On-shell Massless-Massive Correspondence: A Framework to Construct Massive Amplitudes

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paper in preparation

Outline

1 Background & Motivation

2 Bottom-up massive amplitudes

3 Top-down massless amplitudes

4 On-shell matching from UV to IR

5 Conclusion

- Why massive amplitudes?

Massive particles W, Z, h, t.

Higgs factory (CEPC, etc.)

- On-shell technique

Momentum + Little group

the representation of Poincare group $\begin{cases} p^2 = 0, \ \mathrm{U}(1)_{\mathrm{LG}} \\ \mathbf{p}^2 \neq 0, \ \mathrm{SU}(2)_{\mathrm{LG}} \end{cases}$

N. Arkani-Hamed, et al. 1709.04891

massless particle (helicity h) $\begin{cases} (\lambda_\alpha)^{2h} \equiv (|p\rangle_{\dot{\alpha}})^{2h}, h > 0 \\ (\tilde{\lambda}^{\dot{\alpha}})^{2h} \equiv (|p]^{\dot{\alpha}})^{2h}, h < 0 \end{cases}$

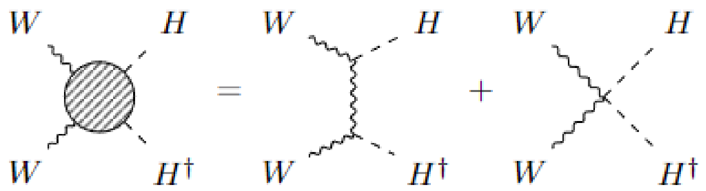
$p_{\alpha\dot{\alpha}} \equiv \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}},$

massive particle (spin s) $(\lambda_\alpha^I)^{2s+i}(\tilde{\lambda}^{\dot{\alpha}I})^{2s-i} \equiv (|p\rangle_\alpha^I)^{2s+i}(|p]^{\dot{\alpha}I})^{2s-i}$

$\mathbf{p}_{\alpha\dot{\alpha}} \equiv \lambda_\alpha^I \tilde{\lambda}_{I\dot{\alpha}},$

,where I is $\mathrm{SU}(2)_{\mathrm{LG}}$ indices

- Constructing amplitude



amplitude = factorized term + contact term

For massless amplitudes, $\lim_{P^2\to 0} P^2\mathcal{A} = \sum_{h_P} A^L \times A^R$

Massless contact term vanishes in the gauge-independence description MA Huber, SD Angelis 2108.03669

For massive amplitudes, $\lim_{\mathbf{P}^2\to\mathbf{m}^2} (\mathbf{P}^2 - \mathbf{m}^2)\mathcal{M} = M^L \otimes M^R$

Massive contact term need other information

- The total unitarity ? D. Liu, Z. Yin 2204.13119
  It cannot work in general field theory.
- UV information (gauge symmetry)

- Massive spinor → Massless spinor

$$h=+\frac{1}{2} \qquad h=-\frac{1}{2}$$

$$\lambda_\alpha^I = \underline{\eta_\alpha}\zeta^{+I} - \underline{\lambda_\alpha}\zeta^{-I},$$
$$\tilde{\lambda}_{\dot\alpha}^I = \underline{\tilde{\eta}_{\dot\alpha}}\zeta^{-I} + \underline{\tilde{\lambda}_{\dot\alpha}}\zeta^{+I}.$$

$$\mathbf{p}_{\alpha\dot\alpha} = p_{\alpha\dot\alpha} + \eta_{\alpha\dot\alpha} = \lambda_\alpha\tilde{\lambda}_{\dot\alpha} + \eta_\alpha\tilde{\eta}_{\dot\alpha}$$

$$m=\langle p\eta\rangle, \quad \tilde{m}=[\eta p] \qquad m\tilde{m}=\mathbf{m}^2$$

For real momentum, $m=\tilde{m}=\mathbf{m}$

- Spin → Helicity $\quad \mathcal{S}=(s_1,\cdots,s_n) \rightarrow \mathcal{H}=(h_1,h_2,\cdots,h_n)$

$$\mathcal{M}^{\mathcal{S}} = \sum_{\mathcal{H}}\prod_{i=1}^{n}\left((\zeta_i^+)^{s_i+h_i}(\zeta_i^-)^{s_i-h_i}\right)\mathcal{M}^{\mathcal{H}}(\lambda,\tilde{\lambda},\eta,\tilde{\eta})$$

- η-expansion $\quad (\lambda_{\alpha\dot\alpha} \gg \eta_{\alpha\dot\alpha})$

$$\mathcal{M}^{\mathcal{H}} = \boxed{\frac{1}{\mathbf{m}^a}}\sum_k \eta^k\frac{\partial^k}{\partial\eta^k}\left(\mathbf{m}^a\mathcal{M}^{\mathcal{H}}\right)$$

$1/\mathbf{m}$-singularity

- Power counting

SM: $$\frac{1}{\mathbf{m}^a}\eta^k\frac{\partial^k}{\partial\eta^k}\left(\mathbf{m}^a\mathcal{M}^{\mathcal{H}}\right)\sim E^{4-n}\left(\frac{v}{E}\right)v^{2k-a}$$

$$\mathcal{M}^{\mathcal{H}}\overset{H.E.}{\rightarrow}\sum_l[\mathcal{M}^{\mathcal{H}}]_l\sim\sum_l E^{4-n}\left(\frac{v}{E}\right)^l$$

$$\mathcal{A}_l^{\text{unbroken}}\sim E^{4-n-l}$$

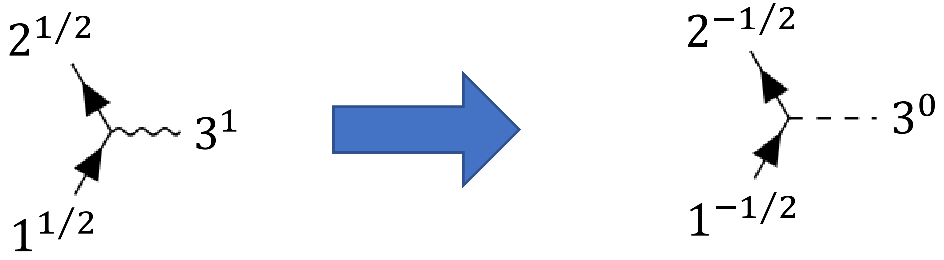
$$\mathcal{A}_l^{\text{broken}}\sim v^l E^{4-n-l}$$

- Massless-massive correspondence

$$\mathcal{M}^{\mathcal{H}}\rightarrow[\mathcal{M}^{\mathcal{H}}]_0+[\mathcal{M}^{\mathcal{H}}]_1+[\mathcal{M}^{\mathcal{H}}]_2+\cdots$$

$$\mathcal{A}_0^{\text{broken}}\quad\mathcal{A}_1^{\text{broken}}\quad\mathcal{A}_2^{\text{broken}}$$

- Goldstone Equivalence Theorem (GET)



$$X_f^1 \frac{\langle \mathbf{13}\rangle[\mathbf{23}]}{\mathbf{m}_3} + X_f^2 \frac{\langle \mathbf{23}\rangle[\mathbf{13}]}{\mathbf{m}_3}$$

$$X_f^1 \frac{\langle 13\rangle[\eta_2 3]}{\mathbf{m}_3} + X_f^2 \frac{\langle 23\rangle[\eta_1 3]}{\mathbf{m}_3} = \left(-X_f^1 \frac{\tilde{m}_2}{\mathbf{m}_3} + X_f^2 \frac{\tilde{m}_1}{\mathbf{m}_3}\right)\langle 12\rangle$$

- The difficulty of the bottom-up approach

  1. At subleading order $(l > 0)$, the GET does not work.
  2. The $(n + l)$-point massless pole structure cannot match to n-point massive pole structure .

$$\mathcal{A}_l^{\text{unbroken}} \overset{①}{\Rightarrow} \mathcal{A}_l^{\text{broken}} \overset{②}{\Rightarrow} [\mathcal{M}]_l \overset{③}{\Rightarrow} \mathcal{M}$$

① gauge symmetry breaking

amplitude = gauge × kinematic
- Gauge: unbroken ⇒ broken
- Kinematic: distinguish Higgs and Goldstone bosons

② pole structure

$l = 0$ For $\mathcal{A}$ with external gauge boson, isolate pole structure to unify UV amplitudes with different helicity categories.

$l > 0$ On-shell Higgsing: reduce $(n + l)$-point pole structure to l-point pole structure

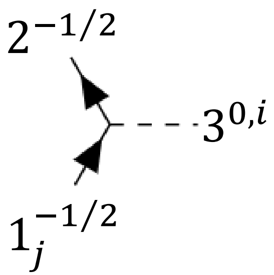
③ recover little-group covariance

Bold spinors $\begin{cases} \lambda, \eta \Rightarrow \boldsymbol{\lambda} \\ \tilde{\lambda}, \tilde{\eta} \Rightarrow \tilde{\boldsymbol{\lambda}} \end{cases}$ IR deform denominator $p^2 \Rightarrow \boldsymbol{p}^2 - m^2$

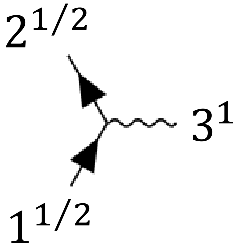
Bold mass $m, \tilde{m} \Rightarrow \mathbf{m}$

- Higgs mechanism

$$Y_j^i\langle 12\rangle$$



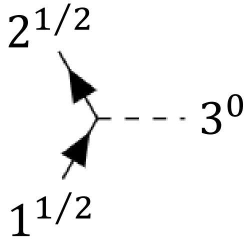
$Y_j^{i=1} \Rightarrow \boldsymbol{X}$



$$\boldsymbol{X}\left(-c_1\frac{[2\eta_2]\langle 12\rangle}{\widetilde{m}_2}-c_2\frac{[1\eta_1]\langle 12\rangle}{\widetilde{m}_1}\right)$$

$$\Rightarrow \boldsymbol{X}\left(c_1\frac{\langle\mathbf{13}\rangle[\mathbf{32}]}{\mathbf{m}_2}-c_2\frac{\langle\mathbf{23}\rangle[\mathbf{31}]}{\mathbf{m}_1}\right)$$

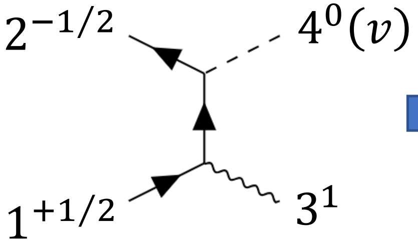
$Y_j^{i=2} \Rightarrow \boldsymbol{Y}$



$$\boldsymbol{Y}\langle\mathbf{12}\rangle$$

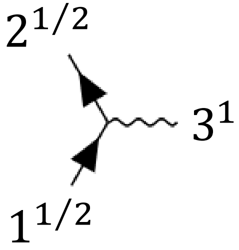
- On-shell Higgsing R. Balkin, et al. 2112.09688

$$gy\frac{\langle 1\eta_3\rangle[34]\langle 42\rangle}{m_3 s_{24}}$$



$$p_4 \Rightarrow \eta_2$$

$$\frac{1}{s_{24}} \Rightarrow \frac{1}{\mathbf{m}_2^2}$$



$$gyv\frac{\langle 1\eta_3\rangle[3|\eta_2|2\rangle}{m_3\mathbf{m}_2^2} \Rightarrow g\frac{\langle\mathbf{13}\rangle[\mathbf{32}]}{\mathbf{m}_3}$$

- Massive ansatz

$$\mathcal{M} \equiv \mathcal{I}(\mathbf{p},\mathbf{m}) \odot \mathcal{E}(\boldsymbol{\lambda},\tilde{\boldsymbol{\lambda}})$$

$\mathrm{SU(2)_{LG}}$ invariant $\mathrm{SU(2)_{LG}}$ covariant

$$\mathcal{M} \supset \sum_{a,b} \frac{\mathbf{c}^{b,a}\mathbf{p}^b\mathbf{m}^a}{(\mathbf{p}^2-\mathbf{m}^2)^d} \odot \mathcal{E}$$

unitarity
locality
scaling information (from lower-point amplitudes)

- The leading order for the term with a given power a

$$\frac{\mathbf{p}^b\mathbf{m}^a}{(\mathbf{p}^2-\mathbf{m}^2)^d} \odot \mathcal{E} \overset{H.E.}{\rightarrow} \boxed{E^{4-n}\left(\frac{v}{E}\right)^{a-n_L}} + \mathcal{O}\left(v^{a-n_L-2}\right)$$

$$\mathcal{A}^{\text{broken}}_{l=a-n_L}$$

$a = 2$

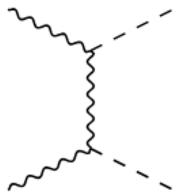
$n_L \backslash l$	0	1	2	$\cdots$
0			•	
1		•		
2	•			

- Order-by-order MMC

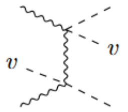
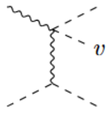
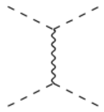
W^+W^-hh amplitude

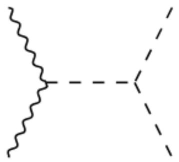
ansatz

$$\left(\frac{\mathbf{c}_1^{2,0}\mathbf{p}^2+\mathbf{c}_1^{0,2}\mathbf{m}^2}{\mathbf{P}_{13}^2-\mathbf{m}_W^2}+\frac{\mathbf{c}_2^{2,0}\mathbf{p}^2+\mathbf{c}_2^{0,2}\mathbf{m}^2}{\mathbf{P}_{14}^2-\mathbf{m}_W^2}+\frac{\mathbf{c}_3^{0,2}\mathbf{m}^2}{\mathbf{P}_{12}^2-\mathbf{m}_h^2}+\mathbf{c}_4^{0,0}\right)\odot\frac{\boldsymbol{\lambda}_1\tilde{\boldsymbol{\lambda}}_1}{\mathbf{m}_1}\frac{\boldsymbol{\lambda}_2\tilde{\boldsymbol{\lambda}}_2}{\mathbf{m}_2}$$

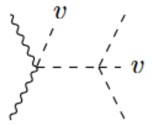


$\Leftrightarrow$

n_L \ l	0	1	2	$\cdots$
0				
1				
2				



$\Leftrightarrow$

n_L \ l	0	1	2	$\cdots$
0				
1				
2				

final result

$$\mathcal{M}(W^+W^-hh)=\frac{\mathbf{g}^{W^+W^-}\mathbf{g}^{W^+W^-}}{2m_W^4}\frac{\langle\mathbf{1}|\mathbf{p}_3|\mathbf{1}]\langle\mathbf{2}|\mathbf{p}_4|\mathbf{2}]+2m_W^2[\mathbf{12}]\langle\mathbf{21}\rangle}{\mathbf{P}_{13}^2-m_W^2}+\frac{\mathbf{g}^{W^+W^-}\mathbf{g}^{W^+W^-}}{2m_W^4}\frac{\langle\mathbf{1}|\mathbf{p}_4|\mathbf{1}]\langle\mathbf{2}|\mathbf{p}_3|\mathbf{2}]+2m_W^2[\mathbf{12}]\langle\mathbf{21}\rangle}{\mathbf{P}_{14}^2-m_W^2}+\frac{\mathbf{g}^{W^+W^-}\lambda_3}{m_W^2}\frac{[\mathbf{12}]\langle\mathbf{21}\rangle}{\mathbf{P}_{12}^2-m_h^2}+\frac{\mathbf{g}^{W^+W^-}\mathbf{g}^{W^+W^-}}{2m_W^4}[\mathbf{12}]\langle\mathbf{21}\rangle$$

1. We found an expansion with correct power counting for the massive amplitudes. Using this expansion, we found each order contribution have a clear UV origin in the top-down approach. The higher-order contribution can be explained by the On-shell Higgsing.

2. Based on this expansion, we propose a bottom-up inspired top-down matching to construct massive amplitudes. In this method, the massive amplitude can be constructed order by order and each massive diagram satisfy the unitarity.

3. Our method can be generalized to other massive field theories with spontaneously symmetry breaking (e.g. SMEFT and other BSM models) in both tree and loop level.