Lattice QCD study of heavy quark diffusion

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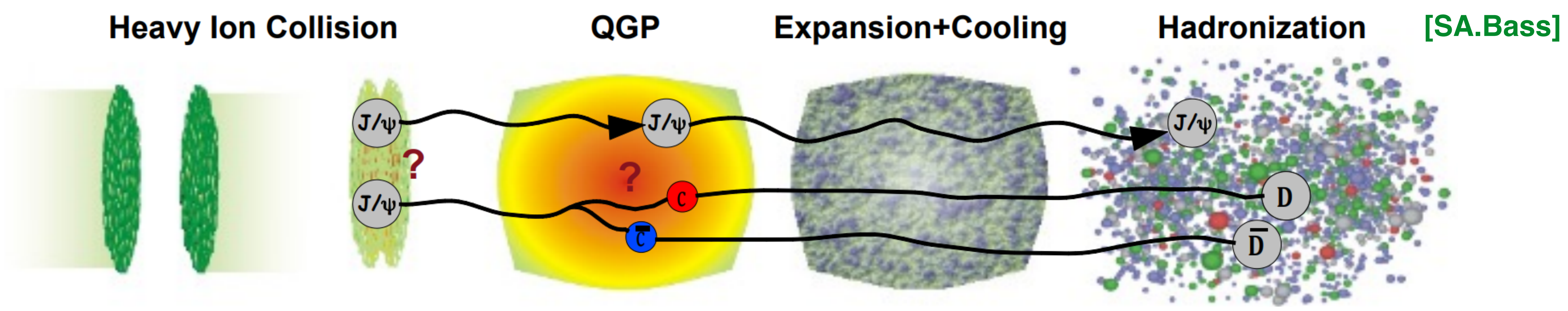
第十四届全国粒子物理学术会议

Aug. 13 - 18, 2024, Qingdao, Shandong

Based on:
[PRD 103(2021) 1, 014511]
[PRL 130 (2023) 23, 231902]
[PRL 132 (2024) 5, 051902]
[PRD 109 (2024) 11, 114505]

Heavy quark diffusion in HICs



Release constituents equilibrate via diffusion process

how fast do heavy quarks equilibrate?

- Perturbative estimates: [G. Moore and D. Teaney, PRC.71.064904]

  $\tau_{\mathrm{kin,charm}} \sim 6\ \mathrm{fm/c} \gg \tau_{\mathrm{kin,light}} \sim 1\ \mathrm{fm/c}$

- Experimental estimates (RHIC): [STAR Collaboration, PRL,106 (2011) 159902]

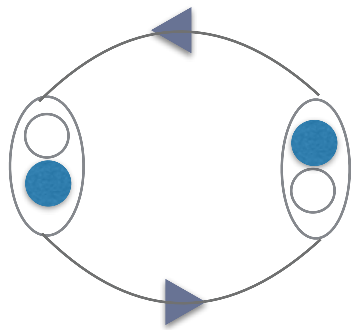
  $\tau_{\mathrm{kin,charm}} \approx \tau_{\mathrm{kin,light}}$

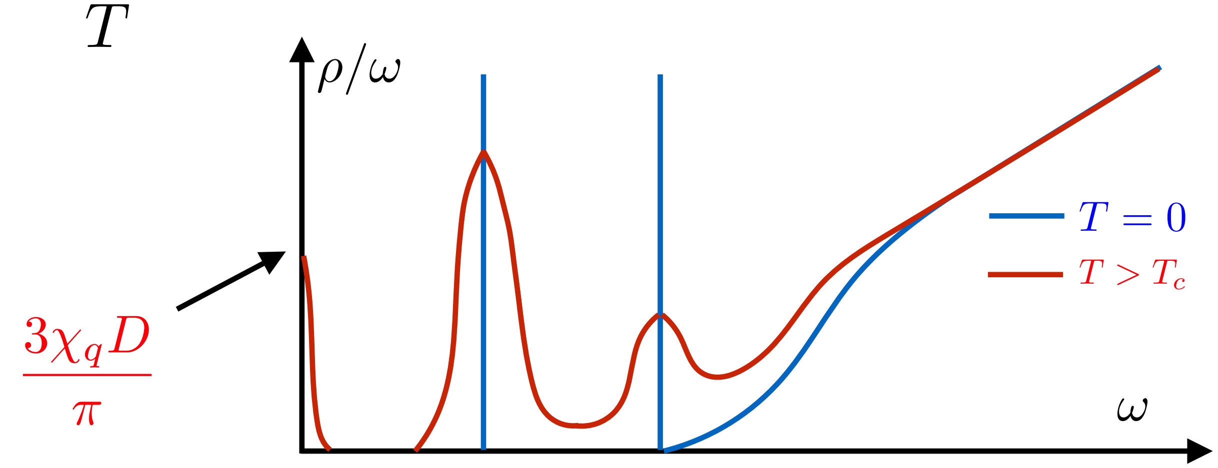
Need non-perturbative ab-initio determination for equilibration time!

Traditional method via meson correlation function

Equilibration time ~ HQ diffusion coefficient

$$\tau_{\rm kin} = \frac{1}{\eta_D} = \frac{DM}{T}$$

$J_H(0,\mathbf{0})$  $J_H(\tau,\boldsymbol{x})$



$$G_H(\tau,\vec{p}) = \sum_{x,y,z} exp(-i\,\vec{p}\cdot\vec{x})\langle J_H(0,\vec{0})J_H^{\dagger}(\tau,\vec{x})\rangle = \int \frac{d\omega}{2\pi}\frac{\cosh(\omega(\tau T-1/2)/T)}{\sinh(\omega/2T)}\rho_H(\omega,\vec{p},T)$$

ill-posed

$K(\omega,\tau,T)$

- HQ diffusion embedded in meson spectral function
- Complicated structure of meson spectral function

PRD 97, 094503

PRD 104 (2021) 11, 114508

Easier alternative: infinite heavy quark mass limit

$$\partial_t p_i = -\eta_D p_i + \xi_i(t) \qquad \langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t')$$

- Mass dependent **momentum** diffusion coefficient

$$\kappa^{(M)} \equiv \frac{M^2\omega^2}{3T\chi_q}\sum_i \frac{2T\rho_V^{ii}(\omega)}{\omega}\Big|^{\eta \ll |\omega| \lesssim \omega_{\rm UV}}$$

- Large quark mass limit in HQ effective field theory

$$\kappa \equiv \frac{\beta}{3}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty}\frac{M^2}{\chi_q}\int_{-\infty}^{\infty}\mathrm{d}t\, e^{i\omega(t-t')}\int \mathrm{d}^3\vec{x}\,\left\langle \frac{1}{2}\{\mathcal{F}^i(t,\vec{x}),\mathcal{F}^i(0,\vec{0})\}\right\rangle\right]$$

$$\partial_t \mathbf{p} = q(\mathbf{E}+\mathbf{v}\times\mathbf{B}) = \mathbf{F}$$

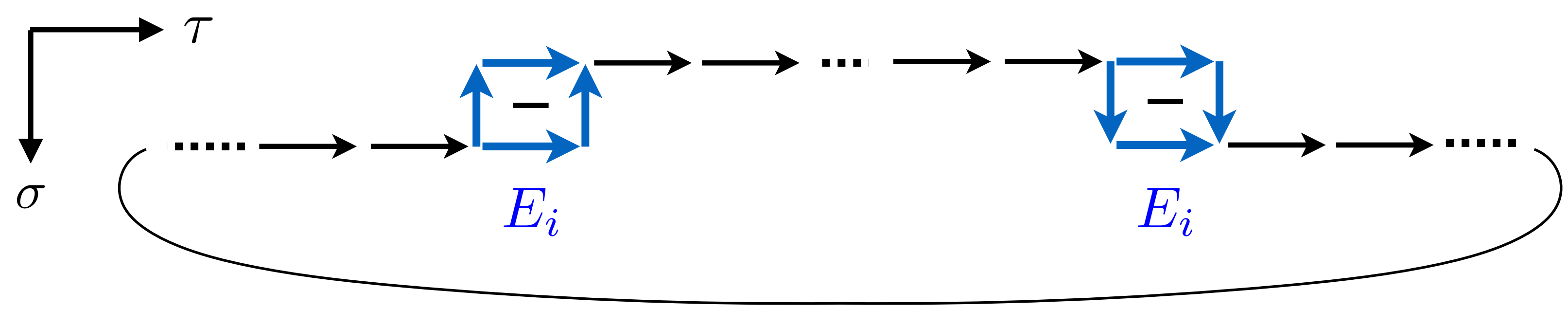
J. Casalderrey-Solana and D. Teaney, PRD 74, 085012

S. Caron-Huot et al., JHEP 0904 (2009) 053

A. Bouttefeux, M. Laine, JHEP 12 (2020) 150

Heavy quark momentum diffusion on the lattice

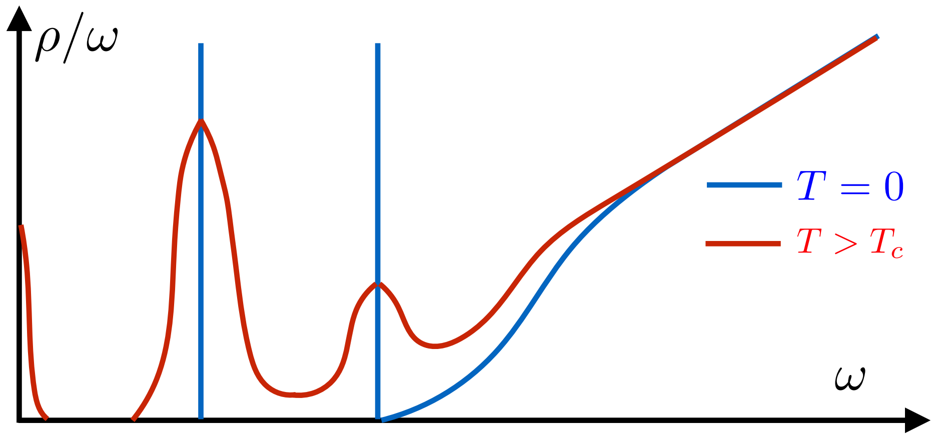
$$\langle \mathcal{F}(t')\mathcal{F}(t)\rangle = q^2\Big\{\langle E_i(t')E_j(t)\rangle + \text{mass correction}\Big\}$$

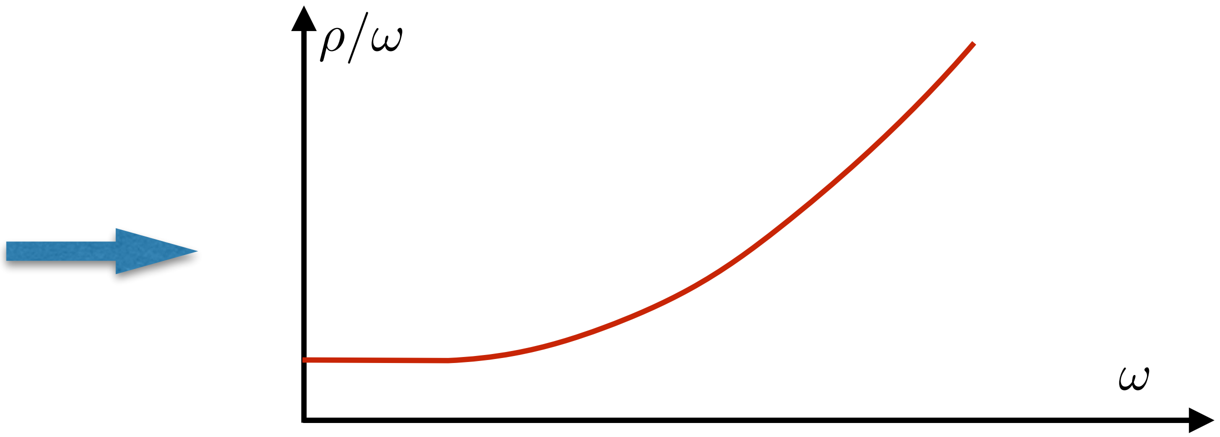


Color-electric field correlators: cheap to measure on the lattice !

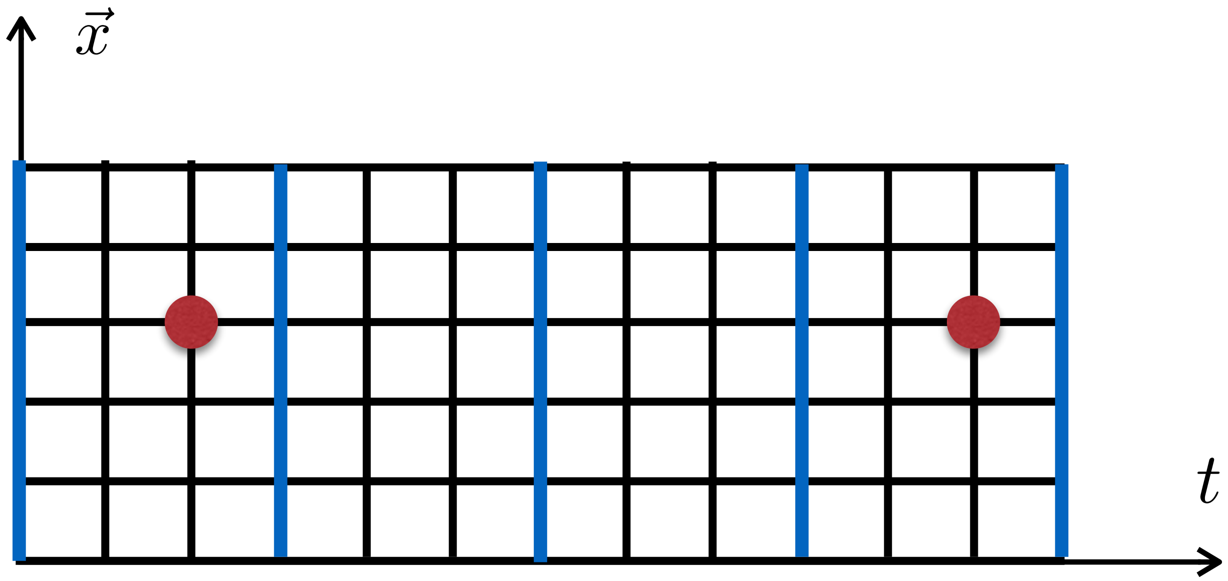
$$G(\tau,T) = \int \frac{d\omega}{\pi} K(\omega,\tau,T)\rho(\omega,T)$$

$$M_Q \to \infty: \quad \frac{1}{2\pi T D} = \frac{\kappa}{4\pi T^3} = \frac{1}{2\pi T^2}\lim_{\omega\to 0}\frac{\rho(\omega)}{\omega}$$





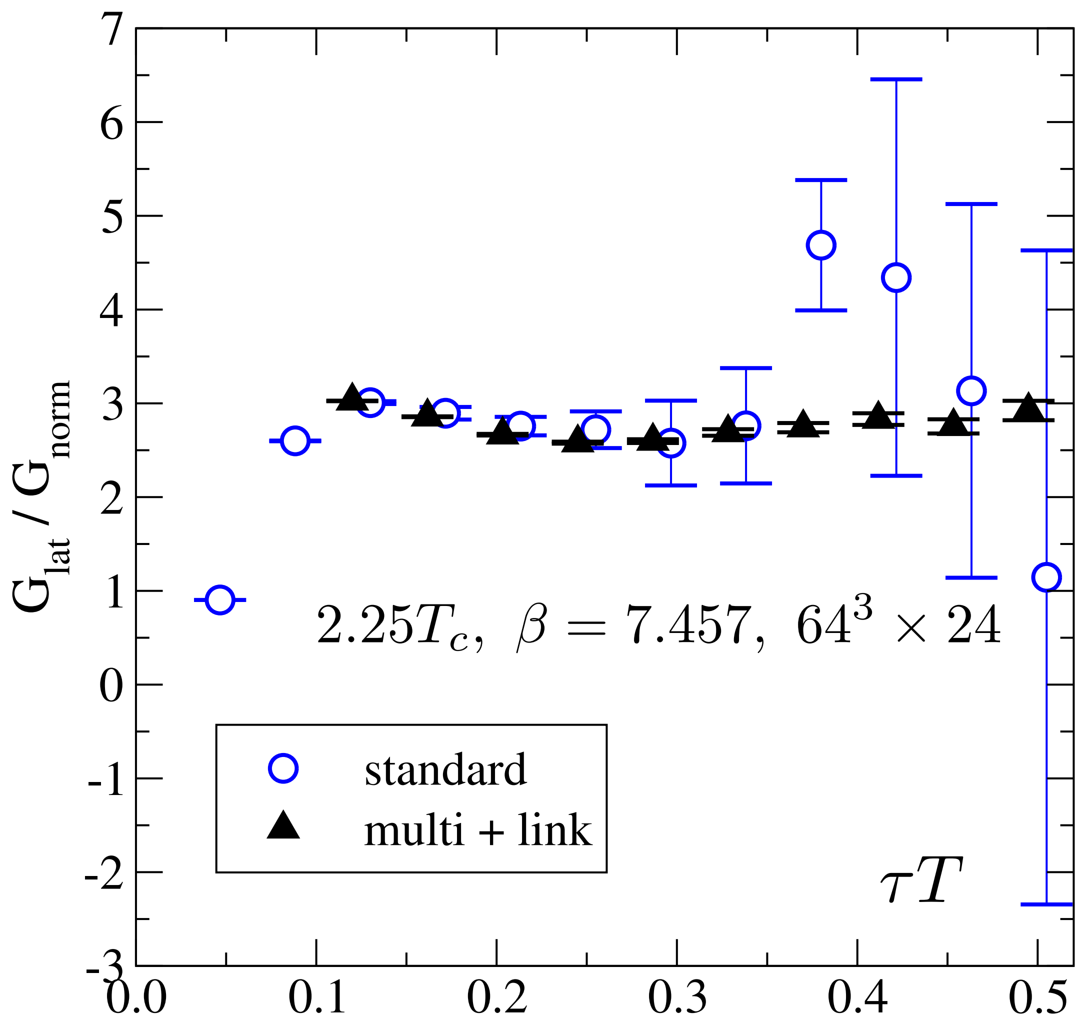
sketch of multi-level algorithm



Independent updates in each sub-lattice followed by a measuring of operator

[M. Luscher and P. Weisz, JHEP 09 (2001) 010]

G_{EE} from multi-level and link-integration:



[A. Francis, et al, PRD92 (2015)116003]

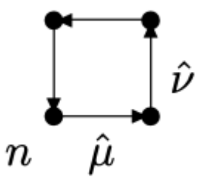
- Multi-level method reduces noise in correlators
- Multi-level is only applicable in quenched approximation

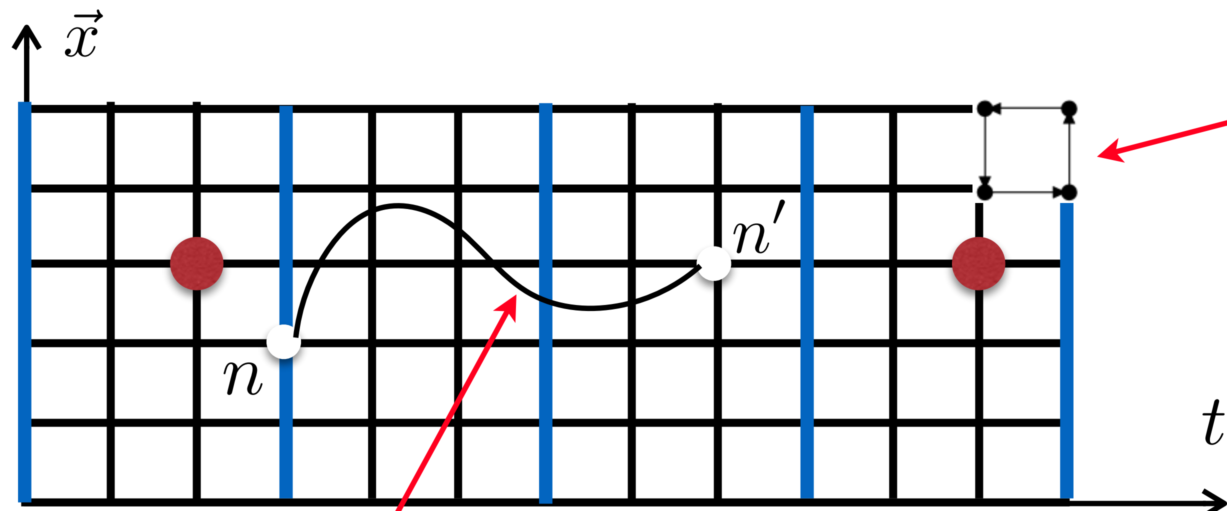
Breaking down of Multi-level algorithm in QCD

$$\mathcal{Z}(V,T) = \int [dU][d\psi][d\bar{\psi}]e^{-S_G[U]-S_F[U,\psi,\bar{\psi}]}$$

$$S_G[U] = \frac{1}{2g^2}\sum_{n,\mu,\nu} 2\mathrm{Tr}[1-P_{\mu\nu}(n)] \qquad S_F[u,\psi,\bar{\psi}] = \int dV \sum_{q=u,d,s,\ldots} \log(\det M_q[U])$$

Action **local in quenched QCD** (sum of plaq.): $P_{\mu\nu}(n) = U_\mu(n)U_\nu(n+\hat{\mu})U^\dagger_\mu(n+\hat{\nu})U^\dagger_\nu(n) =$





Action **non-local in full QCD** (connection between any two sites):

$$M_q(n,n';i,j)[U] = \hat{m}_q\delta_{n,n'}\delta_{ij} + \frac{1}{2}\sum_\mu \eta_\mu(n)\left((U_\mu(n))_{i,j}\delta_{n',n+\hat{\mu}} - (U^\dagger_\mu(n))_{i,j}\delta_{n,n'+\hat{\mu}}\right)$$

A long-standing problem

Journal of High Energy Physics

A way to estimate the heavy quark thermalization rate from the lattice

To cite this article: Simon Caron-Huot *et al* JHEP04(2009)053

View the article online for updates and enhancements.

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  Tomoki Hori, Takuya Mabuchi, Ikuya Kinefuchi et al.

A noise problem stuck for ~15 years!

Gradient flow

Evolve fields according to diffusion equations:

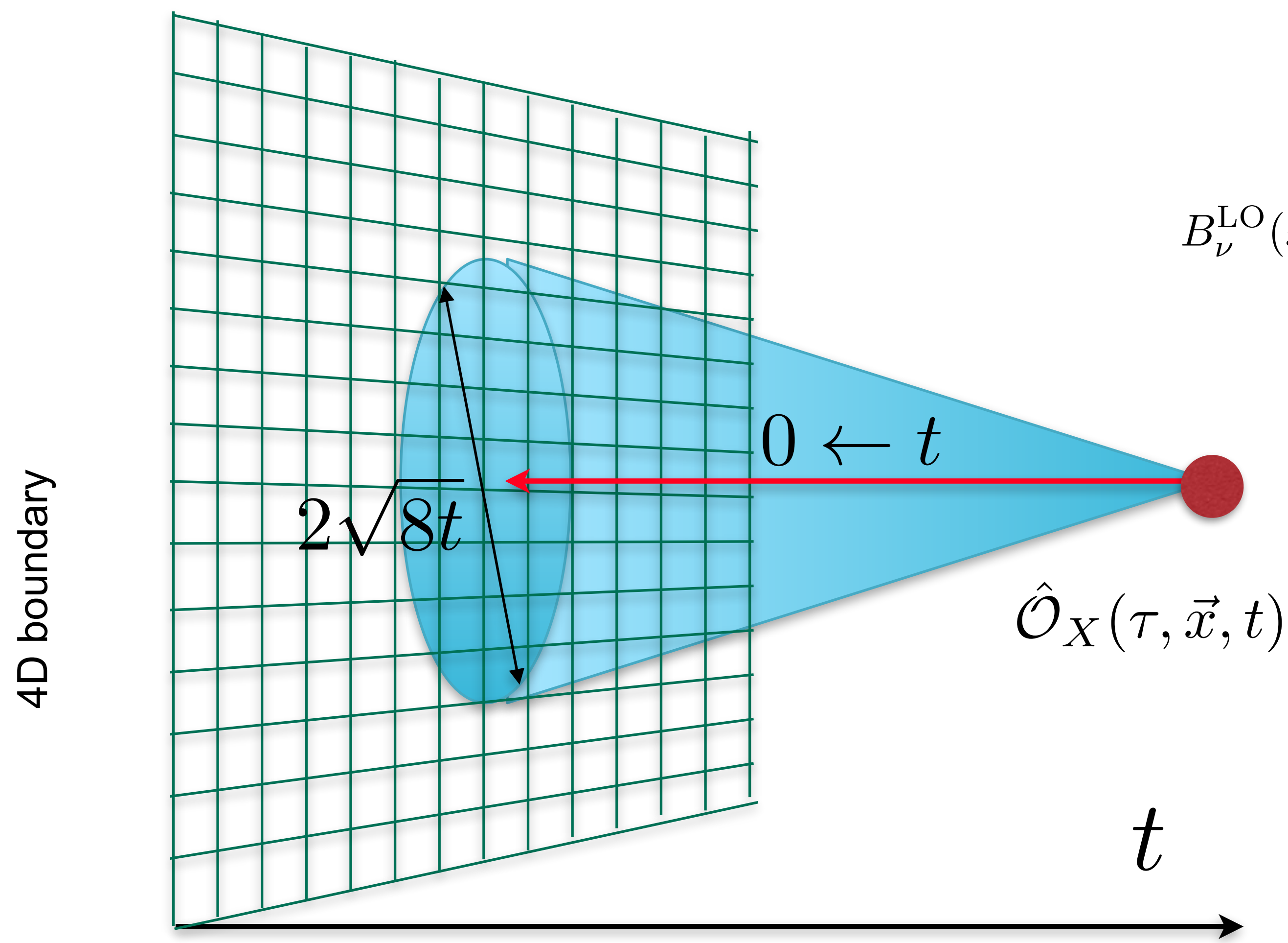
Luscher & Weisz, JHEP1102(2011)051
Narayanan & Neuberger, JHEP0603(2006)064

$$\frac{\mathrm{d}B_\mu(x,t)}{\mathrm{d}t} \sim -\frac{\delta S_G[B_\mu(x,t)]}{\delta B_\mu(x,t)} \sim D_\nu G_{\nu\mu}(x,t)$$

$$B_\nu(x,t)|_{t=0} = A_\nu(x)$$

$$\partial_t \chi(t,x) = [\Delta - \alpha_0 \partial_\mu B_\mu(t,x)]\chi(t,x), \qquad \chi(t=0,x) = \psi(x)$$

$$\partial_t \bar{\chi}(t,x) = \bar{\chi}(t,x)[\overleftarrow{\Delta} + \alpha_0 \partial_\mu B_\mu(t,x)], \qquad \bar{\chi}(t=0,x) = \bar{\psi}(x)$$



smearing radius:

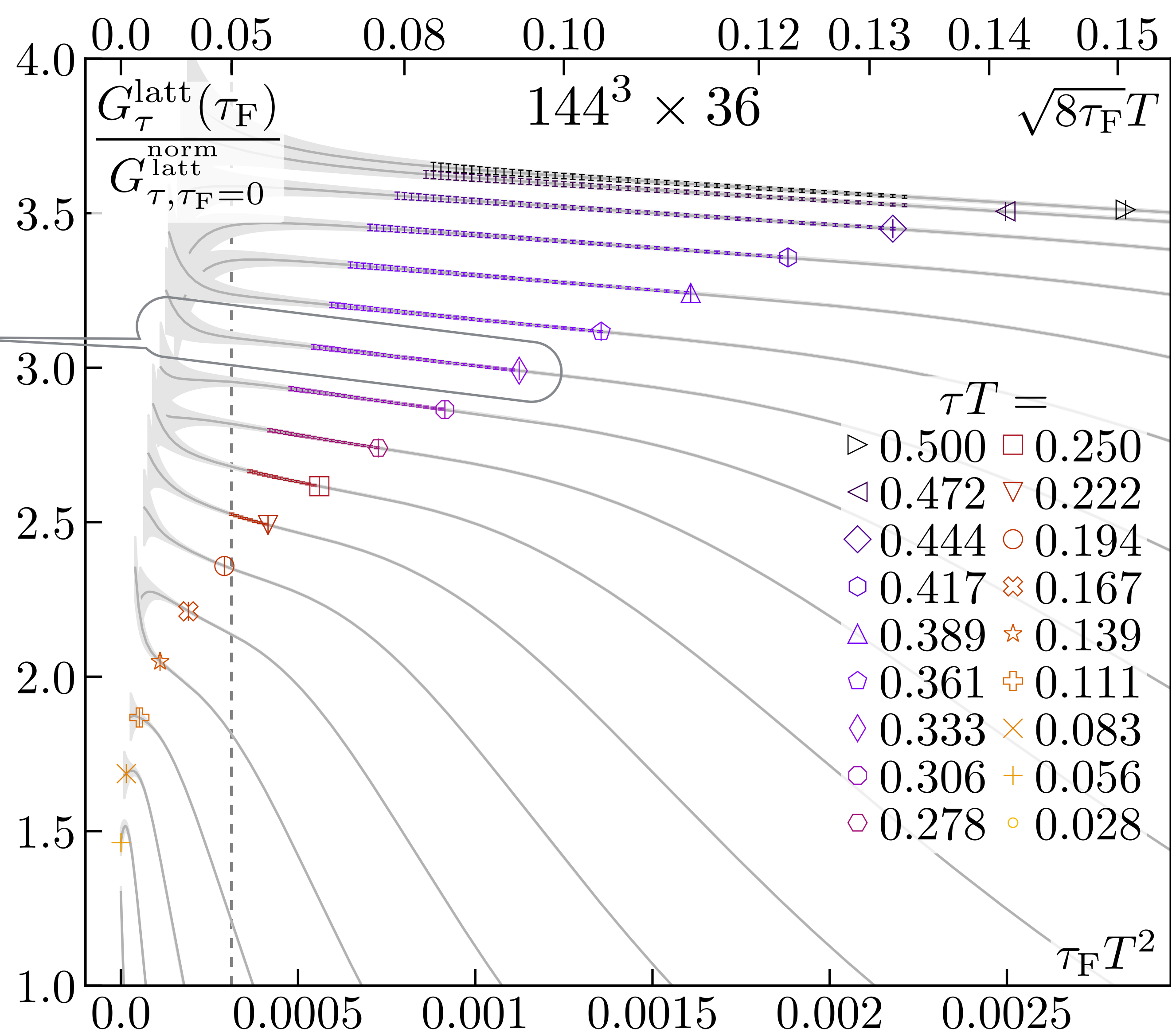
$$B_\nu^{\mathrm{LO}}(x,t) = \int \mathrm{d}y(\sqrt{2\pi}\sqrt{8t}/2)^{-4} \exp\left(\frac{-(x-y)^2}{\sqrt{8t}^2/2}\right) B_\nu(y)$$

The only solution for now!

- Smear the fields along a 5th dimension — flow time *t*
- Good signal at finite *t*
- Back to 4D space defined at zero flow (keeping good signal)

Smearing effects on the correlators

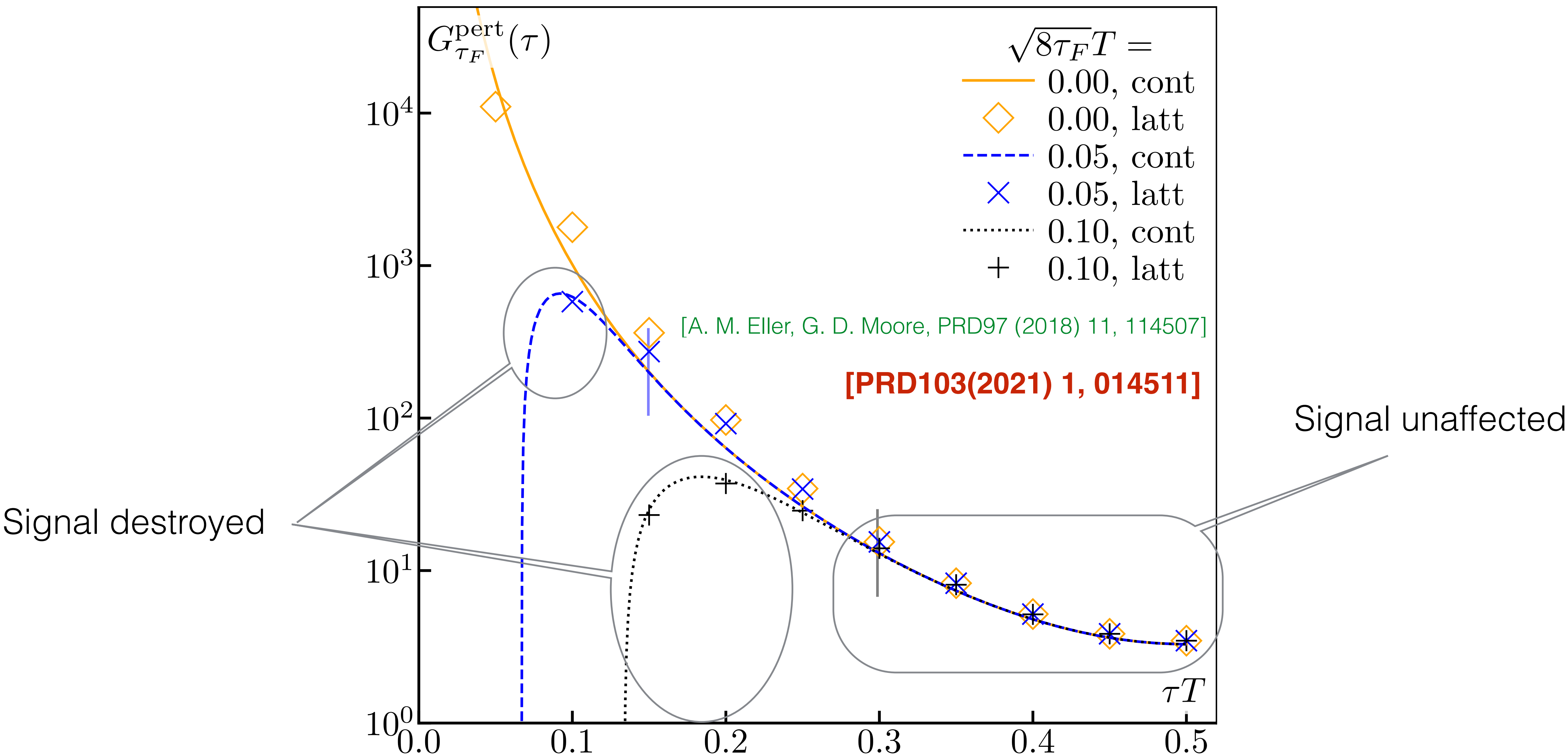
Rapid decrease of error bar



[PRD103(2021) 1, 014511]

- Significant reduction of noise along the flow

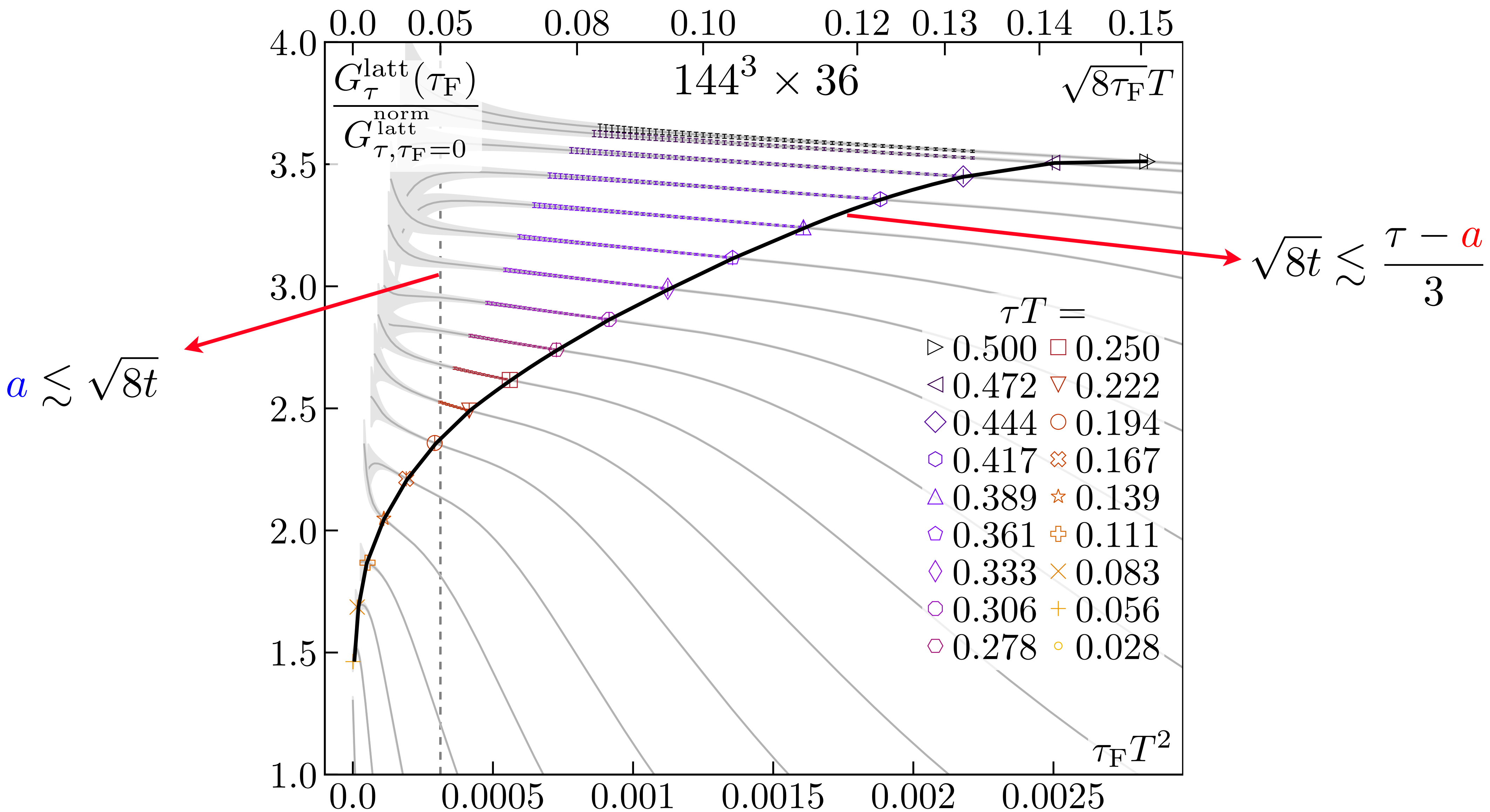
Flow time window from pert. analysis



- At most 1% deviation determines the maximum flow time:

$$a \lesssim \sqrt{8t} \lesssim \frac{\tau - a}{3}$$

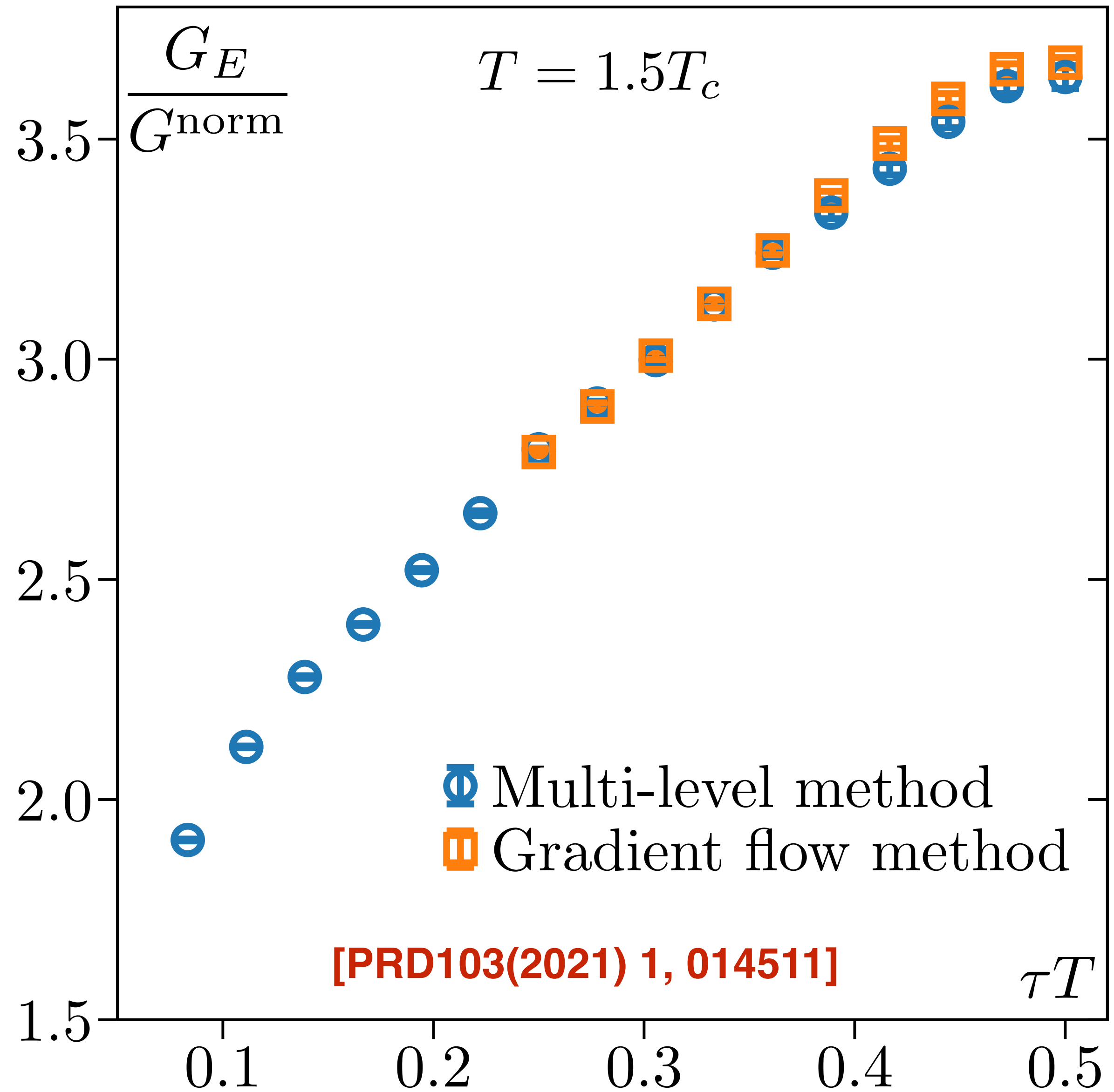
Bring correlators back to 4D space



[PRD103(2021) 1, 014511]

- Linear flow time extrapolation within flow time window

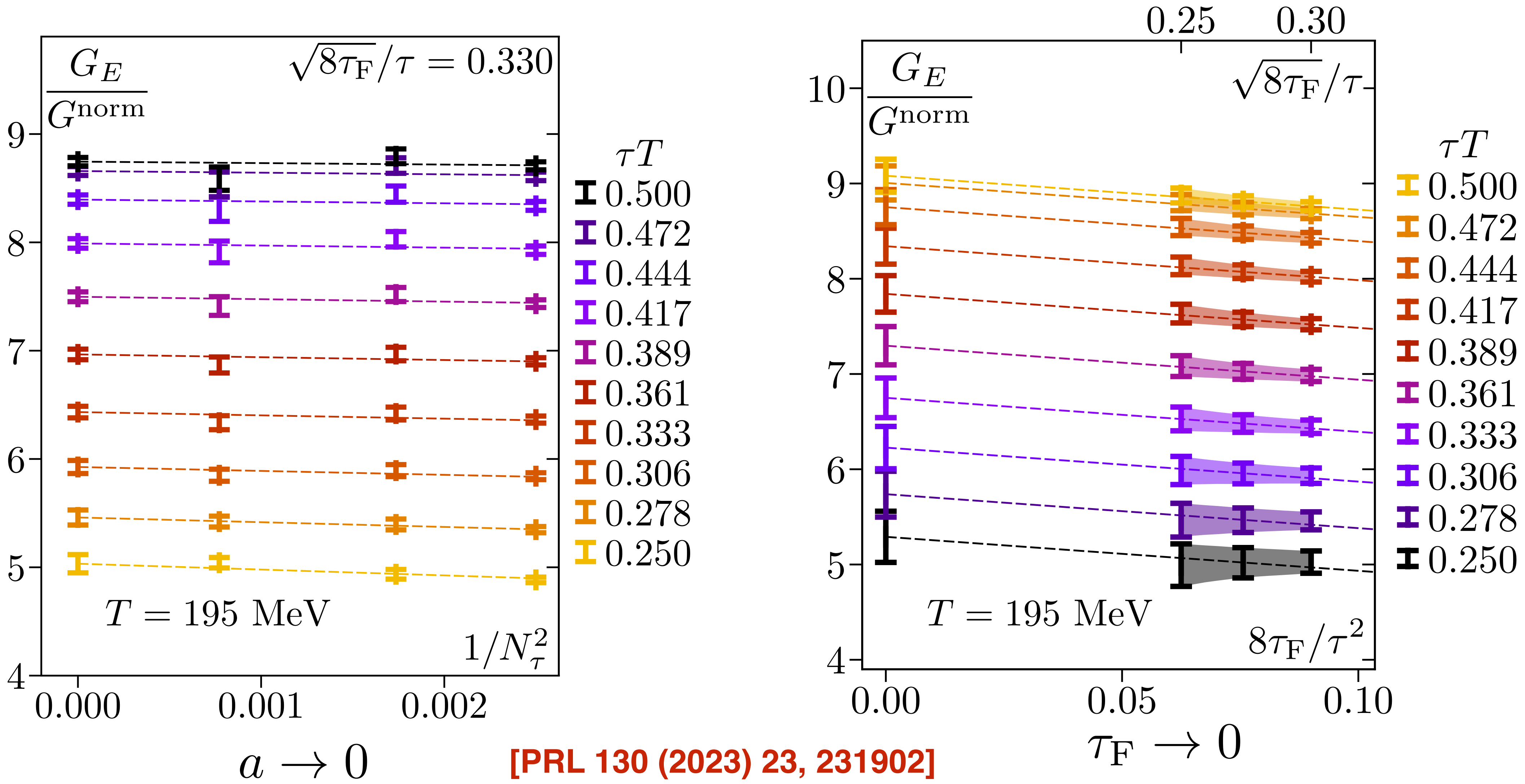
Develop methodology in quenched approximation



- Consistent results from ML & GF
- Gradient flow paves the way to full QCD

Extension to QCD: double extrapolation

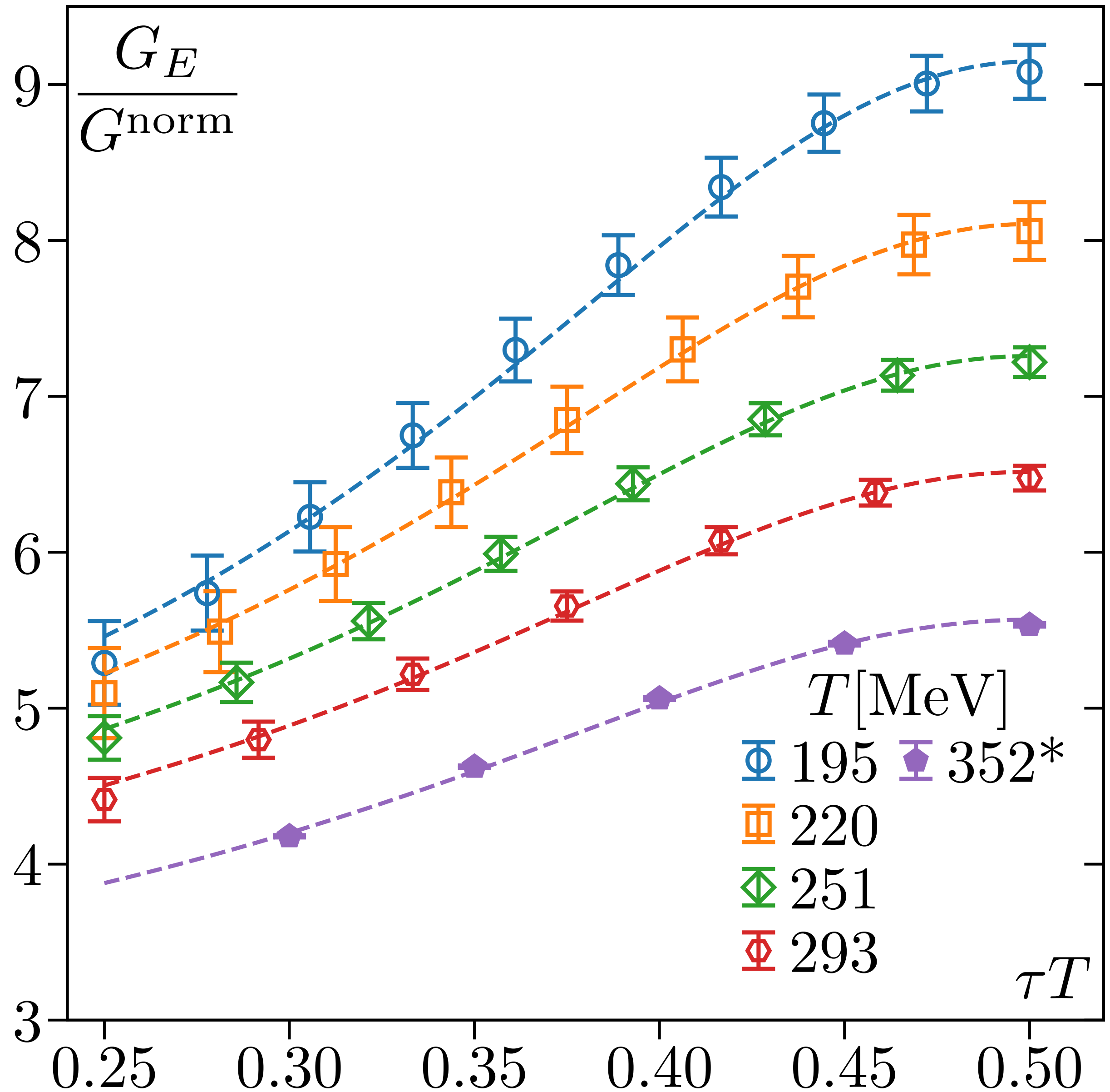
First full QCD calculation of kappa (u+d+s quarks in the sea)



[PRL 130 (2023) 23, 231902]

- Wide temperature range (195 MeV - 352 MeV) with Mpion=320 MeV
- Extrapolation Ansatz describes lattice data well

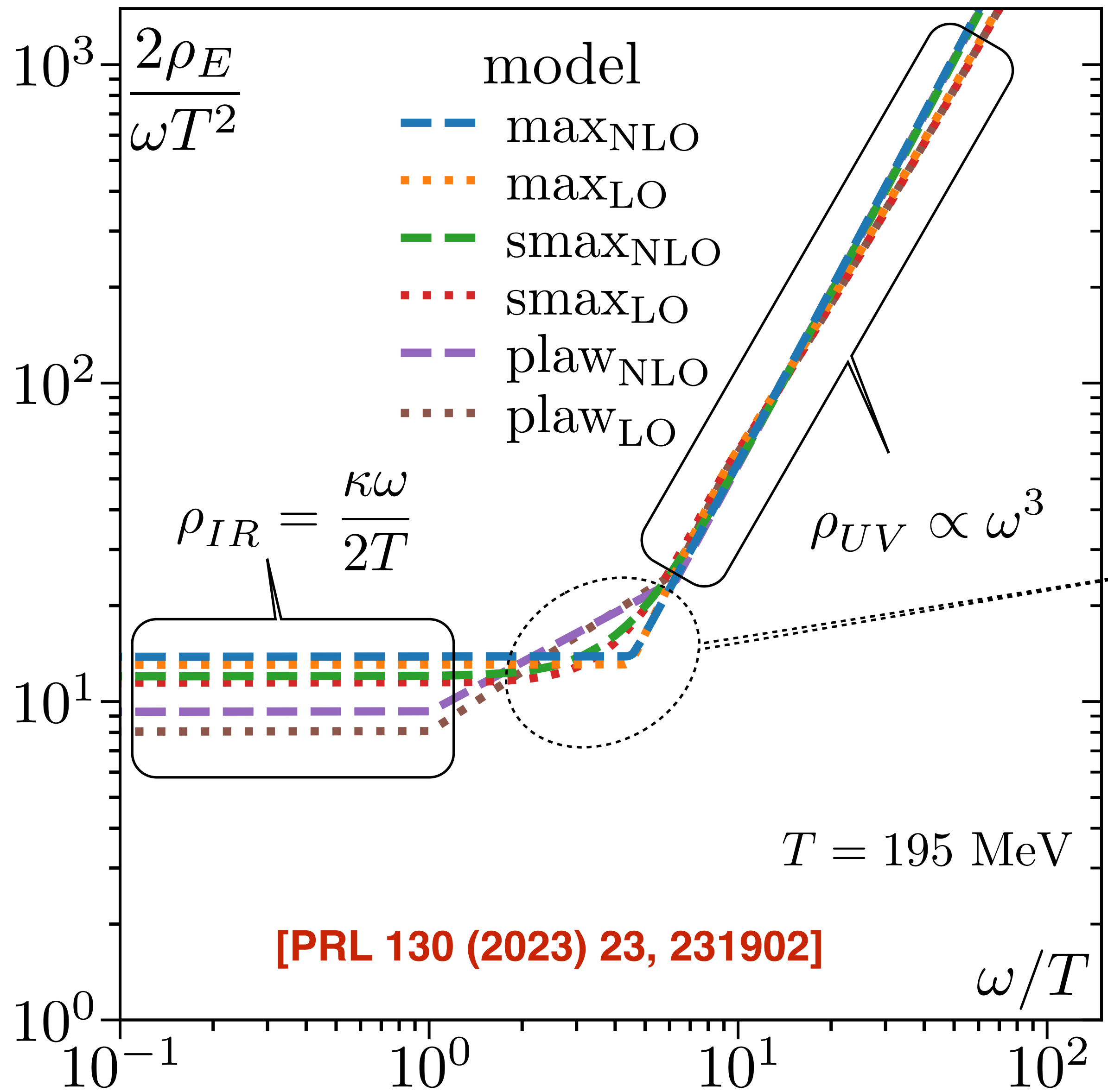
Thermal effects on the final correlators



[PRL 130 (2023) 23, 231902]

- Significant temperature dependence of correlators (what about kappa?)

Spectra analysis

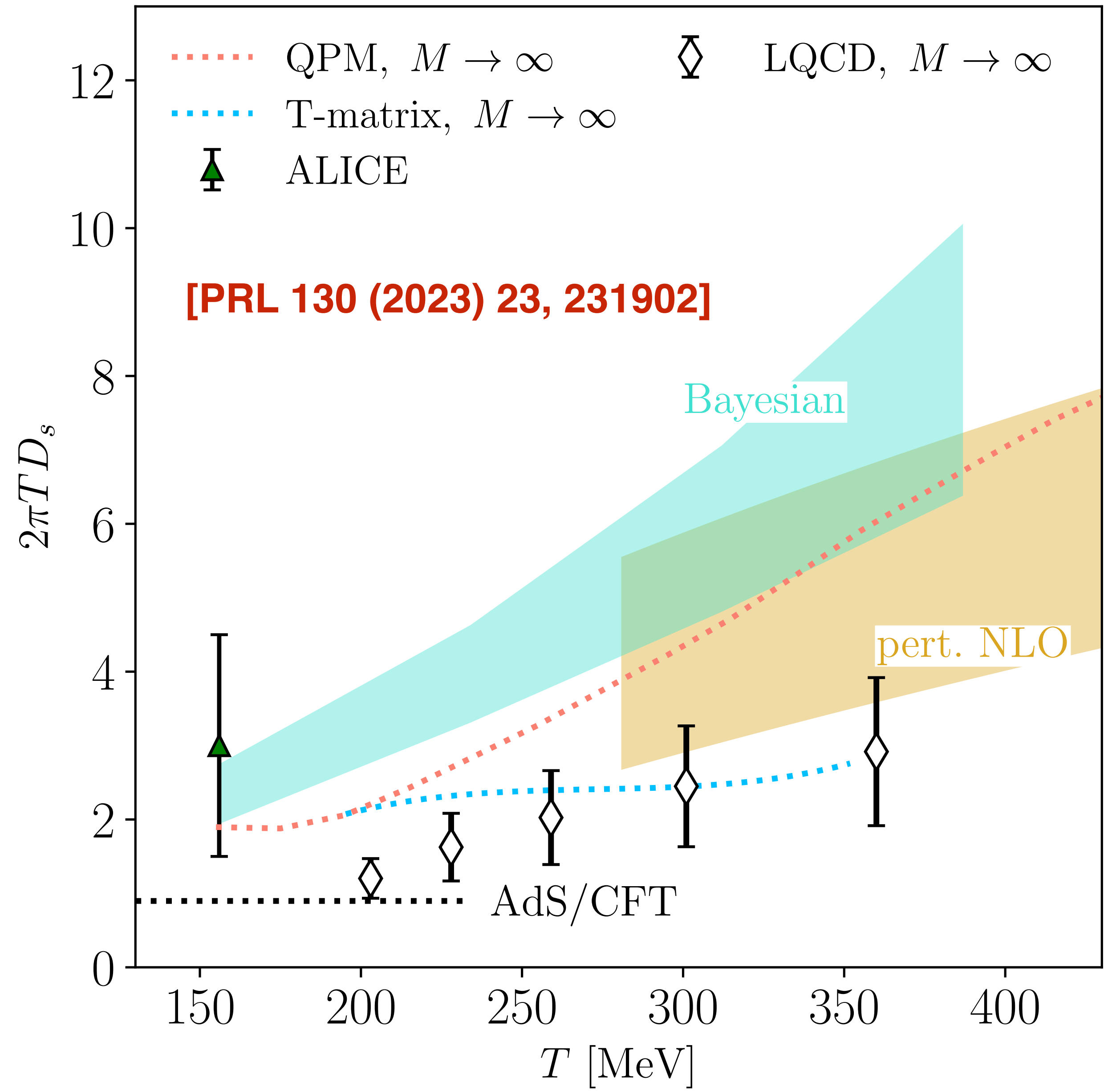


$$\begin{cases} \rho_{\rm max} \equiv \max(\phi_{\rm IR}, \phi_{\rm UV}) \\ \rho_{\rm smax} \equiv \sqrt{\phi_{\rm IR}^2 + \phi_{\rm UV}^2} \\ \rho_{\rm plaw} \equiv \begin{cases} \phi_{\rm IR} & \omega \le \omega_{\rm IR}, \\ a\omega^b \quad \text{for} & \omega_{\rm IR} < \omega < \omega_{\rm UV}, \\ \phi_{\rm UV} & \omega \ge \omega_{\rm UV}, \end{cases} \end{cases}$$

$$G(\tau, T) = \int \frac{d\omega}{\pi} K(\omega, \tau, T)\rho(\omega, T)$$

- Simple structure of spectra —> robust estimate of kappa

HQ diffusion coefficient at HQ mass limit



$$2\pi T D = \frac{4\pi}{\kappa/T^3}$$

- First full QCD results for Kappa
- Agree with AdS/CFT at ~Tc (rapid equilibrium <—> QGP is near perfect fluid)
- Agree with T-matrix estimate at moderate T
- Agree with NLO perturbative estimate at large T
- Mild temperature dependence

Finite mass correction

Physical charm & bottom quark not infinitely heavy!

$$M_c : \sim 1.3 \text{ GeV}$$

$$M_b : \sim 4.5 \text{ GeV}$$

D. Guazzini, et al., JHEP 10 (2007) 081

$$\kappa_E : \; M_Q \to \infty$$

$$\langle \mathcal{F}(t')\mathcal{F}(t)\rangle = q^2 \Big\{ \langle E_i(t')E_j(t)\rangle + \frac{1}{3}\langle \mathbf{v}^2\rangle \langle \delta_{ij} B_k(t')B_k(t) - B_j(t')B_i(t)\rangle \Big\}$$

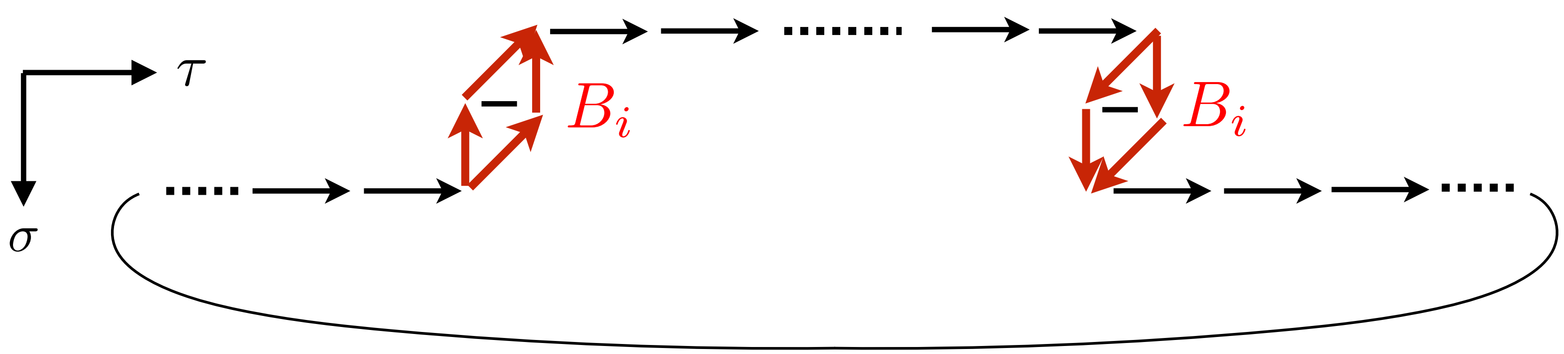
Infinite heavy approx.

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v^2}\rangle \kappa_{\mathbf{B}}$$

Finite mass correction

2/3<v²> at all T:
charm: 18.1~30.0%
bottom: 7.4% ~ 13.1%

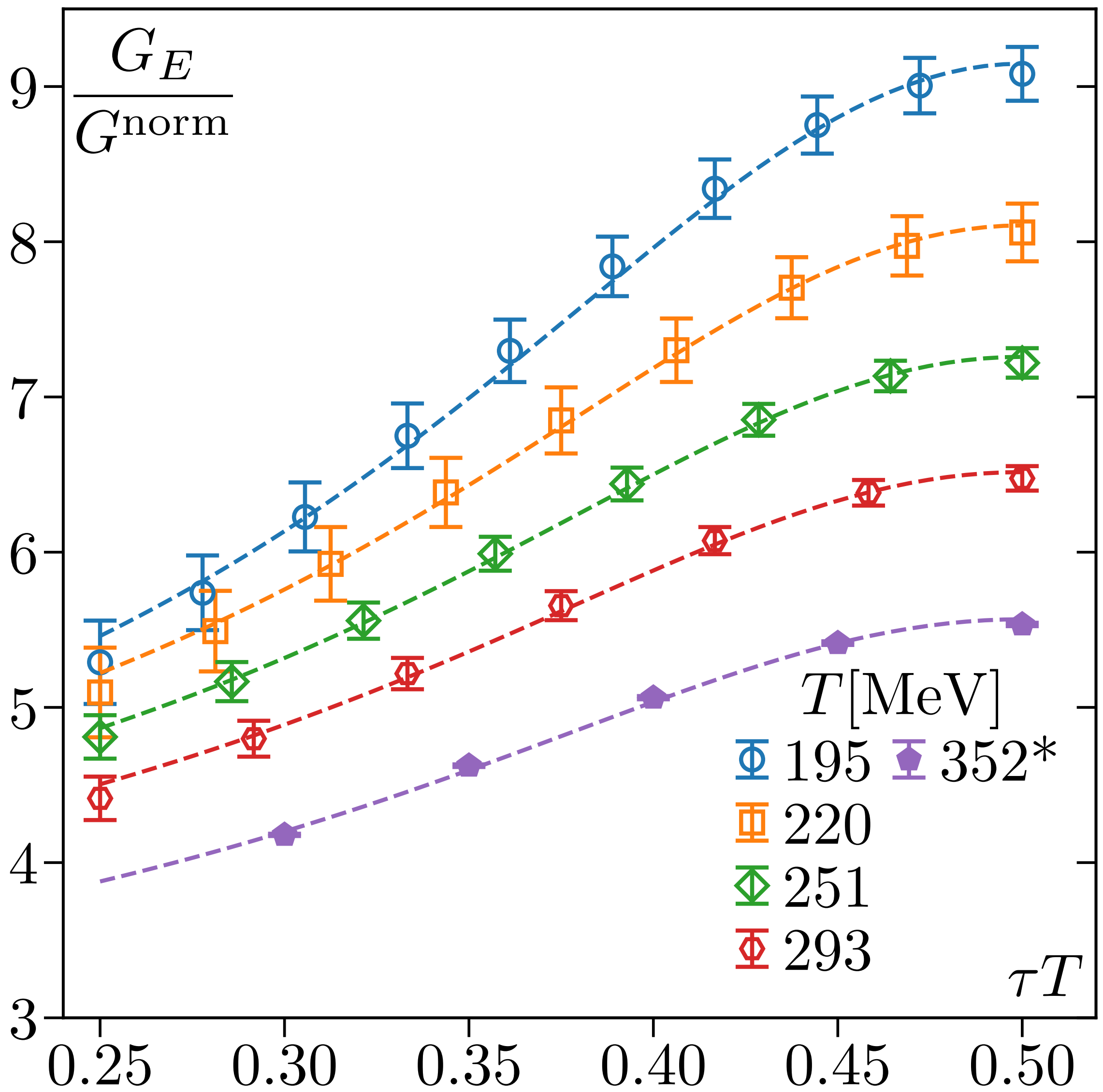
$$D_s = \frac{2T^2}{\kappa} \cdot \frac{\langle p^2\rangle}{3MT}$$



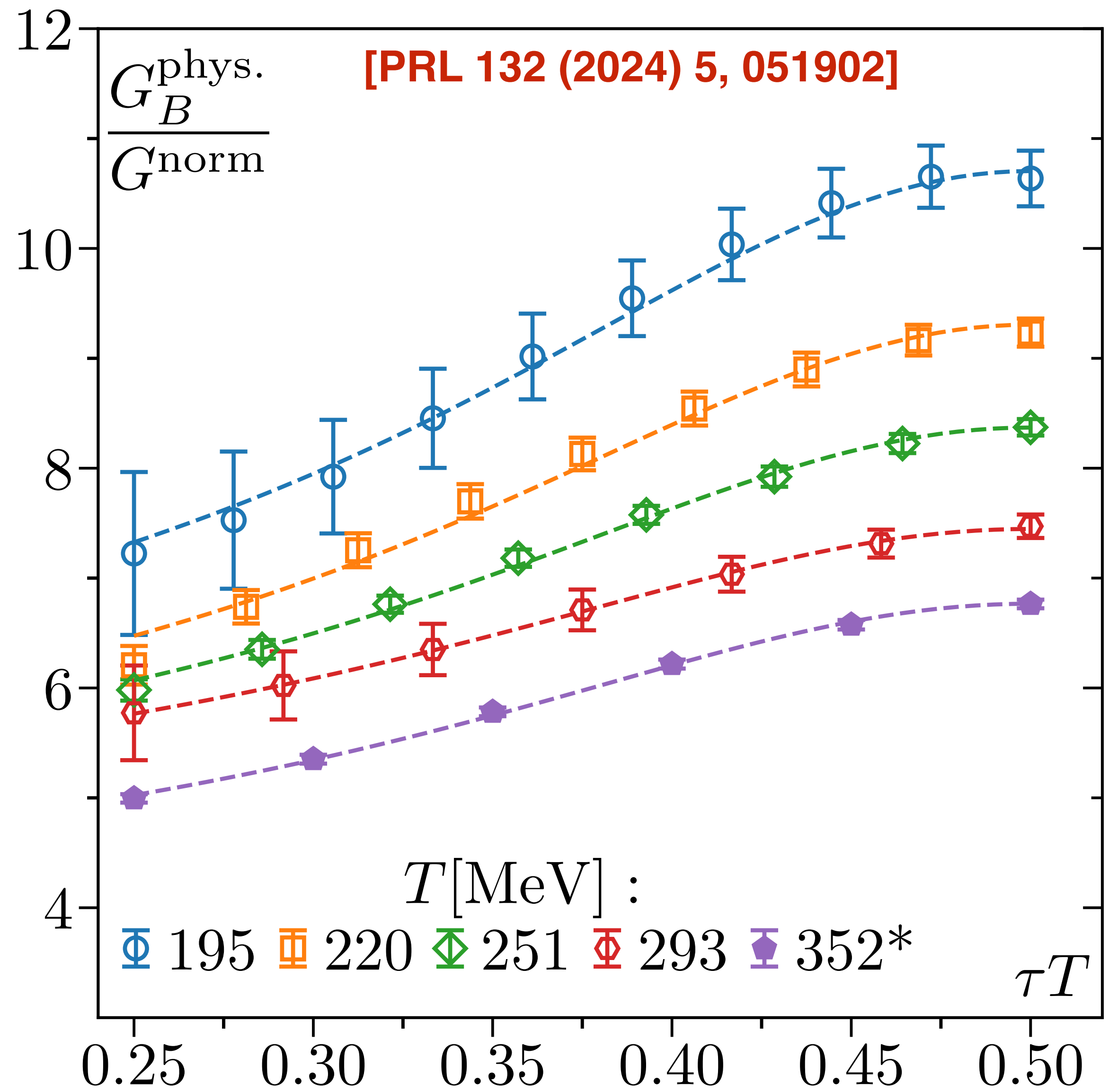
Color-magnetic field correlation function

A. Bouttefeux, M. Laine, JHEP 12 (2020) 150

Color-magnetic correlators



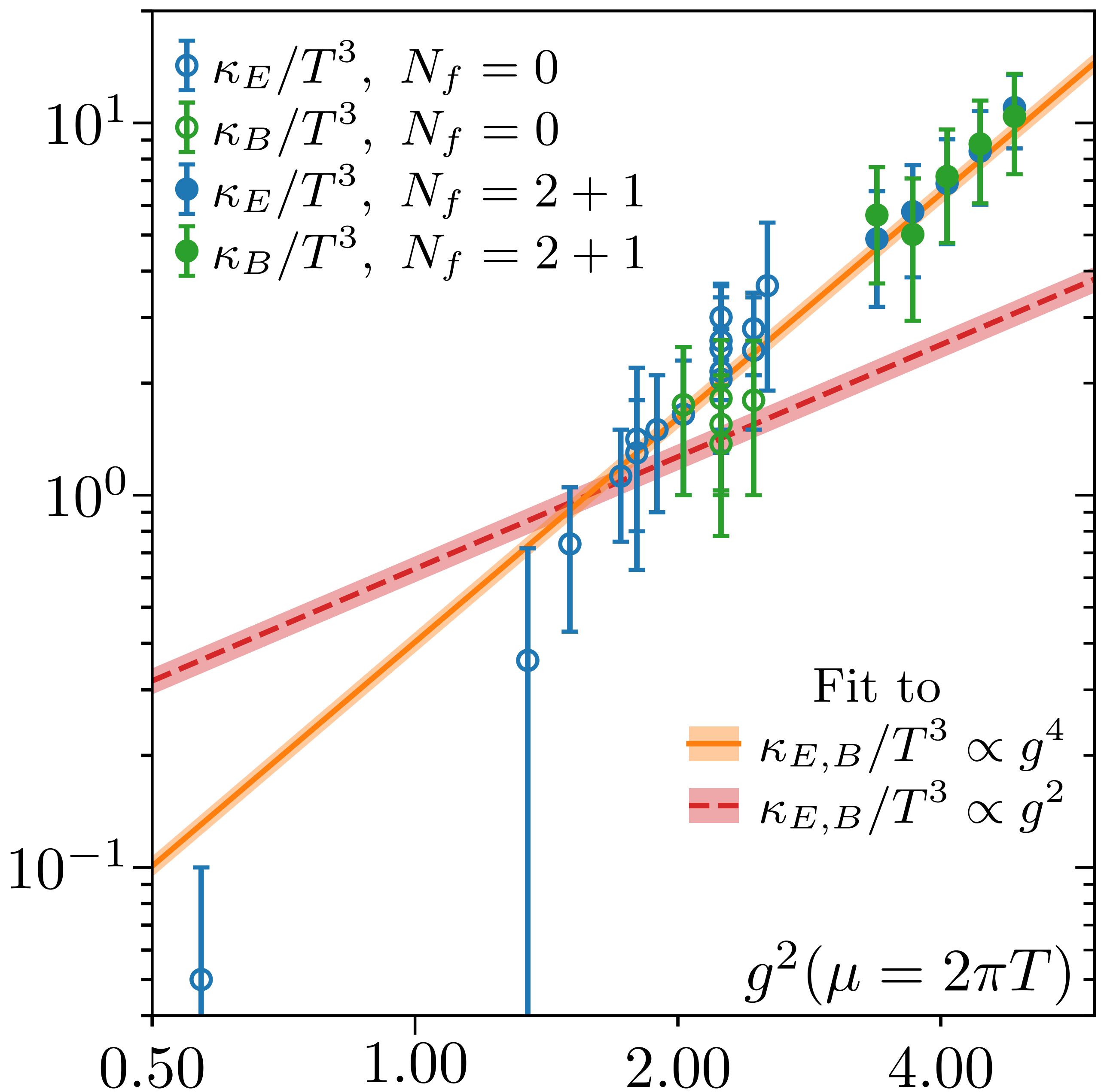
Color-electric field correlators



Color-magnetic field correlators

- Similar size of correlators for *E* and *B*
- Similar spectra modeling methodology

Kappa_E v.s. Kappa_B



==> S. Caron-Huot and G. D. Moore, PRL. 100, 052301 (2008)

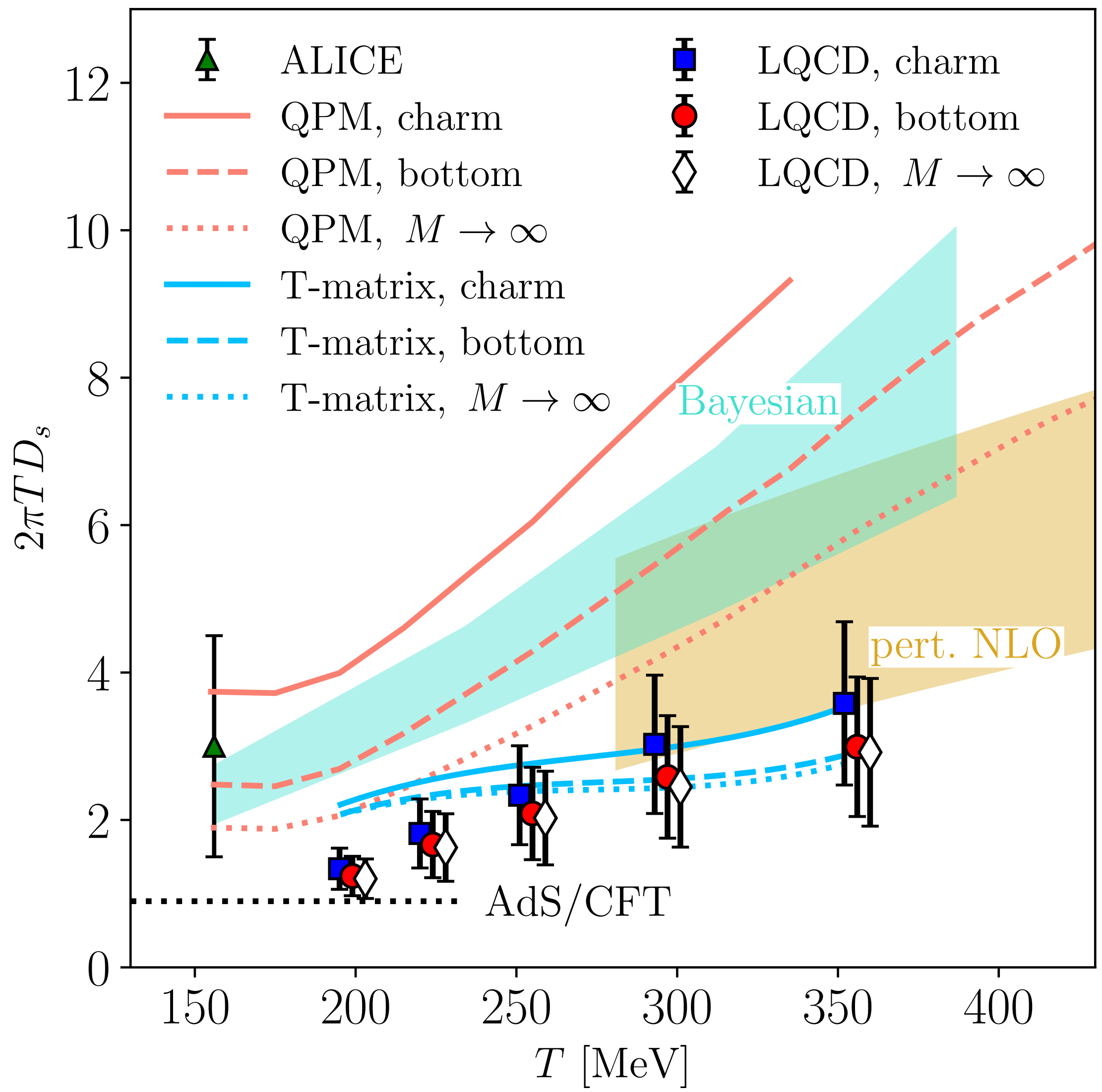
[PRD 109 (2024) 11, 114505]

Quenched results from:

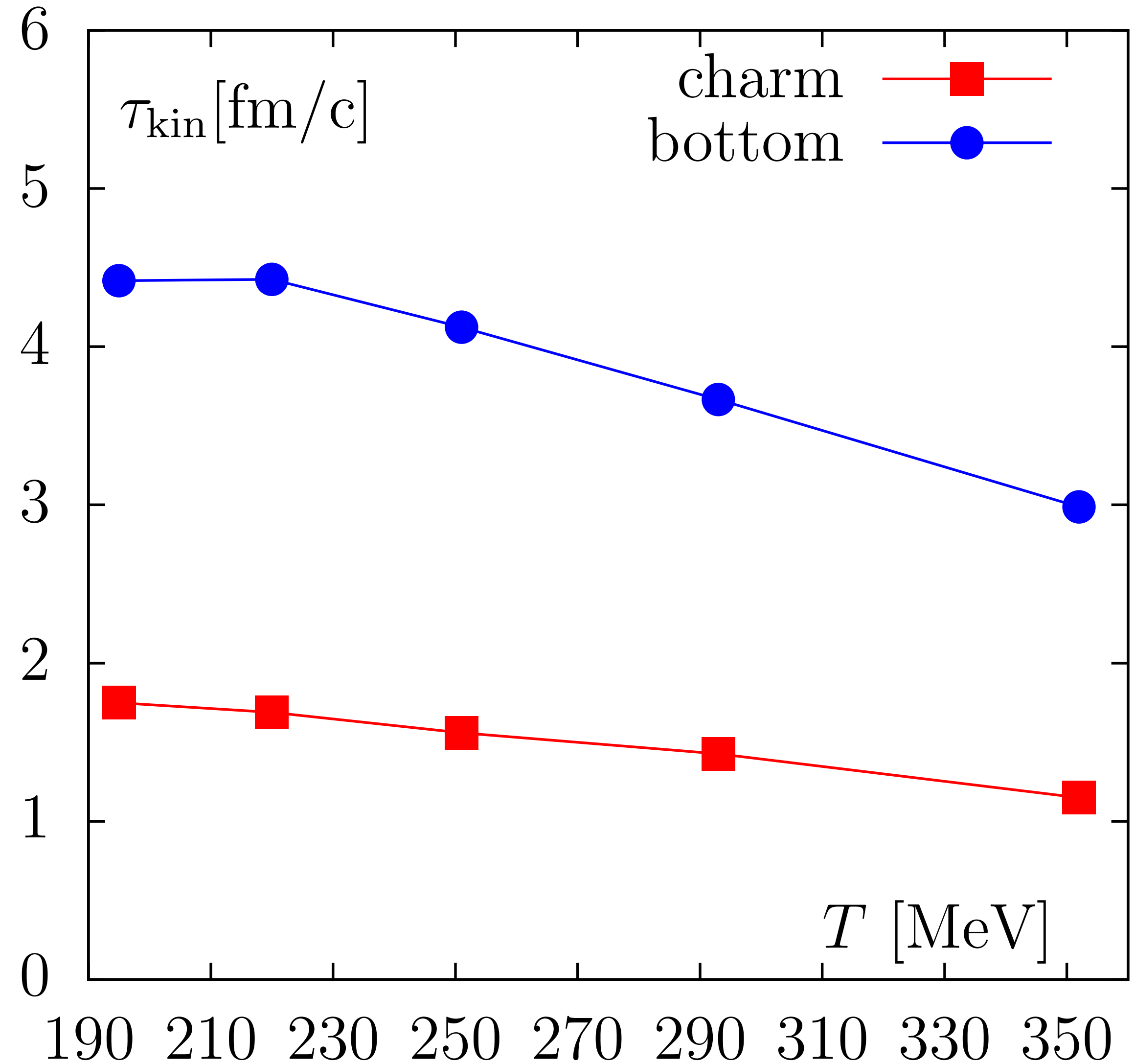
A. Francis, et al., PRD92, 116003
B. L. Altenkort, et al., PRD103,014511
D. Banerjee, et al., Nucl.Phys.A.2023.122721
D. Banerjee, et al., JHEP 08 (2022) 128
N. Brambilla, et al., PRD107, 054508

- Similar magnitude for Kappa_E and Kappa_B in full QCD & quenched
- Smooth connection between quenched and full QCD in temperature
- Lattice results confirms the form suggested by pert. computations

Summary



[PRL 132 (2024) 5, 051902]



$$\tau_{\rm kin} = \frac{1}{\eta_D} = \frac{1}{\kappa/T^3}\left(\frac{T_c}{T}\right)^2\left(\frac{M}{1.5\ \rm GeV}\right)\frac{3\ \rm GeV}{T_c^2}$$

- First lattice study of HQ mass dependence of HQ diffusion: mild
- Universal 2piTD change pattern with quark mass
- Quark mass dependence is weak in LQCD & T-matrix
- Weaker quark mass dependence than QMP calculations
- Equilibration time of charm quark favors the experimental estimate (~1 fm/c for all)

Backup: identify the heavy quark diffusion

Phenomenological diffusion picture of classical particle

Equilibrium -> Relaxation -> Equilibrium

$$\langle A(\mathbf{x})\rangle_{\text{eq}} = 0 \qquad \partial_t \langle A(\mathbf{x},t)\rangle = D\nabla^2 \langle A(\mathbf{x},t)\rangle$$

Solution:

$$\langle A(\mathbf{k},\omega)\rangle = \frac{i}{\omega + iD\mathbf{k}^2}\langle A(\mathbf{k},t=0)\rangle$$

Linear response theory

Perturbation to Hamiltonian:

$$H(t) = H_0 - \int \mathrm{d}\mathbf{x}\; A(\mathbf{x})h(x)e^{\epsilon t}\Theta(-t)$$

Solution:

$$\frac{\partial}{\partial t}\Big(\delta\langle A(\mathbf{k},t=0)\rangle\Big) = -\frac{G_R(\mathbf{k},t)}{\chi_q(\mathbf{k})}\delta\langle A(\mathbf{k},0)\rangle$$

Kubo formula: $$G_R(\mathbf{k},\omega) = \frac{iD\mathbf{k}^2}{\omega + iD\mathbf{k}^2}\chi_q(\mathbf{k}) \sim \rho(\vec{k},\omega)$$

$$A \to J^\mu = \bar{\psi}\gamma^\mu\psi$$

$$D = \frac{1}{3\chi_q}\lim_{\omega\to 0}\sum_{i=1}^{3}\frac{\rho^{ii}(\omega)}{\omega}$$

Backup: full QCD setup

$$N_f = 2+1,\ \text{HISQ},\ m_\pi = 320\ \text{MeV}$$

T [MeV]	β	am_s	am_l	N_σ	N_τ	# conf.
195	7.570	0.01973	0.003946	64	20	5899
	7.777	0.01601	0.003202	64	24	3435
	8.249	0.01011	0.002022	96	36	2256
220	7.704	0.01723	0.003446	64	20	7923
	7.913	0.01400	0.002800	64	24	2715
	8.249	0.01011	0.002022	96	32	912
251	7.857	0.01479	0.002958	64	20	6786
	8.068	0.01204	0.002408	64	24	5325
	8.249	0.01011	0.002022	96	28	1680
293	8.036	0.01241	0.002482	64	20	6534
	8.147	0.01115	0.002230	64	22	9101
	8.249	0.01011	0.002022	96	24	688
352	8.249	0.01011	0.002022	96	20	2488

- Wide temperature range
- Different lattice spacings
- Large lattices towards thermodynamic limit

Backup: anomalous dimension of B-field

- Anomalous dimension in MSbar-scheme

$$Z_B = 1 + \frac{g^2 C_A}{(4\pi)^2}\left[\frac{1}{\epsilon} + 2\ln\left(\frac{\bar{\mu}e^{\gamma_E}}{4\pi T}\right) - 2\right] + \mathcal{O}(g^4)$$

- Gradient flow-scheme —> MSbar-scheme —> physical values
- Scale dependence must go for "WeWant" and $\langle BB\rangle_{\tau_F}$

$$Z^2 = \left(1 - 2\frac{g^2 C_A}{16\pi^2}\ln(\mu^2\tau_F)\right)\left(1 + 2K\frac{g^2 C_A}{16\pi^2}\right) \equiv Z_f^2 Z_K^2$$

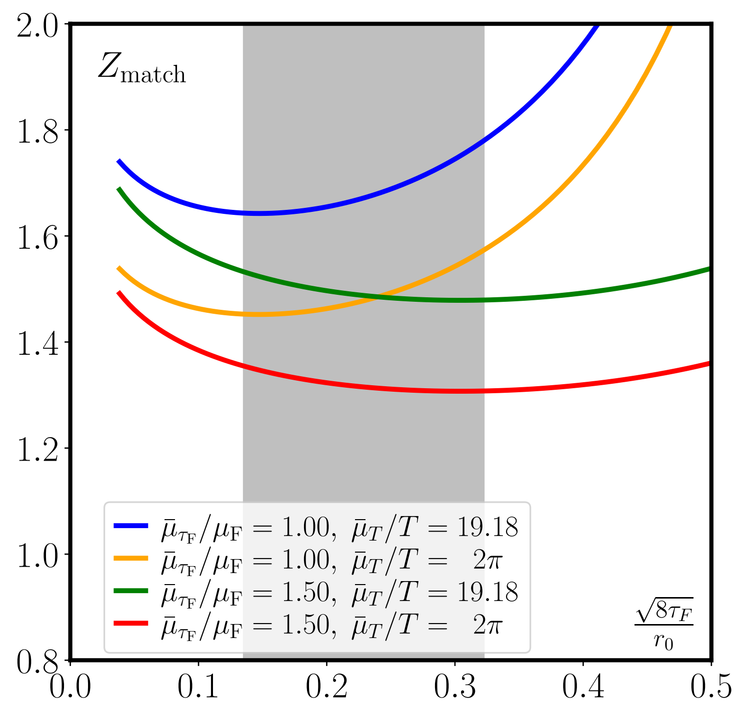
$$\mathrm{WeWant} = Z_B^2\langle BB\rangle_{\mathrm{MS}}$$

$$\langle BB\rangle_{\tau_F} \equiv Z^{-2}\langle BB\rangle_{\mathrm{MS}}$$

$$\mathrm{WeWant} = Z_B^2 Z^2\langle BB\rangle_{\tau_F}$$

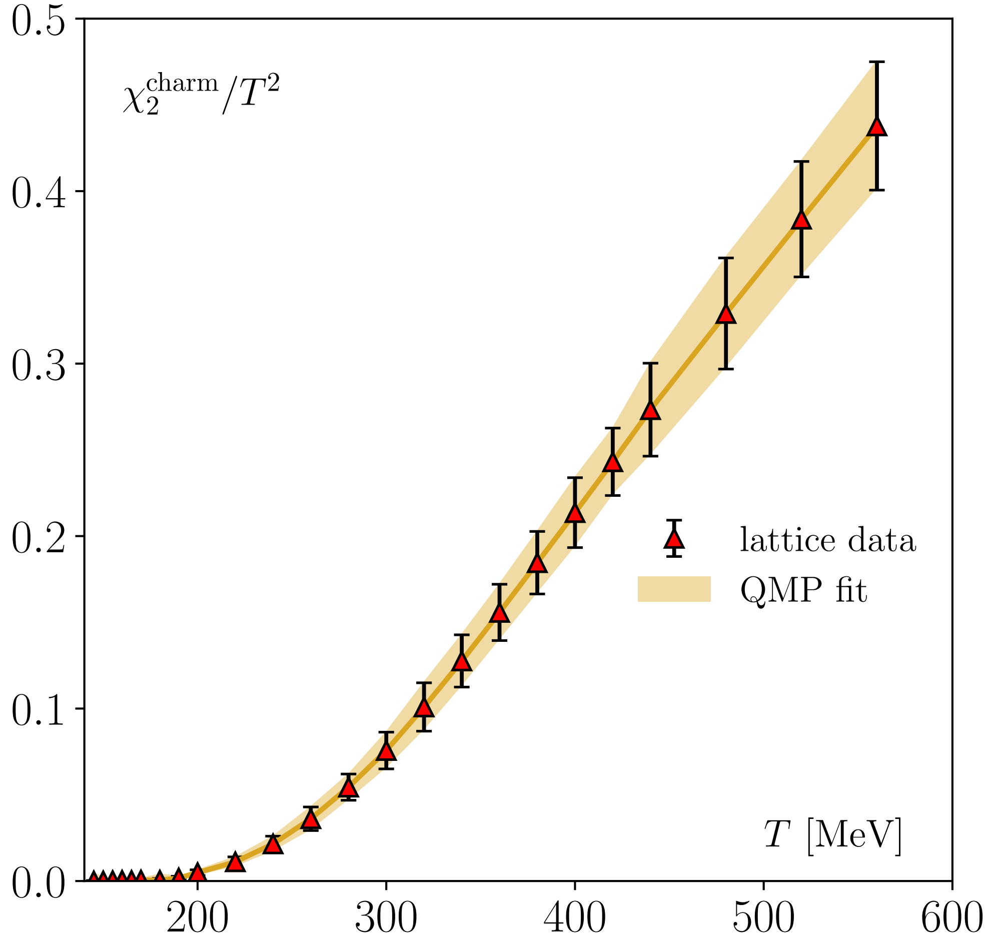
- Determination of the matching factor

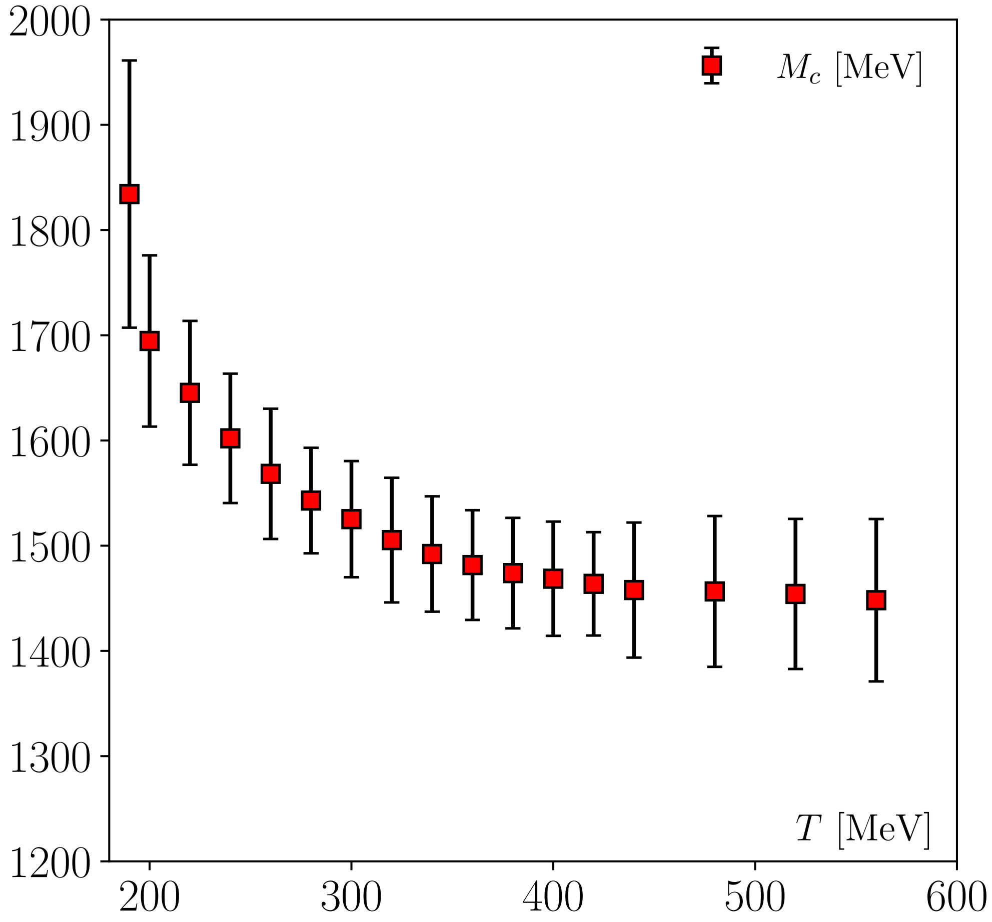
$$\ln Z_{\mathrm{match}} = \int_{\bar{\mu}_T^2}^{\bar{\mu}_{\tau_{\mathrm{F}}}^2} \gamma_0 g^2_{\overline{\mathrm{MS}}}(\bar{\mu})\frac{d\bar{\mu}^2}{\bar{\mu}^2} + \gamma_0 g^2_{\overline{\mathrm{MS}}}(\bar{\mu}_T)\left[\ln\frac{\bar{\mu}_T^2}{(4\pi T)^2} - 2 + 2\gamma_{\mathrm{E}}\right] - \gamma_0 g^2_{\overline{\mathrm{MS}}}(\bar{\mu}_{\tau_{\mathrm{F}}})\left[\ln\frac{\bar{\mu}^2_{\tau_{\mathrm{F}}}}{4\,\mu_{\mathrm{F}}^2} + \gamma_{\mathrm{E}}\right]$$



[PRL 132 (2024) 5, 051902]

Backup: T-dependent charm quark mass





[PRL 132 (2024) 5, 051902]

$$\frac{\chi_2^{\rm charm}}{T^2} = \frac{4N_c}{(2\pi T)^3}\int d^3p\, e^{-E_p/T}$$

$$E_p^2(T) = m^2(T) + p^2$$

$\longrightarrow m(T)$

$$\langle v^2\rangle = \left(\int d^3p\, \frac{p^2}{E_p^2} e^{-E_p/T}\right) / \left(\int d^3p\, e^{-E_p/T}\right)$$

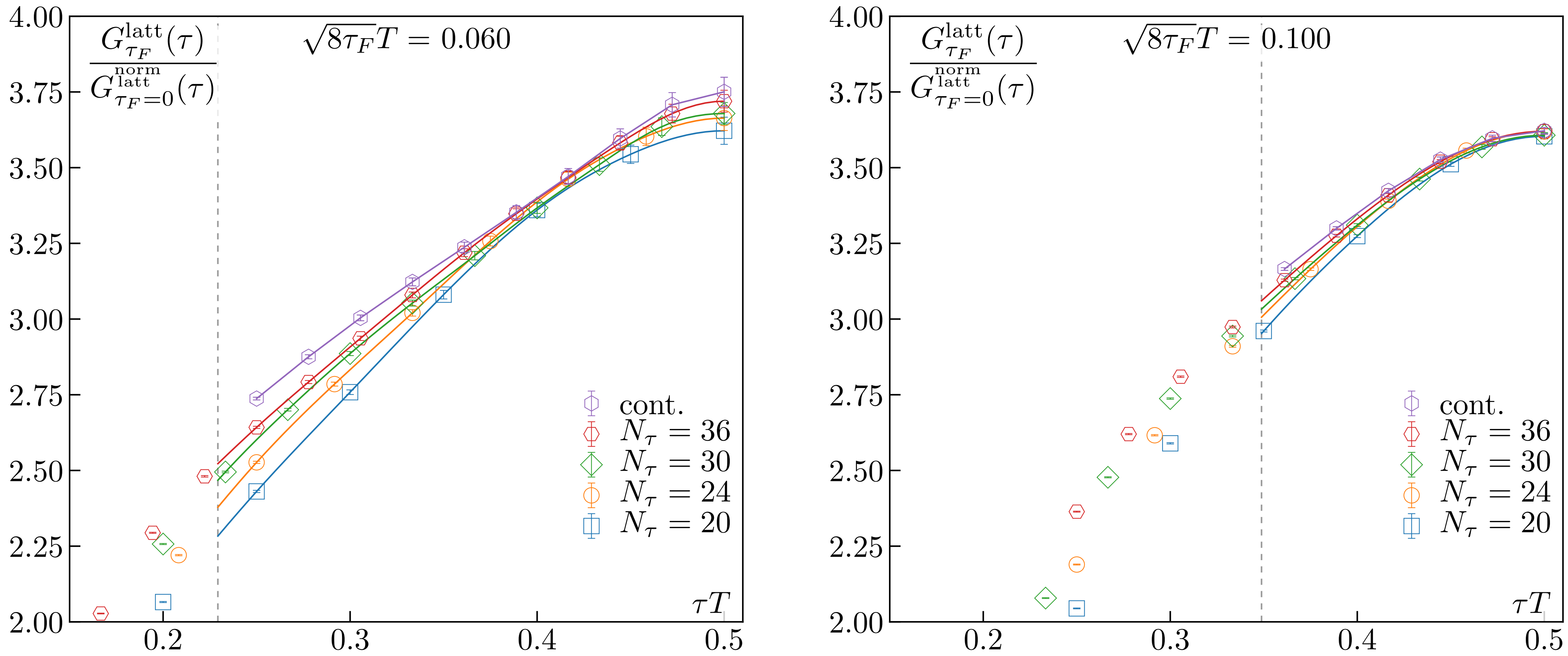
$$\langle p^2\rangle = \left(\int d^3p\, p^2 e^{-E_p/T}\right) / \left(\int d^3p\, e^{-E_p/T}\right)$$

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v^2}\rangle \kappa_{\mathbf{B}}$$

$$D_s = \frac{2T^2}{\kappa}\cdot\frac{\langle p^2\rangle}{3MT}$$

Backup: smearing effects of gradient flow

[HTS et al., PRD103(2021) 1, 014511]



- Gradient flow reduces the noise in correlators
- Gradient flow removes the lattice effects (disordering)
- Need proper flow time range

Backup: scattering from various models