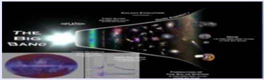
Angular correlation and deformed Hellings-Downs curve by spin-2 ultralight dark matter

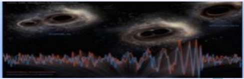
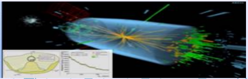
Speaker: Yun-Long Zhang (NAOC)

National Astronomical Observatories,
Chinese Academy of Sciences

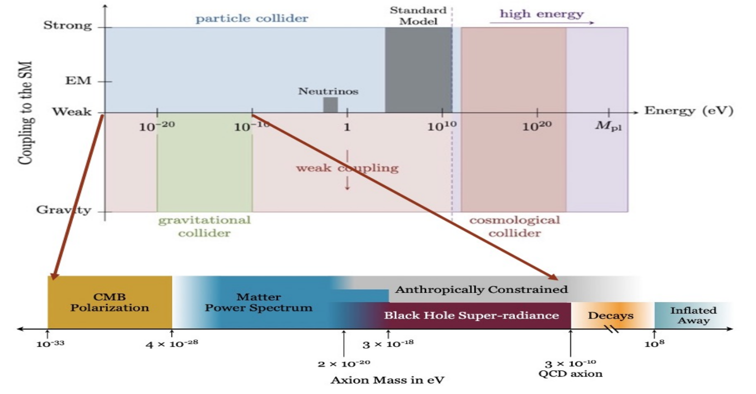
Based on: PRD('24) by R.G. Cai(ITP), J.R. Zhang(HIAS) & Y.L. Zhang
Phys.Rev.D 106, 066006, by S. Sun(BIT), X.Y. Yang(KIAS) & Y.L. Zhang

2024, Aug 16@Qingdao [email: zhangyunlong@nao.cas.cn]

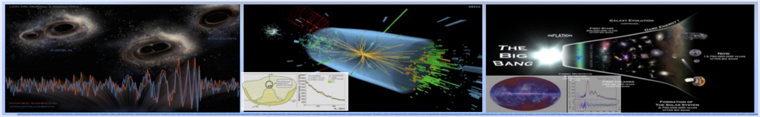


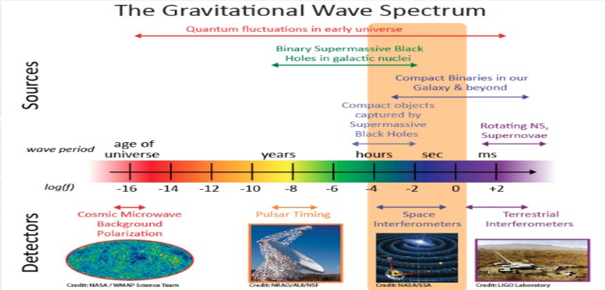
Motivation: new physics in ultra-low energy

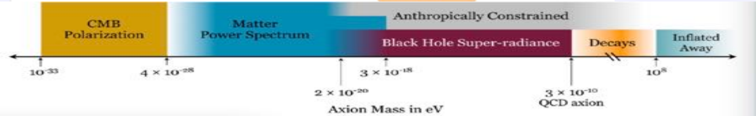


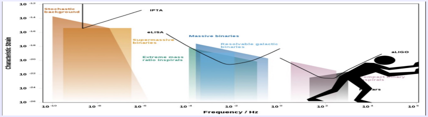
[cf. Baumann-Chia-Porto-Stout, Gravitational Collider Physics, 2019]



Spectrum of gravitational wave and axion mass

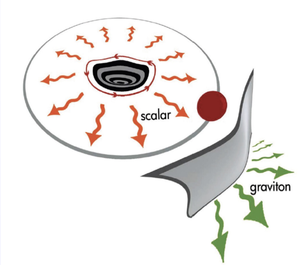


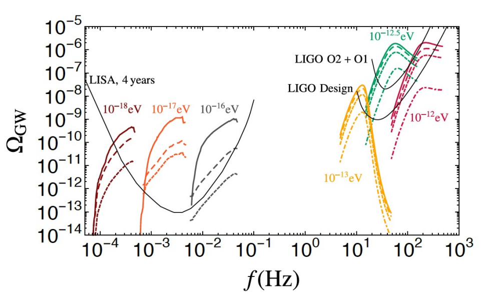




[cf. LISA/Ultra-High-Frequency Gravitational Waves Initiative]

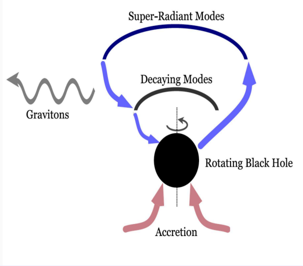
Axion annihilation and Stochastic GWs

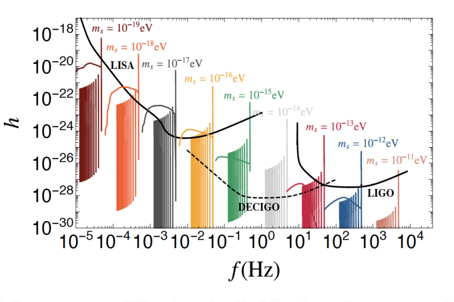




- Axion annihilation $\vartheta + \vartheta \to hh$, Strain $h \sim 10^{-21} - 10^{-32}$.
- Stochastic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

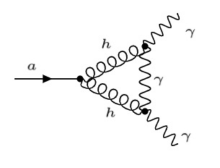
Energy level transition and Monochromic GW

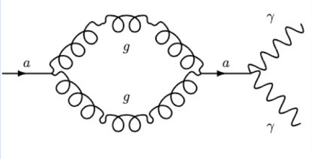




- Energy transition $\vartheta^+ \to \vartheta^- + h$, Strain $h \sim 10^{-19} - 10^{-27}$
- Monochromic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

Branch Ratio of EM & GW ($\vartheta \to \gamma\gamma$ & $\vartheta \to hh$)

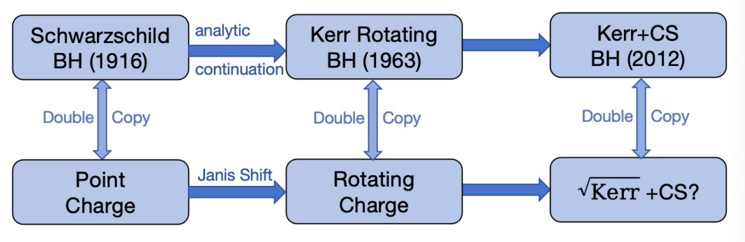
- The triangle Feynman diagram: where the axion-photon coupling is generated from Chern-Simon gravity coupling.
- $\mathcal{L}_{\vartheta F\tilde{F}} = -\frac{\alpha_\gamma}{4}\vartheta F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad \mathcal{L}_{\vartheta R\tilde{R}} = \frac{\alpha_g}{4}\vartheta R^{\beta}{}_{\alpha\gamma\delta}\tilde{R}^{\alpha}{}_{\beta}{}^{\gamma\delta}.$
- 



- The triangle diagram is divergent as $\alpha_\gamma \sim \alpha_g(\Lambda_{cs}/M_{pl})^4$, where Λ_{cs} is the cut-off for Chern-Simons theory.
- Two powers of M_{pl} from $h_{\mu\nu}T^{\mu\nu}$ coupling.

[cf. S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps, PRD'21]

Double Copy Relation



- Weyl double copy: Gravity theory=(Gauge theory)^2
- Correspondence: Gravity solutions $\leftrightarrow$ Gauge solutions

[cf. Y. R. Liu, J. R. Zhang, Y. L. Zhang, "Slowly rotating charges from Weyl double copy for Kerr black hole with Chern-Simons correction," CTP(2024)]

Joint analysis & Branch ratio ($\vartheta \to \gamma\gamma$ & $\vartheta \to hh$)

The interaction

- $S_{total} = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa_4}R - \frac{1}{4}F^2 + \mathcal{L}_\vartheta + \mathcal{L}_{\vartheta F\tilde{F}} + \mathcal{L}_{\vartheta R\tilde{R}}\right)$
- EM field $\mathcal{L}_\vartheta = -\frac{1}{2}(\partial\vartheta)^2 - \frac{1}{2}m_\vartheta^2\vartheta^2$.
- $\mathcal{L}_{\vartheta F\tilde{F}} = -\frac{\alpha_\gamma}{4}\vartheta F_{\mu\nu}\tilde{F}^{\mu\nu}$, $\mathcal{L}_{\vartheta R\tilde{R}} = \frac{\alpha_g}{4}\vartheta R^{\beta}{}_{\alpha\gamma\delta}\tilde{R}^{\alpha}{}_{\beta}{}^{\gamma\delta}$.

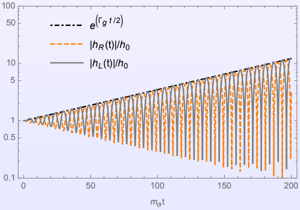
Equation of motion

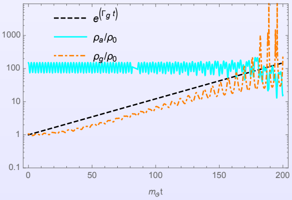
- $\Box h_{ij} =$
  $\kappa_4\alpha_g\tilde{\epsilon}^{pk}{}_{(i}\left[\dot{\vartheta}(\partial_p\Box h_{j)k}) - \ddot{\vartheta}(\partial_p\partial_t h_{j)k})\right] - 2\kappa_4\left(T_{ij}^{(\gamma)} + T_{ij}^{(\vartheta)}\right)$
- axion like field $(\Box - m_\vartheta^2)\vartheta = \frac{\alpha_\gamma}{4}F\tilde{F} - \frac{\alpha_g}{4}R\tilde{R}$
- Polarization $\nabla_\mu F^{\mu\nu} = -\alpha_\gamma\partial_\mu\vartheta\tilde{F}^{\mu\nu}$

Fast GW burst from axion decay $\vartheta \to hh$

EOM of GW($I = L, R$)

- Coupling term $\mathcal{L}_{\vartheta R\tilde{R}} = \frac{\alpha_g}{4}\vartheta R^{\beta}{}_{\alpha\gamma\delta}\tilde{R}^{\alpha}{}_{\beta}{}^{\gamma\delta}$,
- $\left[\ddot{h}_I(t) + k^2 h_I(t)\right]\left[1 - \varepsilon_I \kappa_4 \alpha_g k \dot{\vartheta}(t)\right] = \varepsilon_I \kappa_4 \alpha_g k \ddot{\vartheta}(t)\dot{h}_I(t)$.
- Factor $e^{\Gamma_g t_g}$, $\frac{\Gamma_g}{m_\vartheta} \sim \left(\frac{\kappa_4 \alpha_g}{1\mathrm{eV}^{-3}}\right)\left(\frac{m_\vartheta}{10^{-9}\mathrm{eV}}\right)^2\left(\frac{\vartheta_0}{10^9\mathrm{GeV}}\right)$.



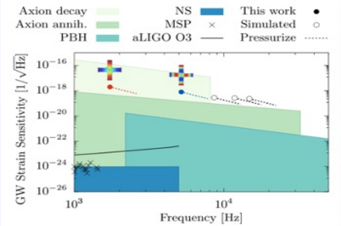


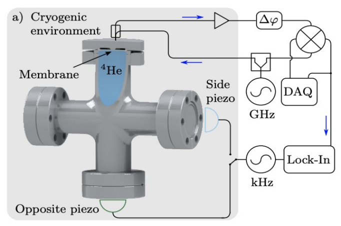
[cf. S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps, PRD'21]

GW burst & high frequency detection

Branch Ratio and GWs

- $\frac{\mathrm{Br}(\vartheta\to gg)}{\mathrm{Br}(\vartheta\to\gamma\gamma)} \simeq \frac{\alpha_g^2}{\alpha_\gamma^2} \simeq \left(\frac{M_{pl}}{\Lambda_{cs}}\right)^8$, (Power of FRB $P_{(\gamma)} \sim 10^{42}$ergs/s).
- High frequency $h_{(g)} \sim 10^{-26}\left(\frac{1\mathrm{GHz}}{\nu}\right)\left(\frac{P_{(g)}}{P_{(\gamma)}}\right)^{1/2}\left(\frac{1\mathrm{kpc}}{L}\right)$
- Low freq. $h_{(g)} \sim 10^{-21}\left(\frac{10^{-2}\mathrm{Hz}}{\nu}\right)^{1/2}\left(\frac{M_{\mathrm{BH}}}{10^7 M_\odot}\right)^{1/2}\left(\frac{1\mathrm{kpc}}{L}\right)$





cf. PRD'21, S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps.
PRD'21, V. Vadakkumbatt et al, Prototype superfluid gravitational wave detector.
L.H. Du's group, Evidence for chiral graviton modes in fractional quantum Hall liquids

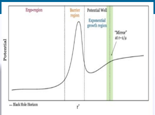
Ultralight dark matter and gravitational wave

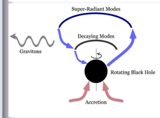
GW & EM signals from axion like DM

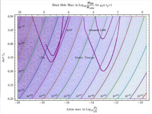
- Axion annihilation $\vartheta + \vartheta \to h$(Stochastic GW)
  Energy transition $\vartheta^{+} \to \vartheta^{-} + h$(Monochromic)
- Superradiance $\alpha \equiv \frac{R_{BH}}{\lambda_\vartheta} \simeq \left(\frac{M_{BH}}{M_\odot}\right)\left(\frac{m_\vartheta}{10^{-10}\text{eV}}\right)$
- Fast Radio Burst from Axion $\sim \vartheta F\tilde{F}$ $(\vartheta \to \gamma\gamma)$
- GW burst from Axion $\sim \vartheta R\tilde{R}$ $(\vartheta \to hh)$

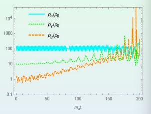
GW detection and Ultra-light DM

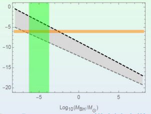
- Tabletop exp: QCD axion & GW burst ($\sim$ GHz)
- LISA & LVK: BH Superradiance ($\sim$ mHz - kHz)
- PTA & SKA : Ultra-light DM ($\sim$ nHz)
- LSS & CMB : DE & Modified gravity ($\sim$ nnHz)







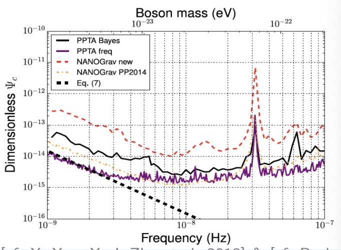


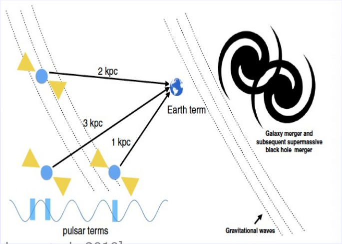


Timing residuals & Ultralight dark matter

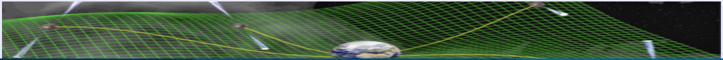
PTA & Ultra light dark matter

- Fuzzy like DM $f_c = \frac{m}{\pi} \simeq 4.8\,\text{nHz}\left(\frac{m}{10^{-23}\text{eV}}\right)$
- Coherence length: $\lambda_{\text{dB}} = \frac{2\pi\hbar}{mv} \simeq 4\text{kpc}\left(\frac{10^{-23}\text{eV}}{m}\right)\left(\frac{10^{-3}}{v}\right)$
- Pulsar timing residuals: $R_c(f) = \frac{1}{\sqrt{3}}\frac{h_c(f)}{2\pi f}\left(\frac{f_s}{f}\right)^{1/2}$

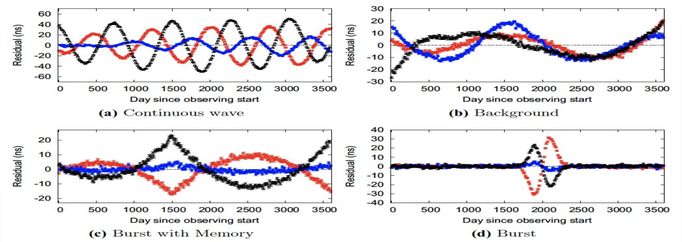


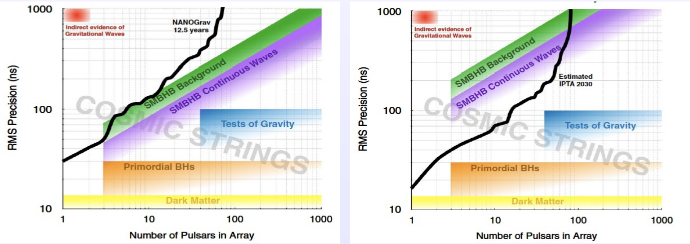


[cf. X. Xue, X. J. Zhu et al. 2018] & [cf. Burke-Spolaor, et al. 2019]



The timing residuals of nHz gravitational waves





[cf. Burke-Spolaor, *et al.*, "The astrophysics of nanohertz gravitational waves"]

Oscillation of fuzzy dark matter

DM oscillation induced time residual

- Metric: $ds^2 = -(1+2\Phi)\,\mathrm{d}t^2 + [(1-2\Psi)\,\delta_{ij} + h_{ij}]\,\mathrm{d}x^i\mathrm{d}x^j$.
- e.g. the scalar field $\phi(x,t) = \phi(x)\cos[mt + \theta_0(x)]$,
- Oscillating potential $\Psi \simeq \bar{\Psi}(x) + \Psi_\phi \cos[2(mt + \theta_0(x))]$
- Doppler effect: $z_\phi(t) \equiv \frac{\omega_0 - \omega_\phi(t)}{\omega_0} \simeq \Psi(x_\phi, t_\phi) - \Psi(x_0, t_0)$.
- Timing residual in the pulse $R_\phi(t) = \int_0^t z_\phi(t')dt'$
- **Strain** $h_\phi = 2\sqrt{3}\,\Psi_\phi = \frac{\sqrt{3}}{4M_{pl}^2}\frac{\rho_\phi}{m^2} \simeq 5.2\times 10^{-17}\alpha_0\left(\frac{f_{yr}}{f}\right)^2$,
- **GW Timing residual** $R_c(f) \equiv \sqrt{\frac{S_c(f)}{T_s}} = \frac{1}{\sqrt{3}}\frac{h_c(f)}{2\pi f}\left(\frac{f_s}{f}\right)^{1/2}$

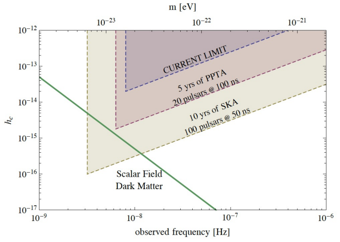
[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]

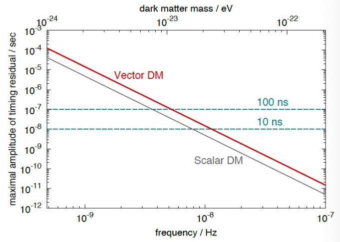
Pulsar timing constraints on ULDM

DM oscillation induced time residual

- Spin-0: massive scalar field $\mathcal{L}_{(0)} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$
- Spin-1: massive vector field $\mathcal{L}_{(1)} = -\frac{1}{4}F^2 - \frac{1}{4}m^2A^2$

[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]



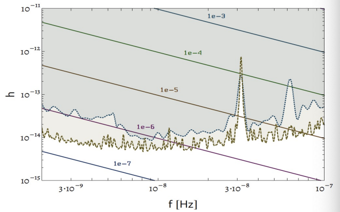


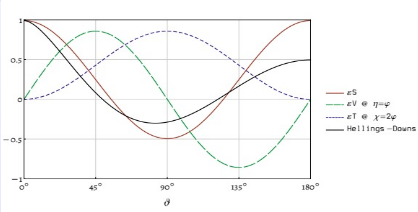
[cf. Nomura-Itoy-Soda, "Pulsar timing residual induced by ultralight vector DM" PRD(2020)]

Pulsar timing constraints on spin-2 ULDM

spin-2 ultralight fields

- Spin-2: massive tensor field(Fierz-Pauli): Bi-metric gravity,
$$\mathcal{L}_{(2)} = \frac{1}{2} M_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma} M_{\rho\sigma} - \frac{1}{4} m^2 \left(M_{\mu\nu}M^{\mu\nu} - M^2\right)$$
- The oscillating solution $M_{ij} = \mathcal{M} \cos\left[mt + \theta_2(x)\right]\varepsilon_{ij}$
- Effective metric perturbations: $\tilde{g}_{ij} = \delta_{ij} + \frac{\alpha_2}{M_{pl}} M_{ij}$
- The redshift $z(t) = \frac{\omega(t)-\omega_0}{\omega_0} = \frac{\alpha_2}{2M_{pl}} \int dt \omega_0 \partial_t M_{ij} n^i n^j$



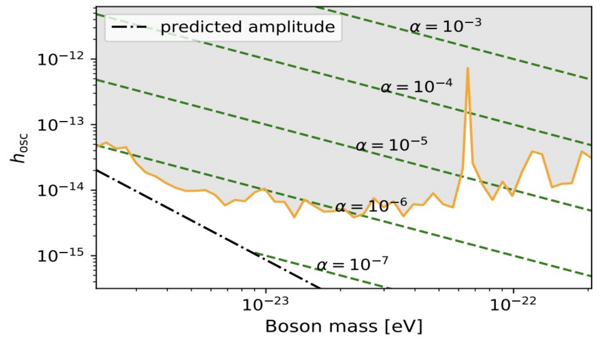


[cf. Armaleo-Nacir-Urbanb, "Pulsar timing array constraints on spin-2 ULDM" JCAP(2020)
Gromov-Son, "Bimetric Theory of Fractional Quantum Hall States" PRX(2017)]

Gravitational effects & coupling effects

$$S=\frac{M_{\rm Pl}^2}{1+\alpha^2}\int d^4x\Big[\sqrt{|g|}R(g)+\alpha^2\sqrt{|f|}R(f)-2\frac{\alpha^2M_{\rm Pl}^2}{1+\alpha^2}\sqrt{|g|}V(g,f;\beta_n)\Big]+\int d^4x\sqrt{|g|}\mathcal{L}_{\rm m}(g,\Psi)$$

$$S^{(2)}=\int d^4x\sqrt{|\bar{g}|}\Big[\mathcal{L}^{(2)}_{\rm GR}(\mathcal{G})+\mathcal{L}^{(2)}_{\rm FP}(M)+-\frac{1}{M_{\rm Pl}}(\mathcal{G}_{\mu\nu}-\alpha M_{\mu\nu})T^{\mu\nu}(\Psi)\Big]\ ,$$



[cf. Y. M. Wu, Z. C. Chen, Q. G. Huang, JCAP **09**, 021 (2023)

"Pulsar timing residual induced by ultralight tensor dark matter,"]

Angular correlation & Hellings-Downs curves

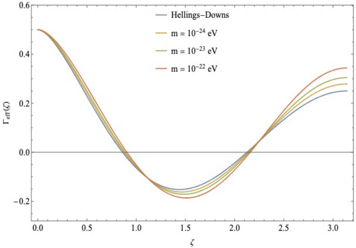


FIG. 2: Effective cross-correlation curves with $\alpha = 10^{-6}$ and mass ranging from 10^{-24} to 10^{-22}. It can be seen that in this range, the deformation of spin-2 dark matter on the Hellings-Downs curve is relatively small.

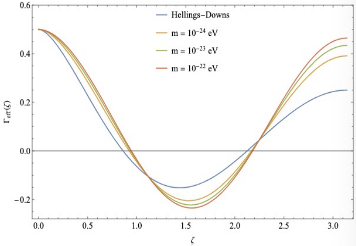


FIG. 3: Effective cross-correlation curves with $\alpha = 10^{-5.5}$. The deformation is very strong in this range, suggesting that if the coupling constant α is above this magnitude, existing ultralight mass spin-2 dark matter would have considerable effects on the deformation of the Hellings-Downs curve at corresponding frequency.

$$S = \frac{M_{\rm Pl}^2}{1+\alpha^2}\int d^4x\Big[\sqrt{|g|}R(g) + \alpha^2\sqrt{|f|}R(f) - 2\frac{\alpha^2 M_{\rm Pl}^2}{1+\alpha^2}\sqrt{|g|}V(g,f;\beta_n)\Big] + \int d^4x\sqrt{|g|}\mathcal{L}_{\rm m}(g,\Psi)$$

$$S^{(2)} = \int d^4x\sqrt{|\bar{g}|}\Big[\mathcal{L}^{(2)}_{\rm GR}(\mathcal{G}) + \mathcal{L}^{(2)}_{\rm FP}(M) + - \frac{1}{M_{\rm Pl}}(\mathcal{G}_{\mu\nu} - \alpha M_{\mu\nu})T^{\mu\nu}(\Psi)\Big] \;,$$

[cf. R.G.Cai, J.R.Zhang, Y.L. Zhang, arXiv: 2402.03984,

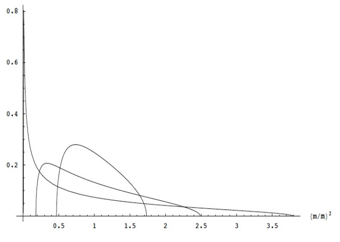
Angular correlation and deformed Hellings-Downs curve by spin-2 ultralight dark matter]

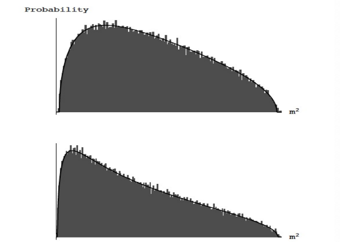
Multi fields: Marcenko-Pastur distribution

Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2-m_-^2)(m_+^2-m^2)}}{2\pi\beta m_0^2 m^2}$,
- Energy density: $\rho_\phi \equiv \int dm\tilde{\rho}(m) = \int dm\frac{1}{2}m^2\tilde{\phi}(m)^2 P(m)$.
- Convenient choice: $\tilde{\rho}(m) \simeq \rho_\phi P(m), \quad \int dmP(m) = 1$.

[cf. Marcenko-Pastur, "Distributions of Eigenvalues for Some Sets of Random Matrices,"(1967)]



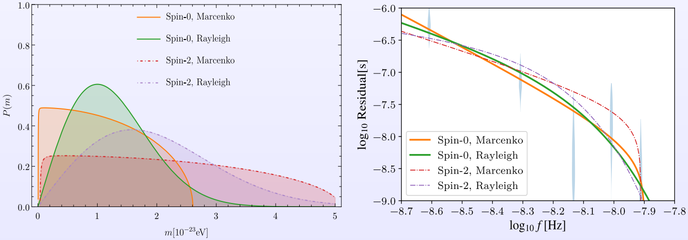


[cf. Easthera-McAllister, "Random Matrices and the Spectrum of N-flation" JCAP(2006)
Cai-Hu-Piao, "Entropy Perturbations in N-flation" PRD(2009)]

Phenomenological fitting results

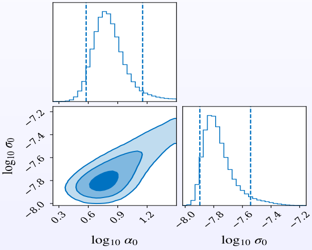
Mass spectrum and ultralight fields

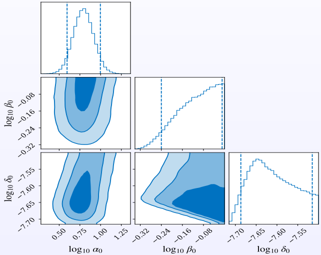
- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2-m_-^2)(m_+^2-m^2)}}{2\pi\beta m_0^2 m^2}$,
- Rayleigh distribution: $P_\sigma(m) = \frac{m}{\sigma^2} e^{-\frac{m^2}{2\sigma^2}}$.

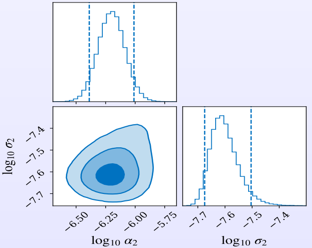


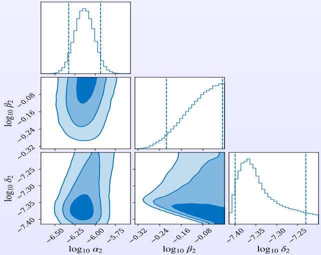
[Sun-Yang-Zhang, PRD(2022) "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter"]

Corner Figures of Bayesian Fitting









Phenomenological fitting results

The effective strain

- $h_c^{\phi}(f) = \frac{\alpha_0}{M_{pl}^2} \frac{\sqrt{3}\rho_{DM}}{4\pi f} P(\pi f)$
- $h_c^{M}(f) = \frac{\alpha_2}{M_{pl}} \frac{m\mathcal{M}P(m)}{\sqrt{5}} = \frac{\alpha_2}{M_{pl}} \frac{2\sqrt{\rho_M}}{\sqrt{5}} P(2\pi f).$

	Parameters	spin-0	spin-1	spin-2
Marcenko	α_i	$5.9^{+1.9}_{-1.3}$	$\sim 3\alpha_0$	$7.6^{+2.2}_{-1.7} \times 10^{-7}$
	$m_-^i/(10^{-23}\mathrm{eV})$	$2.9^{+3.6}_{-0.3} \times 10^{-3}$	$\sim \delta_0(1-\sqrt{\beta_0})$	$6.3^{+6.0}_{-1.7} \times 10^{-3}$
	$m_+^i/(10^{-23}\mathrm{eV})$	$2.61^{+0.21}_{-0.01}$	$\sim \delta_0(1+\sqrt{\beta_0})$	$5.08^{+0.02}_{-0.01}$
Rayleigh	α_i	$5.6^{+3.8}_{-1.0}$	$\sim 3\alpha_0$	$6.1^{+2.1}_{-1.3} \times 10^{-7}$
	$\sigma_i/(10^{-23}\mathrm{eV})$	$1.0^{+0.4}_{-0.1}$	$\sim \sigma_0$	$1.6^{+0.3}_{-0.1}$

[Sun-Yang-Zhang, PRD(2022), "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter"]

Summary and Outlook

ULDM with spin 0,1,2 & GW effects

- Spin-2 ultralight dark matter & Bi-metric gravity
- **Wideband mass spectrum** extension of ULDM
- Search/constraint for ULDM with GW detection
- Angular correlation & **Deformed Hellings-Downs**

GW detection & Ultra-light DM with multi band

- Tabletop exp: GW burst & axion ($\sim$ kHz - GHz)
- LVK & LISA: BH Superradiance ($\sim$ mHz - kHz)
- PTA & SKA : Fuzzy dark matter ($\sim$ nHz)
- LSS & CMB : DE & Modified gravity ($\sim$ nnHz)