A review of QCD phase diagram in functional QCD approaches

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fQCD collaboration :
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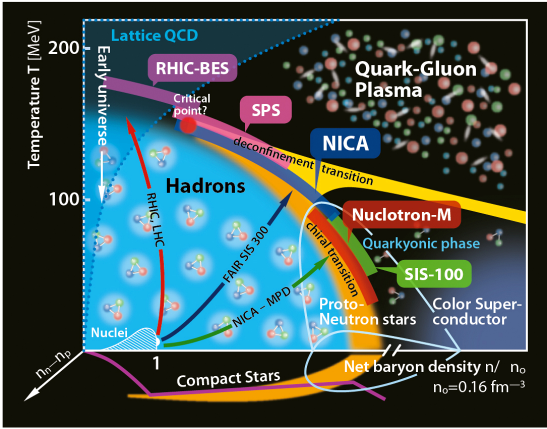
QCD in heavy ion collisions:

Roughly speaking, a transition between hadrons and asymptotic quarks/gluons.

- Transitions coming from the mass scale change of quark and gluon:
  Quark mass generation=Chiral PT
  Gluon mass generation ≈ Deconfinement
- The chiral phase transition dominates the phenomenology in HIC, and connected with crossover at low density by critical end point (CEP)
- Fine structures to classify phases like chiral spin symmetric phase, inhomogeneous phase/Moat regime, quarkyonic phase, color superconductivity phase



The principle of fQCD: We don't do models, we do simplification.

QCD in vacuum:

Cyrol, Mitter, Pawlowski, Strodhoff, PRD 97 (2018) 5, 054006.

Binosi, Chang, Papavassiliou, Qin, Roberts, PLB 742, (2015) 183

Williams, Fischer, Heupel, PRD 93, (2016)034026.

Mitter, Pawlowski, Strodthoff, PRD 91, (2015)054035.

Qin, Chang, Liu, Roberts, Schmidt, PLB 722 (2013) 384

Chang, Roberts, PRL 106 (2011) 072001 ...

Yang-Mills sector:

Eichmann, Pawlowski, Silva, PRD 104 (2021) 11, 114016

Aguilar, Ferreira, Papavassiliou, PRD 105 (2022) 1, 014030

Huber, PR 879, 1 (2020)

Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

Aguilar, Binosi, Papavassiliou, PRD 86 (2012) 014032 ...

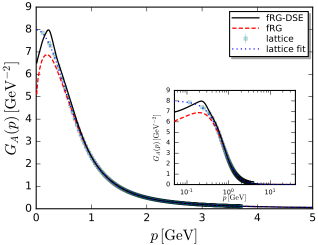
Phase Structure: Fu, Pawlowski, Renneke, PRD 101 (2020) 5, 054032; Gao, Chen, Liu, Roberts, Schmidt, PRD 93 (2016) 9, 094019; Fischer, PPNP 105,(2019)1;Fischer, Luecker, Welzbacher, PRD 90 (2014) 034022 Isserstedt, Buballa, Fischer, PRD 100 (2019) 7, 074011; Qin, Chang, Chen, Liu, Roberts, PRL106 (2011) 172301...

The minimal requirements for a truncation scheme that describes QCD:

- *Describe the running mass of quark and gluon*
- *Describe the running of the coupling*

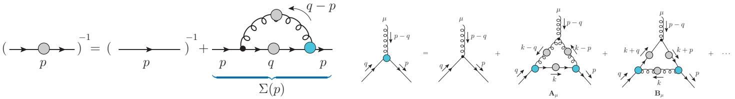
The minimal scheme

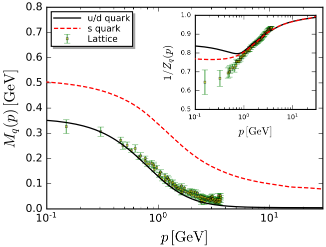
The Yang-Mills sector is relatively separable. One can apply the data in vacuum and compute the difference between finite T/μ and vacuum.



Lattice:
A. G. Duarte et al, PRD 94, 074502 (2016),
P. Boucaud et al, PRD 98, 114515 (2018),
S. Zafeiropoulos et al, PRL122, 162002 (2019)
fRG:
W.-j. Fu et al, PRD 101, 054032 (2020)
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

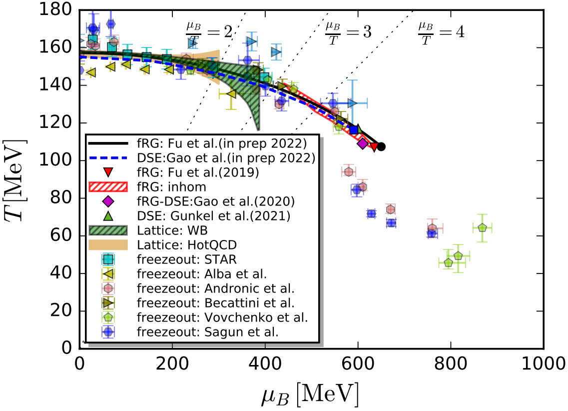
Solve the DSEs of quark propagator and quark gluon vertex:





lattice: P. O. Bowman et al, PRD71, 054507 (2005) **fRG**: W.-j. Fu et al, PRD 101, 054032 (2020) **DSE**: FG et al, PRD 103, 094013(2021)

Phase diagram in temperature-chemical potential region for 2+1 flavour QCD



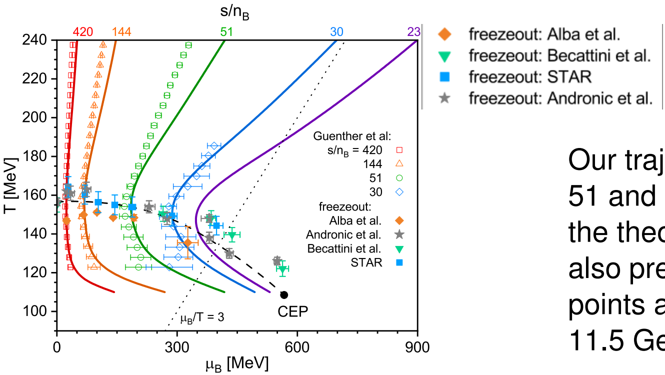
The fQCD computations of chiral phase transition are converging:

- $T_c = 155$ MeV and $\kappa \sim 0.016$
- Estimated range of CEP:
  $T \in (100, 110)$ MeV
  $\mu_B \in (600, 700)$ MeV
- $\sqrt{s_{NN}} \approx 3 - 5$GeV

W.-j. Fu et al, PRD 101, 054032 (2020)
FG and J. Pawlowski, PRD 102, 034027 (2020)
FG and J. Pawlowski, PLB 820, 136584(2021)
P.J. Gunkel, C. S. Fischer, PRD 104, 054022 (2021).

QCD thermodynamic properties

isentropic trajectories in the up to date scheme:



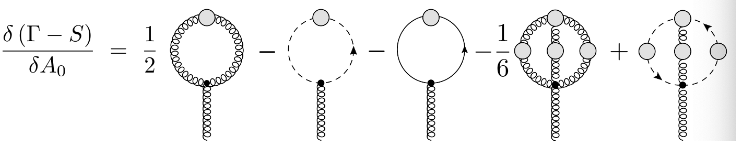
Our trajectories for $s/n_B = 420, 144,$ 51 and 30 which values are chosen in the theoretical studies,
also precisely meet with the freezeout points at $\sqrt{s_{\rm NN}} = 200, 62.4, 19.6$ and 11.5 GeV, respectively.

At CEP, $s/n_B \approx 10.$

Polyakov loop:

- Related to the $Z(N_c)$ center symmetry
- Nontrivial stationary point in gauge field potential.
- equivalent to A_0 condensate in background field, which can be computed via

The DSE of Polyakov loop:



A_0^a condensate= colored imaginary chemical potential

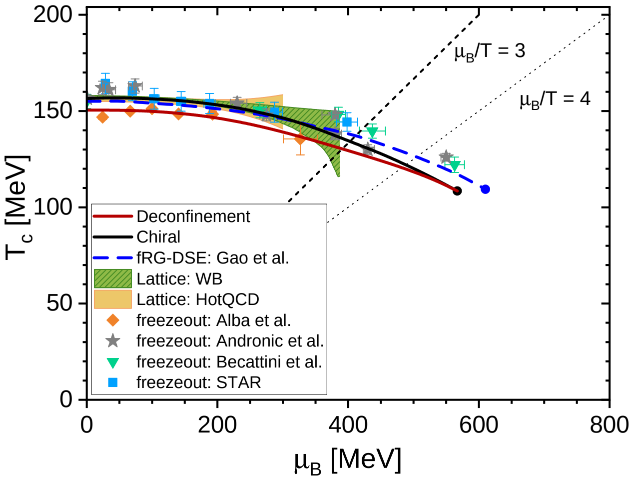
A_0 feeds back on the QCD thermodynamic functions via the quark propagator:

$$G_q^{-1}(p) = i(\omega_p + i\mu_q + gA_0)\gamma_4 Z_q^E(p) + i\vec{\gamma}\cdot\vec{p}\, Z_q^M(p) + Z_q^E(p) M_q(p).$$

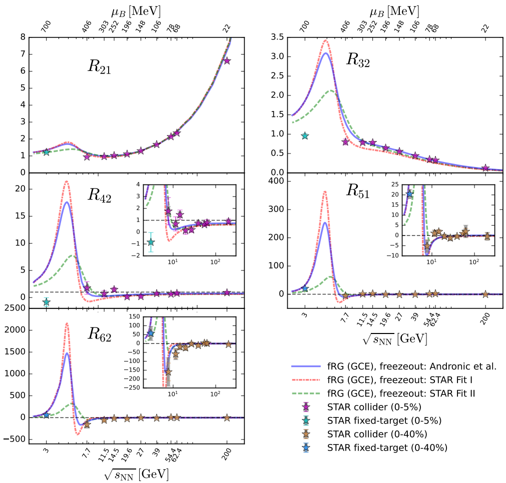
C. S. Fischer et al, PLB 732, 273(2014)

Yi Lu, FG, J. Pawlowski, Yuxin Liu, in preparation

The deconfinement characterized by Polyakov loop is in agreement with Chiral PT with the same CEP location.



The comparisons of kurtosis between fQCD and BESI

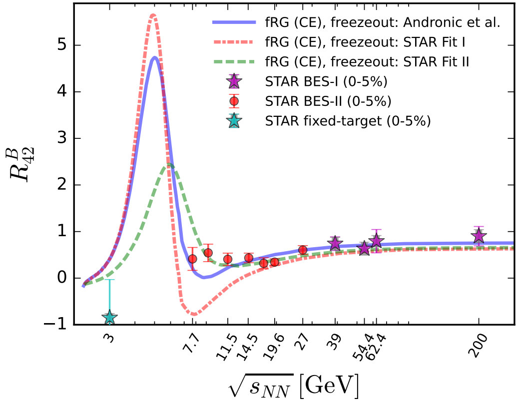


$$R_{nm} = \frac{\chi_n^B}{\chi_m^B}$$

- Good agreements between R_{32} and R_{21} for $\sqrt{s_{\rm NN}} \geq 11.5$ GeV.
- For lower col. energy, the non-equilibrium and finite volume effect on R_{32} and R_{21} becomes sizable.
- No wiggling behavior/peak structure in the BES energy range in R_{42} from fQCD since the CEP is at $\sqrt{s_{\rm NN}} \approx 3 - 5$ GeV.

W. Fu, X. Luo, J. Pawlowski, F. Rennecke, S. Yin, arXiv:2308.15508. Y Shen, W Chen, X Wu, K Xu, M Huang, arXiv:2404.02397

The comparisons of the cumulants between fQCD and BESII



W. Fu, X. Luo, J. Pawlowski, F. Rennecke, S. Yin, arXiv:2308.15508.

CEP estimation from fQCD:
$\mu_B \in (600 - 700)$ *MeV*
$\sqrt{s_{NN}} \approx 3 - 5$ *GeV*

W.-j. Fu et al, PRD 101, 054032 (2020)

FG and Pawlowski, PRD 102, 034027 (2020)

FG and Pawlowski, PLB 820, 136584(2021)

Yi Lu et al, arXiv:2310.18383 (2023)

Huiwen Zheng et al, arXiv:2312.00382 (2023).

...

Consistent results also from Holographic QCD (S. He, D. Li, M. Huang) and Lee-Yang zero extrapolation from lattice QCD.

Chiral PT and the measured cumulants

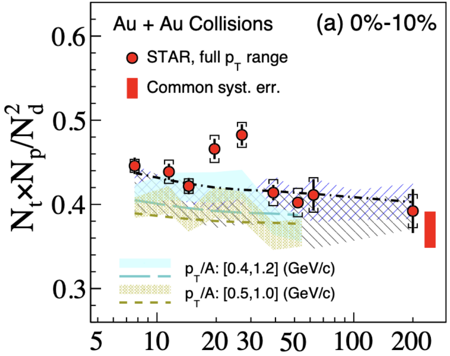
Consistences:

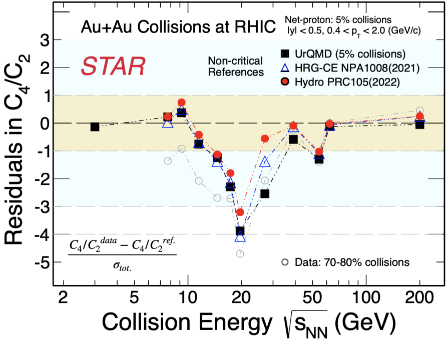
- The measured cumulants in BESII are consistent with the computation from fQCD which estimates both the CEP of Chiral and deconfinement PT at $\sqrt{s_{\mathrm{NN}}} \approx 3 - 5$ GeV;
- No critical behavior observed in RHIC; no critical behavior from Chiral and deconfinement PT at $\sqrt{s_{\mathrm{NN}}} > 7.7$ GeV.

However, finer comparison reveals some inconsistencies at $\sqrt{s_{\mathrm{NN}}} \approx 20 - 30$ GeV:

- Triton yield ration $N_t \times N_p / N_d^2$ even from BESI (STAR, PRL 130, 202301(2023));
- Kurtosis in BESII deviates from UrQMD, HRG and Hydro simulations.

Finer structure in the measurement



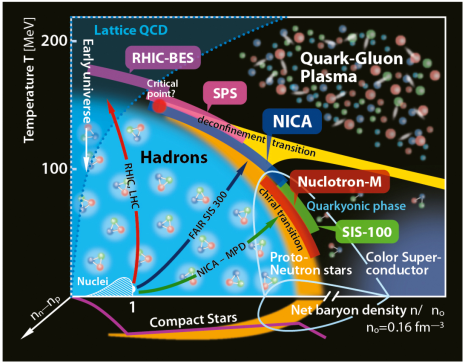


- Density Inhomogeneity from first order phase transition can enhance the Triton yield ratio (Kaijia Sun et al, arXiv: 2205.11010) and also affects the fluctuations.
- Cannot be Chiral and deconfinement PT because the col. energy is too large.

If something happens at $\sqrt{s} \sim 20 - 30$ GeV, what would it be?

Besides of two phase transitions, QCD has rich phases above phase transition T_c:

- Strongly coupled quark gluon plasma (sQGP) at low μ
- The counterparts of sQGP at large μ: inhomogeneous phase/Moat regime, quarkyonic phase, color superconductivity (CSC) phase
- enhanced chiral spin symmetry, small viscosities, exotic behaviors of thermodynamic quantities....



How the sQGP evolves into CSC phase with increasing μ?
Is there another phase transition there?

The conventional CSC phase

The conventional CSC phase can only exist at high chemical potential and hence low temperature.

The Cooper pair Δ in conventional CSC is generated through the gap equation as:

$$\Delta = g^2 T \sum_m \int d^3\vec{q} \frac{\Delta}{q^2} G(p-q)$$

This type of propagator gives a gap that is proportional to chemical potential μ as $\Delta \sim \mu \, e^{-\frac{\mathrm{const}}{g}}$ in weak coupling limit. (D. Son, PRD 59, 094019 (1999); R. Pisarski, D. Rischke, PRD 61, 074017 (2000))

The conventional CSC in QCD is based on the Abelian approximation (bare or BC type vertex)and thus obtains the same type of pairing as in QED.

see Review in M. G. Alford, et al, RMP 80, 1455 (2008).

and Refs. A. Schmitt, Q. Wang, D. Rischke, PRL 91 (2003) 242301; M. Huang, P. Zhuang, W. Chao, PRD 67 (2003) 065015; L. He, M. Jin, P. Zhuang, PRD 71 (2005) 116001; D. Hou, Q. Wang, D. Rischke, PRD 69 (2004) 071501; I. Giannakis, D. Hou, H. Ren, D. Rischke PRL 93 (2004) 232301;D. Nickel, et al, PRD 73, 114028 (2006)

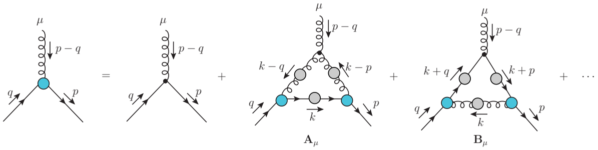
The DSEs in Nambu Gorkov basis

To study the quark pairing in QCD, one needs to compute the gap equation i.e. the quark propagator Dyson-Schwinger equation, in the Nambu-Gorkov basis. It is to extend the fermion field as:

$$\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix}, \qquad \overline{\Psi} = (\overline{\psi}, \overline{\psi}_C)\,,$$

The quark gluon vertex is essential input for the quark pairing gap equation.

In vertex DSE, diagram A is non Abelian diagram and diagram B is the Abelian diagram similar to QED.



The quark gluon vertex in NG basis

The dynamics related to diagram A is very different from that from diagram B.

With diagram A:

- In ultraviolet region with for instance $p \to 0$, the term $Z_1\Delta$ is dominant as Z_1 is proportional to $1/p^2$ and leads to the coefficient of quark gluon vertex $t_4 \sim \Delta/p^2$.
- In the infrared limit, Z_1 and Z are finite constants. Considering Δ to be small, the two solutions become $t_4 = Z_1\Delta$ and $t_4 = \frac{1}{Z\Delta}$.

With diagram B:

- one only gets the first solution $t_4 \propto \Delta$;

- *The second solution of quark gluon vertex is unique in non-Abelian theory.*

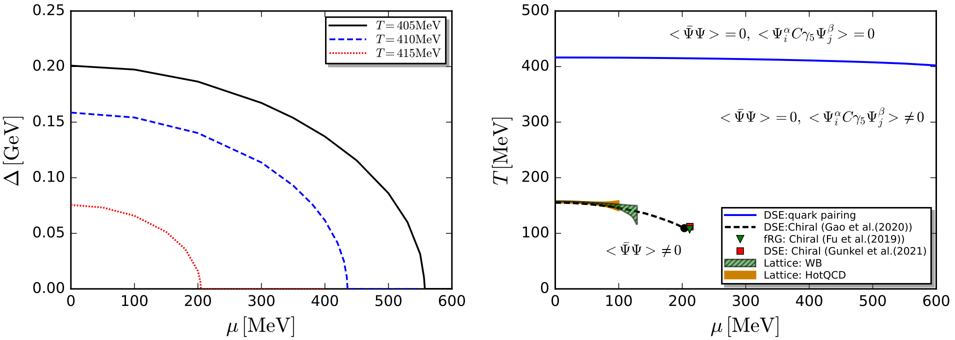
- With $t_4 \propto \frac{1}{\Delta}$, the gap equation becomes: $\Delta \propto \int \frac{Z}{k^2\Delta} G(\bar{k}^2)$

For $Z > 0$, a finite solution for Δ; For $Z < 0$, the trivial solution as $\lambda_4 = \Delta = 0$.

- With $t_4 \propto \Delta$, the gap equation becomes: $\Delta \propto \int_k \frac{\Delta}{k^2} G(\bar{k}^2)$

which gives the conventional CSC gap and proportional to chemical potential μ.

Phase diagram of the pairing



The pairing phase in $T - \mu$ plane:

- Represents a color deconfined phase above the chiral phase transition;
- Quarks are confined into colored bound states as a partial deconfined phase;
- Temperature range $T \in [T_c, T_\Delta \approx 2 - 3\,T_c]$, overlaping with Chiral Spin Symmetric phase and the other conjectured strongly coupled states in sQGP.

Combination of fQCD computations and BES measurements

The conclusions and discussions:

- The chiral PT and deconfinement is with a CEP at $\mu_B \approx 600 - 700$ MeV/ $\sqrt{s_{\rm NN}} \approx 3 - 5$ GeV, which is consistent with the current measurements in BESII.
- The deviation of the measurements and the transport model/hydro-simulations at $\sqrt{s_{\rm NN}} \approx 20 - 30$ GeV, cannot be explained through the critical behavior of chiral phase transition and deconfinement.
- The phase structures near above T_c are rich and can be possibly unified by a quark pairing gap (No first order phase transition though).

Is there something new in heavy ion collisions at $\sqrt{s_{\rm NN}} \approx 20 - 30$ GeV?

The answer is possibly in the phases in the temperature region $T \in [T_c, 3T_c]$.

Thank you!