



Nucleon Energy Correlators for the Color Glass Condensate

Hao-yu Liu (刘昊昱)

(Beijing University of Chemical Technology)

With Xiaohui Liu, Ji-chen Pan, Feng Yuan and Hua Xing Zhu

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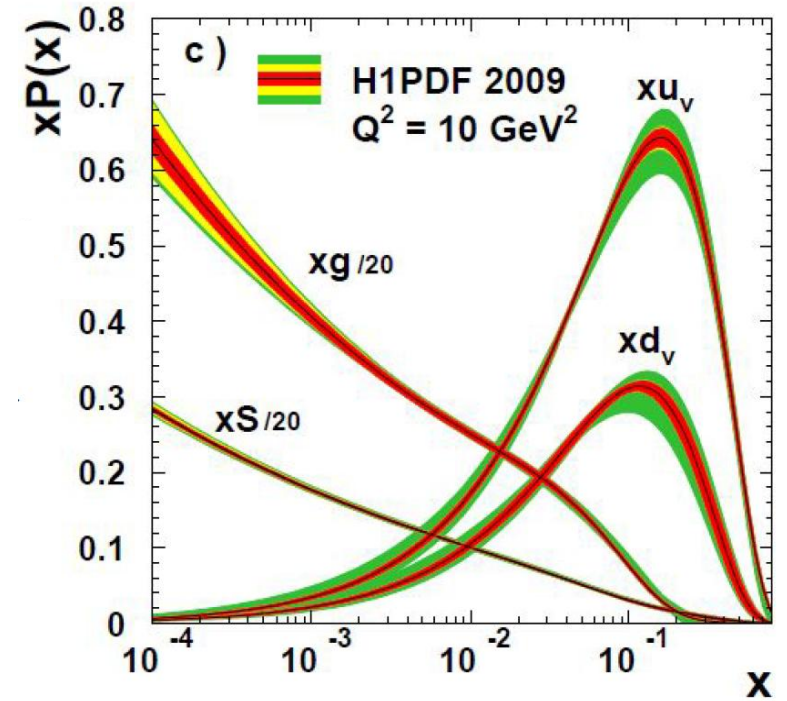
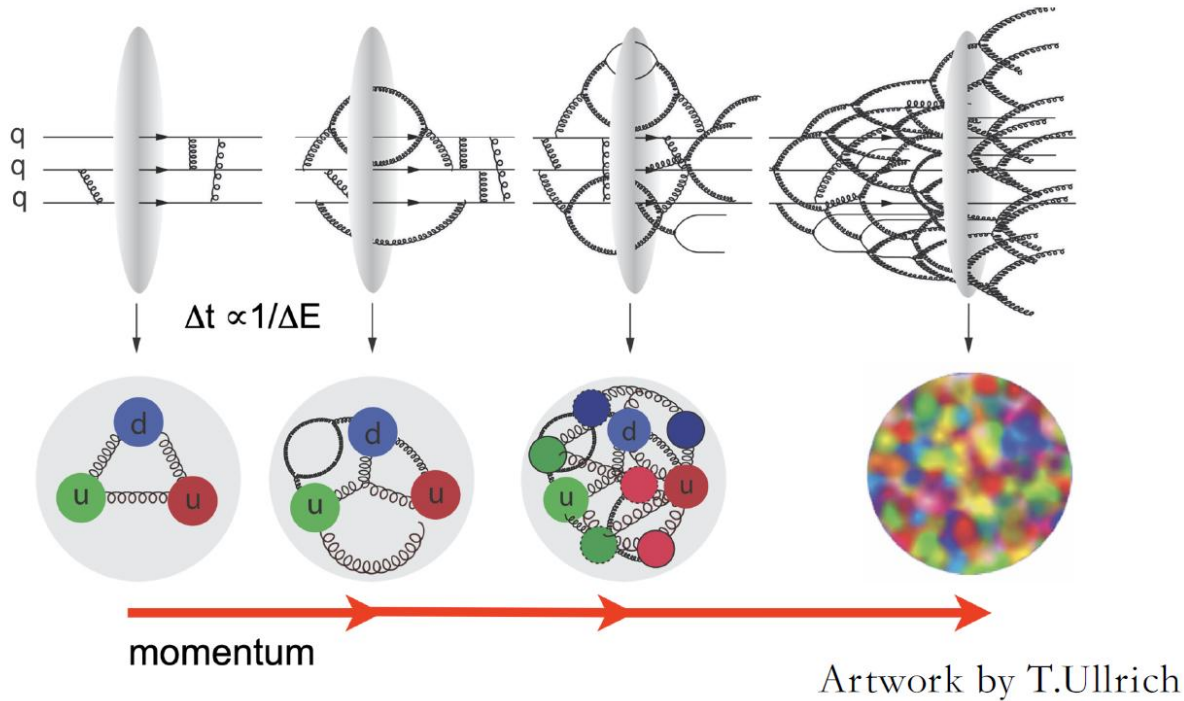
Based on [[HL, X.Liu, J.Pan, F.Yuan, H.Zhu, arXiv:2301.01788](#)]

Email address: lhy1991dbc@126.com

Outline

- Review of CGC effective theory
- Nucleon energy correlator
- Nucleon energy correlator for small x
- Summary

Gluon Saturation



[Gelis, Iancu, Jalilian-Marian, Venugopalan, 2010]

The gluon density increases with Bjorken x decreases



Gluon saturation

splitting \longleftrightarrow recombination

balance

Unitarity

Typical Transverse Momentum

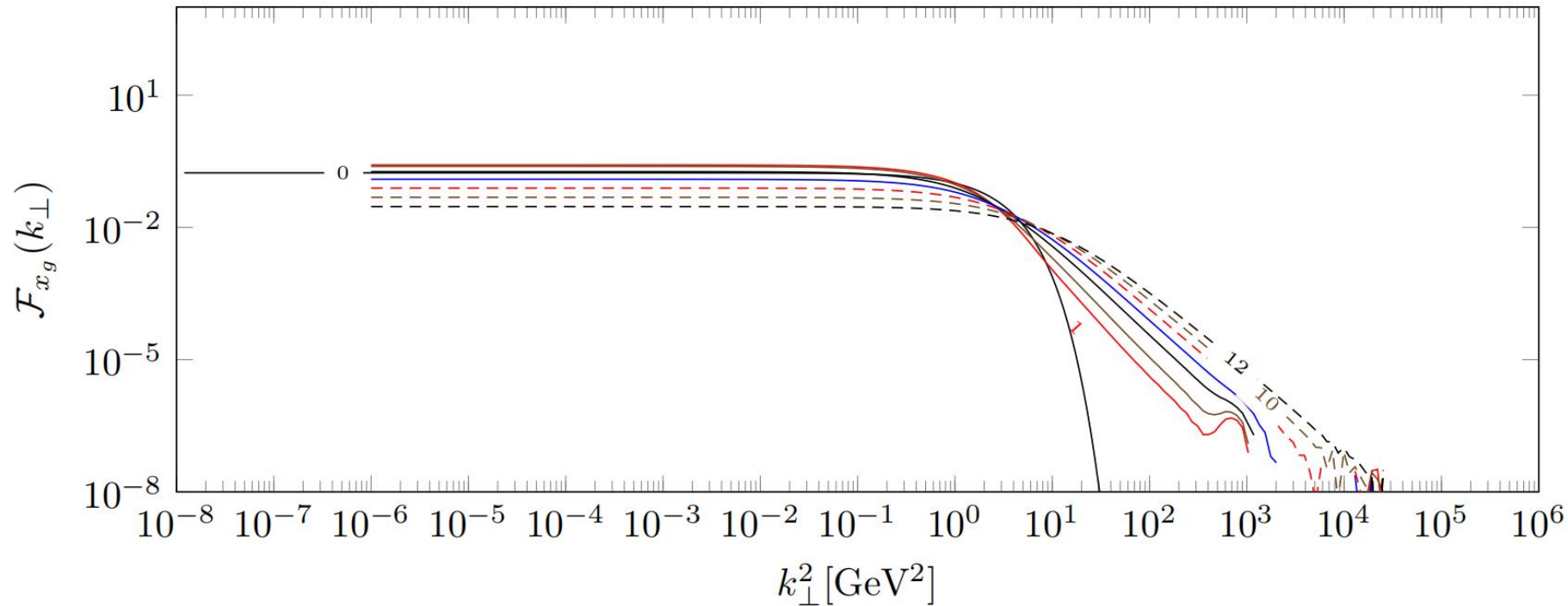
Saturation scale

Dilute \longrightarrow Collinear factorization

Dense \longrightarrow ?

2-4 GeV $Q_s \gg \Lambda_{\text{QCD}}$

Gluon Distribution in CGC



[D. Zaslavsky, 1409.8259]

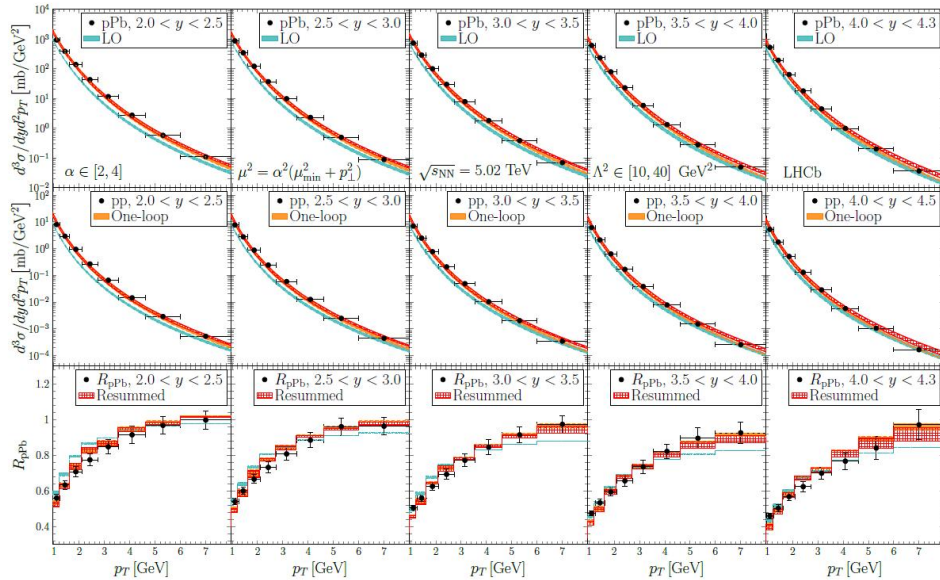
$$\mathcal{F}_{x_g}(\mathbf{k}_\perp) = \int \frac{d^2\mathbf{x}_\perp d^2\mathbf{y}_\perp}{(2\pi)^2} e^{-i\mathbf{k}_\perp \cdot (\mathbf{x}_\perp - \mathbf{y}_\perp)} S_{x_g}^{(2)}(\mathbf{x}_\perp, \mathbf{y}_\perp)$$

Transverse Momentum distribution by **CGC (Color Glass Condensate)**

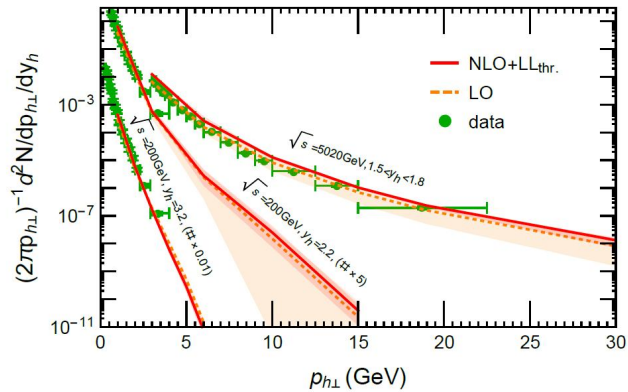
Effective theory of QCD

One of the most appropriate theories for saturation

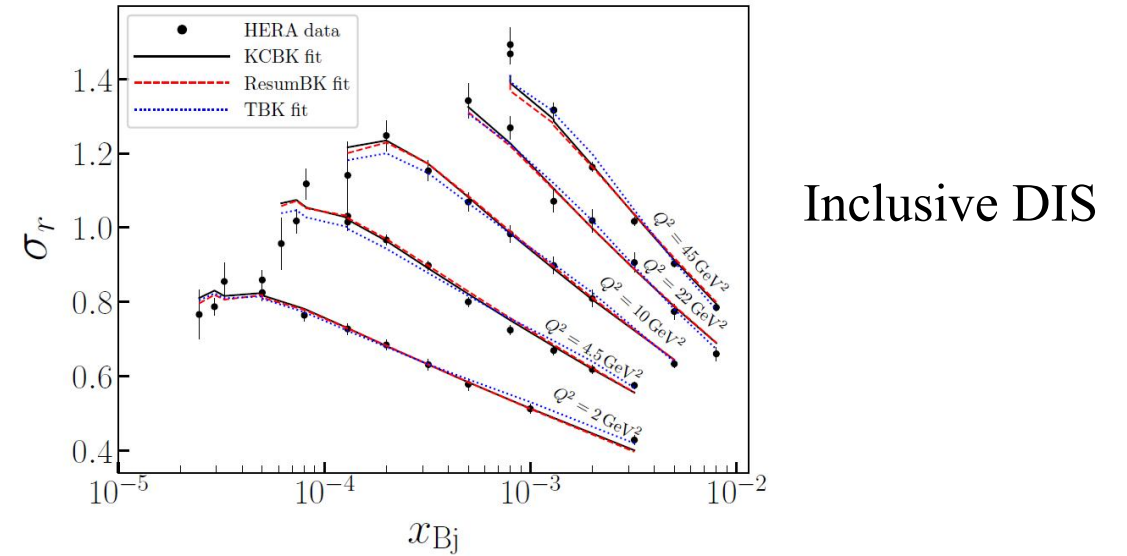
Searching for deterministic evidence of saturation



Single hadron production in pA



- [G.Chirilli, B.Xiao, F.Yuan, 2012]
- [Y.Shi, L.Wang, S.Wei, B.Xiao, 2021]
- [Iancu, Mueller, Triantafyllopoulos, 2016]
- [HL, Y.Ma, K.Chao, 2019]
- [HL, Z.Kang, X.Liu, 2020]



[B. Ducloué, H. Hänninen, T. Lappi, Y. Zhu, 2017]

[G. Beuf, H. Hänninen, T. Lappi, H. Mäntysaari, 2020]

Search for saturation in pA collisions and DIS,
many hints, **no solid evidence**

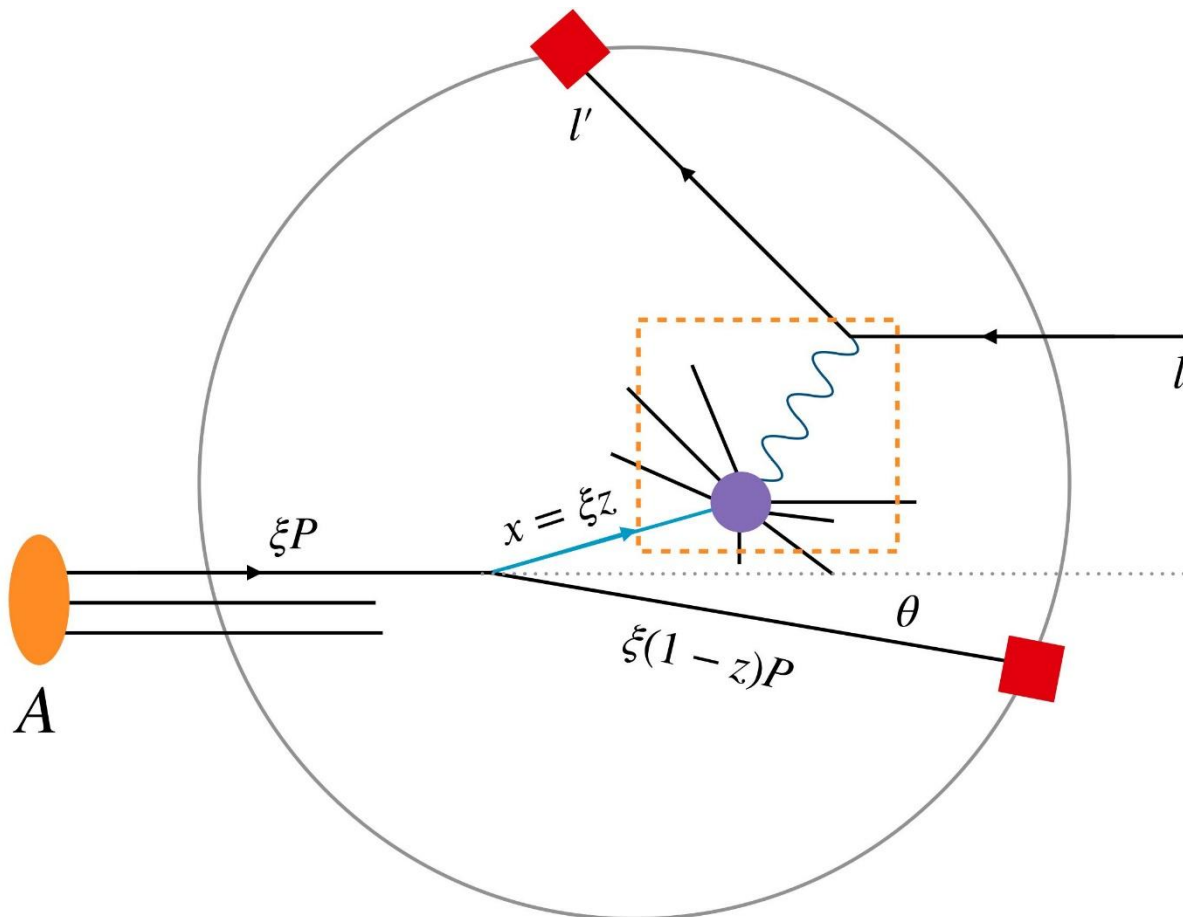
One of the main objects for EIC

Any new prob?

The measurement of Nucleon energy correlator

[X.Liu, H.Zhu, 2022] [H.Cao, X.Liu, H.Zhu, 2023]

Similar to normal DIS



One detector for lepton $x_B = \frac{-q^2}{2P \cdot q}$ $Q^2 = -q^2$

Another detector for the energy deposits at θ

$$\sum_i E_i$$

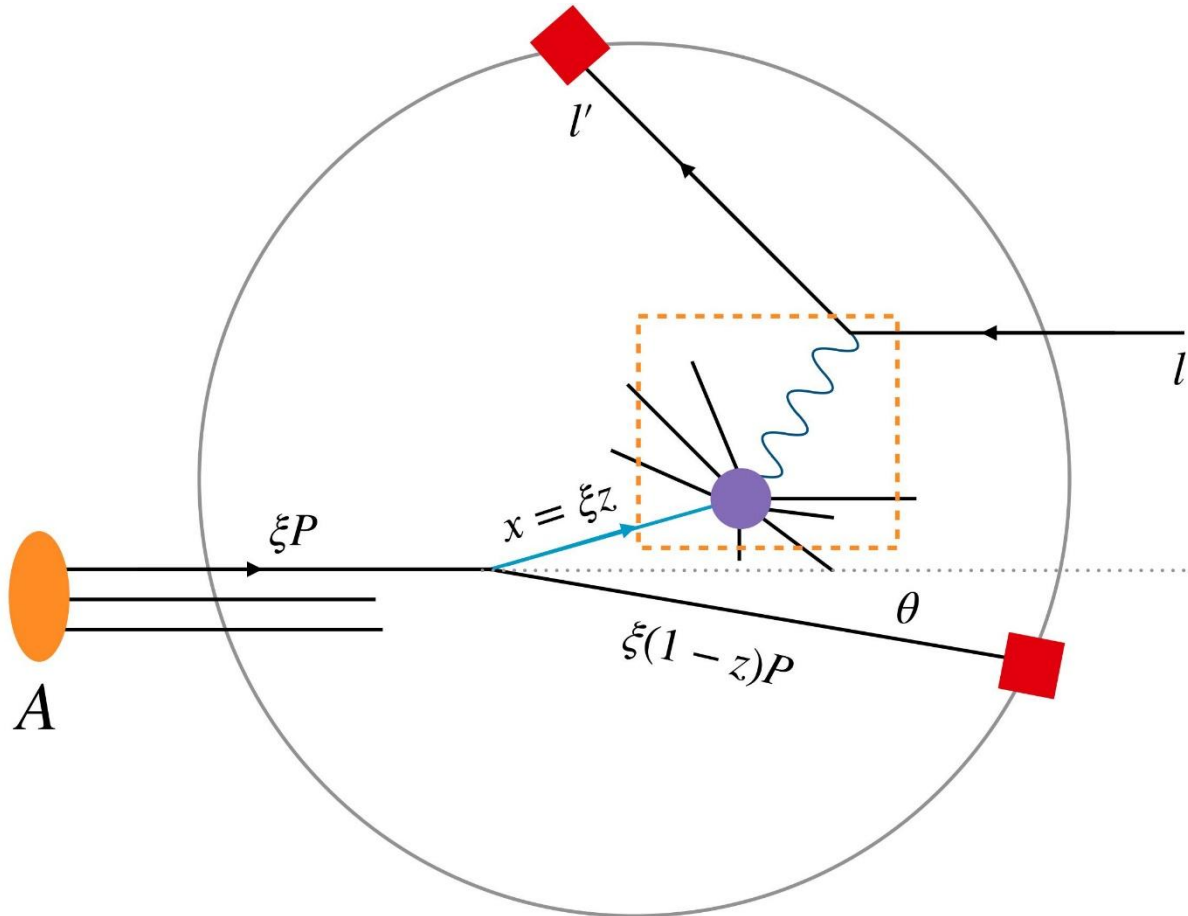
The weighted cross section

$$\Sigma(Q^2, x_B, \theta) = \sum_i \int d\sigma(x_B, Q^2, p_i) \frac{E_i}{E_A} \delta(\theta^2 - \theta_i^2)$$

The energy weight suppresses the soft contributions

No Sudakov logs suppression

The factorization of the weighted cross section



Factorization when $\Lambda_{\text{QCD}} \ll \theta Q \ll Q$

$$\Sigma(Q^2, x_B, \theta) = \int \frac{dx}{x} \hat{\sigma}_{i,\text{DIS}}\left(\frac{x_B}{x}, Q\right) f_{i,\text{EEC}}(x, \theta)$$

Nucleon energy correlator

inclusive partonic DIS cross section

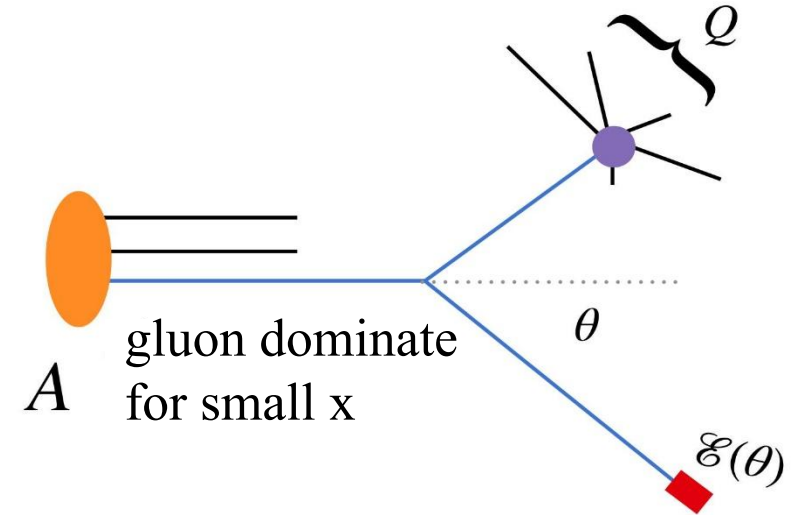
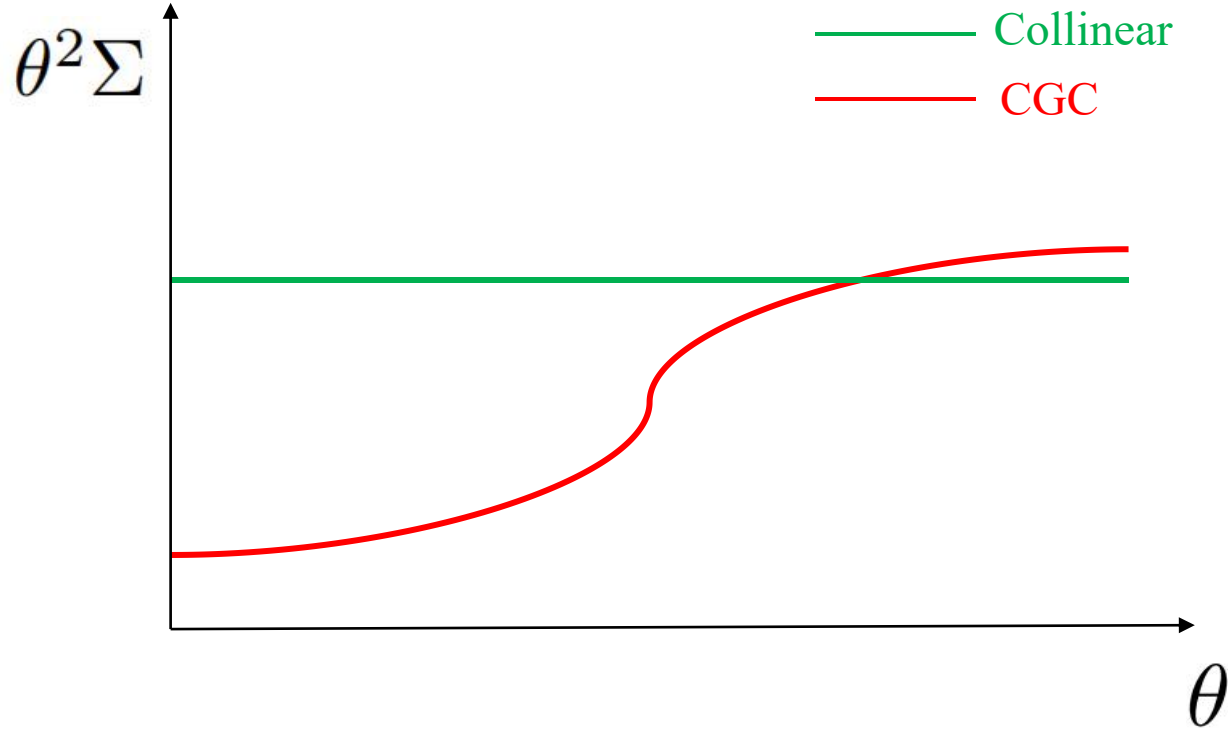
Suppose collinear factorization **still holds** for small x

$$f_{q,\text{EEC}}(x, \theta) = \frac{\alpha_s T_R}{2\pi\theta^2} \int_x^1 \frac{d\xi}{\xi} (1-\xi)(\xi^2 + (1-\xi)^2) \left[\frac{x}{\xi} f_g\left(\frac{x}{\xi}\right) \right]$$

collinear PDF

$\frac{1}{\theta^2}$ scaling $\theta^2 \Sigma$ insensitive to θ

Naïve analysis



$q_t \sim \Lambda_{\text{QCD}}$ for collinear



Splitting function



$\frac{1}{\theta^2}$ scaling

insensitive

Leading order

No Sudakov suppression

Higher order

$q_t \sim Q_s$



hardly hits the detector for $\theta Q \ll Q_s$



suppression

Similar to the collinear case for $\theta Q \gg Q_s$



insensitive

$\theta^2 \Sigma$

Nucleon energy correlator by CGC

$F_{g,x_B}(\vec{g}_t)$ gluon transverse momentum distribution in CGC

$$f_{q,\text{EEC}}(x_B, \theta) = \frac{N_C S_\perp}{8\pi^4} \int d^2 \vec{g}_t \int_{\xi_{\text{cut}}}^1 \frac{d\xi}{\xi} \mathcal{A}_{qg}(\xi, \theta, \vec{g}_t) F_{g,x_B}(\vec{g}_t) \quad \theta^2 \Sigma$$

$$\mathcal{A}_{qg}(\xi, \theta, \vec{g}_t) = \frac{1}{\theta^2} (1 - \xi) \vec{k}_t^2 (\vec{k}_t - \vec{g}_t)^2 \quad \mathcal{A}_{qg}(\xi, \theta, \vec{g}_t) \xrightarrow{\theta Q \ll Q_s} \theta^0 \quad \text{Suppression}$$

$$\times \left| \frac{\vec{k}_t}{\xi \vec{k}_t^2 + (1 - \xi) (\vec{k}_t - \vec{g}_t)^2} - \frac{\vec{k}_t - \vec{g}_t}{(\vec{k}_t - \vec{g}_t)^2} \right|^2 \quad \mathcal{A}_{qg}(\xi, \theta, \vec{g}_t) \xrightarrow{\theta Q \gg Q_s} \frac{1}{\theta^2} \quad \text{insensitive to } \theta$$

$$g_t \sim Q_s \quad k_t = \frac{1-\xi}{\xi} \frac{Q}{2} \theta$$

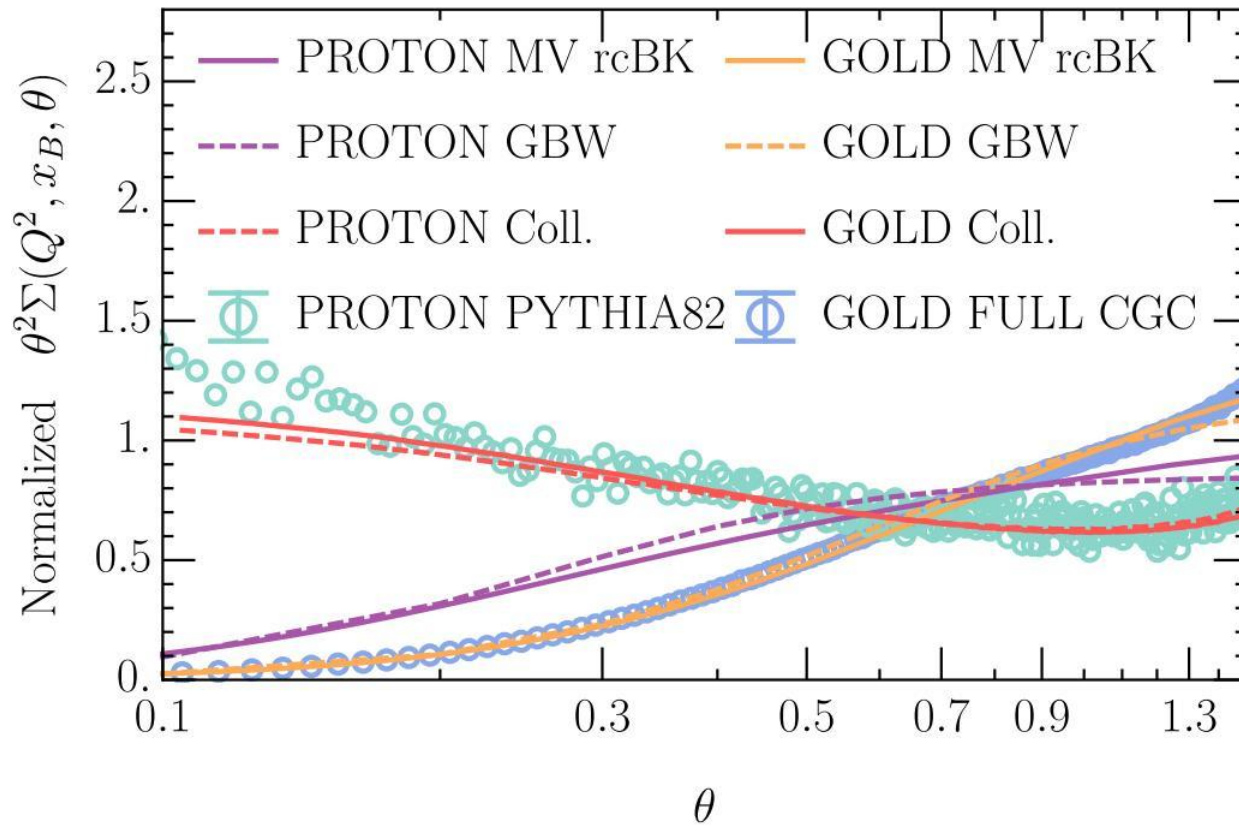
Expansion under different approximations

Different scaling of θ

Consistent with the naïve argument

Comparing CGC and collinear result

$$x_B = 3 \times 10^{-3}, Q^2 = 25 \text{ GeV}^2, \sqrt{s} = 105 \text{ GeV}$$



CGG result **is suppressed** for small θ

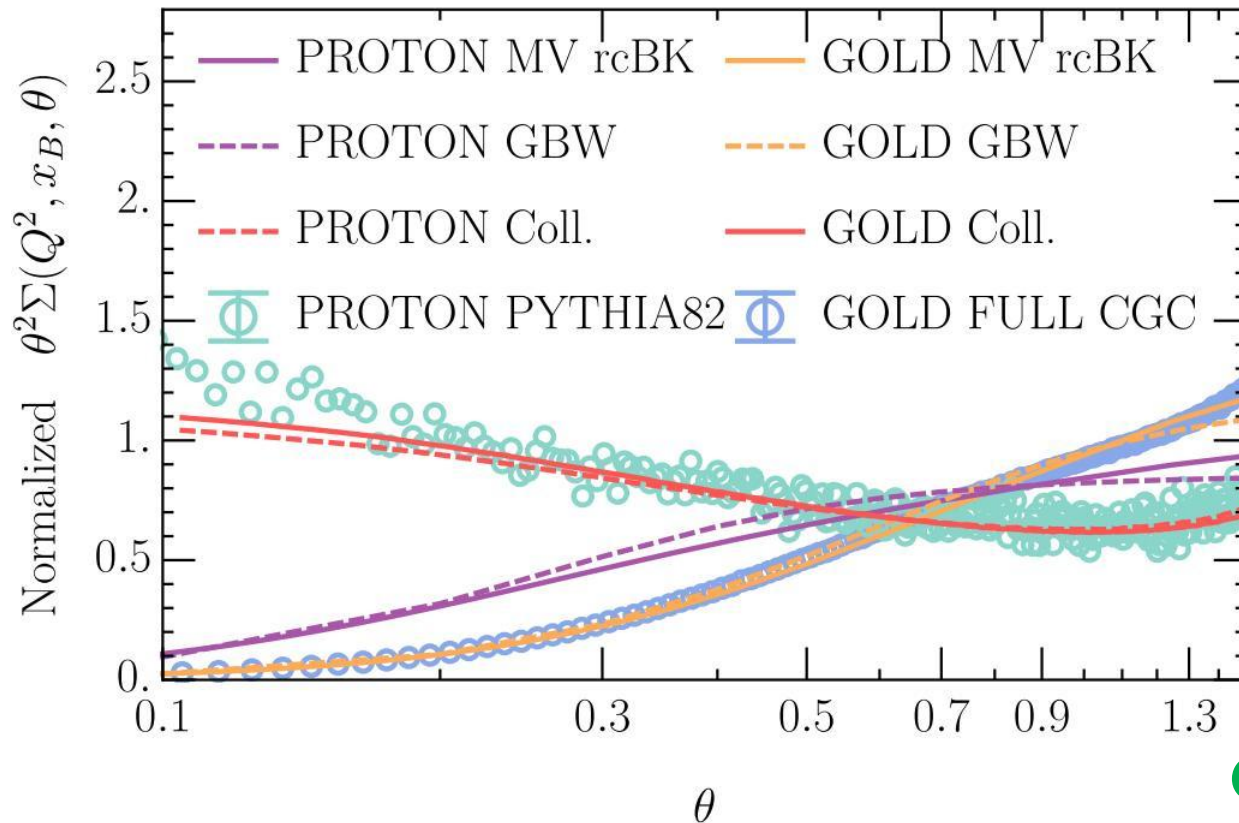
Collinear factorization **can't give this suppression** because of the **absence of Sudakov factor**

NEEC is **a clean probe** for small-x phenomena

Collinear and CGC prediction for HERA kinematics

Comparing CGC and collinear result

$$x_B = 3 \times 10^{-3}, Q^2 = 25 \text{ GeV}^2, \sqrt{s} = 105 \text{ GeV}$$



The spectrum becomes flat for $\theta Q \gg Q_s$

We can estimate Q_s by the tuning point

For the proton

$$\theta \sim 0.15 - 0.2 \quad Q_s \sim \theta Q \sim 0.75 - 1.0 \text{ GeV}$$

For the gold nucleus

$$\theta \sim 0.4 - 0.5 \quad Q_s \sim \theta Q \sim 2 - 2.5 \text{ GeV}$$

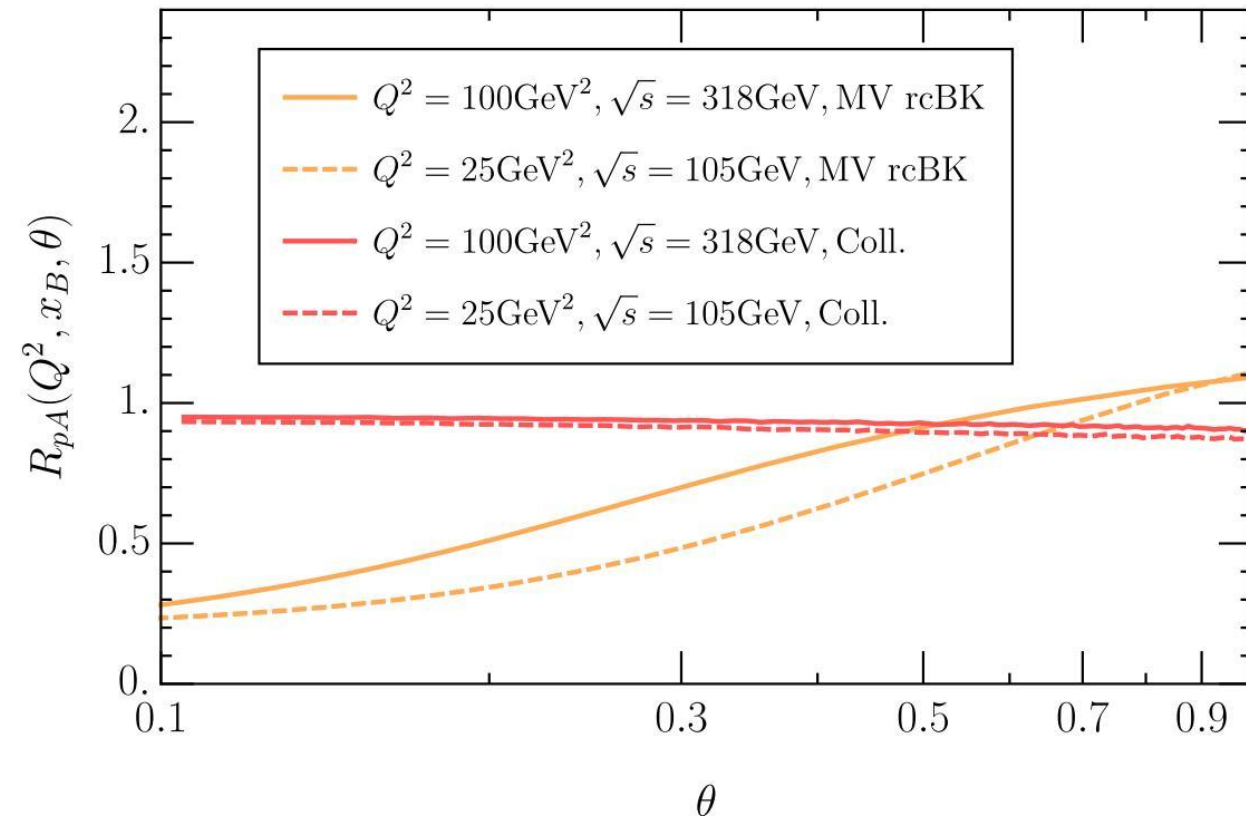
Consistent with the input Q_s parameter

A probe for the onset of gluon saturation

Nuclear Modification Factor

$$R_{pA} = \frac{A^{-1} \Sigma_A(Q^2, x_B, \theta)}{\Sigma_p(Q^2, x_B, \theta)}$$

Take ratios to reduce the systematics



Q_s is larger for large nuclear

$\theta^2 \Sigma(Q^2, x_B, \theta)$ is suppressed comparing to proton

There is no suppression for the collinear case

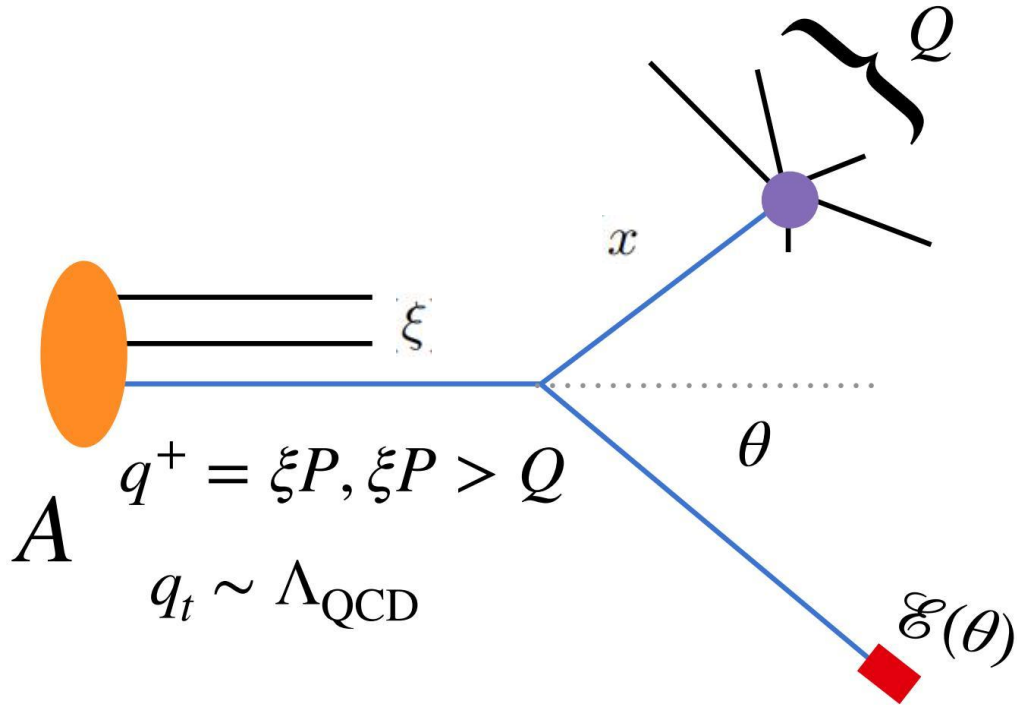
Summary and Outlook

- Nucleon energy correlator is **a new probe** of **the gluon saturation** phenomenon in DIS **at the future electron-ion colliders**. The θ -shape of the weighted cross section **behaves differently** in the collinear factorization theorem and the CGC.
- The difference is due to the **intrinsic transverse momentum** of order Q_s induced by the non-linear small- x dynamics. The nucleon energy correlator offer **a great opportunity to pin down the onset of the gluon saturation phenomenon** in eA collisions.
- This probe is **fully inclusive** and does not involve fragmentation functions or jet clustering, it is **both theoretically and experimentally clean**.
- **Extensions to other observables**, such as measuring nucleon energy correlator in **prompt photon production**, can also be carried out at the LHC.

Thank You!

The Back up

Definition of Nucleon energy correlator



x the momentum fraction that
 initiates a scattering process

Quark contribution

$$\begin{aligned}
 f_{q,\text{EEC}}(x, \theta) &= \int \frac{dy^-}{4\pi E_A} e^{-ixP y^-} \\
 &\quad \times \gamma^+ \langle A | \bar{\psi}(y^-) \mathcal{L}^\dagger(y^-) \mathcal{E}(\theta) \mathcal{L}(0) \psi(0) | A \rangle \\
 \mathcal{E}(\theta) | X \rangle &= \sum_{i \in X} E_i \delta(\theta_i^2 - \theta^2) | X \rangle
 \end{aligned}$$

When $\theta E_A \gg \Lambda_{\text{QCD}}$, $f_{\text{EEC}}(x, \theta)$ can be further factorized

$$f_{i,\text{EEC}}(x, \theta) = \int \frac{d\xi}{\xi} I_{ij} \left(\frac{x}{\xi}, \theta \right) [\xi f_{j/A}(\xi)]$$

$f_{j/A}(\xi)$ collinear PDF

$I_{ij} \left(\frac{x}{\xi}, \theta \right)$ matching coefficient

Determined by the splitting function

Nucleon energy correlator in collinear factorization

By SCET

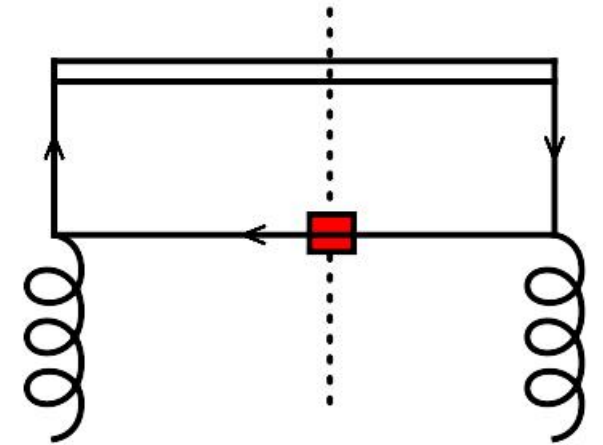
$$\begin{aligned}
 f_{q,\text{EEC}}(N, \theta) &= \int dz z^{N-1} \frac{1}{2} \int \frac{dq^+ d^2 q_t}{q^+ (2\pi)^3} \frac{q^+}{P^+} \delta((1-z)P^+ - q^+ - p_X^+) \delta(\theta_q^2 - \theta^2) \\
 &\quad \times g_s^2 \text{Tr}[T_a T_a] \frac{1}{2} \text{Tr}[\gamma_t^\mu \gamma_{t,\mu} \not{q} \gamma^+] \left(\frac{-\vec{q}_t^2}{(q^+)^2} + \frac{-\vec{p}_t^2}{(p^+)^2} \right) \left(\frac{p^+}{p^2} \right)^2 \\
 &\quad \times \frac{1}{2} \frac{1}{N_C^2 - 1} \sum_X \langle P | A_{t,a,\mu} | X \rangle \langle X | A_t^{a,\mu} | P \rangle \int d\xi P^+ \delta((1-\xi)P^+ - p_X^+)
 \end{aligned}$$

$$l_t \sim \Lambda_{\text{QCD}} \quad p^2 = (l - q)^2 = -2l \cdot q = -\frac{l^+}{q^+} \vec{q}_t^2$$

$$q_t = q_z \sin \theta = \frac{q^+}{2} \theta_q$$

Origin of $\frac{1}{\theta^2}$ scaling

$$f_{q,\text{EEC}}(N, \theta) = \int dx x^{N-1} (1-x) \frac{\alpha_s}{2\pi} T_R \left\{ \frac{1}{\theta^2} [x^2 + (1-x)^2] \right\} \int d\xi \xi^N f_{g/P}(\xi)$$



Evolution of Nucleon energy correlator

By the factorization

$$\Sigma(Q^2, x_B, \theta) = \int \frac{dx}{x} \hat{\sigma}_{i,\text{DIS}} \left(\frac{x_B}{x}, Q \right) f_{i,\text{EEC}}(x, \theta)$$

$$d\Sigma/d \ln \mu = 0$$

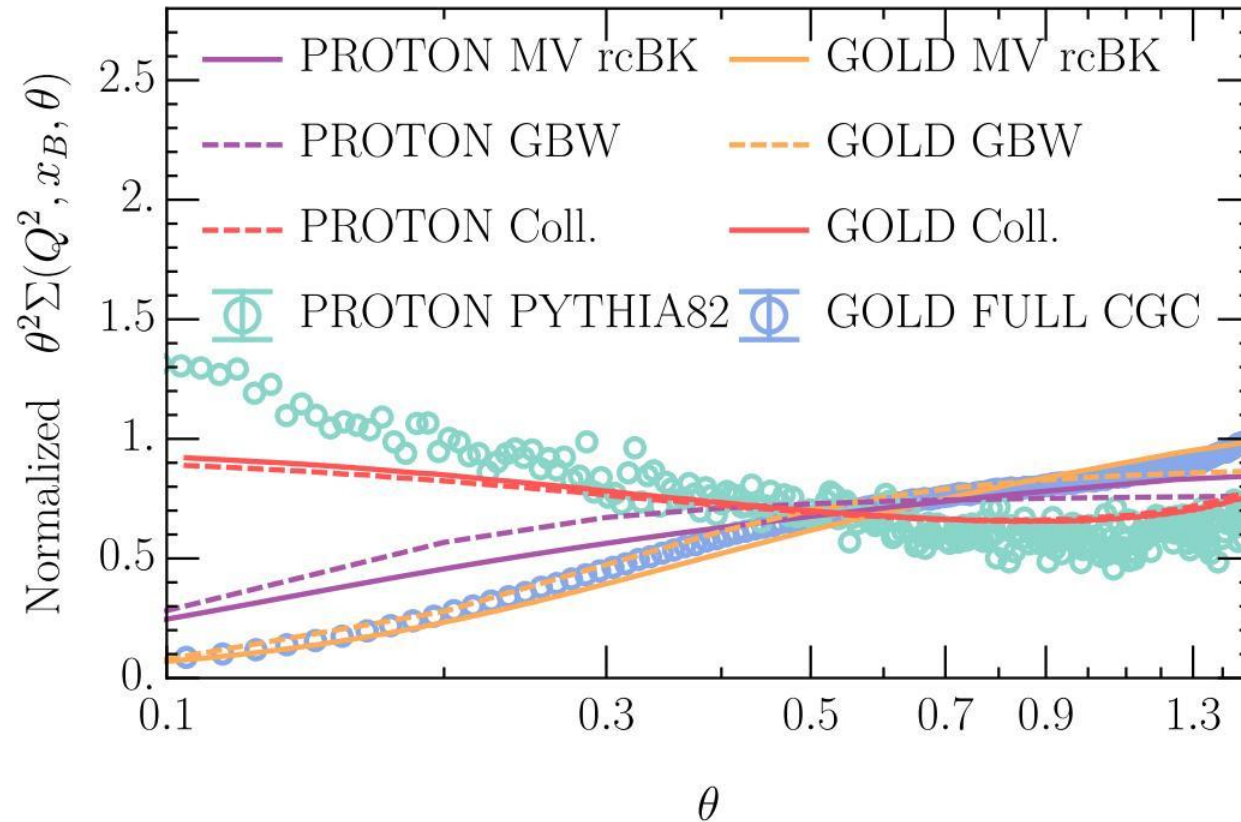
$$d\hat{\sigma}_{i,\text{DIS}}/d \ln \mu = -P_{ji} \otimes \hat{\sigma}_{j,\text{DIS}}$$



$$\frac{df_{i,\text{EEC}}(x, \theta)}{d \ln \mu} = P_{ij} \otimes f_{j,\text{EEC}}$$

Comparing CGC and collinear result

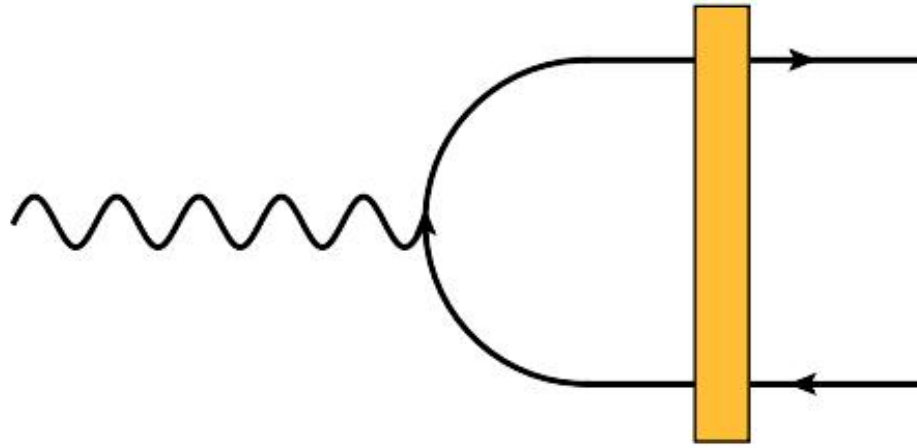
$$x_B = 3 \times 10^{-3}, Q^2 = 100 \text{ GeV}^2, \sqrt{s} = 318 \text{ GeV}$$



EIC kinematics

Full CGC

$\theta^2 \Sigma$ by direct CGC calculation



$$\Sigma_t^{\gamma^*}(Q^2, x_B, \theta) = \sum_q \frac{2N_c \alpha^2 e_q^2}{\pi^2 x_B Q^2} S_\perp \int dz d^2 \vec{k}_t \frac{d^2 \vec{l}_t}{(2\pi)^2} F_{g, x_B}(\vec{l}_t) [z^2 + (1-z)^2] \left| \frac{\vec{k}_t}{\vec{k}_t^2 + \Delta^2} - \frac{\vec{k}_t - \vec{l}_t}{(\vec{k}_t - \vec{l}_t)^2 + \Delta^2} \right|^2$$

$$\times \left[\frac{\vec{k}_t^2 + (1-z)^2 Q^2}{(1-z)Q} \frac{x_B}{Q} \right] \frac{1}{2\theta} \delta \left(\theta - \tan^{-1} \frac{2k_t(1-z)Q}{k_t^2 - (1-z)^2 Q^2} \right) \theta \left(\frac{\vec{k}_t^2 + (1-z)^2 Q^2}{(1-z)Q} < x_B \frac{Q}{x_B} \right)$$