中国物理学会高能物理分会
HIGH ENERGY PHYSICS BRANCH OF CPS

山东大学
SHANDONG UNIVERSITY

第十四届全国粒子物理学术会议

top-pair production at the high-energy colliders

Xing-Gang Wu, Chongqing University

Related publications for top-pair production

- S.~J.~Brodsky and X.~G.~Wu, Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the Principle of Maximum Conformality, PRL109, 042002 (2012)
- S.~J.~Brodsky and X.~G.~Wu, Application of the Principle of Maximum Conformality to Top-Pair Production, PRD86, 014021 (2012)
- S.~Q.~Wang, X.~G.~Wu, Z.~G.~Si and S.~J.~Brodsky, Predictions for the Top-Quark Forward-Backward Asymmetry at High Invariant Pair Mass Using the Principle of Maximum Conformality, PRD93, 014004 (2016)
- S.~Q.~Wang, X.~G.~Wu, Z.~G.~Si and S.~J.~Brodsky, A precise determination of the top-quark pole mass, EPJC78, 237 (2018)
- S.~Q.~Wang, X.~G.~Wu, J.~M.~Shen and S.~J.~Brodsky, Reanalysis of the top-quark pair hadroproduction and a precise determination of the top-quark pole mass at the LHC, CPC45, 113102 (2021)

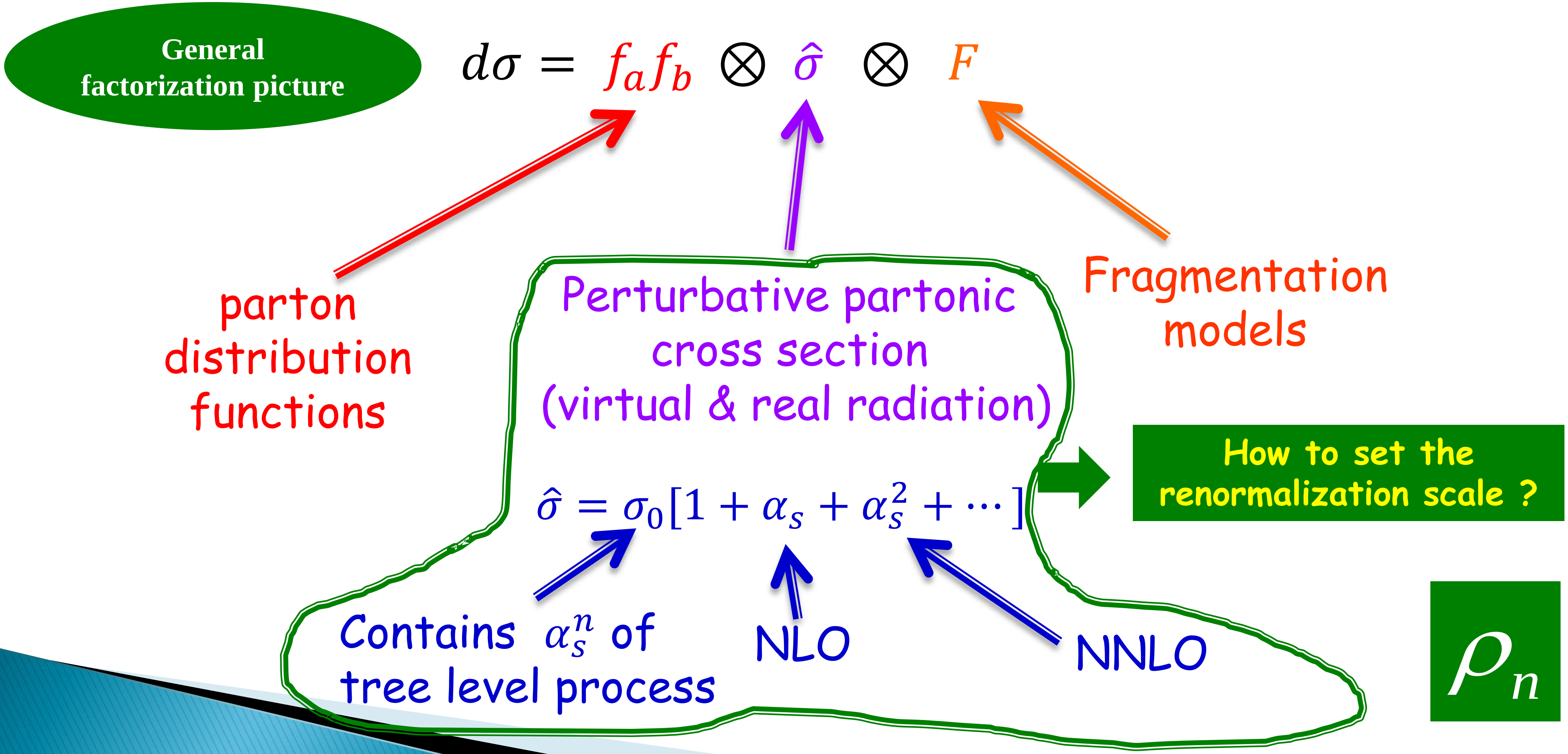
- S.~Q.~Wang, S.~J.~Brodsky, X.~G.~Wu, L.~Di Giustino and J.~M.~Shen, Renormalization scale setting for heavy quark pair production in e^+e^- annihilation near the threshold region, PRD102, 014005 (2020)
- J.~Yan, X.~G.~Wu, Z.~F.~Wu, J.~H.~Shan and H.~Zhou, Reanalysis of the top-quark pair production via the e+e\ensuremath{-} annihilation near the threshold region up to N3LO QCD corrections, PLB853, 138664 (2024)

OUTLINE

- Perturbative QCD scale-setting problem
- Top-pair production via hadron-hadron collisions
- Top-pair production via e+e- collisions near the threshold region
- Summary and outlook

- Perturbative QCD scale-setting problem

Fixed-order predictions such as those at the hadronic colliders



General factorization picture

$$d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$$

parton distribution functions

Perturbative partonic cross section (virtual & real radiation)

Fragmentation models

$$\hat{\sigma} = \sigma_0[1 + \alpha_s + \alpha_s^2 + \cdots]$$

Contains α_s^n of tree level process

NLO

NNLO

How to set the renormalization scale ?

ρ_n

Standard RGI

Equivalence to: $\frac{\partial \rho_n}{\partial \mu_R} \neq 0; \frac{\partial \rho_n}{\partial \mathrm{R}} \neq 0;$ $n-$perturbative order, R-scheme

At any fixed-order, QCD series is non-conformal, the prediction is scale and scheme dependent due to mismatching of α_s with its coefficients for an arbitrary choice of scale.

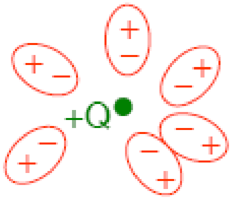
Initial perturbative series 原始微扰序列

But how about the improved series ?

Brodsky-Lepage-Mackenzie (BLM)

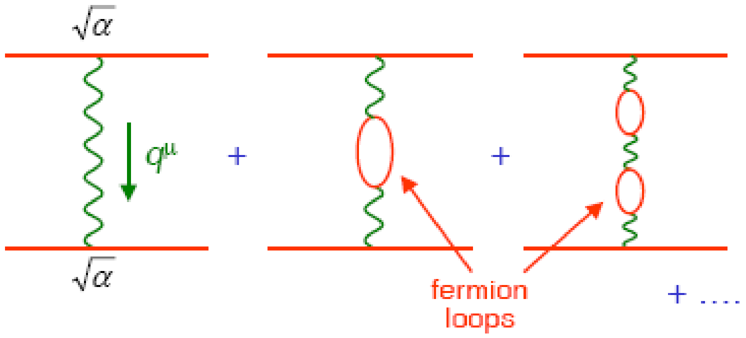
Running Couplings

- QED:



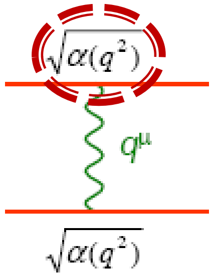
"Bare" electric charge is screened by "halo" of e⁺e⁻ pairs

In terms of Feynman diagrams:



→ infinite series of diagrams which can be (approximately) summed:

$$\alpha(q^2)=\alpha(q_0^2)\Big/\left[1-\frac{\alpha(q_0^2)}{3\pi}\ln\left(\frac{q^2}{q_0^2}\right)\right]$$



→ replace by single diagram with running coupling

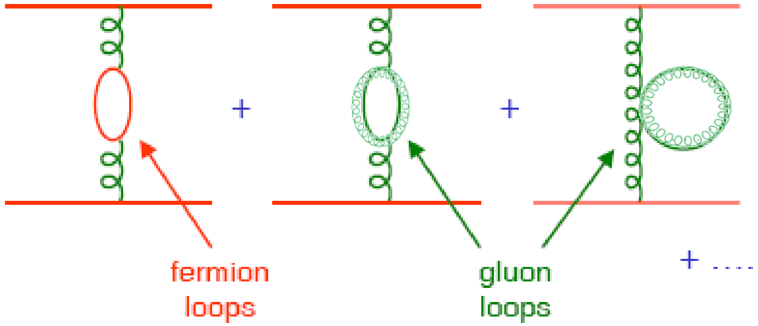
Q0 – Initial scale

Resummed to achieve a scale-invariant prediction independent to Q0

--链圈图近似--

类比找思路

- QCD: similar, but now have:



Extra diagrams produce anti-screening

$$\alpha_s(q^2)=\alpha_s(q_0^2)\Big/\left[1+B\alpha_s(q_0^2)\ln\left(\frac{q^2}{q_0^2}\right)\right]$$

where $B=\frac{11N-2N_f}{12\pi}$ with $N_c=3$ $N_f=6$

GM-L：重求和泡泡图可获得准确的α值 → **BLM：重求和泡泡图可获得准确的α_s值**

PMC tries to solve scheme-and-scale ambiguities simultaneously

Basis—renormalization group equation (RGE)

Phys.Rev.D86,054018 (2012)

$\frac{\partial\rho_n}{\partial\mu_R}\neq 0;\frac{\partial\rho_n}{\partial \mathrm{R}}\neq 0;$ We cannot get exact constraints from those inequalities

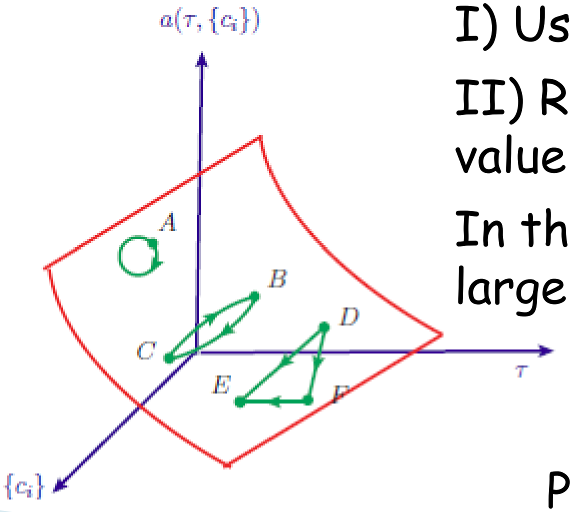
Key idea of PMC: we can only get the answer from RGE itself, which can be used to determine the running behavior of coupling constant, thus fixing scale ambiguity.

I) Using RGE to determine the beta-terms at each order.

II) Resumming all the same type beta-terms, determining the exact value for exch perturbative order.

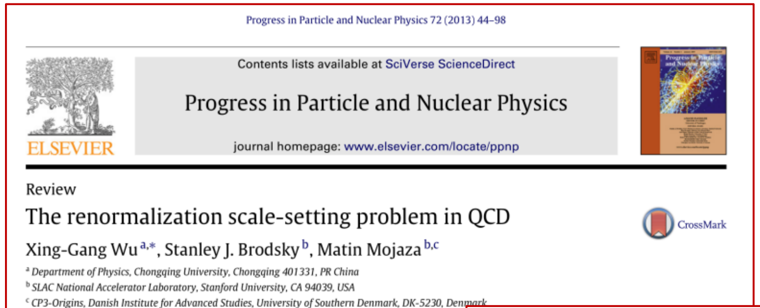
In this sense, PMC is similar to resummation, which resum a kind of large log-terms to form a steady prediction.

But different to a pure resummation to improve the reliability, PMC tends to solve the scale-setting problem.



PMC satisfies RGE-properties: symmetry, reflecxity,transitivity

QCD scale-setting problem
Several reviews of PMC


Progress in Particle and Nuclear Physics 72 (2013) 44–98

Contents lists available at SciVerse ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

ELSEVIER

Review

The renormalization scale-setting problem in QCD

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ARTICLE INFO

Keywords:
Renormalization group
Renormalization scale
BLM/PMC
QCD

ABSTRACT

A key problem in making precise pertu
malization scale of the running couplin
an arbitrary range and an arbitrary sys
fact, this *ad hoc* procedure gives result
tion scheme, and it is in conflict with
Predictions for physical results should b
theoretical conventions. We review cu
the renormalization scale ambiguity a

2013


IOP Publishing

Reports on Progress in Physics

Rep. Prog. Phys. **78** (2015) 126201 (15pp)

doi:10.1088/0034-4885/78/12/126201

Review

Renormalization group invariance and optimal QCD renormalization scale-setting: a key issues review

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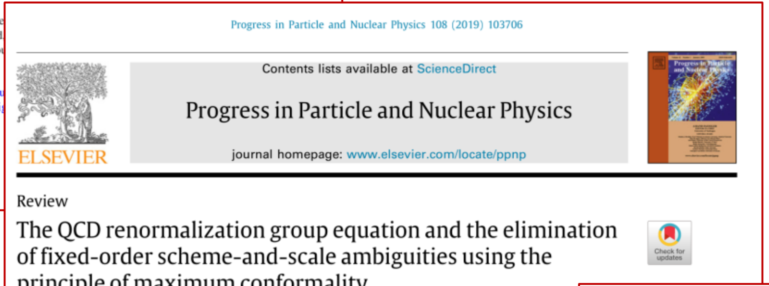
2015


Progress in Particle and Nuclear Physics 108 (2019) 103706

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ELSEVIER

Review

The QCD renormalization group equation and the elimination of fixed-order scheme-and-scale ambiguities using the principle of maximum conformality

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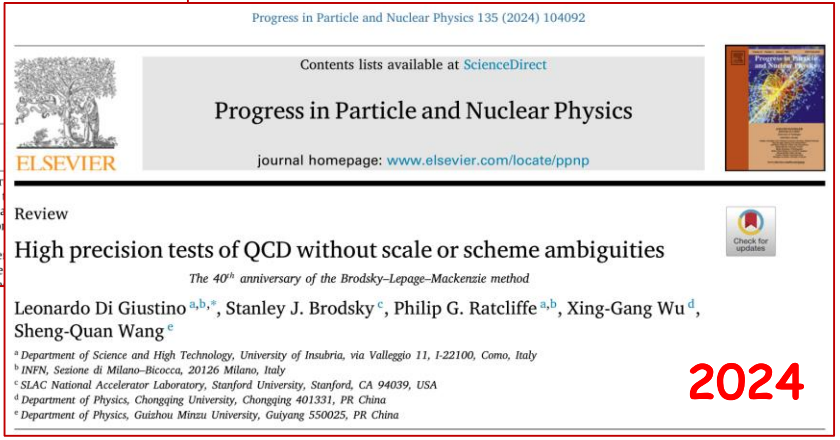
Principle of maximum conformality

ABSTRACT

The conventional scale setting approach to fixed-order per
tions is based on a guessed renormalization scale, usually
the large log-terms of the pQCD series, together with an
uncertainty. This *ad hoc* assignment of the renormalization
of the QCD running coupling at each perturbative order
the choices of both the renormalization scale and the re
leads to conventional renormalization scheme-and-scale

2019


Progress in Particle and Nuclear Physics 135 (2024) 104092

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journal homepage: www.elsevier.com/locate/ppnp

ELSEVIER

Review

High precision tests of QCD without scale or scheme ambiguities

The 40th anniversary of the Brodsky–Lepage–Mackenzie method

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2024

了解PMC进展及应用

初始微扰序列存在硬伤：能标相关、方案相关、收敛性不可控

Comparing with initial series, the PMC series has advantages

Better convergence; More accurate without scheme-and-scale dependence; The coefficients have no RGE-relations; …

Thus it has good potential to do the estimation; especially it can achieve more precise prediction with less given orders.

微扰区域

The perturbative series is meaningful
(Feasible，Reliability，Precision、Predictive)

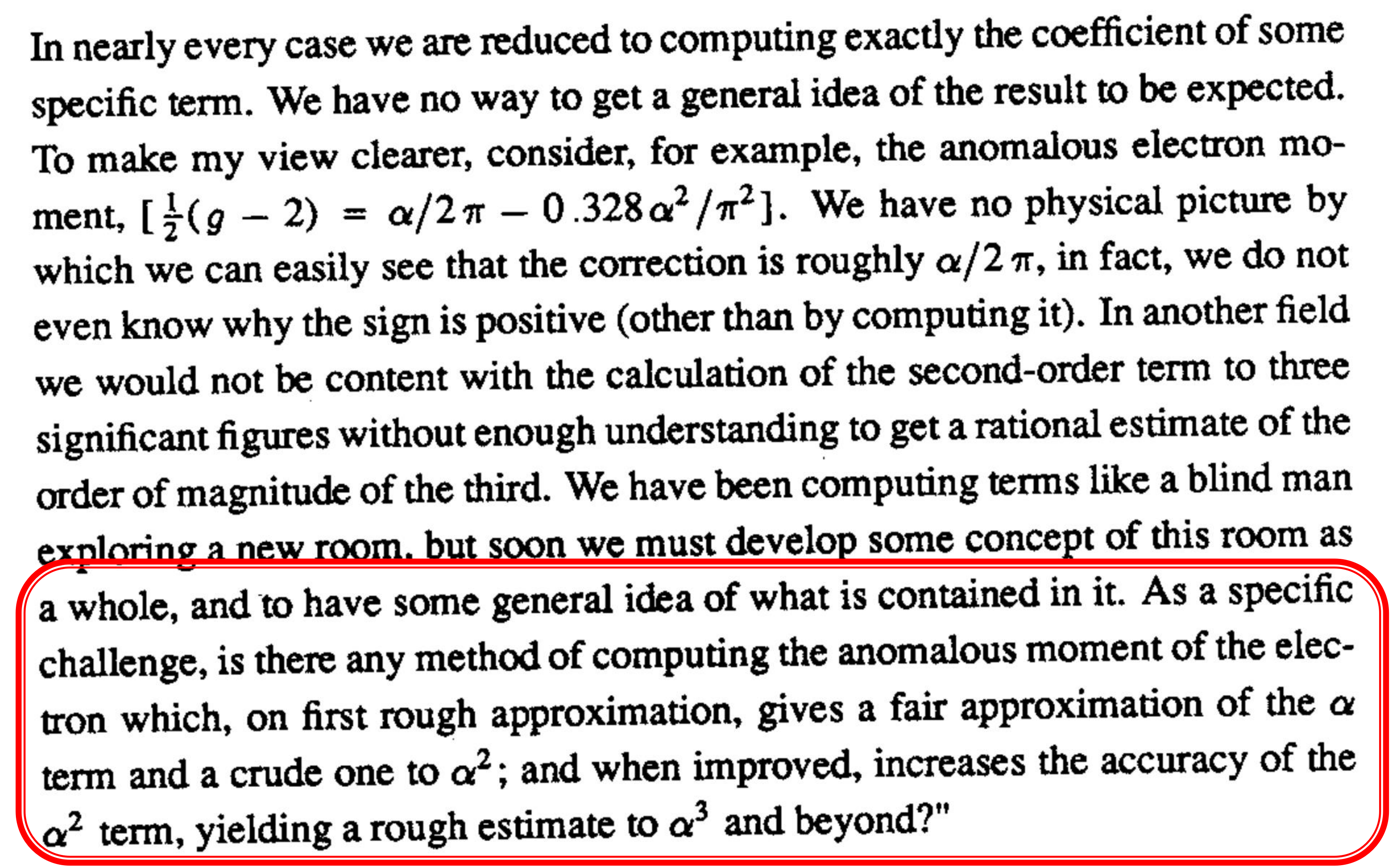
渐近自由　　可重整性　　重求和（各种类型）　估算未知高阶

The fixed-order series cannot solve all things, in principal, one can finish enough higher-order terms, and etc.;

And after removing scale and scheme ambiguity, there is still residual scale dependence due to unknown higher-order (UHO) terms, and what's the magnitude of the UHO-terms !

注：传统讨论误差，实际并不完整 — 极少估算未知高阶项大小并将其作为误差

Feynman, 1959, ``How to estimate higher order terms in the perturbation series without having to laboriously calculate Feynman diagrams"

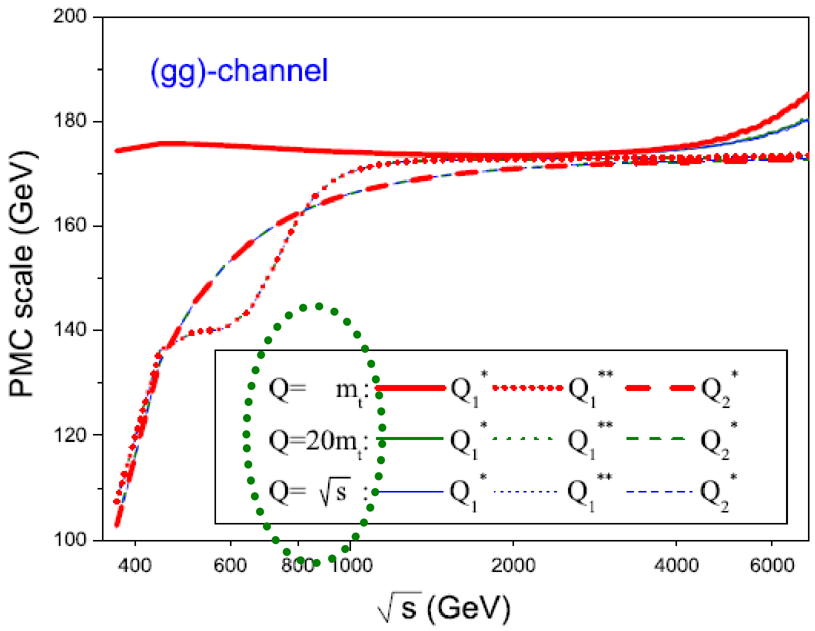


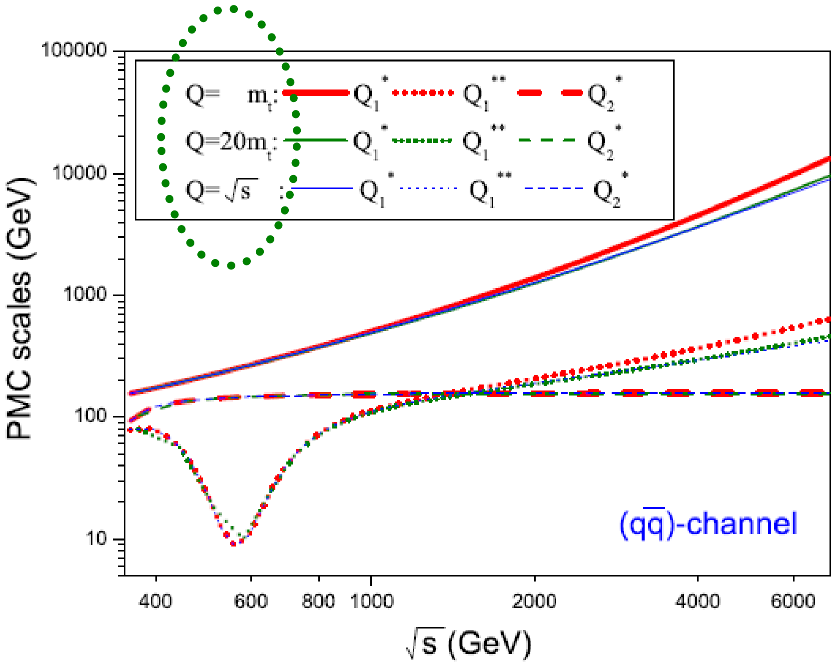
In nearly every case we are reduced to computing exactly the coefficient of some specific term. We have no way to get a general idea of the result to be expected. To make my view clearer, consider, for example, the anomalous electron moment, $[\frac{1}{2}(g - 2) = \alpha/2\pi - 0.328\alpha^2/\pi^2]$. We have no physical picture by which we can easily see that the correction is roughly $\alpha/2\pi$, in fact, we do not even know why the sign is positive (other than by computing it). In another field we would not be content with the calculation of the second-order term to three significant figures without enough understanding to get a rational estimate of the order of magnitude of the third. We have been computing terms like a blind man exploring a new room, but soon we must develop some concept of this room as a whole, and to have some general idea of what is contained in it. As a specific challenge, is there any method of computing the anomalous moment of the electron which, on first rough approximation, gives a fair approximation of the α term and a crude one to α^2; and when improved, increases the accuracy of the α^2 term, yielding a rough estimate to α^3 and beyond?"

➢ Top-pair production via hadron-hadron collisions

顶夸克对强产生过程的两圈QCD修正

$$\sigma_{t\bar{t}} = \sum_{i,j}\int_{4m_t^2}^{S} ds\,\mathcal{L}_{ij}(s, S, \mu_f)\hat{\sigma}_{ij}(s, \alpha_s(\mu_R), \mu_R, \mu_f),$$



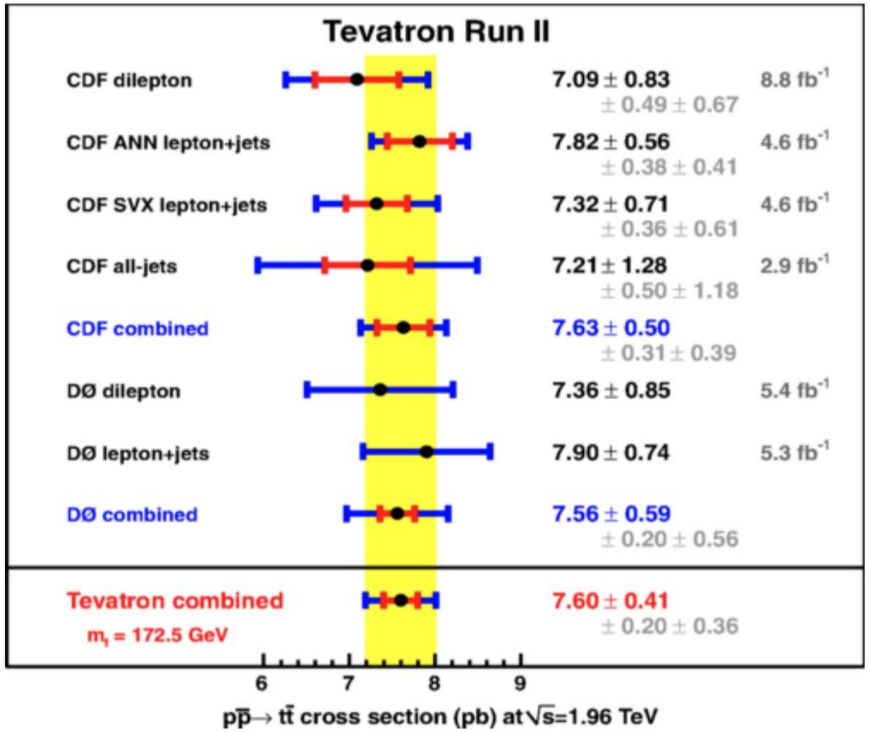


每一微扰阶数下的PMC能标随子过程质心对撞能量的变化
与初始重整化能标的选择无关 - 残留能标依赖性小
- - 如，即使初始能标变化20倍，PMC能标变化也很小 - -

顶夸克对强产生过程的两圈QCD修正

The NNLO top-pair total hadroproduction cross-section almost unchanged !

	PMC scale-setting					Conventional scale-setting		
	$Q = m_t/4$	$Q = m_t/2$	$Q = m_t$	$Q = 2m_t$	$Q = 4m_t$	$\mu_r \equiv m_t/2$	$\mu_r \equiv m_t$	$\mu_r \equiv 2m_t$
Tevatron (1.96 TeV)	7.620(5)	7.622(5)	7.626(3)	7.622(6)	7.623(6)	7.742(5)	7.489(3)	7.199(5)
LHC (7 TeV)	171.6(1)	171.7(1)	171.8(1)	171.7(1)	171.7(1)	168.8(1)	164.6(1)	157.5(1)
LHC (14 TeV)	941.8(8)	941.9(8)	941.3(5)	941.4(8)	941.4(8)	923.8(7)	907.4(4)	870.9(6)



Kentaro KAWADE
Nagoya University
2016
On behalf of ATLAS, CDF, CMS,
DØ, and LHCb collaborations

3%-4%

$10m_t$ ($20m_t$) => 15% (19%)

顶夸克对强产生前后不对称性

A consistent perturbative-order-analysis of the asymmetry

$NLO-q\bar{q}$ $NNLO-q\bar{q}$

$$A_{FB} = \frac{\alpha_s^3 N_1 + \alpha_s^4 N_2 + \mathcal{O}(\alpha_s^5)}{\alpha_s^2 D_0 + \alpha_s^3 D_1 + \alpha_s^4 D_2 + \mathcal{O}(\alpha_s^5)}$$

$$= \frac{\alpha_s}{D_0}\left[N_1 + \alpha_s\left(N_2 - \frac{D_1 N_1}{D_0}\right) + \alpha_s^2\left(\frac{D_1^2 N_1}{D_0^2} - \frac{D_1 N_2}{D_0} - \frac{D_2 N_1}{D_0}\right) + \cdots\right]$$

total LO *total NLO* *total NNLO*

$[\alpha_s^2 D_0 : \alpha_s^3 D_1 : \alpha_s^4 D_2 \simeq 1 : 18\% : 12\%]$

$[\alpha_s^3 N_1 : \alpha_s^4 N_2 \sim 1 : 50\%]$

same importance

$N_1 D_1 / D_0$ term and the N_2 term

通常方案下的SM－NLO QCD结果

$$A_{FB} = \frac{N_1}{D_0}\alpha_s.$$

$$[\alpha_s^2 D_0 : \alpha_s^3 D_1 : \alpha_s^4 D_2 \simeq 1 : 41\% : 2\%]$$
$$[\alpha_s^3 N_1 : \alpha_s^4 N_2 \sim 1 : 3\%]$$

NNLO-terms N_2, D_2 are highly suppressed and negligible

$$A_{FB} = \frac{\alpha_s}{D_0}\left[N_1 - \alpha_s\left(\frac{D_1 N_1}{D_0}\right) + \alpha_s^2\left(\frac{D_1^2 N_1}{D_0^2}\right)\right]$$

we just call it approximate NNLO asymmetry

It is natural to assume all the higher orders are also negligible

resummed

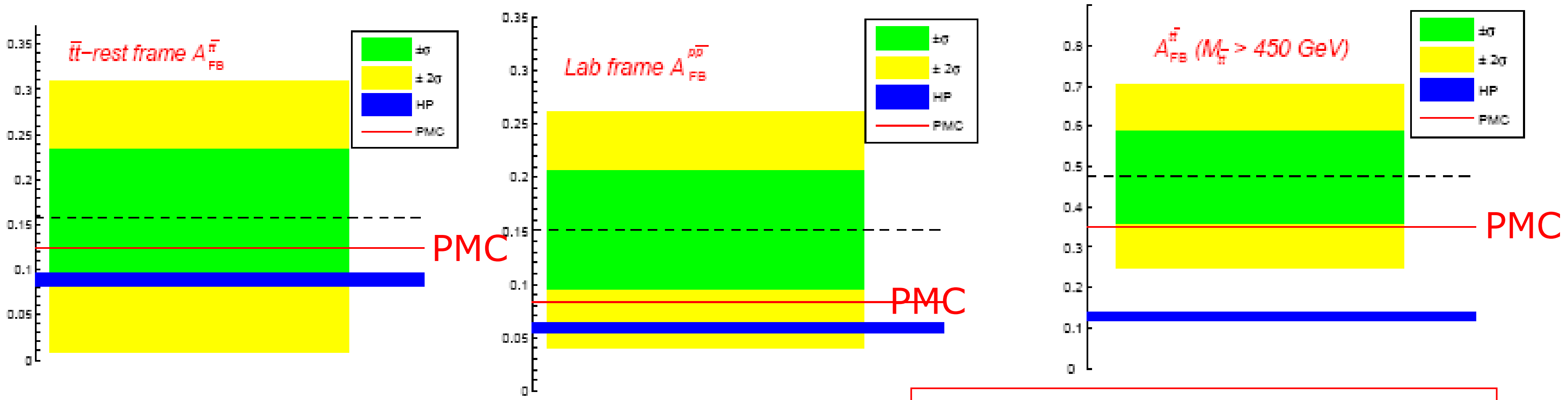
$$A_{FB} = \frac{\alpha_s^3 N_1}{\alpha_s^2 D_0 + \boxed{\alpha_s^3 D_1}}$$

include the electro-weak

$$A_{FB} = \frac{\alpha_s^3 N_1 + \alpha_s^2 \alpha \tilde{N}_1 + \alpha^2 \tilde{N}_0}{\alpha_s^2 D_0 + \alpha_s^3 D_1}$$

Tevatron

around 1σ-error is obtained



$$A_{FB}^{t\bar{t},\mathrm{PMC}} \simeq 12.7\% \; ; \quad A_{FB}^{p\bar{p},\mathrm{PMC}} \simeq 8.39\%$$

$$\overline{\alpha}_s\left(\overline{\mu}_R^{\mathrm{PMC,NLO}}\right) = 0.1460$$

$$\overline{\mu}_R^{\mathrm{PMC,NLO}} \sim \exp(-19/10)m_t \simeq 26 \text{ GeV}.$$

$$A_{FB}^{t\bar{t},PMC}(M_{t\bar{t}} > 450 \text{ GeV}) \simeq 35.0\%$$

Hollik and Pagani, Phys.Rev.D84, 093003 (2011)

HP

$$A_{FB}^{t\bar{t}} = (9.7, 8.9, 8.3)\%, \qquad A_{FB}^{p\bar{p}} = (6.4, 5.9, 5.4)\%.$$

$$A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV}) = (13.9, 12.8, 11.9)\%,$$

CDF

$$A_{FB}^{t\bar{t},\mathrm{CDF}} = (15.8 \pm 7.5)\% \quad A_{FB}^{p\bar{p},\mathrm{CDF}} = (15.0 \pm 5.5)\%$$

$$A_{FB}^{t\bar{t}}(M_{t\bar{t}} > 450 \text{ GeV}) = (47.5 \pm 11.4)\%$$

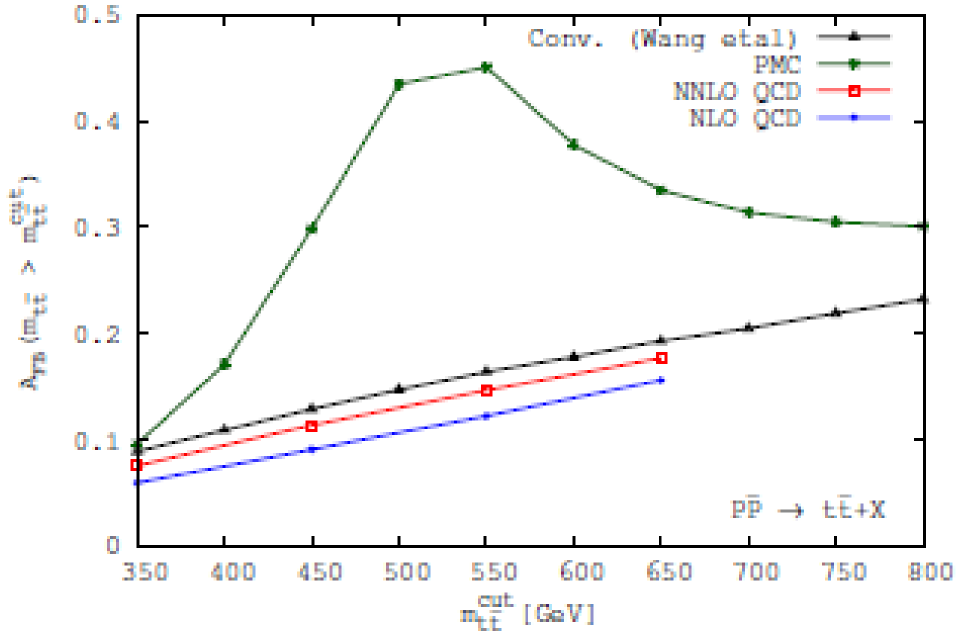


Figure 11. Predictions for the $m_{t\bar{t}}$ cumulative asymmetry: pure QCD at NLO and NNLO (as derived in this work), NLO prediction of Ref. [11] including EW corrections, as well as the PMC scale-setting prediction of Ref. [11].

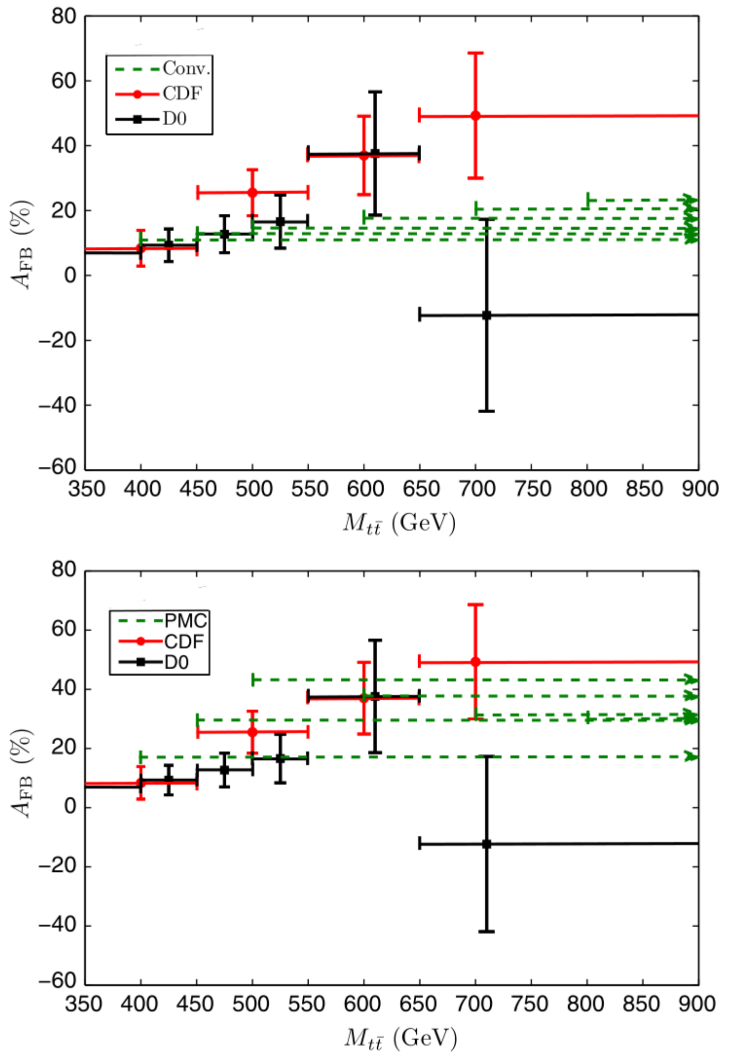


FIG. 2. A comparison of SM predictions of A_{FB} using Conv. and PMC scale settings with the CDF [6] and D0 [9] measurements. The Conv. predictions are for the NLO pQCD predictions with $\mathcal{O}(\alpha_s^2\alpha)$ and the $\mathcal{O}(\alpha^2)$ electroweak contributions and the PMC predictions are calculated by Eq. (5). The upper diagram is for conventional scale setting, and the lower one is for PMC scale setting. The initial scale is taken as $\mu_r = m_t$.

LHC—误差太大，仍能与标准模型的预言相符合
Atlas, arXiv:1604.05538, A^{ttbar}\in [0.5,5.7]%

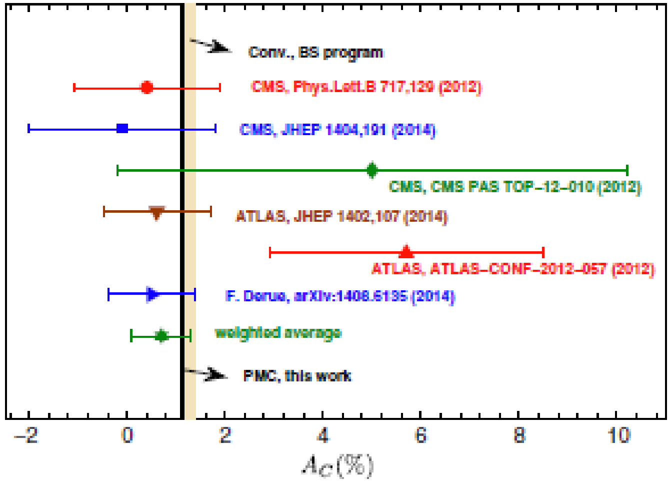


FIG. 1 (color online). The top-quark charge asymmetry A_C assuming conventional scale setting (Conv.) and PMC scale setting for $\sqrt{S}=7$ TeV; the error bars are for $\mu_r^{\rm init} \in [m_t/2, 2m_t]$ and $\mu_f \in [m_t/2, 2m_t]$. As a comparison, the experimental results [49–54] are also presented.

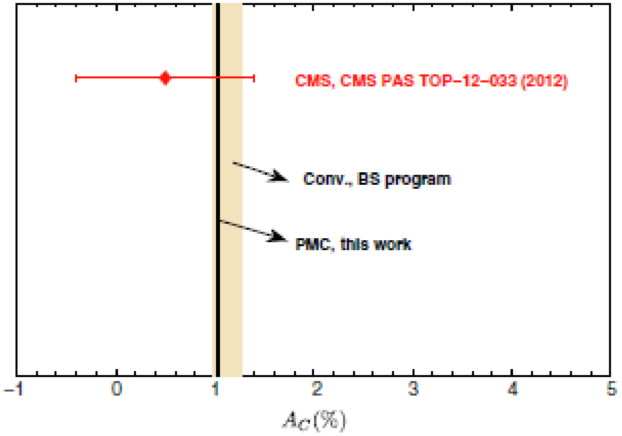


FIG. 2 (color online). The top-quark charge asymmetry A_C assuming conventional scale setting (Conv.) and PMC scale setting for $\sqrt{S}=8$ TeV; the error bars are for $\mu_r^{\rm init} \in [m_t/2, 2m_t]$ and $\mu_f \in [m_t/2, 2m_t]$. The CMS measurement [80] is also presented.

采用司宗国教授计算的数据完成通常方案下的结果分析

Wang,Wu,Si,Brodsky, Phys.Rev.D90(2014)114034

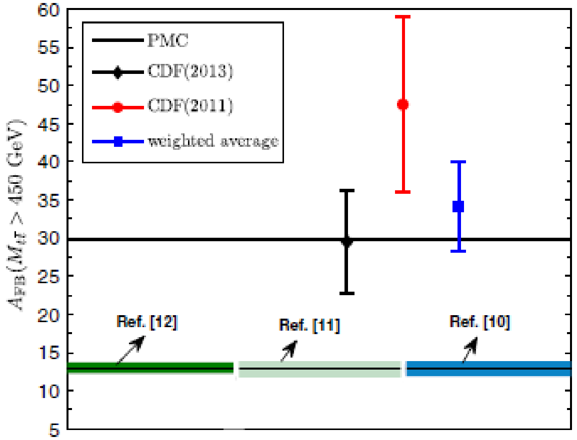
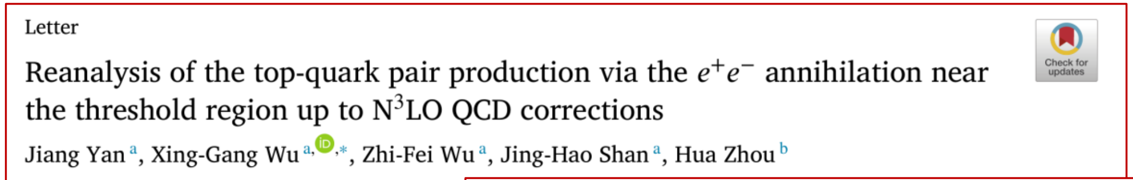


FIG. 1. Comparison of the PMC prediction for the top-pair asymmetry $A_{\rm FB}(M_{t\bar{t}} > 450$ GeV$)$ with the CDF measurement [5,6]. The NLO results predicted by Ref. [12], Ref. [11], and Ref. [10] under conventional scale setting are presented as a comparison, and are shown by shaded bands.

- Top-pair production via e+e- collisions near the threshold region



Letter

Reanalysis of the top-quark pair production via the e^+e^- annihilation near the threshold region up to N^{3}LO QCD corrections

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Chinese Physics C Vol. 48, No. 4 (2024) 043104

Revisiting the top-quark pair production at future e^+e^- colliders*

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Precise determination of the top-quark on-shell mass M_t via its scale-invariant perturbative relation to the top-quark $\overline{\mathrm{MS}}$ mass $\overline{m}_t(\overline{m}_t)$*

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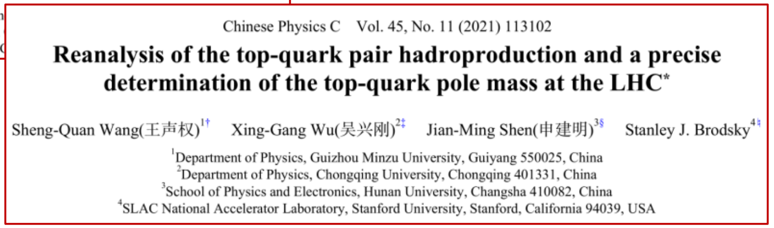

Chinese Physics C Vol. 45, No. 11 (2021) 113102

Reanalysis of the top-quark pair hadroproduction and a precise determination of the top-quark pole mass at the LHC*

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CPC (2021)
CPC (2024)
CPC (2024)
PLB (2024)

NLO QCD correction

J. Jersak, E. Laermann, and P. M. Zerwas, Phys. Rev. D 25, 1218 (1982)

N^{2}LO QCD correction

J. Gao and H. X. Zhu, Phys. Rev. Lett. 113, 262001 (2014)

L. Chen, O. Dekkers, D. Heisler, W. Bernreuther, and Z. G. Si, J. High Energy Phys. 12 (2016) 098

N^{3}LO QCD correction

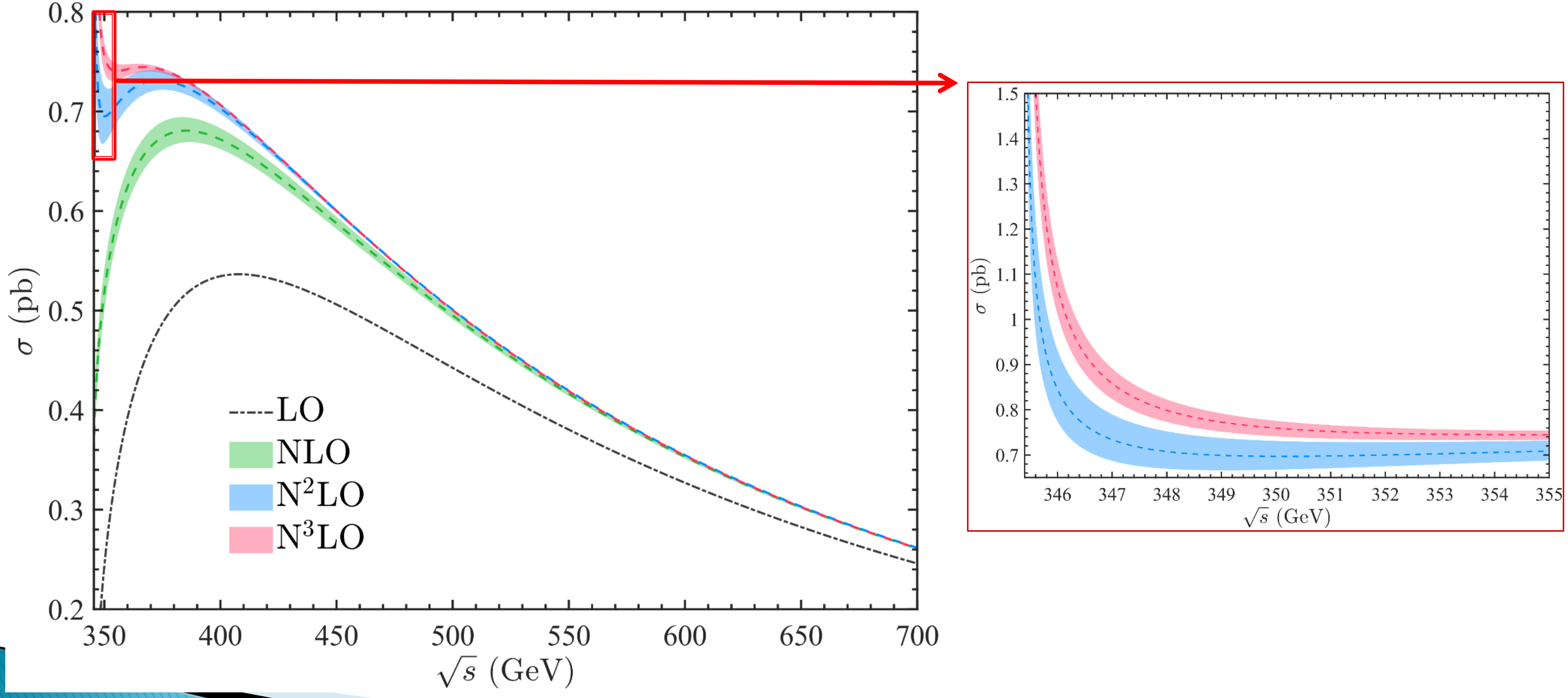
M. Fael, F. Lange, K. Schönwald, and M. Steinhauser, Phys. Rev. Lett. 128, 172003 (2022)

M. Fael, F. Lange, K. Schönwald, and M. Steinhauser, Phys. Rev. D 106, 034029 (2022)

L. Chen, X. Chen, X. Guan and Y. Q. Ma, Phys. Rev. Lett. 132, 10 (2024)

$e^+e^- \to \gamma^* \to t\bar{t}$

$\sqrt{s} \approx 2m_t$



Up to N^{3}LO, total cross section of $e^+e^- \to \gamma^* \to t\bar{t}$ can be written as

$$\sigma = \sigma_0\left(1 + r_1\alpha_s + r_2\alpha_s^2 + r_3\alpha_s^3 + \cdots\right)$$

$$\sigma_0 = N_C\frac{4\pi\alpha^2}{3s}\frac{v(3-v^2)}{2}e_t^2$$

$$r_1 = \frac{1}{v}r_{1,v} + r_{1,+}$$

$$r_2 = \frac{1}{v^2}r_{2,v^2} + \frac{1}{v}r_{2,v} + r_{2,+}$$

$$r_3 = \frac{1}{v^2}r_{3,v^2} + \frac{1}{v}r_{3,v} + r_{3,+}$$

only numerical results !

$$\boldsymbol{v} = \sqrt{\mathbf{1} - \frac{\mathbf{4}\boldsymbol{m_t^2}}{\boldsymbol{s}}}$$ **the velocity of produced quarks**

PSLQ algorithm

we schematically factorize total cross section as the product of non-Coulomb and Coulomb parts

$$\sigma = \sigma_0 \times \mathcal{R}_{\mathrm{NC}} \times \mathcal{R}_{\mathrm{C}}$$

Coulomb part

reconstructing analytic form !

Non-Coulomb part

PolyLogTools: C. Duhr and F. Dulat, JHEP 08, 135 (2019)

H. R. P. Ferguson, and D. H. Bailey, RNR Technical Report RNR-91-032 (1992)

H. R. P. Ferguson, D. H. Bailey, and S. Arno, Math. Comp. 68, 351 (1999)

The QCD coupling $\alpha_s^{\mathrm{V}}(\mathbf{q}^2)$ has been introduced for describing the interaction of the non-relativistic heavy quark-antiquark pair, which is defined as the effective charge in the following Coulomb-like potential:

$$V(\mathbf{q}^2) = -4\pi C_F \frac{\alpha_s^{\mathrm{V}}(\mathbf{q}^2)}{\mathbf{q}^2},$$

where $\alpha_s^{\mathrm{V}}(\mathbf{q}^2)$ absorbs all the higher-order QCD corrections, which is related to the $\overline{\mathrm{MS}}$-scheme coupling via the following way

$$\alpha_s^{\mathrm{V}}(\mathbf{q}^2) = \alpha_s(\mu^2) + \left(a_1 - \beta_0 \ln\frac{\mathbf{q}^2}{\mu^2}\right)\alpha_s^2(\mu^2) + \left(a_2 - (2a_1\beta_0 + \beta_1)\ln\frac{\mathbf{q}^2}{\mu^2} + \beta_0^2 \ln^2\frac{\mathbf{q}^2}{\mu^2}\right)\alpha_s^3(\mu^2) + \cdots$$

$$a_1 = \frac{1}{4\pi}\left(\frac{31}{9}C_A - \frac{20}{9}T_F n_l\right)$$

$$a_2 = \frac{1}{(4\pi)^2}\left[\left(\frac{4343}{162} + 4\pi^2 - \frac{\pi^2}{4} + \frac{22}{3}\zeta_3\right)C_A^2 - \left(\frac{1798}{81} + \frac{56}{3}\zeta_3\right)C_A T_F n_l - \left(\frac{55}{3} - 16\zeta_3\right)C_F T_F n_l + \left(\frac{20}{9}T_F n_l\right)^2\right]$$

T. Appelquist, M. Dine and I. J. Muzinich, Phys. Lett. B 69, 231 (1977)
W. Fischler, Nucl. Phys. B 129, 157 (1977)

M. Peter, Phys. Rev. Lett. 78, 602 (1997)
Y. Schroder, Phys. Lett. B 447, 321 (1999)

$$\sigma = \sigma_0 \times \mathcal{R}_{\rm NC} \times \mathcal{R}_{\rm C}$$

exactly non-conformal term

$$\mathcal{R}_{\rm C} = 1 + C_F \frac{\pi}{2v}\alpha_s^{\rm V}(sv^2) + C_F^2\frac{\pi^2}{12v^2}\alpha_s^{\rm V,2}(sv^2) + \boxed{C_F\left(\frac{\pi^3}{3v}\beta_0^2 - C_F\frac{2\zeta_3}{v^2}\beta_0\right)}\alpha_s^{\rm V,3}(sv^2) + \cdots$$

PMC

$$\mathcal{R}_{\rm C}\Big|_{\rm PMC} = 1 + C_F \frac{\pi}{2v}\alpha_s^{\rm V}(Q_{*,\rm C}^2) + C_F^2\frac{\pi^2}{12v^2}\alpha_s^{\rm V,2}(Q_{*,\rm C}^2) + \boxed{0\times}\alpha_s^{\rm V,3}(Q_{*,\rm C}^2) + \cdots$$

resum

$$\frac{X}{1-\exp(-X)}$$

Sommerfeld-Gamow-Sakharov factor

$$X = \pi\, C_F \frac{\alpha_s^{\rm V}(Q_{*,\rm C}^2)}{v}$$

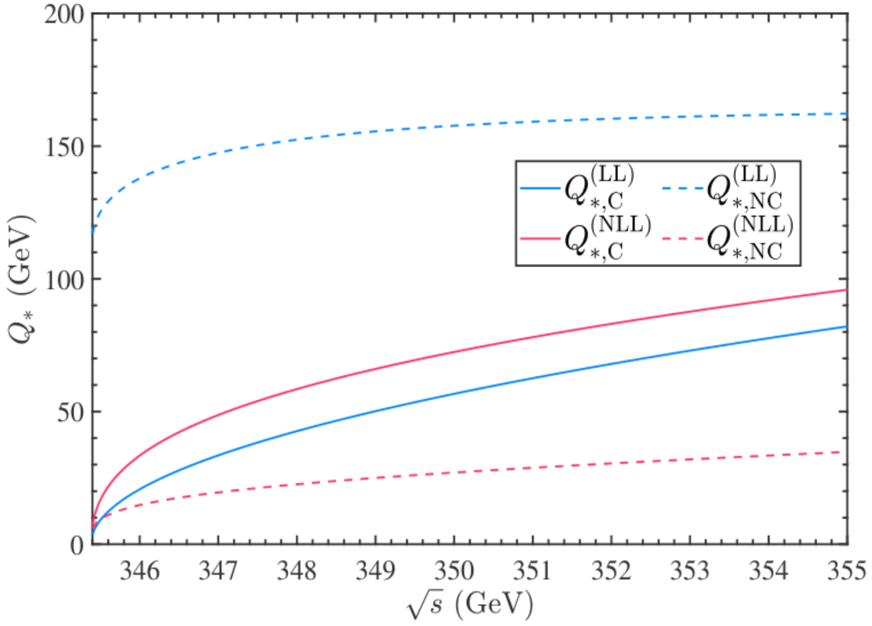
Obtained by solving NR Schrödinger equation

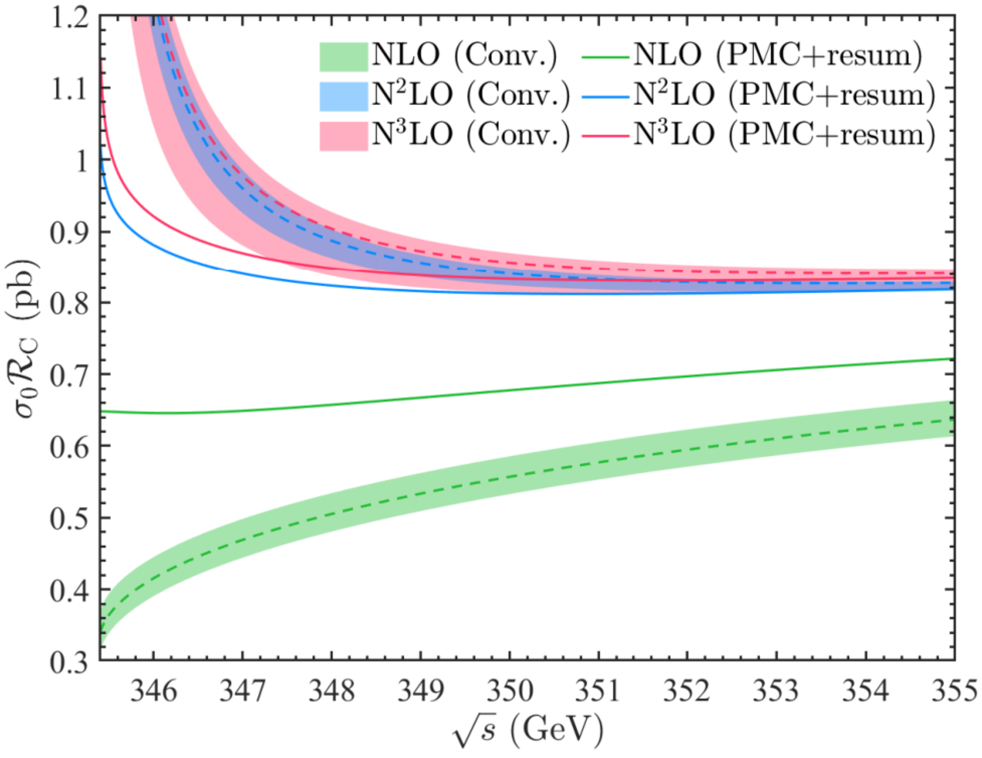
A. D. Sakharov, Zh. Eksp. Teor. Fiz. 18, 631 (1948)

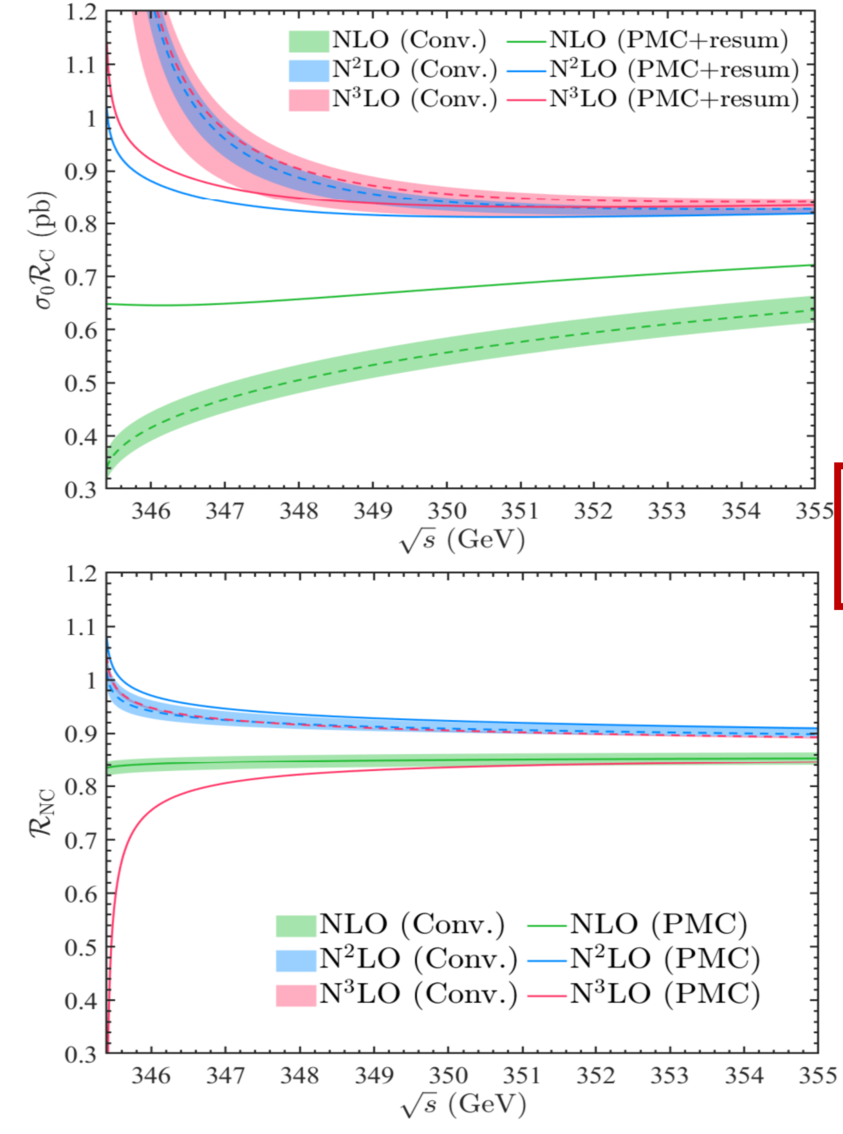
$$\frac{X}{1-\exp(-X)} = 1 + \frac{X}{2} + \frac{X^2}{12} - \frac{X^4}{720} + \cdots$$

The X^3-coefficient is exactly zero!

$$\lim_{v\to 0^+} v\,\frac{\pi C_F \alpha_s^V/v}{1-\exp(-\pi C_F \alpha_s^V/v)} = \pi C_F \alpha_s^V \text{ is a finite value}$$







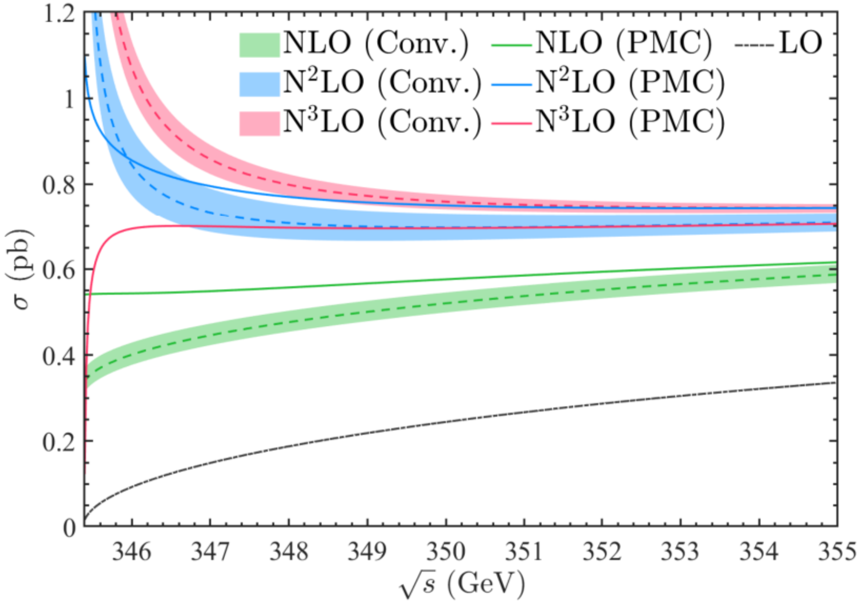
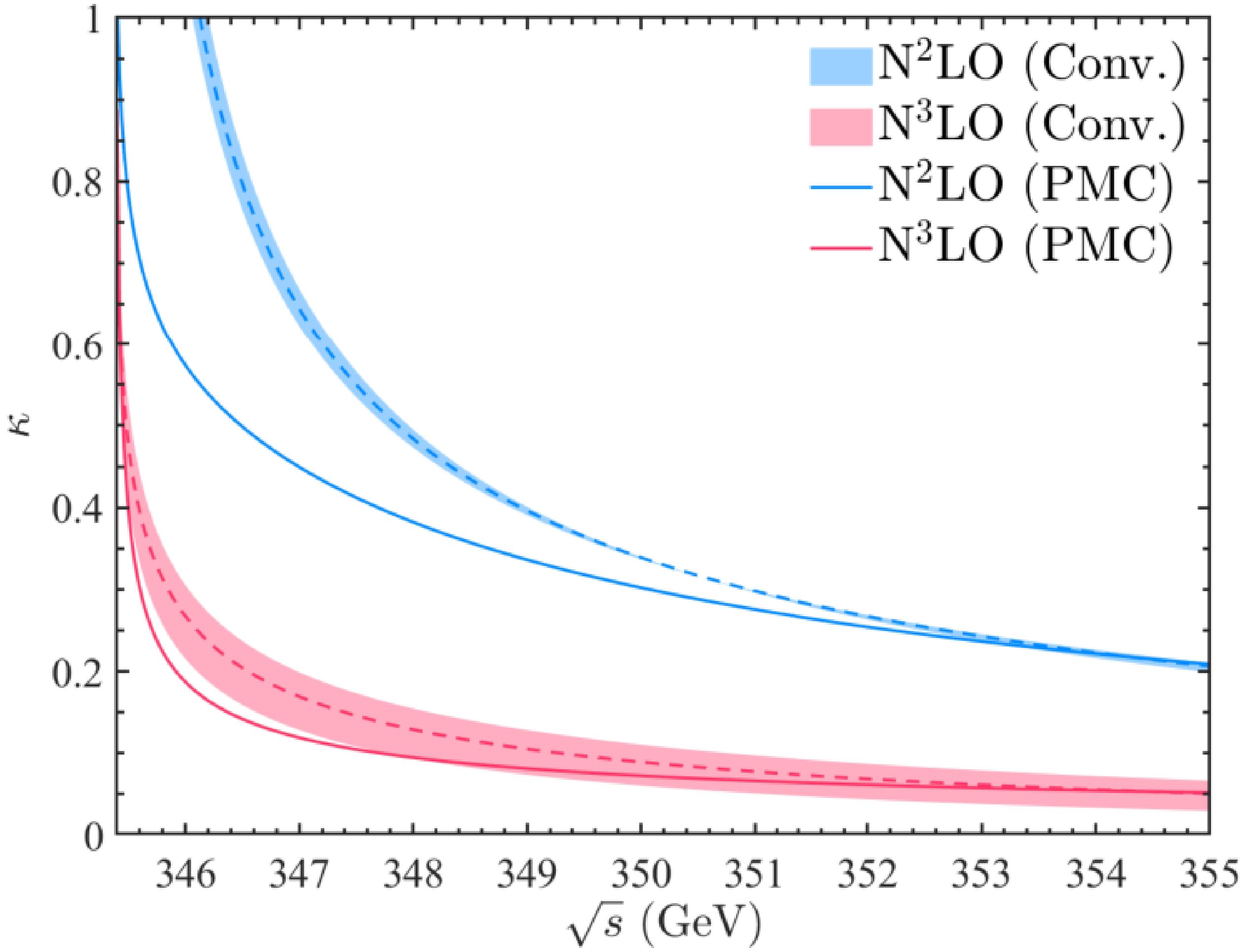


FIG. 3. (Color online) Total cross section $\sigma_{t\bar{t}}$ with different QCD corrections under conventional (dashed line) and PMCs (solid line) scale-setting approaches, respectively.

$$\kappa\left(\mathrm{N^iLO}\right) = \left|\frac{\sigma\left(\mathrm{N^iLO}\right) - \sigma\left(\mathrm{N^{i-1}LO}\right)}{\sigma\left(\mathrm{N^{i-1}LO}\right)}\right|$$



After applying the PMC, the scale dependence is removed and the pQCD convergence near the threshold region is improved.

Usual way for UHO-terms

- **Conventional:**

Varying scale — Rough order estimation and cannot estimate conformal contribution

- **Convervative:**

The one-order higher shall always be smaller than the given order

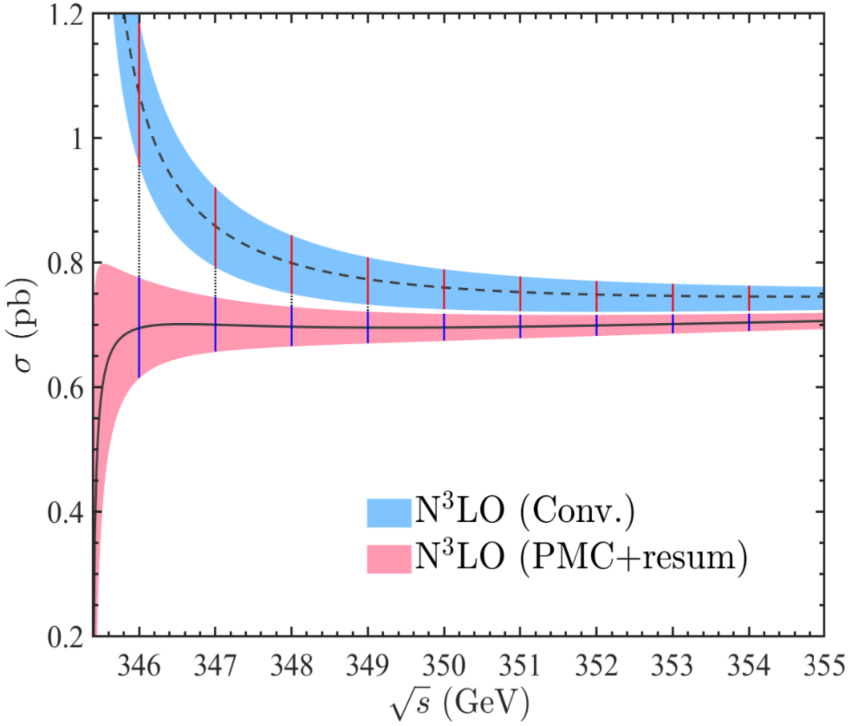
- **Resummation:**

Find a proper generating function – Pade approximation

- **Probability analysis:**

Bayesian analysis
LRTO analysis – in preparation

Bayesian analysis



Providing more reliable foundation for constraining predictions of UHO contributions

By applying PMC, uncertainties caused by the UHO-terms become smaller. These results confirm the importance of a proper scale-setting approach.

➢ Summary and outlook

圈技术大步往前之时，可同时考虑在已知有限阶下获得最精确的结果

At fixed-order, guessing/using typical momentum flow as the scale, one cannot get precise value for all-orders, and also for each order, thus it becomes an important systematic error

PMC is not simply chosen “special/effective scale”, but basing on RGE and standard RGI and using general way to set the optimal scale such that to achieve precise prediction at any fixed order

更收敛、更精确的序列是估算未知高阶项贡献的基石

与Stevenson, Kataev等的论战还是继续

Great thanks !