The Quantum Simulation on a (1+1)D Sphaleron Model

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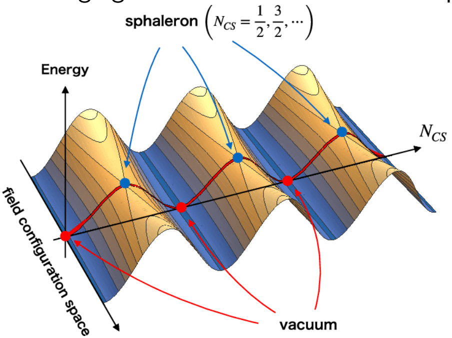
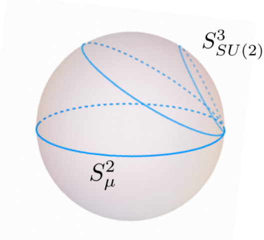
The Sphaleron Solution

- The *static unstable* vacuum solution at which the non-contractible loop in the field configuration space of the gauge field reaches its *saddle point* is the generalized sphaleron solution.
- It is a physically static solution governing the thermal equilibrium dynamics between vacuum states with different N_{CS}. Its dynamics are crucial but challenging to simulate on classical computers.



The Sphaleron Solution

- The sphaleron solution in the *Weinberg-Salam model* is given by Manton which reach its saddle point at $\mu = \pi/2$ has important implications in theories of baryogenesis.



$$
\begin{aligned}
E = \int \Bigg\{ & \frac{4}{g^2 r^2} \left[\left(\frac{df}{dr} \right)^2 \sin^2 \mu + \frac{2}{r^2} [f(1-f)]^2 \sin^4 \mu \right] \\
& + \frac{v^2}{2} \left[\left(\frac{dh}{dr} \right)^2 \sin^2 \mu + \frac{2}{r^2} \{ [(h(1-f))^2 \sin^2 \mu \right. \\
& - 2fh(1-f)(1-h) \cos^2 \mu \sin^2 \mu + [f(1-h)]^2 \cos^2 \mu \sin^2 \mu \\
& + \frac{\lambda v^4}{4} (h^2 - 1)^2 \sin^4 \mu \Bigg\} 4\pi r^2 \, dr
\end{aligned}
$$

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Quantum Computing Applied to High-Energy Physics

- Quantum computing capabilities:
  - Experimental data analysis
  - Lattice gauge calculations
- Physical properties:
  - Intrinsic physical properties align with particle physics to address classical computation blind spots (e.g., sign problem)
- High-energy physics simulations:
  - Simulating field structures
  - Modeling evolution behavior

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Quantum Simulation Challenges

- Immature technology:
  - Theoretical error-correction structure undeveloped
  - Extremely limited number of available qubits
- Theoretical challenges
  - Variety of particles and multi-level symmetries
  - Dependence on algorithm design
  - Truncating infinite-dimensional Hilbert space to finite spaces
- Current status
  - Utilizing low-dimensional models
  - Techniques for truncating Hilbert space
  - Analyzing feasibility of quantum simulation

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The (1+1)D Sphaleron Model

- $O(3)$ non-linear sigma model:

$$S_0 = \frac{1}{2g^2}\int d^2x\mathcal{L}_0 = \frac{1}{2g^2}\int d^2x(\partial_\mu\vec{n})^2$$

$$|\vec{n}|^2 = 1$$

- Extra symmetral breaking term:

$$\mathcal{L} = \mathcal{L}_0 - \mathcal{L}' = \frac{1}{2g^2}(\partial_\mu\vec{n})^2 - \frac{\omega^2}{g^2}(1+n_z)$$

- Field configuration:

$$\vec{n}(\xi(x),\eta) = (\sin\eta\sin\xi(x), \sin\eta\cos\eta(1-\cos\xi(x)), -\sin^2\eta\cos\xi(x)-\cos^2\eta)$$

$$\lim_{x\to-\infty}\xi(x) = 0 \quad \lim_{x\to+\infty}\xi(x) = 2\pi$$

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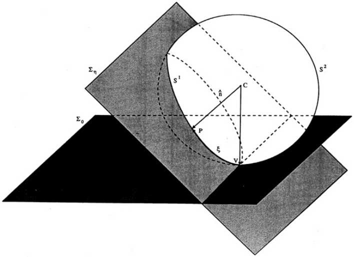
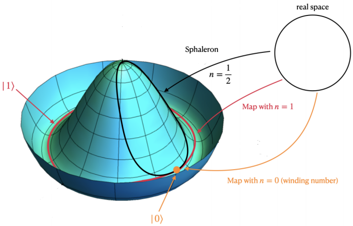
The (1+1)D Sphaleron Model





The sphaleron solution:

$$E = \frac{\sin^2\eta}{g^2}\int dx\left(\frac{1}{2}\left(\frac{\partial\xi}{\partial x}\right)^2 + \omega^2(1-\cos\xi)\right)$$

$$\xi = 2\arcsin(\operatorname{sech}(\omega x)) \quad \eta = \frac{\pi}{2} \quad E_{\text{sph}}\Big|_{\eta=\frac{\pi}{2}} = \frac{8\omega}{g^2}$$

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- Discretization:

$$H = \frac{g^2}{2a}\sum_{k=-N}^{N}\vec{L}_k^2 - \frac{1}{ag^2}\sum_{k=-N}^{N-1}\vec{n}_k\vec{n}_{k+1} + \frac{a\omega^2}{g^2}\sum_{k=-N}^{N} n_k^z,$$

- Change the form:

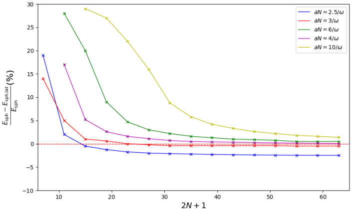
$$H = \frac{\sin^2\eta_0}{ag^2}\sum_{k=-N}^{N}\left(a^2\omega^2(1-\cos\xi_k) + (1-\cos(\xi_k-\xi_{k+1}))\right)$$

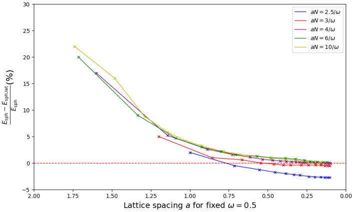
- Boundary condition and Sphaleron solution:

$$\eta_0 = \frac{\pi}{2}, \xi_0 = \pi, \xi_{-N} = 0, \xi_N = 2\pi$$

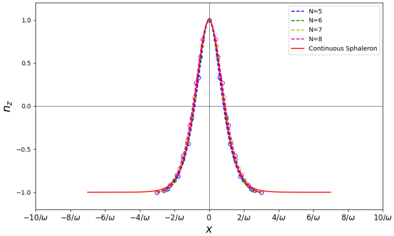
$$E_{\text{sph,lat}}\Big|_{\eta=\frac{\pi}{2}} \overset{aN\to\infty, a\to 0}{\longrightarrow} E_{\text{sph}}\Big|_{\eta=\frac{\pi}{2}}$$

- The choosen of finite volume:





- The field configuration at volume of $\frac{6}{\omega}$:



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Constructing quantum gates and quantum algorithms

- Hamiltonian in the form of Pauli quantum gates:

$$|n\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

$$\langle n|\hat{\sigma}_i|n\rangle = n_i.$$

$$\mathcal{H} = -\frac{1}{ag^2}\sum_{k=-N}^{N}\vec{\sigma}_k\vec{\sigma}_{k+1} + \frac{a\omega^2}{g^2}\sum_{k=-N}^{N}\sigma_k^z$$

- Boundary condition

$$\theta_k^{\rm sph} = \xi_k^{\rm sph} - \pi, \varphi_k^{\rm sph} = {\rm Const}$$

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- Initial Hamiltonian

$$\mathcal{H}_{\text{in}} = \sum_{k=-N}^{N} \frac{a\omega^2}{g^2} \sigma_k^z$$

- Initial States: The central qubit on $(0, 0, 1)$, the rest on $(0, 0, -1)$
- Boundary Condition

$$\theta_0 = 0, \theta_{-N} = -\pi, \theta_N = \pi$$

- Adiabatic algorithms for $\bar{g} = \frac{1}{ag^2}, \bar{\omega} = \frac{a\omega^2}{g^2}$

$$\begin{aligned} U(t) = \mathcal{T}\left[e^{-i\int_0^t d\tau H(\tau)}\right] &\approx \prod_{s=0}^{S} e^{i\left(H_g\left(\frac{st}{S}\right)+H_\omega\right)\frac{t}{S}} \\ &\approx \prod_{s=0}^{S} \left(e^{-i\sum_{i=-N}^{N-1}(\sigma_i^x\sigma_{i+1}^x+\sigma_i^y\sigma_{i+1}^y)\bar{g}\frac{st}{S}\frac{t}{2S}} e^{i\sum_{i=-N}^{N-1}(\sigma_i^z\sigma_{i+1}^z)\bar{g}\frac{st}{S}\frac{t}{2S}} e^{-i\bar{\omega}\frac{t}{S}\sum_{i=-N}^{N}\sigma_i^z} \right. \\ &\qquad \left. e^{i\sum_{i=-N}^{N-1}(\sigma_i^z\sigma_{i+1}^z)\bar{g}\frac{st}{S}\frac{t}{2S}} e^{i\sum_{i=-N}^{N-1}(\sigma_i^x\sigma_{i+1}^x+\sigma_i^y\sigma_{i+1}^y)\bar{g}\frac{st}{S}\frac{t}{2S}} \right) \end{aligned}$$

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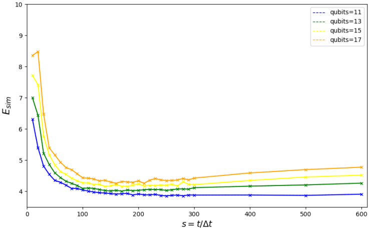
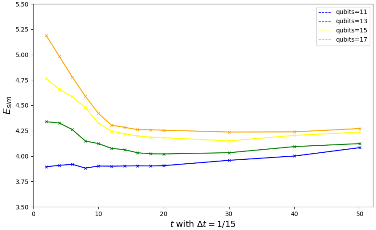
Simulation Results



The energy of simulation final states with fixed $t = 15$



The energy of simulation final states with fixed $\Delta t = \frac{1}{15}$

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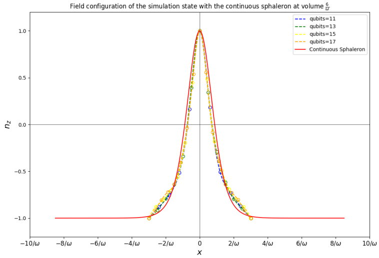
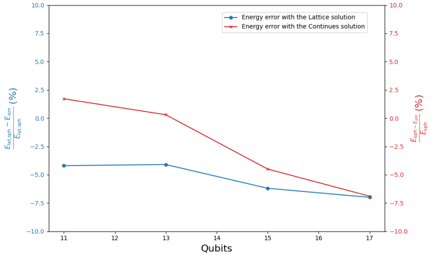
Simulation Results



The configuration of simulation states with boundary conditions matching to the therotical states



The energy error of simulation states with boundary conditions matching to the therotical states

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Referance I

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Thank You!