of the traced nucleons finally emitted at mid-rapidity ($|y_0| < 0.1$) with different impact parameters.



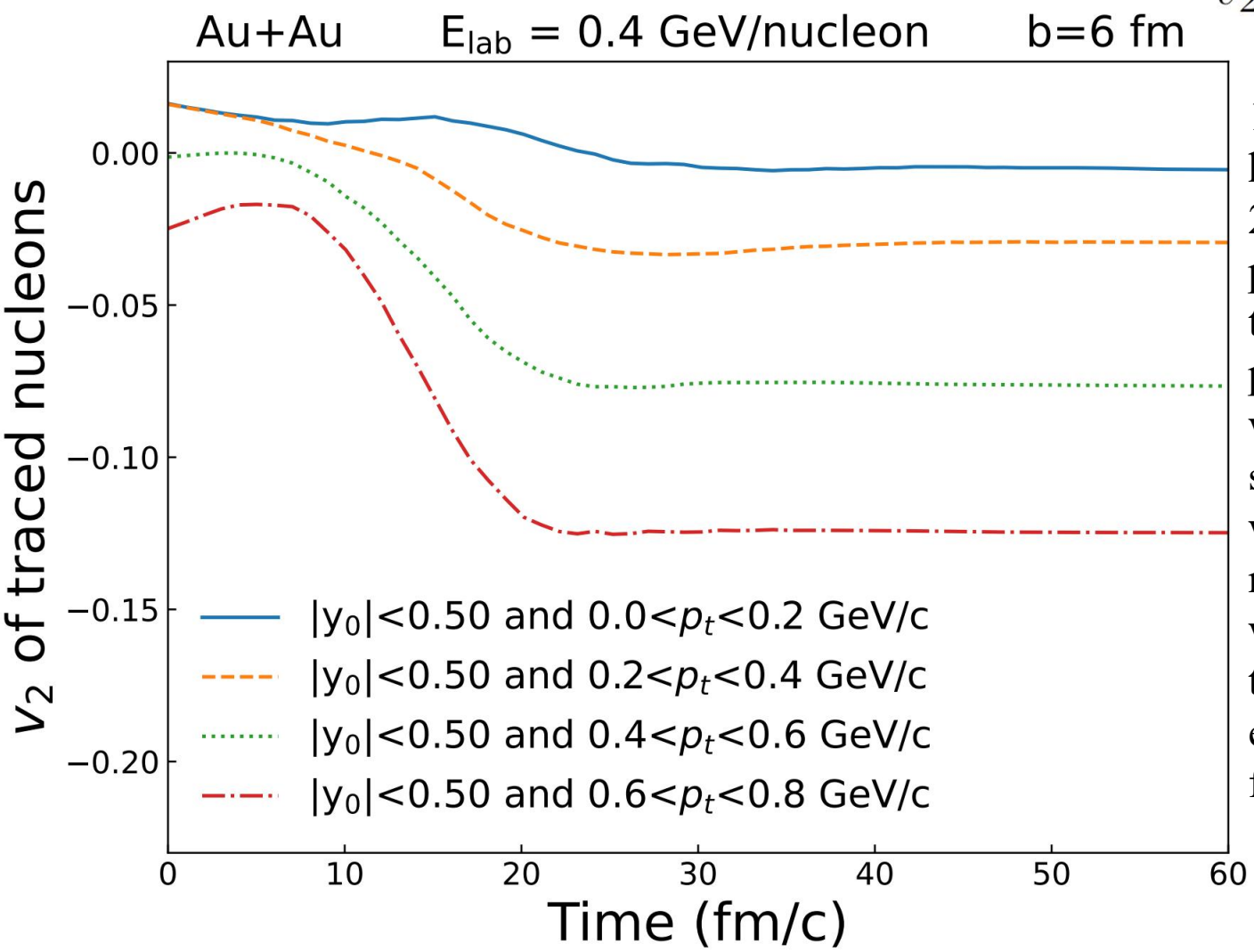
1. At final stage, elliptic flow increases will increasing b.

2. At the initial time, v_2 is non-zero, indicating that those nucleons **are not randomly "selected"** out of the uniform distribution in the initial state. This is especially so for large b where v_2(t=0 fm/c) is positive because nucleons with larger |px| and smaller |py| tend to have higher probability to experience NN scattering and finally emerge at midrapidity.

Gao B, **Wang Y**, Gao Z, et al. Phys. Lett. B, 838 (2023) 137685.

Time evolution of v_2 of the traced nucleons finally emitted at mid-rapidity ($|y_0| < 0.5$) with different transverse momentum.

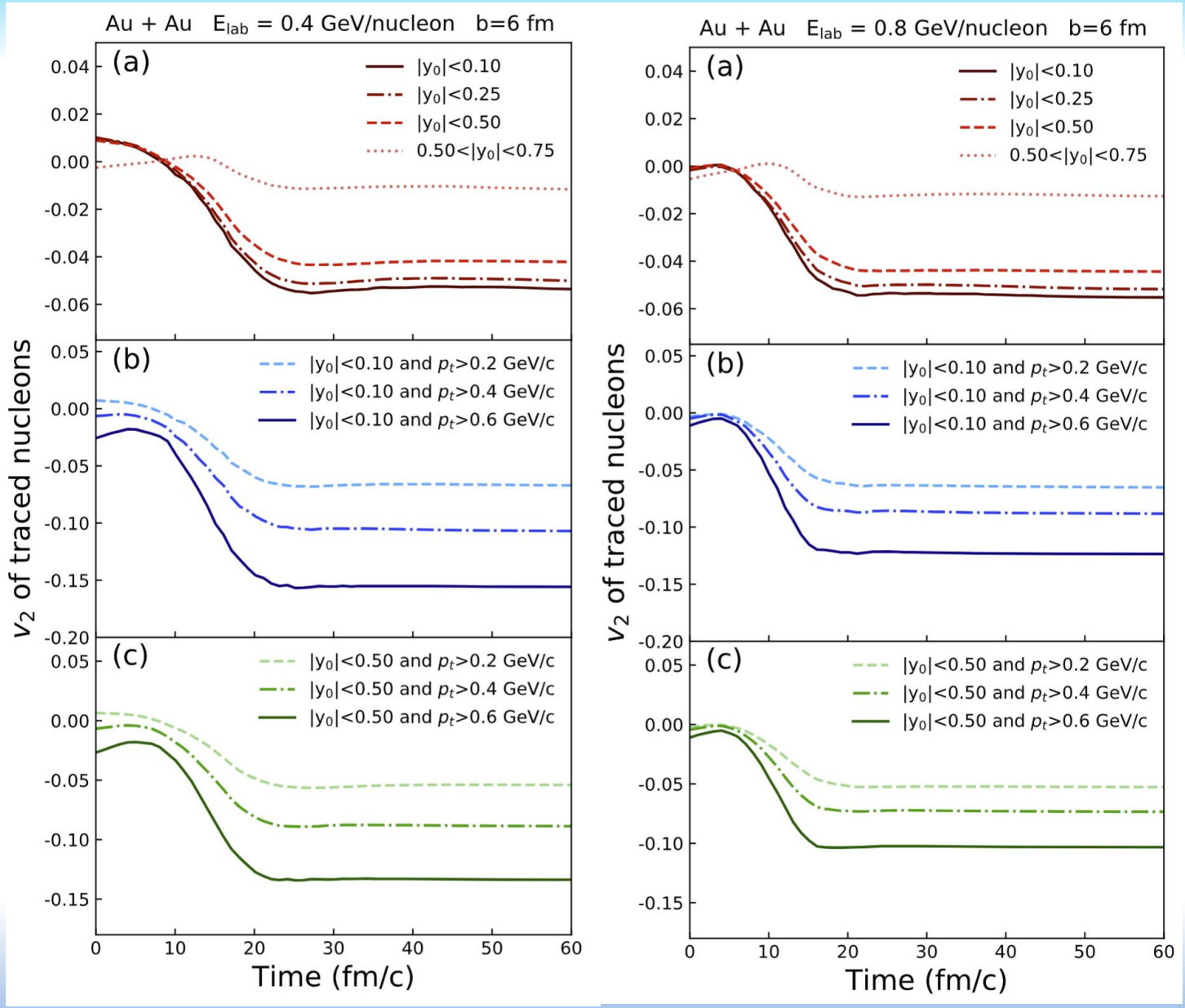
$$v_2 = \langle V_2 \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_t^2} \right\rangle$$



1. v_2 is more negative for high p_t nucleons.
2. Nucleons with more positive V_2 at the initial time have higher probability to be emitted with lower p_t at the final state. Because nucleons with positive V_2 are more likely to encounter with spectator nucleons; their p_t are then reduced easily via interactions from spectator nucleons.

Mid-rapidity nucleons **are not randomly "selected"** out of the uniform distribution in the initial state, indicating the momentum correlation between the initial and final stages.

Time evolution of v_2 of the traced nucleons finally emitted at mid-rapidity ($|y_0| < 0.1$) with different transverse momentum.



The momentum correlation becomes very weak at 800 MeV/nucleon.

The trivial contribution of the initial (Fermi) momentum to the total momentum.

More violent nucleon-nucleon collisions may wash out the effects of the initial momentum.

Quantitative attribution of MF and NN effects to v_2 evolution

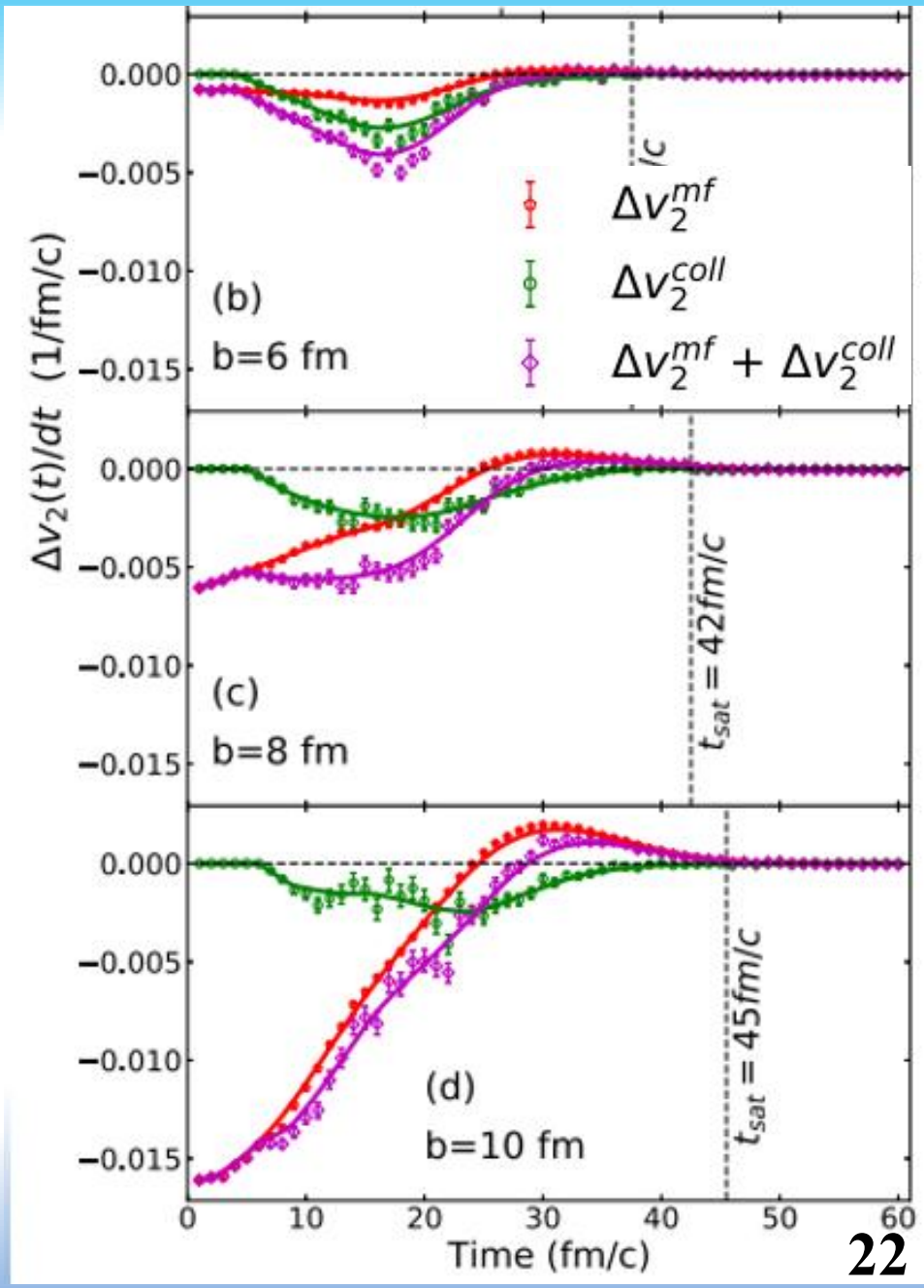
$$v_2 = \langle V_2 \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_t^2} \right\rangle.$$

$$\Delta v_2^{\mathrm{coll}}(t) = \left\langle V_2^{\mathrm{aft.coll}}(t) - V_2^{\mathrm{bef.coll}}(t) \right\rangle.$$

$$\Delta v_2^{\mathrm{mf}}(t) = \left\langle V_2^{\mathrm{aft.mf}}(t + \Delta t) - V_2^{\mathrm{bef.mf}}(t) \right\rangle.$$

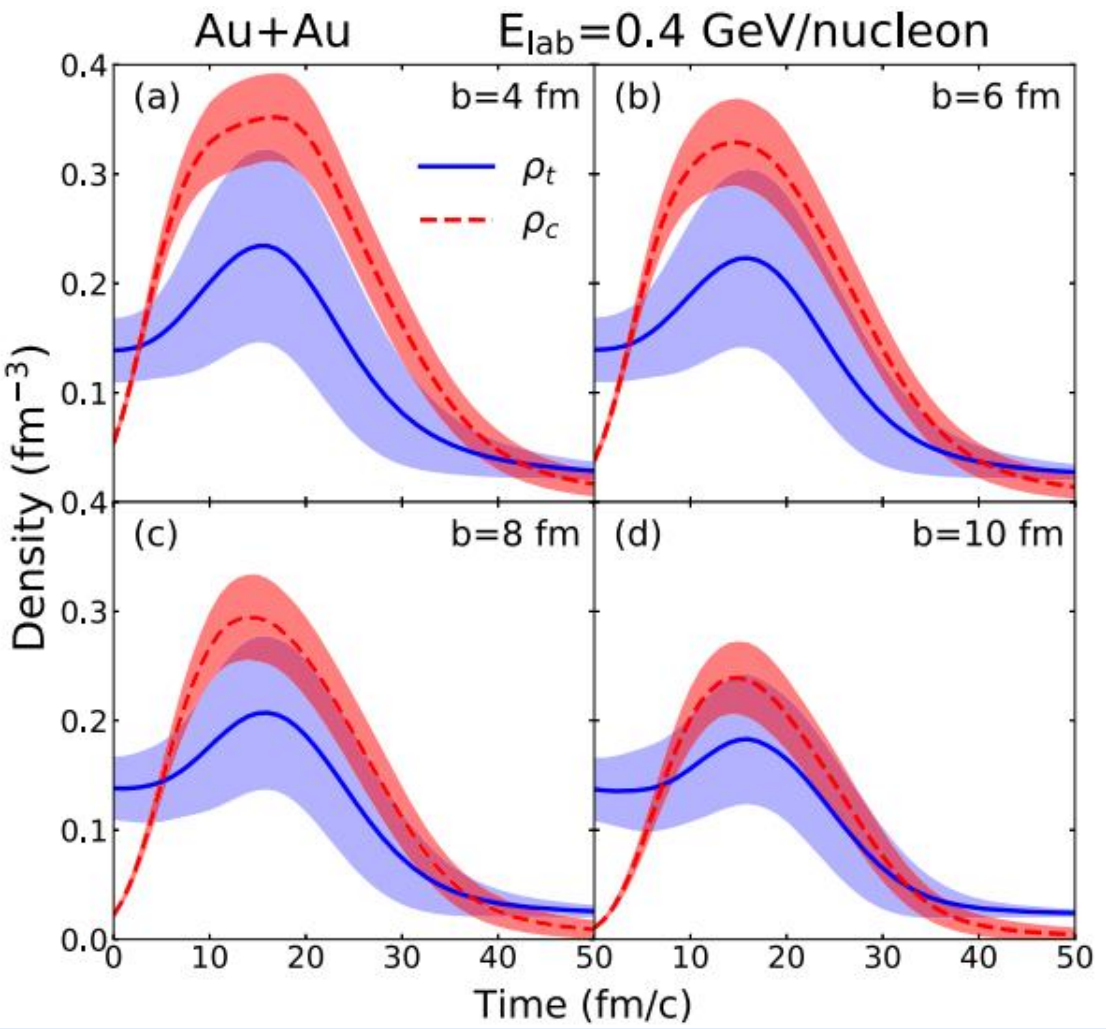
$\Delta v_2^{\mathrm{coll}}(t)$ is always negative.

$\Delta v_2^{\mathrm{mf}}(t)$ can be either negative or positive, depending on the impact parameter and time.



The density at which the EOS is probed by elliptic flow

$$\langle \rho \rangle_{v_2} = \frac{\int_0^{t_{sat}} |\Delta v_2(t)| \rho_t(t) dt}{\int_0^{t_{sat}} |\Delta v_2(t)| dt}.$$



ρ_c denotes the density at the central region.

ρ_t denotes the density experienced by the traced nucleons.

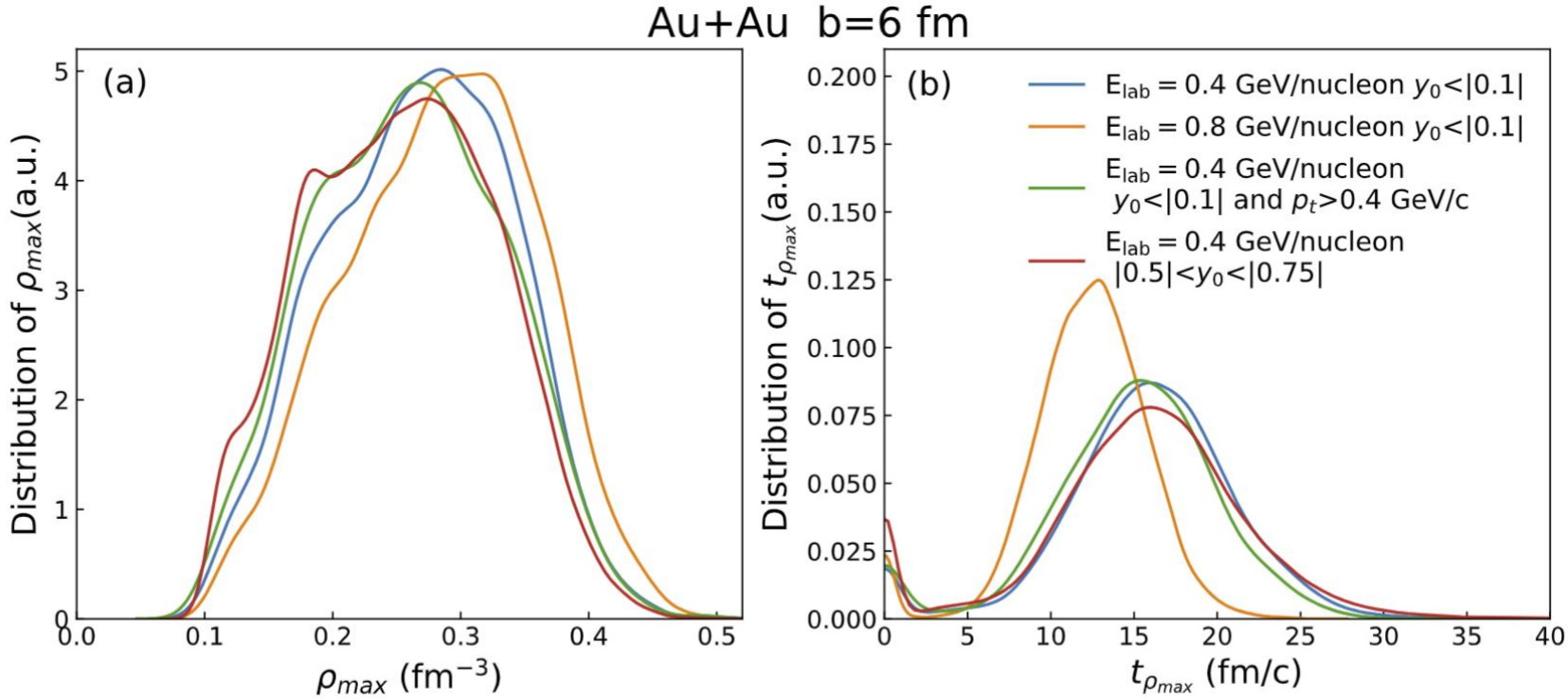
The most relevant density probed by v_2 of nucleons at mid-rapidity is about 60% of the maximum density reached during the collisions.

The density at which the EOS is probed by elliptic flow

"The high-density EoS affects the compression and determines the timing and speed of the expansion, while the elliptic flow develops after the time lag." From anonymous referee.

$$\rho(\mathbf{r};t) = \sum_{i=1}^{N} \frac{1}{(2\pi L)^{3/2}} \exp\left(-\frac{(\mathbf{r}-\mathbf{r}_i(t))^2}{2L}\right).$$

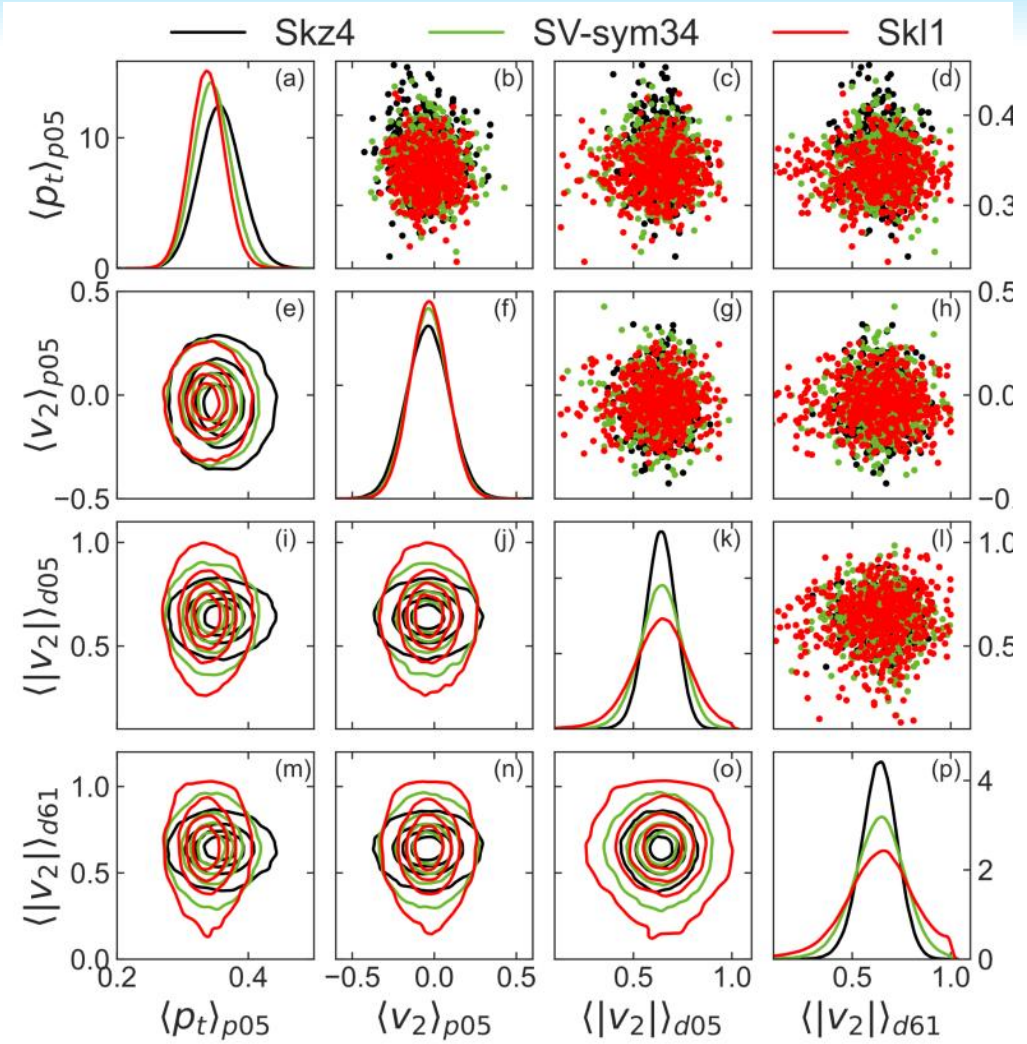
Density defined for each traced nucleons. Different nucleons reach their maximum density (ρ_{max}) at different times.



The average value of ρ_{max} can be used to estimate the upper limit density that v_2 can reflect. $<\rho_{max}>$ is about 80% of the maximum density created in the central region.

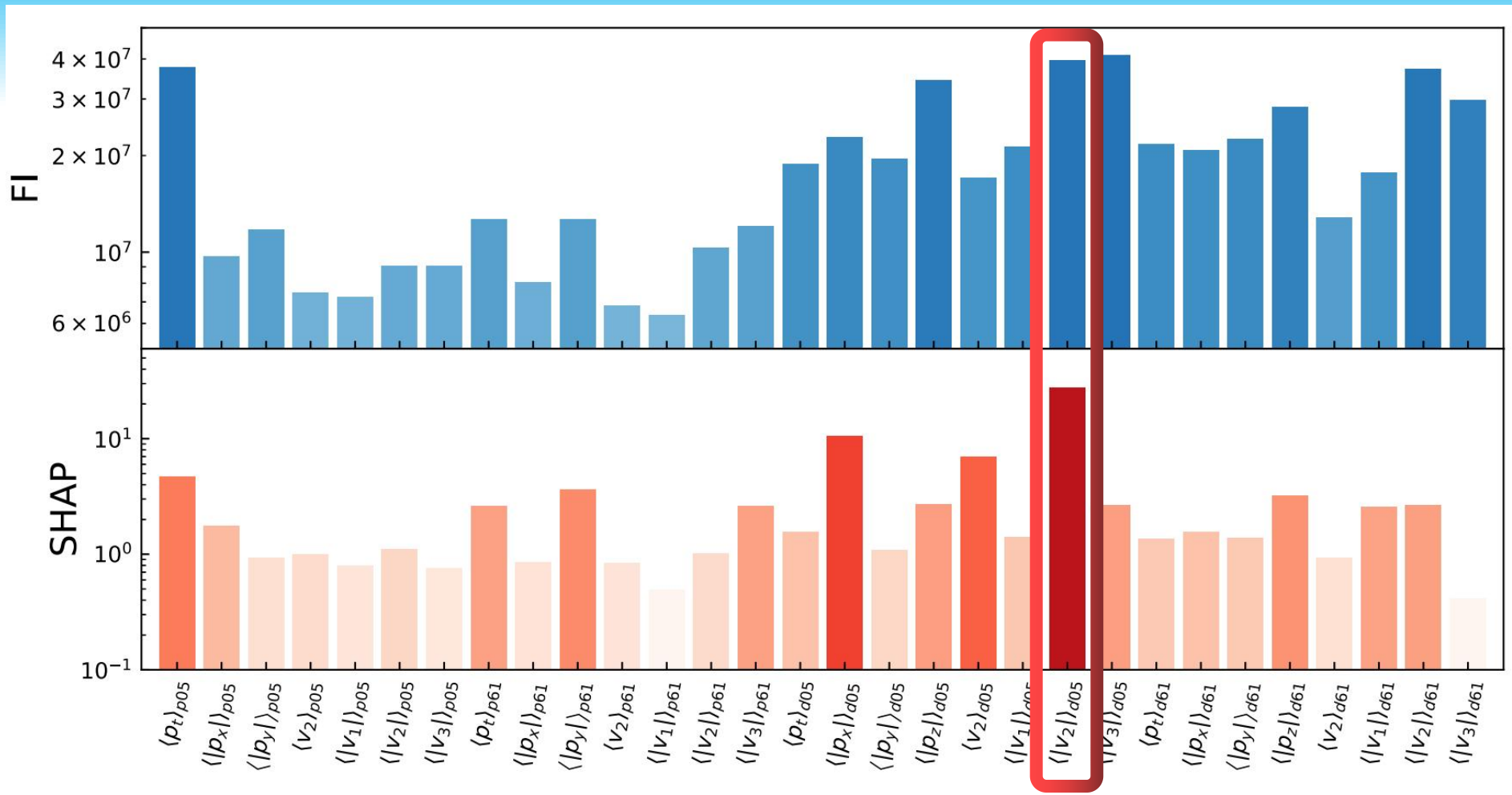
Results from machine learning

Particles	Rapidity window	Feature	Description
Free protons	$\|y_0\| < 0.5$	$\langle p_t \rangle_{p05}$	Mean value of p_t
		$\langle \|p_x\| \rangle_{p05}$	Mean value of $\|p_x\|$
		$\langle \|p_y\| \rangle_{p05}$	Mean value of $\|p_y\|$
		$\langle v_2 \rangle_{p05}$	Mean value of v_2
		$\langle \|v_1\| \rangle_{p05}$	Mean value of $\|v_1\|$
		$\langle \|v_2\| \rangle_{p05}$	Mean value of $\|v_2\|$
		$\langle \|v_3\| \rangle_{p05}$	Mean value of $\|v_3\|$
	$0.6 < \|y_0\| < 1.0$	$\langle p_t \rangle_{p61}$	Mean value of p_t
		$\langle \|p_x\| \rangle_{p61}$	Mean value of $\|p_x\|$
		$\langle \|p_y\| \rangle_{p61}$	Mean value of $\|p_y\|$
		$\langle v_2 \rangle_{p61}$	Mean value of v_2
		$\langle \|v_1\| \rangle_{p61}$	Mean value of $\|v_1\|$
		$\langle \|v_2\| \rangle_{p61}$	Mean value of $\|v_2\|$
		$\langle \|v_3\| \rangle_{p61}$	Mean value of $\|v_3\|$
Deuterons	$\|y_0\| < 0.5$	$\langle p_t \rangle_{d05}$	Mean value of p_t
		$\langle \|p_x\| \rangle_{d05}$	Mean value of $\|p_x\|$
		$\langle \|p_y\| \rangle_{d05}$	Mean value of $\|p_y\|$
		$\langle \|p_z\| \rangle_{d05}$	Mean value of $\|p_z\|$
		$\langle v_2 \rangle_{d05}$	Mean value of v_2
		$\langle \|v_1\| \rangle_{d05}$	Mean value of $\|v_1\|$
		$\langle \|v_2\| \rangle_{d05}$	Mean value of $\|v_2\|$
		$\langle \|v_3\| \rangle_{d05}$	Mean value of $\|v_3\|$
	$0.6 < \|y_0\| < 1.0$	$\langle p_t \rangle_{d61}$	Mean value of p_t
		$\langle \|p_x\| \rangle_{d61}$	Mean value of $\|p_x\|$
		$\langle \|p_y\| \rangle_{d61}$	Mean value of $\|p_y\|$
		$\langle \|p_z\| \rangle_{d61}$	Mean value of $\|p_z\|$
		$\langle v_2 \rangle_{d61}$	Mean value of v_2
		$\langle \|v_1\| \rangle_{d61}$	Mean value of $\|v_1\|$
		$\langle \|v_2\| \rangle_{d61}$	Mean value of $\|v_2\|$
		$\langle \|v_3\| \rangle_{d61}$	Mean value of $\|v_3\|$



Yongjia Wang, et al., Physics Letters B 822 (2021) 136669;
Yongjia Wang, et al., Physics Letters B 835 (2022) 137508.

Results from machine learning



The Feature_importance (FI) technology of LightGBM and Shapley additive explanations (SHAP) are two opular feature attribution methods to identify the most important features that drive predictions.

Yongjia Wang, et al., Physics Letters B 822 (2021) 136669;
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Summary

- ✓ By reversely tracing nucleons finally emitted at mid-rapidity in the entire reaction process, the contributions of MF and NN collisions to the elliptic flow evolution can be studied.
- ✓ The momentum correlation between the initial and final stages can be seen at 0.4A GeV but not at 0.8A GeV.
- ✓ The density at which the EOS is probed by elliptic flow is about 60~80% of the maxmium density created in the central region.

Thanks for your attention.