N^3LO QCD predictions for top-quark decay

王烨凡 (南京师范大学)

in collaboration with 陈龙斌, 李海涛, 李钊, 王健, 吴泉锋

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Editors' Suggestion **Letter**

Analytic third-order QCD corrections to top-quark and semileptonic $b \to u$ decays

Long-Bin Chen, Hai Tao Li, Zhao Li, Jian Wang, Yefan Wang, and Quan-feng Wu

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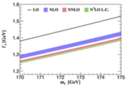


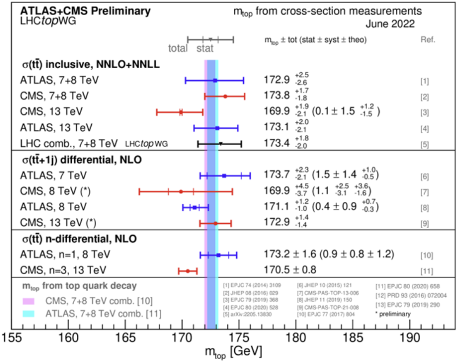
The authors compute the leading color contribution to the third-order QCD correction to the top quark decay width analytically. They additionally obtain the leading color third-order QCD correction to the inclusive semileptonic $b \to u$ decay.

Show Abstract +

Motivation

Top-quark mass is one of the fundamental parameters in the Standard Model.

Summary of the top-mass analyses at the LHC.

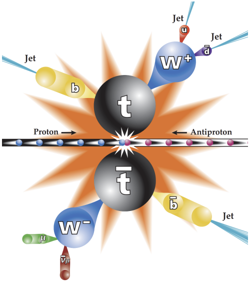


Motivation

Top decay width Γ_t is one of the fundamental properties of top-quark.

Due to its large mass, Γ_t is expected to be very large.

The measurement of Γ_t could hint new physics.



[Denisov, Vellidis 2015]

Motivation

The top-quark decays almost exclusively to Wb. $\Gamma_t = \Gamma_t(t \to Wb)$.

$$\left|V_{\mathrm{CKM}}\right| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$

[PDG 2022]

The most precise measurement for Γ_t by now is given by CMS

$\Gamma_t = 1.36 \pm 0.02\ (\mathrm{stat.})^{+0.14}_{-0.11}\ (\mathrm{syst.})$ GeV [CMS, 2014].

In the future e^+e^- collider, Γ_t can be measured with an uncertainty of 30 MeV [Martinez, Miquel 2019].

Theoretical Development

NLO QCD corrections [Jezabek, Kuhn 1989, Czarnecki 1990, Li, Oakes, Yuan 1991]

NLO EW corrections [Denner, Sack 1991, Eilam, Mendel, Migneron, Soni 1991]

Numerical result of full NNLO QCD corrections [Gao, Li, Zhu 2013, Brucherseifer, Caola, Melnikov 2013]

Numerical result of full N^3LO QCD corrections [Chen, Chen, Guan, Ma 2023]

Analytic results of NNLO and N^3LO QCD corrections [Chen, Li, Wang, Wang 2022, Chen, Li, Li, Wang, Wang, Wu 2023]

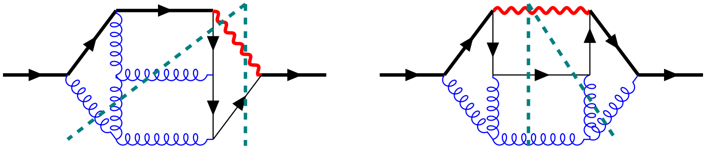
PMC scale settings up to N^3LO QCD [Jiang, Wu, Zhou, Li, Shan 2024]

Optical Theorem

The forward scattering amplitudes Σ for $t \to Wb \to t$

$$\Gamma_t = \frac{\text{Im}(\Sigma)}{m_t} \tag{1}$$

For N^3LO QCD corrections, the self-energy four-loop diagrams are considered. For example,



The imaginary part comes from cut diagrams.

The complicated phase space integration can be avoided.

Calculation Method

1. Generating diagrams and amplitudes.
2. Reducing amplitudes to master integrals by FIRE [Smirnov, Chuharev 2020].
3. Analytically calculating the master integrals.

Master Integrals Calculations

Method: canonical differential equations. For example,

$$\frac{\partial F_4(w,\epsilon)}{\partial w} = \frac{\epsilon\,(F_5 - 2F_4)}{w-1} - \frac{\epsilon\,(F_4 + F_5)}{w}, \qquad w = \frac{m_W^2}{m_t^2}, \quad D = 4 - 2\epsilon \tag{2}$$

Two important ingredients: – see Yang Zhang's talk.

1. Construct canonical form (ϵ form, $d\log$ form) – `Libra` [Lee 2021]

2. Boundary conditions – `AMFlow` [Liu, Ma 2022] and PSLQ method [Ferguson, Beiley, Arno 1992 1999]

Results: harmonic polylogarithms (HPLs).

Analytic calculations of four loop master integrals are non-trivial.

Leading Color Contribution at NNLO

QCD corrections of Γ_t.

$$\Gamma(t \to Wb) = \Gamma_0 \left[X_0 + \frac{\alpha_s}{\pi} X_1 + \left(\frac{\alpha_s}{\pi}\right)^2 X_2 + \left(\frac{\alpha_s}{\pi}\right)^3 X_3 \right], \quad (3)$$

$$\Gamma_0 = \frac{G_F m_t^3 |V_{tb}|^2}{8\sqrt{2}\pi}. \quad (4)$$

$$\begin{aligned} X_2 &= C_F \left[C_F X_F + C_A X_A + T_R n_l X_l + T_R n_h X_h \right] \\ &= C_F \left[N_c \left(X_A + \frac{X_F}{2} \right) + \frac{n_l}{2} X_l - \frac{1}{2N_c} X_F + \frac{n_h}{2} X_h \right] \\ X_{2,\text{L.C}} &= N_c \left(X_A + \frac{X_F}{2} + \frac{n_l}{2} X_l \right) \end{aligned} \quad (5)$$

In the top quark decay, $N_C = 3, n_l = 5$, $X_{2,\text{L.C}}/X_2 > 95\%$.

Analytic Calculations at N^{3}LO

According to NNLO, the leading color contributions are dominant.

We concentrate on the leading color contributions at N^{3}LO.

$$X_3 = C_F\left[N_c^2 Y_A + \widetilde{Y}_A + \frac{\overline{Y}_A}{N_c^2} + n_l n_h Y_{lh} + n_l\left(N_c Y_l + \frac{\widetilde{Y}_l}{N_c}\right) + n_l^2 Y_{l2} + n_h\left(N_c Y_h + \frac{\widetilde{Y}_h}{N_c}\right) + n_h^2 Y_{h2}\right]. \tag{6}$$

$$X_{3,\mathrm{L.C}} = C_F\left[N_c^2 Y_A + n_l N_c Y_l + n_l^2 Y_{l2}\right]. \tag{7}$$

In leading color approximation, there are still 408 Feynman diagrams and 185 master integrals need to be calculated.

Analytic Results of N^{3}LO

$$X_{3,\text{L.C}} = C_F\Big[N_c^2 Y_A + n_l N_c Y_l + n_l^2 Y_{l2}\Big], \qquad w = \frac{m_W^2}{m_t^2} \tag{8}$$

$$\begin{aligned}
Y_l = {} & (8w^3+12w^2+43w+8)\Big(H(0,0,0,0,1,w) - H(0,0,0,1,0,w)\Big) \\
& + (-2w^3+51w^2+86w+7)\,H(0,0,0,1,1,w) + (2w^3-15w^2-20w-1)\,H(0,0,1,0,1,w) \\
& + (-2w^3-45w^2-80w-9)\,H(0,0,1,1,0,w) - 2\,(6w^3-33w^2-46w-1)\,H(0,0,1,1,1,w) \\
& + (6w^3+3w^2+14w+5)\,H(0,1,0,0,1,w) + 2\,(6w^3+21w^2+50w+8)\,H(0,1,0,1,1,w) \\
& - 2\,(2w^3+27w^2+56w+6)\,H(0,1,1,0,1,w) + 2\,(2w^3-27w^2-40w-3)\,H(0,1,1,1,0,w) \\
& - (2w+1)(w-1)^2\Big(2H(0,1,0,1,0,w) - H(1,0,0,0,1,w) + H(1,0,0,1,0,w) \\
& + 4H(1,0,0,1,1,w) - 2H(1,0,1,0,1,w) - 2H(1,0,1,1,0,w) \\
& - 4H(1,1,0,0,1,w) + 4H(1,1,0,1,0,w)\Big) + \cdots
\end{aligned} \tag{9}$$

Numerical Results

Input parameters from [P.D.G 2022]

$$
\begin{aligned}
m_t &= 172.69 \text{ GeV}, \quad m_b = 4.78 \text{ GeV}, \\
m_W &= 80.377 \text{ GeV}, \quad \Gamma_W = 2.085 \text{ GeV}, \\
m_Z &= 91.1876 \text{ GeV}, \quad G_F = 1.16638 \times 10^{-5} \text{ GeV}^{-2}, \\
|V_{tb}| &= 1, \quad \alpha_s(m_Z) = 0.1179.
\end{aligned} \tag{10}
$$

$\Gamma_t^{(0)} = 1.486$ GeV with $m_b = 0$ and on-shell W.

$$
\begin{aligned}
\Gamma_t = \Gamma_t^{(0)}[&(1 + \delta_b^{(0)} + \delta_W^{(0)}) \\
&+ (\delta_b^{(1)} + \delta_W^{(1)} + \delta_{\rm EW}^{(1)} + \delta_{\rm QCD}^{(1)}) \\
&+ (\delta_b^{(2)} + \delta_W^{(2)} + \delta_{\rm EW}^{(2)} + \delta_{\rm QCD}^{(2)} + \delta_{\rm EW\times QCD}^{(2)}) \\
&+ (\delta_b^{(3)} + \delta_W^{(3)} + \delta_{\rm EW}^{(3)} + \delta_{\rm QCD}^{(3)} + \delta_{\rm EW\times QCD}^{(3)})],
\end{aligned} \tag{11}
$$

Numerical Results

Corrections in percentage (%) normalized by the LO width $\Gamma_t^{(0)} = 1.486$ GeV with $m_b = 0$ and on-shell W. ($m_t = 172.69$ GeV)

	$\delta_b^{(i)}$	$\delta_W^{(i)}$	$\delta_{\rm EW}^{(i)}$	$\delta_{\rm QCD}^{(i)}$	Γ_t [GeV]
LO	-0.273	-1.544	—	—	1.459
NLO	0.126	0.132	1.683	-8.575	$1.361^{+0.0091}_{-0.0130}$
NNLO	≈ 0.03	0.030	*	-2.070	$1.331^{+0.0055}_{-0.0051}$
$\rm N^3LO$	*	0.009	*	-0.667	$1.321^{+0.0025}_{-0.0021}$

QCD corrections are dominant.

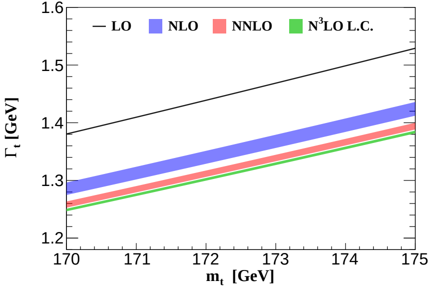
$X_{3,\rm L.C}/X_3 \approx 95\%$ compared with full numerical result. [Chen, Chen, Guan, Ma 2023]

NLO EW correction is 1.683%. $\delta_b^{(2)}$ from [Chen, Chen, Guan, Ma 2023]

The off-shell W boson effect at NNLO and N3LO are further suppressed.

Numerical Results

m_t varies from 170 GeV to 175 GeV. μ varies from $\frac{m_t}{2}$ to $2m_t$.



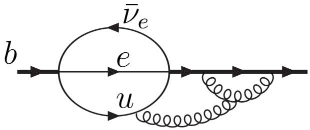
The scale uncertainty at N^{3}LO is reduced to $\pm 0.2\%$, only half of that at NNLO.

It is very convenient to use fitted function within this range

$$\Gamma_t(m_t) = 0.027037 \times m_t - 3.34801 \text{ GeV} \tag{12}$$

Relations With Other Process

Integrating w $(w = m_W^2/m_t^2)$ from 0 to 1, we obtain $\mathrm{N^3LO}$ QCD corrections in semileptonic decay $\Gamma(b \to X_u e\bar{\nu}_e)$. The $\mathrm{N^3LO}$ correction is about 6% at $\mu = 4.78$ GeV.



$$\Gamma(b \to X_u e\bar{\nu}_e) = \frac{G_F^2 |V_{ub}|^2 m_b^5}{192\pi^3}\left[1 + \sum_{i=1}\left(\frac{\alpha_s}{\pi}\right)^i b_i\right]. \tag{13}$$

$$\begin{aligned} b_3 = C_F\Bigg[& N_c^2\left(\frac{9651283}{82944} - \frac{1051339\pi^2}{62208} - \frac{67189\zeta(3)}{864} + \frac{4363\pi^4}{6480} + \frac{59\pi^2\zeta(3)}{32} + \frac{3655\zeta(5)}{96}\right. \\ & \left. - \frac{109\pi^6}{3780}\right) + n_l N_c\left(-\frac{729695}{27648} + \frac{48403\pi^2}{15552} + \frac{1373\zeta(3)}{108} + \frac{133\pi^4}{1728} - \frac{13\pi^2\zeta(3)}{72} - \frac{125\zeta(5)}{24}\right) \\ & + n_l^2\left(\frac{24763}{20736} - \frac{1417\pi^2}{15552} - \frac{37\zeta(3)}{216} - \frac{121\pi^4}{6480}\right) + \text{subleading color}\Bigg] \\ = & \, (-195.3 \pm 9.8) C_F\,. \end{aligned} \tag{14}$$

Summary

We provide the first leading color QCD correction at N^{3}LO analytically, which can be used to perform fast numerical evaluations.

The NNLO and N^{3}LO QCD corrections decrease the LO result by -2.07% and -0.667% with $m_t = 172.69$ GeV and $\mu = m_t$

We derive the analytic N^{3}LO QCD leading color predictions for the semileptonic $b \to u$ decay width.

Thanks !

Expansion series near $w = 0$

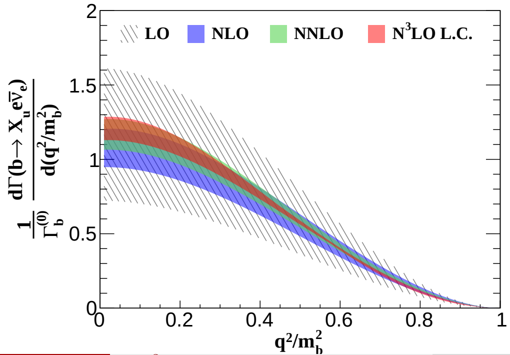
$$
\begin{aligned}
Y_A =& \left[\frac{203185}{41472} - \frac{12695\pi^2}{1944} - \frac{4525\zeta(3)}{576} - \frac{1109\pi^4}{25920} + \frac{37\pi^2\zeta(3)}{36} + \frac{1145\zeta(5)}{96} + \frac{47\pi^6}{2835} - \frac{3\zeta(3)^2}{4}\right] \\
&+ w\left[-\frac{157939}{2304} + \frac{140863\pi^2}{20736} + \frac{5073\zeta(3)}{64} - \frac{14743\pi^4}{6480} - \frac{169\pi^2\zeta(3)}{72} - \frac{45\zeta(5)}{16} + \frac{3953\pi^6}{22680}\right. \\
&\left. - \frac{15\zeta(3)^2}{4}\right] + w^2\left[\log(w)\left(\frac{851099}{27648} - \frac{5875\pi^2}{2304} - \frac{33\zeta(3)}{8} + \frac{\pi^4}{10}\right) - \frac{82610233}{331776} + \frac{799511\pi^2}{27648}\right. \\
&\left. + \frac{4093\zeta(3)}{32} - \frac{5987\pi^4}{2880} - \frac{91\pi^2\zeta(3)}{16} - \frac{275\zeta(5)}{8} + \frac{347\pi^6}{3024} - \frac{9\zeta(3)^2}{8}\right] + \mathcal{O}(w^3), \\
Y_{l2} =& \left[-\frac{695}{2592} - \frac{91\pi^2}{972} + \frac{11\zeta(3)}{36} - \frac{2\pi^4}{405}\right] + w\left[\frac{245}{144} - \frac{73\pi^2}{648} - \frac{\zeta(3)}{3}\right] \\
&+ w^2\left[\log(w)\left(\frac{245}{432} - \frac{\pi^2}{72}\right) - \frac{791}{162} + \frac{85\pi^2}{432} + \frac{3\zeta(3)}{4} + \frac{2\pi^4}{135}\right] + \mathcal{O}(w^3), \qquad (15)
\end{aligned}
$$

The dilepton invariant mass spectrum for $b \to X_u e\bar{\nu}_e$ up to N^{3}LO

The dilepton invariant mass spectrum in the on-shell mass scheme

$$\frac{d\Gamma(b \to X_u e\bar{\nu}_e)}{dq^2} = \Gamma_b^{(0)} \sum_{i=0} \left(\frac{\alpha_s}{\pi}\right)^i X_i \left(\frac{q^2}{m_b^2}\right), \qquad (16)$$

with $\Gamma_b^{(0)} = G_F^2 |V_{ub}|^2 m_b^3 / 96\pi^3$. The scale uncertainties are around $\pm 12\%$, $\pm 9\%$, and $\pm 6\%$ at NLO, NNLO, and N^{3}LO, respectively, at $q^2/m_b^2 = 0.2$.



Mathematica program

TopWidth.m can be downloaded from https://github.com/haitaoli1/TopWidth. The package HPL is required [Maitre 2006]. It has been used in [Jiang, Wu, Zhou, Li, Shan 2024]

```
<< TopWidth`
        (****** TopWidth-1.0 ******)
    Authors: Long-Bin Chen, Hai Tao Li, Jian Wang, YeFan Wang
    TopWidth[QCDorder, mbCorr, WwidthCorr, EWcorr, mu] is provided for top width calculations
    Please cite the paper for reference: arXiv:2212.06341

*-*-*-*-*-* HPL 2.0 *-*-*-*-*-*

Author: Daniel Maitre, University of Zurich

(* SetParameters[mt,mb,mw,Wwidth, GF] *)
SetParameters[17269/100, 478/100, 80377/1000, 2085/1000, 911876/10000, 11663788×10^-12];
TopWidth[3, 1, 1, 1, 17269/100]

1.32073
```