第十四届全国粒子物理学术会议

Semi-inclusive DIS in the Target Fragmentation Region

陈开宝

山东建筑大学

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青岛

Based on:

K.B. Chen, J.P. Ma and X.B. Tong, JHEP 05 (2024) 298

K.B. Chen, J.P. Ma and X.B. Tong, PRD 108, 094015 (2023)

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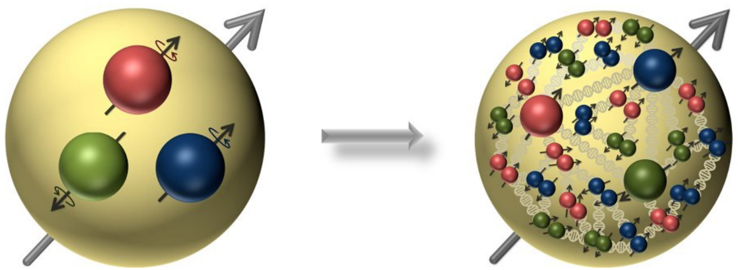
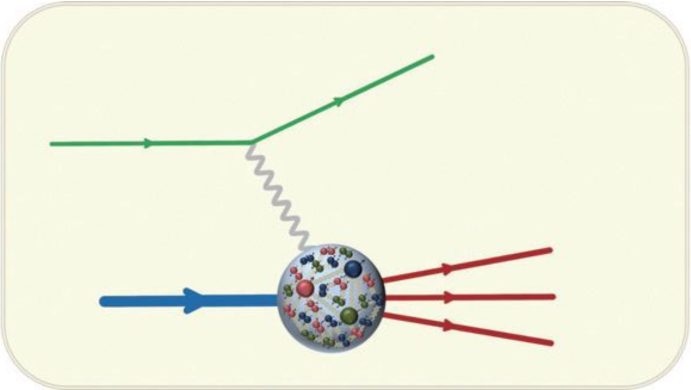
Introduction

■ Probing nucleon structure



Nucleon structure



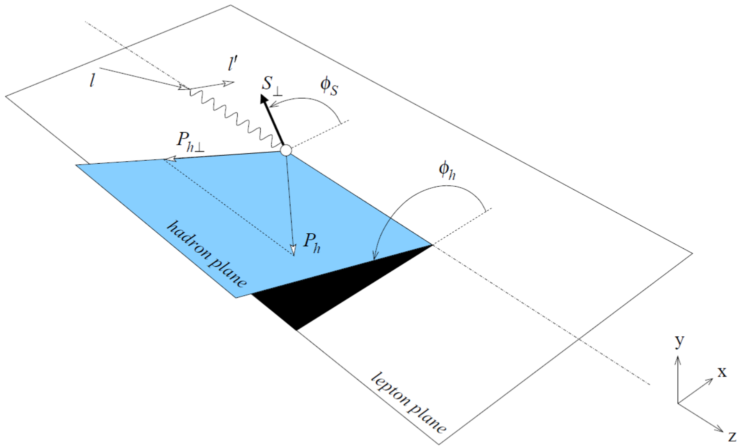
Lepton-nucleon deep inelastic scattering (DIS)

Images taken from
Front. Phys. 16(6), 64701

Why nucleon structure important?

- Fundamental components of the structure of matter
- Insights into the properties of strong interaction
- Inputs for describing high energy reactions
- Prerequisite for doing new physics researches
- …… ……

■ **Semi-inclusive DIS (SIDIS)**



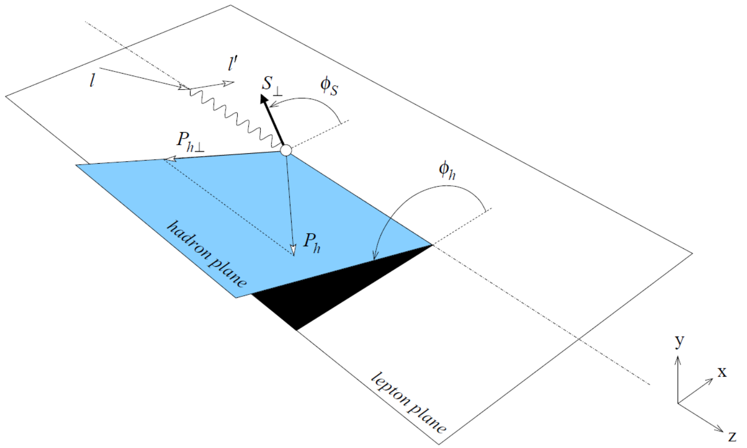
A. Bacchetta et al., JHEP 02, 093 (2007)

18 structure functions for SIDIS with polarized lepton and nucleon

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} &= \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\\
&\Big\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}\\
&+ \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}\\
&+ S_{\parallel}\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\Big]\\
&+ S_{\parallel}\lambda_e\Big[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big]\\
&+ |\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)\\
&\qquad + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}\\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]\\
&+ |\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}\\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$

Introduction

■ Semi-inclusive DIS (SIDIS)



A. Bacchetta et al., JHEP 02, 093 (2007)

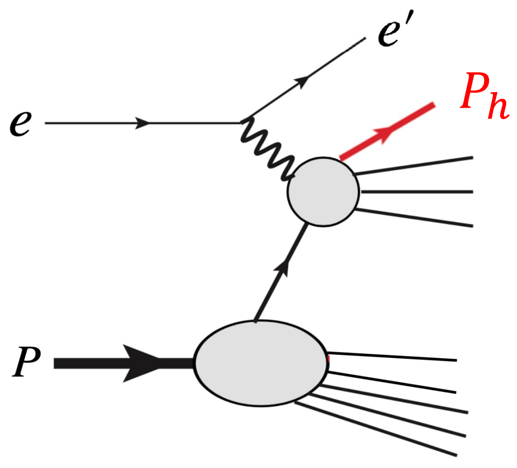
$$P_{h\perp} \sim M \ll Q$$

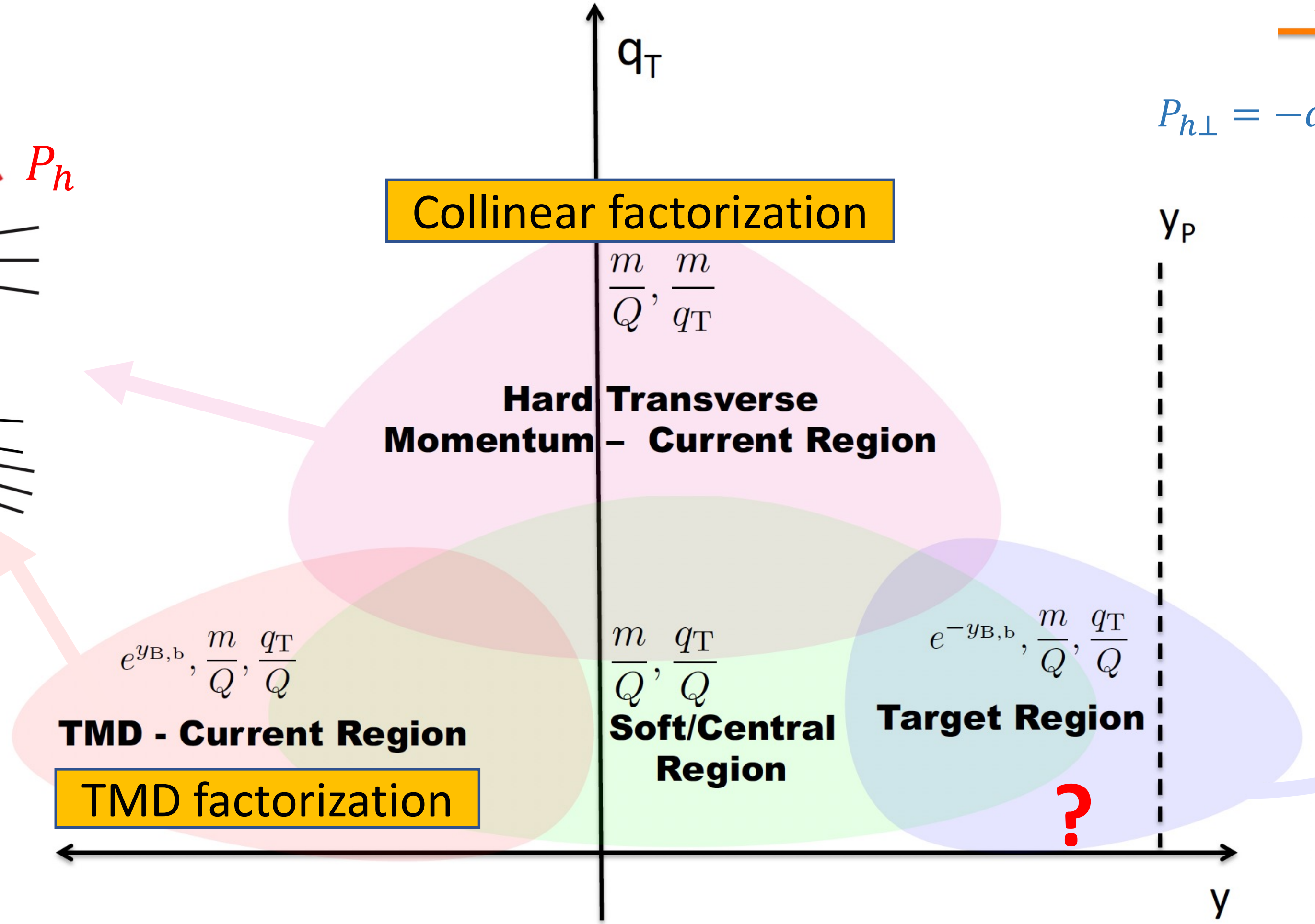
TMD factorization applies.

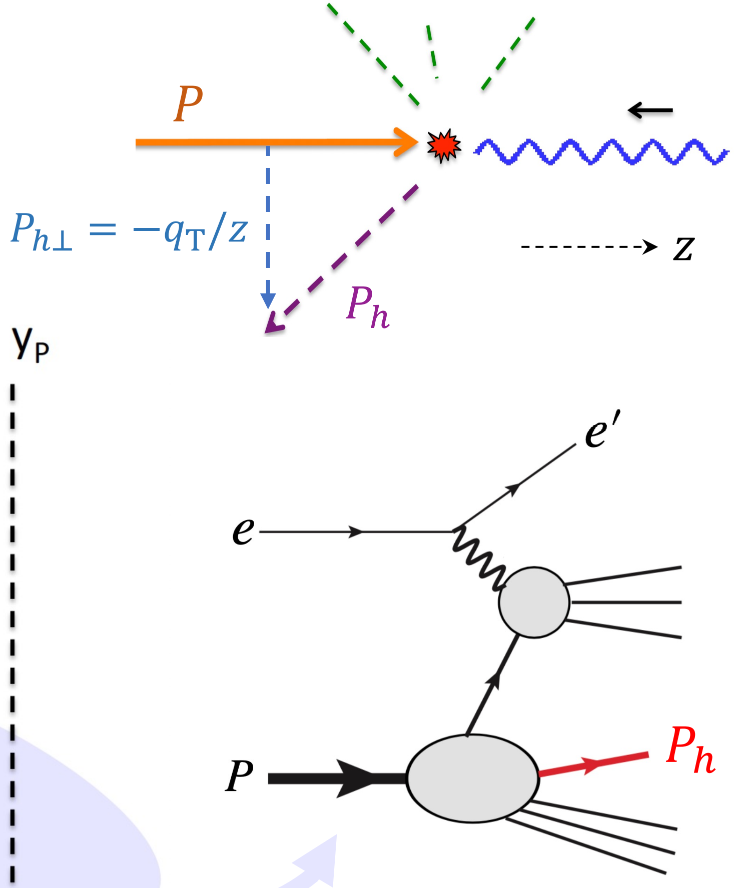
Structure functions are expressed by the convolution of TMD PDFs and FFs.

$F_{UU,T} \sim f_1 \otimes D_1$	Number density
$F_{LL} \sim g_{1L} \otimes D_1$	Helicity
$F_{UT,T}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^\perp \otimes D_1$	Sivers
$F_{LT}^{\cos(\phi_h-\phi_S)} \sim g_{1T} \otimes D_1$	Worm-gear L-T
$F_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1 \otimes H_1^\perp$	Transversity
$F_{UU}^{\cos 2\phi_h} \sim h_1^\perp \otimes H_1^\perp$	Boer-Mulders
$F_{UL}^{\sin 2\phi_h} \sim h_{1L}^\perp \otimes H_1^\perp$	Worm-gear T-L
$F_{UT}^{\sin(3\phi_h-\phi_S)} \sim h_{1T}^\perp \otimes H_1^\perp$	Pretzelosity
$\vdots$	

■ **Kinematic regions of SIDIS**







Mapping kinematical regions of SIDIS in terms of the produced hadron's **rapidity** and **transverse momentum** in the Breit frame

M. Boglione et al. JHEP 10 (2019) 122

■ SIDIS in the target fragmentation region (TFR)

- TFR SIDIS events were found at HERA.

M. Derrick et al. (ZEUS Collaboration), Phys. Lett. B346, 399 (1995).

T. Ahmed et al. (H1 Collaboration), Phys. Lett. B348, 681 (1995).

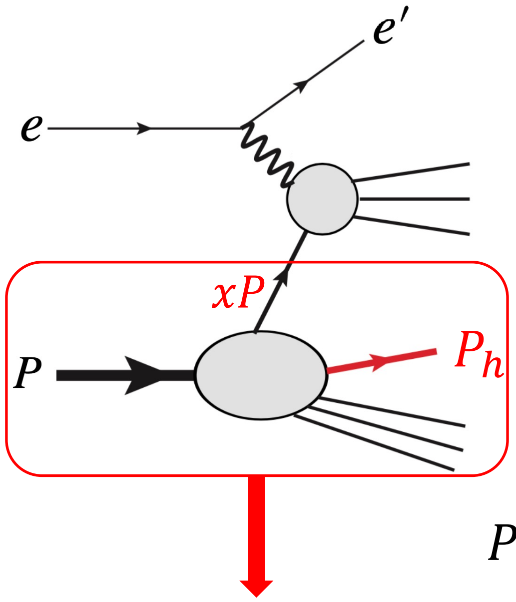
- Theoretical description: the concept of fracture function

$F(x,\xi_h)$: Fracture function

Parton distributions in the presence of an almost collinear particle observed in the final state.

(conditional probability, hybrid of PDFs and FFs)

L. Trentadue and G. Veneziano, Phys. Lett. B323, 201 (1994)



$P_h \approx \xi_h P$

$d\sigma \sim F(x,\xi_h) \otimes H$

SIDIS in the target fragmentation region (TFR)

Only four structure functions at twist-2 and the tree level.

- Structure functions in the TFR

$$\frac{d\sigma^{\text{TFR}}}{dx_B\, dy\, d\zeta\, d^2\boldsymbol{P}_{h\perp}\, d\phi_S} = \frac{2\alpha_{\text{em}}^2}{Q^2 y}\Bigg\{\left(1-y+\frac{y^2}{2}\right)\sum_a e_a^2\left[\underbrace{M(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)}_{F_{UU,T}} - |\boldsymbol{S}_\perp|\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\underbrace{M_T^h(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)}_{F_{UT,T}^{\sin(\phi_h-\phi_S)}}\sin(\phi_h-\phi_S)\right]$$

$$+\lambda_l y\left(1-\frac{y}{2}\right)\sum_a e_a^2\left[S_\parallel \underbrace{\Delta M_L(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)}_{F_{LL}} + |\boldsymbol{S}_\perp|\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\underbrace{\Delta M_T^h(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)}_{F_{LT}^{\cos(\phi_h-\phi_S)}}\cos(\phi_h-\phi_S)\right]\Bigg\}.$$

Anselmino, Barone, and Kotzinian, Phys. Lett. B699, 108 (2011); Phys. Lett. B706, 46 (2011).

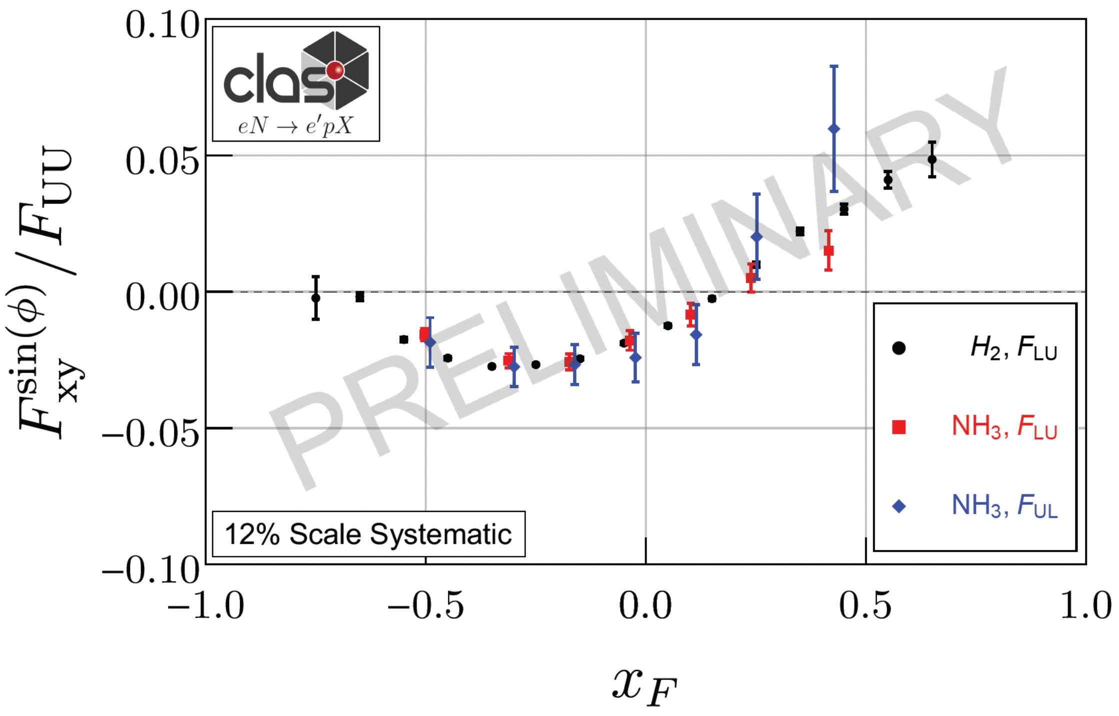
Can we go beyond twist-2 or tree level? For:

1. More accurate description of the process

2. Novel azimuthal/spin asymmetries

■ SIDIS in the target fragmentation region (TFR)

- Preliminary results from JLab



A. Accardi et al., arXiv:2306.09360.

CLAS12 beam-spin asymmetry results show different behavior in CFR and TFR !

Sensitive to $F_{LU}^{\sin\phi}$, $F_{UL}^{\sin\phi}$.

Structure functions beyond twist-2 !

Contents

Twist-3 contributions for TFR SIDIS

Kinematics

$$e(l,\lambda_e) + h_A(P,S) \rightarrow e(l') + h(P_h) + X$$

$$\frac{d\sigma}{dx_B dy d\xi_h d\psi d^2 P_{h\perp}} = \frac{\alpha^2 y}{4\xi_h Q^4} L_{\mu\nu}(l,\lambda_e,l') W^{\mu\nu}(q,P,S,P_h)$$

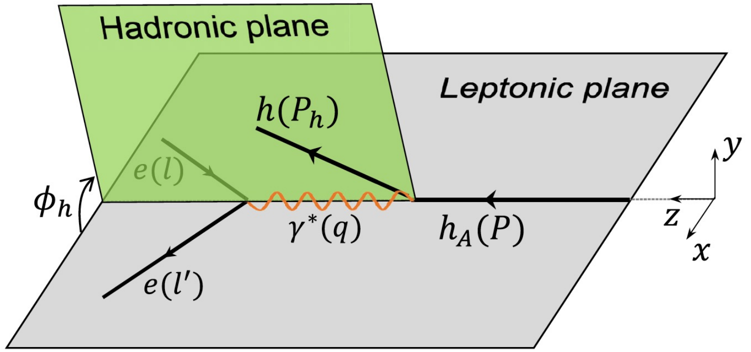
$$L^{\mu\nu}(l,\lambda_e,l') = 2(l^\mu l'^\nu + l^\nu l'^\mu - l\cdot l' g^{\mu\nu}) + 2i\lambda_e \epsilon^{\mu\nu\rho\sigma} l_\rho l'_\sigma$$

$$W^{\mu\nu}(q,P,S,P_h) = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{iq\cdot x} \langle S; h_A | J^\mu(x) | hX \rangle \langle Xh | J^\nu(0) | h_A; S \rangle$$

$$P^\mu \approx (P^+, 0,0,0) \qquad l^\mu = \left(\frac{1-y}{y} x_B P^+, \frac{Q^2}{2x_B y P^+}, \frac{Q\sqrt{1-y}}{y}, 0\right)$$

$$P_h^\mu = (P_h^+, P_h^-, \vec{P}_{h\perp}) \qquad q^\mu = \left(-x_B P^+, \frac{Q^2}{2x_B P^+}, 0,0\right)$$

$$S^\mu = \left(\frac{S_L P^+}{M}, -\frac{S_L M}{2P^+}, \vec{S}_\perp\right)$$



$$Q^2 = -q^2,$$

$$x_B = \frac{Q^2}{2P\cdot q},$$

$$y = \frac{P\cdot q}{P\cdot l},$$

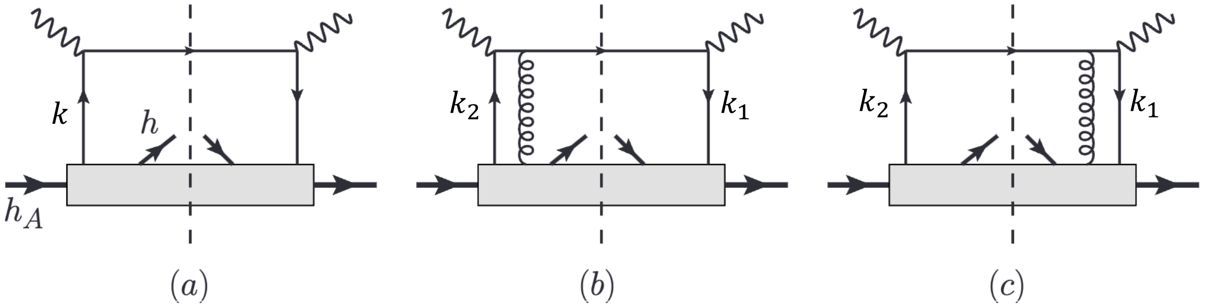
$$\xi_h = \frac{P_h\cdot q}{P\cdot q} \approx \frac{P_h^+}{P^+},$$

$$\cancel{z_h = \frac{P\cdot P_h}{P\cdot q}} \ll 1.$$

Twist-3 contributions for TFR SIDIS

■ Hadronic tensor

Hadronic tensor at the tree level



… …

Multiple gluon scattering in the final state.

$$W^{\mu\nu}\Big|_a = \int \frac{d^3k}{(2\pi)^3}\left[\left(\gamma^\mu(\not{k}+\not{q})\gamma^\nu\right)_{ij} 2\pi\delta\left((k+q)^2\right)\right]\sum_X \int \frac{d^3\eta}{(2\pi)^4} e^{-ik\cdot\eta}\langle h_A|\bar{\psi}_i(\eta)|hX\rangle\langle Xh|\psi_j(0)|h_A\rangle,$$

$$W^{\mu\nu}\Big|_b = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\left[\left(\gamma^\mu(\not{k}_1+\not{q})\gamma_\alpha \frac{i(\not{k}_2+\not{q})}{(k_2+q)^2+i\epsilon}\gamma^\nu\right)_{ij} 2\pi\delta\left((k_1+q)^2\right)\right]$$
$$\times(-ig_s)\sum_X \int \frac{d^3\eta d^3\eta_1}{(2\pi)^4} e^{-ik_1\cdot\eta}e^{i(k_1-k_2)\cdot\eta_1}\langle h_A|\bar{\psi}_i(\eta)|hX\rangle\langle Xh|G^\alpha(\eta_1)\psi_j(0)|h_A\rangle,$$

$k_i \sim Q(1,\lambda^2,\lambda)$, with $\lambda \sim \frac{\Lambda_{QCD}}{Q}$

$$W^{\mu\nu}\Big|_c = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\left[\left(\gamma^\mu \frac{i(\not{k}_1+\not{q})}{(k_1+q)^2-i\epsilon}\gamma_\alpha(\not{k}_2+\not{q})\gamma^\nu\right)_{ij} 2\pi\delta\left((k_2+q)^2\right)\right]$$
$$\times(-ig_s)\sum_X \int \frac{d^3\eta d^3\eta_1}{(2\pi)^4} e^{-ik_1\cdot\eta}e^{i(k_1-k_2)\cdot\eta_1}\langle h_A|\bar{\psi}_i(\eta)G^\alpha(\eta_1)|hX\rangle\langle Xh|\psi_j(0)|h_A\rangle,$$

Carrying out the collinear expansion procedures … …

Twist-3 contributions for TFR SIDIS

■ Hadronic tensor

twist-2

twist-3

$$W^{\mu\nu} = \xi_h(\gamma^\mu\gamma^+\gamma^\nu)_{ij}\mathcal{M}_{ji}(x_B) + \left[\frac{-i\xi_h}{2q^-}(\gamma^\mu\gamma^+\gamma_{\perp\alpha}\gamma^-\gamma^\nu)_{ij}\mathcal{M}^\alpha_{\partial,ji}(x_B) + (\mu\leftrightarrow\nu)^*\right]$$
$$+\left\{\frac{-i\xi_h}{2q^-}(\gamma^\mu\gamma^+\gamma_{\perp\alpha}\gamma^-\gamma^\nu)_{ij}\int dx_2\left[\mathrm{P}\frac{1}{x_2-x_B} - i\pi\delta(x_2-x_B)\right]\mathcal{M}^\alpha_{F,ji}(x_B,x_2) + (\mu\leftrightarrow\nu)^*\right\}$$

$$\boxed{\begin{aligned}
\mathcal{M}_{ij}(x) &= \int\frac{d\eta^-}{2\xi_h(2\pi)^4}e^{-ixP^+\eta^-}\sum_X\langle h_A|\bar\psi_j(\eta^-)\mathcal{L}_n^\dagger(\eta^-)|hX\rangle\langle Xh|\mathcal{L}_n(0)\psi_i(0)|h_A\rangle,\\
\mathcal{M}^\alpha_{\partial,ij}(x) &= \int\frac{d\eta^-}{2\xi_h(2\pi)^4}e^{-ixP^+\eta^-}\sum_X\langle h_A|\bar\psi_j(\eta^-)\mathcal{L}_n^\dagger(\eta^-)|hX\rangle\langle Xh|\partial_\perp^\alpha(\mathcal{L}_n\psi_i)(0)]|h_A\rangle,\\
\mathcal{M}^\alpha_{F,ij}(x_1,x_2) &= \int\frac{d\eta^-d\eta_1^-}{4\pi\xi_h(2\pi)^4}e^{-ix_1P^+\eta^- - i(x_2-x_1)P^+\eta_1^-}\sum_X\langle h_A|\bar\psi_j(\eta^-)|hX\rangle\langle Xh|g_sF^{+\alpha}(\eta_1^-)\psi_i(0)|h_A\rangle.
\end{aligned}}$$

Gauge invariant matrix elements!

$$\mathcal{L}_n(x) = \mathcal{P}\exp\left\{-ig_s\int_0^\infty d\lambda\, G^+(\lambda n + x)\right\}$$

$$g_sF^{+\alpha} = g_s[\partial^+G_\perp^\alpha - \partial_\perp^\alpha G^+] + \mathcal{O}(g_s^2)$$

Twist-3 contributions for TFR SIDIS

■ **Fracture functions defined via the correlation matrices**

$$\mathcal{M}_{ij}(x) = \frac{(\gamma_\rho)_{ij}}{2N_c}\left[\bar{n}^\rho\left(u_1 - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}u_{1T}^h\right) + \frac{1}{P^+}\left(P_{h\perp}^\rho u^h - M\tilde{S}_\perp^\rho u_T - S_L\tilde{P}_{h\perp}^\rho u_L^h - \frac{P_{h\perp}^{\langle\rho}P_{h\perp}^{\beta\rangle}}{M}\tilde{S}_{\perp\beta}u_T^h\right)\right]$$
$$- \frac{(\gamma_\rho\gamma_5)_{ij}}{2N_c}\left[\bar{n}^\rho\left(S_L l_{1L} - \frac{P_{h\perp}\cdot S_\perp}{M}l_{1T}^h\right) + \frac{1}{P^+}\left(\tilde{P}_{h\perp}^\rho l^h + MS_\perp^\rho l_T + S_L P_{h\perp}^\rho l_L^h - \frac{P_{h\perp}^{\langle\rho}P_{h\perp}^{\beta\rangle}}{M}S_{\perp\beta}l_T^h\right)\right] + \cdots,$$

$$\mathcal{M}_{\partial,ij}^\alpha(x) = \frac{(\gamma^-)_{ij}}{2N_c}i\left(-P_{h\perp}^\alpha u_\partial^h + M\tilde{S}_\perp^\alpha u_{\partial T} + S_L\tilde{P}_{h\perp}^\alpha u_{\partial L}^h + \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}\tilde{S}_{\perp\beta}u_{\partial T}^h\right)$$
$$+ \frac{(\gamma^-\gamma_5)_{ij}}{2N_c}i\left(\tilde{P}_{h\perp}^\alpha l_\partial^h + MS_\perp^\alpha l_{\partial T} + S_L P_{h\perp}^\alpha l_{\partial L}^h - \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}S_{\perp\beta}l_{\partial T}^h\right) + \cdots,$$

$$\mathcal{M}_{F,ij}^\alpha(x_1,x_2) = \frac{(\gamma^-)_{ij}}{2N_c}\left(P_{h\perp}^\alpha w^h - M\tilde{S}_\perp^\alpha w_T - S_L\tilde{P}_{h\perp}^\alpha w_L^h - \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}\tilde{S}_{\perp\beta}w_T^h\right)$$
$$- \frac{(\gamma^-\gamma_5)_{ij}}{2N_c}i\left(\tilde{P}_{h\perp}^\alpha v^h + MS_\perp^\alpha v_T + S_L P_{h\perp}^\alpha v_L^h - \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}S_{\perp\beta}v_T^h\right) + \cdots,$$

Fracture functions up to twist-3

$$\tilde{a}_\perp^\mu \equiv \varepsilon_\perp^{\mu\nu}a_{\perp\nu}$$

$$P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle} \equiv P_{h\perp}^\alpha P_{h\perp}^\beta + g_\perp^{\alpha\beta}\vec{P}_{h\perp}^2/2$$

- Red: twist-2.
- Green: twist-3
- Functions of $(x, \xi_h, P_{h\perp}^2)$

- Only chiral-even terms contribute
- Similar form ($k_\perp \to P_{h\perp}$) and naming rules as TMDs

See e.g., S.Y. Wei, Y.K. Song, KBC, Z.T. Liang, PRD 95, 074017

Twist-3 contributions for TFR SIDIS

■ Fracture functions defined via the correlation matrices

The twist-3 fracture functions are not independent from each other!

Applying QCD equation of motion $i\gamma \cdot D\psi = 0$, gives:

$$x[u_S^K(x) + il_S^K(x)]$$
$$= u_{\partial S}^K(x) + il_{\partial S}^K(x) + i\int dy\left[\mathrm{P}\frac{1}{y-x} - i\pi\delta(y-x)\right][w_S^K(x,y) - v_S^K(x,y)]$$

The relations take a unified form!

The hadronic tensor will be expressed by those fracture functions defined only from the quark-quark correlator $\mathcal{M}_{ij}$.

Four sets of equations

S	K
null	h
L	h
T	null
T	h

Twist-3 contributions for TFR SIDIS

Results for structure functions and azimuthal asymmetries

Structure functions

Azimuthal/spin asymmetries

Twist-2

$$F_{UU,T} = x_B u_1, \qquad F_{UT,T}^{\sin(\phi_h-\phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B u_{1T}^h,$$

$$F_{LL} = x_B l_{1L}, \qquad F_{LT}^{\cos(\phi_h-\phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B l_{1T}^h.$$

$$\langle \sin(\phi_h-\phi_S)\rangle_{UT} = \frac{|\vec{P}_{h\perp}|}{2M}\frac{u_{1T}^h}{u_1},$$

$$\langle \cos(\phi_h-\phi_S)\rangle_{LT} = \frac{|\vec{P}_{h\perp}|C(y)}{2MA(y)}\frac{l_{1T}^h}{u_1}.$$

Twist-3

$$F_{UU}^{\cos\phi_h} = -\frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 u^h, \qquad F_{LU}^{\sin\phi_h} = \frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 l^h,$$

$$F_{UL}^{\sin\phi_h} = -\frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 u_L^h, \qquad F_{LL}^{\cos\phi_h} = -\frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 l_L^h,$$

$$F_{UT}^{\sin\phi_S} = -\frac{2M}{Q} x_B^2 u_T, \qquad F_{LT}^{\cos\phi_S} = -\frac{2M}{Q} x_B^2 l_T,$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} = -\frac{\vec{P}_{h\perp}^2}{QM} x_B^2 u_T^h, \qquad F_{LT}^{\cos(2\phi_h-\phi_S)} = -\frac{\vec{P}_{h\perp}^2}{QM} x_B^2 l_T^h.$$

$$\langle \cos\phi_h\rangle_{UU} = -\frac{|\vec{P}_{h\perp}|}{Q}\frac{B(y)}{A(y)}\frac{x_B u^h}{u_1}, \qquad \langle \sin\phi_h\rangle_{LU} = \frac{|\vec{P}_{h\perp}|}{Q}\frac{D(y)}{A(y)}\frac{x_B l^h}{u_1},$$

$$\langle \sin\phi_h\rangle_{UL} = -\frac{|\vec{P}_{h\perp}|}{Q}\frac{B(y)}{A(y)}\frac{x_B u_L^h}{u_1}, \qquad \langle \cos\phi_h\rangle_{LL} = -\frac{|\vec{P}_{h\perp}|}{Q}\frac{D(y)}{A(y)}\frac{x_B l_L^h}{u_1},$$

$$\langle \sin\phi_S\rangle_{UT} = -\frac{M}{Q}\frac{B(y)}{A(y)}\frac{x_B u_T}{u_1}, \qquad \langle \cos\phi_S\rangle_{LT} = -\frac{M}{Q}\frac{D(y)}{A(y)}\frac{x_B l_T}{u_1},$$

$$\langle \sin(2\phi_h-\phi_S)\rangle_{UT} = -\frac{\vec{P}_{h\perp}^2}{2MQ}\frac{B(y)}{A(y)}\frac{x_B u_T^h}{u_1}, \qquad \langle \cos(2\phi_h-\phi_S)\rangle_{LT} = -\frac{\vec{P}_{h\perp}^2}{2MQ}\frac{D(y)}{A(y)}\frac{x_B l_T^h}{u_1}.$$

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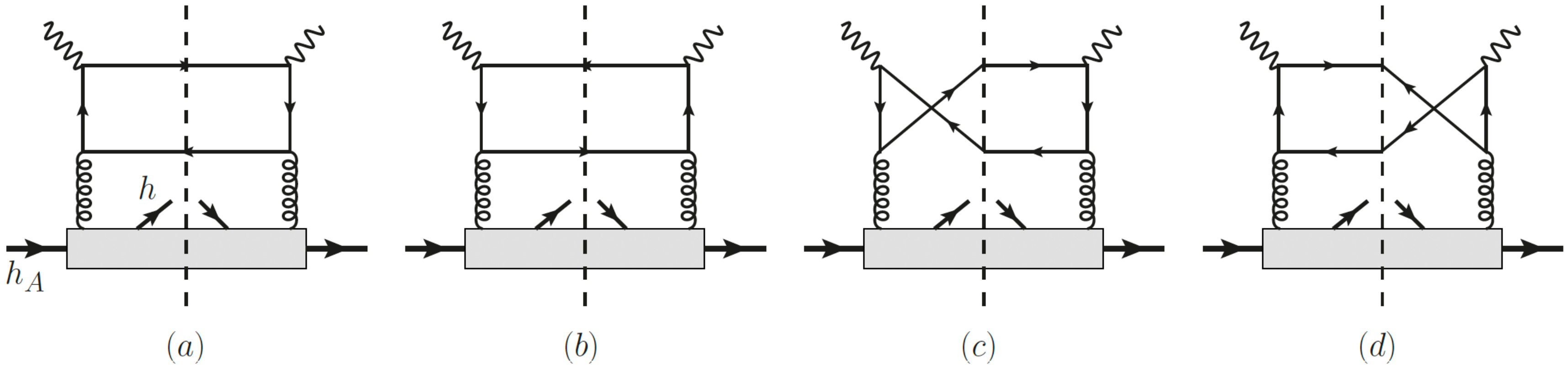
Gluonic contribution



$$W^{\mu\nu}(q,P,S,P_h) = \alpha_s T_F \sum_f e_f^2 \int \frac{dx}{x} \int d\Phi_{k_1 k_2} H^{\mu\nu\alpha\beta}(k_g,k_1,k_2)\mathcal{M}_{G,\alpha\beta}(x,\xi_h,P_{h\perp}),$$

$$\mathcal{M}_G^{\alpha\beta}(x,\xi_h,P_{h\perp}) = \frac{1}{2\xi_h(2\pi)^3}\frac{1}{xP^+}\int \frac{d\lambda}{2\pi} e^{-i\lambda x P^+} \sum_X \langle h_A(P)|(G^{+\alpha}(\lambda n)\mathcal{L}_n^\dagger(\lambda n))^a|Xh(P_h)\rangle$$
$$\times \langle h(P_h)X|(\mathcal{L}_n(0)G^{+\beta}(0))^a|h_A(P)\rangle,$$

Gluonic contribution

Gluon fracture functions

$$\mathcal{M}_G^{\alpha\beta} = -\frac{1}{2-2\epsilon} g_\perp^{\alpha\beta} u_{1g} + \frac{1}{2M^2}\left(P_{h\perp}^\alpha P_{h\perp}^\beta + \frac{1}{2-2\epsilon} g_\perp^{\alpha\beta} P_{h\perp}^2\right) t_{1g}^h + S_L\left[i\frac{\varepsilon_\perp^{\alpha\beta}}{2} l_{1gL} + \frac{\tilde{P}_{h\perp}^{\{\alpha} P_{h\perp}^{\beta\}}}{4M^2} t_{1gL}^h\right]$$
$$+ \frac{g_\perp^{\alpha\beta}}{2-2\epsilon}\frac{P_{h\perp}\cdot\tilde{S}_\perp}{M} u_{1gT}^h + \frac{P_{h\perp}\cdot S_\perp}{M}\left[i\frac{\varepsilon_\perp^{\alpha\beta}}{2} l_{1gT}^h - \frac{\tilde{P}_{h\perp}^{\{\alpha} P_{h\perp}^{\beta\}}}{4M^2} t_{1gT}^{hh}\right] + \frac{\tilde{P}_{h\perp}^{\{\alpha} S_\perp^{\beta\}} + \tilde{S}_\perp^{\{\alpha} P_{h\perp}^{\beta\}}}{8M} t_{1gT}^h + \cdots.$$

The hard parts

$$H_{(a)+(b)}^{\mu\nu\alpha\beta}(k_g,k_1,k_2) = \mathrm{Tr}\left[\not{k}_1\gamma^\nu \frac{(\not{k}_g-\not{k}_2)}{(k_g-k_2)^2}\gamma^\beta \not{k}_2\gamma^\alpha \frac{(\not{k}_g-\not{k}_2)}{(k_g-k_2)^2}\gamma^\mu\right] + (k_1\leftrightarrow k_2),$$

$$H_{(c)+(d)}^{\mu\nu\alpha\beta}(k_g,k_1,k_2) = \mathrm{Tr}\left[\not{k}_1\gamma^\nu \frac{(\not{k}_g-\not{k}_2)}{(k_g-k_2)^2}\gamma^\beta \not{k}_2\gamma^\mu \frac{(\not{k}_1-\not{k}_g)}{(k_1-k_g)^2}\gamma^\alpha\right] + (k_1\leftrightarrow k_2).$$

One-loop contributions for TFR SIDIS

■ Gluonic contribution

$$F_{UU}^{\cos 2\phi_h} = -\frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}^2}{2M^2} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} z^2 t_{1g}^h(x_B/z, \xi_h, P_{h\perp}),$$

$$F_{UL}^{\sin 2\phi_h} = \frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}^2}{2M^2} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} z^2 t_{1gL}^h(x_B/z, \xi_h, P_{h\perp}),$$

$$F_{UT}^{\sin(3\phi_h-\phi_s)} = \frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}^3}{4M^3} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} z^2 t_{1gT}^{hh}(x_B/z, \xi_h, P_{h\perp}),$$

$$F_{UT}^{\sin(\phi_h+\phi_s)} = \frac{\alpha_s T_F}{2\pi}\frac{P_{h\perp}}{2M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} z^2 \left[t_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + \frac{P_{h\perp}^2}{2M^2} t_{1gT}^{hh}(x_B/z, \xi_h, P_{h\perp})\right].$$

Four structure functions generated uniquely by the gluon fracture functions!

One-loop contributions for TFR SIDIS

Quark and gluon contribution

$$F_{UU,L} = \frac{\alpha_s}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \left[4T_F z\bar{z} u_{1g}(x_B/z, \xi_h, P_{h\perp}) + 2C_F z u_1(x_B/z, \xi_h, P_{h\perp}) \right],$$

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = \frac{\alpha_s}{2\pi} \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \left[4T_F z\bar{z} u_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + 2C_F z u_{1T}^h(x_B/z, \xi_h, P_{h\perp}) \right].$$

Twist-4 if at the tree level

$$F_{UU,T} = x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \left[\mathcal{H}_g(z) u_{1g}(x_B/z, \xi_h, P_{h\perp}) + \mathcal{H}_q(z) u_1(x_B/z, \xi_h, P_{h\perp}) \right],$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \left[\mathcal{H}_g(z) u_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + \mathcal{H}_q(z) u_{1T}^h(x_B/z, \xi_h, P_{h\perp}) \right],$$

$$F_{LL} = x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \left[\Delta\mathcal{H}_g(z) l_{1gL}(x_B/z, \xi_h, P_{h\perp}) + \Delta\mathcal{H}_q(z) l_{1L}(x_B/z, \xi_h, P_{h\perp}) \right],$$

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^{1} \frac{dz}{z} \left[\Delta\mathcal{H}_g(z) l_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + \Delta\mathcal{H}_q(z) l_{1T}^h(x_B/z, \xi_h, P_{h\perp}) \right],$$

LO + NLO corrections

$$\mathcal{H}_q(z) = \delta(\bar{z}) + \frac{\alpha_s}{2\pi} \left\{ P_{qq}(z) \ln\frac{Q^2}{\mu^2} + C_F \left[2\left(\frac{\ln\bar{z}}{\bar{z}}\right)_+ - \frac{3}{2}\left(\frac{1}{\bar{z}}\right)_+ - (1+z)\ln\bar{z} - \frac{1+z^2}{\bar{z}}\ln z + 3 - \left(\frac{\pi^2}{3} + \frac{9}{2}\right)\delta(\bar{z}) \right] \right\},$$

$$\mathcal{H}_g(z) = \frac{\alpha_s}{2\pi} \left[P_{qg}(z) \ln\frac{Q^2\bar{z}}{\mu^2 z} - T_F(1-2z)^2 \right],$$

$$\Delta\mathcal{H}_q(z) = \delta(\bar{z}) + \frac{\alpha_s}{2\pi} \left\{ \Delta P_{qq}(z) \ln\frac{Q^2}{\mu^2} + C_F \left[(1+z^2)\left(\frac{\ln\bar{z}}{\bar{z}}\right)_+ - \frac{3}{2}\left(\frac{1}{\bar{z}}\right)_+ - \frac{1+z^2}{\bar{z}}\ln z + 2 + z - \left(\frac{\pi^2}{3} + \frac{9}{2}\right)\delta(\bar{z}) \right] \right\},$$

$$\Delta\mathcal{H}_g(z) = \frac{\alpha_s}{2\pi} \left[\Delta P_{qg}(z) \left(\ln\frac{Q^2\bar{z}}{\mu^2 z} - 1 \right) + 2T_F\bar{z} \right].$$

One-loop contributions for TFR SIDIS

Structure functions	Twist	Order (α_S)
$F_{UU,T}$	2	0
$F_{UU,L}$		1
$F_{UU}^{\cos\phi_h}$	3	
$F_{UU}^{\cos 2\phi_h}$		1
$F_{LU}^{\sin\phi_h}$	3	
$F_{UL}^{\sin\phi_h}$	3	
$F_{UL}^{\sin 2\phi_h}$		1
F_{LL}	2	0
$F_{LL}^{\cos\phi_h}$	3	

Structure functions	Twist	Order (α_S)
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	2	0
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$		1
$F_{UT}^{\sin(\phi_h+\phi_S)}$		1
$F_{UT}^{\sin\phi_S}$	3	
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	3	
$F_{UT}^{\sin(3\phi_h-\phi_S)}$		1
$F_{LT}^{\cos\phi_S}$	3	
$F_{LT}^{\cos(\phi_h-\phi_S)}$	2	0
$F_{LT}^{\cos(2\phi_h-\phi_S)}$	3	

All 18 structure functions are non-zero up to twist-3 or one-loop level.

Contents

Summary

- TFR SIDIS is factorized with fracture functions. TFR is complementary to CFR for describing the SIDIS process and studying the nucleon structure.
- We calculate the TFR SIDIS up to twist-3 at the tree level of pQCD. Structure functions and azimuthal asymmetries are given using the gauge-invariant fracture functions.
- By adding one-loop contributions at twist-2, all 18 structure functions for TFR SIDIS are non-zero.

谢谢！