Spin hydrodynamics of Dirac fermions consistent with entropy principle

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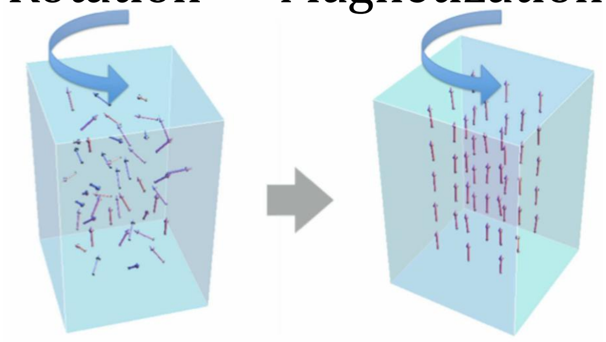
OUTLINE

Polarizations relate to spin

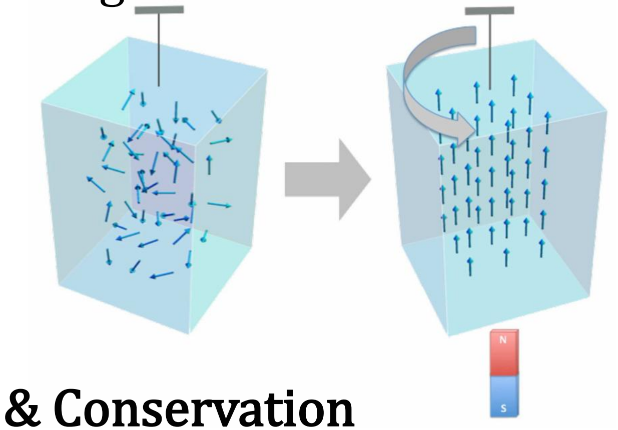
Barnett Effect

Rotation ⇒ Magnetization



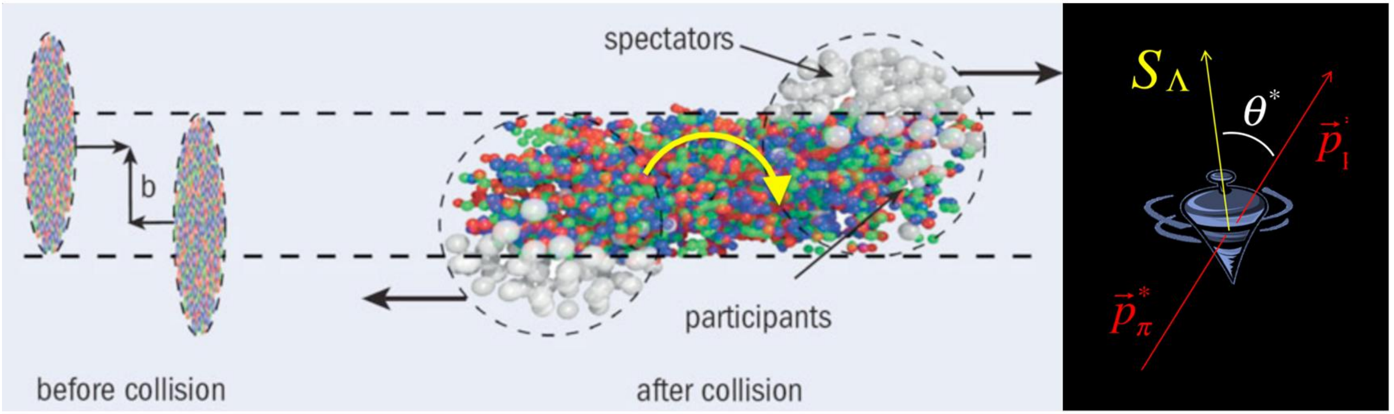
Einstein de-Haas Effect

Magnetization ⇒ Rotation



Spin-Orbit coupling & Conservation

OAM ⟹ Polarizations of Λ hyperons and vector mesons



Liang, Wang, PRL 2005; Betz, Gyulassy, Torrieri, PRC 2007; Becatini, Piccinini, Rizzo, PRC 2008

Global Polarization STAR , Nature 2017

UrQMD, AMPT, PICR hydro, CKT+collisions

Karpenko, Becattini, EPJC 2017; Li, Pang, Wang, Xia PRC 2017

Xie, Wang, Csernai, PRC 2017; Sun, Ko, PRC 2017

Shi, Li, Liao, PLB 2018; Wei, Deng, Huang, PRC 2019

Fu, Xu, Huang, Song, PRC 2021; S. Ryu, V. Jupic, C. Shen, PRC 2021

Y.X. Wu, C. Yi, G.Y. Qin, S. Pu, PRC 2022

Theory=Experiment

Local polarization STAR, PRL 2019

Sign problem **Theory≠Experiment**

Xia, Li, Tang, Wang, PRC 2018; Becattini , Karpenko, PRL 2018

Shear-induced polarization

B. Fu, S. Liu, L. Pang, H. Song, Y. Yin, PRL 2021

S. Liu, Y. Yin, JHEP 2021;

Becattini, Buzzegoli, Inghirami, Karpenko, PRL 2021

Becattini, Buzzegoli, Palermo, PLB 2021;

S. Lin, Z. Wang, JHEP 2022

Eos, Initial condition sensitivity

PRC 104, 064901 2021; PRC 105, 064909 2022

Theoretic approaches to spin hydrodynamics

Microscopic approach

QKT=Boltzmann equation+spin

Pro: systematic treatment of spin transport

Cons: assumes medium weakly coupled, not well-justified for QGP

Macroscopic approach

Phenomenology=hydrodynamics+spin

Pro: consistent with entropy principle

Cons: ambiguity due to pseudo-gauge transformations, frame choices

Quantum statistical density operators

Becattini, Tinti, X. Sheng, et al., (2008-2023); J. Hu, PRD 2021

Quantum kinetic theory(QKT)

Florkowski, Friman, Kumar, Jaiswal, Ryblewski, Speranza, et al., 2017-2021

Weickgenannt, Speranza, Sheng, Wang, Rischke, PRL, PRD 2019-2021

S. Shi, Gale, Jeon, PRC 2021; S. Lin, Z. Wang, JHEP 2022

Entropy current analysis(phenomenology)

Hattori, Hongo, Huang, Matsuo, Taya, PLB 2019; Fukushima, S. Pu, PLB 2021

Hongo, Huang, Kaminski, Stephanov, Yee, JHEP 2021; Li, Stephanov, Yee, PRL 2021

D. She, A. Huang, D. Hou, J. Liao, Sci.B 2022; D.-L. Wang, S. Fang, S. Pu, PRD 2021

Daher, Das, Florkowski, Ryblewski, et al., PRD, PRC 2023

Effective Lagrangian Montenegro, Tinti, Torrieri, PRD(2017-2020)

Holography Gallegos, Gürsoy, JHEP 2020; Garbiso, Kaminski, JHEP 2020

Equilibrium partition functions Gallegos, Yarom, SPP 2021

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Spin hydrodynamics in the regime $\tau \leq \Gamma_r$

Conserved currents $\Theta^{\mu\nu}, J^{\mu\nu\alpha}$

$\partial_\mu \Theta^{\mu\nu} = 0 \quad \Longleftarrow$ spacetime translations symmetry

$\partial_\mu J^{\mu\nu\alpha} = 0 \quad \Longleftarrow$ Lorentz symmetry(rotation $\nu\alpha$=ij & boost $\nu\alpha$=0i)

$$J^{\mu\nu\alpha} = \Sigma^{\mu\nu\alpha} + (x^\nu \Theta^{\mu\alpha} - x^\alpha \Theta^{\mu\nu}) \quad \Longrightarrow \quad \partial_\mu \Sigma^{\mu\nu\alpha} = \Theta^{\alpha\nu} - \Theta^{\nu\alpha}$$

spin orbital

$$\mathcal{S}^{\mu\nu} = \mathcal{R}^{\mu\nu} + \mathcal{B}^{\mu\nu} = \epsilon^{\mu\nu\alpha\sigma}\mathcal{R}_\alpha u_\sigma + 2u^{[\mu}\mathcal{B}^{\nu]}$$

First law of thermodynamics in local rest frame near local equilibrium

$\varepsilon + p = Ts + \frac{1}{2}\omega_{\mu\nu}\mathcal{S}^{\mu\nu}$ spin density $\mathcal{S}^{\mu\nu} = -u_\lambda \Sigma^{\lambda\mu\nu}$ $\omega_{\mu\nu} - \varpi_{\mu\nu} \sim O(\partial)$ near equilibrium

$\omega_{\mu\nu} = \varpi_{\mu\nu}$ in equilibrium

Second law of thermodynamics

entropy current $s^\mu = p\beta^\mu - \Theta^{\mu\nu}\beta_\nu - \frac{1}{2}\omega_{\nu\alpha}\Sigma^{\mu\nu\alpha} + G^\mu$

entropy principle $\partial_\mu s^\mu \geq 0$

pseudo-gauge — Gauthier-Villars, Paris, 1960; Becattini, Tinti, PRD 2011

entropy-gauge — Becattini, Daher, X. Sheng, PLB 2024

Entropy current analysis of Dirac fermions

Spin tensor

$$\Sigma^{\mu\nu\alpha} = \epsilon^{\mu\nu\alpha\sigma}(\mathcal{R}_\sigma + \widetilde{\mathcal{R}}u_\sigma) \quad \text{totally anti-symmetric}$$

Hongo, Huang, Kaminski, Stephanova, Yee, JHEP 2021

$$\Sigma'^{\mu\nu\alpha} = u^\mu \mathcal{S}^{\nu\alpha} + \widetilde{\Sigma}'^{\mu\nu\alpha} = u^\mu\left(\epsilon^{\nu\alpha\rho\sigma}\mathcal{R}_\rho u_\sigma + 2u^{[\nu}\mathcal{B}^{\alpha]}\right) + \widetilde{\Sigma}'^{\mu\nu\alpha}$$

Hattori, Hongo, Huang, Matsuo, Taya, PLB 2019

Entropy current analysis

$$\varepsilon + p = Ts + \frac{1}{2}\omega_{\mu\nu}\mathcal{R}^{\mu\nu}$$

$$s^\mu = su^\mu + \tilde{s}^\mu + s_\delta^\mu \qquad \tilde{s}^\mu = -\beta_\nu\widetilde{\Theta}^{\mu\nu} - \frac{1}{2}\beta\omega_{\nu\alpha}\widetilde{\Sigma}^{\mu\nu\alpha}$$

$$\Theta^{\mu\nu} = \varepsilon u^\mu u^\nu + p\Delta^{\mu\nu} + \widetilde{\Theta}^{\mu\nu} + \Theta_\delta^{\mu\nu} \qquad \widetilde{\mathcal{R}} \to \widetilde{\mathcal{R}} + \mathcal{R}_\delta$$

$$\partial_\mu s^\mu = (\beta\omega_{\mu\nu} - \partial_\mu\beta_\nu)\widetilde{\Theta}^{\mu\nu} - \frac{1}{2}\partial_\mu(\beta\omega_{\nu\alpha})\epsilon^{\mu\nu\alpha\sigma}\widetilde{\mathcal{R}}u_\sigma$$

$$+ \partial_\alpha(\mathcal{R}^{\alpha\mu}u^\nu)\beta\omega_{\mu\nu}$$

$$+ \partial_\mu s_\delta^\mu + \partial_\mu\Theta_\delta^{\mu\nu}\beta_\nu - \frac{1}{2}\partial_\mu(\epsilon^{\mu\nu\alpha\sigma}\mathcal{R}_\delta u_\sigma)\beta\omega_{\nu\alpha}$$

$$\varepsilon + p = Ts + \frac{1}{2}\omega_{\mu\nu}\mathcal{S}^{\mu\nu}$$

$$s'^\mu = su^\mu + \tilde{s}'^\mu \qquad \tilde{s}'^\mu = -\beta_\nu\widetilde{\Theta}'^{\mu\nu} - \frac{1}{2}\beta\omega_{\nu\alpha}\widetilde{\Sigma}'^{\mu\nu\alpha}$$

$$\Theta'^{\mu\nu} = \varepsilon u^\mu u^\nu + p\Delta^{\mu\nu} + \widetilde{\Theta}'^{\mu\nu}$$

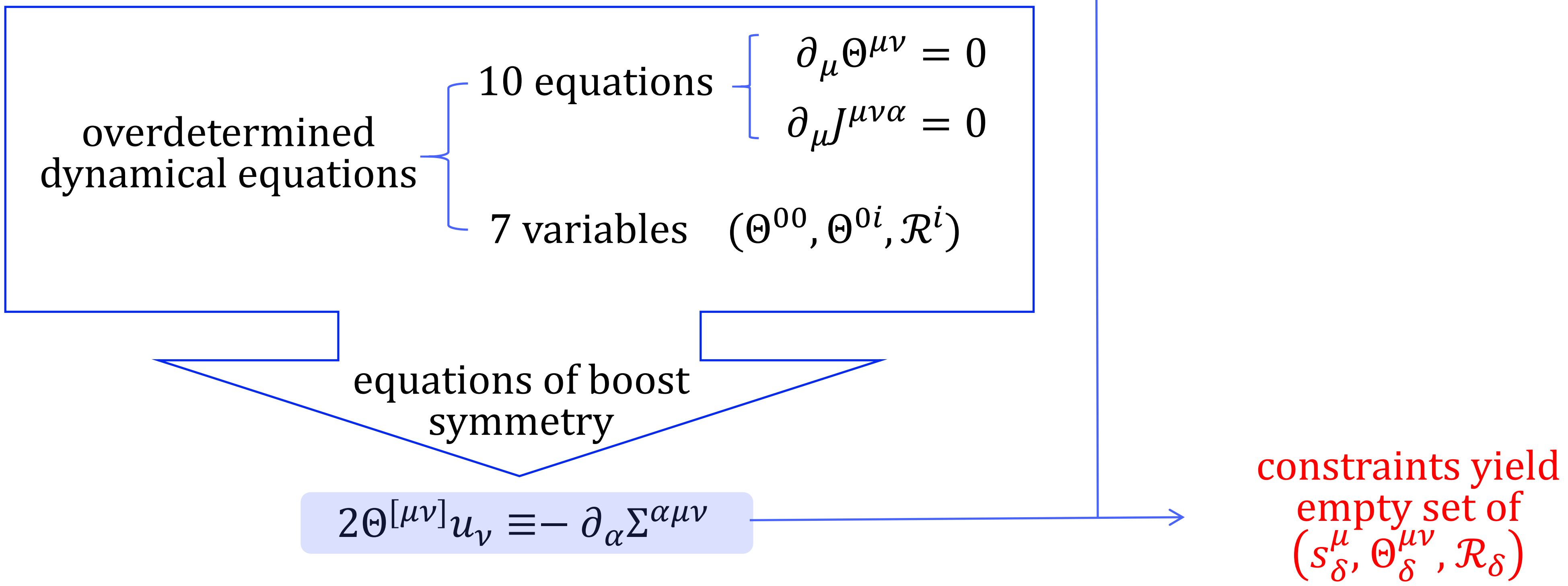
$$\partial_\mu s'^\mu = (\beta\omega_{\mu\nu} - \partial_\mu\beta_\nu)\widetilde{\Theta}'^{\mu\nu} - \frac{1}{2}\partial_\mu(\beta\omega_{\nu\alpha})\widetilde{\Sigma}'^{\mu\nu\alpha}$$

Entropy principle issue in the absence of $\mathcal{B}$

Constraints from nondissipative transports with general $\omega_{\mu\nu}$

$$\partial_\alpha(\mathcal{R}^{\alpha\mu}u^\nu)\beta\omega_{\mu\nu} + \partial_\mu s^\mu_\delta + \partial_\mu \Theta^{\mu\nu}_\delta \beta_\nu - \frac{1}{2}\partial_\mu(\epsilon^{\mu\nu\alpha\sigma}\mathcal{R}_\delta u_\sigma)\beta\omega_{\nu\alpha} = 0$$

Constraints from the absence of $\mathcal{B}$



Pseudo-gauge solution in the presence of $\mathcal{B}$

Pseudo-gauge solution

$$\Theta'^{\mu\nu}, s'^{\mu}, \Sigma'^{\mu\nu\alpha} = u^{\mu}\mathcal{S}^{\nu\alpha} + \widetilde{\Sigma}'^{\mu\nu\alpha}$$

Hattori, Hongo, Huang, Matsuo, Taya, PLB 2019

$$\Theta_{\delta}^{\mu\nu} =- \partial_{\alpha}(\mathcal{S}^{\alpha\mu}u^{\nu}) - \frac{1}{2}\epsilon^{\mu\nu\alpha\sigma}\partial_{\alpha}(\mathcal{R}_{\delta}u_{\sigma}),\ s_{\delta}^{\mu} = 0$$

Daher, Das, Florkowski, Ryblewski, PRC 2023

$\Theta'^{\mu\nu}, s'^{\mu}, \Sigma'^{\mu\nu\alpha}$ → pseudo-gauge transformation → $\Theta^{\mu\nu}, s^{\mu}, \Sigma^{\mu\nu\alpha}$ totally anti-symmetric

similar to nondissipative transports in conventional hydrodynamics

↓

$\Theta'^{\mu\nu}, s'^{\mu}, \Sigma'^{\mu\nu\alpha}$ → pseudo-gauge transformation → $\Theta^{\mu\nu} = \Theta^{\nu\mu}, s^{\mu}, \Sigma^{\mu\nu\alpha} = 0$

Li, Stephanov, Yee, PRL 2021

Linear mode analysis: Dirac fermions

- One pair of sound modes: $\omega_{\text{sound}}(\boldsymbol{k}) = \pm c_s|\boldsymbol{k}| - \frac{i}{2}\gamma_{||}\boldsymbol{k}^2 \mp c_s^3\kappa_s' \frac{k^3}{\Gamma_b} + O(\boldsymbol{k}^3)$
- One longitudinal spin-boost mode: $\omega_{\text{spin,b},||} = -i\Gamma_b - ic_s^2\kappa_s'\boldsymbol{k}^2 + O(\boldsymbol{k}^2)$
- One longitudinal spin-rotation mode: $\omega_{\text{spin,r},||} = -i\Gamma_r + O(\boldsymbol{k}^2)$
- Two transverse spin-boost modes: $\omega_{\text{spin,b},\perp} = -i\Gamma_b + O(\boldsymbol{k}^2)$
- Two shear modes: $\omega_{\text{shear}}(\boldsymbol{k}) = -i\gamma_\perp\boldsymbol{k}^2 + O(\boldsymbol{k}^3)$
- Two transverse spin-rotation modes: $\omega_{\text{spin,r},\perp} = -i\Gamma_r - i\gamma_s\boldsymbol{k}^2 + O(\boldsymbol{k}^2)$

Linear mode analysis: general spin particles

- One pair of sound modes: $\omega_{\text{sound}}(\boldsymbol{k}) = \pm c_s|\boldsymbol{k}| - \frac{i}{2}\gamma_{||}\boldsymbol{k}^2 \mp 2c_s^3\kappa_s'\frac{k^3}{\Gamma_b} + O(\boldsymbol{k}^3)$
- One longitudinal spin-boost mode: $\omega_{\text{spin,b,}||} = -i\Gamma_b - 3ic_s^2\kappa_s'\boldsymbol{k}^2 + O(\boldsymbol{k}^2)$
- One longitudinal spin-rotation mode: $\omega_{\text{spin,r,}||} = -i\Gamma_r + O(\boldsymbol{k}^2)$
- Two transverse spin-boost modes: $\omega_{\text{spin,b,}\perp} = -i\Gamma_b + O(\boldsymbol{k}^2)$
- Two shear modes: $\omega_{\text{shear}}(\boldsymbol{k}) = -i\gamma_\perp\boldsymbol{k}^2 + O(\boldsymbol{k}^3)$
- Two transverse spin-rotation modes: $\omega_{\text{spin,r,}\perp} = -i\Gamma_r - i\gamma_s\boldsymbol{k}^2 + O(\boldsymbol{k}^2)$

Summary&Outlook

Summary

Spin hydro of Dirac fermions need boost variables in general

Dispersion relations for spin hydro of Dirac fermions & general spin particles

Outlook

Causality and stability analysis with nondissipative transports

Possible solution with $\omega_{\mu\nu} \to r_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} r^{\rho} u^{\sigma}$?

Thanks for your attention!

Spin polarization

Global Polarization

AMPT, UrQMD, PICR hydro, CKT+collisions...

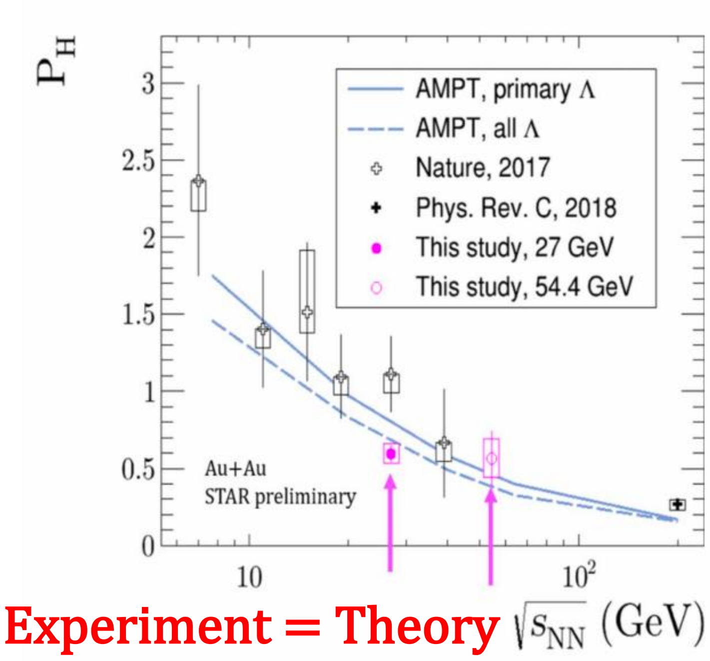
Karpenko, Becattini, EPJC 2017; Li, Pang, Wang, Xia PRC 2017

Xie, Wang, Csernai, PRC 2017; Sun, Ko, PRC 2017

Shi, Li, Liao, PLB 2018; Wei, Deng, Huang, PRC 2019

Fu, Xu, Huang, Song, PRC 2021; S. Ryu, V. Jupic, C. Shen, PRC 2021

Y.X. Wu, C. Yi, G.Y. Qin, S. Pu, PRC 2022...

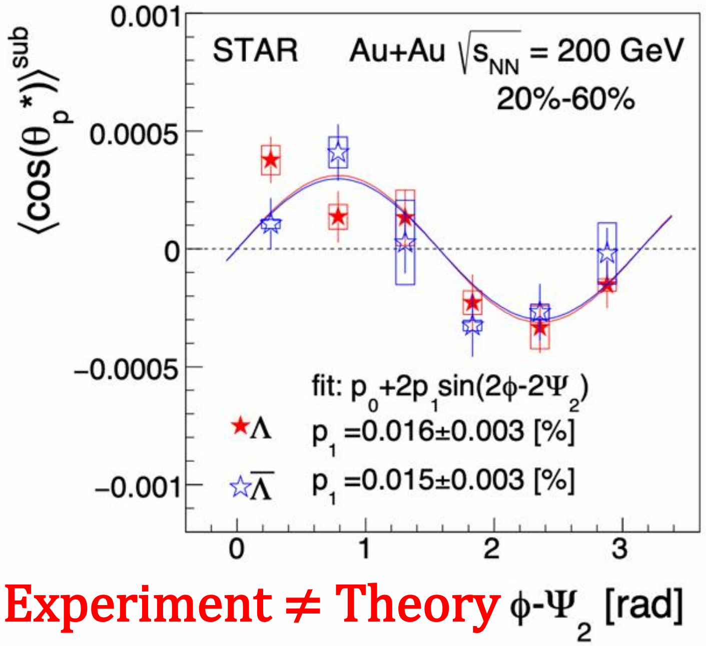


Experiment = Theory

Local polarization

Sign problem

PRL123, 132301 2019



Experiment ≠ Theory

Shear-induced polarization, Eos, Initial condition sensitivity...

PRL 127, 142301 2021 JHEP 07, 188 2021;

PRL 127, 272302 2021 PLB 820,136519 2021;

PRC 104, 064901 2021; PRC 105, 064909 2022...