



# Polarized TMD FFs

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2024/08/15

Y.Gao, K.B.Chen, **YKS**, S.Y.We, ArXiv: 2403.06133

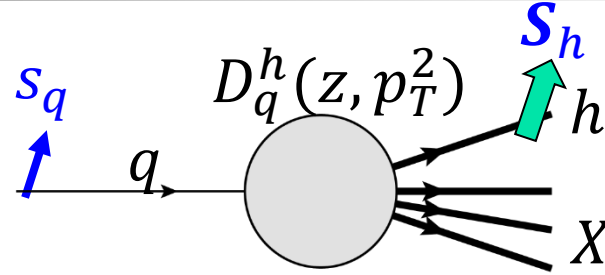
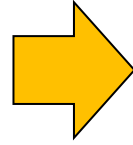
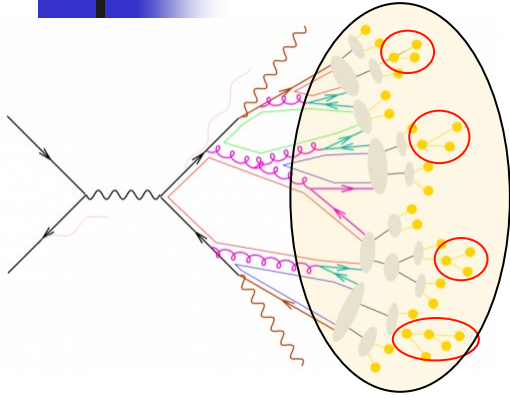
Y.L.Pan, K.B.Chen, **YKS**, S.Y.We, PLB 850 (2024) 138509

K.B.Chen, Z.T.Liang, **YKS**, S.Y.We, PRD 105 (2022) 034027

K.B.Chen, Z.T.Liang, Y.L.Pan, **YKS**, S.Y.We, PLB 816 (2021) 136217

- I. Introduction
- II. Transverse  $\Lambda$  polarization in  $e^+e^-$ , ep/eA and hadronic collisions ( $D_{1T}^\perp$ )
- III. Longitudinal and transverse  $\Lambda$  polarization in  $e^+e^-$ , ep/eA collisions ( $H_{1T}$ ,  $H_{1T}^\perp$ ,  $H_{1L}^\perp$ )
- IV. Weak decay contributions to FFs  $\tilde{D}_{1L}$ ,  $\tilde{G}_1$
- V. Conclusion and outlook

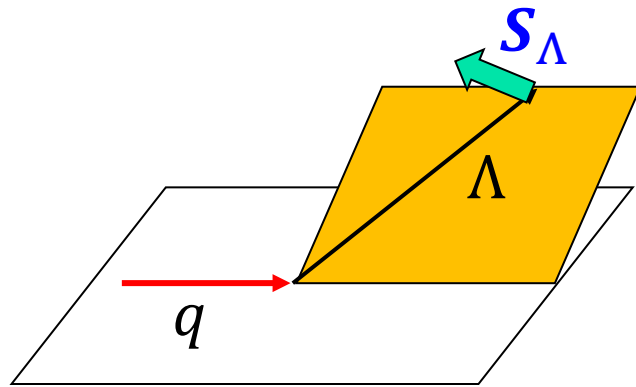
# Introduction



TMD handbook  
2304.03302

Polarized transverse-momentum-dependent fragmentation functions (**Polarized TMD FFs**)

- For unpolarized reactions, without consider W/Z-exchange, **quarks are unpolarized**



$$D_{1T}^\perp(z, p_T^2)$$

$$S_\Lambda \cdot (k \times p_\Lambda)$$

Leading Quark TMDFFs



		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons	L	$D_1 = \text{circle with red dot}$ Unpolarized		$H_1^\perp = \text{circle with red dot} - \text{circle with red dot}$ Collins
	T	$D_{1T}^\perp = \text{circle with red dot} - \text{circle with red dot}$ Polarizing FF	$G_1 = \text{circle with red arrow} - \text{circle with red arrow}$ Helicity	$H_1 = \text{circle with red arrow} - \text{circle with red arrow}$ Transversity $H_{1T}^\perp = \text{circle with red arrow} - \text{circle with red arrow}$

# Belle data and parametrizations

➤ Belle collaboration PRL 122 (2019) 042001

1. Inclusive process in thrust frame

$$e^+e^- \rightarrow \Lambda(\bar{\Lambda})X$$

2. Semi-inclusive process

$$e^+e^- \rightarrow \Lambda(\bar{\Lambda})hX, \quad h = \pi^\pm, K^\pm$$

➤  $P_\Lambda$  for  $\Lambda\pi^+$  and  $\Lambda\pi^-$  are of opposite sign with  $0.2 < z_\Lambda < 0.4$

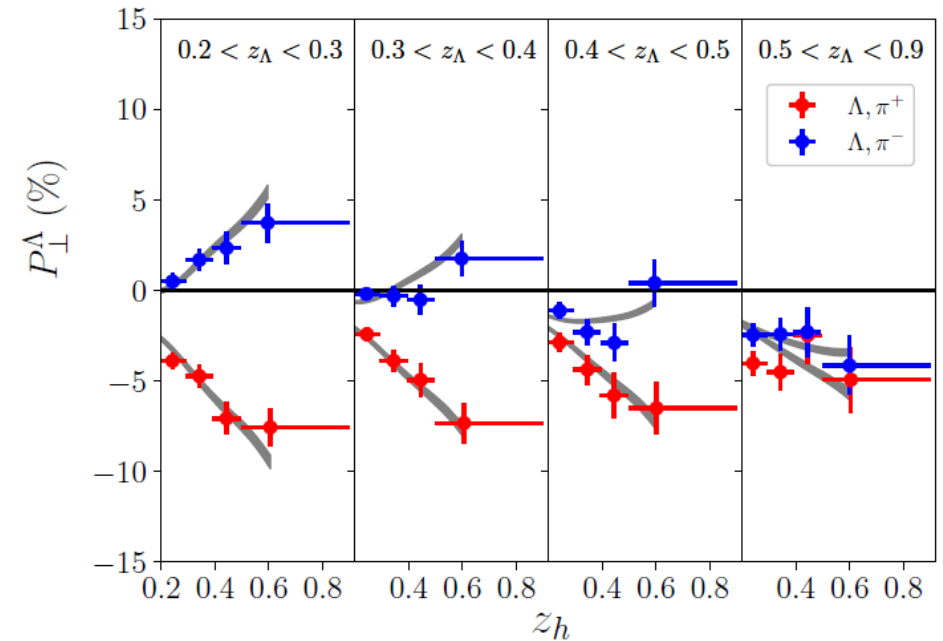
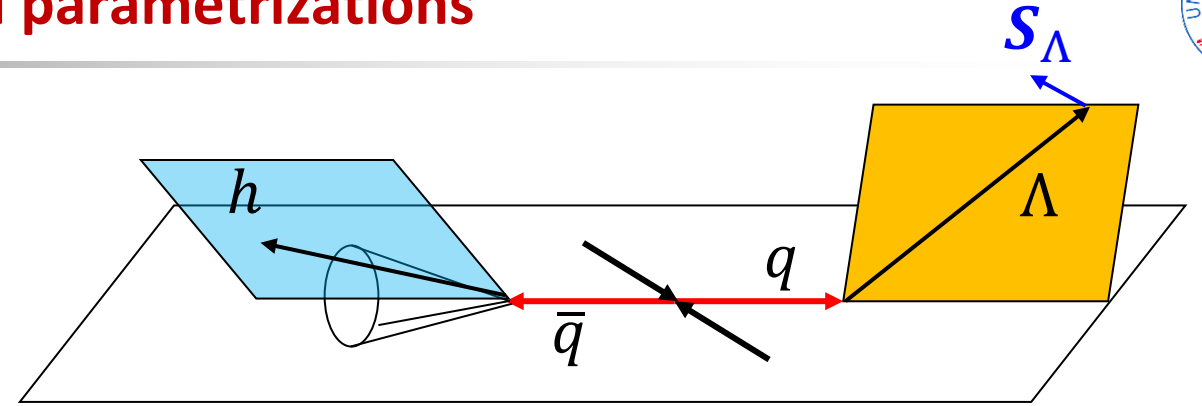
$$e^+e^- \rightarrow \Lambda(uds)\pi^+(u\bar{d})X, \quad e^+e^- \rightarrow \Lambda(uds)\pi^-(d\bar{u})X$$

$$P_\Lambda \propto \sum_q e_q^2 D_{1T,q}^{\perp\Lambda} \Rightarrow D_{1T,u}^{\perp\Lambda} \sim -D_{1T,d}^{\perp\Lambda} ???$$

➤ Parametrizations with  $D_{1T,u}^{\perp\Lambda} \neq D_{1T,d}^{\perp\Lambda}$

U.D'Alesio, F.Murgia, M.Zaccheddu (DMZ), PRD 102 (2020) 05400

D.Callos, Z.B.Kang, J.Terry (CKT), PRD 102 (2020) 096007



CKT

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# Transverse $\Lambda$ polarization in $e^+e^-$ -annihilations ( $D_{1T}^\perp$ )

➤ However, all q's carry same color charges, and

(1)  $m_u \sim m_d \sim$  several MeV

(2)  $\Lambda$  is a isospin singlet with  $I = 0$

Isospin symmetry should apply to  $D_q^\Lambda$ , i.e.,  $D_u^\Lambda = D_d^\Lambda$

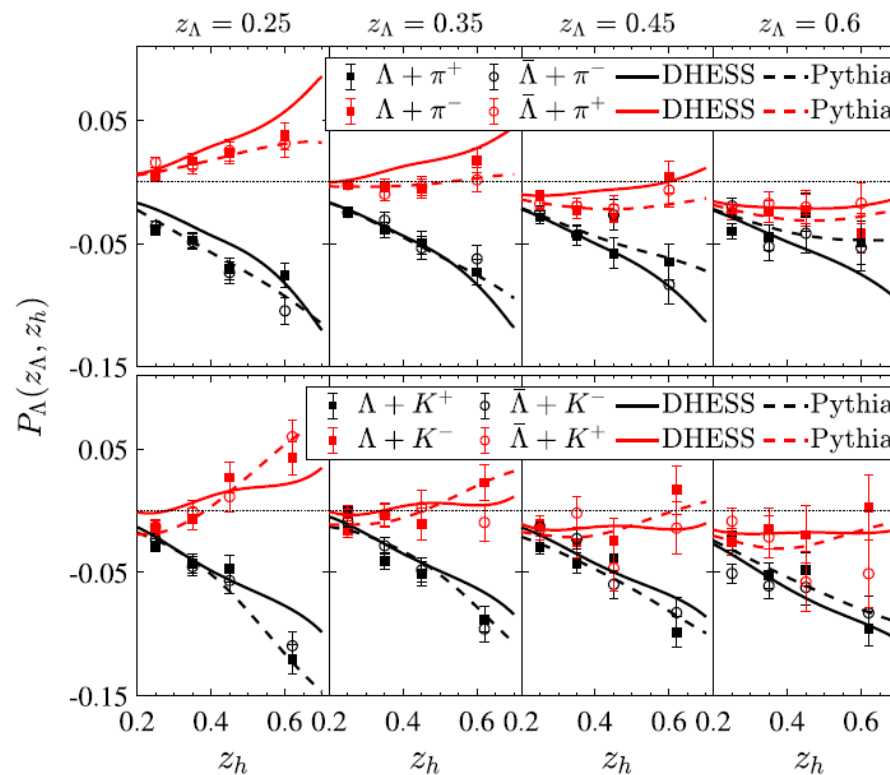
➤ Based on an **isospin symmetric** formalism, we fit the Belle data well using **CLPSW** parametrizations.

K.B.Chen, Z.T.Liang, Y.L.Pan, YKS, S.Y.We, PLB 816 (2021) 136217

$$D_{1Tu}^\perp = D_{1Td}^\perp,$$

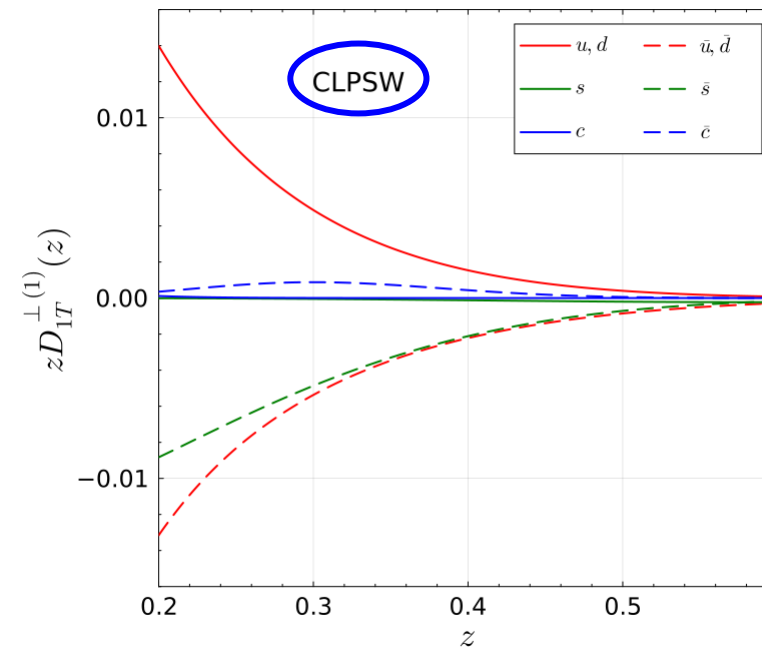
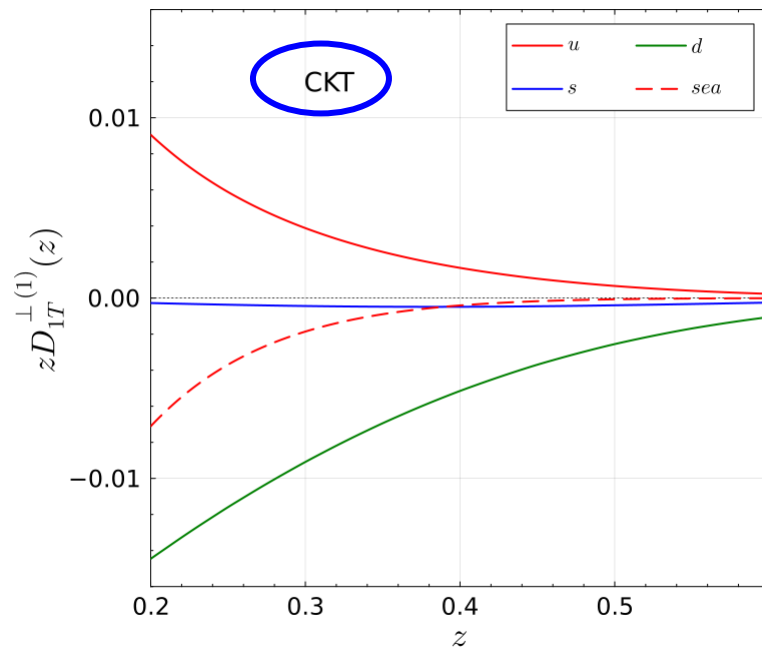
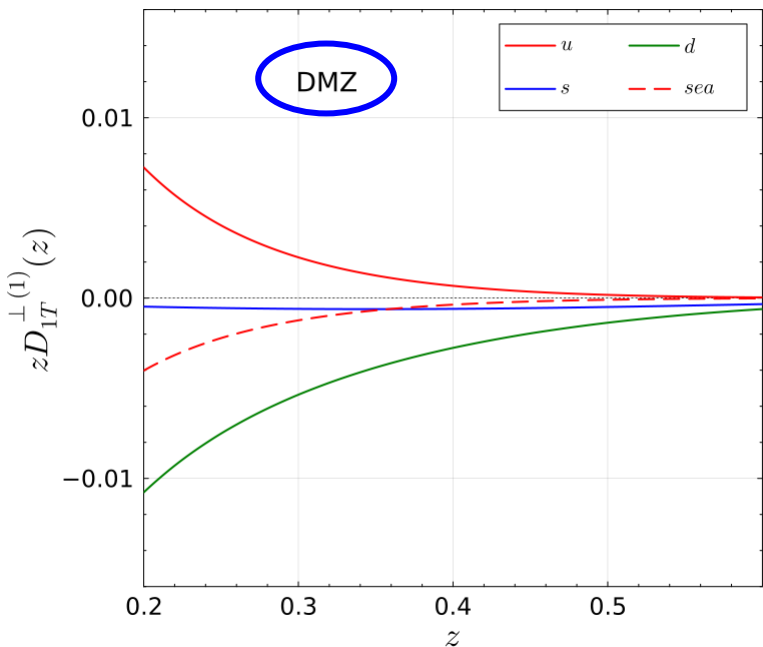
$$D_{1T\bar{u}}^\perp = D_{1T\bar{d}}^\perp,$$

$$D_{1Ts}^\perp, D_{1T\bar{s}}^\perp, D_{1Tc}^\perp, D_{1T\bar{c}}^\perp$$



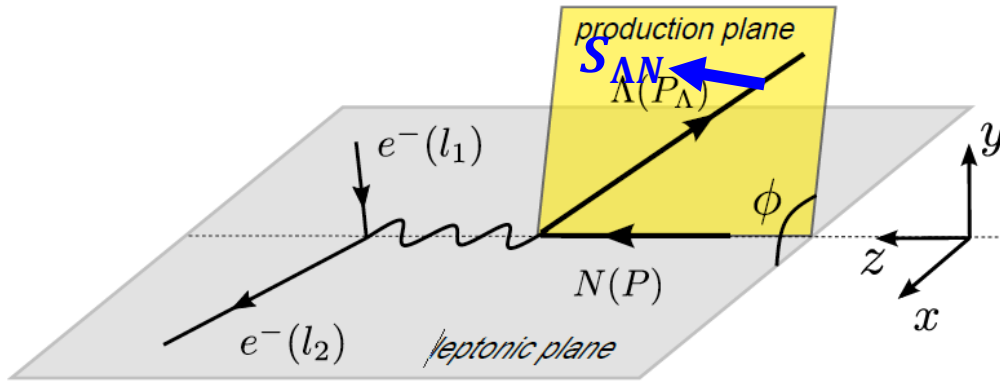
# Comparison of different parametrizations

## ➤ Comparison of three parametrizations

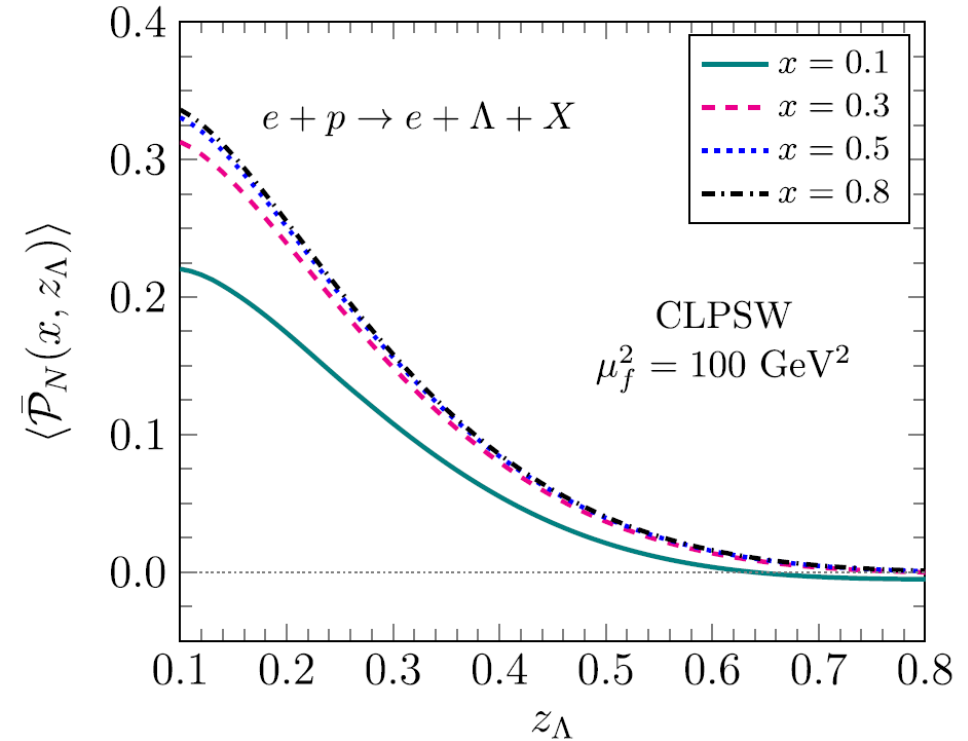


How to decipher the **flavor structure** (Isospin symmetry) of the polarized FFs  $D_{1T,q}^{\perp\Lambda}$ ?

# Transverse polarization of $\Lambda$ in $ep/eA$ collisions ( $D_{1T}^\perp$ )



$$\langle \bar{\mathcal{P}}_N(x, z_\Lambda) \rangle = \frac{\sqrt{\pi} \kappa_3(z_\Lambda) \sum_q e_q^2 x f_{1q}(x) D_{1Tq}^\perp(z_\Lambda)}{2z_\Lambda \sum_q e_q^2 x f_{1q}(x) D_{1q}^\Lambda(z_\Lambda)}$$



K.B.Chen, Z.T.Liang, **YKS**, S.Y.We, PRD 105 (2022) 034027

See also

Z.B.Kang, K.Lee, D.Y.Shao, F.Zhao, JHEP 11 (2021) 005

Z.B.Kang, J.Terry, A.Vossen, Q.H.Xu, J.L.Zhang, PRD 105 (2022) 094033

U.D'Alesio, L.Gamberg, F.Murgia, M.Zaccheddu, PRD 108 (2023) 094004

Z. Ji, X.Y.Zhao, A.Q.Guo, Q.H.Xu, J.L.Zhang, Nucl.Sci.Tech. 34 (2023) 155

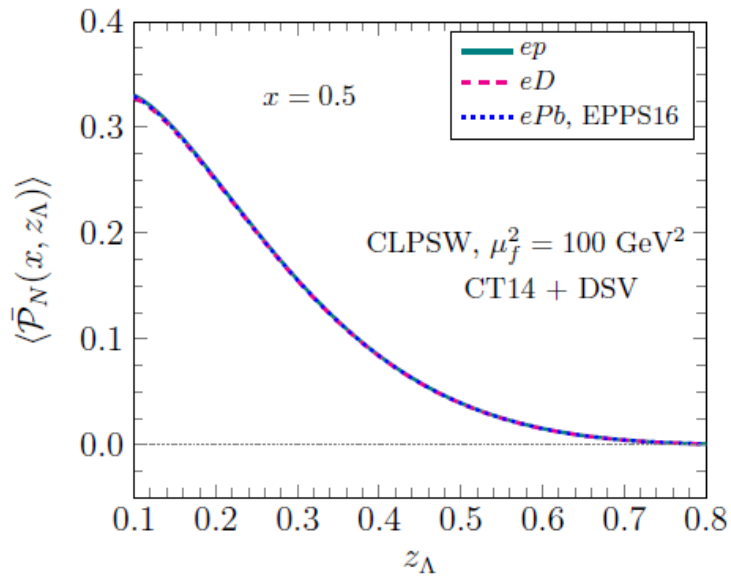


# Test of Isospin symmetry at the EIC with $\mathcal{P}_N$ for SIDIS

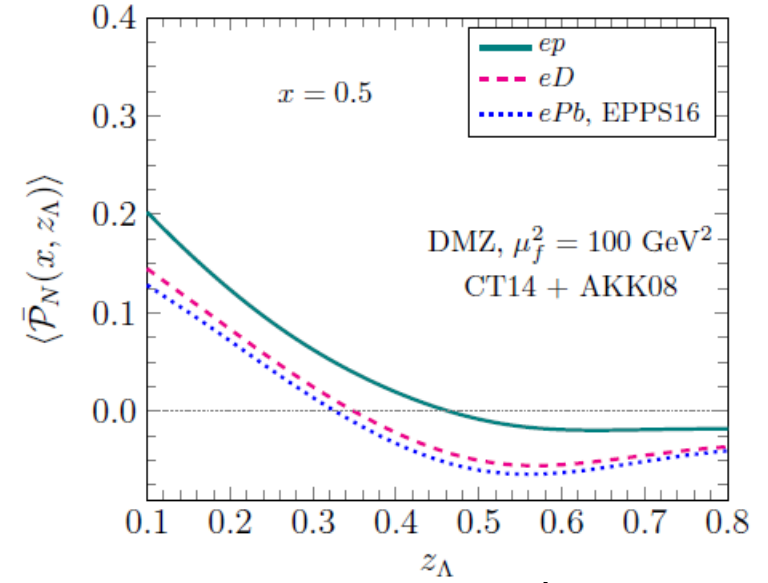
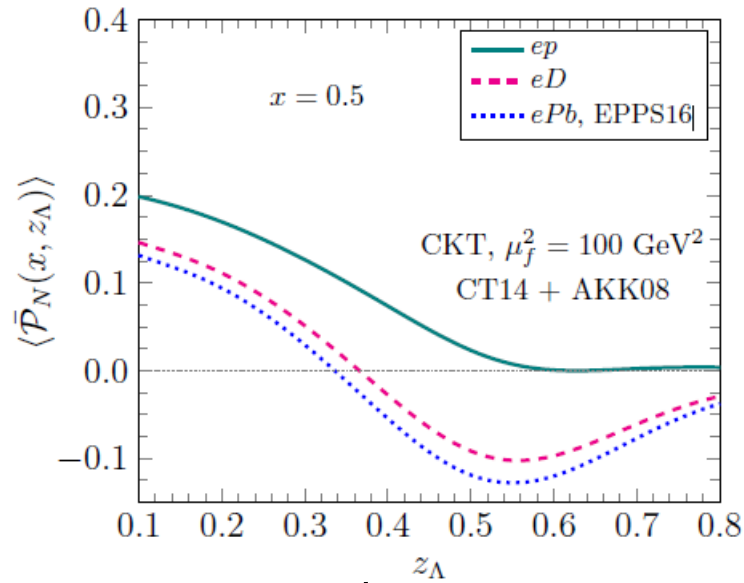
$$ep/eD/ePb \rightarrow e\Lambda X$$

Different u/d ratio  $\rightarrow$   $\begin{cases} \text{same } \mathcal{P}_N, & (D_{1u}^\perp = D_{1d}^\perp), \text{ CLPSW} \\ \text{different } \mathcal{P}_N, & (D_{1u}^\perp \neq D_{1d}^\perp), \text{ CKT, DMZ} \end{cases}$

**EPPS16:** Eskola, Paakkinen, Paukkunen, Salgado, Eur.Phys.J.C 77 (2017) 163



**Isospin symmetric parametrization**



**Isospin symmetry violating parametrizations**

K.B.Chen, Z.T.Liang, YKS, S.Y.Wei, PRD 105 (2022) 034027

# Transverse $\Lambda$ production in hadronic collisions ( $D_{1T}^{\perp\Lambda}$ )

- A wealth of data from hadronic collisions, e.g.,  $pp, p\bar{p}, pA, AA, \gamma A(UPC), \dots$
- Direct extension with  $pp \rightarrow \Lambda hX$  suffer from **violation of QCD factorization theorem**

J. Collins, J.W.Qiu, PRD 75 (2007) 114014

- “Hadron inside jets” proposed to study TMD JFFs in hadronic collisions

F.Yuan, PRL 100 (2008) 032003

Z. B. Kang, X. Liu, F. Ringer and H. Xing, JHEP 11 (2017), 068

Z. B. Kang, K. Lee and F. Zhao, PLB 809 (2020), 135756

- (1) Reconstruct jets from pp collisions
- (2) Measure the  $p_T$  distribution of hadrons with respect to jet axis.

To explore the **potential for flavor separation** for  $D_{1T,q}^{\perp\Lambda}$ , we perform a detailed **phenomenological analysis** on various hadronic collisions

Y.Gao, K.B.Chen, **YKS**, S.Y.We, ArXiv: 2403.06133

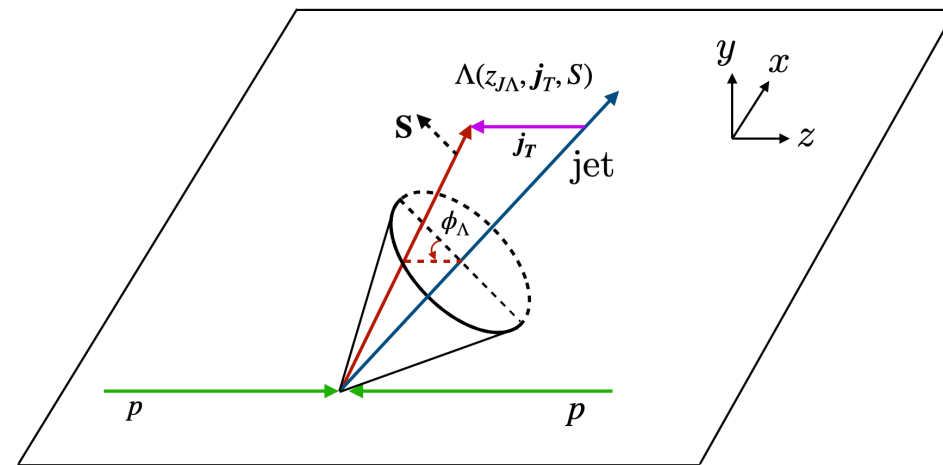


Figure from STAR

# $\Lambda$ inside jets in hadronic collisions

(1)  $p + p \rightarrow j + X$ , with jet reconstructed

$$\frac{d\sigma_{pp \rightarrow jX}}{dy d^2\mathbf{k}_T} = \sum_{abc} \int dy_2 x_1 f_a(x_1, \mu_f) x_2 f_b(x_2, \mu_f) \frac{1}{\pi} \frac{d\hat{\sigma}_{ab \rightarrow jc}}{d\hat{t}}$$

$$R_j(y, k_T) \equiv \frac{d\sigma_{pp \rightarrow jX}}{\sum_i d\sigma_{pp \rightarrow iX}}$$

(2)  $j \rightarrow \Lambda + X'$ , described by TMD jet FFs

$$\frac{d\sigma_{pp \rightarrow j(\rightarrow \Lambda)X}}{dy d^2k_T dz d^2p_{\Lambda T}} = \sum_j \frac{d\sigma_{pp \rightarrow jX}}{dy d^2\mathbf{k}_T} \left( D_{1j}^{\Lambda}(z, p_{\Lambda T}) + \frac{\varepsilon_{\perp}^{\rho\sigma} p_{T\rho} S_{\Lambda T\sigma}}{zM} D_{1T,j}^{\perp\Lambda}(z, p_{\Lambda T}) \right)$$

$$P_{\Lambda} = \frac{\sqrt{\pi} \Delta_{\Lambda} \sum_j R_j(y, k_T) D_{1T,j}^{\perp\Lambda}(z)}{2zM \sum_j R_j(y, k_T) D_{1j}^{\Lambda}(z)}$$

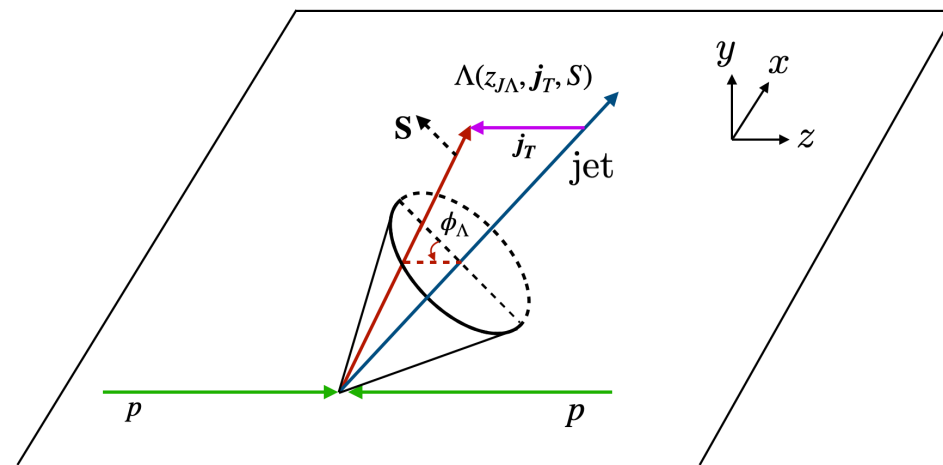
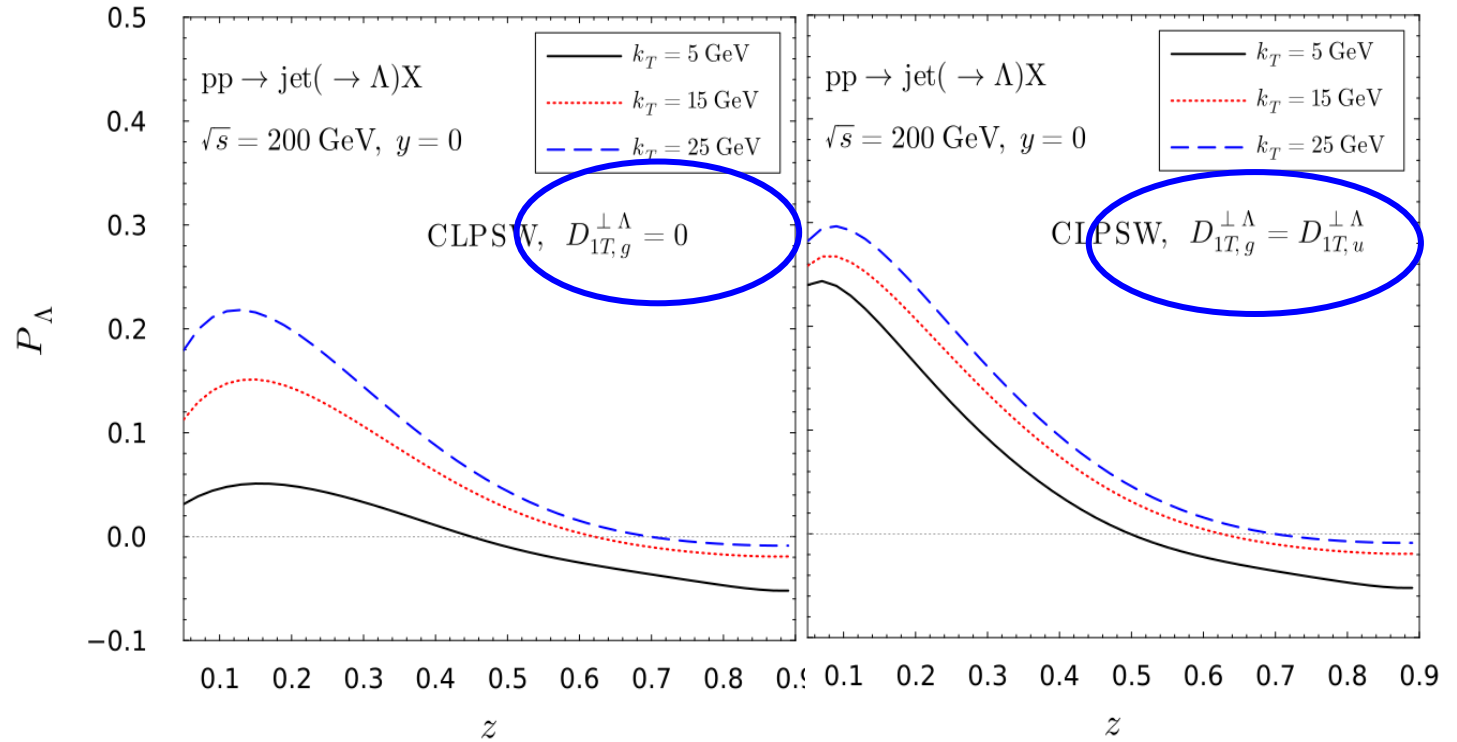
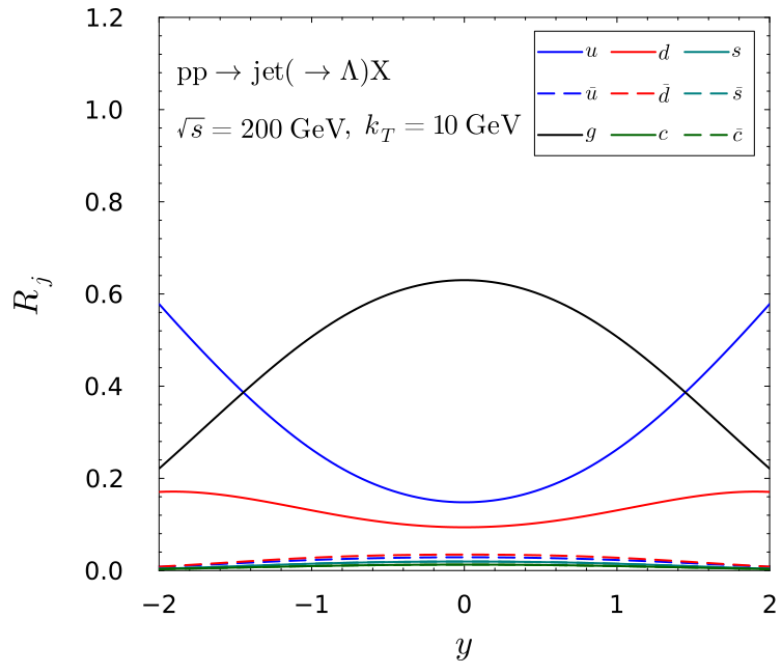


Figure from STAR

- We have quite different quark/gluon production  $R_j(y, k_T)$  in different reaction/kinematic regions, which provide the potential for deciphering the flavor structure.



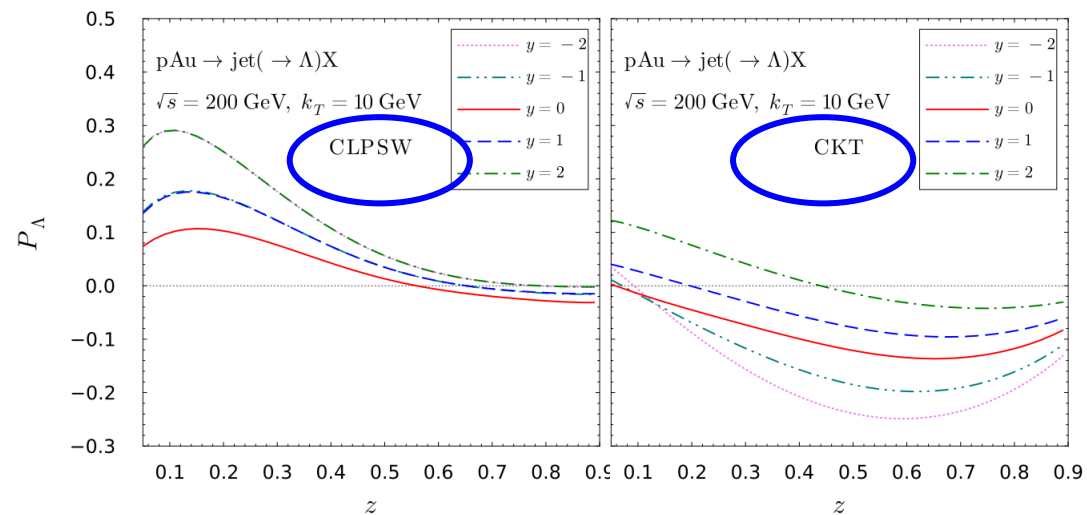
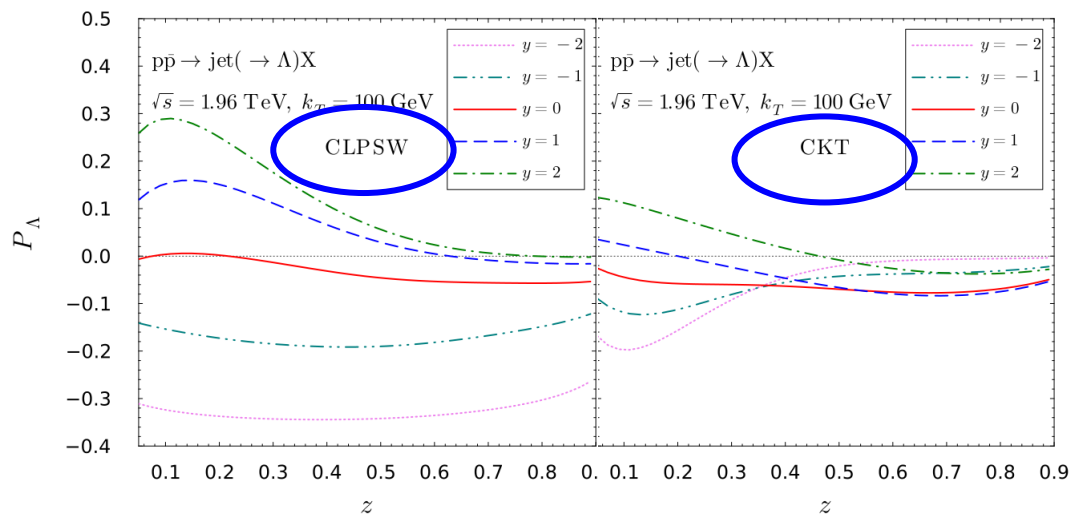
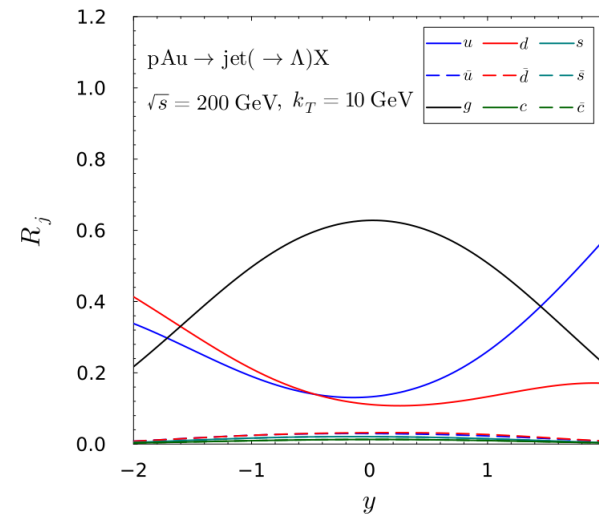
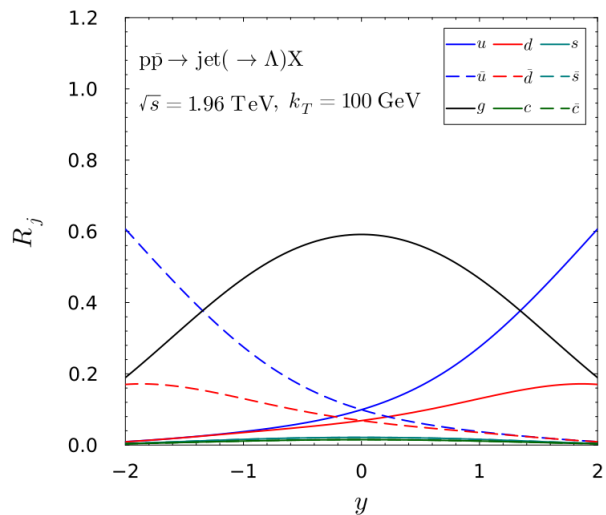
Central rapidity & small  $k_T$  region, **gluon dominate!**

$\Rightarrow$  a nice place to study the **gluon polarized FF  $D_{1T,g}^{\perp\Lambda}$**

CT18 PDF, DSV FF  $D_1^\Lambda$ , CLPSW  $D_{1T}^\perp$

$p\bar{p}$

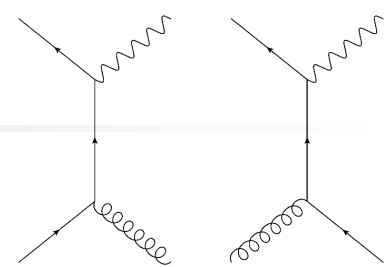
$pA$



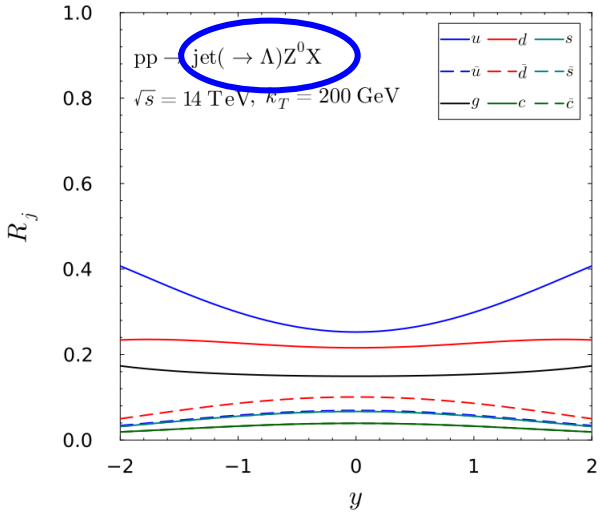
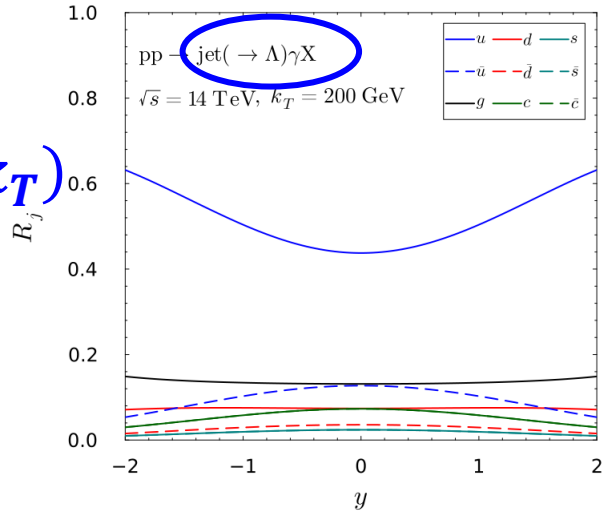
Forward rapidity region, **u** quark dominate;  
backward rapidity region, **u-bar** quark dominate

Forward rapidity region, **u** quark dominate;  
backward rapidity region, **u + d** quark dominate

# $\gamma/Z^0$ -associated $\Lambda$ production



$R_j(y, k_T)$

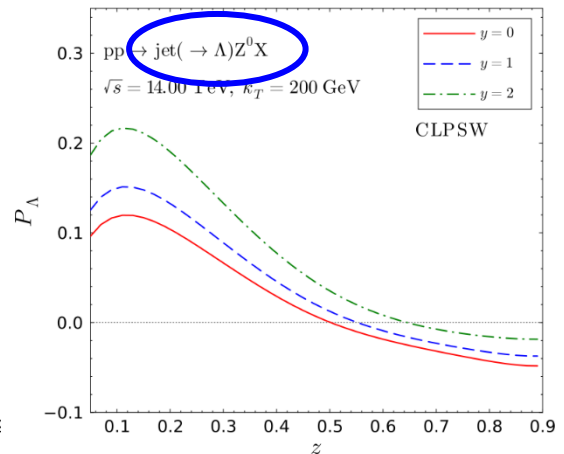
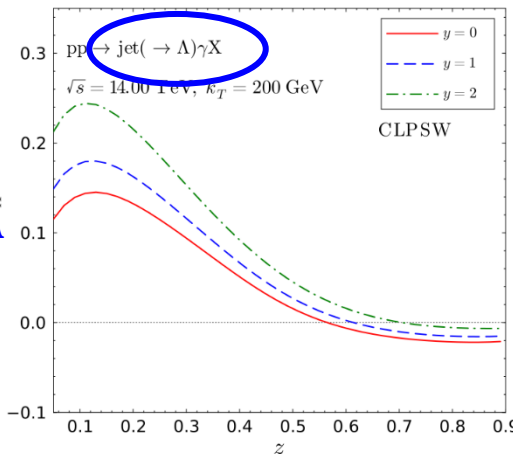


- Quarks dominate over gluons
- $u$  dominate in  $\gamma$ -associated process, while  $u \sim d$  in  $Z$ -associated process
- ⇒ a complementary place to study the difference between  $D_{1T,u}^{\perp\Lambda}$  and  $D_{1T,\bar{u}}^{\perp\Lambda}$

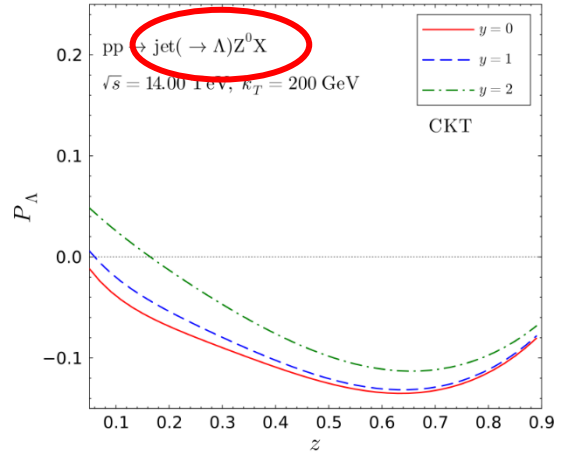
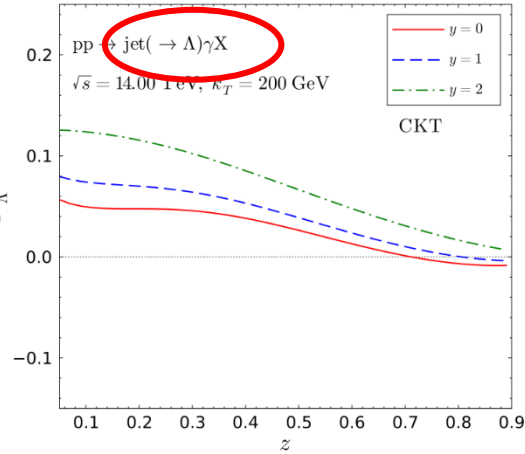
$$D_{1T,u}^{\perp\Lambda} = D_{1T,d}^{\perp\Lambda}$$

$$D_{1T,u}^{\perp\Lambda} \sim -D_{1T,d}^{\perp\Lambda}$$

$P_{\Lambda}^{\uparrow}$



$P_{\Lambda}^{\downarrow}$



CLPSW

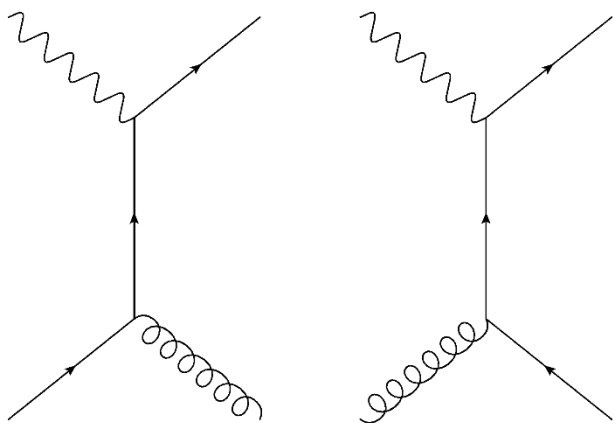
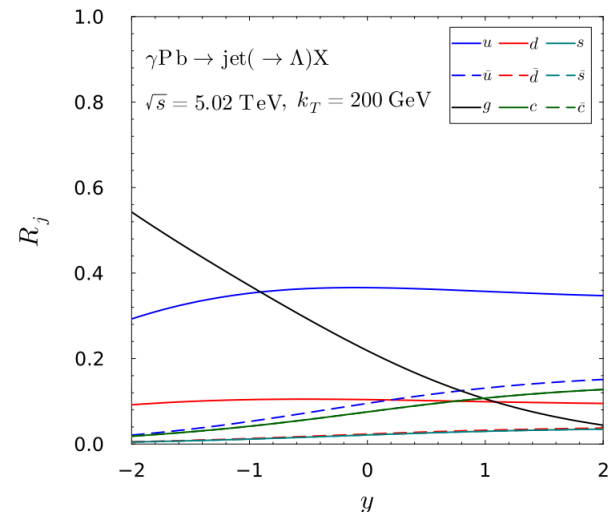
CKT

# Λ production in UPC

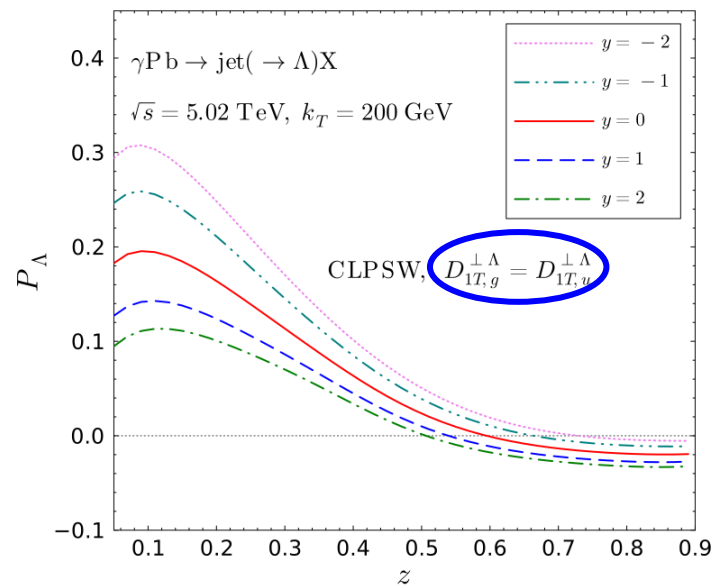
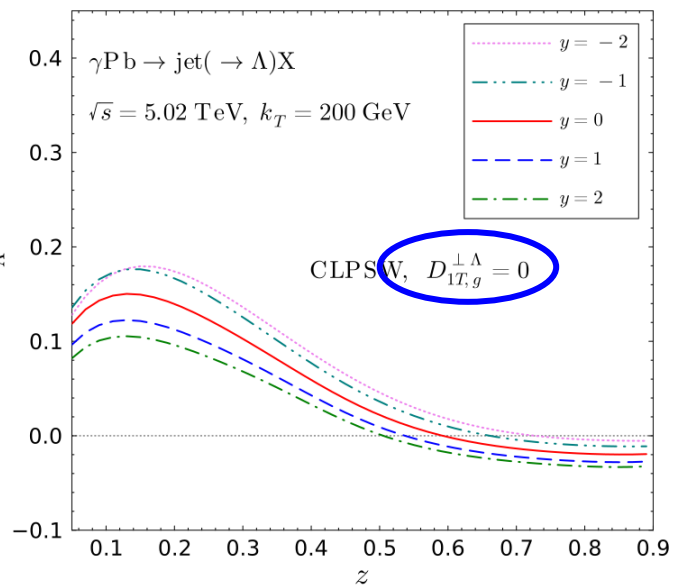
- Highly energetic nucleus  $\Leftrightarrow$  quarks and gluons inside the nucleus  
 + quasi-real photons surrounding the nucleus
- Equivalent Photon approximation (EPA)

$$xf_\gamma(x) = \frac{2Z^2\alpha}{\pi} \left[ \zeta K_0(\zeta)K_1(\zeta) - \frac{\zeta^2}{2} (K_1^2(\zeta) - K_0^2(\zeta)) \right]$$

$$R_j(y, k_T)$$



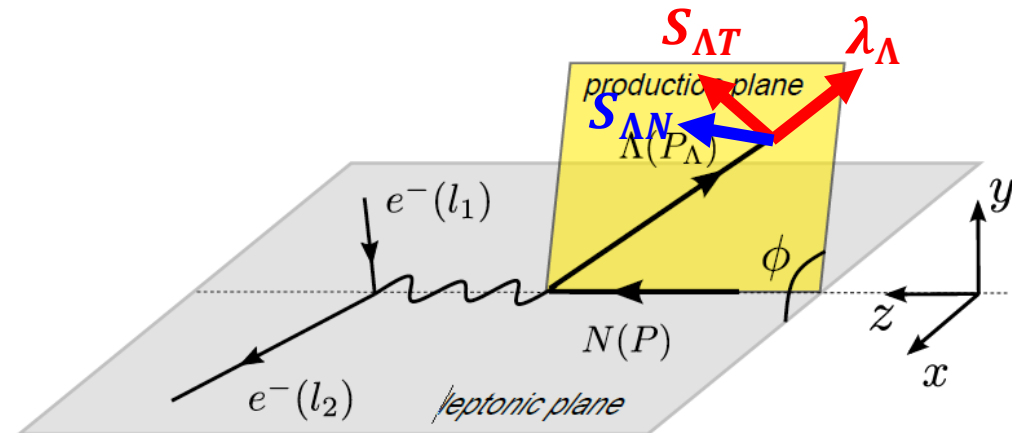
$$P_\Lambda$$



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# Longitudinal and transverse $\Lambda$ polarization in ep/eA collision ( $H_{1T}$ , $H_{1T}^\perp$ , $H_{1L}^\perp$ )



Leading Quark TMDFFs ○ → Hadron Spin ⊙ → Quark Spin

TMD handbook  
2304.03302

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Polarized Hadrons	Unpolarized (or Spin 0) Hadrons	$D_1 = \text{○}$ Unpolarized		$H_1^\perp = \text{⊙} - \text{⊙}$ Collins
	L		$G_1 = \text{⊙} \rightarrow - \text{⊙}$ Helicity	$H_{1L}^\perp = \text{⊙} \rightarrow - \text{⊙}$
Polarized Hadrons		$D_{1T}^\perp = \text{⊙} - \text{⊙}$ Polarizing FF	$G_{1T}^\perp = \text{⊙} - \text{⊙}$	$H_1 = \text{⊙} - \text{⊙}$ Transversity
				$H_{1T}^\perp = \text{⊙} - \text{⊙}$

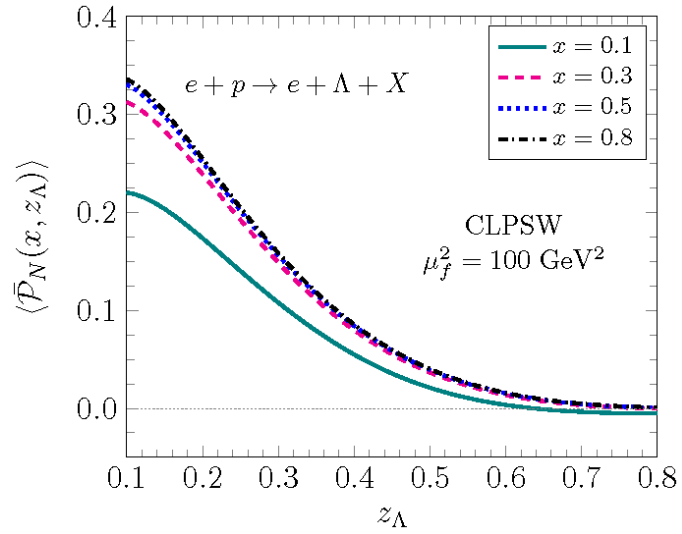
$$S_\Lambda \cdot (k \times p_\Lambda)$$

$$(s_{qT} \cdot p_{\Lambda T})(S_\Lambda \cdot p_\Lambda)$$

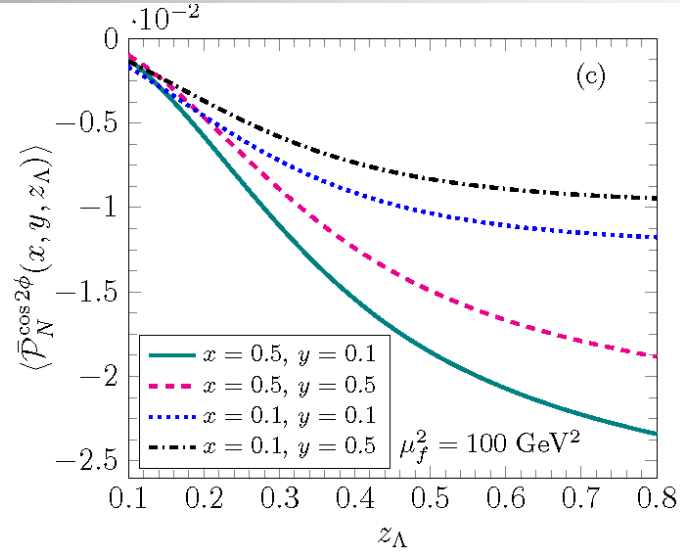
$$s_{qT} \cdot S_{\Lambda T}$$

$$(s_{qT} \cdot p_{\Lambda T})(S_{\Lambda T} \cdot p_{\Lambda T})$$

# Longitudinal and transverse $\Lambda$ polarization in ep/eA collision ( $H_{1T}, H_{1T}^\perp, H_{1L}^\perp$ )

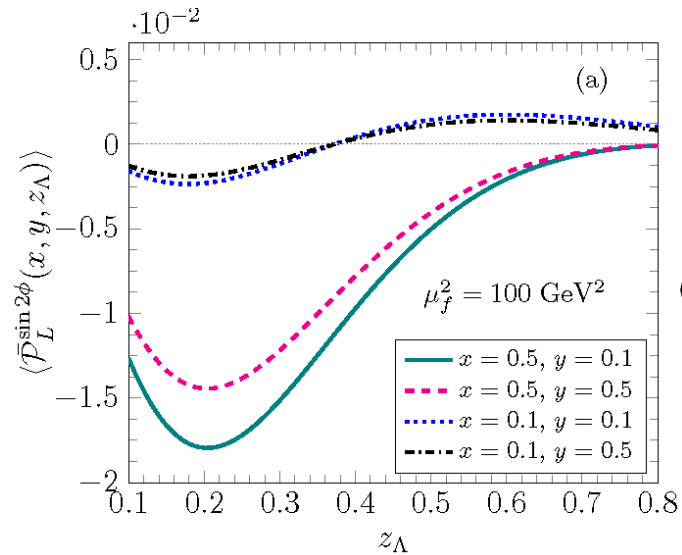


$$\langle \bar{\mathcal{P}}_N \rangle \propto \frac{f_{1q} D_{1T}^\perp}{f_1 D_1}$$

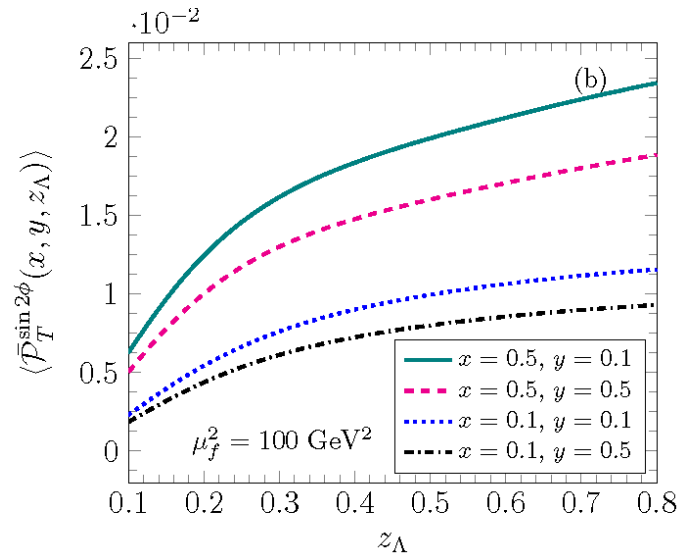


$$\langle \bar{\mathcal{P}}_N^{\cos 2\phi} \rangle \propto \frac{h_1^\perp (-H_{1T} + \kappa_5 H_{1T}^\perp)}{f_1 D_1}$$

Assuming  $H_{1L}^\perp \sim H_{1T}^\perp \sim (D_{1T}^\perp)/z$  and  $H_{1T} \sim G_{1L}$



$$\langle \bar{\mathcal{P}}_L^{\sin 2\phi} \rangle \propto \frac{h_1^\perp H_{1L}^\perp}{f_1 D_1}$$



$$\langle \bar{\mathcal{P}}_T^{\sin 2\phi} \rangle \propto \frac{h_1^\perp (-H_{1T} + \kappa_4 H_{1T}^\perp)}{f_1 D_1}$$

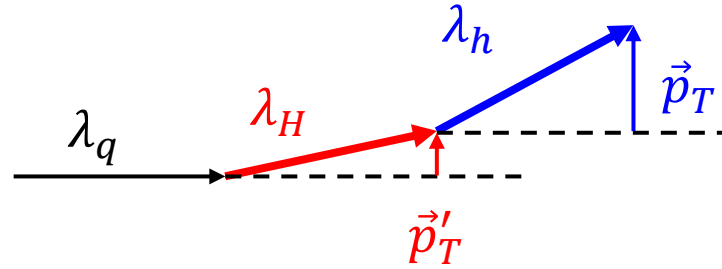
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# Weak decay contributions to TMD FFs $\tilde{D}_{1L}, \tilde{G}_1$


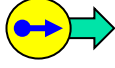
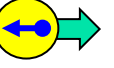
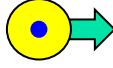


$$q \rightarrow H + X$$

$$\downarrow$$

$$h + X'$$



$$D_{\lambda_q}^{\lambda_h} \quad D_+^+, D_-^+, D_+^-, D_-^- \rightarrow \begin{cases} D_1 = \frac{1}{2} (D_+^+ + D_+^- + D_-^+ + D_-^-) \\ G_{1L} = \frac{1}{2} (D_+^+ - D_+^- - D_-^+ + D_-^-) \\ \tilde{D}_{1L} = \frac{1}{2} (D_+^+ - D_+^- + D_-^+ - D_-^-) \\ \tilde{G}_1 = \frac{1}{2} (D_+^+ + D_+^- - D_-^+ - D_-^-) \end{cases}$$

$D_1$	$G_{1L}$	$\tilde{D}_{1L}$	$\tilde{G}_1$
	 - 		 - 

$\tilde{D}_{1L}$ : Difference of absolute values of hadron polarizations

$\tilde{G}_1$ : Difference of number of hadrons

} in jets initiated by helicity + and - quarks

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## Weak decay contributions to TMD FFs $\tilde{D}_{1L}, \tilde{G}_1$

- QCD  $\theta$ -vacuum breaks parity invariance  $\Rightarrow$  non-zero parity-odd FFs [Kang, Kharzeev 2011]

$$\mathcal{L} = \mathcal{L} + \frac{g^2}{32\pi^2} \theta(x, t) F_a^{\mu\nu} \tilde{F}_{\mu\nu}^a \quad \Rightarrow \quad \Xi(z) \sim \gamma_\mu p^\mu (D_1 + \lambda_h \tilde{D}_{1L}) + \gamma_\mu \gamma_5 p^\mu (\lambda_h G_{1L} + \tilde{G}_1)$$

- The  $\theta$ -parameter induce parity-odd FFs  $\tilde{D}_{1L}, \tilde{G}_1$  with different signs in each event, hard to probe in exps.
- Hadrons detected in exps may contain **weak decay** contributions, thus violating parity invariance.
- It is not an easy task to subtract all decay contributions, leaving room for P-odd FFs
- **We perform a detailed calculation of weak decay contributions to P-odd FFs, and estimate the magnitudes of their observables in exps.**

$$D_q^{h,H}(\lambda_q, \lambda_h; z, p_T) = \sum_{\lambda_H} \int dz' d^2 p'_T \frac{dN(\lambda_h, \lambda_H)}{dz d^2 p_T} D_q^H(\lambda_q, \lambda_H; z', p'_T)$$

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# Weak decay contributions to TMD FFs $\tilde{D}_{1L}, \tilde{G}_1$

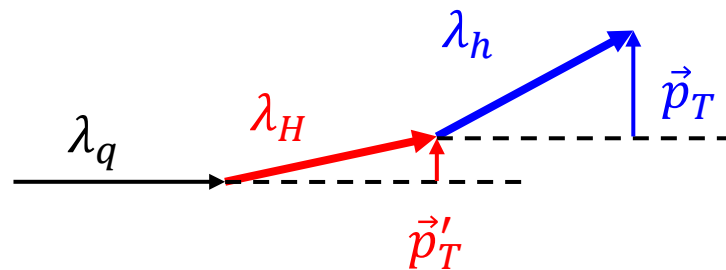
$$D_q^h = D_q^{h,\text{dir}} + \sum_H D_q^{h,H}$$

- $D_1^{h,\text{dir}}$ : directly produced part, free of parity violation
- $D_1^{h,H}$ : decay contribution from parent hadron H

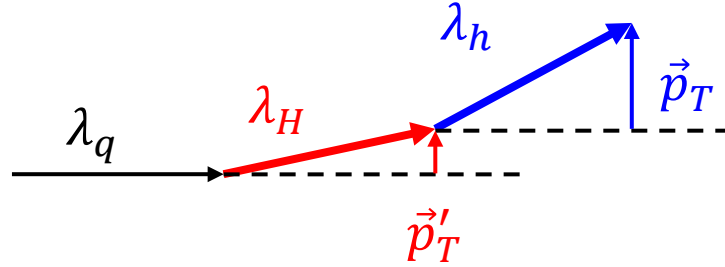
$$H \rightarrow h + X$$

Where X can be a single particle (2-body decay) or several particles (3-body decays etc.)

$$D_q^{h,H}(\lambda_q, \lambda_h; z, p_T) = \sum_{\lambda_H} \int dz' d^2 p_T' \frac{dN(\lambda_h, \lambda_H)}{dz d^2 p_T} D_q^H(\lambda_q, \lambda_H; z', p_T')$$



# Weak decay contributions to TMD FFs $\tilde{D}_{1L}, \tilde{G}_1$



$\frac{dN(\lambda_h, \lambda_H)}{dzd^2p_T}$  contain information of weak interactions. We calculate them via helicity amplitudes.

➤  $\frac{1}{2} \rightarrow \frac{1}{2} + 0$

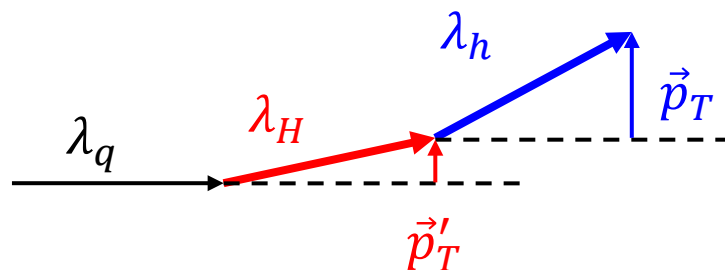
$$\frac{dN(\lambda_h, \lambda_H)}{dzd^2p_T} = \frac{1}{8\pi} \frac{2M_H}{z|\vec{p}_h^*|} \delta[(p_H - p_h)^2 - M_X^2] [1 + \gamma \lambda_H \lambda_h \omega_i \cdot \omega_f + (1 - \gamma) \lambda_H \lambda_h (\omega_i \cdot \hat{p}_h^*) (\omega_f \cdot \hat{p}_h^*) + \alpha (\lambda_H \omega_i \cdot \hat{p}_h^* + \lambda_h \omega_f \cdot \hat{p}_h^*) + \beta \lambda_H \lambda_h \hat{p}_h^* \cdot (\omega_i \times \omega_f)]$$

➤  $\frac{3}{2} \rightarrow \frac{1}{2} + 0$

$$\frac{dN(\lambda_h, \lambda_H)}{dzd^2p_T} = \frac{1}{8\pi} \frac{2M_H}{z|\vec{p}_h^*|} \delta[(p_H - p_h)^2 - M_X^2] [1 + \alpha \lambda_h \omega_f \cdot \hat{p}_h^*]$$

➤ For strong decays  $\alpha = \beta = 0, \gamma = \pm 1$

# Weak decay contributions to TMD FFs $\tilde{D}_{1L}, \tilde{G}_1$



- By inserting  $\frac{dN(\lambda_h, \lambda_H)}{dz d^2 p_T}$  into the decay contributions to P-odd FFs, we finally obtain

$$\tilde{D}_{1L}^{h,H}(z) = \frac{M_H}{2|\mathbf{p}_h^*|} \int \frac{dz'}{z'} d^2 p'_T D_{1q}^H(z', p'_T) K_{U \rightarrow L}$$

$$\tilde{G}_1^{h,H}(z) = \frac{M_H}{2|\mathbf{p}_h^*|} \int \frac{dz'}{z'} d^2 p'_T G_{1L,q}^H(z', p'_T) K_{L \rightarrow U}$$

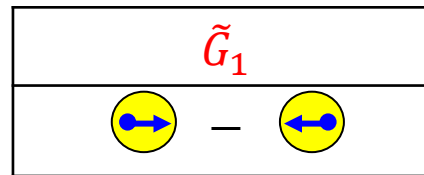
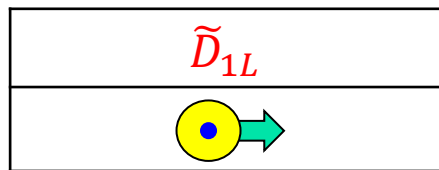
where the kernel functions are given by

$$K_{U \rightarrow L} = \alpha \frac{M_H E_h E_h^* - E_H m_h^2}{M_H |\mathbf{p}_h| |\mathbf{p}_h^*|}, \quad K_{L \rightarrow U} = \alpha \frac{M_H E_h - E_H E_h^*}{|\mathbf{p}_H| |\mathbf{p}_h^*|}$$



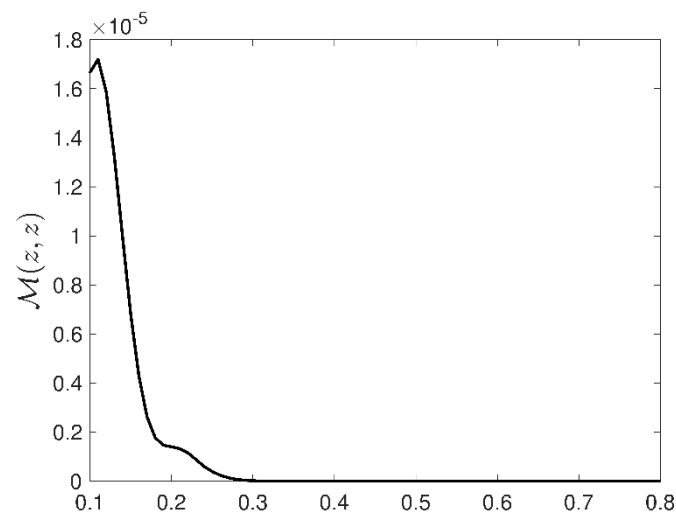
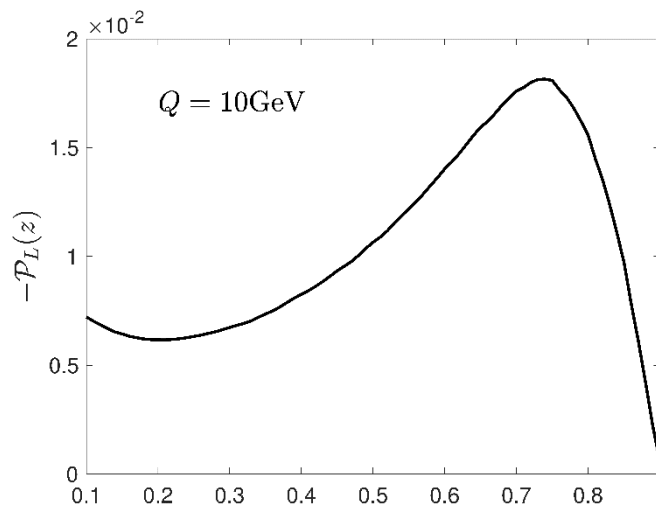
# Observables of $\tilde{D}_{1L}, \tilde{G}_1$

1. Spontaneous  $\Lambda$  polarizations in  $e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow \Lambda X$
2. Modifications to the di-hadron cross sections in  $e^+e^- \rightarrow h_1 h_2 X$



$$\mathcal{P}_L(z) = \frac{\sum_q [\Delta\omega_q G_{1L,q}(z) + \omega_q \tilde{D}_{1L,q}(z)]}{\sum_q \omega_q D_{1,q}(z)}$$

$$\mathcal{M}(z_{h_1}, z_{h_2}) \equiv \frac{\sum_q \tilde{G}_{1q}^{h_1}(z_{h_1}) \tilde{G}_{1\bar{q}}^{h_2}(z_{h_2})}{\sum_q D_{1q}^{h_1}(z_{h_1}) D_{1\bar{q}}^{h_2}(z_{h_2})}$$



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## Conclusions and outlook

- $D_{1T}^\perp$ : Belle data, three group of parametrizations, Flavor structure (isospin symmetry)
  - ep/eA
  - Hadronic collisions
  - $D_{1T,g}^\perp$  in  $pp$  collisions, and in  $\gamma A$  collisions (UPC)
  - $D_{1T,u}^\perp$  v.s.  $D_{1T,\bar{u}}^\perp$  in  $p\bar{p}$  collision
  - $D_{1T,u}^\perp$  v.s.  $D_{1T,d}^\perp$  in  $pA$  collisions, and in  $\gamma/Z^0$ -associated process (Isospin symmetry)
- $H_{1T}, H_{1T}^\perp, H_{1L}^\perp$ : rough estimates in  $e^+e^-$ , ep/eA collisions
- $\tilde{D}_{1L}, \tilde{G}_1$ : non-negligible magnitudes, probable to measure in exp.

A garden with a wealth of polarized TMD FFs unexplored, calling for experimental efforts from  $e^+e^-$ -annihilation, EIC, hadron collider et al.

*Thanks for you attention!*