A systematic investigation on dark matter-electron scattering in effective field theories

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arXiv: 2405. 04855; *JHEP* 07 (2024) 279

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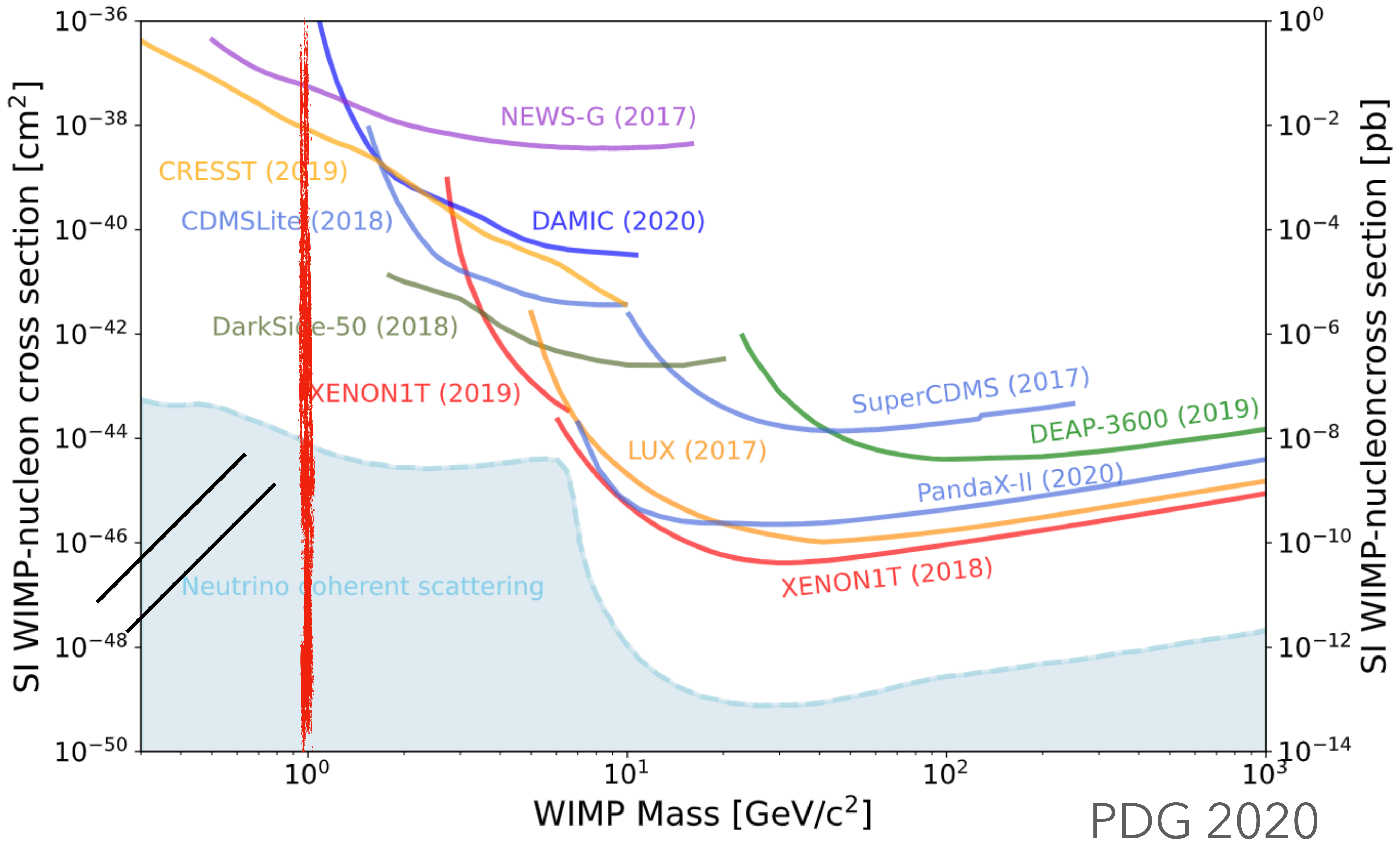
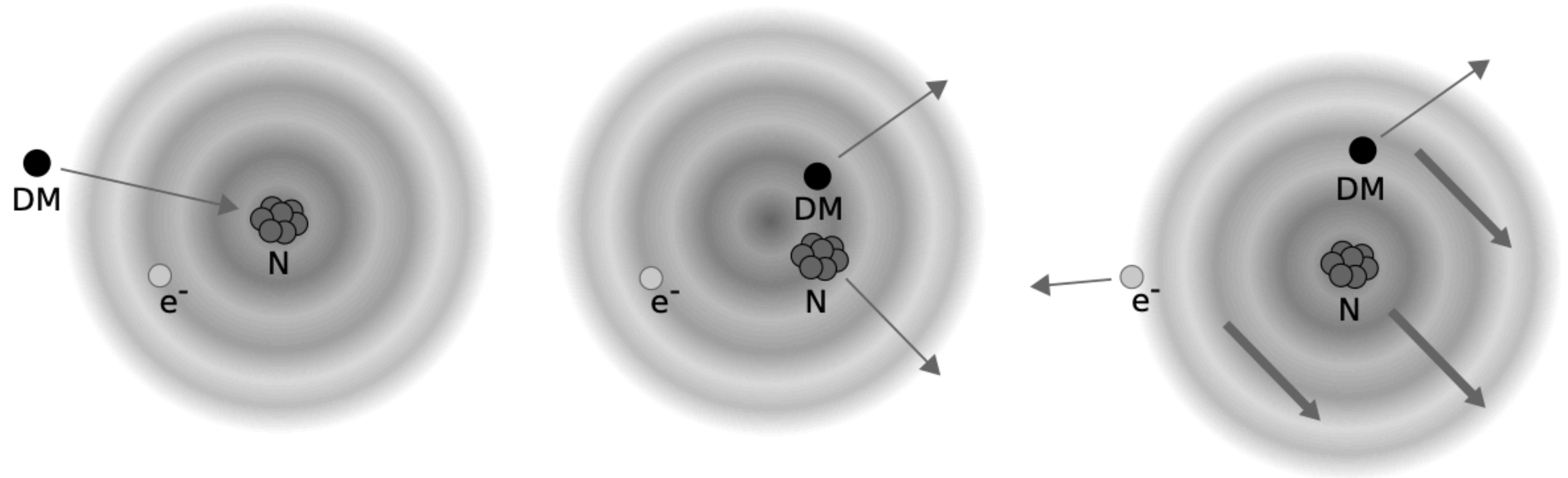
DM direct detection (DMDD) and electron recoil (ER)

- Nuclear recoil
  - ☑ Conventional direct DM searches
  - ☑ Lose sensitivity to DM masses below ~ 1 GeV
- Migdal effects
  - ☑ Observe the electron ionised after DM induced but unobservable nuclear recoil
  - ☑ The Migdal effect has never been observed
- **Electron recoil**
  - ☑ Sensitive to sub-MeV DM
- …

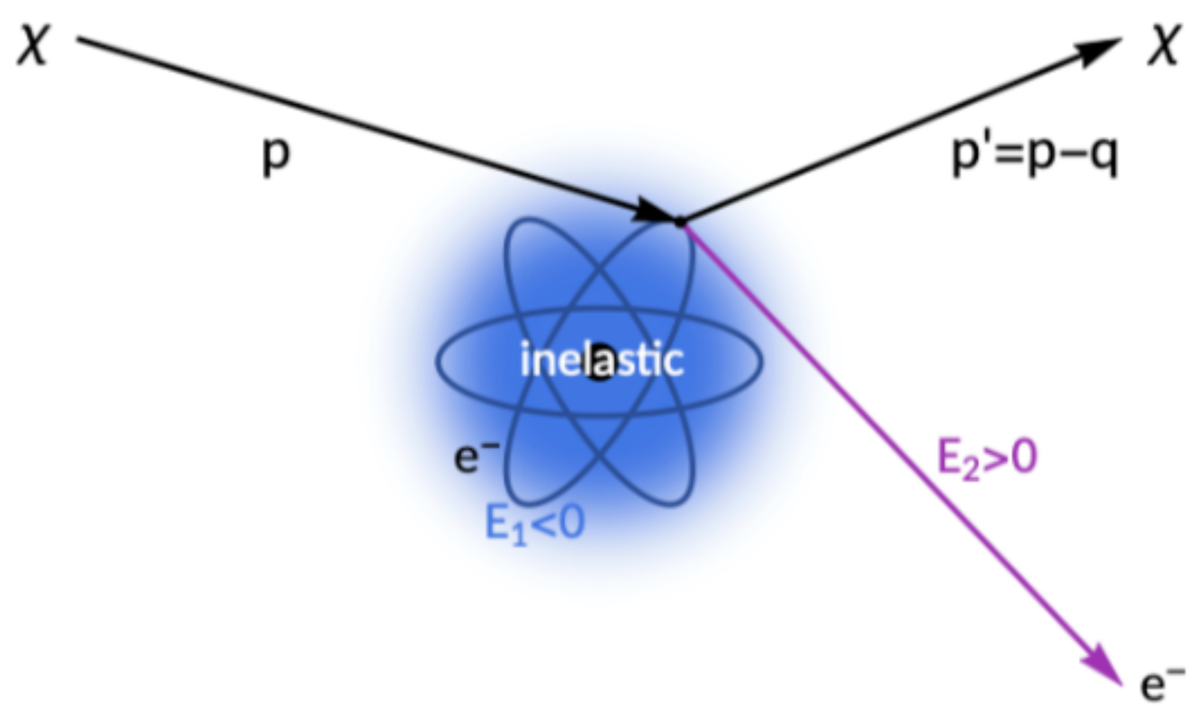


PDG 2020



1711.09906

Formalism for DM-atom scattering



Catena et al., 1912.08204

$$\frac{\mathrm{d}\mathcal{R}_{\rm ion}^{n\ell}}{\mathrm{d}\ln E_e} = \frac{n_\chi}{128\pi m_\chi^2 m_e^2}\int \mathrm{d}q\, q \int \frac{\mathrm{d}^3\mathbf{v}}{v} f_\chi(\mathbf{v})\Theta(v - v_{\rm min})\overline{\left|\mathcal{M}_{\rm ion}^{n\ell}\right|^2}$$

Summing over the quantum numbers of the initial and final state electrons

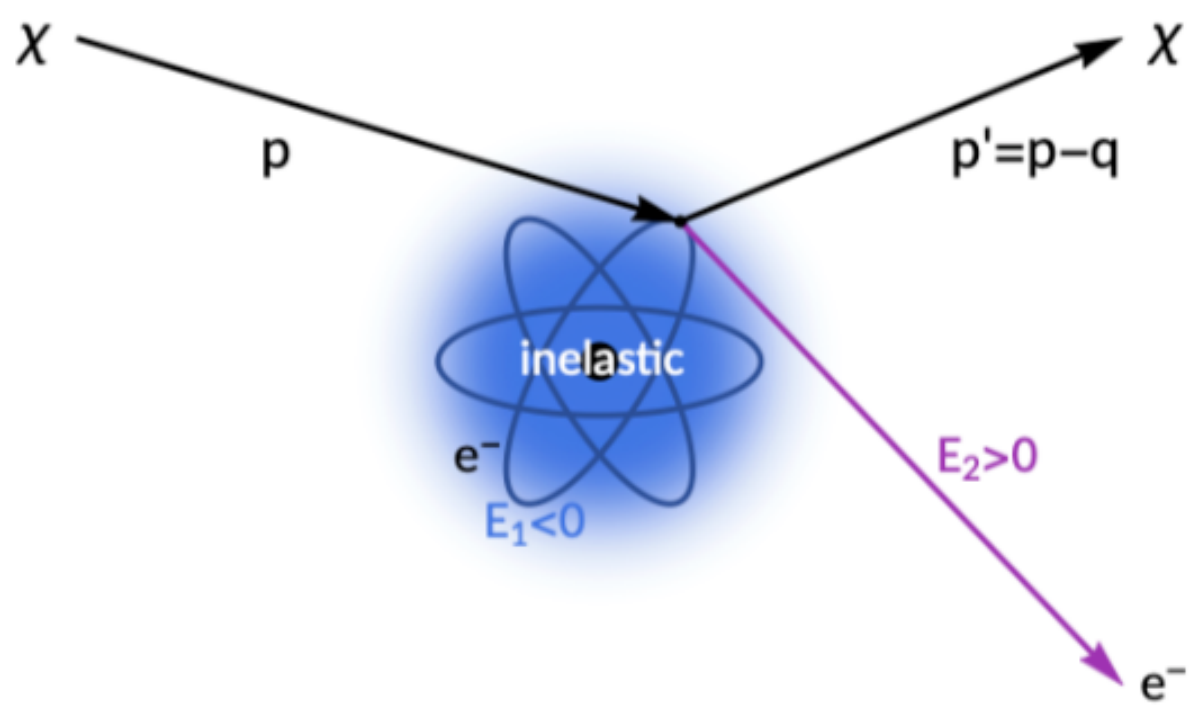
$$\overline{\left|\mathcal{M}_{\rm ion}^{n\ell}\right|^2} \equiv \frac{4Vk'^3}{(2\pi)^3}\sum_{m=-\ell}^{\ell}\sum_{\ell'=0}^{\infty}\sum_{m'=-\ell'}^{\ell'}\overline{\left|\mathcal{M}_{1\to 2}\right|^2}$$

$$\mathcal{M}_{1\to 2} = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\tilde{\psi}_2^*(\mathbf{k}+\mathbf{q})\mathcal{M}(\mathbf{q},\mathbf{v}_{\rm el}^{\perp})\tilde{\psi}_1(\mathbf{k})$$

Free electron-DM amplitude

Atomic wave-functions

Formalism for DM-atom scattering



Catena et al., 1912.08204

$$\frac{\mathrm{d}\mathcal{R}_{\mathrm{ion}}^{n\ell}}{\mathrm{d}\ln E_e} = \frac{n_x}{128\pi m_x^2 m_e^2}\int \mathrm{d}q\, q \int \frac{\mathrm{d}^3\mathbf{v}}{v} f_x(\mathbf{v})\Theta(v - v_{\mathrm{min}}) \overline{\left|\mathcal{M}_{\mathrm{ion}}^{n\ell}\right|^2}$$

$$\mathcal{M}_{1\to 2} = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \tilde{\psi}_2^*(\mathbf{k}+\mathbf{q})\mathcal{M}(\mathbf{q}, \mathbf{v}_{\mathrm{el}}^{\perp})\tilde{\psi}_1(\mathbf{k})$$

Free electron-DM amplitude

Atomic wave-functions

$$\mathcal{M}(\mathbf{q}, \mathbf{v}_{\mathrm{el}}^{\perp}) \to \mathcal{M}(\mathbf{q}) \quad \rightarrow \quad \mathcal{M}_{1\to 2} = \mathcal{M}(q = \alpha m_e)\frac{\mathcal{M}(q)}{\mathcal{M}(q=\alpha m_e)} \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\tilde{\psi}_2^*(\mathbf{k}+\mathbf{q})\tilde{\psi}_1(\mathbf{k})$$

$F_{\mathrm{DM}}(q)$ $\quad f_{1\to 2}(\mathbf{q})$

$$\rightarrow \quad \frac{d\mathcal{R}_{\mathrm{ion}}^{n\ell}}{d\ln E_e} = \frac{n_\chi \bar{\sigma}_e}{8\mu_{\chi e}} \times \int q\mathrm{d}q\, \eta(v_{\mathrm{min}}) |f_{\mathrm{ion}}^{n\ell}(q)|^2 |F_{\mathrm{DM}}(q)|^2$$

J. Kopp et al., 0907.3159,

R. Essig, et al., 1108.5383,

R. Essig, et al., 1206.2644

$$\eta(v_{\mathrm{min}}) \equiv \int \mathrm{d}^3\vec{v} f_\chi(\vec{v})\frac{1}{v}\Theta(v - v_{\mathrm{min}})$$

$$|f_{\mathrm{ion}}^{n\ell}(q)|^2 \propto \sum_{l'm'm} |f_{1\to 2}(\mathbf{q})|^2$$

How about the general case?

Based on the EFT approach

$v_{\rm DM} \sim 10^{-3}, v_e \sim \alpha_{\rm em} \sim 10^{-2}$ ➜ **Non-relativistic (NR) EFT framework**

$$\{\mathbb{1}_e, \boldsymbol{S}_e\} \otimes \{\mathbb{1}_x, \boldsymbol{S}_x, \tilde{\boldsymbol{S}}_x\} \otimes \{i\boldsymbol{q}, \boldsymbol{v}_{\rm el}^\perp\}$$

$$\mathbf{v}_{\rm el}^\perp = \mathbf{v} - \mathbf{q}/(2\mu_{xe}) - \mathbf{k}/m_e$$

NR operators	Power counting	DM type		
		scalar	fermion	vector
$\mathcal{O}_1 = \mathbb{1}_x \mathbb{1}_e$	1	✓	✓	✓
$\mathcal{O}_3 = \mathbb{1}_x \left(\frac{i\boldsymbol{q}}{m_e} \times \boldsymbol{v}_{\rm el}^\perp\right) \cdot \boldsymbol{S}_e$	qv	✓	✓	✓
$\mathcal{O}_4 = \boldsymbol{S}_x \cdot \boldsymbol{S}_e$	1	–	✓	✓
$\mathcal{O}_5 = \boldsymbol{S}_x \cdot \left(\frac{i\boldsymbol{q}}{m_e} \times \boldsymbol{v}_{\rm el}^\perp\right) \mathbb{1}_e$	qv	–	✓	✓
$\mathcal{O}_6 = \left(\boldsymbol{S}_x \cdot \frac{\boldsymbol{q}}{m_e}\right)\left(\frac{\boldsymbol{q}}{m_e} \cdot \boldsymbol{S}_e\right)$	q^2	–	✓	✓
$\mathcal{O}_7 = \mathbb{1}_x \boldsymbol{v}_{\rm el}^\perp \cdot \boldsymbol{S}_e$	v	✓	✓	✓
$\mathcal{O}_8 = \boldsymbol{S}_x \cdot \boldsymbol{v}_{\rm el}^\perp \mathbb{1}_e$	v	–	✓	✓
$\mathcal{O}_9 = -\boldsymbol{S}_x \cdot \left(\frac{i\boldsymbol{q}}{m_e} \times \boldsymbol{S}_e\right)$	q	–	✓	✓
$\mathcal{O}_{10} = \mathbb{1}_x \frac{i\boldsymbol{q}}{m_e} \cdot \boldsymbol{S}_e$	q	✓	✓	✓
$\mathcal{O}_{11} = \boldsymbol{S}_x \cdot \frac{i\boldsymbol{q}}{m_e} \mathbb{1}_e$	q	–	✓	✓
$\mathcal{O}_{12} = -\boldsymbol{S}_x \cdot (\boldsymbol{v}_{\rm el}^\perp \times \boldsymbol{S}_e)$	v	–	✓	✓
$\mathcal{O}_{13} = (\boldsymbol{S}_x \cdot \boldsymbol{v}_{\rm el}^\perp)\left(\frac{i\boldsymbol{q}}{m_e} \cdot \boldsymbol{S}_e\right)$	qv	–	✓	✓
$\mathcal{O}_{14} = (\boldsymbol{S}_x \cdot \frac{i\boldsymbol{q}}{m_e})(\boldsymbol{v}_{\rm el}^\perp \cdot \boldsymbol{S}_e)$	qv	–	✓	✓
$\mathcal{O}_{15} = \boldsymbol{S}_x \cdot \frac{\boldsymbol{q}}{m_e}\left[\frac{\boldsymbol{q}}{m_e} \cdot (\boldsymbol{v}_{\rm el}^\perp \times \boldsymbol{S}_e)\right]$	q^2v	–	✓	✓
$\mathcal{O}_{17} = \frac{i\boldsymbol{q}}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \boldsymbol{v}_{\rm el}^\perp \mathbb{1}_e$	qv	–	–	✓
$\mathcal{O}_{18} = \frac{i\boldsymbol{q}}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \boldsymbol{S}_e$	q	–	–	✓
$\mathcal{O}_{19} = \frac{\boldsymbol{q}}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \frac{\boldsymbol{q}}{m_e} \mathbb{1}_e$	q^2	–	–	✓
$\mathcal{O}_{20} = -\frac{\boldsymbol{q}}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \left(\frac{\boldsymbol{q}}{m_e} \times \boldsymbol{S}_e\right)$	q^2	–	–	✓
$\mathcal{O}_{21} = \boldsymbol{v}_{\rm el}^\perp \cdot \tilde{\boldsymbol{S}}_x \cdot \boldsymbol{S}_e$	v	–	–	✓
$\mathcal{O}_{22} = \left(\frac{i\boldsymbol{q}}{m_e} \times \boldsymbol{v}_{\rm el}^\perp\right) \cdot \tilde{\boldsymbol{S}}_x \cdot \boldsymbol{S}_e + \boldsymbol{v}_{\rm el}^\perp \cdot \tilde{\boldsymbol{S}}_x \cdot \left(\frac{i\boldsymbol{q}}{m_e} \times \boldsymbol{S}_e\right)$	qv	–	–	✓
$\mathcal{O}_{23} = -\frac{i\boldsymbol{q}}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot (\boldsymbol{v}_{\rm el}^\perp \times \boldsymbol{S}_e)$	qv	–	–	✓
$\mathcal{O}_{24} = \frac{\boldsymbol{q}}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \left(\frac{\boldsymbol{q}}{m_e} \times \boldsymbol{v}_{\rm el}^\perp\right)$	q^2v	–	–	✓
$\mathcal{O}_{25} = \left(\frac{\boldsymbol{q}}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \boldsymbol{v}_{\rm el}^\perp\right)\left(\frac{\boldsymbol{q}}{m_e} \cdot \boldsymbol{S}_e\right)$	q^2v	–	–	✓
$\mathcal{O}_{26} = \left(\frac{\boldsymbol{q}}{m_e} \cdot \tilde{\boldsymbol{S}}_x \cdot \frac{\boldsymbol{q}}{m_e}\right)(\boldsymbol{v}_{\rm el}^\perp \cdot \boldsymbol{S}_e)$	q^2v	–	–	✓

JHEP 07 (2024) 279, JL, YL, XM, and **HW**

How about the general case?

Catena et al., Phys. Rev. Res. 2, 033195 (2020) (105 citations)

$$\mathscr{M}(\mathbf{q}, \mathbf{v}_{\rm el}^{\perp}) = \mathscr{M}(\mathbf{q}, \mathbf{v}_{\rm el}^{\perp})\,|_{\mathbf{k}=0} + \mathbf{k}\cdot\nabla_{\mathbf{k}}\mathscr{M}(\mathbf{q}, \mathbf{v}_{\rm el}^{\perp})$$

→ $$\mathscr{M}_{1\to 2} = f_{1\to 2}(\mathbf{q})\mathscr{M}(\mathbf{q}, \mathbf{v}_{\rm el}^{\perp})\,|_{\mathbf{k}=0} + m_e\,\mathbf{f}_{1\to 2}(\mathbf{q})\cdot\nabla_{\mathbf{k}}\mathscr{M}(\mathbf{q}, \mathbf{v}_{\rm el}^{\perp})\,|_{\mathbf{k}=0}$$

$$f_{1\to 2}(\mathbf{q}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\tilde{\psi}_2^*(\mathbf{k}+\mathbf{q})\tilde{\psi}_1(\mathbf{k})$$

$$\mathbf{f}_{1\to 2}(\mathbf{q}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\tilde{\psi}_2^*(\mathbf{k}+\mathbf{q})\frac{\mathbf{k}}{m_e}\tilde{\psi}_1(\mathbf{k})$$

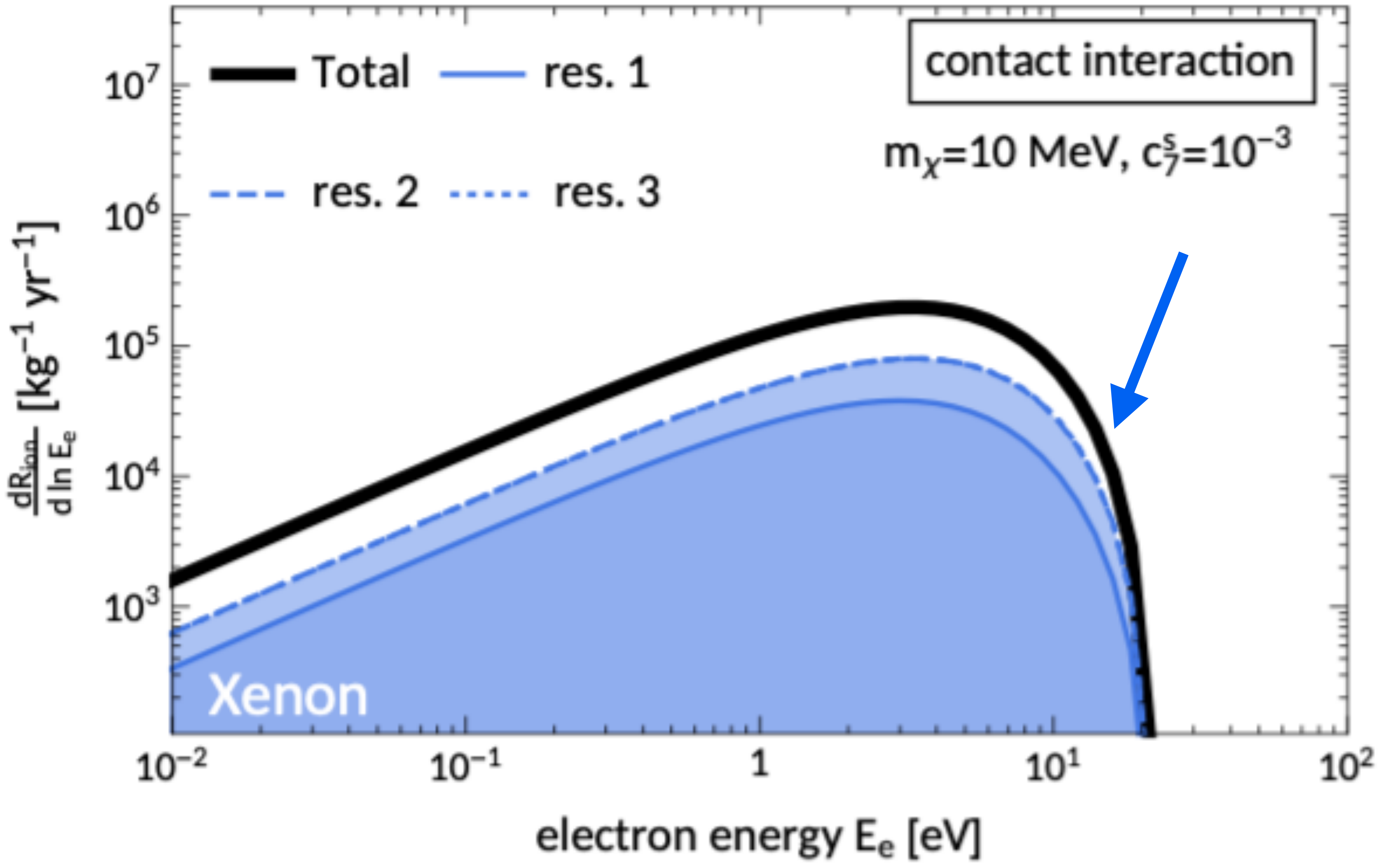
$$\overline{\left|\mathscr{M}_{\rm ion}^{n\ell}\right|^2} = \sum_{i=1}^{4} R_i^{n\ell}(q, v)W_i^{n\ell}(k', q)$$

1 usual $|f_{\rm ion}^{n\ell}(q)|^2$ (W_1) + 3 new atomic response functions $(W_{2,3,4})$

Comparison

$$\mathcal{O}_7 = \mathbb{1}_x \, \boldsymbol{v}_{\rm el}^{\perp} \cdot \boldsymbol{S}_e$$

Contributions from W_2 and W_3 dominate



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$$\rightarrow \quad \mathcal{M}_{1\to2} = f_{1\to2}(\mathbf{q})\mathcal{M}(\mathbf{q}, \mathbf{v}_{\rm el}^{\perp})\,|_{\mathbf{k}=0} + m_e\,\mathbf{f}_{1\to2}(\mathbf{q})\cdot\nabla_{\mathbf{k}}\mathcal{M}(\mathbf{q}, \mathbf{v}_{\rm el}^{\perp})\,|_{\mathbf{k}=0}$$

$$f_{1\to2}(\mathbf{q}) = \int\frac{{\rm d}^3k}{(2\pi)^3}\tilde{\psi}_2^*(\mathbf{k}+\mathbf{q})\tilde{\psi}_1(\mathbf{k})$$

$$\mathbf{f}_{1\to2}(\mathbf{q}) = \int\frac{{\rm d}^3k}{(2\pi)^3}\tilde{\psi}_2^*(\mathbf{k}+\mathbf{q})\frac{\mathbf{k}}{m_e}\tilde{\psi}_1(\mathbf{k})$$

A minus sign is missed

$$\rightarrow \quad \mathbf{f}_{1\to2}(\mathbf{q}) = \int d^3\mathbf{r}\,\psi_2^*(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}\frac{-i\nabla}{m_e}\psi_1(\mathbf{r})$$

$$W_2 \to -W_2$$

$$\overline{\left|\mathcal{M}_{\rm ion}^{n\ell}\right|^2} = \sum_{i=1}^{4} R_i^{n\ell}(q,v)W_i^{n\ell}(k',q)$$

1 usual $|f_{\rm ion}^{n\ell}(q)|^2\,(W_1)$ + 3 new atomic response functions $(W_{2,3,4})$

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$$\mathcal{O}_7 = \mathbb{1}_x \, \boldsymbol{v}_{\rm el}^{\perp} \cdot \boldsymbol{S}_e$$

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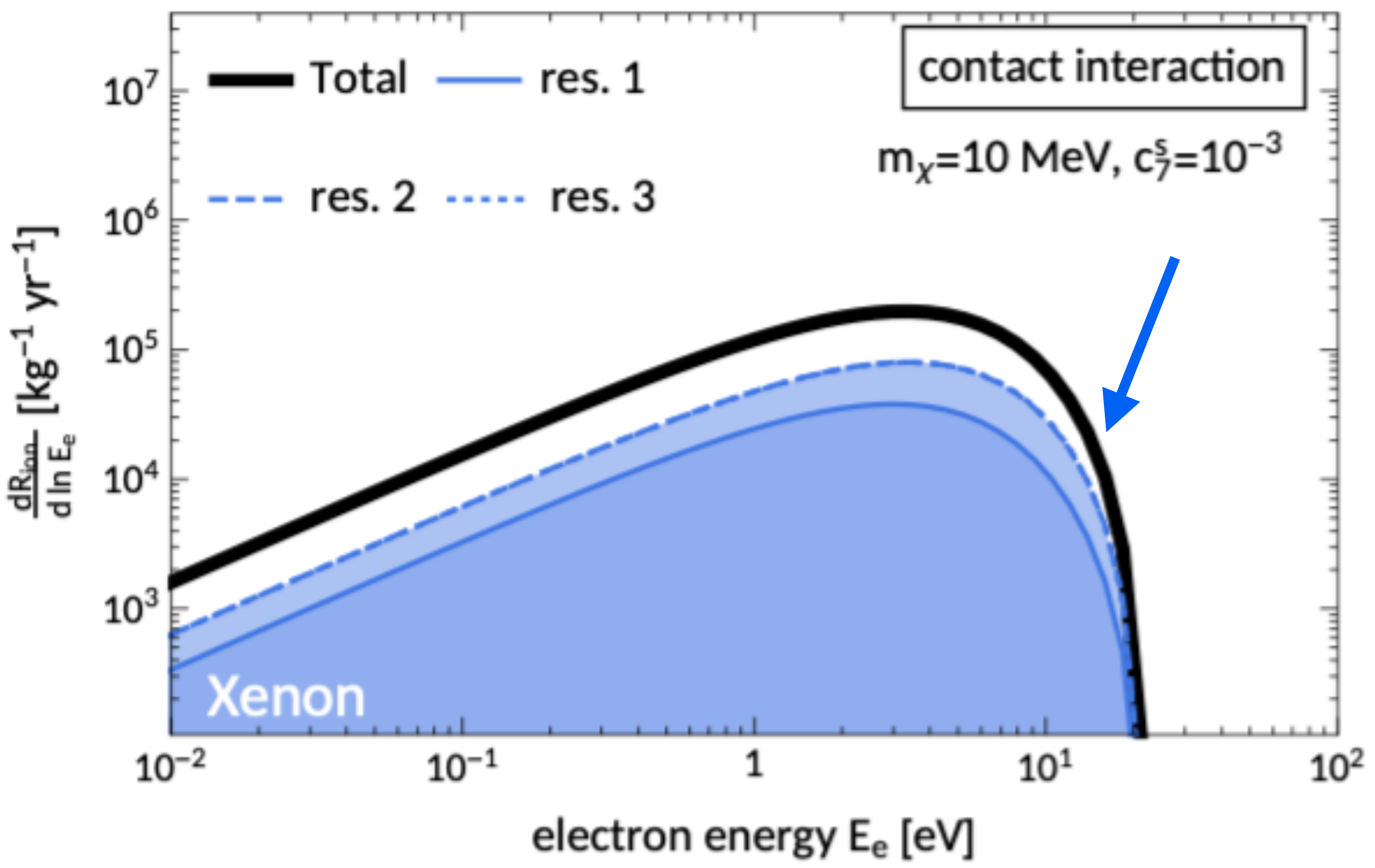

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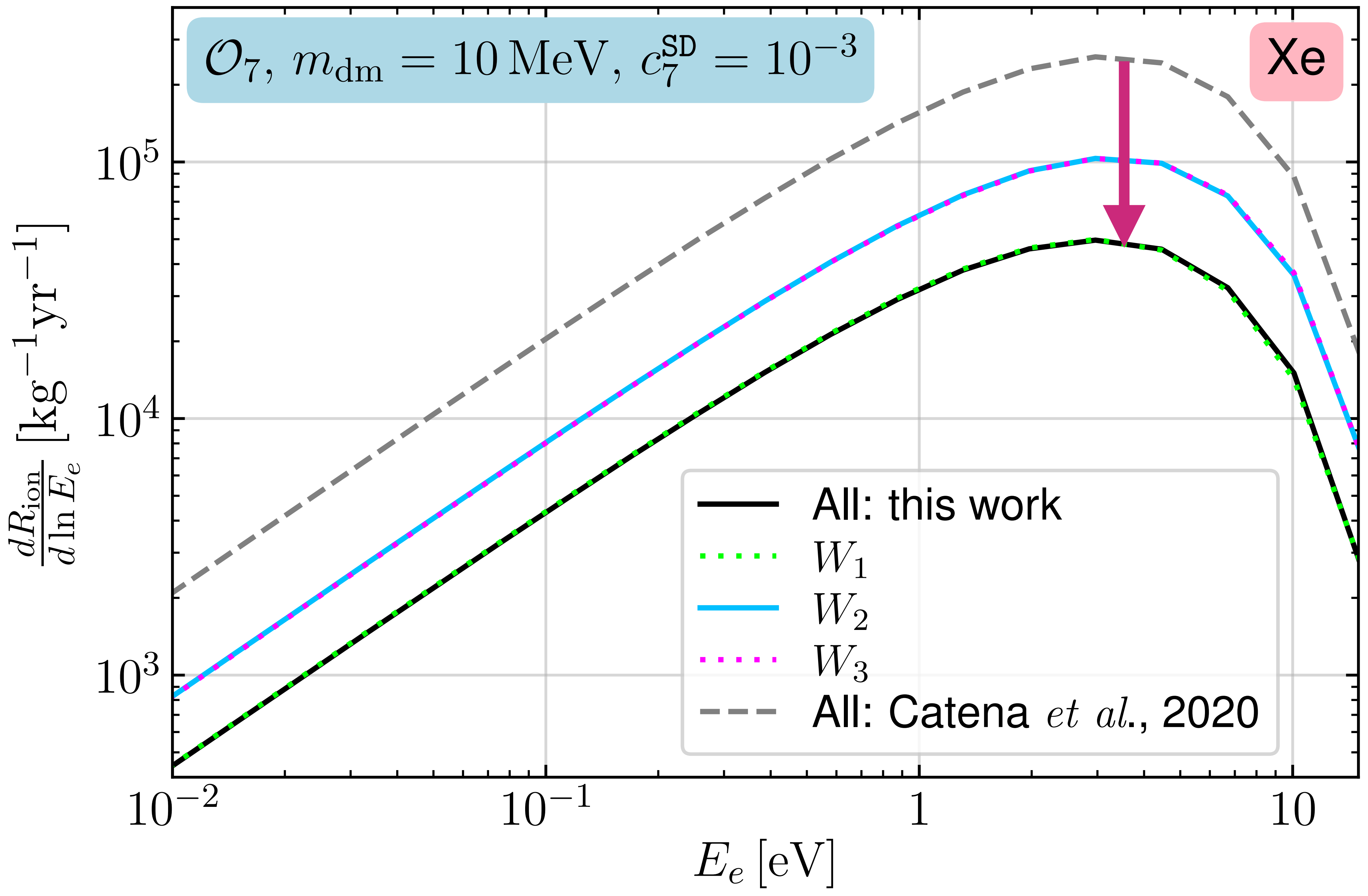
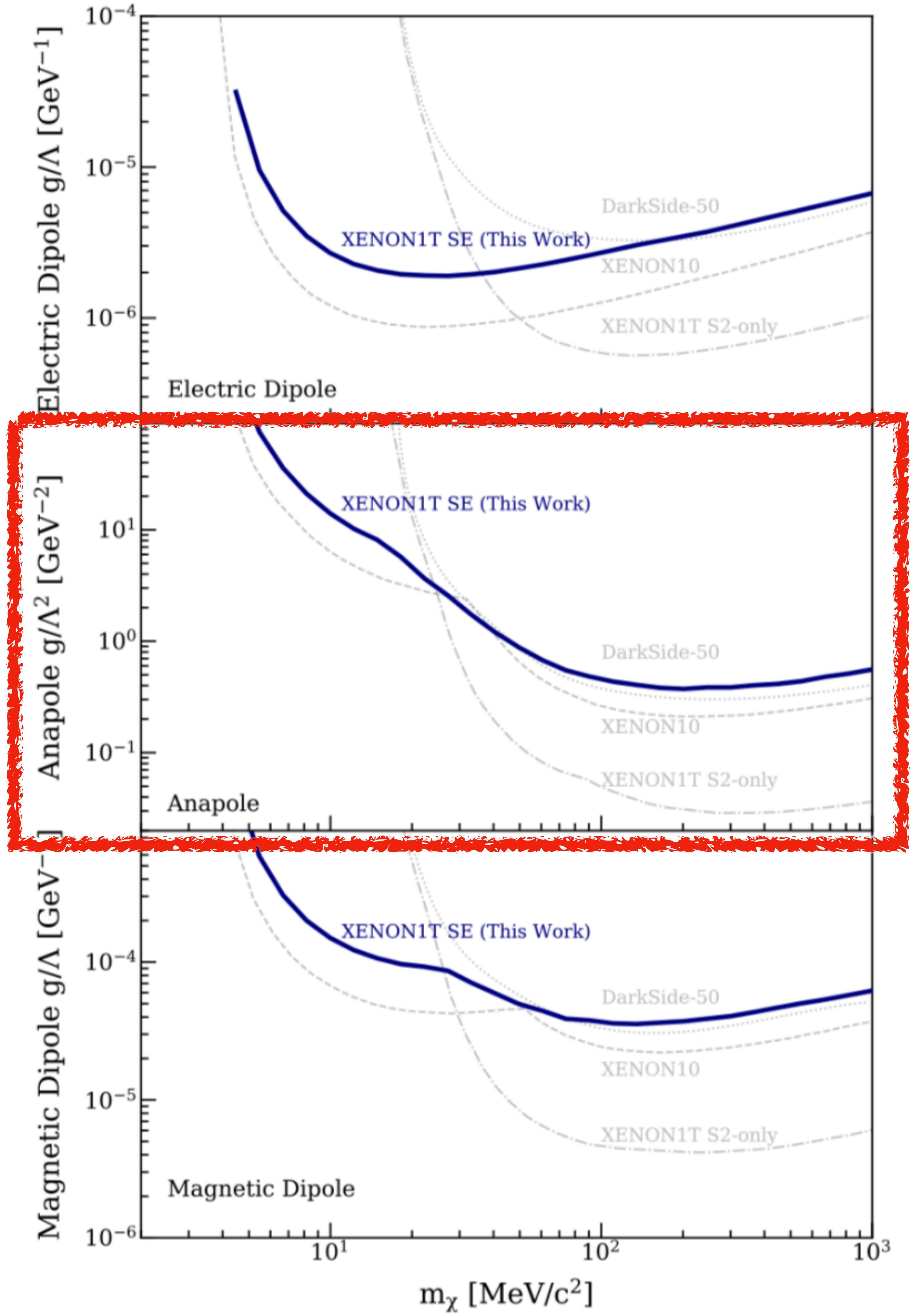
arXiv: *2405. 04855*, JL, YL, XM, and **HW**



- **The sign difference cause a factor of 5 difference in the differential rate**
- **Constructive inteference between W_2 and W_3, leaving W_1 contribution dominant**

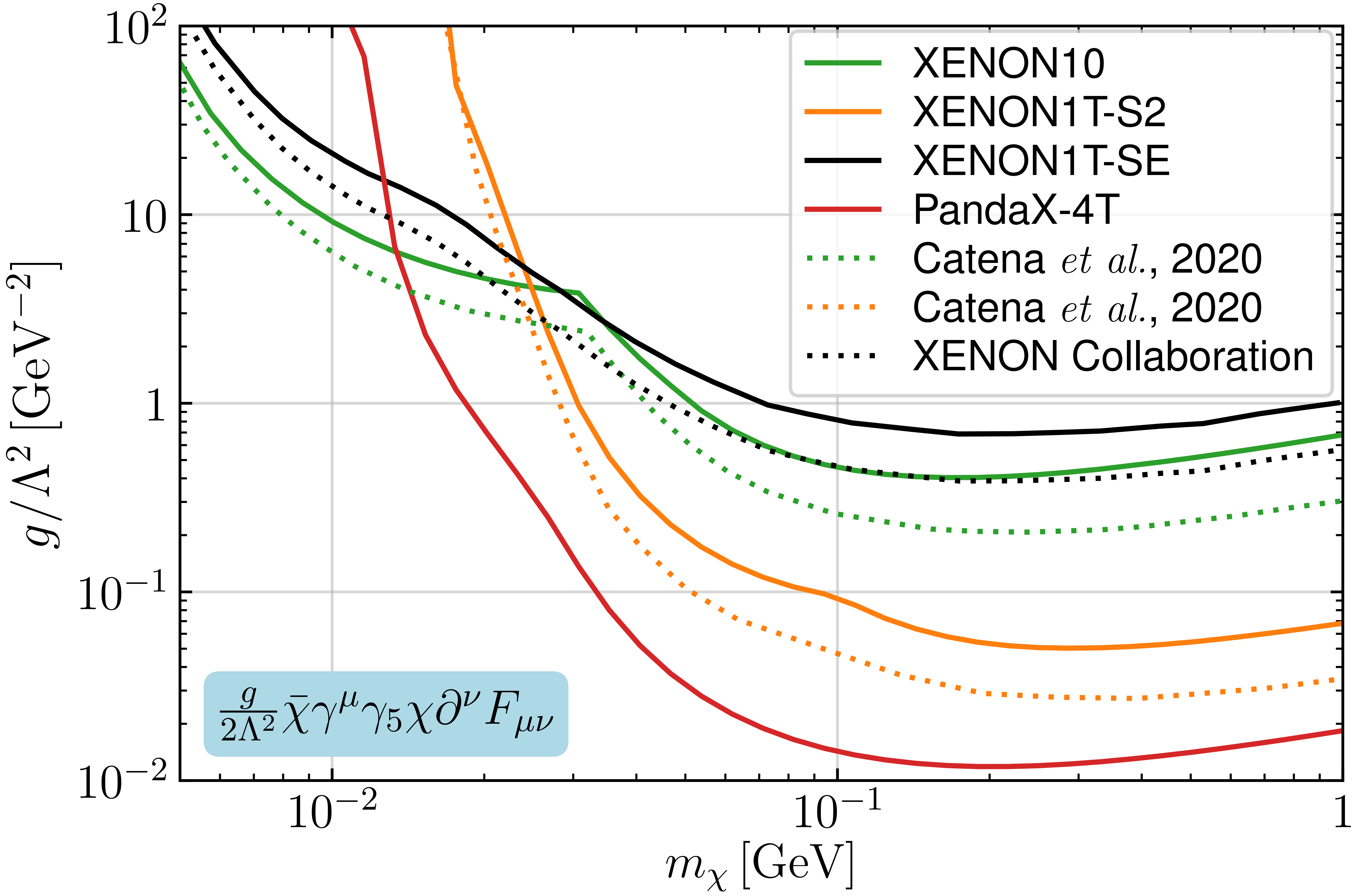
XENON1T SE constraints on EM form factors of DM

XENON Collaboration, 2112.12116



$$a_\chi(\bar{\chi}\gamma^\mu\gamma_5\chi)\partial^\nu F_{\mu\nu} \rightarrow 8a_\chi e\, m_\chi m_e\left(\mathcal{O}_8 - \mathcal{O}_9\right)$$

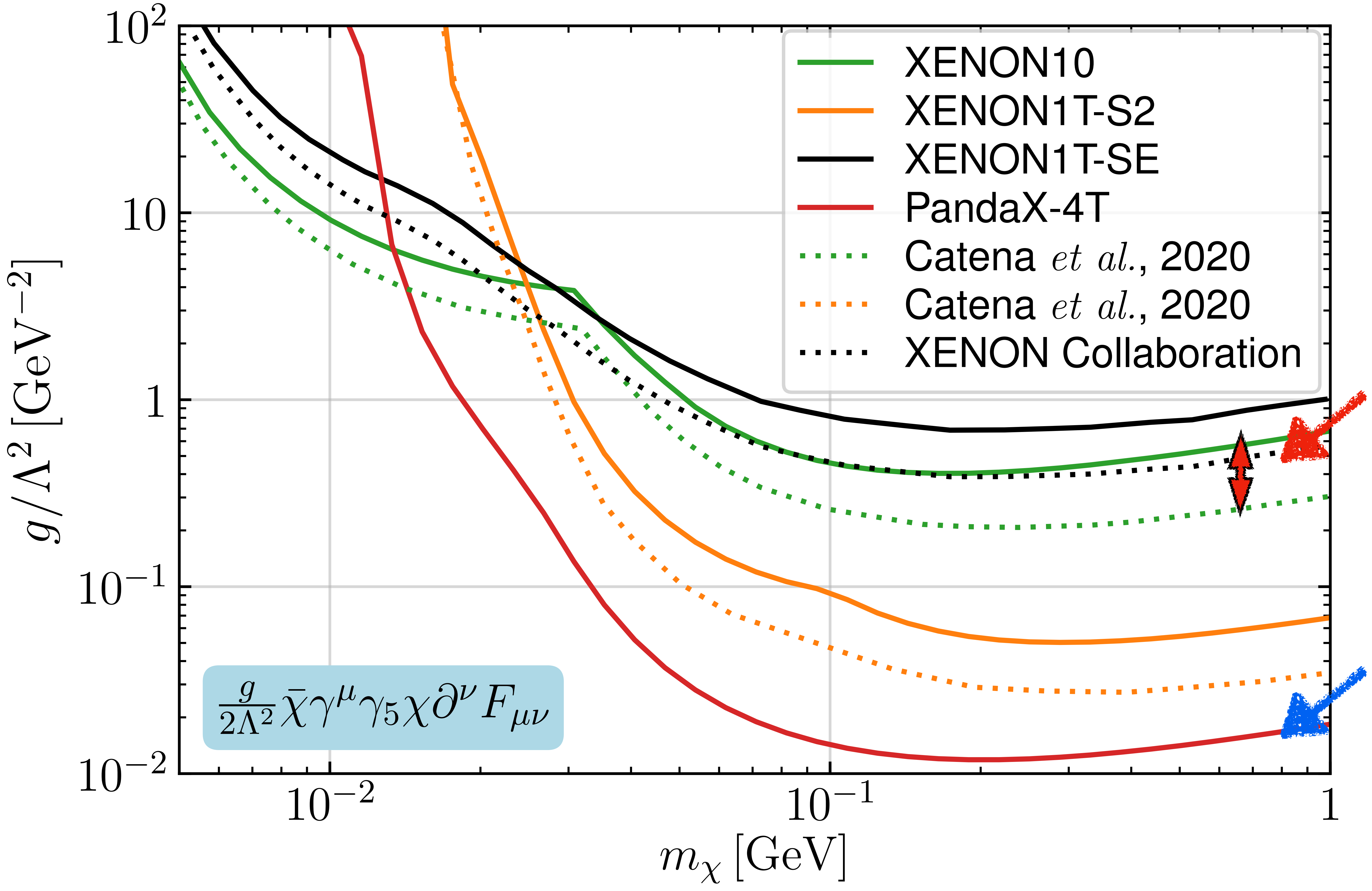
arXiv: *2405. 04855*, JL, YL, XM, and **HW**



XENON1T SE constraints on EM form factors of DM

$$a_\chi(\bar{\chi}\gamma^\mu\gamma_5\chi)\partial^\nu F_{\mu\nu} \to 8a_\chi e\, m_\chi m_e\left(\mathcal{O}_8 - \mathcal{O}_9\right)$$

arXiv: *2405. 04855*, JL, YL, XM, and **HW**



- **The constraints are weakened by a factor of 2 if the sign of W2 is corrected**

- **PandaX-4T set the most stringent constraint when** $m_\chi \gtrsim 20$ MeV

Our formalism

JHEP 07 (2024) 279, JL, YL, XM, and **HW**

Based on the NR operator structure

$$\mathbf{v}_{\rm el}^{\perp} = \mathbf{v} - \mathbf{q}/(2\mu_{xe}) - \mathbf{k}/m_e$$

$$\mathbf{v}_{0}^{\perp} \equiv \mathbf{v} - \mathbf{q}/(2\mu_{xe})$$

$$\mathscr{M}(\mathbf{q}, \mathbf{v}_{\rm el}^{\perp}) = \mathscr{M}(\mathbf{q},0) + \mathbf{v}_{\rm el}^{\perp} \cdot \nabla_{\mathbf{v}_{\rm el}^{\perp}} \mathscr{M}(\mathbf{q}, \mathbf{v}_{\rm el}^{\perp})$$

$$\rightarrow \quad \mathscr{M}_{1\to2} = f_{\rm S}(\mathbf{q})\mathscr{M}_{\rm S} + \mathbf{f}_{\rm V}(\mathbf{q}) \cdot \mathscr{M}_{\rm V}$$

$$f_{\rm S}(\mathbf{q}) = f_{1\to2}(\mathbf{q})$$

$$\mathbf{f}_{\rm V}(\mathbf{q}) = \mathbf{v}_{0}^{\perp} f_{1\to2}(\mathbf{q}) - \mathbf{f}_{1\to2}(\mathbf{q})$$

$$\overline{\left|\mathscr{M}_{\rm ion}^{n\ell}\right|^2} = a_0 \widetilde{W}_0 + a_1 \widetilde{W}_1 + a_2 \widetilde{W}_2$$

We need only three generalized atomic response functions

$$\widetilde{W}_0 = W_1,$$

$$\widetilde{W}_1 = |\boldsymbol{v}_0^{\perp}|^2 W_1 - 2\frac{y_e}{x_e} W_2 + W_3,$$

$$\widetilde{W}_2 = \frac{y_e^2}{x_e} W_1 - 2\frac{y_e}{x_e} W_2 + \frac{1}{x_e} W_4.$$

Our formalism

JHEP 07 (2024) 279, JL, YL, XM, and **HW**

Based on the NR operator structure

$$\mathcal{M}(\mathbf{q}, \mathbf{v}_{\rm el}^{\perp}) = \mathcal{M}(\mathbf{q},0) + \mathbf{v}_{\rm el}^{\perp}$$

$$\rightarrow \quad \mathcal{M}_{1\to 2} = f_S(\mathbf{q})\mathcal{M}_S +$$

$$\overline{\left|\mathcal{M}_{\rm ion}^{n\ell}\right|^2} =$$

We need only three ge

$$\widetilde{W}_0 = W_1,$$

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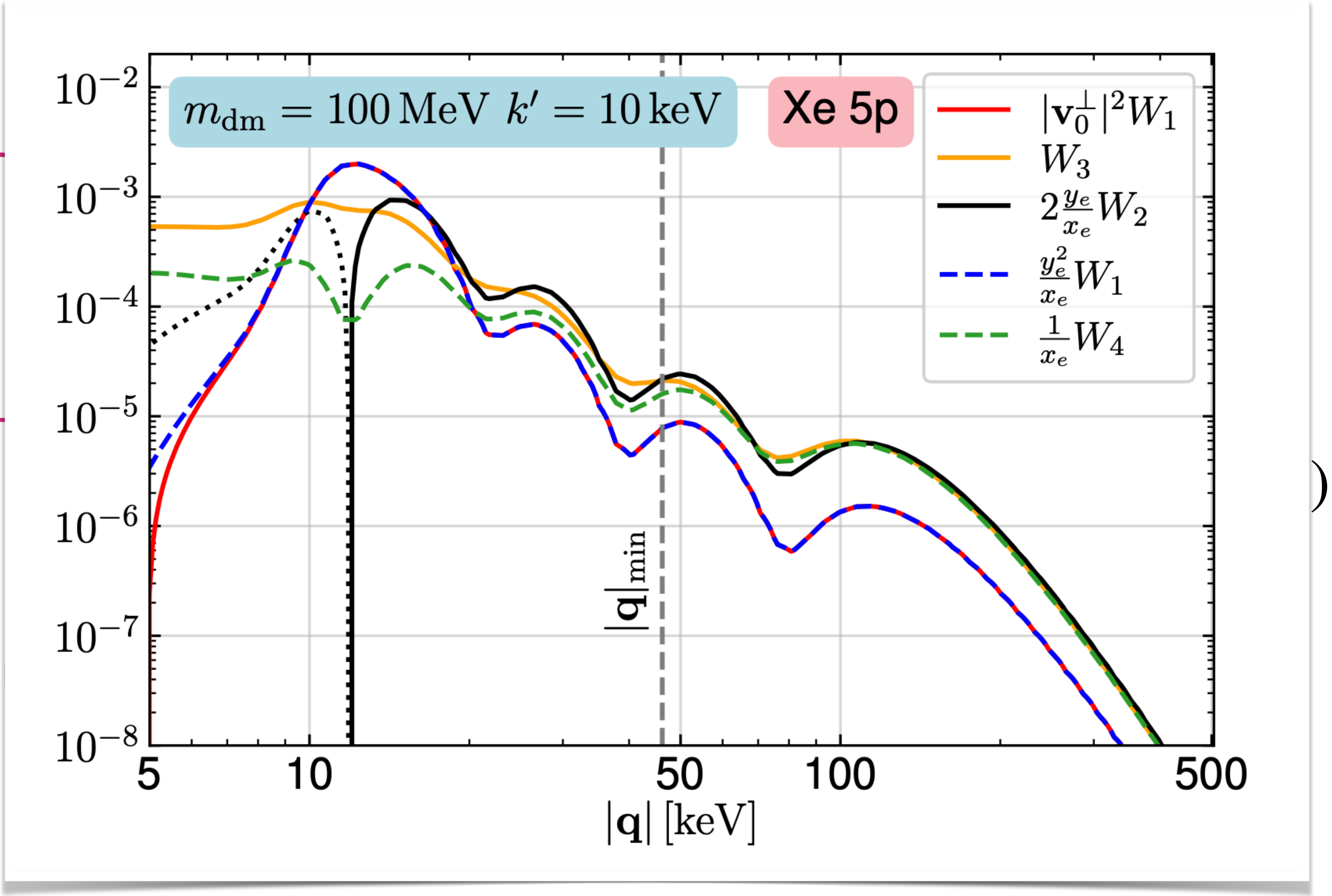
$$\widetilde{W}_2 = \frac{y_e^2}{x_e}W_1 - 2\frac{y_e}{x_e}W_2 + \frac{1}{x_e}W_4.$$



The minus sign leads to a strong cancellation among W_2 and $W_{3,4}$

Our formalism

$$\overline{\left|\mathcal{M}^{n\ell}_{\text{ion}}\right|^2} = a_0 \widetilde{W}_0 + a_1 \widetilde{W}_1 + a_2 \widetilde{W}_2$$

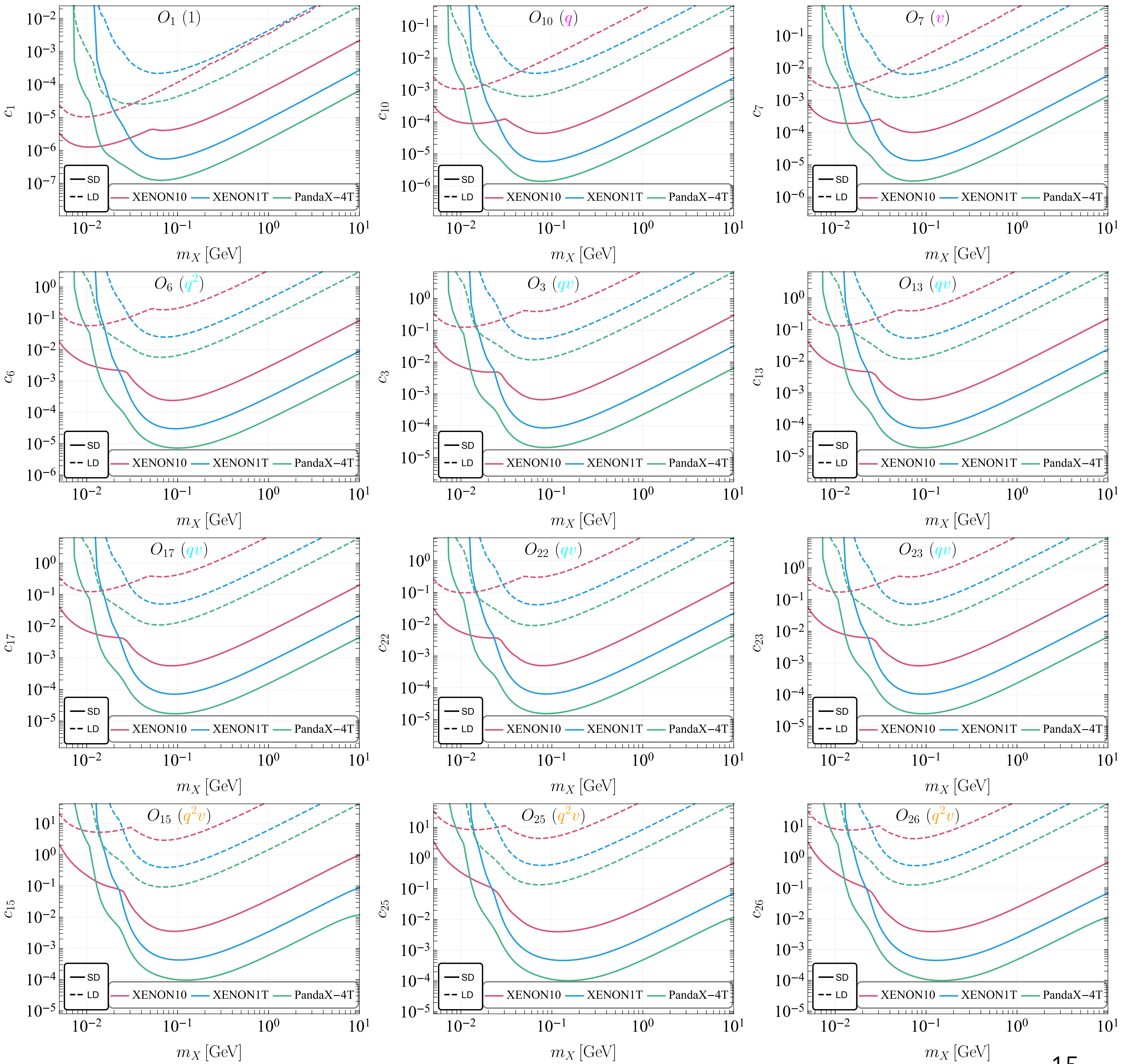
- a_0 and $a_{1,2}$ involve different NR operators
- Clear power counting for q
- Do not contain any atomic properties

$$x_e \equiv q^2/m_e^2$$

Type	DM response functions
Scalar DM	$a_0 = \lvert c_1\rvert^2 + \frac{1}{4}\lvert c_{10}\rvert^2 x_e$ $a_1 = \frac{1}{4}\lvert c_7\rvert^2 + \frac{1}{4}\lvert c_3\rvert^2 x_e$ $a_2 = -\frac{1}{4}\lvert c_3\rvert^2 x_e$
Fermion DM	$a_0 = \lvert c_1\rvert^2 + \frac{3}{16}\lvert c_4\rvert^2 + \left(\frac{1}{8}\lvert c_9\rvert^2 + \frac{1}{4}\lvert c_{10}\rvert^2 + \frac{1}{4}\lvert c_{11}\rvert^2 + \frac{1}{8}\Re[c_4 c_6^*]\right) x_e + \frac{1}{16}\lvert c_6\rvert^2 x_e^2$ $a_1 = \frac{1}{4}\lvert c_7\rvert^2 + \frac{1}{4}\lvert c_8\rvert^2 + \frac{1}{8}\lvert c_{12}\rvert^2 + \left(\frac{1}{4}\lvert c_3\rvert^2 + \frac{1}{4}\lvert c_5\rvert^2 + \frac{1}{16}\lvert c_{13}\rvert^2 + \frac{1}{16}\lvert c_{14}\rvert^2 - \frac{1}{8}\Re[c_{12} c_{15}^*]\right) x_e$ $+\frac{1}{16}\lvert c_{15}\rvert^2 x_e^2$ $a_2 = -\left(\frac{1}{4}\lvert c_3\rvert^2 + \frac{1}{4}\lvert c_5\rvert^2 - \frac{1}{8}\Re[c_{12} c_{15}^*] - \frac{1}{8}\Re[c_{13} c_{14}^*]\right) x_e - \frac{1}{16}\lvert c_{15}\rvert^2 x_e^2$
Vector DM	$a_0 = \lvert c_1\rvert^2 + \frac{1}{2}\lvert c_4\rvert^2 + \left(\frac{1}{3}\lvert c_9\rvert^2 + \frac{1}{4}\lvert c_{10}\rvert^2 + \frac{2}{3}\lvert c_{11}\rvert^2 + \frac{5}{36}\lvert c_{18}\rvert^2 + \frac{1}{3}\Re[c_4 c_6^*]\right) x_e$ $+ \left(\frac{1}{6}\lvert c_6\rvert^2 + \frac{2}{9}\lvert c_{19}\rvert^2 + \frac{1}{12}\lvert c_{20}\rvert^2\right) x_e^2$ $a_1 = \frac{1}{4}\lvert c_7\rvert^2 + \frac{2}{3}\lvert c_8\rvert^2 + \frac{1}{3}\lvert c_{12}\rvert^2 + \frac{5}{36}\lvert c_{21}\rvert^2 + \left(\frac{1}{4}\lvert c_3\rvert^2 + \frac{2}{3}\lvert c_5\rvert^2 + \frac{1}{6}\lvert c_{13}\rvert^2 + \frac{1}{6}\lvert c_{14}\rvert^2 + \frac{1}{6}\lvert c_{17}\rvert^2\right.$ $\left.+\frac{3}{8}\lvert c_{22}\rvert^2 + \frac{7}{72}\lvert c_{23}\rvert^2 - \frac{1}{3}\Re[c_{12} c_{15}^*] + \frac{1}{12}\Re[c_{21} c_{25}^*] - \frac{1}{18}\Re[c_{21} c_{26}^*] + \frac{1}{12}\Re[c_{22} c_{23}^*]\right) x_e$ $+ \left(\frac{1}{6}\lvert c_{15}\rvert^2 + \frac{1}{6}\lvert c_{24}\rvert^2 + \frac{1}{24}\lvert c_{25}\rvert^2 + \frac{1}{18}\lvert c_{26}\rvert^2\right) x_e^2$ $a_2 = -\left(\frac{1}{4}\lvert c_3\rvert^2 + \frac{2}{3}\lvert c_5\rvert^2 - \frac{1}{18}\lvert c_{17}\rvert^2 + \frac{7}{24}\lvert c_{22}\rvert^2 + \frac{1}{72}\lvert c_{23}\rvert^2 - \frac{1}{3}\Re[c_{12} c_{15}^*] - \frac{1}{3}\Re[c_{13} c_{14}^*]\right.$ $\left.-\frac{1}{36}\Re[c_{21} c_{25}^*] - \frac{1}{6}\Re[c_{21} c_{26}^*] + \frac{1}{4}\Re[c_{22} c_{23}^*]\right) x_e$ $- \left(\frac{1}{6}\lvert c_{15}\rvert^2 + \frac{1}{6}\lvert c_{24}\rvert^2 - \frac{1}{72}\lvert c_{25}\rvert^2 - \frac{1}{9}\Re[c_{25} c_{26}^*]\right) x_e^2$

JHEP 07 (2024) 279, JL, YL, XM, and **HW**

Constraints on the NR operators



$$M_i = (c_i^s + c_i^l \frac{q_{\rm ref}^2}{q^2}) < O_i > \qquad q_{\rm ref} = \alpha m_e$$

- The constriants follows consistently with the power counting of q and/or v
- The suppression from v is comparable with that from q

PandaX-4T set the most stringent constraint when $m_X \gtrsim 20$ MeV

JHEP 07 (2024) 279, JL, YL, XM, and **HW**

Constraints on the relativistic operators

All possible LO DM-electron and DM-photon operators

Dim	Relativistic operators	NR reduction
	Scalar case	
dim-5	$\mathcal{O}^S_{\ell\phi} = (\bar{\ell}\ell)(\phi^\dagger\phi)$	$2m_e\mathcal{O}_1$
	$\mathcal{O}^P_{\ell\phi} = (\bar{\ell}i\gamma_5\ell)(\phi^\dagger\phi)$	$-2m_e\mathcal{O}_{10}$
dim-6	$\mathcal{O}^V_{\ell\phi} = (\bar{\ell}\gamma^\mu\ell)(\phi^\dagger i\overleftrightarrow{\partial_\mu}\phi)\,(\times)$	$4m_em_\phi\mathcal{O}_1$
	$\mathcal{O}^A_{\ell\phi} = (\bar{\ell}\gamma^\mu\gamma_5\ell)(\phi^\dagger i\overleftrightarrow{\partial_\mu}\phi)\,(\times)$	$-8m_em_\phi\mathcal{O}_7$
	$\mathcal{L}^Q_\phi = \lvert(\partial_\mu - iQ_\phi eA_\mu)\phi\rvert^2\,(\times)$	$-4Q_\phi e^2\frac{m_em_\phi}{\boldsymbol{q}^2}\mathcal{O}_1$
	$\mathcal{L}^{\rm cr}_\phi = b_\phi(\phi^\dagger i\overleftrightarrow{\partial^\mu}\phi)\partial^\nu F_{\mu\nu}\,(\times)$	$4b_\phi em_em_\phi\mathcal{O}_1$
	Fermion case	
dim-6	$\mathcal{O}^S_{\ell\chi1} = (\bar{\ell}\ell)(\bar{\chi}\chi)$	$4m_em_\chi\mathcal{O}_1$
	$\mathcal{O}^S_{\ell\chi2} = (\bar{\ell}\ell)(\bar{\chi}i\gamma_5\chi)$	$4m_e^2\mathcal{O}_{11}$
	$\mathcal{O}^P_{\ell\chi1} = (\bar{\ell}i\gamma_5\ell)(\bar{\chi}\chi)$	$-4m_em_\chi\mathcal{O}_{10}$
	$\mathcal{O}^P_{\ell\chi2} = (\bar{\ell}i\gamma_5\ell)(\bar{\chi}i\gamma_5\chi)$	$4m_e^2\mathcal{O}_6$
	$\mathcal{O}^V_{\ell\chi1} = (\bar{\ell}\gamma^\mu\ell)(\bar{\chi}\gamma_\mu\chi)\,(\times)$	$4m_em_\chi\mathcal{O}_1$
	$\mathcal{O}^V_{\ell\chi2} = (\bar{\ell}\gamma^\mu\ell)(\bar{\chi}\gamma_\mu\gamma_5\chi)$	$8m_em_\chi(\mathcal{O}_8 - \mathcal{O}_9)$
	$\mathcal{O}^A_{\ell\chi1} = (\bar{\ell}\gamma^\mu\gamma_5\ell)(\bar{\chi}\gamma_\mu\chi)\,(\times)$	$-8m_e(m_\chi\mathcal{O}_7 + m_e\mathcal{O}_9)$
	$\mathcal{O}^A_{\ell\chi2} = (\bar{\ell}\gamma^\mu\gamma_5\ell)(\bar{\chi}\gamma_\mu\gamma_5\chi)$	$-16m_em_\chi\mathcal{O}_4$
	$\mathcal{O}^T_{\ell\chi1} = (\bar{\ell}\sigma^{\mu\nu}\ell)(\bar{\chi}\sigma_{\mu\nu}\chi)\,(\times)$	$32m_em_\chi\mathcal{O}_4$
	$\mathcal{O}^T_{\ell\chi2} = (\bar{\ell}\sigma^{\mu\nu}\ell)(\bar{\chi}i\sigma_{\mu\nu}\gamma_5\chi)\,(\times)$	$8m_e(m_e\mathcal{O}_{10} - m_\chi\mathcal{O}_{11} - 4m_\chi\mathcal{O}_{12})$
	$\mathcal{L}^Q_\chi = \bar{\chi}i\gamma^\mu(\partial_\mu - iQ_\chi eA_\mu)\chi\,(\times)$	$-4Q_\chi e^2\frac{m_em_\chi}{\boldsymbol{q}^2}\mathcal{O}_1$
	$\mathcal{L}^{\rm mdm}_\chi = \mu_\chi(\bar{\chi}\sigma^{\mu\nu}\chi)F_{\mu\nu}\,(\times)$	$4\mu_\chi e\left(m_e\mathcal{O}_1 + 4m_\chi\mathcal{O}_4 + \frac{4m_e^2m_\chi}{\boldsymbol{q}^2}(\mathcal{O}_5 - \mathcal{O}_6)\right)$
	$\mathcal{L}^{\rm edm}_\chi = d_\chi(\bar{\chi}i\sigma^{\mu\nu}\gamma_5\chi)F_{\mu\nu}\,(\times)$	$d_\chi e\frac{16m_e^2m_\chi}{\boldsymbol{q}^2}\mathcal{O}_{11}$
	$\mathcal{L}^{\rm cr}_\chi = b_\chi(\bar{\chi}\gamma^\mu\chi)\partial^\nu F_{\mu\nu}\,(\times)$	$4b_\chi e\,m_em_\chi\mathcal{O}_1$
	$\mathcal{L}^{\rm anap.}_\chi = a_\chi(\bar{\chi}\gamma^\mu\gamma_5\chi)\partial^\nu F_{\mu\nu}$	$8a_\chi e\,m_em_\chi(\mathcal{O}_8 - \mathcal{O}_9)$

Dim	Relativistic operators	NR reduction
	Vector case A	
dim-5	$\mathcal{O}^{\tt S}_{\ell X} = (\bar{\ell}\ell)(X^\dagger_\mu X^\mu)$	$-2m_e\mathcal{O}_1$
	$\mathcal{O}^{\tt P}_{\ell X} = (\bar{\ell}i\gamma_5\ell)(X^\dagger_\mu X^\mu)$	$2m_e\mathcal{O}_{10}$
	$\mathcal{O}^{\tt T}_{\ell X1} = \frac{i}{2}(\bar{\ell}\sigma^{\mu\nu}\ell)(X^\dagger_\mu X_\nu - X^\dagger_\nu X_\mu),\ (\times)$	$-4m_e\mathcal{O}_4$
	$\mathcal{O}^{\tt T}_{\ell X2} = \frac{1}{2}(\bar{\ell}\sigma^{\mu\nu}\gamma_5\ell)(X^\dagger_\mu X_\nu - X^\dagger_\nu X_\mu),\ (\times)$	$-m_e(\mathcal{O}_{11} + 4\mathcal{O}_{12}) + 4\frac{m_e^2}{m_X}\left(\frac{1}{3}\mathcal{O}_{10} - \mathcal{O}_{18}\right)$
dim-6	$\mathcal{O}^{\tt V}_{\ell X1} = \frac{1}{2}[\bar{\ell}\gamma_{(\mu}i\overleftrightarrow{D}_{\nu)}\ell](X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu)$	$m_e^2\mathcal{O}_1$
	$\mathcal{O}^{\tt V}_{\ell X2} = (\bar{\ell}\gamma_\mu\ell)\partial_\nu(X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu)$	$-4m_e^2(\mathcal{O}_{17} + \mathcal{O}_{20}) + \frac{4}{3}m_e(i\boldsymbol{q}\cdot\boldsymbol{v}^\perp_{\rm el})\mathcal{O}_1$
	$\mathcal{O}^{\tt V}_{\ell X3} = (\bar{\ell}\gamma_\mu\ell)(X^\dagger_\rho\overleftrightarrow{\partial_\nu}X_\sigma)\epsilon^{\mu\nu\rho\sigma}$	$-4m_em_X(\mathcal{O}_8 - \mathcal{O}_9)$
	$\mathcal{O}^{\tt V}_{\ell X4} = (\bar{\ell}\gamma^\mu\ell)(X^\dagger_\nu i\overleftrightarrow{\partial_\mu}X^\nu),\ (\times)$	$-4m_em_X\mathcal{O}_1$
	$\mathcal{O}^{\tt V}_{\ell X5} = (\bar{\ell}\gamma_\mu\ell)i\partial_\nu(X^{\mu\dagger}X^\nu - X^{\nu\dagger}X^\mu),\ (\times)$	$2m_e^2\left(\mathcal{O}_5 - \mathcal{O}_6 - \frac{m_e}{m_X}\mathcal{O}_{19}\right) + 2\boldsymbol{q}^2\mathcal{O}_4 + \frac{2}{3}\frac{m_e}{m_X}\boldsymbol{q}^2\mathcal{O}_1$
	$\mathcal{O}^{\tt V}_{\ell X6} = (\bar{\ell}\gamma_\mu\ell)i\partial_\nu(X^\dagger_\rho X_\sigma)\epsilon^{\mu\nu\rho\sigma},\ (\times)$	$-2m_e^2\mathcal{O}_{11}$
	$\mathcal{O}^{\tt A}_{\ell X1} = \frac{1}{2}[\bar{\ell}\gamma_{(\mu}\gamma_5 i\overleftrightarrow{D}_{\nu)}\ell](X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu)$	$-2m_e^2\left(\frac{m_e}{m_X}\mathcal{O}_9 - 4\mathcal{O}_{21} + \frac{4}{3}\mathcal{O}_7\right)$
	$\mathcal{O}^{\tt A}_{\ell X2} = (\bar{\ell}\gamma_\mu\gamma_5\ell)\partial_\nu(X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu)$	$-8m_e^2\left(\frac{1}{3}\mathcal{O}_{10} - \mathcal{O}_{18}\right)$
	$\mathcal{O}^{\tt A}_{\ell X3} = (\bar{\ell}\gamma_\mu\gamma_5\ell)(X^\dagger_\rho\overleftrightarrow{\partial_\nu}X_\sigma)\epsilon^{\mu\nu\rho\sigma}$	$8m_em_X\mathcal{O}_4$
	$\mathcal{O}^{\tt A}_{\ell X4} = (\bar{\ell}\gamma^\mu\gamma_5\ell)(X^\dagger_\nu i\overleftrightarrow{\partial_\mu}X^\nu)$	$8m_em_X\mathcal{O}_7$
	$\mathcal{O}^{\tt A}_{\ell X5} = (\bar{\ell}\gamma_\mu\gamma_5\ell)i\partial_\nu(X^{\mu\dagger}X^\nu - X^{\nu\dagger}X^\mu),\ (\times)$	$4m_e^2\mathcal{O}_9$
	$\mathcal{O}^{\tt A}_{\ell X6} = (\bar{\ell}\gamma_\mu\gamma_5\ell)i\partial_\nu(X^\dagger_\rho X_\sigma)\epsilon^{\mu\nu\rho\sigma},\ (\times)$	$4m_e^2\left(\mathcal{O}_{14} - \frac{m_e}{m_X}\mathcal{O}_{20}\right)$
	$\mathcal{L}_{\kappa_\Lambda} = i\frac{\kappa_\Lambda}{2}(X^\dagger_\mu X_\nu - X^\dagger_\nu X_\mu)F^{\mu\nu}\,(\times)$	$-2e\kappa_\Lambda\left[\frac{m_e}{m_X}\left(\frac{1}{3}\mathcal{O}_1 - \frac{m_e^2}{\boldsymbol{q}^2}\mathcal{O}_{19}\right) - \mathcal{O}_4 - \frac{m_e^2}{\boldsymbol{q}^2}(\mathcal{O}_5 - \mathcal{O}_6)\right]$
	$\mathcal{L}_{\tilde{\kappa}_\Lambda} = i\frac{\tilde{\kappa}_\Lambda}{2}(X^\dagger_\mu X_\nu - X^\dagger_\nu X_\mu)\tilde{F}^{\mu\nu}\,(\times)$	$2e\tilde{\kappa}_\Lambda m_e^2\frac{1}{\boldsymbol{q}^2}\mathcal{O}_{11}$
dim-6	$\mathcal{O}_{X\gamma1} = \epsilon^{\mu\nu\rho\sigma}\left(X^\dagger_\rho\overleftrightarrow{\partial_\nu}X_\sigma\right)\partial^\lambda F_{\mu\lambda}$	$-4em_em_X(\mathcal{O}_8 - \mathcal{O}_9)$
	$\mathcal{O}_{X\gamma2} = \epsilon^{\mu\nu\rho\sigma}i\partial_\nu\left(X^\dagger_\rho X_\sigma\right)\partial^\lambda F_{\mu\lambda}\,(\times)$	$-2em_e^2\mathcal{O}_{11}$
	$\mathcal{O}_{X\gamma3} = \left(X^\dagger_\nu i\overleftrightarrow{\partial^\mu}X^\nu\right)\partial^\lambda F_{\mu\lambda}$	$-4em_em_X\mathcal{O}_1$
	$\mathcal{O}_{X\gamma4} = \partial_\nu(X^{\mu\dagger}X^\nu + X^{\nu\dagger}X^\mu)\partial^\lambda F_{\mu\lambda}$	$4e\,m_e\left[\frac{1}{3}(i\boldsymbol{q}\cdot\boldsymbol{v}^\perp_{\rm el})\mathcal{O}_1 - m_e(\mathcal{O}_{17} + \mathcal{O}_{20})\right]$
	$\mathcal{O}_{X\gamma5} = i\partial_\nu(X^{\mu\dagger}X^\nu - X^{\nu\dagger}X^\mu)\partial^\lambda F_{\mu\lambda}\,(\times)$	$e\left[2m_e^2\left(\mathcal{O}_5 - \mathcal{O}_6 - \frac{m_e}{m_X}\mathcal{O}_{19}\right) + 2\boldsymbol{q}^2\mathcal{O}_4 + \frac{2}{3}\frac{m_e}{m_X}\boldsymbol{q}^2\mathcal{O}_1\right]$
	Vector case B	
dim-7	$\tilde{\mathcal{O}}^{\tt S}_{\ell X1} = (\bar{\ell}\ell)X^\dagger_{\mu\nu}X^{\mu\nu}$	$4m_em_X^2\mathcal{O}_1$
	$\tilde{\mathcal{O}}^{\tt S}_{\ell X2} = (\bar{\ell}\ell)X^\dagger_{\mu\nu}\tilde{X}^{\mu\nu}$	$4m_e^2m_X\mathcal{O}_{11}$
	$\tilde{\mathcal{O}}^{\tt P}_{\ell X1} = (\bar{\ell}i\gamma_5\ell)X^\dagger_{\mu\nu}X^{\mu\nu}$	$-4m_em_X^2\mathcal{O}_{10}$
	$\tilde{\mathcal{O}}^{\tt P}_{\ell X2} = (\bar{\ell}i\gamma_5\ell)X^\dagger_{\mu\nu}\tilde{X}^{\mu\nu}$	$4m_e^2m_X\mathcal{O}_6$
	$\tilde{\mathcal{O}}^{\tt T}_{\ell X1} = \frac{i}{2}(\bar{\ell}\sigma^{\mu\nu}\ell)(X^\dagger_{\mu\rho}X^\rho_\nu - X^\dagger_{\nu\rho}X^\rho_\mu),\ (\times)$	$4m_em_X^2\mathcal{O}_4$
	$\tilde{\mathcal{O}}^{\tt T}_{\ell X2} = \frac{1}{2}(\bar{\ell}\sigma^{\mu\nu}\gamma_5\ell)(X^\dagger_{\mu\rho}X^\rho_\nu - X^\dagger_{\nu\rho}X^\rho_\mu),\ (\times)$	$\frac{1}{3}m_em_X[3m_X(\mathcal{O}_{11} + 4\mathcal{O}_{12}) - 4m_e(2\mathcal{O}_{10} + 3\mathcal{O}_{18})]$
dim-6	$\tilde{\mathcal{O}}_{X\gamma1} = i(X^\dagger_{\mu\rho}X^\rho_\nu - X^\dagger_{\nu\rho}X^\rho_\mu)F^{\mu\nu}\,(\times)$	$2e\Big[\frac{2}{3}m_X(m_e + m_X)\mathcal{O}_1 + 2m_X^2\mathcal{O}_4 + \frac{1}{\boldsymbol{q}^2}\left(2m_e^2m_X^2(\mathcal{O}_5 - \mathcal{O}_6) - 2m_e^2m_X(m_X - 2m_e)\mathcal{O}_{19}\right)\Big]$
	$\tilde{\mathcal{O}}_{X\gamma2} = i(X^\dagger_{\mu\rho}X^\rho_\nu - X^\dagger_{\nu\rho}X^\rho_\mu)\tilde{F}^{\mu\nu}\,(\times)$	$-4em_e^2m_X^2\frac{1}{\boldsymbol{q}^2}\mathcal{O}_{11}$

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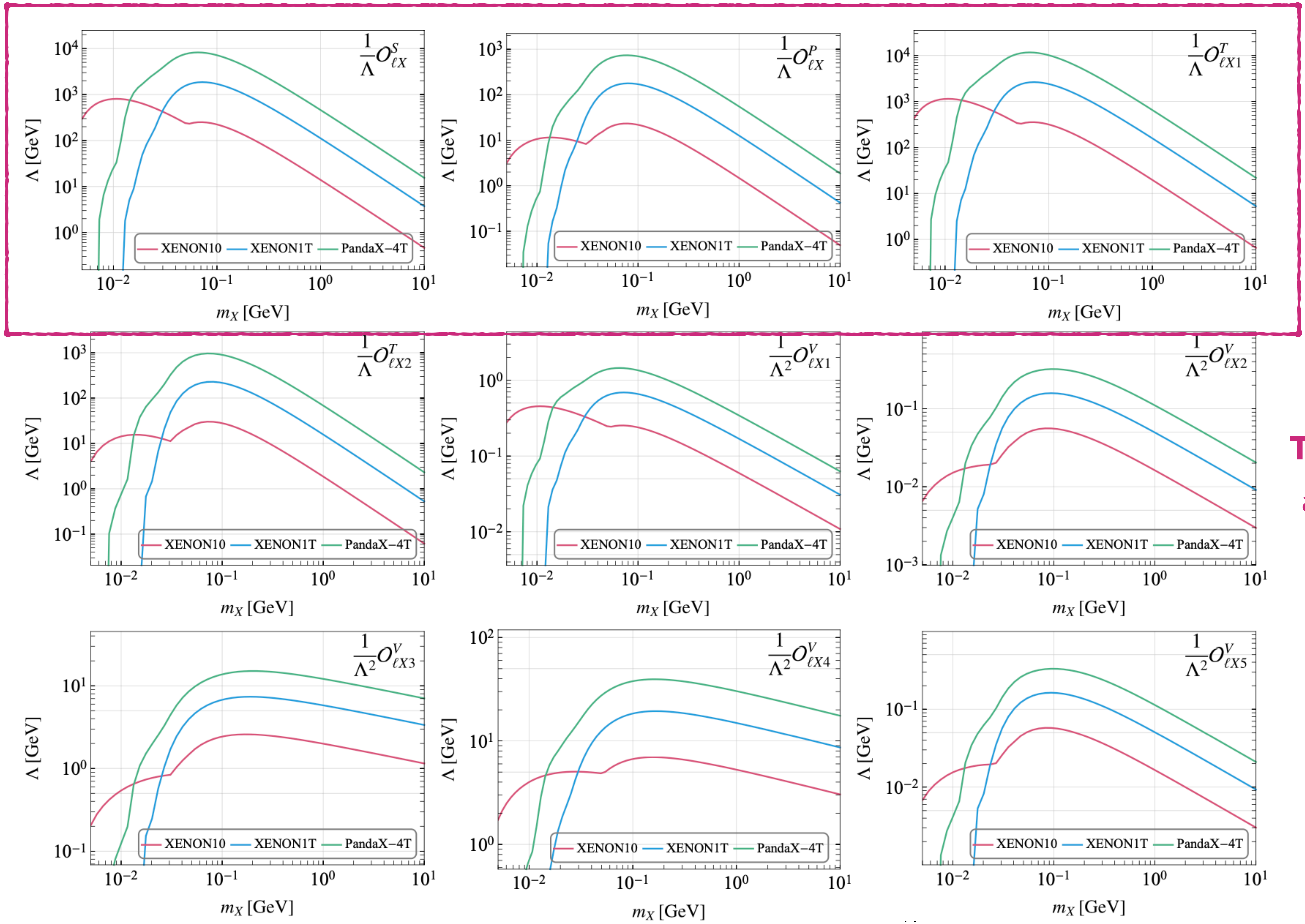
Constraints on the relativistic operators

– Vector DM case



$\mathcal{O}^{\mathrm{S}}_{\ell X} = (\bar{\ell}\ell)(X^{\dagger}_{\mu}X^{\mu})$

$\mathcal{O}^{\mathrm{P}}_{\ell X} = (\bar{\ell}i\gamma_5\ell)(X^{\dagger}_{\mu}X^{\mu})$

$\mathcal{O}^{\mathrm{T}}_{\ell X1} = \frac{i}{2}(\bar{\ell}\sigma^{\mu\nu}\ell)(X^{\dagger}_{\mu}X_{\nu} - X^{\dagger}_{\nu}X_{\mu})$, $(\times)$

The PandaX-4T constraints on $\mathcal{O}^{S}_{\ell X}$ and $\mathcal{O}^{T}_{\ell X1}$ can up to about 10 TeV

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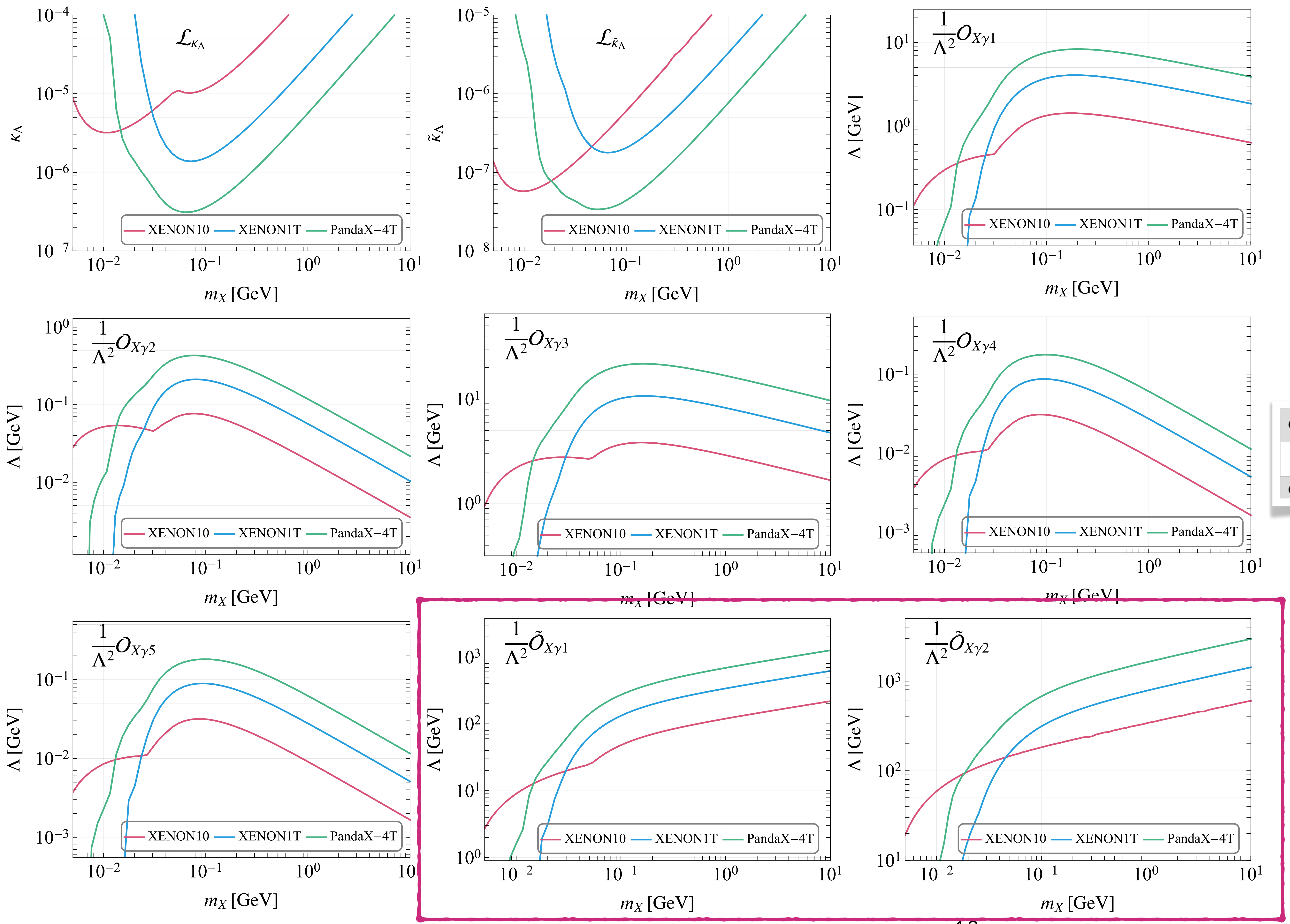
Constraints on the relativistic operators

– Vector DM case



$\tilde{\mathcal{O}}_{X\gamma1} = i(X^\dagger_{\mu\rho}X^\rho_\nu - X^\dagger_{\nu\rho}X^\rho_\mu)F^{\mu\nu}\ (\times)$	$2e\Big[\frac{2}{3}m_X(m_e+m_X)\mathcal{O}_1 + 2m_X^2\mathcal{O}_4$
	$+\frac{1}{q^2}\left(2m_e^2m_X^2(\mathcal{O}_5-\mathcal{O}_6) - 2m_e^2m_X(m_X-2m_e)\mathcal{O}_{19}\right)\Big]$
$\tilde{\mathcal{O}}_{X\gamma2} = i(X^\dagger_{\mu\rho}X^\rho_\nu - X^\dagger_{\nu\rho}X^\rho_\mu)\tilde{F}^{\mu\nu}\ (\times)$	$-4em_e^2m_X^2\frac{1}{q^2}\mathcal{O}_{11}$

Become stronger as m_X increases

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Summary

- We find a crucial minus sign was missed for W_2 in 1912.08204, which has significant phenomenological consequences on some specific DM scenarios.
- A more compact amplitude squared is provided for the general DM-electron interactions for three DM scenarios.
- A matching dictionary between the relativistic and NR operators is given.
- The constraints from the xenon target experiments were studied, and we find the PandaX-4T set the most stringent constraints on the effective operators when $m_{\rm DM} \gtrsim 20\ \rm MeV$.

Thank You!