Rotating gluon system and color confinement

http://arxiv.org/abs/2312.06166

Yin Jiang

Beihang University

Rotation and magnetic field

Magnetic field	Rotation
Pseduo-vector	Pseduo-vector
Polarize J	Polarize J
Chiral transportation	Chiral transportation
Anomalous effects	Anomalous effects
Chiral catalysis	Chiral inhibition
Inverse chiral catalysis	?

Quark fluctuation In Gluon propagator

Understand confinement with KvBLL CALORON

- BPS dyon is good solution at finite temperature because 1) nonzero topological charge; 3) confinement; 4) periodic along imaginary time axis.
- Focus on the SU(2) gauge group case from now on. There are two kinds of dyons, namely M and L dyon and their anti-dyon. For M and Mbar dyon $F^a_{\mu\nu} = \pm\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^a_{\alpha\beta}$ or $E^a_i = \pm B^a_i$.

$$A^a_4 = \pm n_a(\frac{1}{r} - \rho\ \coth(\rho r))$$

$$A^a_m = \epsilon_{amk} n_k(\frac{1}{r} - \rho\,\mathrm{csch}(\rho r)).$$

Dyon	Action	$A^3_4(\infty)$	$\Phi(\infty)$	q_E	q_M
M	ρ_M	$\rho_M - \omega$	$\rho_M - \omega n_3$	+	+
$\bar{M}$	ρ_M	$\rho_{\bar{M}} + \omega$	$\rho_{\bar{M}} + \omega n_3$	+	-
L	$2\pi T - \rho_L$	$\rho_L + \omega$	$\rho_L + \omega n_3$	-	-
$\bar{L}$	$2\pi T - \rho_{\bar{L}}$	$\rho_{\bar{L}} - \omega$	$\rho_{\bar{L}} - \omega n_3$	-	+

- ρ is a free parameter characterizing the size of the dyon.
- Dyon carries nonzero color magnetic and electric charge. Not good enough.

Understand confinement with KvBLL CALORON

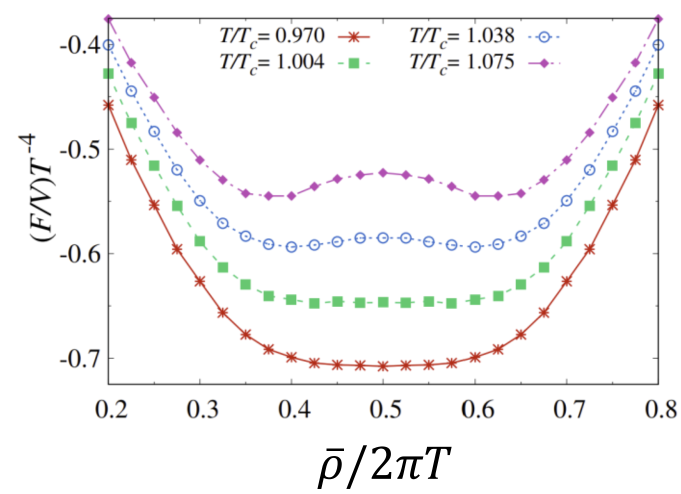
- As a solution of the Yang-Mills equation, KvBLL caloron is good because 1) color neutrality; 2) nonzero topological charge; 3) confinement; 4) periodic along imaginary time axis.
- Constructed with dyons by the AHDM construction. A potential non-trivial gluon field responsible for the confinement. Far from the cores of dyons

$$A_4^{caloron} = \frac{\tau_3}{2}(\bar{\rho} + \frac{1}{r} + \frac{1}{s})$$

$$A_\phi^{caloron} = -\frac{\tau_3}{2}(\frac{1}{r} + \frac{1}{s})\sqrt{\frac{(r_{LM} - r + s)(r_{LM} + r - s)}{(r_{LM} + r + s)(r + s - r_{LM})}}$$

$$F_p(T,\omega) = \frac{1}{3(2\pi)^2 T}\bar{\rho}^2(2\pi T - \bar{\rho})^2.$$

$$F_{np}(T) = -c[|\bar{\rho}|^3(\frac{\Lambda}{\pi T})^{\frac{22|\bar{\rho}|}{6\pi T}} + |2\pi T - \bar{\rho}|^3(\frac{\Lambda}{\pi T})^{\frac{22|2\pi T - \bar{\rho})|}{6\pi T}}]$$



$$L = \mathcal{P}e^{i\int_0^\beta dx_4 A_4} \quad : \frac{\bar{\rho}}{2\pi T} = 0.5, Tr(L) = 0$$ confinement

Spin the pure gluon system

- Real angular velocity.
- Caloron and anti-caloron as the background color field.
- Hard boundary. Finite size effect in the perturbative part.
- Running coupling.

Yang-Mills equation under rotation

- Yang-Mills field under rotation

$$g_{00} = 1 - \vec{v}^2,\ g_{mn} = -\delta_{mn},\ g_{0m} = g_{m0} = -v_m.$$

$$S = -\frac{1}{4g^2}\int d^4x \sqrt{|\det(g)|} F^a_{\mu\nu} F^{\mu\nu a}$$

- Redefine $A_0(new) = A_0(old) + v^j A_j(old)$ and $A_j(new) = A_j(old)$

$$S = -\frac{1}{4g^2}\int d^4x(-2G^a_{0i}G^a_{0i} + G^a_{mn}G^a_{mn})$$

$$\begin{aligned} G^a_{0m} &= \partial_0 A^a_m - \partial_m A^a_0 + f^{abc} A^b_0 A^c_m \\ &+ A^a_n \partial_n v_m - v_n \partial_n A^a_m \\ G^a_{mn} &= \partial_m A^a_n - \partial_n A^a_m + f^{abc} A^b_m A^c_n. \end{aligned}$$

Complex for real velocity at finite temperature

- Find a solution like (use an imaginary velocity temporarily)

$$A^{\text{full}} = (A_4^{\text{static}}(t, \vec{x}; \rho) + \delta A_4,\ \vec{A}^{\text{static}}(t, \vec{x}; \rho)).$$ **GET** $$\delta A_4 = \mp \omega_a \frac{\tau^a}{2}.$$

Yang-Mills equation under rotation

- Gauge transformation is changed

$$A_0 \to UA_0U^\dagger + iU(\partial_0 + v^j\partial_j)U^\dagger,$$
$$A_i \to UA_iU^\dagger + iU\partial_iU^\dagger.$$

- Polyakov loop is changed

$$L = \mathcal{P}e^{i\int_0^\beta dx_4(A_4 + v_m A_m)}.$$

- It can be checked explicitly the caloron's asymptotic solution satisfies the equation automatically, which means there is no polarization term for the caloron. Thus the order parameter is still $\bar{\rho}$

$$A_4^{caloron} = \frac{\tau_3}{2}(\bar{\rho} + \frac{1}{r} + \frac{1}{s}) \qquad A_\phi^{caloron} = -\frac{\tau_3}{2}(\frac{1}{r} + \frac{1}{s})\sqrt{\frac{(r_{LM} - r + s)(r_{LM} + r - s)}{(r_{LM} + r + s)(r + s - r_{LM})}}$$

Yang-Mills equation under rotation

- The physical meaning of the good analytical result is we have actually switched to the local inertial frame with the so-called vierbein fields

$$e^{\mu}_{\mu} = 1, e^{m}_{0} = -v^{m};$$
$$\xi^{\mu}_{\mu} = 1, \xi^{m}_{0} = v^{m}.$$

- The new solution leads to the extra field strength tensor going to zero

$$\delta G^{a}_{4m} \quad = -\partial_m \delta A^{a}_{4} + A^{a}_{n}\partial_n v_m - v_n \partial_n A^{a}_{m} + \epsilon^{abc}\delta A^{b}_{4} A^{c}_{m}$$

- This means although the gauge field is changed by the rotation, the strength tensor is the same as that in static case.

Dyon	Action	$A_4^3(\infty)$	$\Phi(\infty)$	q_E	q_M
M	ρ_M	$\rho_M - \omega$	$\rho_M - \omega n_3$	+	+
$\bar{M}$	ρ_M	$\rho_{\bar{M}} + \omega$	$\rho_{\bar{M}} + \omega n_3$	+	-
L	$2\pi T - \rho_L$	$\rho_L + \omega$	$\rho_L + \omega n_3$	-	-
$\bar{L}$	$2\pi T - \rho_{\bar{L}}$	$\rho_{\bar{L}} - \omega$	$\rho_{\bar{L}} - \omega n_3$	-	+

The popular twisted boundary condition

- The twisted boundary condition along the imaginary temporal axis works because the extra velocity related terms can be reduced as

$$A_n^a \partial_n v_m - v_n \partial_n A_m^a = (\vec{A}^a \cdot \nabla)\vec{v} - (\vec{v} \cdot \nabla)\vec{A}^a$$
$$= -\omega(\partial_\phi A_\varrho^a \hat{\varrho} + \partial_\phi A_\phi^a \hat{\phi} + \partial_\phi A_z^a \hat{z})$$

- If we have a solution in static case, the following gauge field is also a solution for the rotational equation with an imaginary velocity

$$A = A(x_4, \varrho, \phi + \omega x_4, z),$$

$$A(x_4 + \beta, \varrho, \phi + \omega x_4 + \omega\beta, z) = A(x_4, \varrho, \phi + \omega x_4, z)$$

- It is not necessarily satisfied(especially for real velocity) because the field profile should satisfy

$$A(x_4 + \beta, \varrho, \phi, z) = A(x_4, \varrho, \phi, z).$$

NOT USED in this work

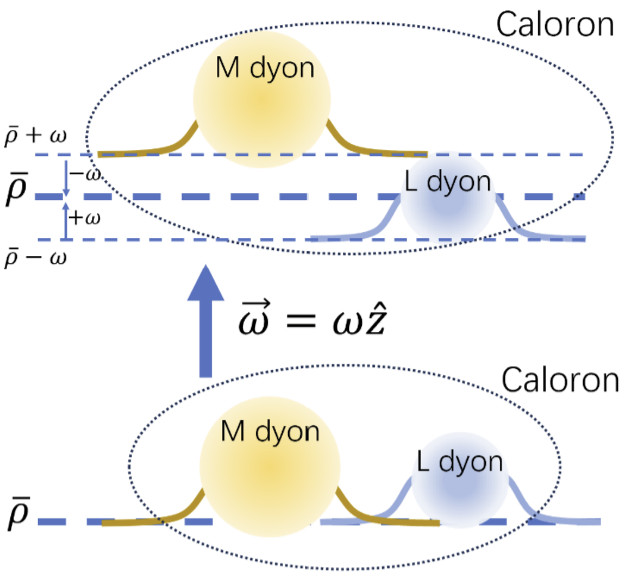
Construct KvBLL CALORON with dyons

- As a solution of the Yang-Mills equation, single dyon is NOT good because it is color charged;
- Combine several dyons, i.e. M and L in SU(2) case.

1) Comb(gauge transform) dyons to make them have the same asymptotic behavior at spatial infinity.
2) Superpose them using ADHM construction.

Dyon	Action	$A_4^3(\infty)$	$\Phi(\infty)$	q_E	q_M
M	ρ_M	$\rho_M - \omega$	$\rho_M - \omega n_3$	+	+
$\bar{M}$	ρ_M	$\rho_{\bar{M}} + \omega$	$\rho_{\bar{M}} + \omega n_3$	+	-
L	$2\pi T - \rho_L$	$\rho_L + \omega$	$\rho_L + \omega n_3$	-	-
$\bar{L}$	$2\pi T - \rho_{\bar{L}}$	$\rho_{\bar{L}} - \omega$	$\rho_{\bar{L}} - \omega n_3$	-	+

$$A_4^{calron}(r \to +\infty) = \bar{\rho}\frac{\tau_3}{2}$$



Once a good semi-classical solution obtained

- Compute the thermodynamic potential.
- Minimize the potential and compute Polyakov loop.

1. Nonperturbative part from the caloron. Replace the velocity with a real one

$$
\begin{aligned}
F_{np}(T,\omega) \quad &= -\frac{c}{2}[sgn(\bar{\rho})(\bar{\rho}+i\omega)^3(\frac{\Lambda}{\pi T})^{\frac{22sgn(\bar{\rho})(\bar{\rho}+i\omega)}{6\pi T}} \\
&+sgn(\bar{\rho}_c)(\bar{\rho}_c+i\omega)^3(\frac{\Lambda}{\pi T})^{\frac{22sgn(\bar{\rho}_c)(\bar{\rho}_c+i\omega)}{6\pi T}}] \\
&-\frac{c}{2}[sgn(\bar{\rho})(\bar{\rho}-i\omega)^3(\frac{\Lambda}{\pi T})^{\frac{22sgn(\bar{\rho})(\bar{\rho}-i\omega)}{6\pi T}} \\
&+sgn(\bar{\rho}_c)(\bar{\rho}_c-i\omega)^3(\frac{\Lambda}{\pi T})^{\frac{22sgn(\bar{\rho}_c)(\bar{\rho}_c-i\omega)}{6\pi T}}](5
\end{aligned}
$$

Once a good semi-classical solution obtained

2. Perturbative part of the thermodynamic potential.

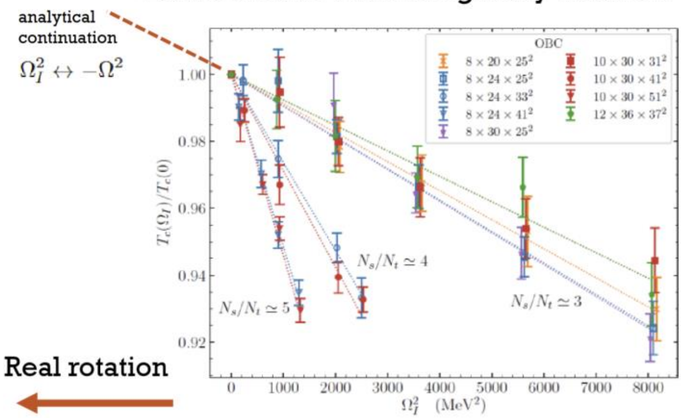
$$F_p^{\omega}(T,\omega) = -\sum_{\substack{s,m=1\\ n=-\infty}}^{+\infty} \frac{e^{\frac{sn\omega}{T}}}{\pi^2 s R^3} \frac{4\xi_n^{(m)} cos(s\frac{\bar{\rho}}{T})}{J_{n+1}(\xi_n^{(m)})^2} K_1(s\frac{\xi_n^{(m)}}{TR})$$

3. Mystical running coupling.

$$g(\omega) = (1 + 0.1\omega/\Lambda)g$$

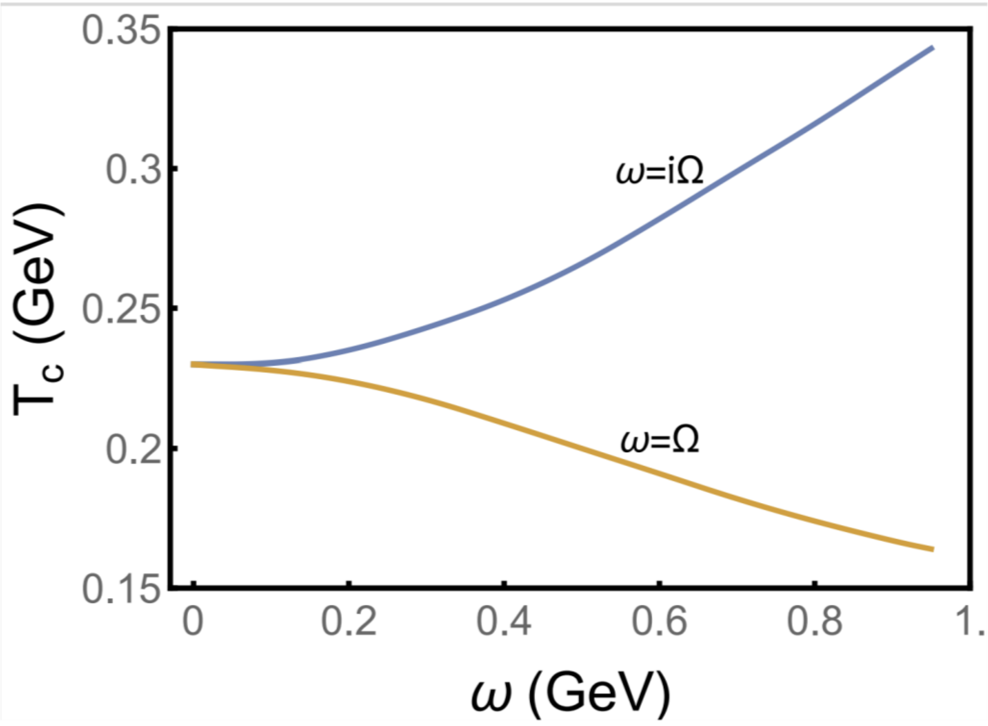
- Which of them gives the vortical catalysis?

▪ Lattice result with imaginary rotation



Braguta V V, Kotov A Y, Kuznedelev D D, et al. arXiv:2110.12302, 2021.

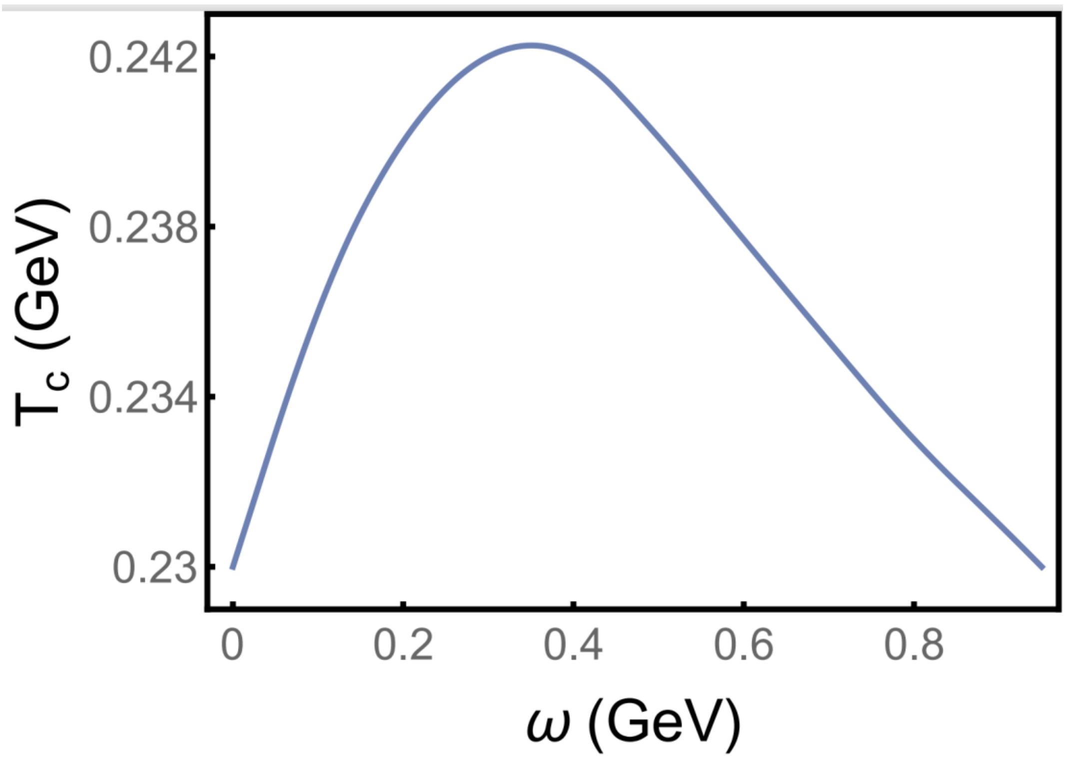
Constant running coupling



Bag is broken by rotation.

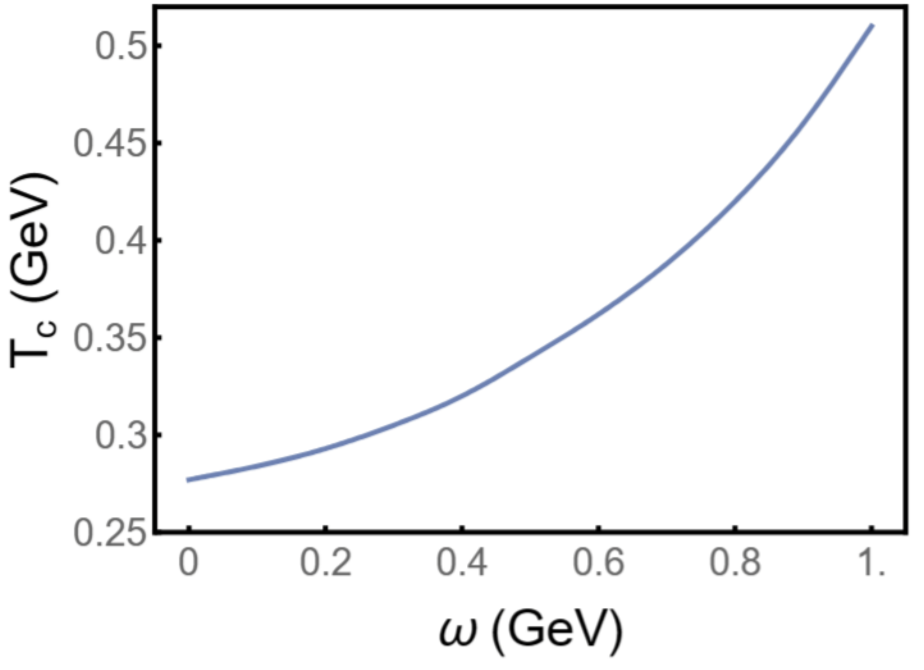
- Rotation helps to free color charge.
- The perturbation part will not be helpful to confine the color charge.
- Finite-size and polarization help to free color charge.

Running coupling constant



- Competition between running coupling and the other two contributions.
- The increasing range is short and unsignificant. It may disappear if the coupling dependence on rotation is weaker.

When the running coupling dominates



- Running coupling helps to confine color charge.
- *The only ambiguity in this computation.*

Outlook

- Achieved in these works
  - Modified QCD vacuum and fluctuation contribution(finite size and polarization) are not powerful enough to enhance the critical temperature.
  - The increase coupling constant may be the only reason to give us vortical catalysis.

- Double check the coupling running behavior.
- Obtain solutions with their center at arbitrary positions. Study the inhomogeneousness of the system.
- Consider dyon ensemble beyond dilute limit.
- Compute spatial dependent results to compare with lattice QCD.

Thank you for your attention!