中国物理学会高能物理分会
第十四届全国粒子物理学术会议

Extended Nambu--Jona-Lasinio model for quark and nuclear matters

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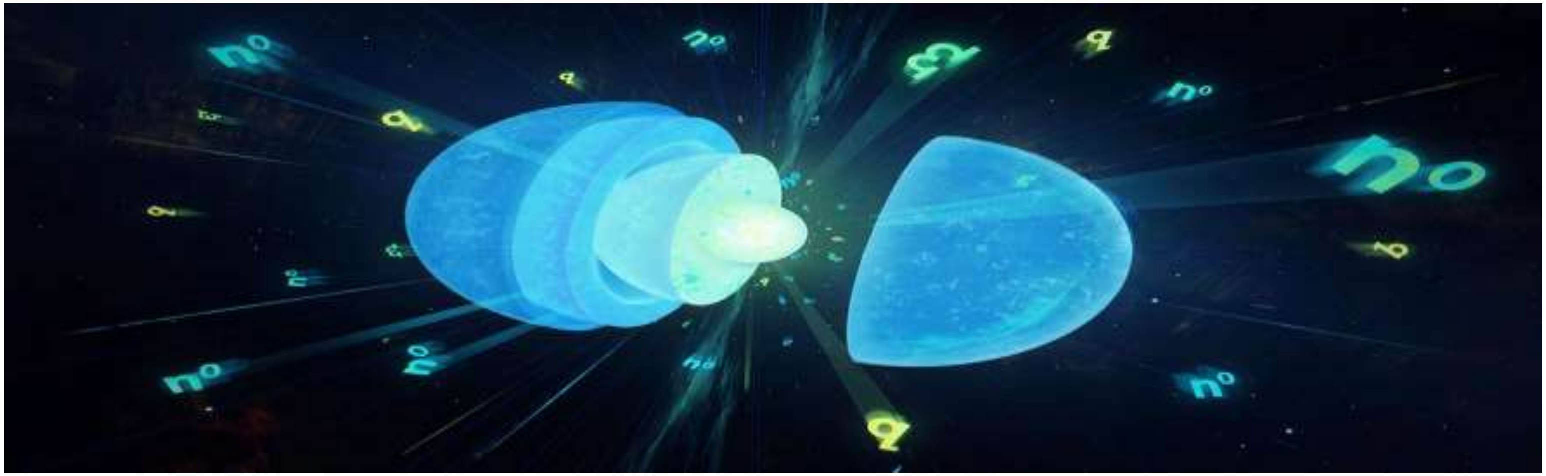
GC, arXiv:2403.19331.



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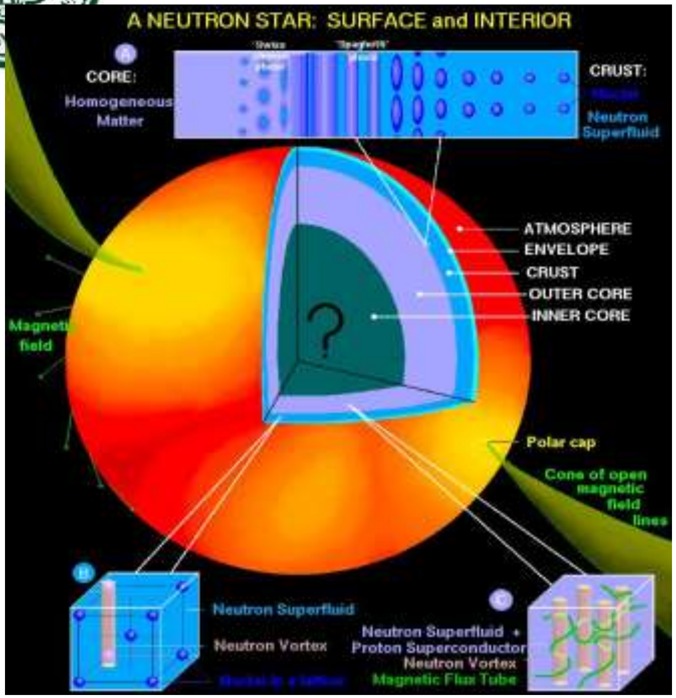
outline

- Internal structure of neutron stars
- quarkyonic matter and QCD phase diagram
- Extended NJL model
- Numerical results and discussions
- Summary and perspective

Internal structure of neutron stars



Atmosphere: plasma

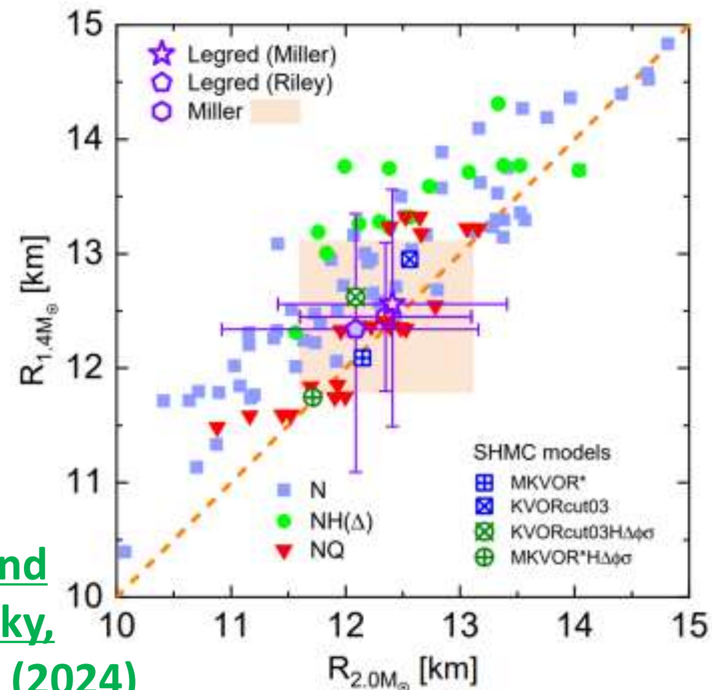
Outer crust： heavy nuclei, e

Inner crust： heavy nuclei, e, n

Outer core： n, p, e

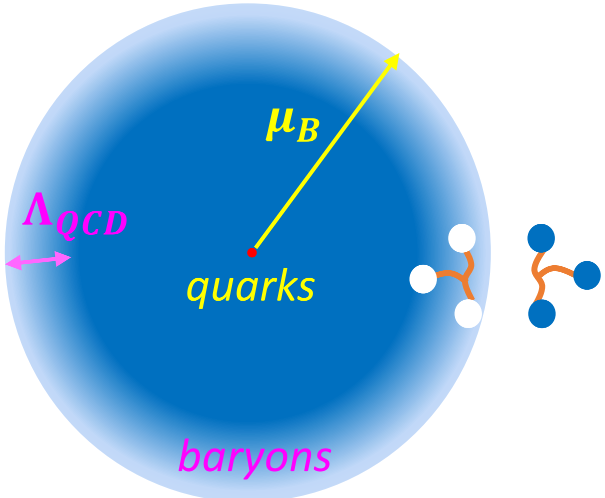
Innter core： n, p, e

~~Hyperon~~, QGP？



E. Kolomeitsev and D. N. Voskresensky, PRC 110, 025801 (2024)

Quarkyonic matter



momentum space

L. McLerran and R. D. Pisarski, NPA 796, 83 (2007).

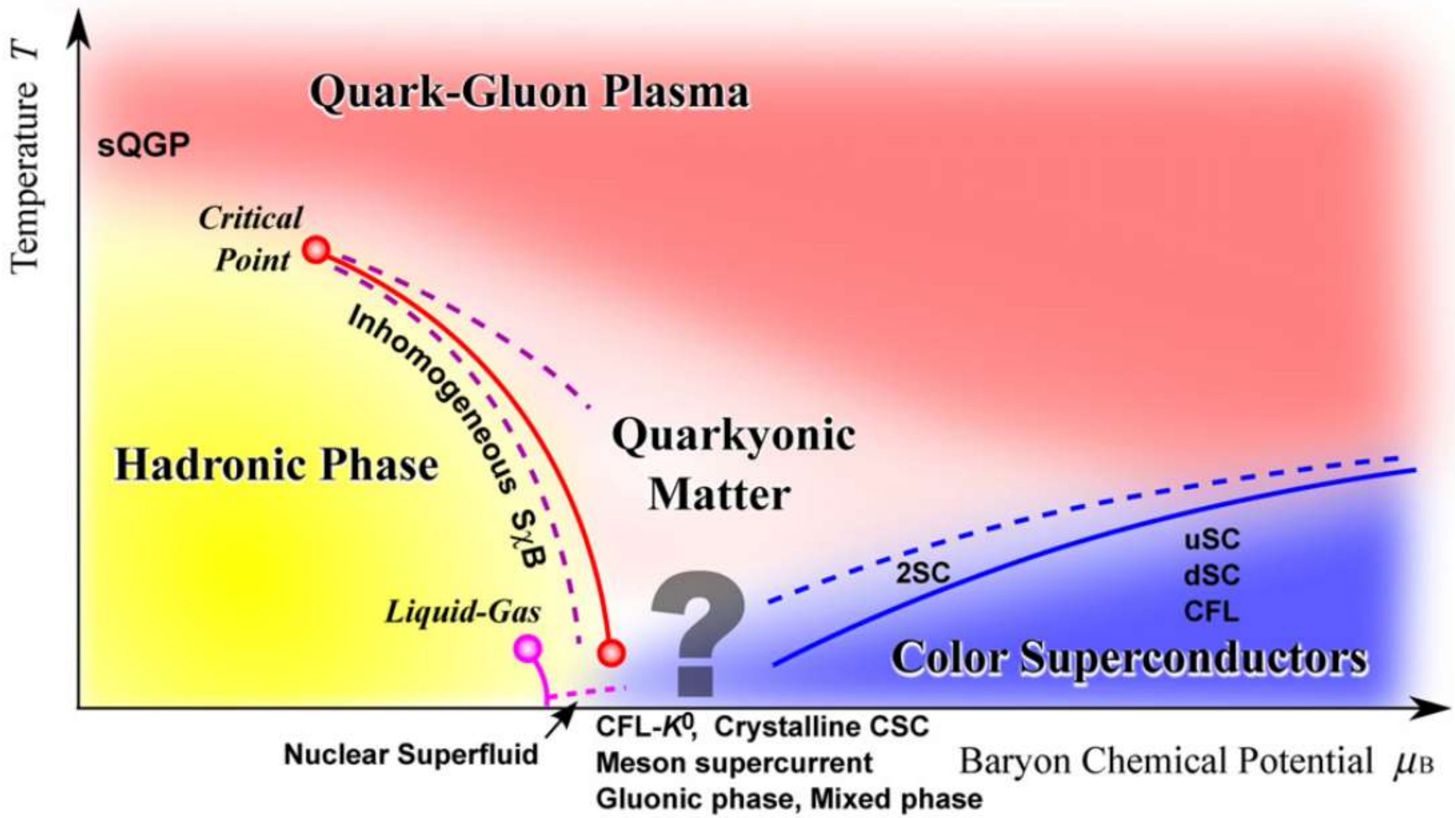
Chiral symmetry restored
+
confinement

1. Fermi sea: nearly free quarks;
2. Fermi surface: mainly confined baryons;
3. Proved in large N_C limit;
4. Found in LQCD simulations with $N_C = 2$

V. V. Braguta et. al., PRD.94.114510 (2016)

All-happy QCD phase diagram

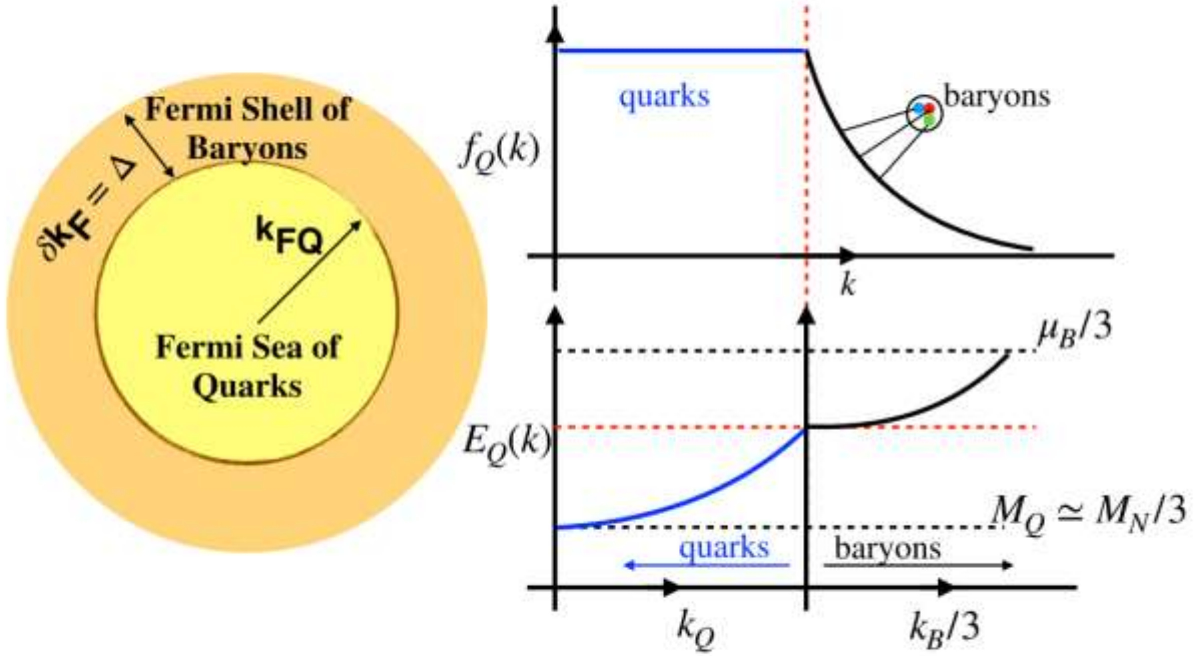
K. Fukushima and T. Hatsuda, Rep. Prog. Phys. 74 (2011) 014001



Chance: lower energy HICs for fixed target

Quarkyonic matter in neutron stars

L. McLerran and S. Reddy, PRL 122, 122701 (2019)

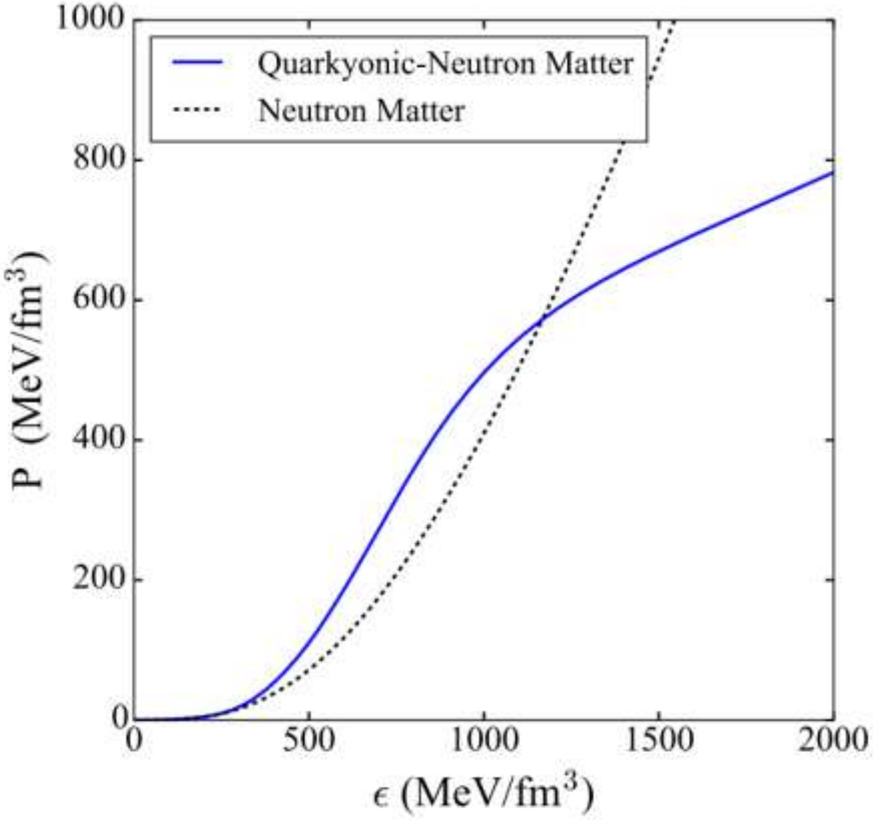


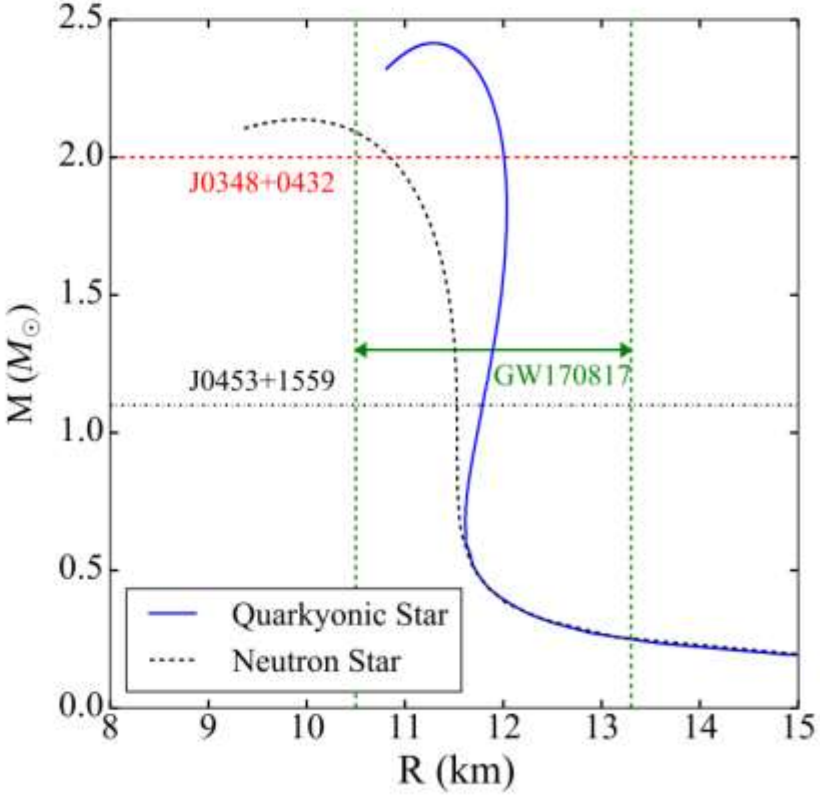
$$n_B = \frac{2}{3\pi^2}\left[k_{FB}^3 - (k_{FB}-\Delta)^3 + k_{FQ}^3\right] \qquad k_{FQ} = \frac{(k_{FB}-\Delta)}{N_c}\Theta(k_{FB}-\Delta)$$

$$\epsilon(n_B) = 4\int_{N_c k_{FQ}}^{k_{FB}} \frac{d^3k}{(2\pi)^3}\sqrt{k^2+M_N^2} + 4N_c\int_0^{k_{FQ}} \frac{d^3k}{(2\pi)^3}\sqrt{k^2+M_Q^2}$$

Neutron potential

$$V_n(n_n) = \tilde{a} n_n \left(\frac{n_n}{n_0}\right) + \tilde{b} n_n \left(\frac{n_n}{n_0}\right)^2$$





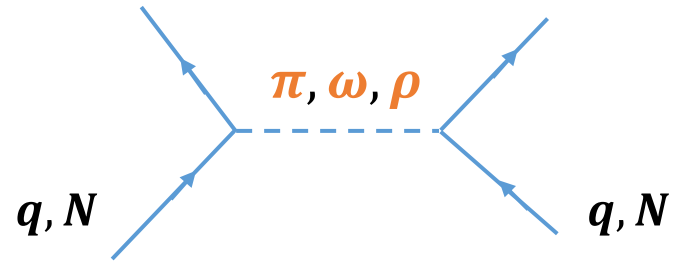
Equation of state

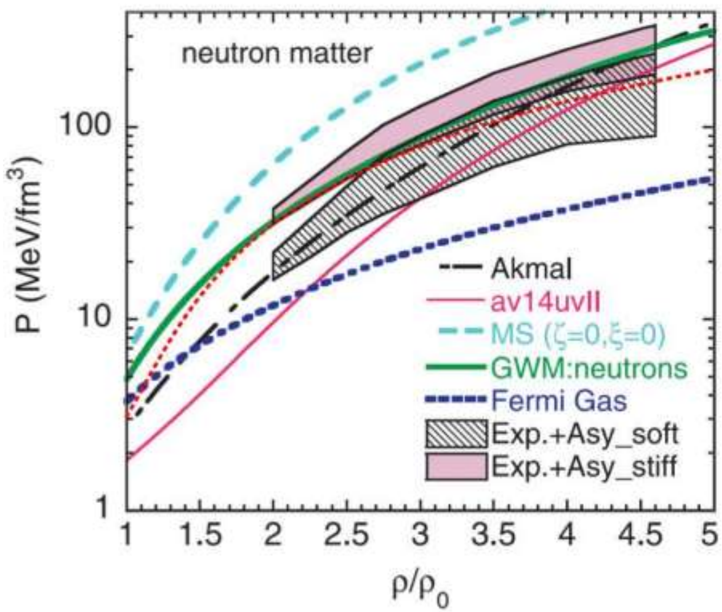
Mass-radius relation

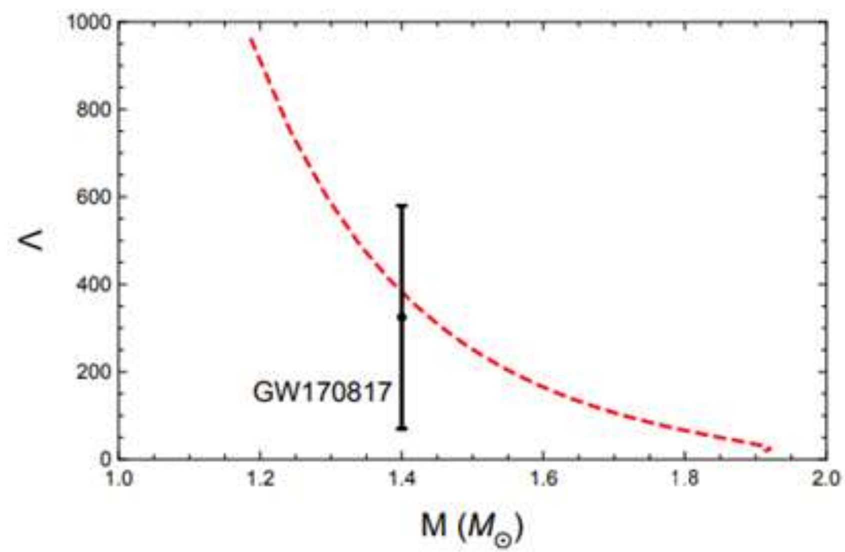
Chiral effective field model

Quark-meson model + Walecka model

$$L = L_q + L_m + L_N$$



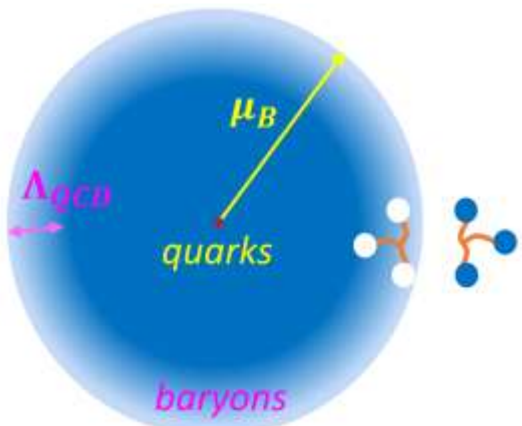




GC and J. Liao, JHEP 10, 168 (2020);
GC, Phys. Rev. D 105, 114020 (2022).

drawback: artificially introduce "blocking" chemical potential for nucleons

$$\mu'_B = \mu_B - (N_c m_q - m_N) \qquad \mu'_B = \sqrt{m_N^2 + (N_c k_F)^2}$$



BCS theory: fermion pairing near the Fermi surface in mean field and random phase approximations

introduce six-quark interaction in NJL model, and three quarks pair into nucleons

Nucleons in terms of quarks

Mesons: $\sigma \sim \bar{\psi}\psi$, $\pi \sim \bar{\psi} i\gamma^5 \tau \psi$

QCD sum rules:

$$p \propto \epsilon_{abc}(u^{aT} C\gamma_\mu u^b)\gamma_5\gamma^\mu d^c,$$
$$n \propto \epsilon_{abc}(d^{aT} C\gamma_\mu d^b)\gamma_5\gamma^\mu u^c$$

B. L. Ioffe, Nucl. Phys. B 188, 317-341 (1981)

$C = i\gamma_2\gamma_0$

Take Fiertz transformations:

$$p \propto \epsilon_{abc}(u^{aT} C\gamma_5 d^b)u^c - \epsilon_{abc}(u^{aT} C d^b)\gamma_5 u^c,$$
$$n \propto \epsilon_{abc}(u^{aT} C\gamma_5 d^b)d^c - \epsilon_{abc}(u^{aT} C d^b)\gamma_5 d^c.$$

X. m. Jin, et. Al., Phys. Rev. C 49, 464-477 (1994).

Lattice QCD: scalar channels dominate

$$N \equiv \begin{pmatrix} p \\ n \end{pmatrix} \propto \psi^c \left(\psi^T C \tau_2 \epsilon_c \gamma_5 \psi\right)$$

Extended NJL model

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ + \gamma^0\frac{\mu_B}{3} - m_0\right)\psi + G_2\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right]$$
$$- \left(\bar{\psi}\tau_2\epsilon_c\gamma_5\psi_C\right)\bar{\psi}^c\tilde{G}_3\left(i\partial\!\!\!/ + \gamma^0\mu_B\right)\psi^{c'}\left(\bar{\psi}_C\tau_2\epsilon_{c'}\gamma_5\psi\right)$$

Principles for the introduction:

1. **Desired three-quark state for nucleons;**
2. **Keep the original symmetries: chiral, Lorentz;**
3. **Bare nucleon term the same as Dirac field;**
4. **Coupling constant captures confinement.**

Beyond mean field approximation

Mean field approximation $\langle\sigma\rangle = m - m_0 \neq 0$

Thermodynamic potential

$$\Omega_0 = \frac{(m-m_0)^2}{4G_2} - 2N_\mathrm{f}N_\mathrm{c}\int^{\Lambda}\frac{\mathrm{d}^3k}{(2\pi)^3}E_\mathbf{k} - 2TN_\mathrm{f}N_\mathrm{c} \times \sum_{t=\pm}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\ln\left[1+e^{-\frac{1}{T}(E_\mathbf{k}+t\,\frac{\mu_\mathrm{B}}{3})}\right].$$

RPA

$$S^{-1}_{\sigma/\boldsymbol{\pi}}(p) = \frac{i}{2G} - \mathrm{Tr}\; S^\mathrm{c}_\mathrm{q}(q)\Gamma_{\sigma/\boldsymbol{\pi}}S^\mathrm{c}_\mathrm{q}(q-p)\Gamma_{\sigma/\boldsymbol{\pi}}$$

Contributions of pions

$$\Omega_\pi = 3T\int\frac{\mathrm{d}^3k}{(2\pi)^3}\ln\left(1-e^{-\frac{E^\pi_\mathbf{k}}{T}}\right)$$

$$E^\pi_\mathbf{k} = (\mathbf{k}^2+m_\pi^2)^{1/2}$$

RPA for nucleons and beyond

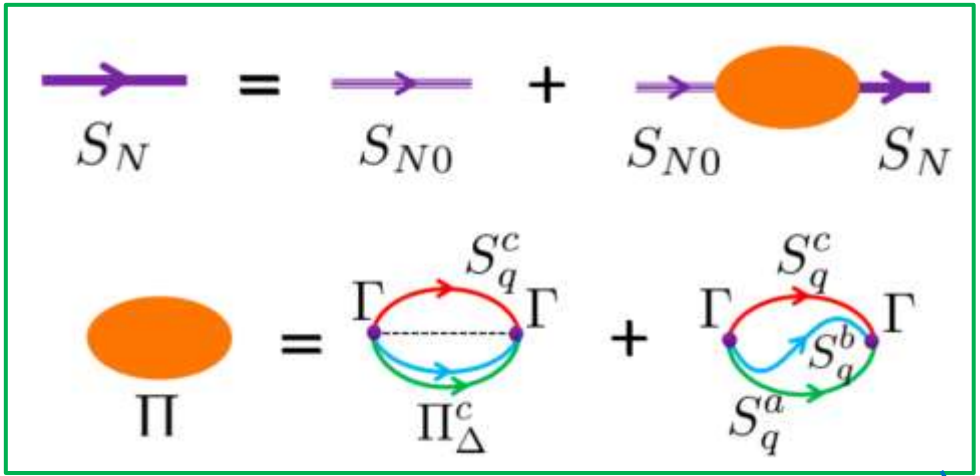
HS transformation

$$N \equiv \sqrt{\tilde{G}_3}\,\psi^{\rm c}\left(\psi^{\rm T} C \tau_2 \epsilon_{\rm c} \gamma_5 \psi\right)$$

$$\mathcal{L} = \bar{N}\, i\, S_{\rm N0}^{-1}\, N - \bar{N}\, i\Gamma\, \psi^{\rm c}\left(\bar{\psi}_C \tau_2 \epsilon_{\rm c} \gamma_5 \psi\right) - \left(\bar{\psi} \tau_2 \epsilon_{\rm c} \gamma_5 \psi_C\right) \bar{\psi}^{\rm c}\, i\Gamma\, N.$$

$$S_{\rm N0} = i/\not{P}$$

RPA scheme for dressed nucleon propagator



$$\Omega_{\rm N} = -2TN_{\rm f} \sum_{t=\pm} \int \frac{{\rm d}^3 k}{(2\pi)^3} \ln\left[1 + e^{-\frac{1}{T}(E^{\rm N}_{\bf k} + t\,\mu_{\rm B})}\right]$$

$$E^{\rm N}_{\bf k} = ({\bf k}^2 + m^2_{\rm N})^{1/2}$$

Coupling and current quark mass

Dressed propagator $$S_{\rm N} = \frac{i}{\gamma^0[\tilde{P}_0 - \bar{\Pi}_0(\tilde{P}_0)\tilde{P}_0^2] - \bar{\Pi}_{\rm s}(\tilde{P}_0)\tilde{P}_0^2}$$

Nucleon vacuum mass is large

$\bar{\Pi}_0, \bar{\Pi}_s \propto \tilde{G}_3$

⇓

Coupling $$\tilde{G}_3 = G_3/|\tilde{P}^2|^d \ (d = 1, 3/2)$$

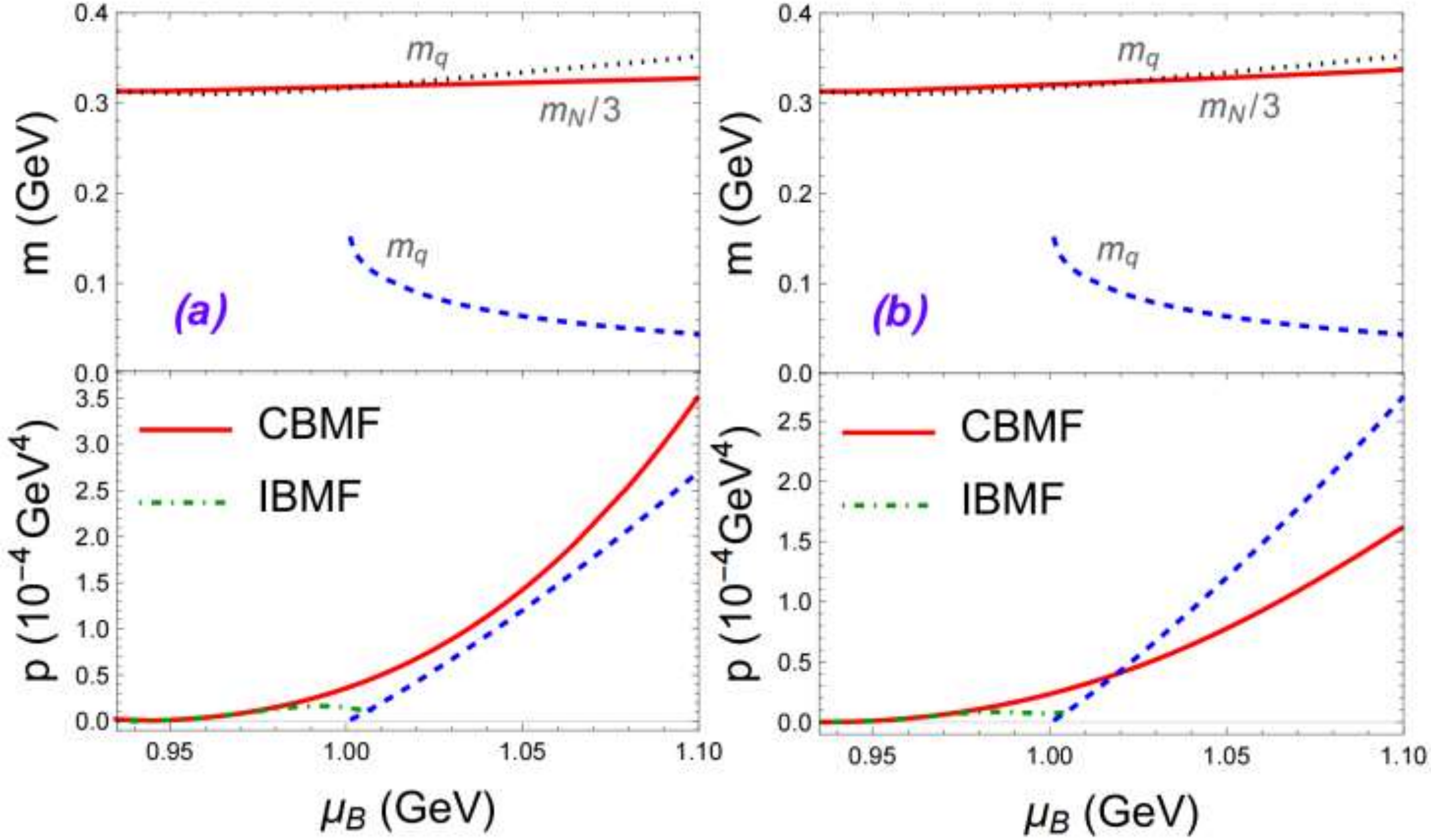
Small quark mass

$$\langle \bar{\rm u}{\rm u}\rangle = \langle \bar{\rm d}{\rm d}\rangle = -(0.25\ {\rm GeV})^3 \quad m_{\rm q} = 0.3135\ {\rm GeV}$$

Large quark mass

$$\langle \bar{\rm u}{\rm u}\rangle = \langle \bar{\rm d}{\rm d}\rangle = -(0.249\ {\rm GeV})^3 \quad m_{\rm q} = 0.320\ {\rm GeV}$$

Results for small quark mass

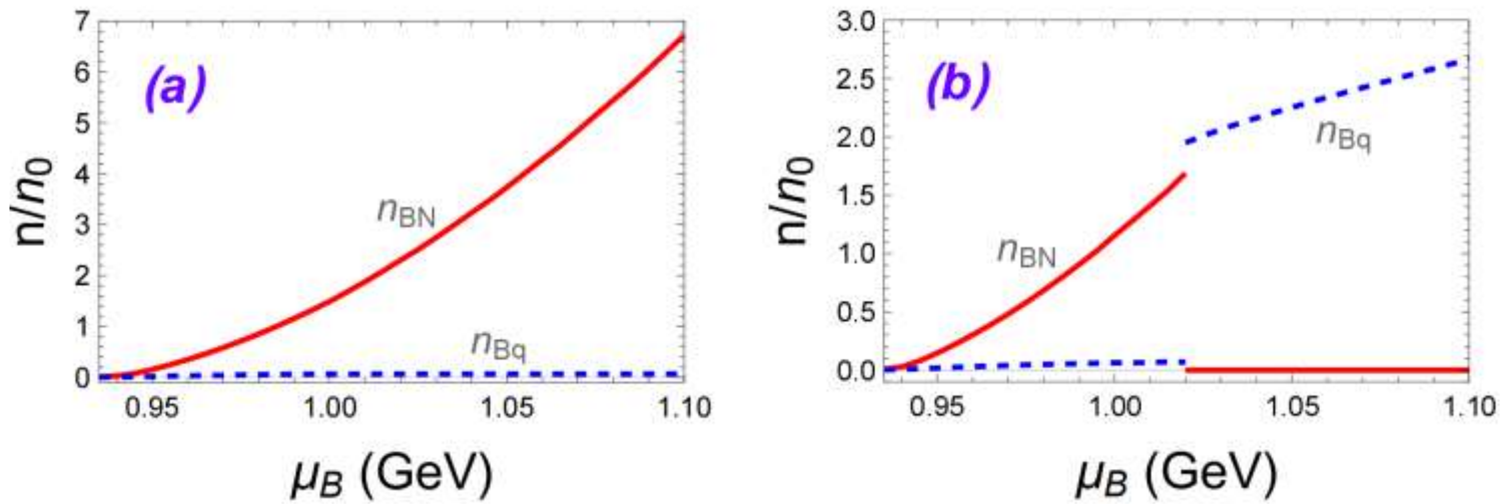


IBMF: dynamical mass m_q with Ω_0

CBMF: dynamical mass m_q with $\Omega_0 + \Omega_N$

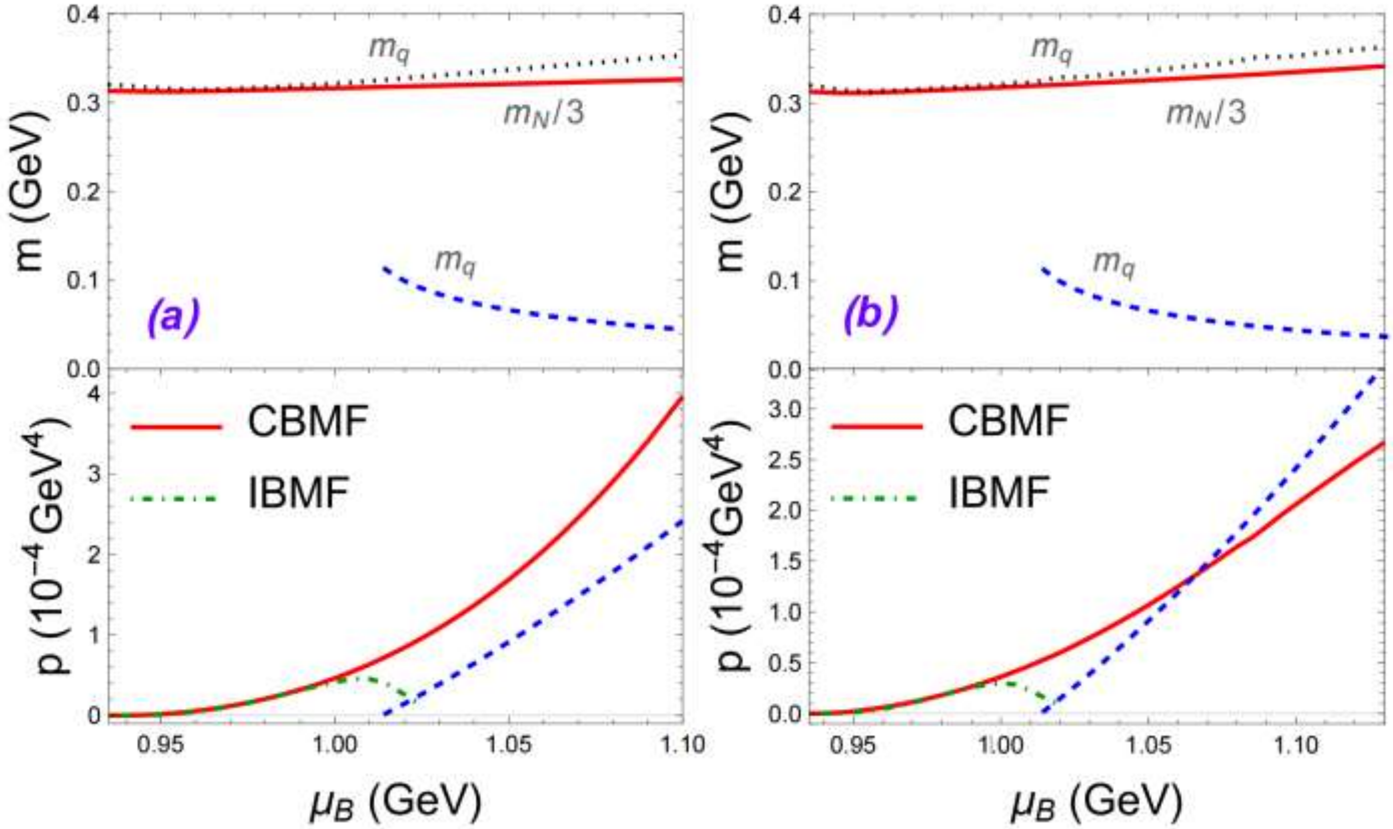
1. For larger d, there is first-order transition between nuclear and quark matters

2. Associated with chiral symmetry restoration

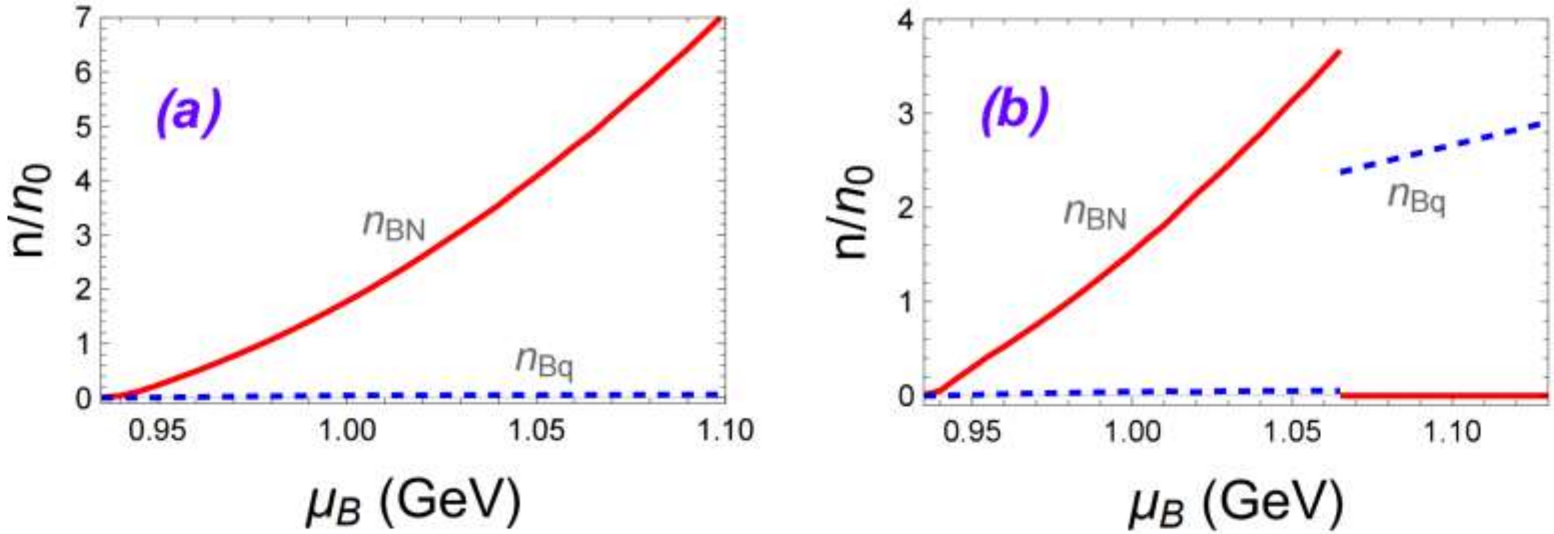


3. no quarkyonic matter

Results for large quark mass



For larger d, there is first-order transition



(b) Baryon density decreases with chemical potential at the transition point

Nonphysical

Summary and perspectives

- Establish a self-consistent NJL model for quark and nuclear matters

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ + \gamma^0\frac{\mu_\mathrm{B}}{3} - m_0\right)\psi + G_2\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right]$$

$$- \left(\bar{\psi}\tau_2\epsilon_\mathrm{c}\gamma_5\psi_C\right)\bar{\psi}^\mathrm{c}\tilde{G}_3\left(i\partial\!\!\!/ + \gamma^0\mu_\mathrm{B}\right)\psi^{\mathrm{c}'}\left(\bar{\psi}_C\tau_2\epsilon_{\mathrm{c}'}\gamma_5\psi\right)$$

- For small quark mass and large d， there is first-order transition between nuclear and quark matters, but no quarkyonic matter

➢ Saturation properties and neutron stars