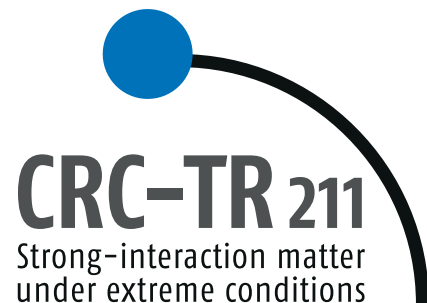


MIXING, MOATS AND MODULATIONS IN DENSE QCD MATTER

Fabian Rennecke



IHIC 2024

TSINGHUA UNIVERSITY, BEIJING - 15/07/2024

PATTERNS

Regularly repeated arrangements; spatial modulations. Ubiquitous in nature

materials



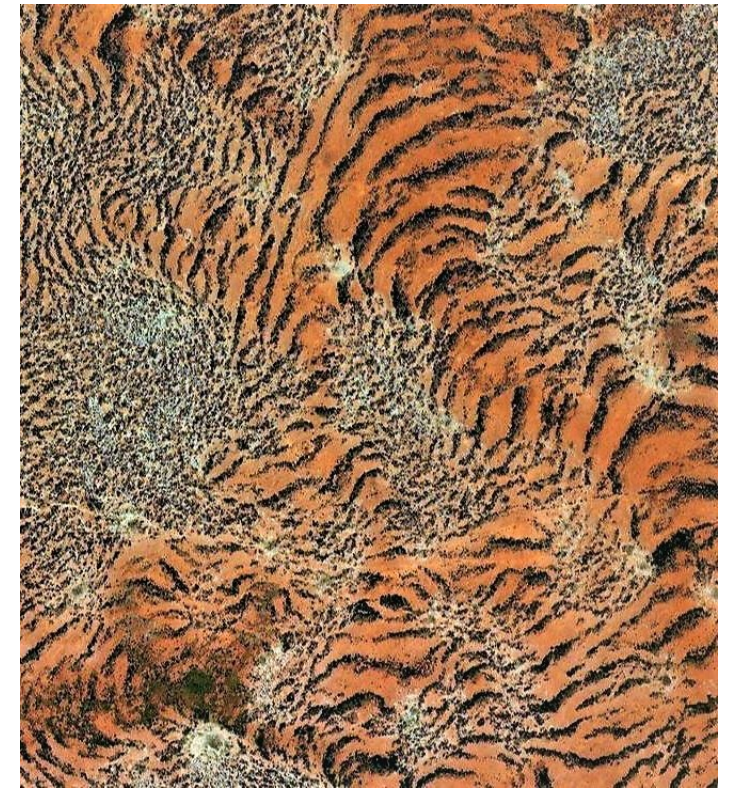
[Robert Fotograf]

animals



[Maksymilian Rose]

vegetation



[Somali Water and Land Information Management]

PATTERN FORMATION

THE CHEMICAL BASIS OF MORPHOGENESIS

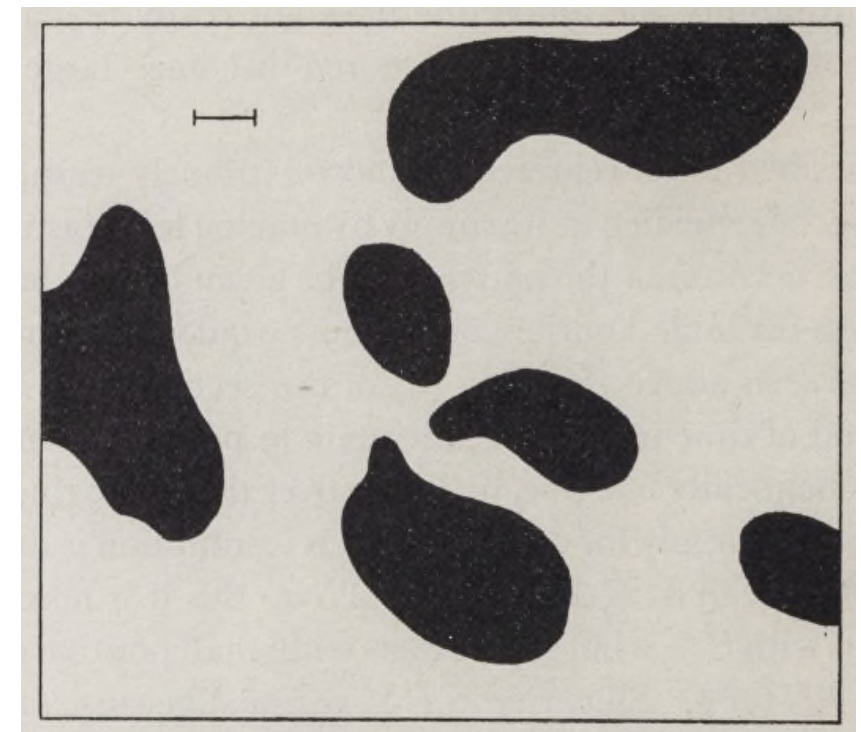
By A. M. TURING, F.R.S. *University of Manchester*

(Received 9 November 1951—Revised 15 March 1952)

It is suggested that a system of chemical substances, called morphogens, reacting together and diffusing through a tissue, is adequate to account for the main phenomena of morphogenesis. Such a system, although it may originally be quite homogeneous, may later develop a pattern or structure due to an instability of the homogeneous equilibrium, which is triggered off by random disturbances. Such reaction-diffusion systems are considered in some detail in the case of an isolated ring of cells, a mathematically convenient, though biologically unusual system.

reaction-diffusion equation

$$\partial_t \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \begin{pmatrix} Z_\phi \nabla^2 + M_\phi^2 & G_1 \\ G_2 & Z_\chi \nabla^2 + M_\chi^2 \end{pmatrix} \cdot \begin{pmatrix} \phi \\ \chi \end{pmatrix}$$

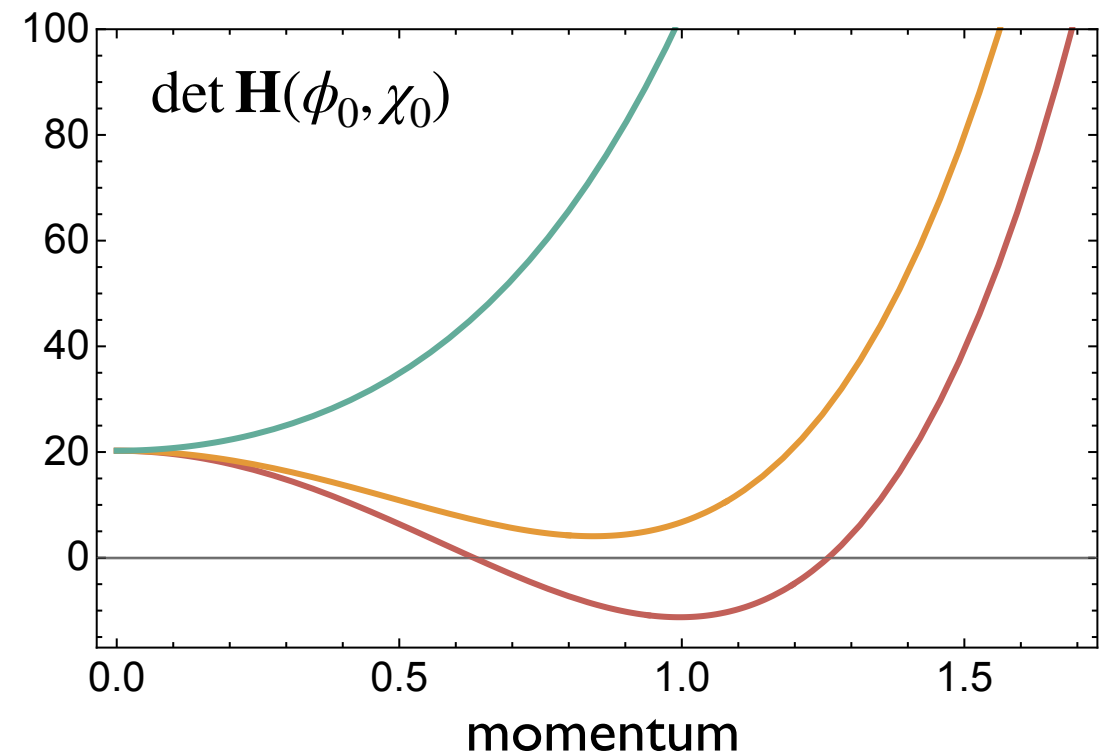


PATTERN FORMATION

Characterize possible solutions through "Hessian" \mathbf{H} .

Example: Brusselator model

$$\partial_t \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \mathbf{H}(\phi, \chi) \cdot \begin{pmatrix} \phi \\ \chi \end{pmatrix}$$

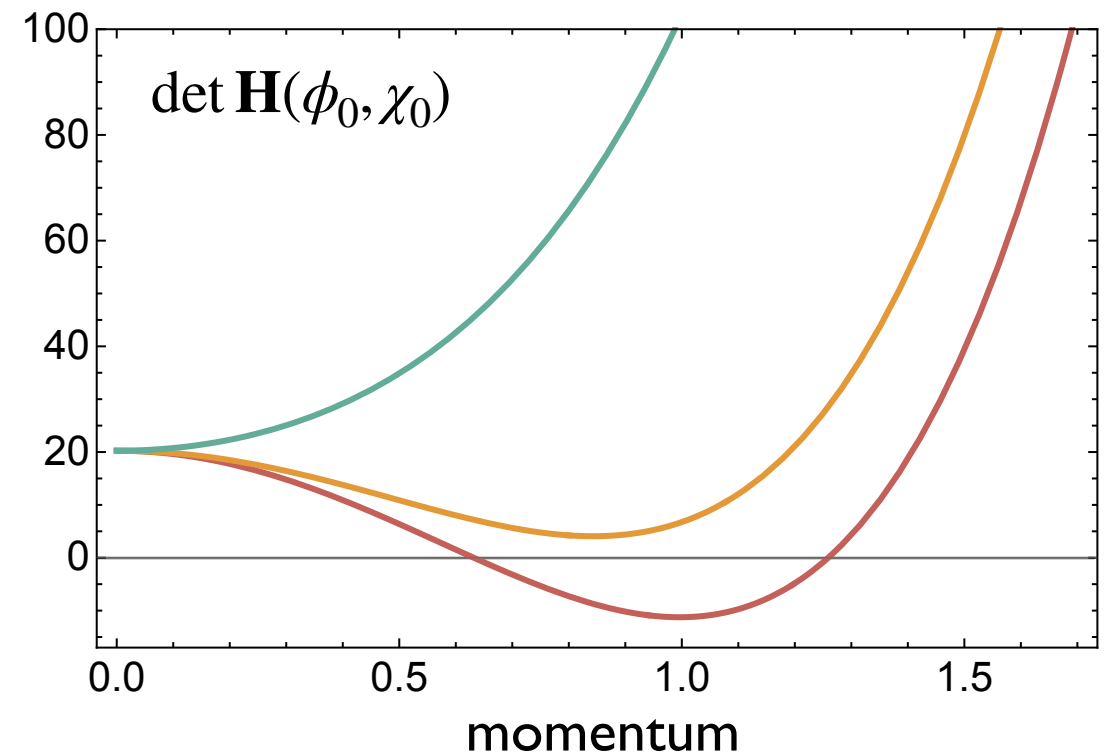


PATTERN FORMATION

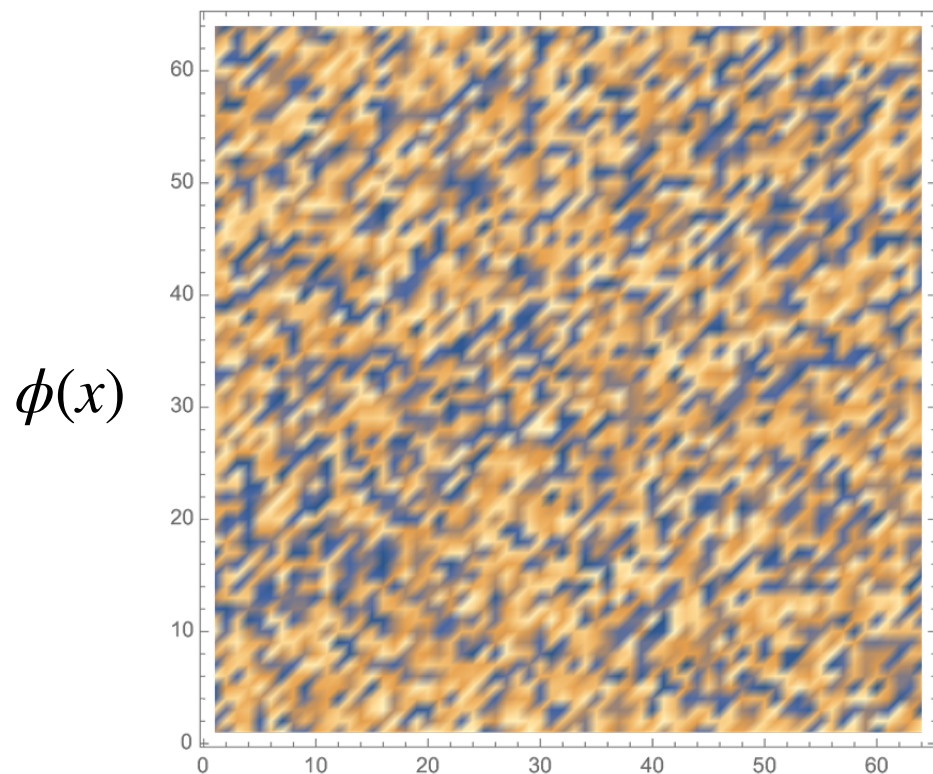
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Example: Brusselator model

$$\partial_t \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \mathbf{H}(\phi, \chi) \cdot \begin{pmatrix} \phi \\ \chi \end{pmatrix}$$



$t = t_0$

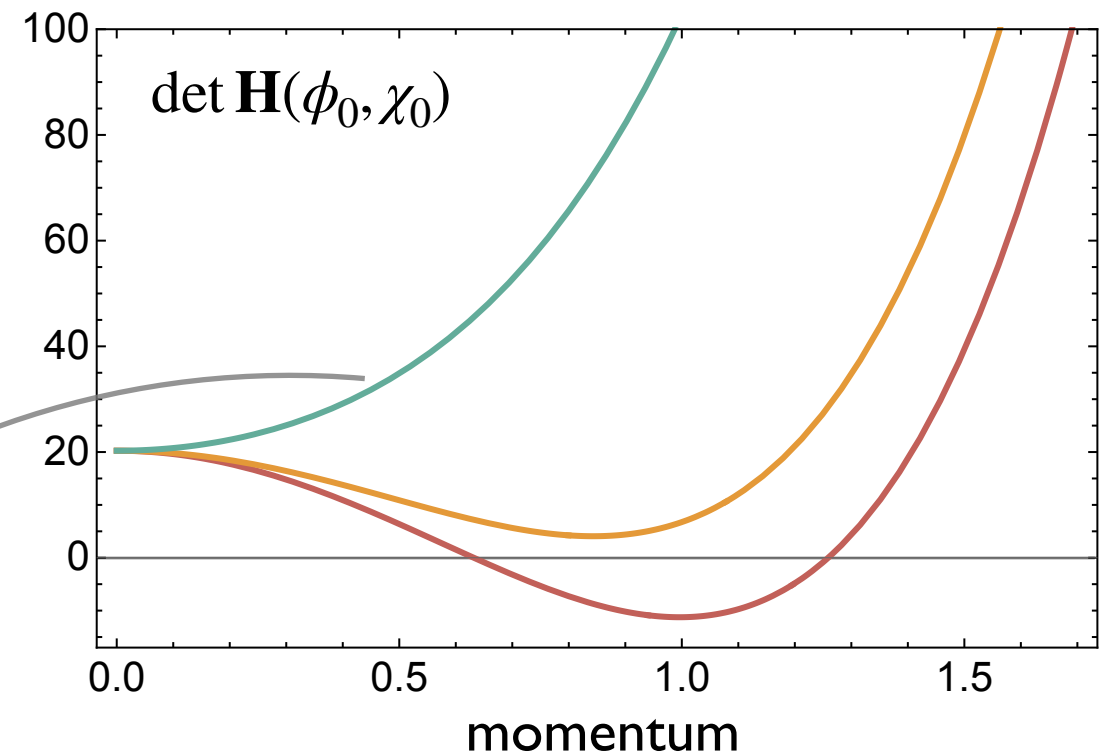


PATTERN FORMATION

Characterize possible solutions through "Hessian" \mathbf{H} .

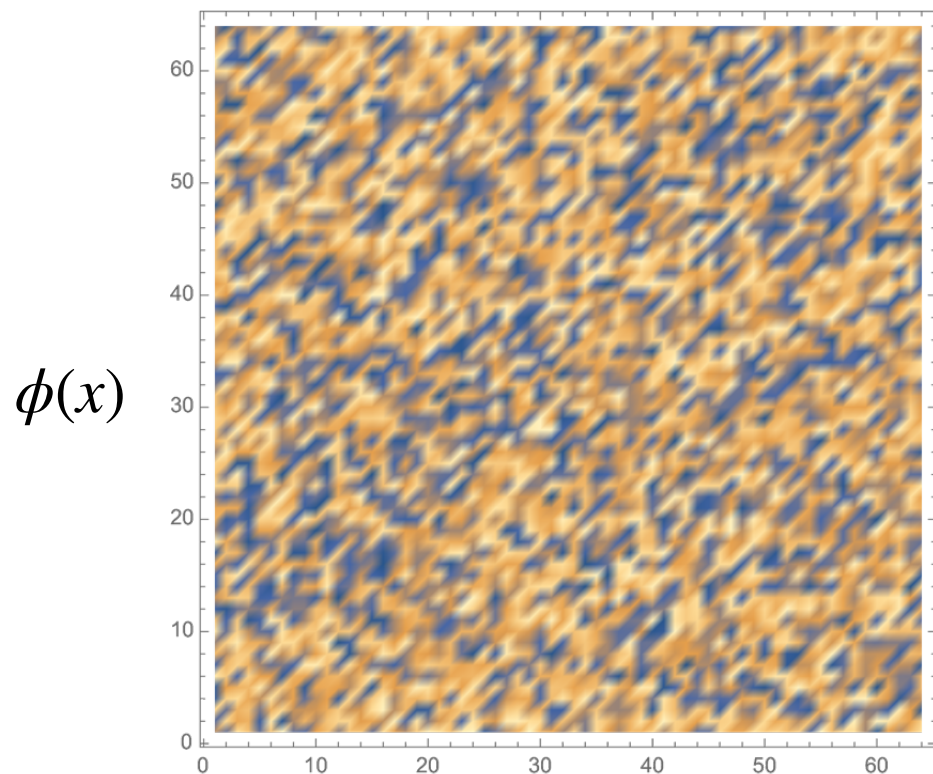
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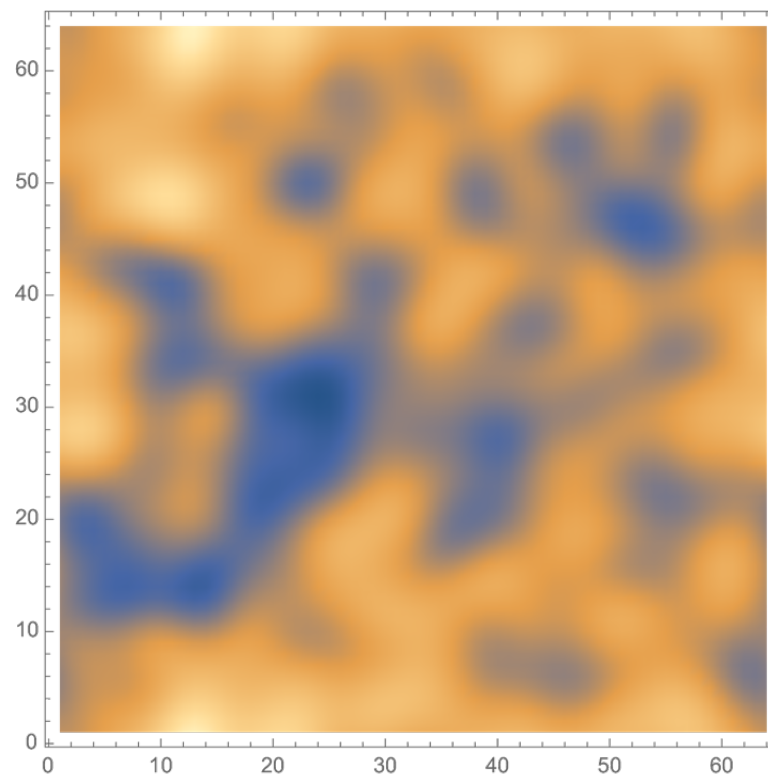


homogeneous phase

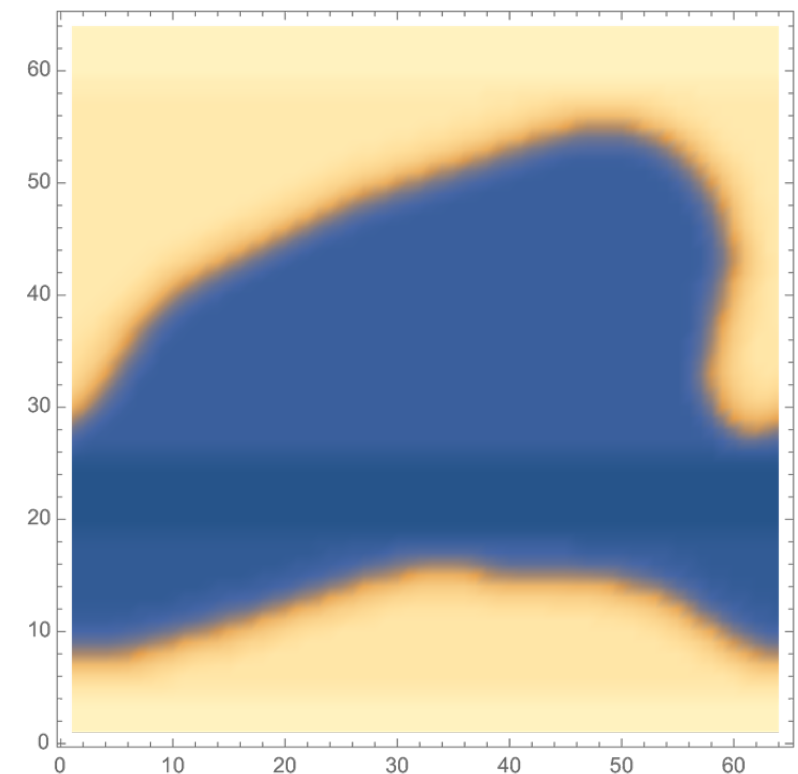
$t = t_0$



$t = t_1$



$t = t_2$

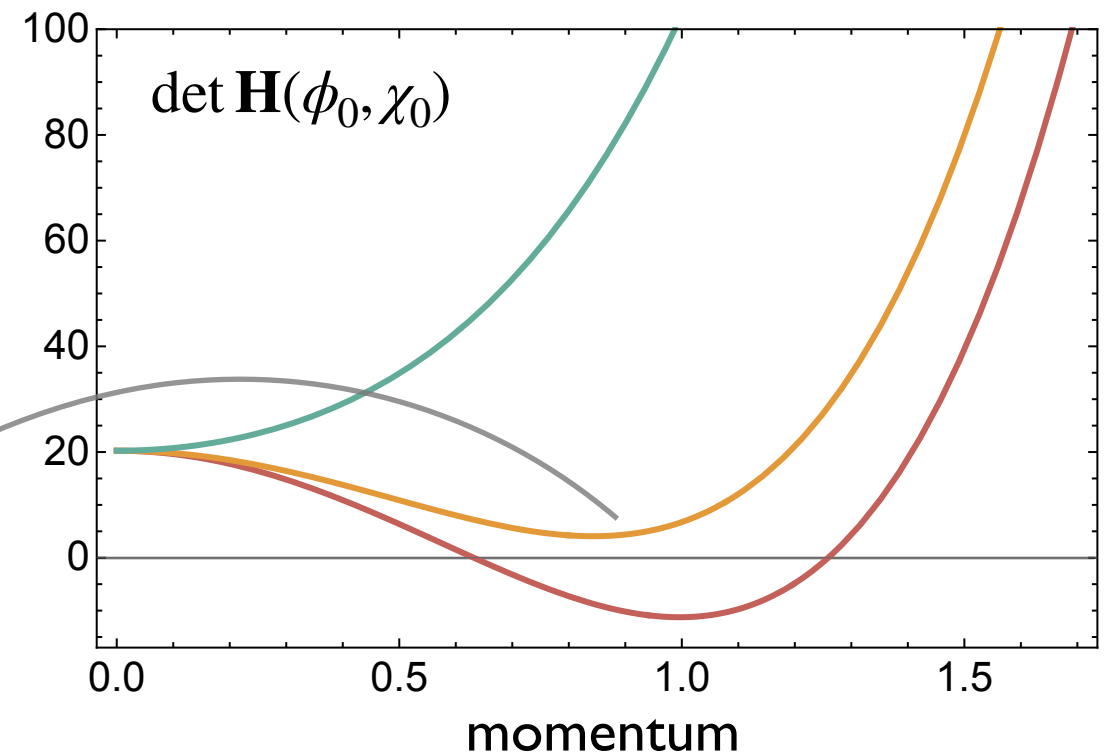


PATTERN FORMATION

Characterize possible solutions through "Hessian" \mathbf{H} .

Example: Brusselator model

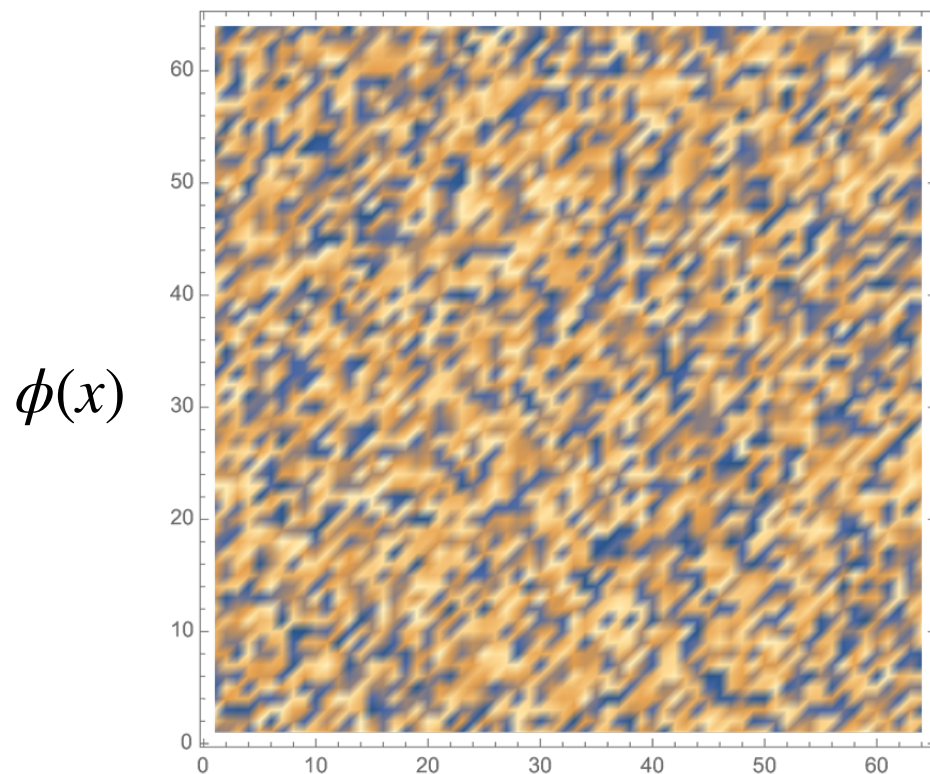
$$\partial_t \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \mathbf{H}(\phi, \chi) \cdot \begin{pmatrix} \phi \\ \chi \end{pmatrix}$$



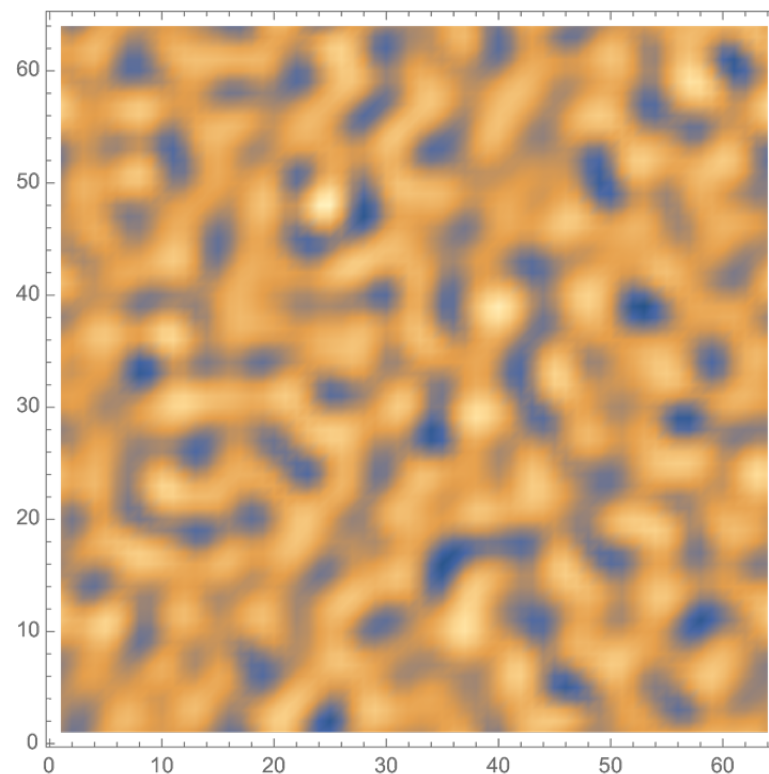
moat regime: "homogeneous pattern"

[Pisarski, FR (2021)]

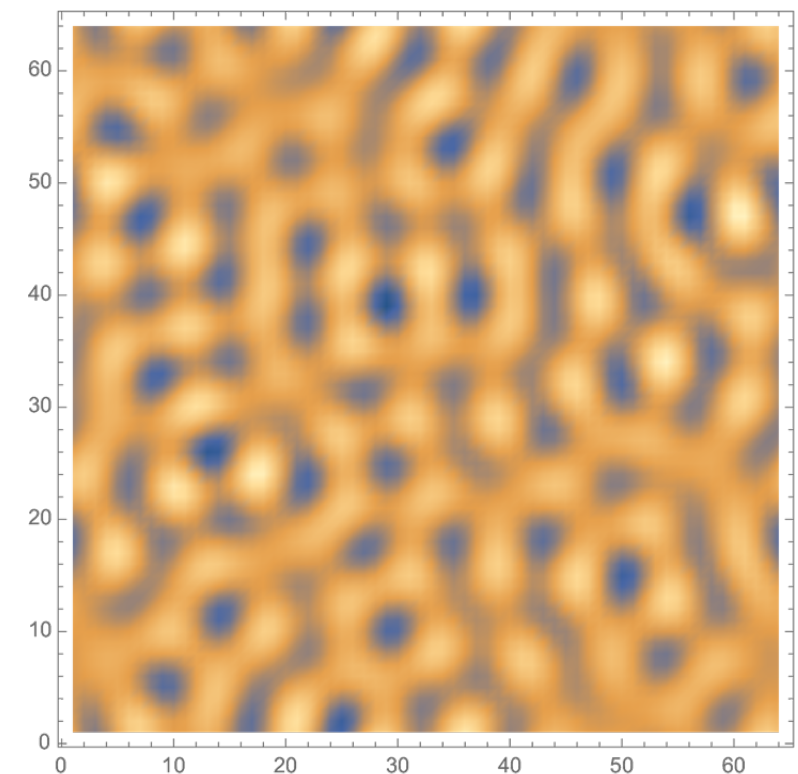
$t = t_0$



$t = t_1$



$t = t_2$

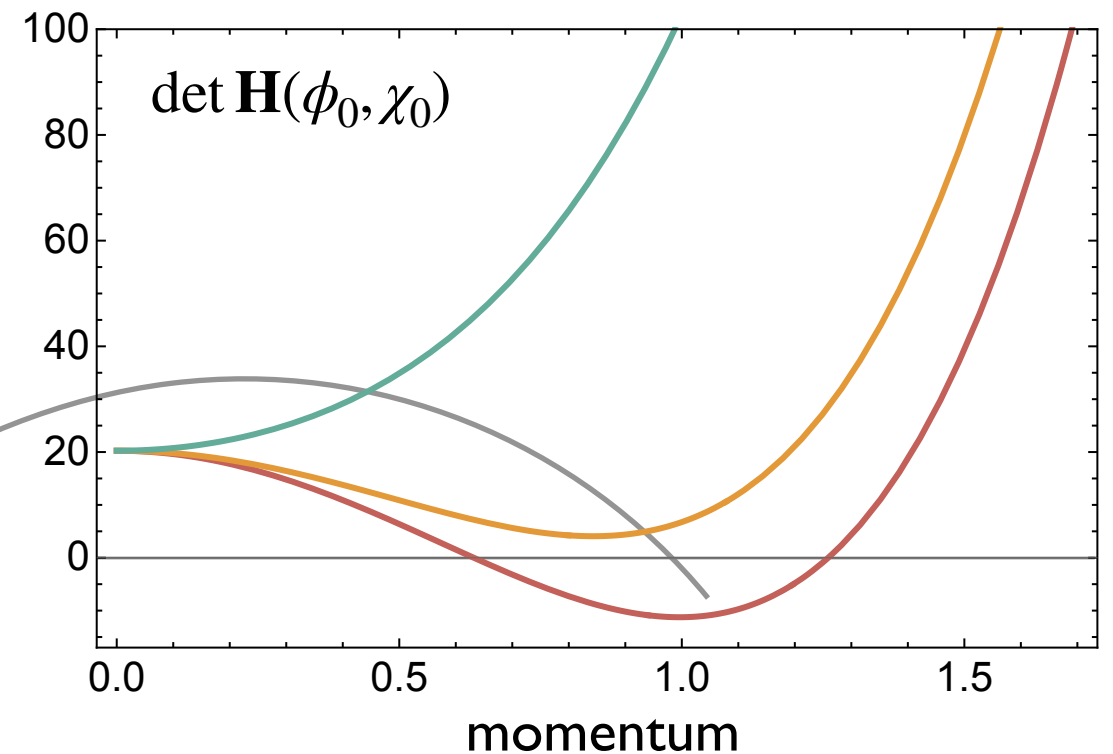


PATTERN FORMATION

Characterize possible solutions through "Hessian" \mathbf{H} .

Example: Brusselator model

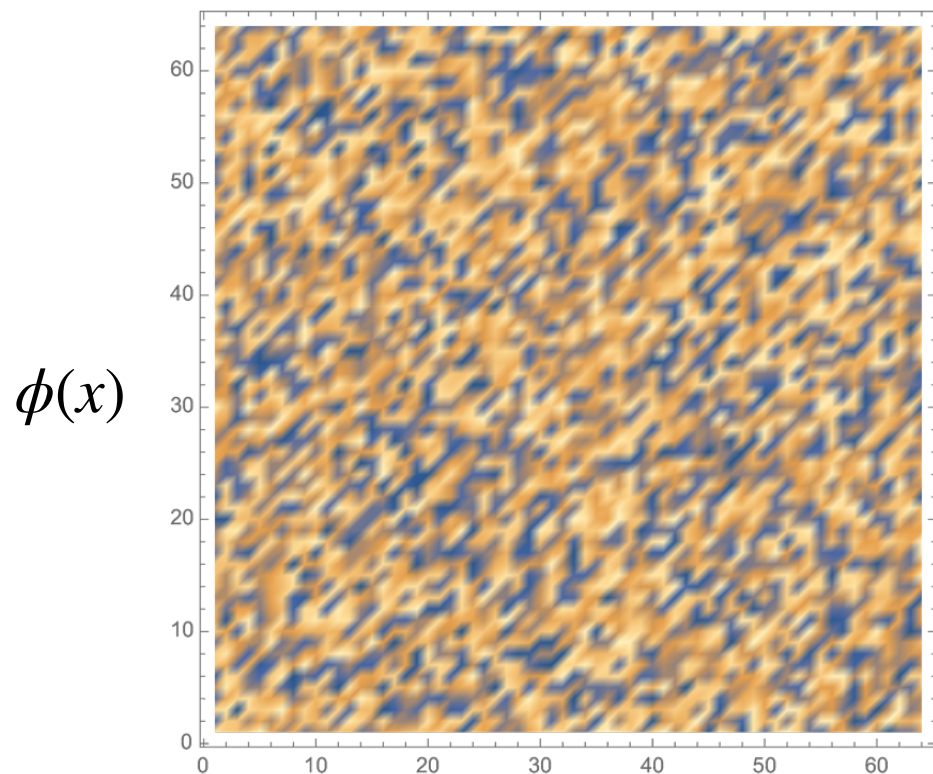
$$\partial_t \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \mathbf{H}(\phi, \chi) \cdot \begin{pmatrix} \phi \\ \chi \end{pmatrix}$$



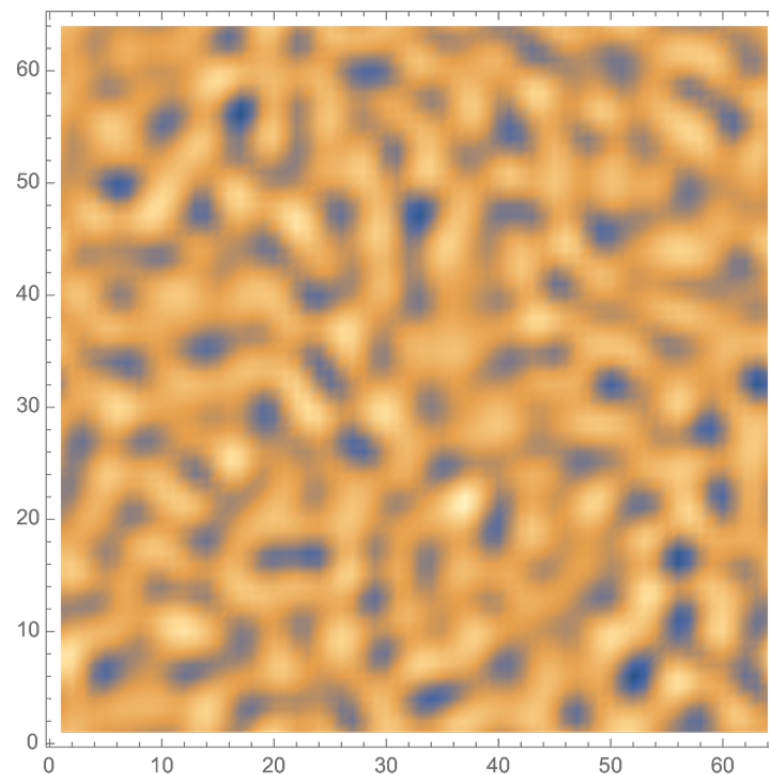
inhomogeneous phase : Turing pattern

instability of homogeneous ground state, $\det \mathbf{H}(\phi_0, \chi_0) \leq 0$

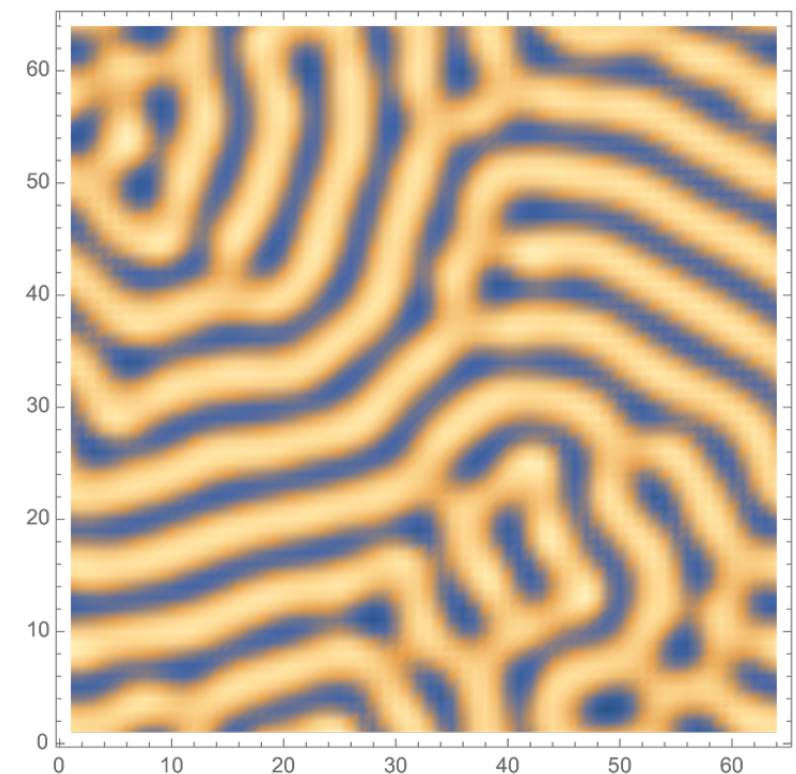
$t = t_0$



$t = t_1$



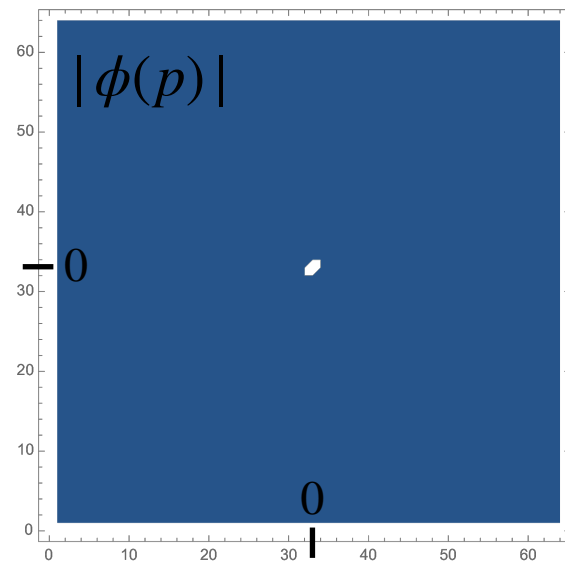
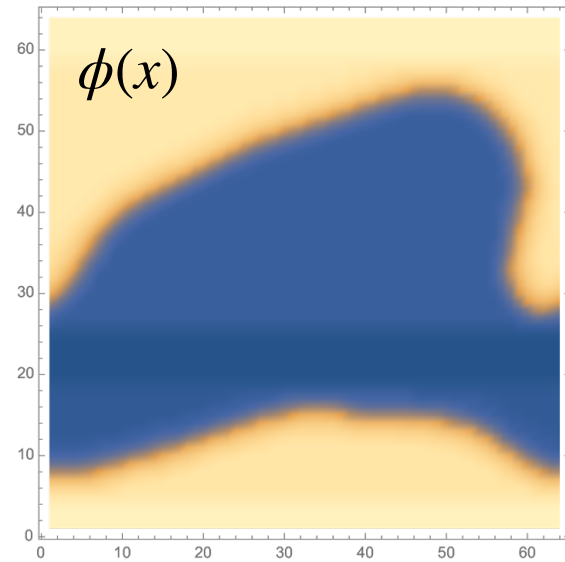
$t = t_2$



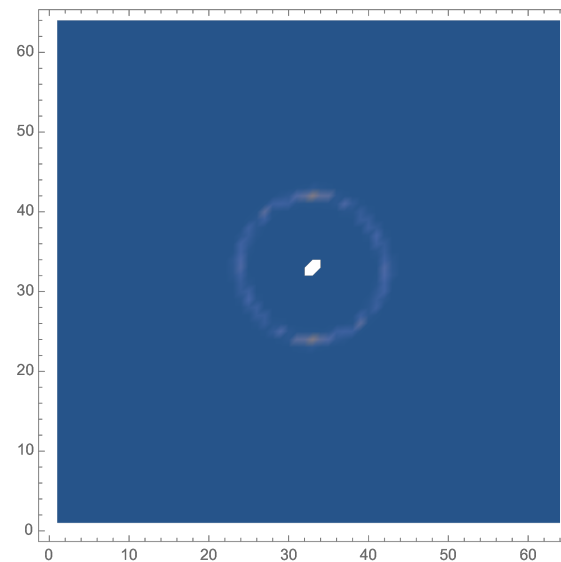
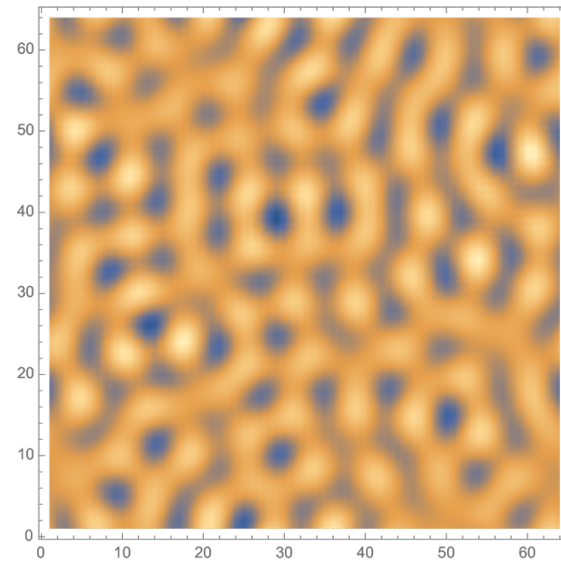
PATTERN FORMATION

in momentum space

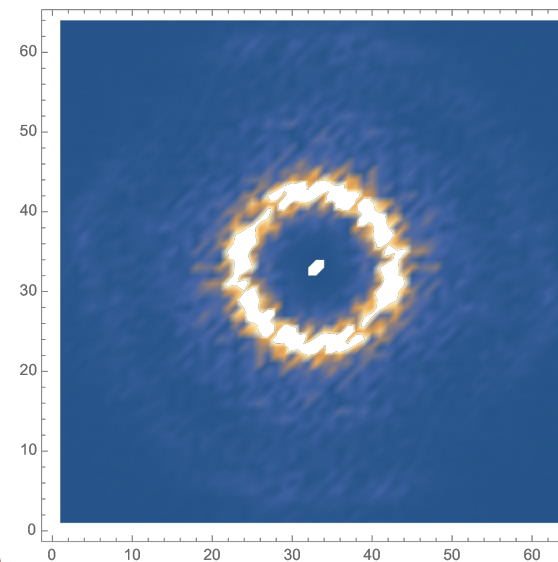
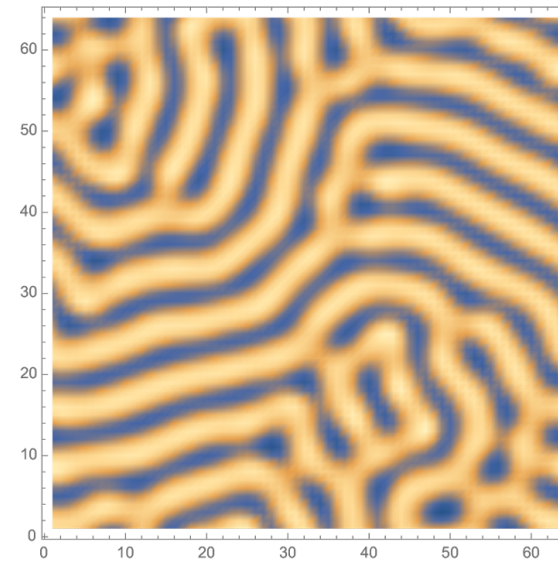
homogeneous



moat



inhomogeneous



patterns

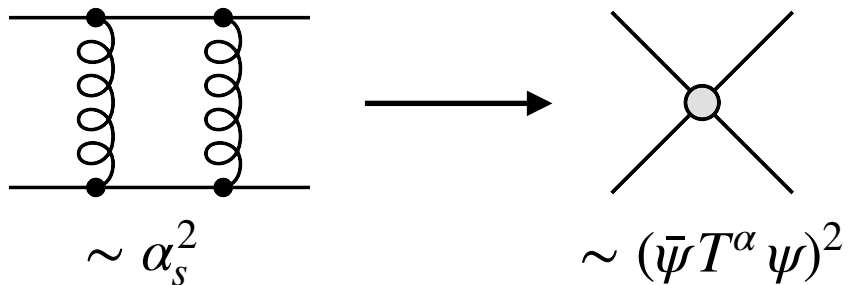


favored momentum
(wavenumber)

PATTERN FORMATION IN QCD

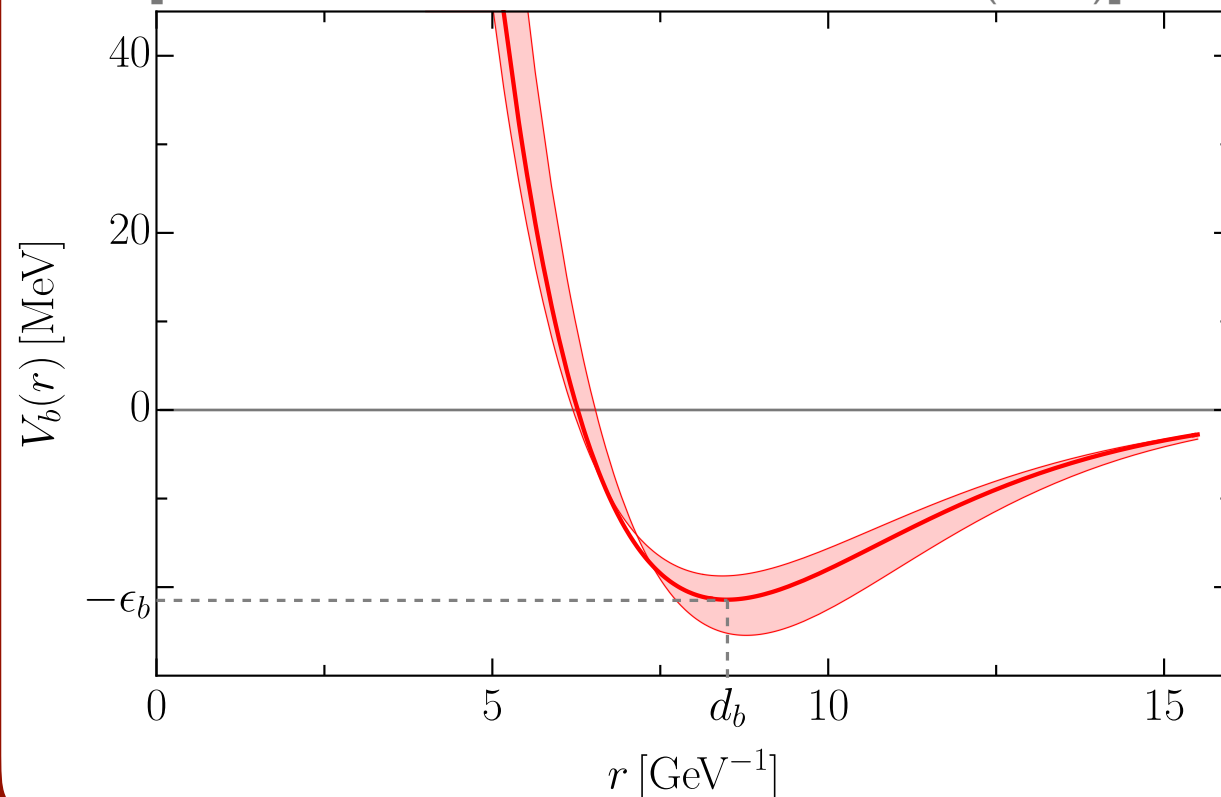
Intuition tells us there are two important ingredients for pattern formation:

competition between attractive and repulsive interactions

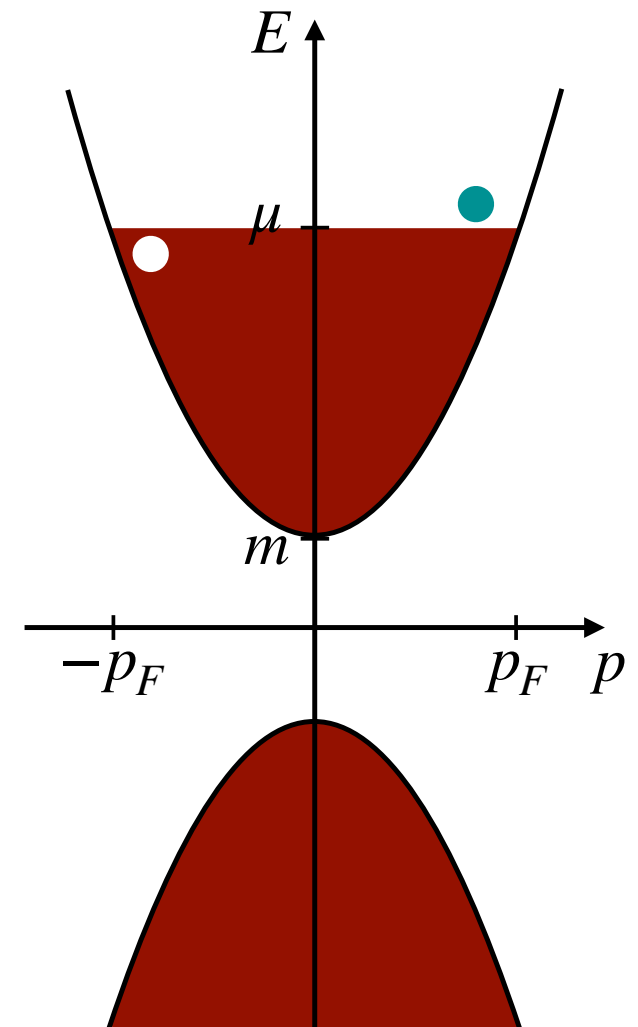


potential between quark-bilinears
in the vector channel, $\bar{\psi} \gamma^0 \psi$

[Fukushima, Horak, Pawłowski, Wink, Zelle (2023)]



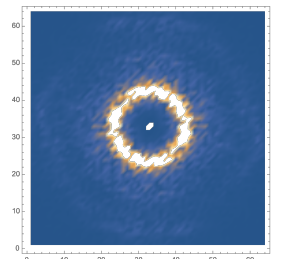
a large Fermi surface



net-momentum excitation around
Fermi-surface

$$\bar{\psi}_+(p_F) \psi_+(-p_F)$$

spatial modulations
(Friedel oscillations)
[FR, Yin (in preparation)]



DENSE QCD MATTER AND CK SYMMETRY

QCD at finite density:

$$\mathcal{L}_{\text{QCD}} = \bar{q} \left(i\gamma^\mu D_\mu - m_q \right) q - \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + i\mu \bar{q} \gamma^0 q$$

- charge conjugation symmetry (C) is broken at finite μ
- retains symmetry under CK (K : complex conjugation)

$$D_\mu = \partial_\mu - igA_\mu^a T^a$$
$$F_{\mu\nu}^a T^a = -\frac{i}{g} [D_\mu, D_\nu]$$

$$\bar{q}q \longrightarrow \bar{q}q$$
$$C: \bar{q}\gamma^\mu q \longrightarrow -\bar{q}\gamma^\mu q$$
$$A^\mu \longrightarrow -A^\mu$$

DENSE QCD MATTER AND CK SYMMETRY

QCD at finite density:

$$\mathcal{L}_{\text{QCD}} = \bar{q} \left(i\gamma^\mu D_\mu - m_q \right) q - \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + i\mu \bar{q} \gamma^0 q$$

$$\begin{aligned} D_\mu &= \partial_\mu - ig A_\mu^a T^a \\ F_{\mu\nu}^a T^a &= -\frac{i}{g} [D_\mu, D_\nu] \end{aligned}$$

- charge conjugation symmetry (C) is broken at finite μ
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$$\begin{aligned} \bar{q}q &\longrightarrow \bar{q}q \\ C: \bar{q}\gamma^\mu q &\longrightarrow -\bar{q}\gamma^\mu q \\ A^\mu &\longrightarrow -A^\mu \end{aligned}$$

Intuition from CK -extended ϕ^4 -theory:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \lambda \phi^4 \\ &\quad - \frac{1}{2} (\partial_i \omega^0)^2 - \frac{1}{2} m_\omega (\omega^0)^2 + ig \phi \omega^0 \end{aligned}$$

- scalar field ϕ , vector field ω^μ
- linear coupling between ϕ and ω^0 : **mixing**
- imaginary coupling ig : **repulsion**
- possesses CK symmetry

Non-relativistic EoM is like the reaction-diffusion equation from earlier!

The system has two peculiar features:

non-Hermitian Hessian ($p_0 = 0$)

$$H = \begin{pmatrix} \vec{p}^2 + m^2 & -ig \\ -ig & \vec{p}^2 + m_\omega^2 \end{pmatrix}$$

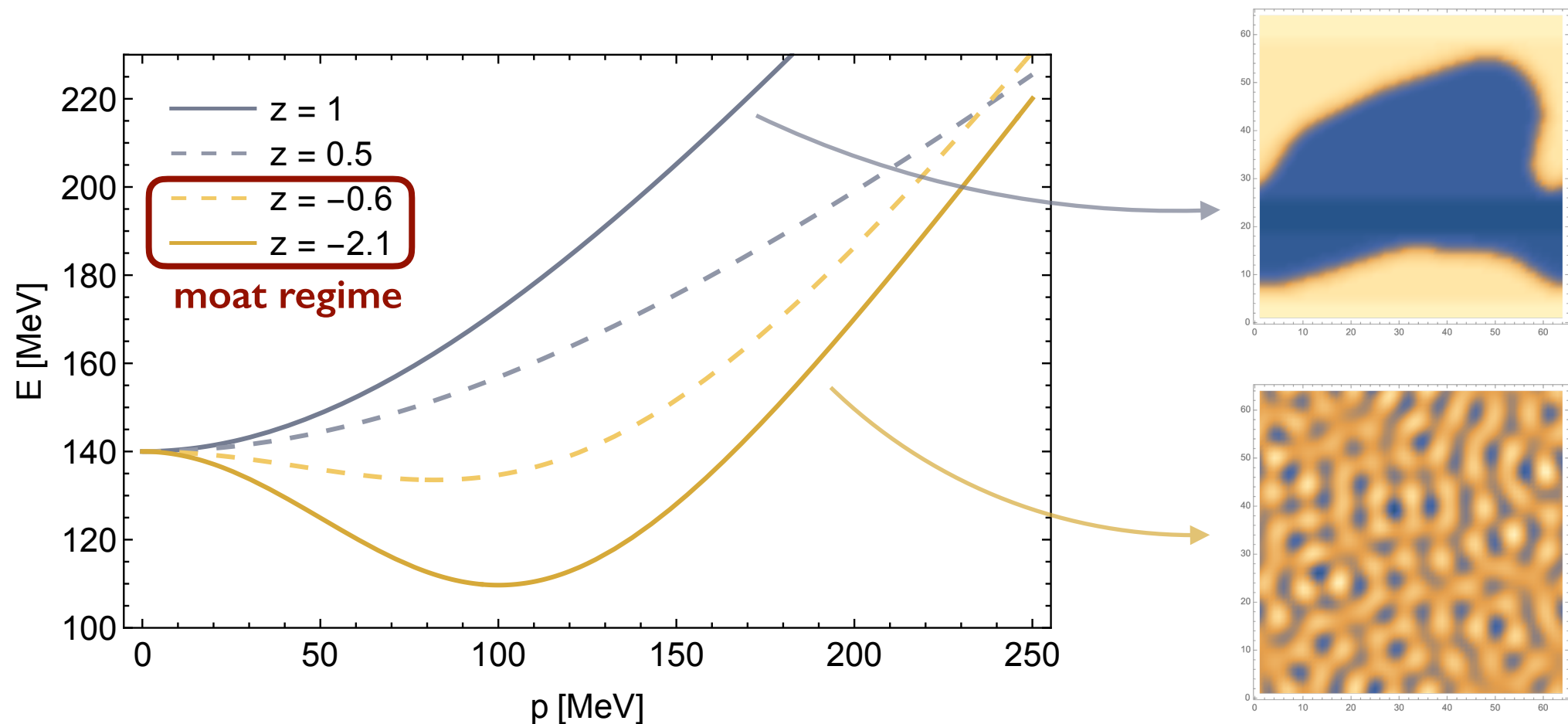
modified dispersion of ϕ
from integrating-out ω^0

$$E^2(\vec{p}^2) = \vec{p}^2 + m^2 + \frac{g^2}{\vec{p}^2 + m_\omega^2}$$

MODIFIED DISPERSION

in the small-momentum regime:

$$E^2(\vec{p}^2) = \underbrace{\left(1 - \frac{g^2}{m_\omega^4}\right)}_z \vec{p}^2 + \frac{g^2}{m_\omega^6} \vec{p}^4 + m^2 + \frac{g^2}{m_\omega^2} + \mathcal{O}(\vec{p}^6)$$



$E^2 = \det H$
 $\longrightarrow z < 0$ for strong repulsive mixing: **moat regime**

[Pisarski, FR (2021)]

MODIFIED HESSIAN

Diagonalize to get physical degrees of freedom

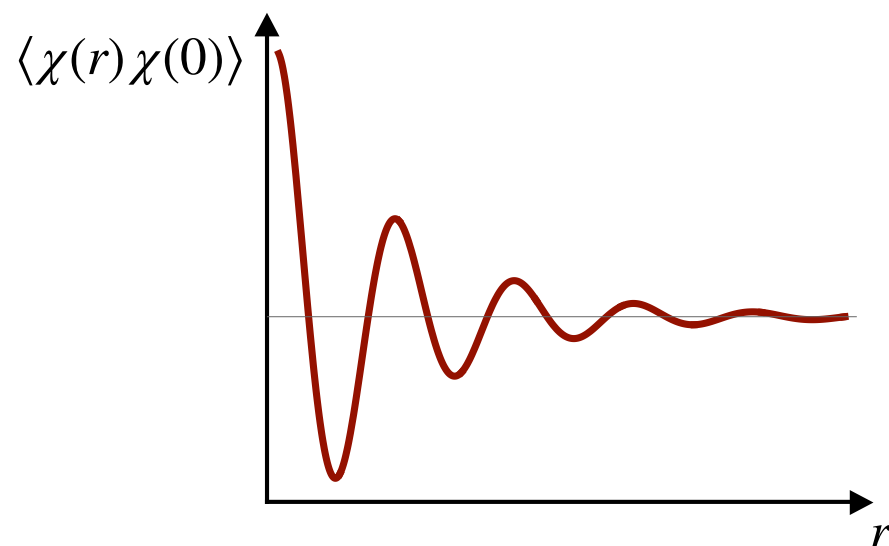
$$H = \begin{pmatrix} \vec{p}^2 + m^2 & -ig \\ -ig & \vec{p}^2 + m_\omega^2 \end{pmatrix} \longrightarrow \begin{pmatrix} \vec{p}^2 + M_+^2 & 0 \\ 0 & \vec{p}^2 + M_-^2 \end{pmatrix}, \quad M_\pm = \frac{m^2 + m_\omega^2}{2} \pm \frac{1}{2} \sqrt{(m^2 - m_\omega^2)^2 - 4g^2}$$

→ strong repulsive mixing: **eigenvalues/masses come in complex conjugate pairs**
(follows from *CK* symmetry)

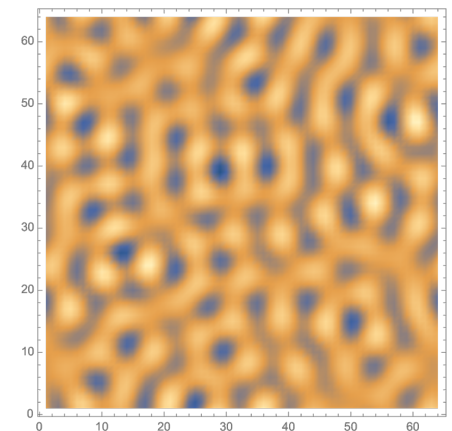
These masses determine screening properties of the physical fields

$$\lim_{r \rightarrow \infty} \langle \chi(r) \chi(0) \rangle \sim e^{-Mr}$$

- real M : ordinary exponential decay of disordered fields
- complex M : **spatial modulations** $\sim e^{-\text{Re}[M]r} \sin(\text{Im}[M]r)$



spatial modulations for
strong repulsive mixing



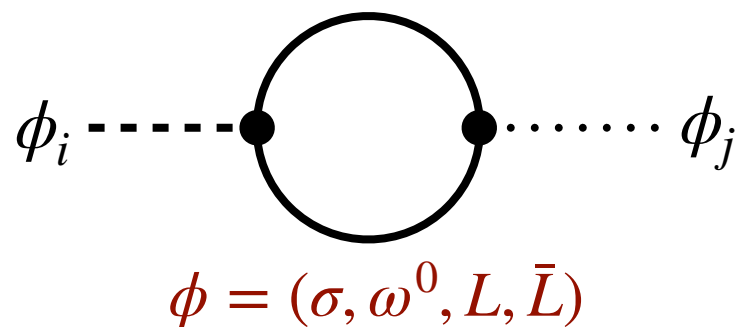
MIXING, MOATS AND MODULATIONS IN QCD

The intuition from this simple model is directly applicable to QCD!

MIXING, MOATS AND MODULATIONS IN QCD

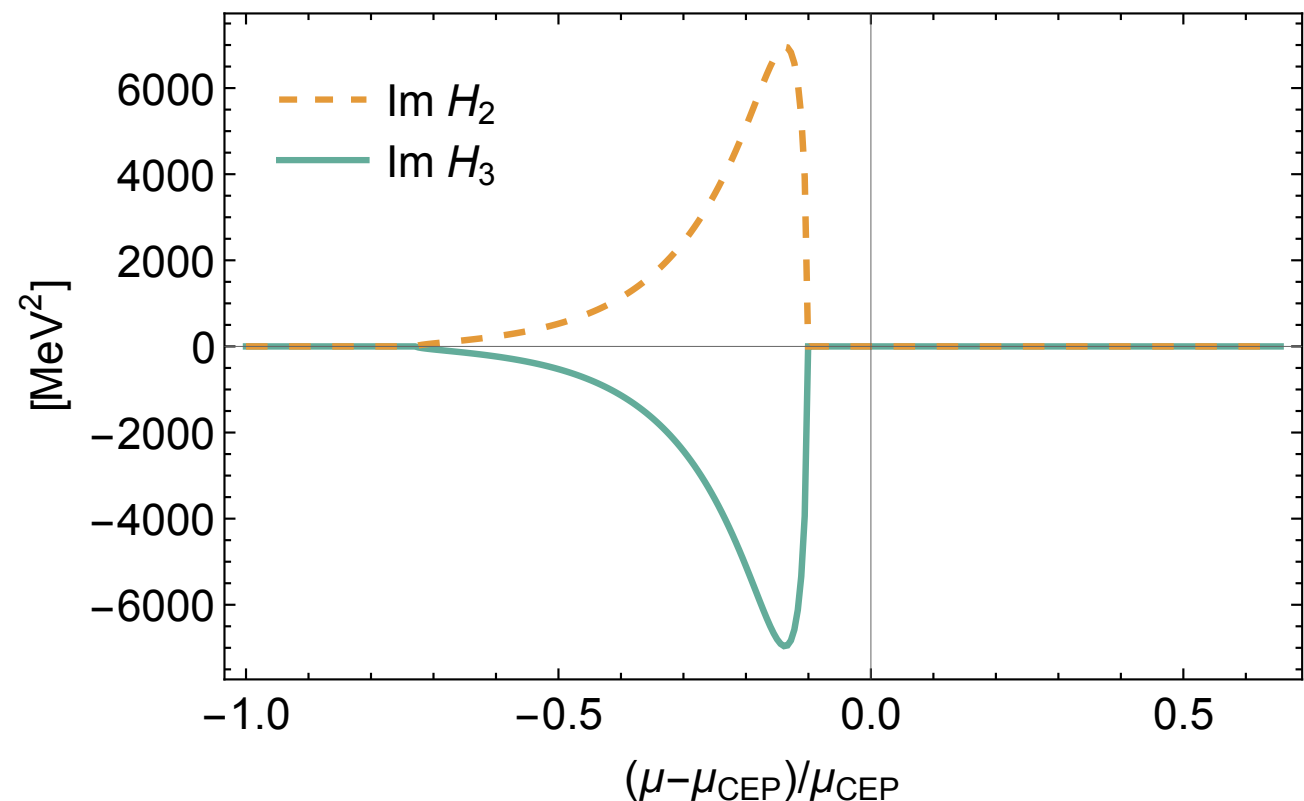
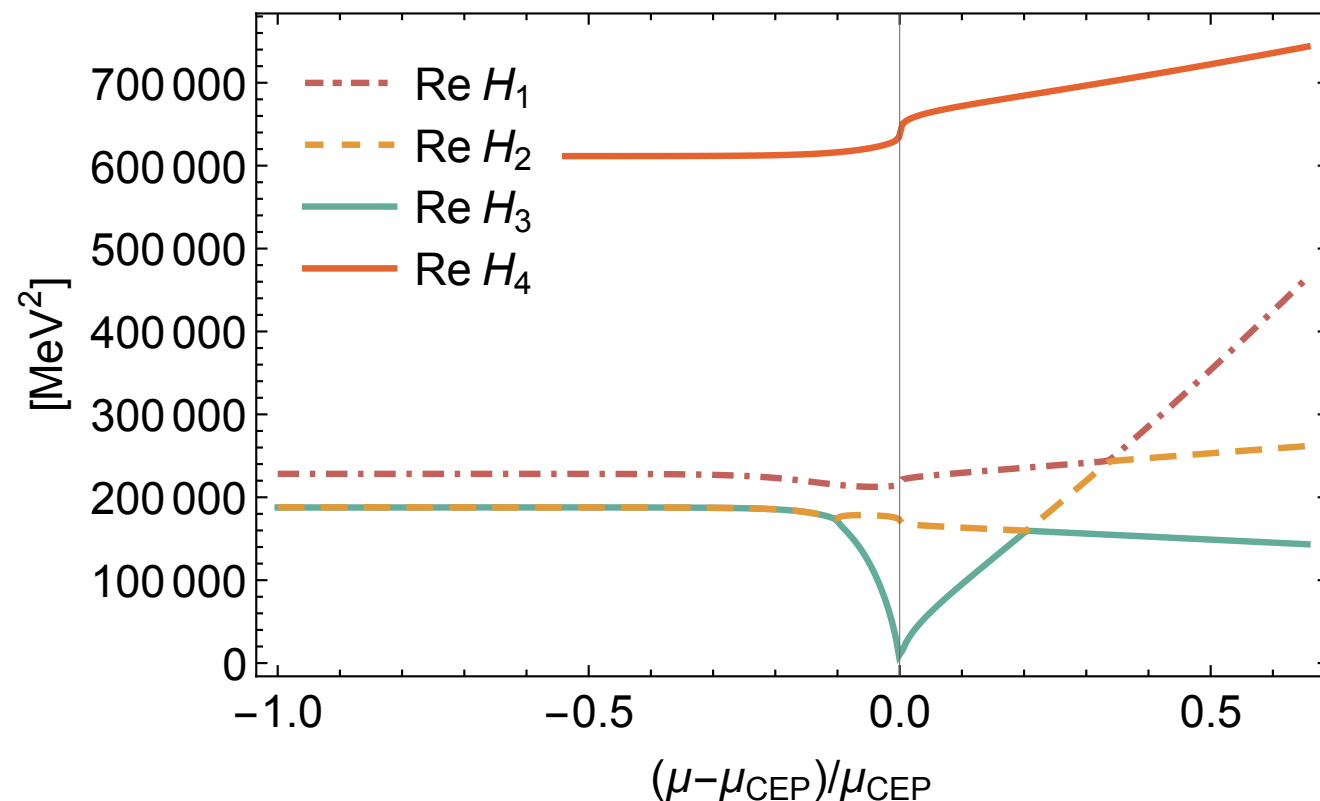
Extensive repulsive mixing in QCD at finite density from fundamental interactions between quarks

[Haensch, FR, von Smekal (2023)]



- chiral condensate σ (chiral symmetry breaking)
- Polyakov loops L, \bar{L} (confinement)
- density mode ω^0 (C -symmetry breaking)

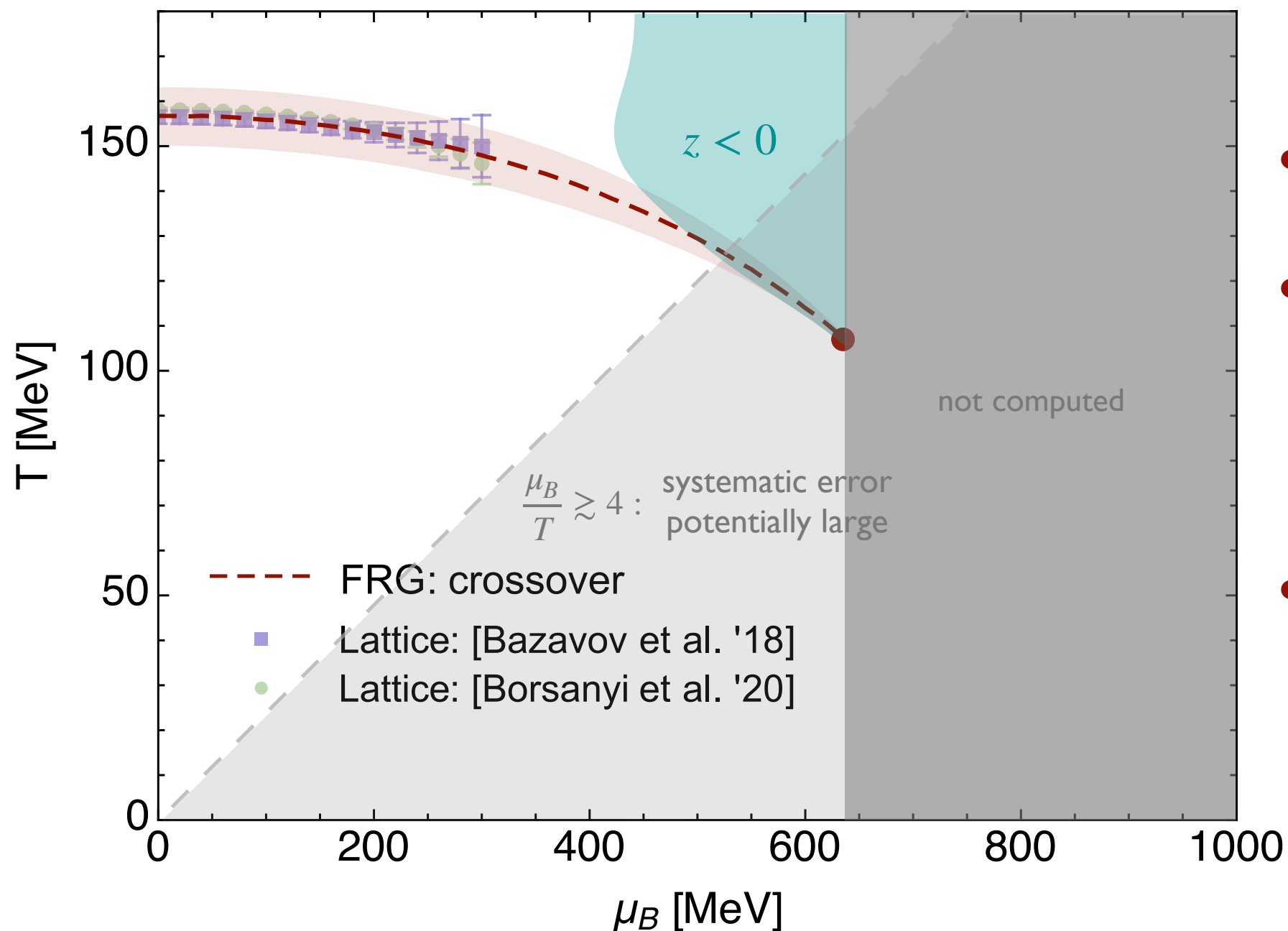
Eigenvalues of the Hessian at $T = T_{\text{CEP}}$ in a low-energy model (PQM model with vector repulsion)



MIXING, MOATS AND MODULATIONS IN QCD

The QCD phase diagram

[Fu, Pawłowski, FR 2019)]

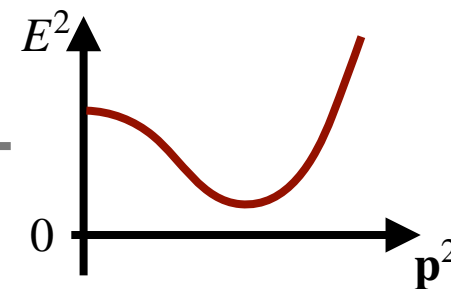
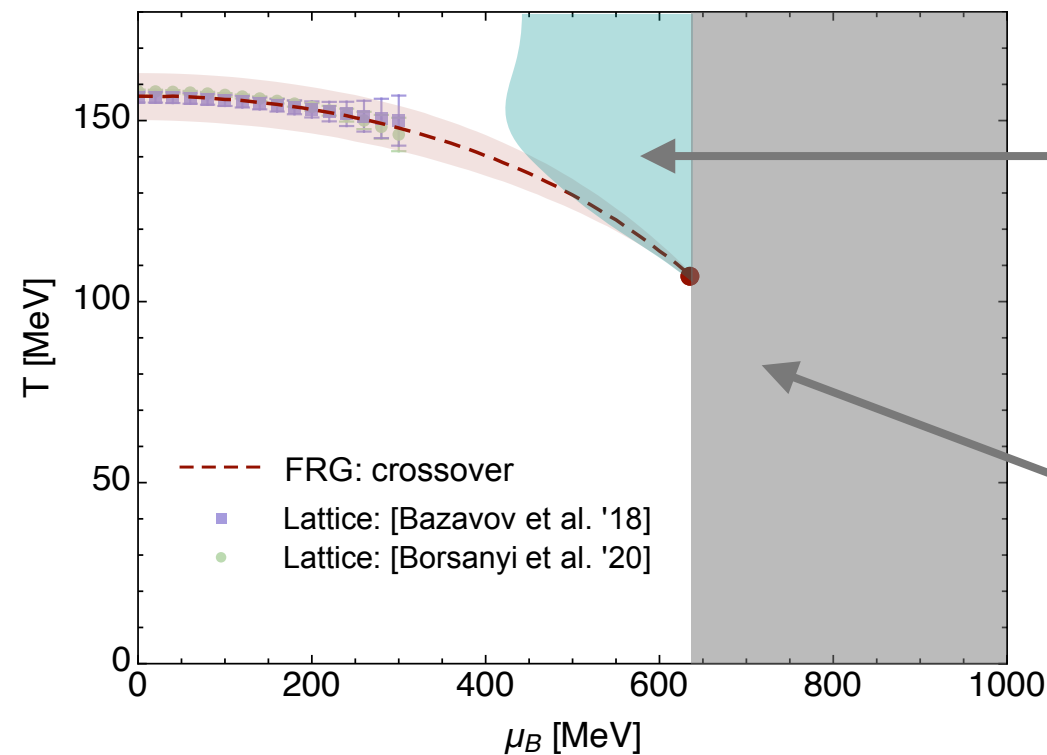


- direct calculation in QCD using the FRG
- CEP at $(T, \mu) = (107, 635)$ MeV consistent with DSE results and extrapolations of lattice data
 - [Gunkel, Fischer (2021)]
 - [Basar et al. (2023)]
 - [Schmidt et al. (2024)]
- need to improve systematics for definitive statements

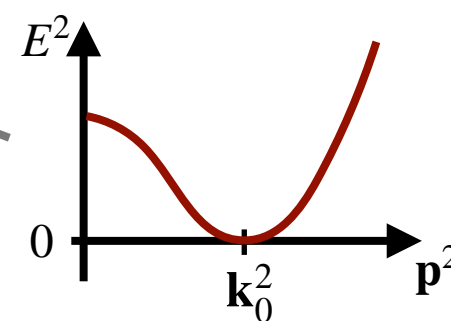
➡ indication for extended region with $z < 0$ in QCD: **moat regime**

FROM HOMOGENEOUS TO TOURING PATTERNS

The energy gap might close at lower T and larger μ_B :



$$E^2 > 0 \text{ for all } p^2$$

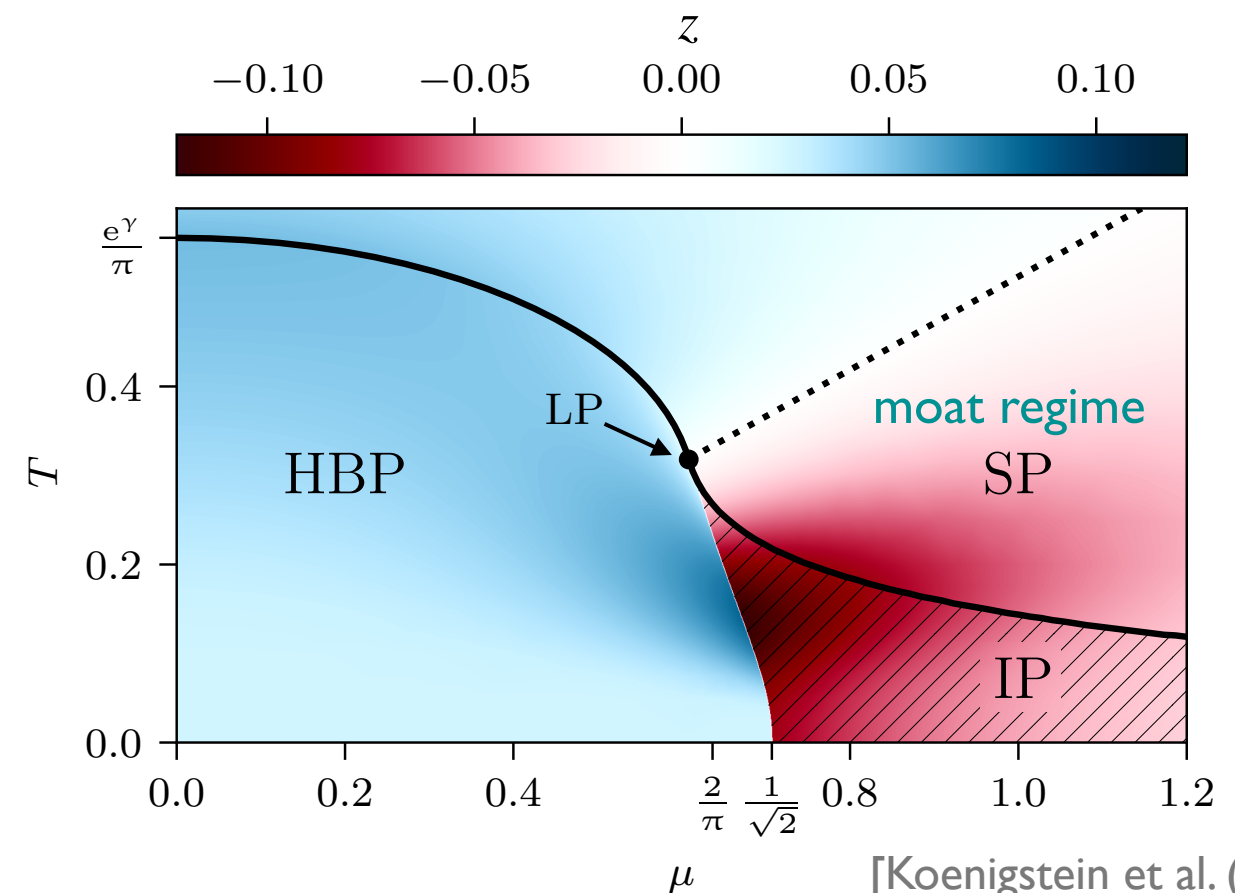


$$E^2 = 0 \text{ at } p^2 > 0:$$

Zero energy cost to condense particles with nonzero momentum k_0

→ instability towards formation of an inhomogeneous condensate

- Example: Gross-Neveu Model in $1+1$ dim. at large N_f



[Koenigstein et al. (2021)]

TYPES OF PATTERNS

Will an inhomogeneous instability automatically lead to Turing patterns?

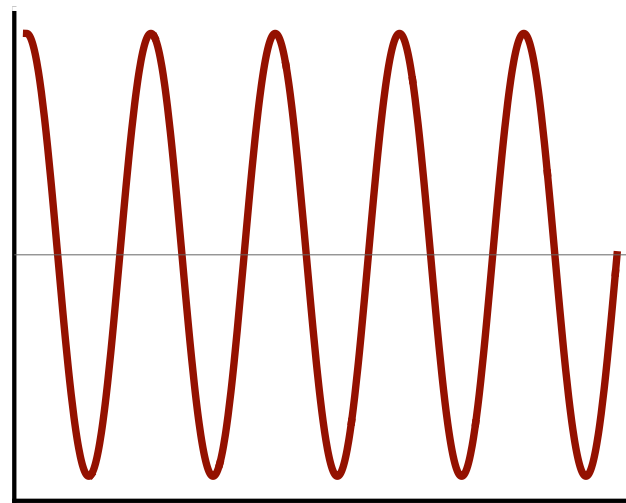
No: formation of inhomogeneous phases depends on dynamics of soft (massless) modes.

- fluctuation-induced instabilities of inhomogeneous phases
- **other types of phases possible** (possibly without long-range order, but always patterned)

inhom. phase

no instability
(typical in mean-field)

$$\langle \phi(r)\phi(0) \rangle \sim \sin(k_0 r)$$

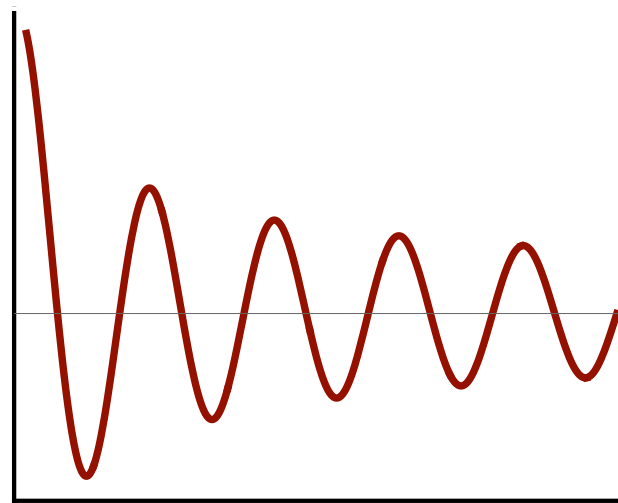


[Fukushima, Hatsuda, RPP 74 (2010)]
[Buballa, Carignano, PPNP 81 (2014)]

liquid crystal

Landau-Peierls instability
(Goldstones from spatial SB)

$$\langle \phi(r)\phi(0) \rangle \sim \sin(k_0 r) r^{-\alpha}$$

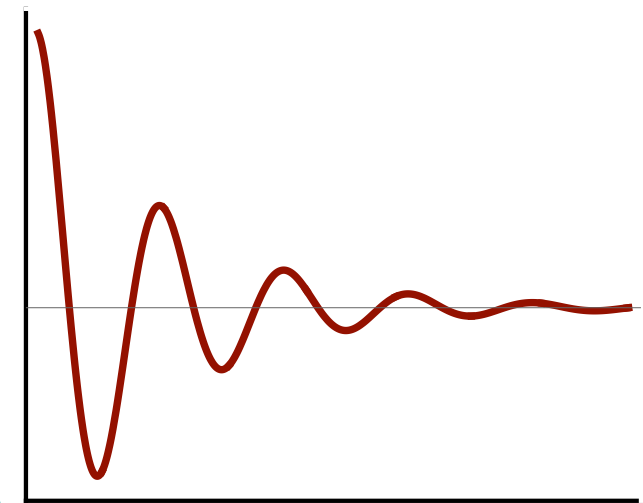


[Landau, Lifshitz, Stat. Phys. I, §137]
[Lee et al., PRD 92 (2015)]
[Hidaka et al., PRD 92 (2015)]

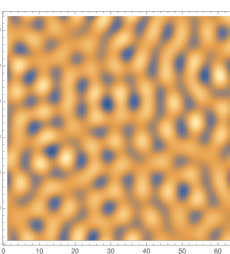
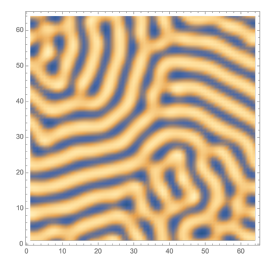
quantum pion liquid

PTV instability
(Goldstones from flavor SB)

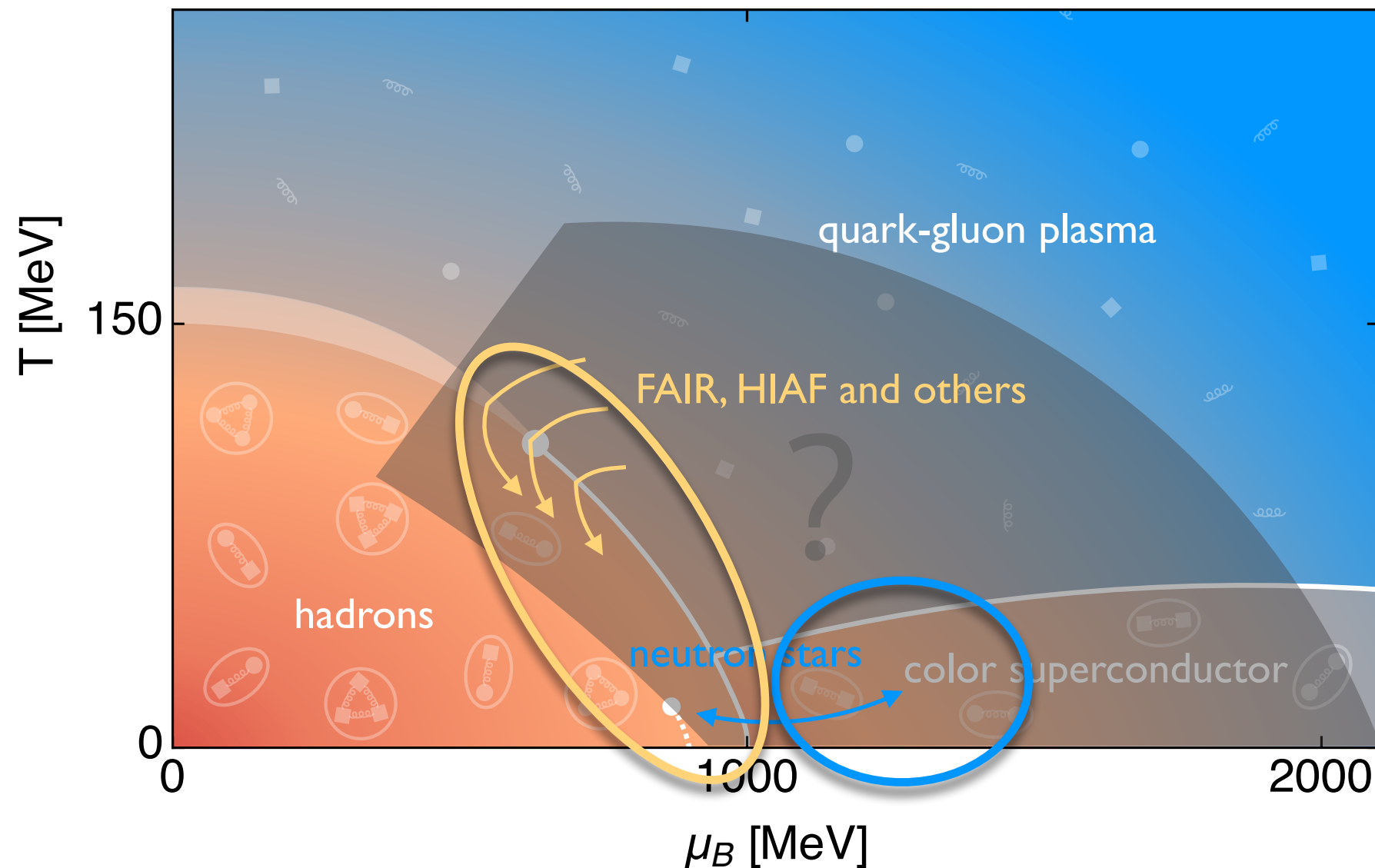
$$\langle \phi(r)\phi(0) \rangle \sim \sin(k_0 r) e^{-mr}$$



[Pisarski, Tselik, Valgushev, PRD 102 (2020)]
[Pisarski, PRD 103 (2021)]
[Valgushev, Winstel (2024)]



WHERE DO WE EXPECT PATTERNS?



patterns are expected in the "unknown" region of the phase diagram

this will be covered by future fixed target experiments

➔ search for patterns in heavy-ion collisions!

SEARCH FOR PATTERNS

intuitive idea: [Pisarski, FR (2021)]

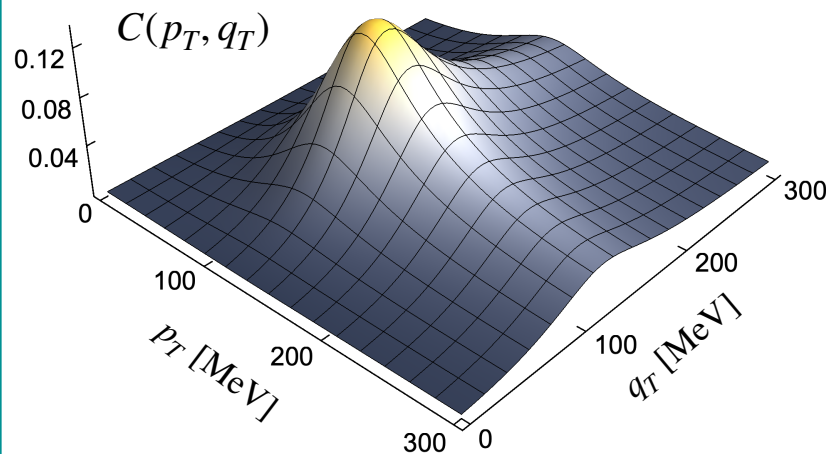
Characteristic feature of patterns: modes with minimal energy at nonzero momentum
⇒ enhanced particle production at nonzero momentum

→ look for signatures in the momentum dependence of particle correlations

- example: two-particle correlations

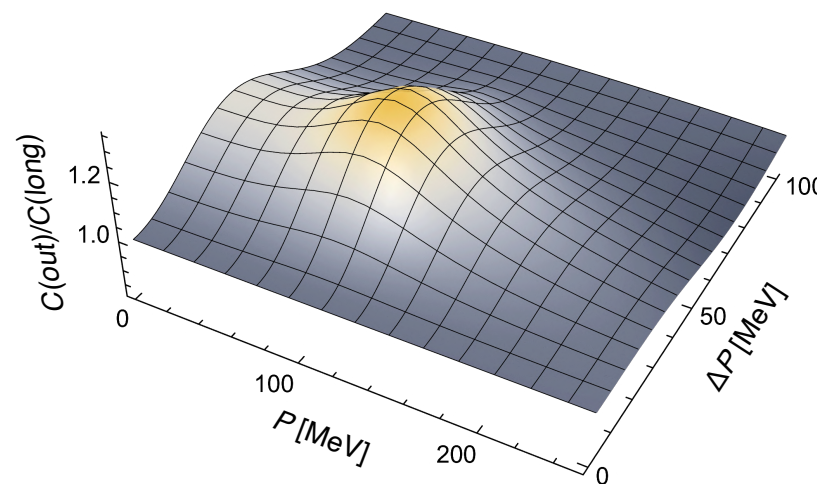
thermodynamic fluctuations in a moat regime

[Pisarski, FR (2021)]



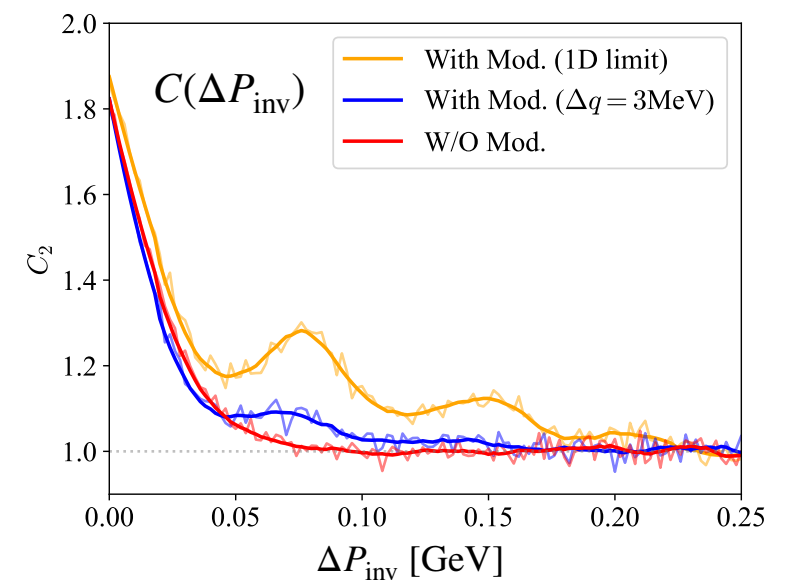
HBT in a moat regime

[FR, Pisarski, Rischke (2023)]



HBT from primordial inhomogeneity

[Fukushima et al. (2023)]



- work in progress: dilepton production [Nussinov, Ogilvie, Pannullo, Pisarski, FR, Schindler, Winstel]

→ peak position related to wavenumber of underlying spatial modulation

SUMMARY

Mixing, moats and modulations likely in dense QCD matter

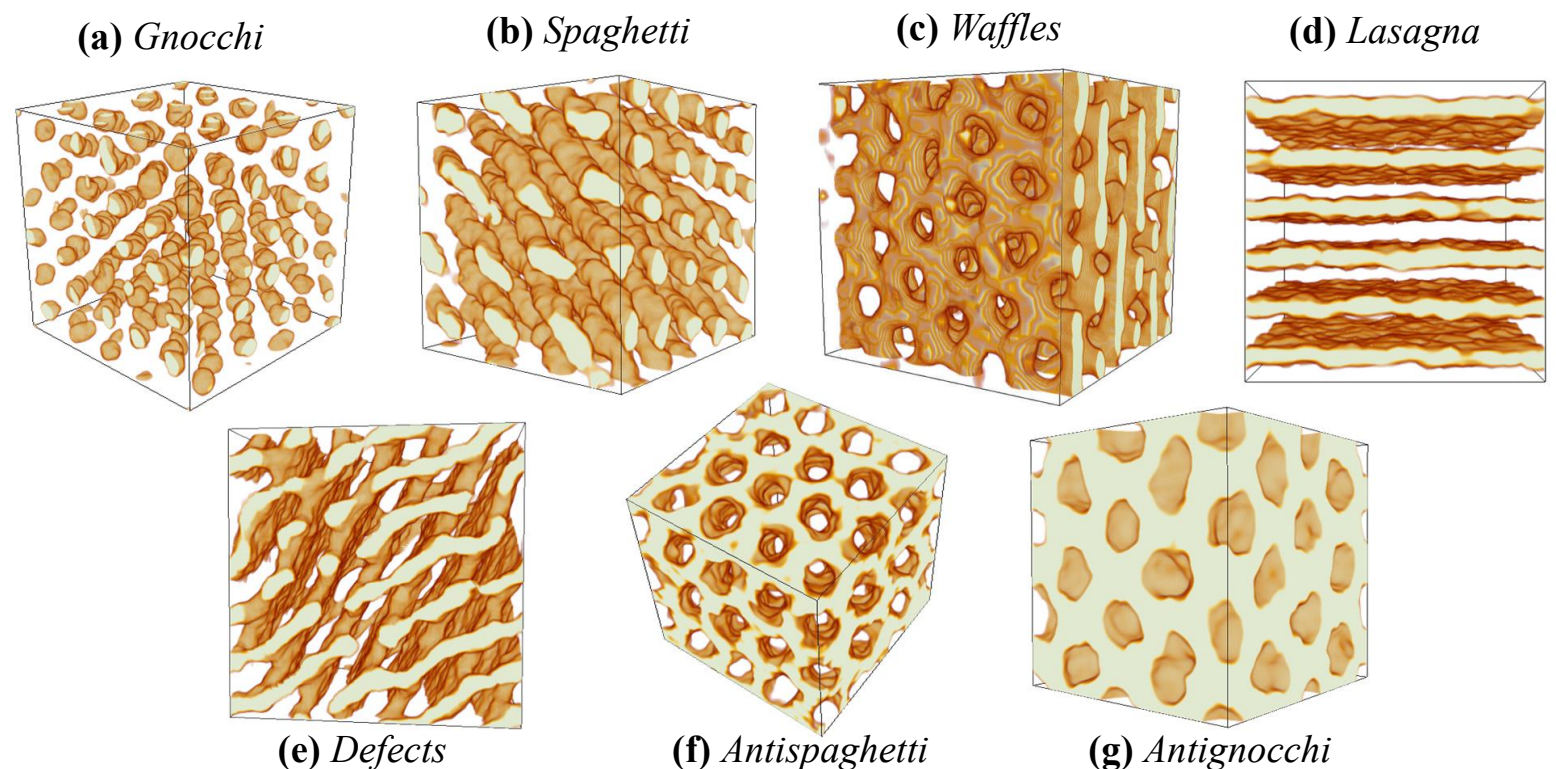
- directly related to pattern formation
- generic features of systems with C -breaking and competing attraction and repulsion

... expected to occur in HIC range

- characteristic enhancement of correlations at finite momentum
- need to understand (and find more) possible experimental signatures

... relevant for neutron stars

nuclear pasta [Caplan, Horowitz (2017)]



... but details not well understood

mostly mean-field results, but fluctuations decide which phases are actually realized