中國科學院高能物理研究所
Institute of High Energy Physics Chinese Academy of Sciences

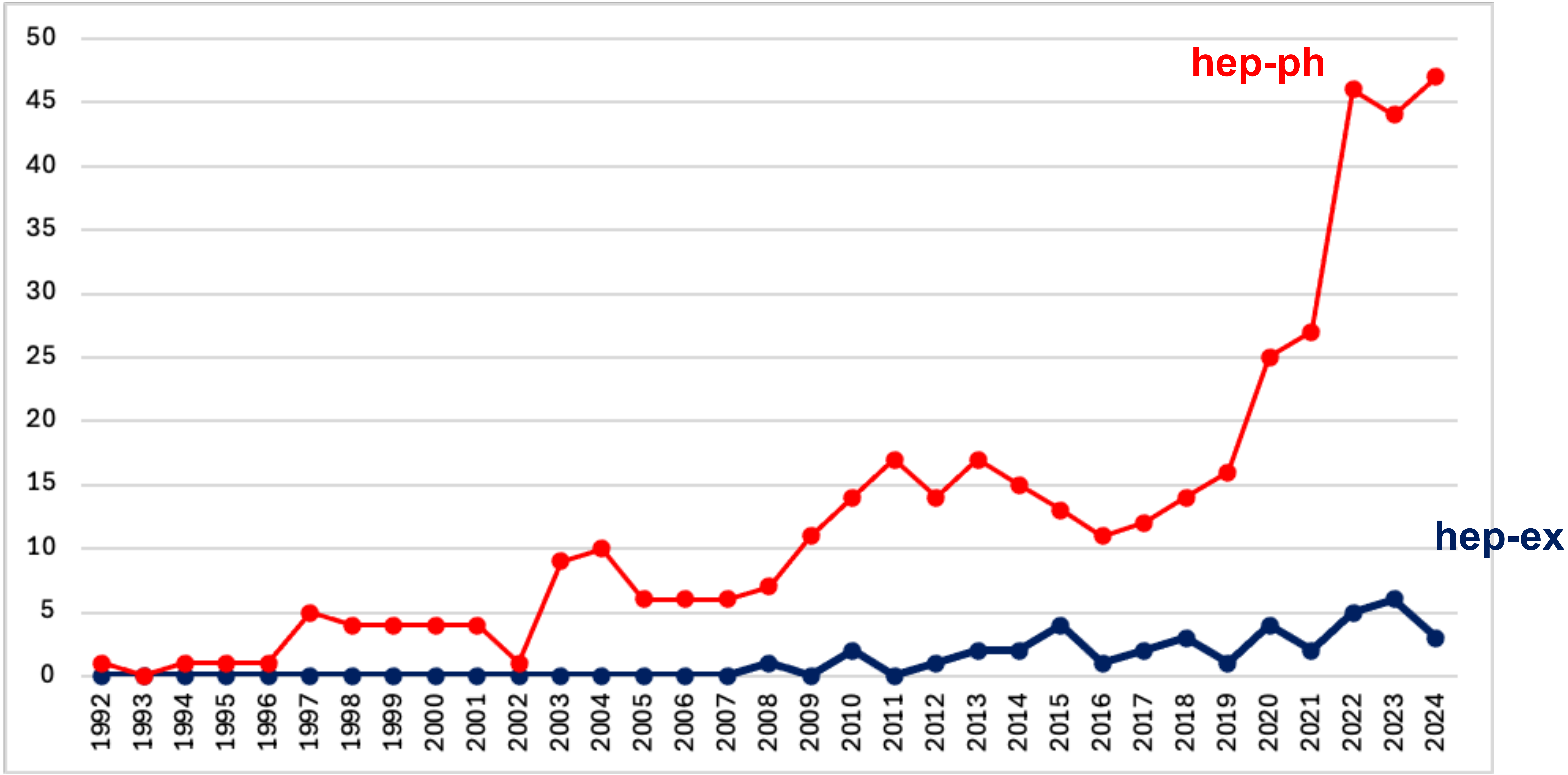
From Quantum Entanglement to Quantum Reality

— Testing Bell inequalities in W boson pair production

Based on Phys. Rev. D **109**, 036022 in collaboration with Qi Bi, Kun Cheng and Qing-Hong Cao

Hao Zhang

Theoretical Physics Division, Institute of High Energy Physics, Chinese Academy of Sciences
For “中国物理学会高能物理分会第十四届全国粒子物理学术会议”, Aug 15th, 2024, Qingdao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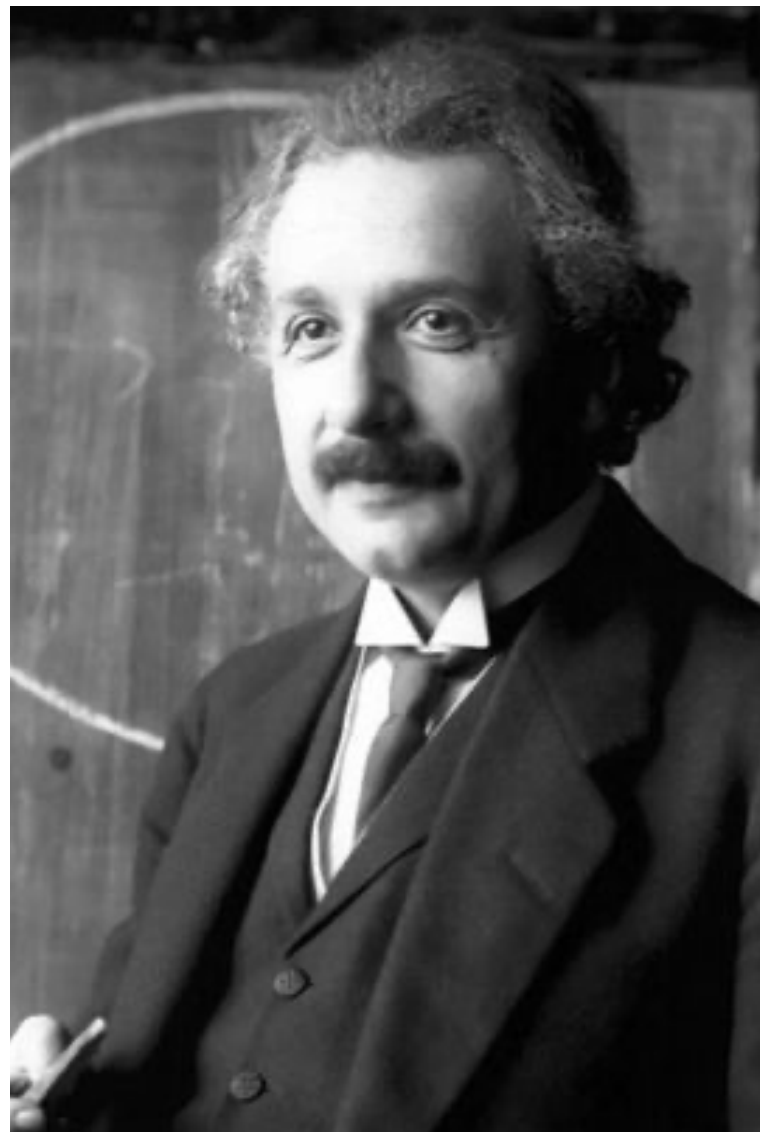
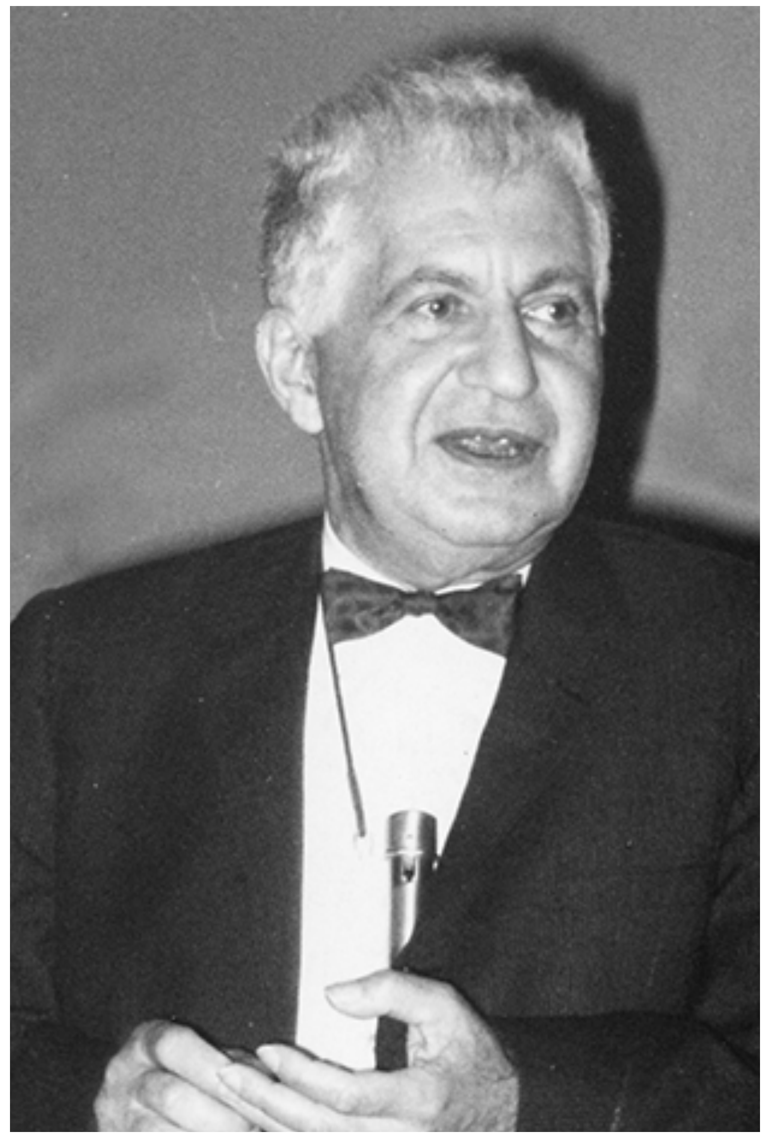
Quantum Reality



Albert Einstein

(1879/03/14-1955/04/18)

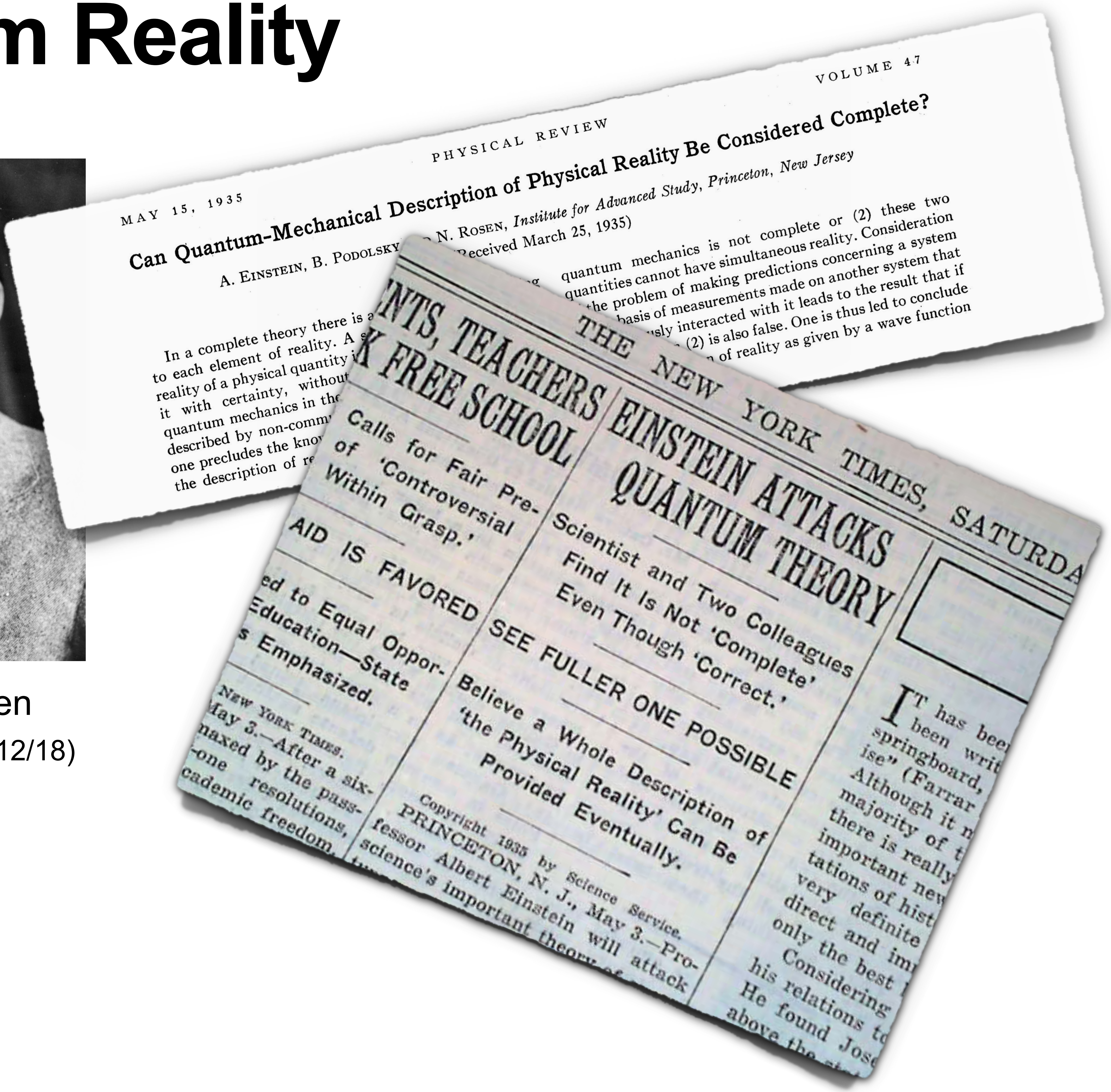


Boris Yakovlevich Podolsky

(1896/06/29-1966/11/28)

Nathan Rosen

(1909/03/22-1995/12/18)



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY … N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

…eceived March 25, 1935)

In a complete theory there is a
to each element of reality. A s
reality of a physical quantity i
it with certainty, without
quantum mechanics in the
described by non-comm
one precludes the know
the description of re

quantum mechanics is not complete or (2) these two
quantities cannot have simultaneous reality. Consideration
the problem of making predictions concerning a system
basis of measurements made on another system that
usly interacted with it leads to the result that if
(2) is also false. One is thus led to conclude
n of reality as given by a wave function

THE NEW YORK TIMES, SATURDA

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K FREE SCHOOL

Calls for Fair Pre-
of 'Controversial
Within Grasp.'

AID IS FAVORED

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Education—State
s Emphasized.

NEW YORK TIMES.
May 3.—After a six-
naxed by the pass-
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EINSTEIN ATTACKS
QUANTUM THEORY

Scientist and Two Colleagues
Find It Is Not 'Complete'
Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of
'the Physical Reality' Can Be
Provided Eventually.

Copyright 1935 by Science Service.
PRINCETON, N. J., May 3.—Pro-
fessor Albert Einstein will attack
science's important theory of

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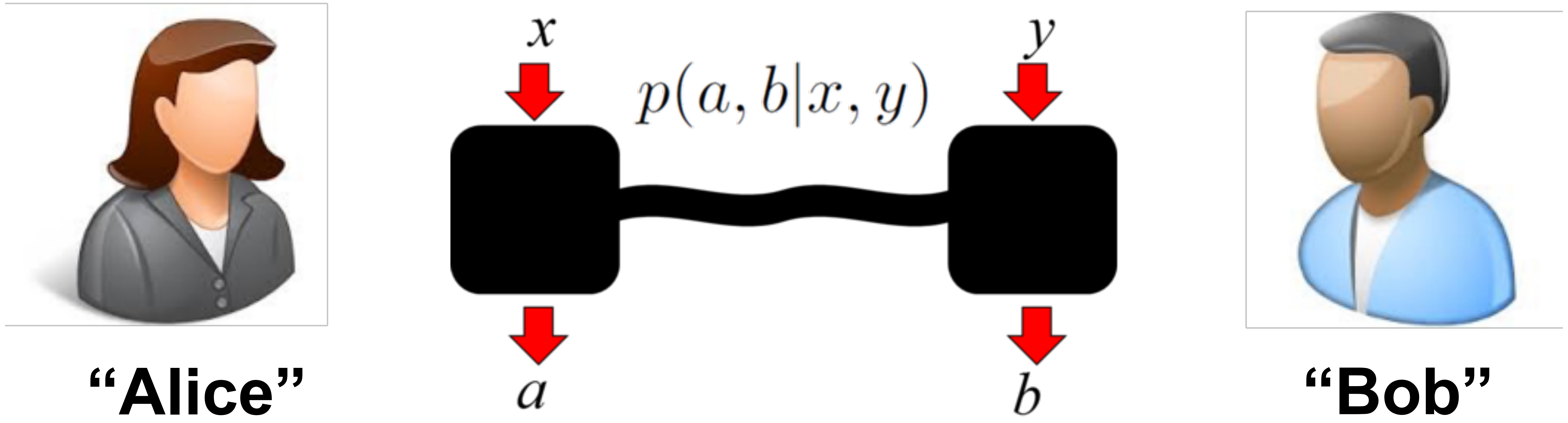
The different correlations

- A question of decomposition



Local correlations

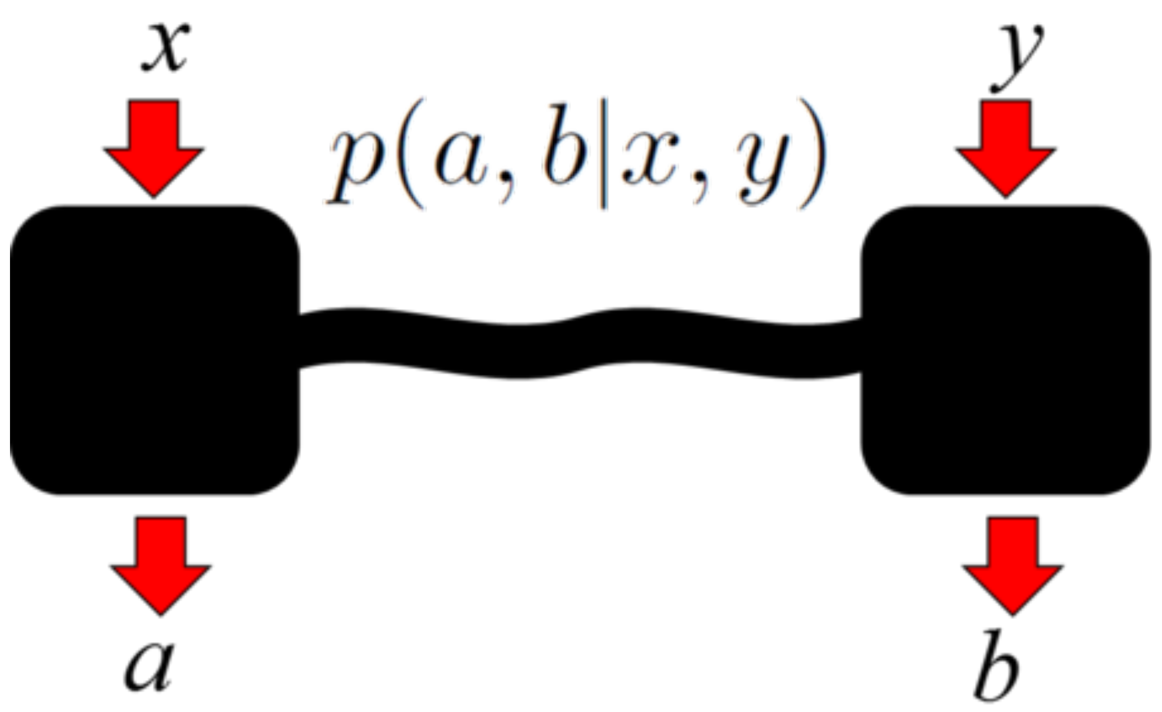
Quantum correlations

No-signaling correlations

N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Rev. Mod. Phys. 86 (2014) 419

Local correlations

“Alice”



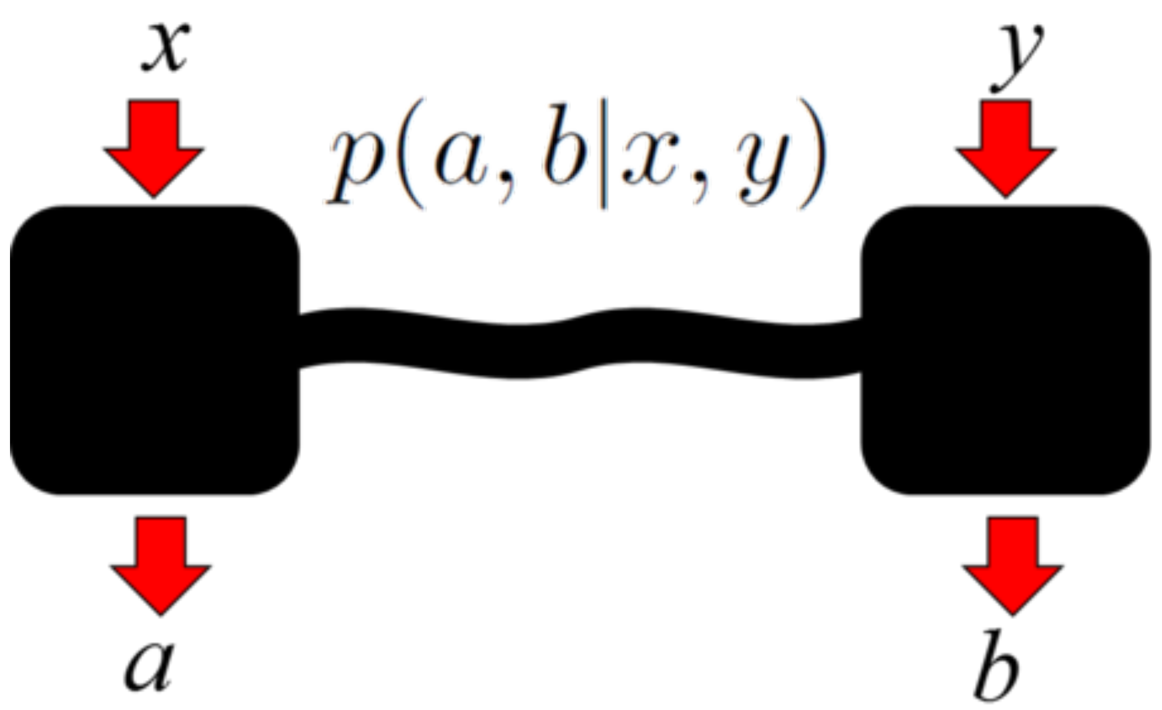
“Bob”

$$p(ab|xy) = \int_\Lambda d\lambda q(\lambda) p(a|x,\lambda) p(b|y,\lambda),$$

N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Rev. Mod. Phys. 86 (2014) 419

Local correlations

“Alice”



“Bob”

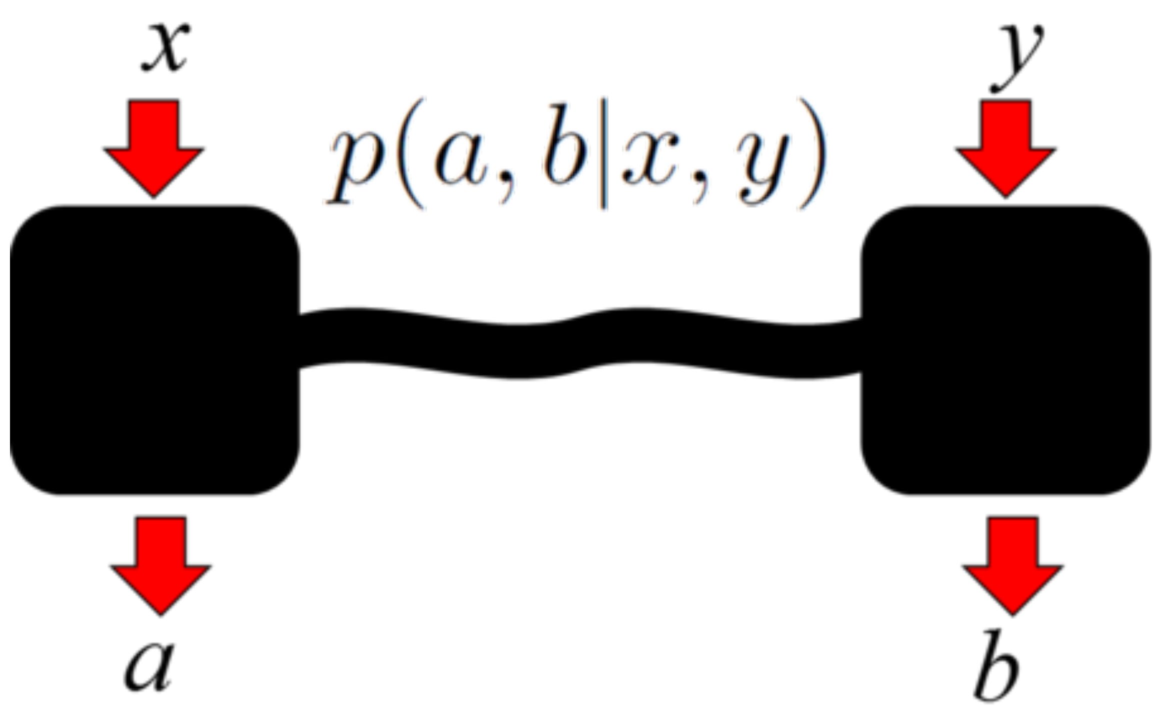
$$p(ab|xy) = \int_{\Lambda} d\lambda q(\lambda) p(a|x,\lambda) p(b|y,\lambda),$$

λ-“Hidden variable(s)”

N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Rev. Mod. Phys. 86 (2014) 419

Quantum correlations-I

“Alice”



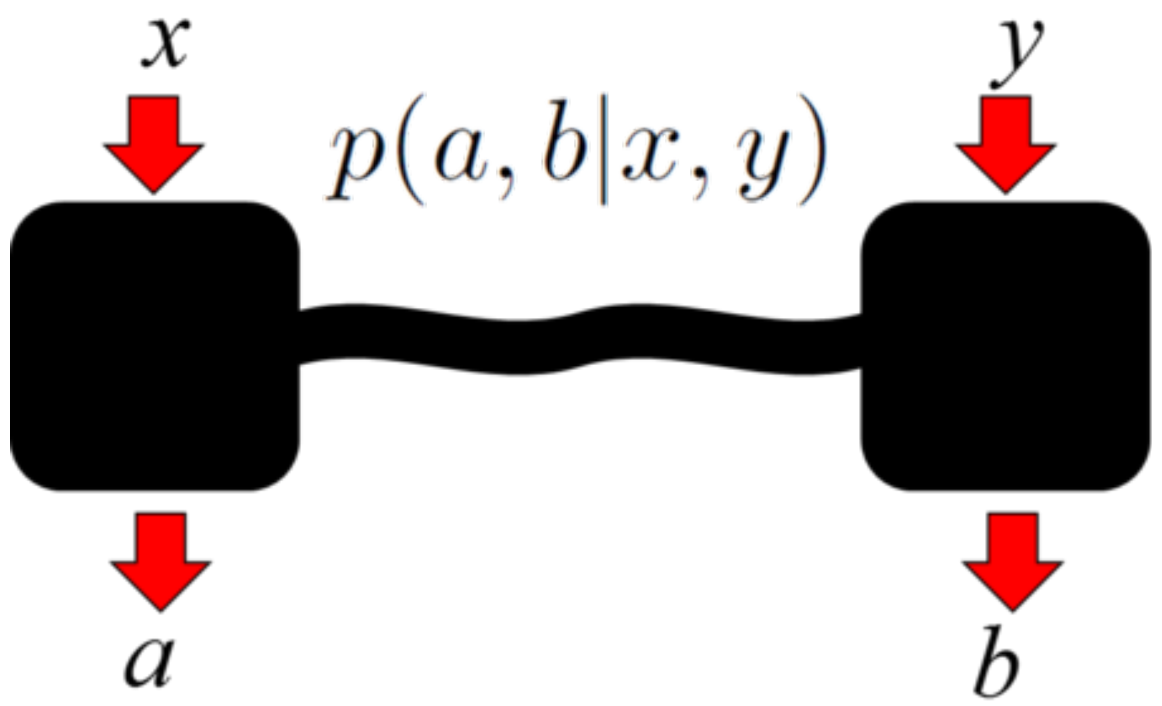
“Bob”

$$p(ab|xy) = \mathrm{tr}\left(\rho_{AB} M_{a|x} \otimes M_{b|y}\right),$$

N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Rev. Mod. Phys. 86 (2014) 419

Quantum correlations-II

“Alice”



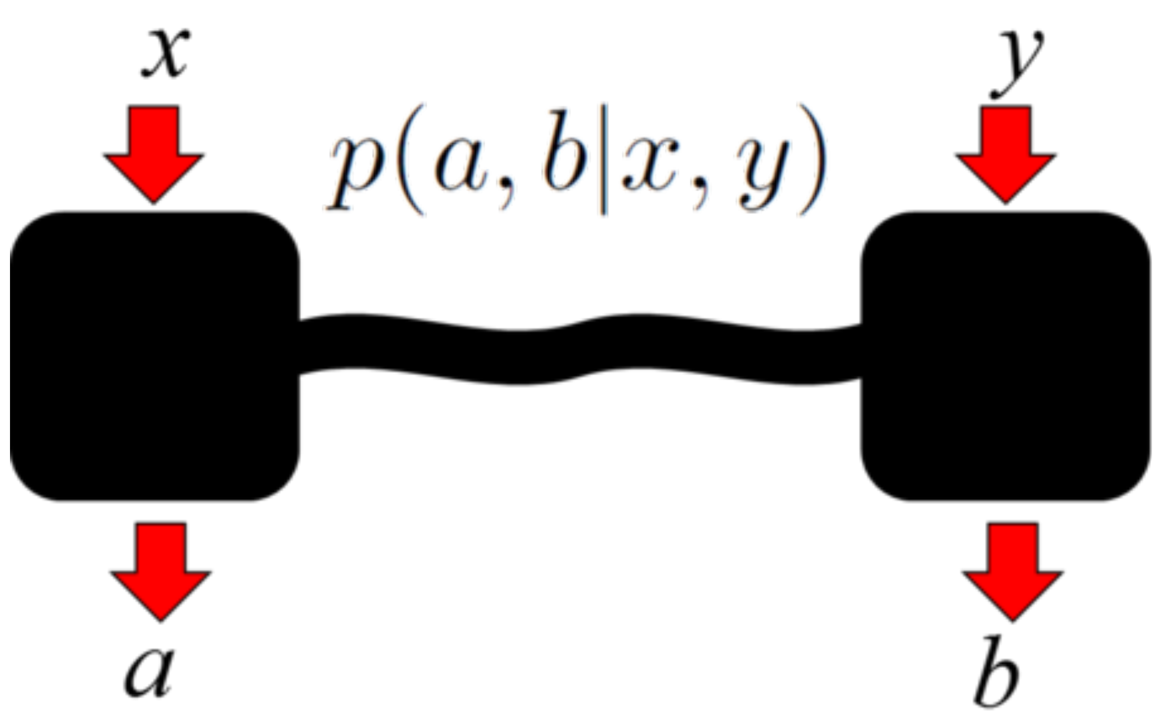
“Bob”

$$p(ab\,|\,xy) = \rho_{AB}(M_{a|x}M_{b|y}),\ [M_{a|x}, M_{b|y}] = 0$$

N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Rev. Mod. Phys. 86 (2014) 419

No-signaling correlations

“Alice”



“Bob”

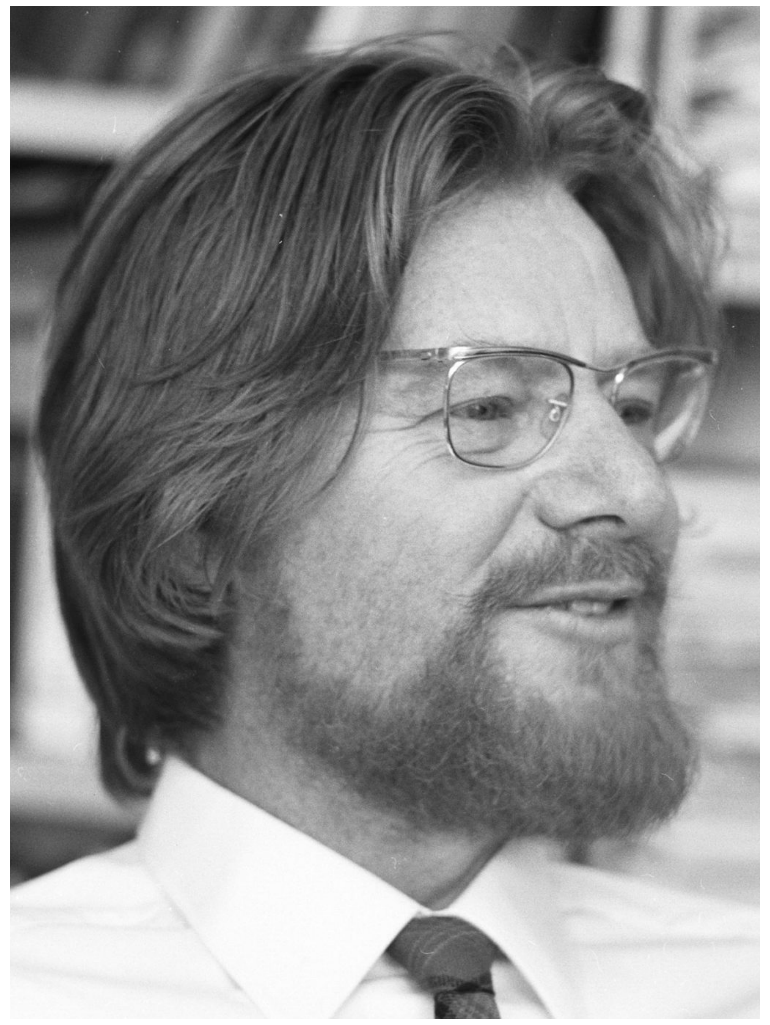
$$\sum_{b=1}^{\Delta} p(ab|xy) = \sum_{b=1}^{\Delta} p(ab|xy'), \quad \text{for all } \ a, x, y, y',$$

$$\sum_{a=1}^{\Delta} p(ab|xy) = \sum_{a=1}^{\Delta} p(ab|x'y), \quad \text{for all } \ b, y, x, x'.$$

N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Rev. Mod. Phys. 86 (2014) 419

The different correlations

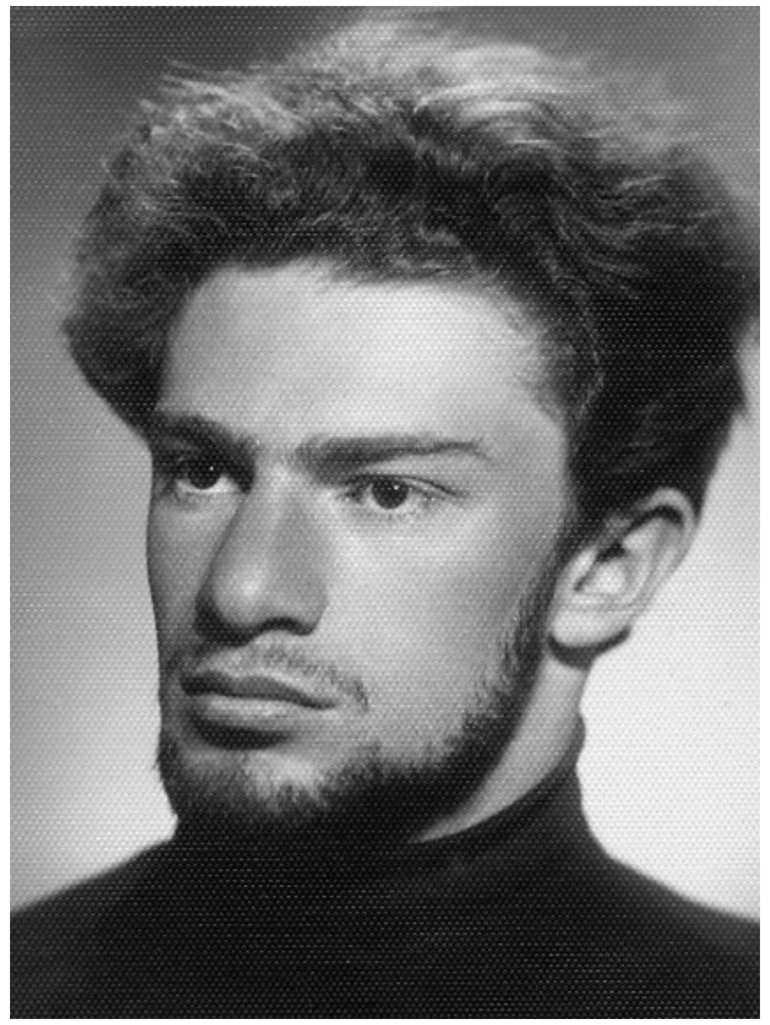
Local correlations $\subset$ **Quantum correlations** $\subset$ **No-signaling correlations**



John Stewart Bell
(1928/07/28-1990/10/01)

Bell inequality

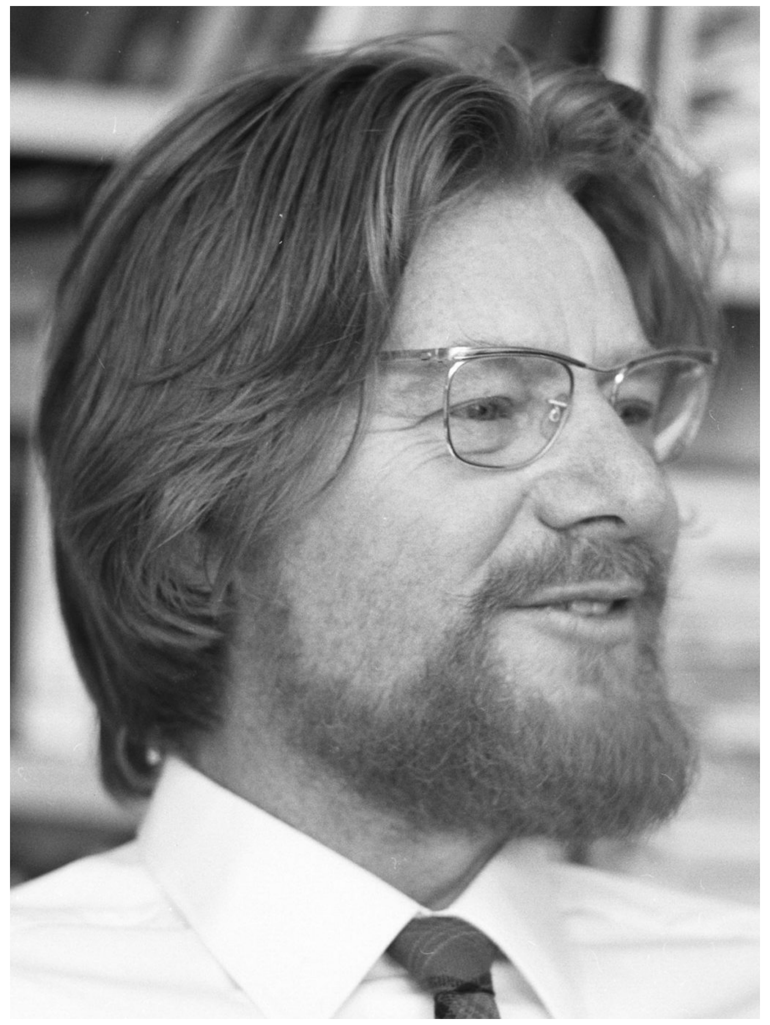
Tsirelson's bound



Boris Semyonovich Tsirelson
(1950/05/04-2020/01/21)

The different correlations

Local correlations $\subset$ **Quantum correlations** $\subset$ **No-signaling correlations**



John Stewart Bell
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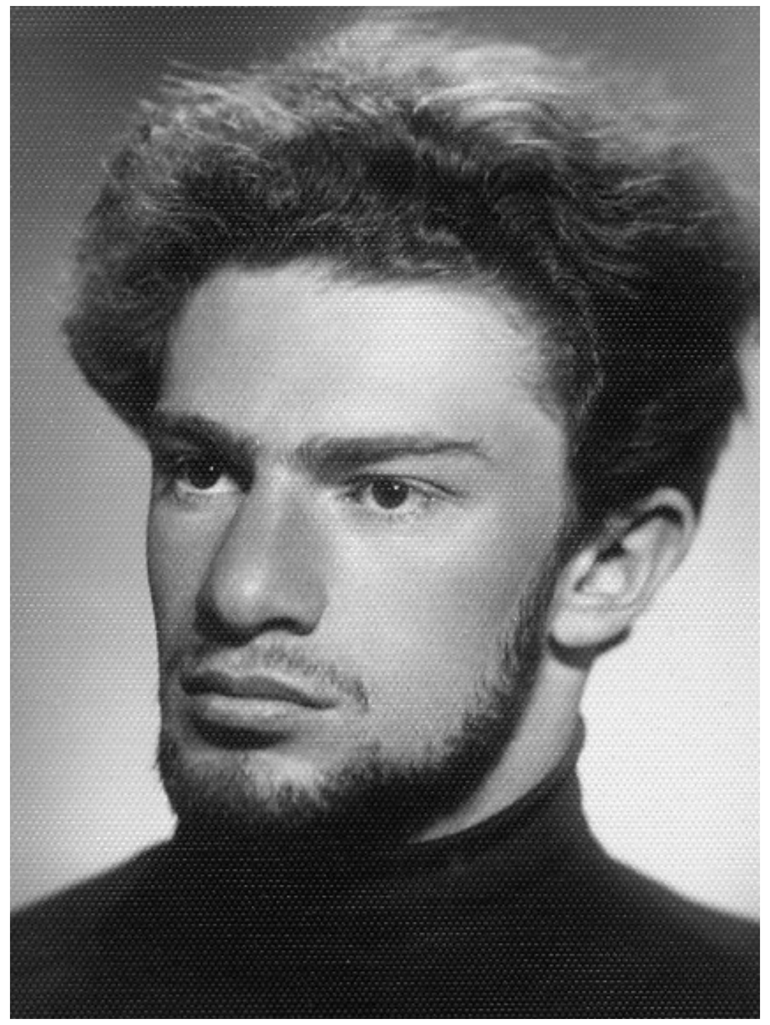
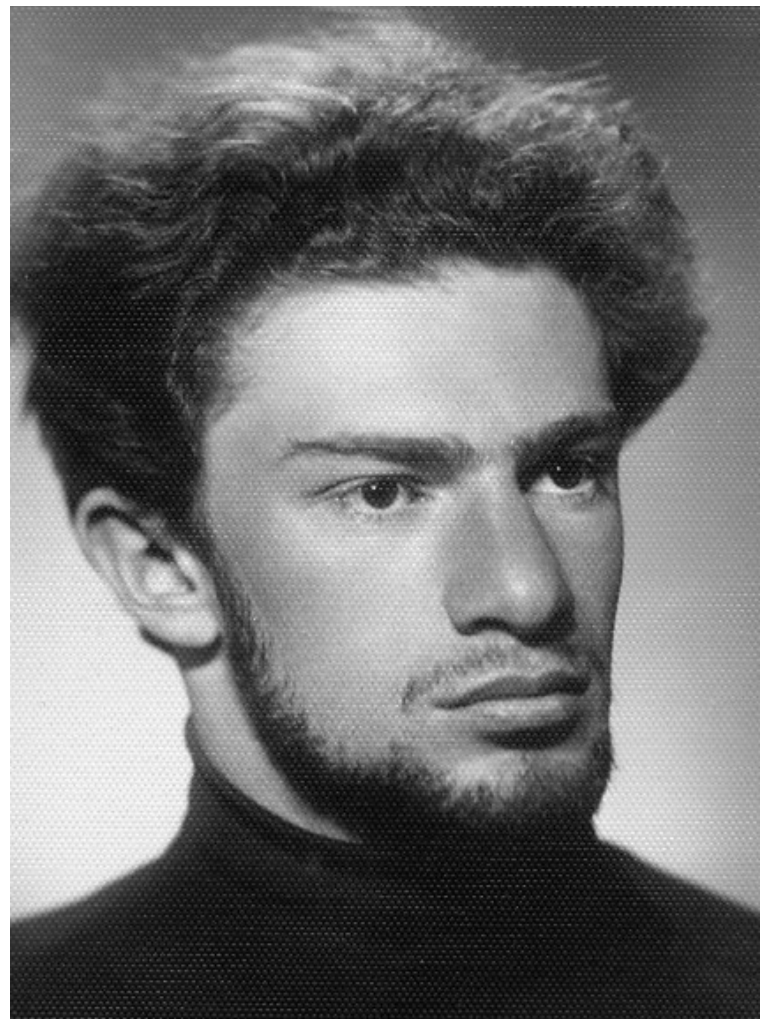
Tsirelson's bound



Boris Semyonovich Tsirelson
(1950/05/04-2020/01/21)

Quantum correlations-I ? **Quantum correlations-II**

The different correlations



Tsirelson’s Problem

(Connes’ embedding problem, Kirchberg’s Conjecture)

Борис Семёнович Цирельсон
בוריס סמיונוביץ' צירלסון
(Boris Semyonovich Cirelson, before 1983
Boris Semyonovich Tsirelson, after 1983)

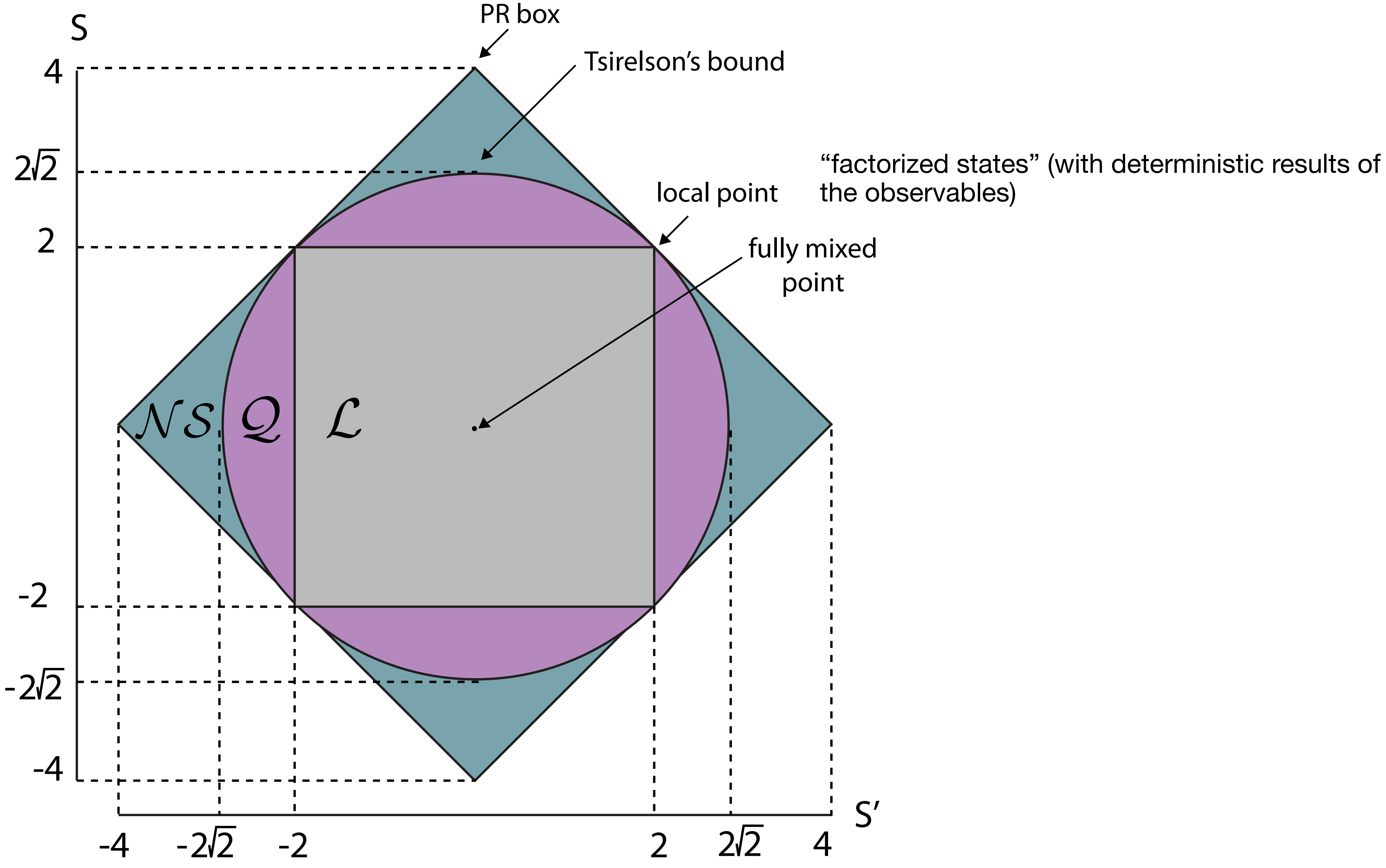
(1950/05/04-2020/01/21)

Local correlations ⊂ **Quantum correlations-I** ⊂ **Quantum correlations-II** ⊂ **No-signaling correlations**

A. Connes, “***Classification of injective factors cases*** II_1, II_∞, III_λ, $\lambda \neq 1$”, Ann. Math. 104 (1976) 73-155;
B. Tsirelson, “***Bell inequalities and operator algebras***”, https://www.tau.ac.il/~tsirel/download/bellopalg.pdf;
W. Slofstra, “***The Set of Quantum Correlations is not Closed***”, Forum of Mathematics, Pi. 2019;7:e1;
Z. Ji, A. Natarajan, T. Vidick, J. Wright, and H. Yuen, “MIP*=RE”, arXiv:2001.04838[quant-ph];
Z. Ji, A. Natarajan, T. Vidick, J. Wright, and H. Yuen, “MIP*=RE”, Commun. ACM 64 (2021) 131.

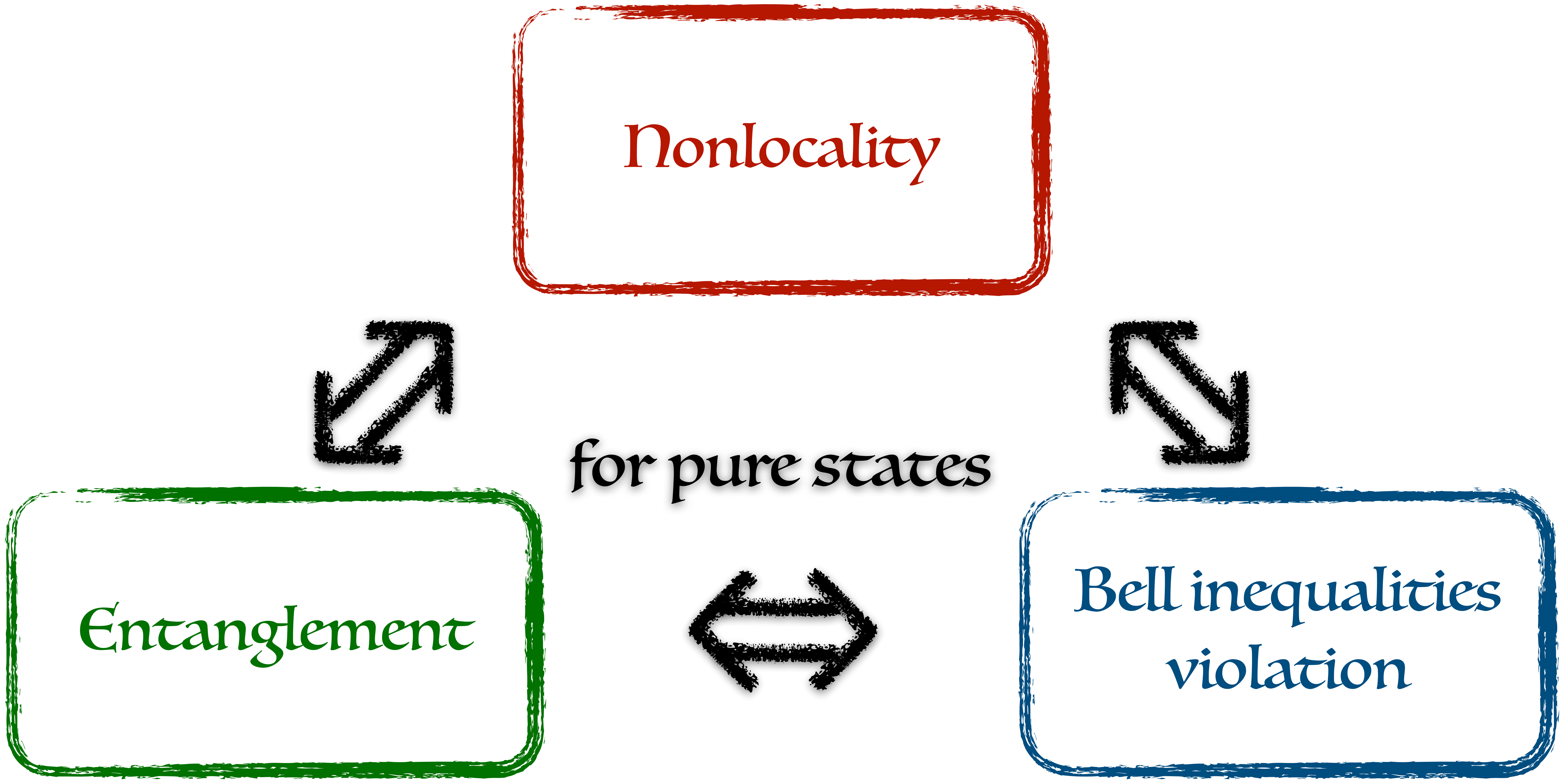
The different correlations

A visualization of the relation between these three kinds of correlation



N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, Rev. Mod. Phys. 86 (2014) 419

The different correlations



Nonlocality

for pure states

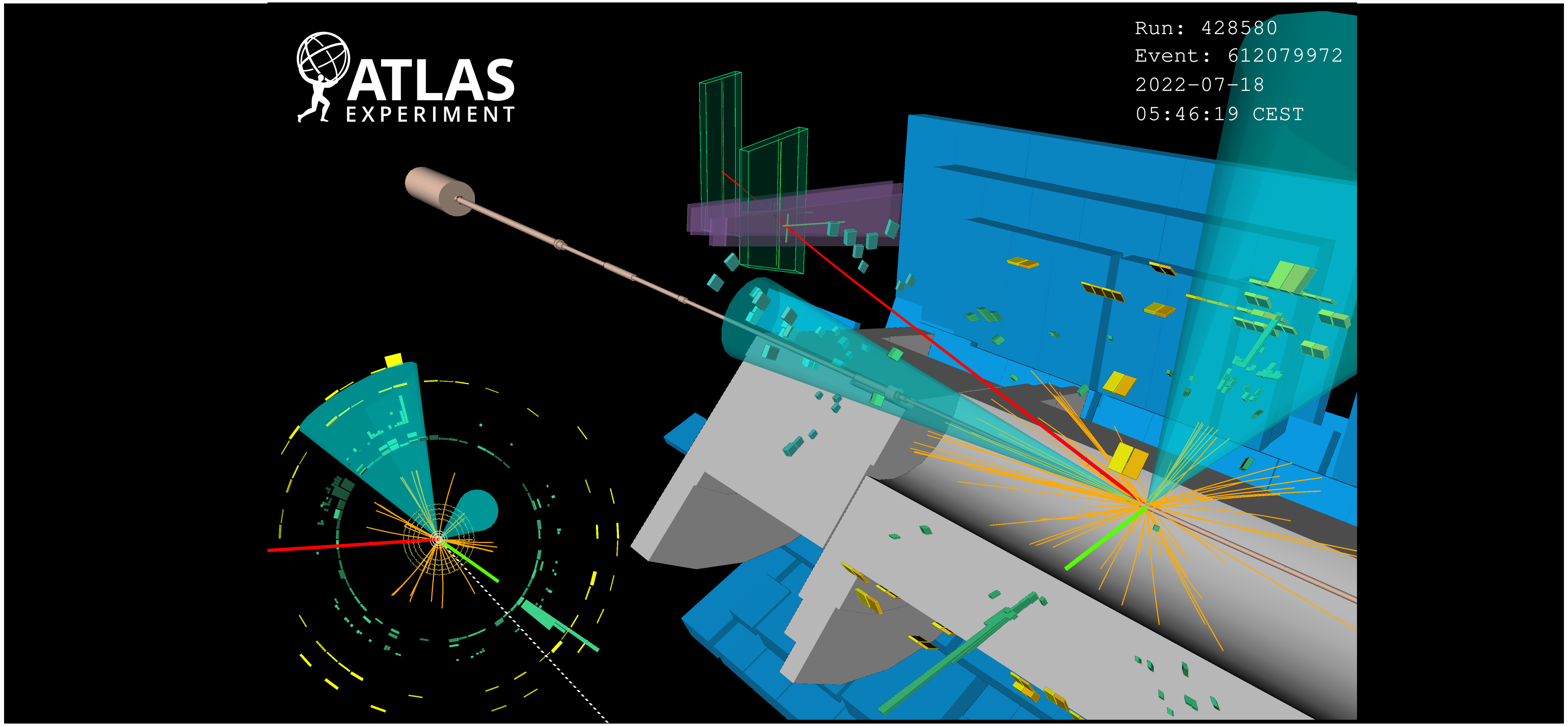
Entanglement

Bell inequalities violation

The Verification in EW scale

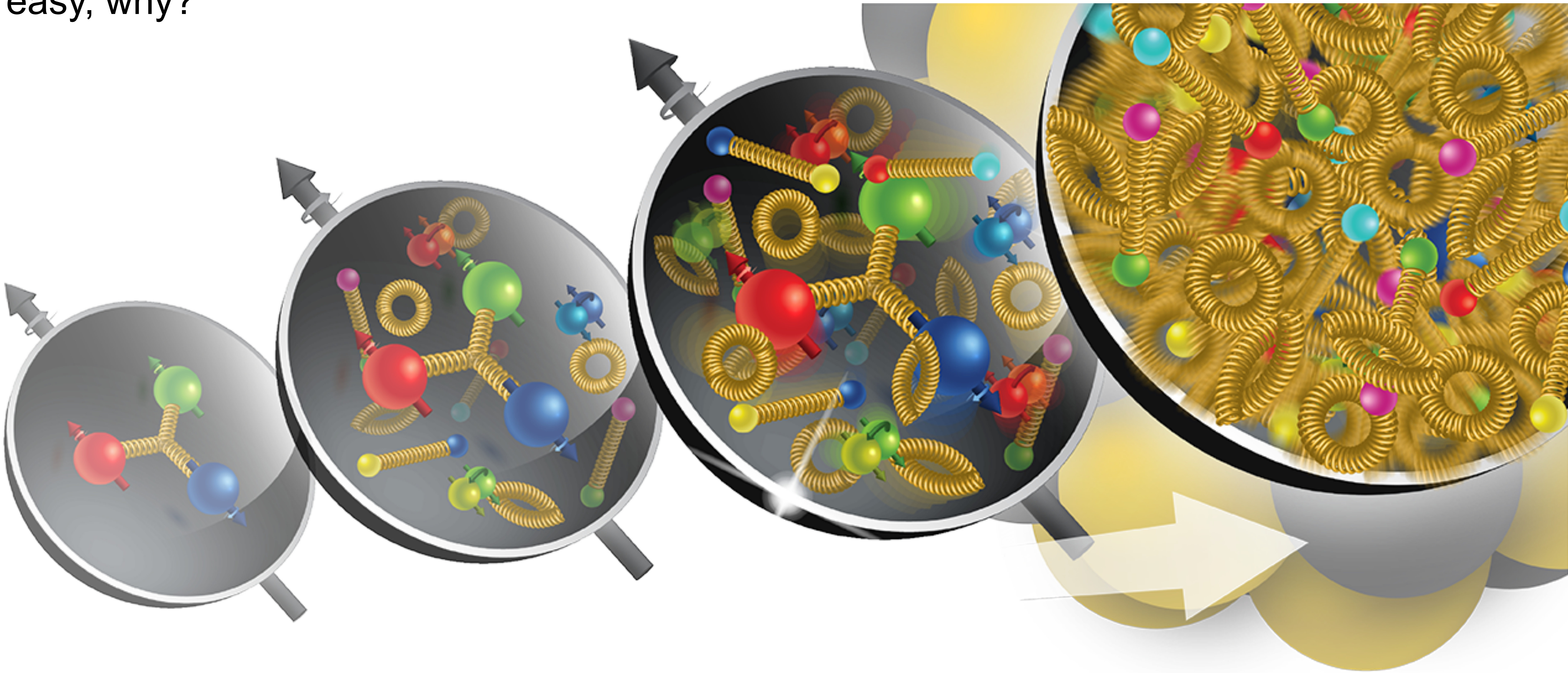
- The most popular topic: $t\bar{t}$ production at the LHC.

- Why?



The Verification at the EW scale

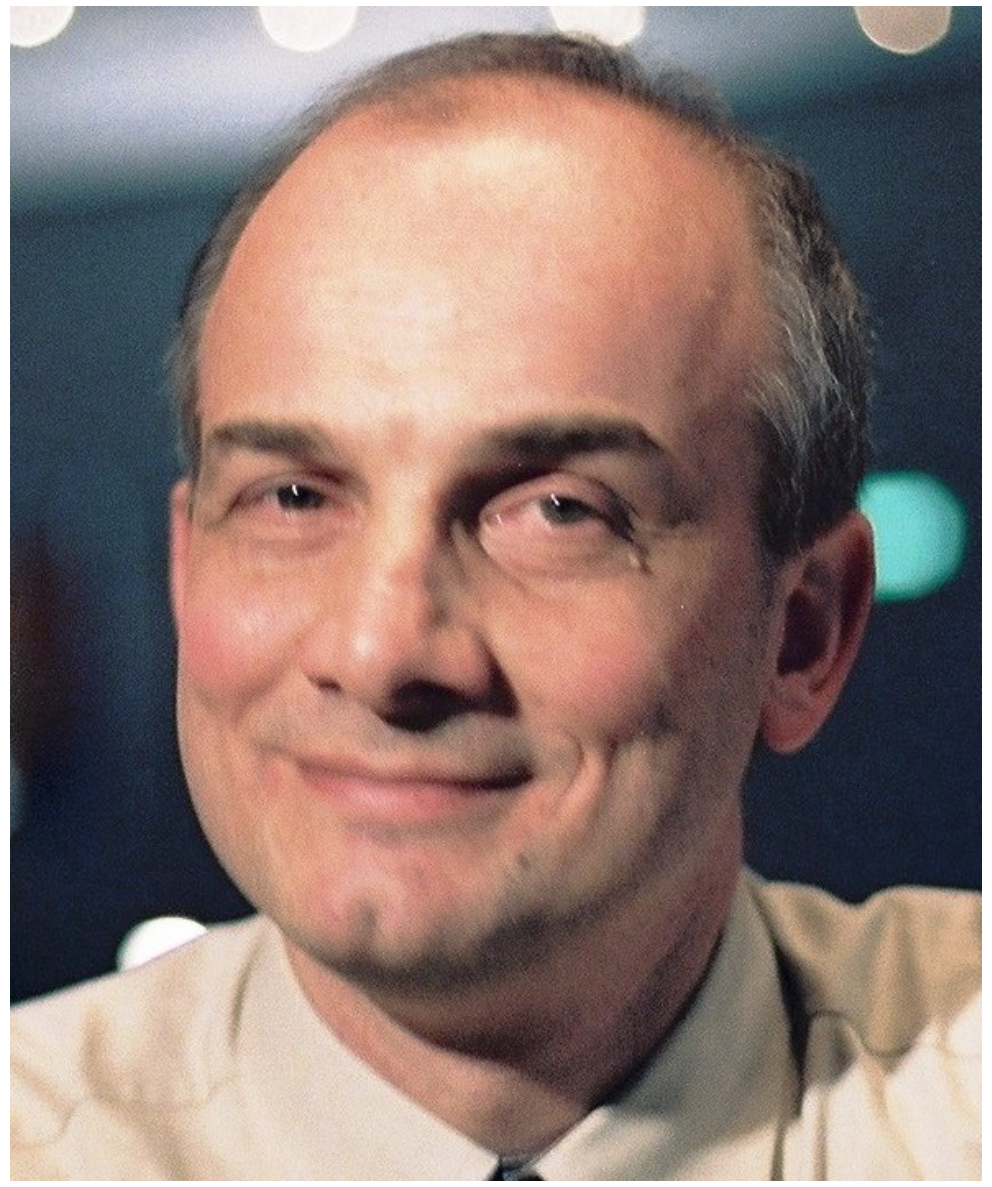
- It is not easy, why?



The Verification at the EW scale

"A quantitatively characterization of the degree of the entanglement between the subsystems of a system in a mixed state, is not unique! "

$$\rho_{AB} \overset{?}{=} \sum_{i=1}^{N} p_i \rho_A^{(i)} \otimes \rho_B^{(i)}, \quad \left(\sum_{i}^{N} p_i = 1,\ p_i > 0 \right)$$



"Finally, we prove that the weak membership problem for the convex set of separable normalized bipartite density matrices is ***NP-HARD***. "

——Leonid Gurvits

L. Gurvits, *"Classical Deterministic Complexity of Edmonds' Problem and Quantum Entanglement"*, 2003

The Verification at the EW scale

- For 2×2 and 2×3 system, it is solved by Peres, and Horodeckis 1996 (Peres-Horodecki criterion, concurrence).

Asher Peres

(1934/01/30-2005/01/01)

Ryszard Horodecki

(1943/09/30-)

Paweł Horodecki

(1971-)

Michał Horodecki

(1973-)

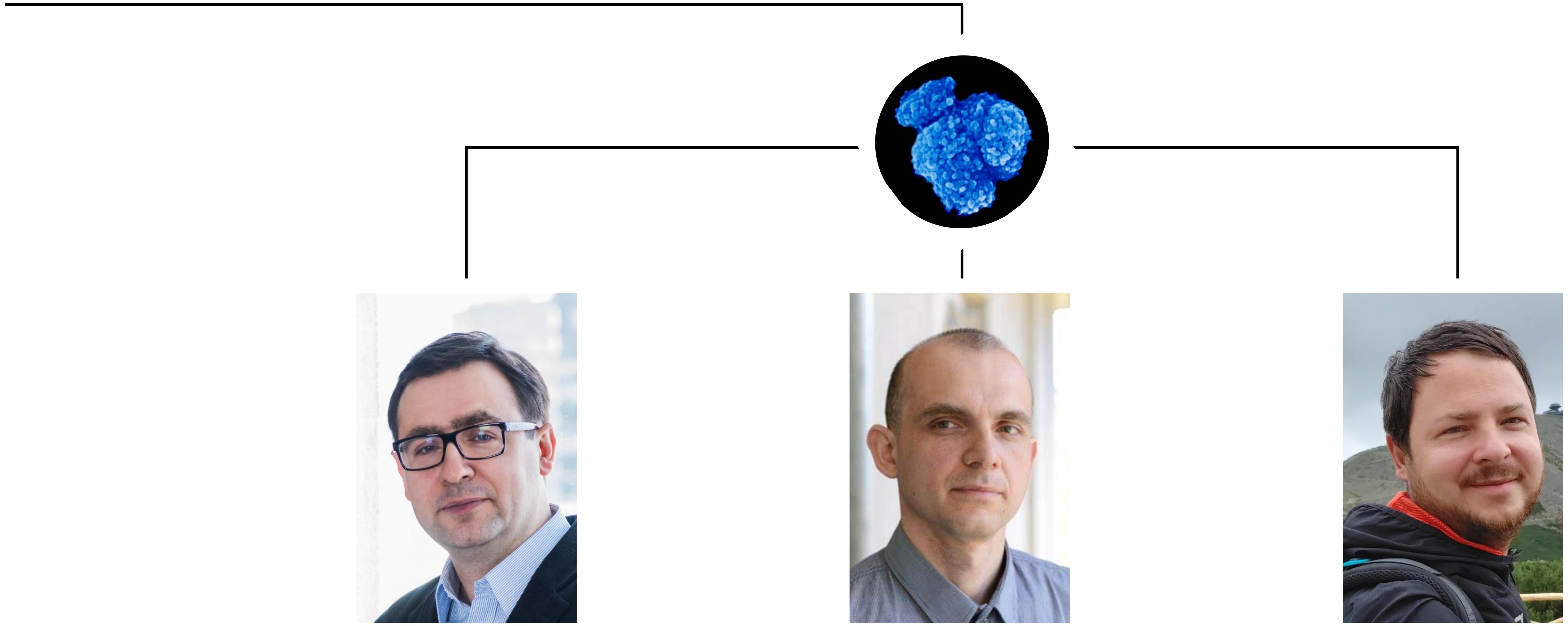
A. Peres, “Separability Criterion for Density Matrices”, Phys. Rev. Lett. 77 (1996) 1413;Michał Horodecki, Paweł Horodecki, Ryszard Horodecki, “Separability of mixed states: necessary and sufficient conditions”, Phys. Lett. A 223 (1996) 1.

The Verification at the EW scale

- For 2×2 and 2×3 system, it is solved by Peres, and Horodeckis 1996 (Peres-Horodecki criterion, concurrence).

Ryszard ***Horodecki***

(1943/09/30-)



Paweł ***Horodecki***

(1971-)

Michał ***Horodecki***

(1973-)

Karol ***Horodecki***

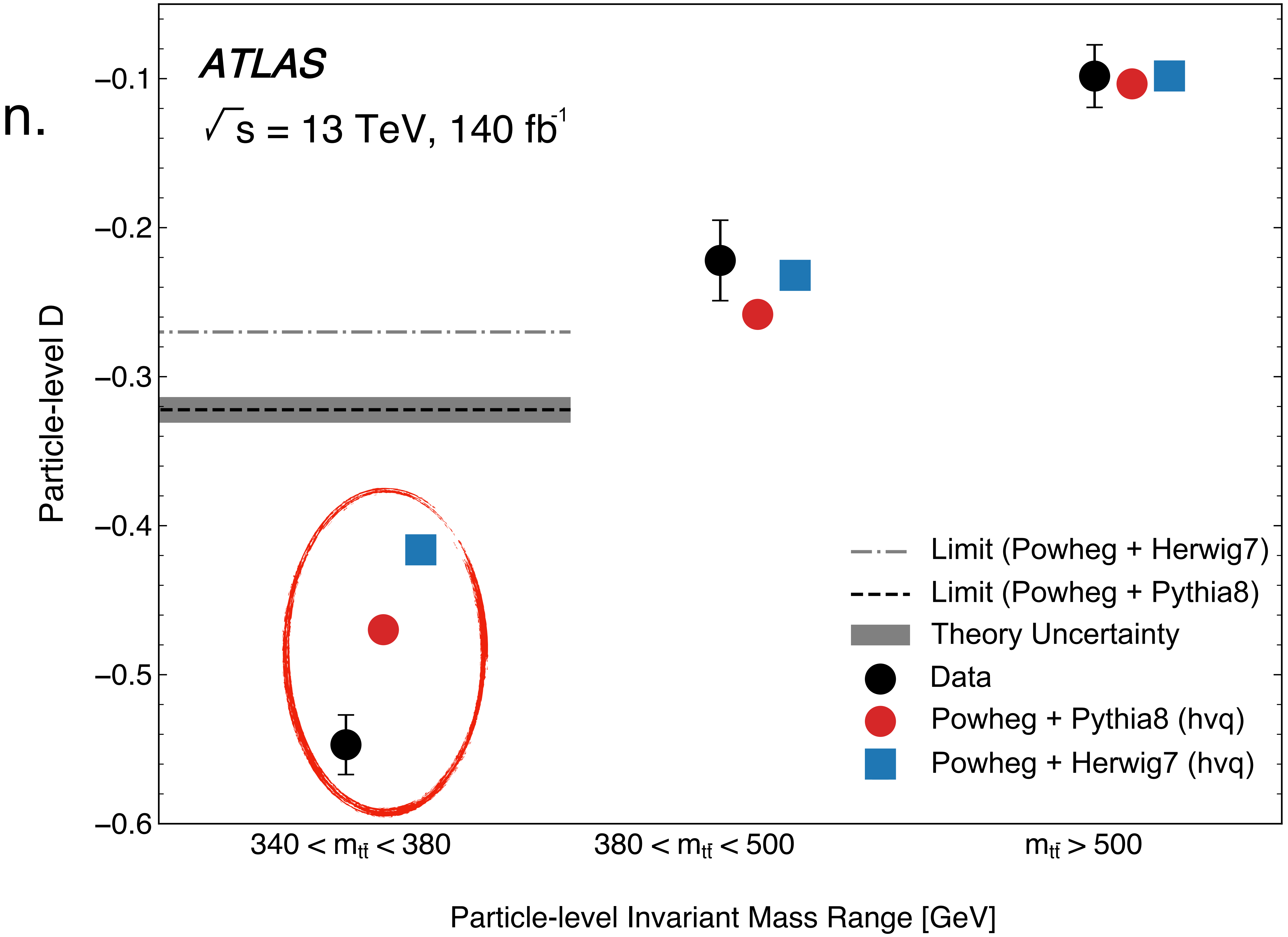
(1981(?)-)

The Verification at the EW scale

- For 2×2 and 2×3 system, it is solved by Peres, and Horodeckis 1996 (Peres-Horodecki criterion, concurrence).

- The result from the ATLAS collaboration.

$$D \equiv -3\langle \cos\varphi(\ell_t^+\ell_{\bar{t}}^-)\rangle$$



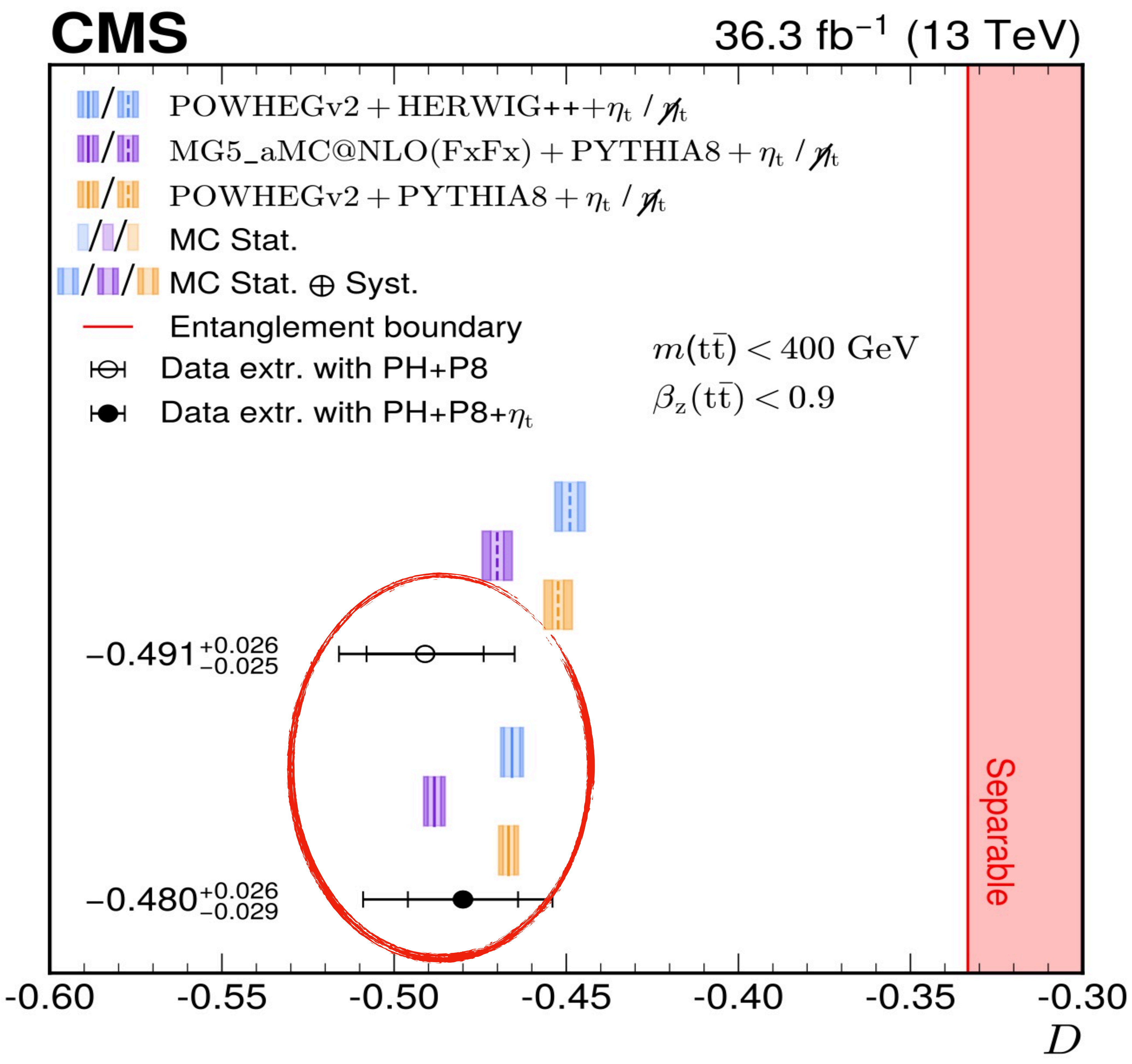
ATLAS Collaboration, arXiv:2311.07288[hep-ex];
Y. Afik and J. R. M. de Nova, Eur. Phys. J. Plus **136** (2021) 907.

The Verification at the EW scale

- For 2×2 and 2×3 system, it is solved by Peres, and Horodeckis 1996 (Peres-Horodecki criterion, concurrence).

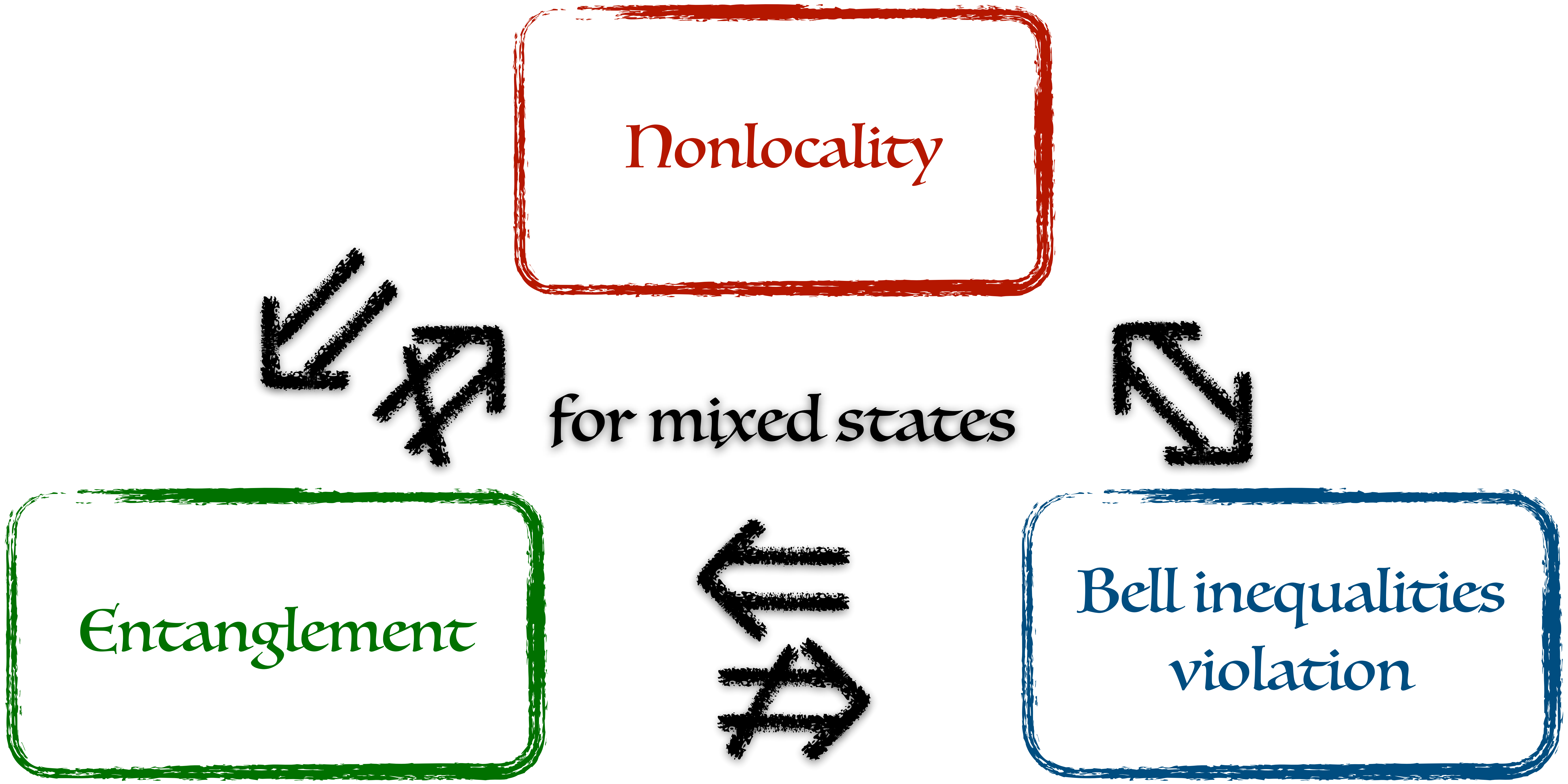
- The result from the CMS collaboration.

$$D \equiv -3\langle \cos\varphi(\ell_t^+ \ell_{\bar{t}}^-)\rangle$$



CMS Collaboration, arXiv:2406.03976[hep-ex].

The Verification at the EW scale



Nonlocality

for mixed states

Entanglement

Bell inequalities violation

WW production at Higgs factory

- The initial state is a mixed state

➡ (Generalized) Bell inequality (but not entanglement) as a test of the quantum reality

“Alice”

$\hat{S}^+_{\mathbf{n}_1}$: W^+ spin along the $\mathbf{n}_1$-direction
$\hat{S}^+_{\mathbf{n}_2}$: W^+ spin along the $\mathbf{n}_2$-direction

$\hat{S}^-_{\mathbf{n}_3}$: W^- spin along the $\mathbf{n}_3$-direction
$\hat{S}^-_{\mathbf{n}_4}$: W^- spin along the $\mathbf{n}_4$-direction

“Bob”

$$\max_{\mathbf{n}_1,\mathbf{n}_2,\mathbf{n}_3,\mathbf{n}_4} \; [p(S^+_{\mathbf{n}_1} = S^-_{\mathbf{n}_3}) + p(S^+_{\mathbf{n}_2} + 1 = S^-_{\mathbf{n}_3}) + p(S^+_{\mathbf{n}_2} = S^-_{\mathbf{n}_4}) + p(S^+_{\mathbf{n}_1} = S^-_{\mathbf{n}_4}) \\ - p(S^+_{\mathbf{n}_1} + 1 = S^-_{\mathbf{n}_3}) - p(S^+_{\mathbf{n}_2} = S^-_{\mathbf{n}_3}) - p(S^+_{\mathbf{n}_2} + 1 = S^-_{\mathbf{n}_4}) - p(S^+_{\mathbf{n}_1} = S^-_{\mathbf{n}_4} + 1)] > 2$$

Collins-Gisin-Linden-Massar-Popescu (CGLMP) inequality

D. Collins, N. Gisin, N. Linden, S. Massar, S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).

WW production at Higgs factory

- The density matrix (some technical details…)

$$\hat{\rho}_{WW} \propto \mathcal{M}(e^+e^- \to W^+W^-)\hat{\rho}_{e^+e^-}\mathcal{M}(e^+e^- \to W^+W^-)^\dagger$$

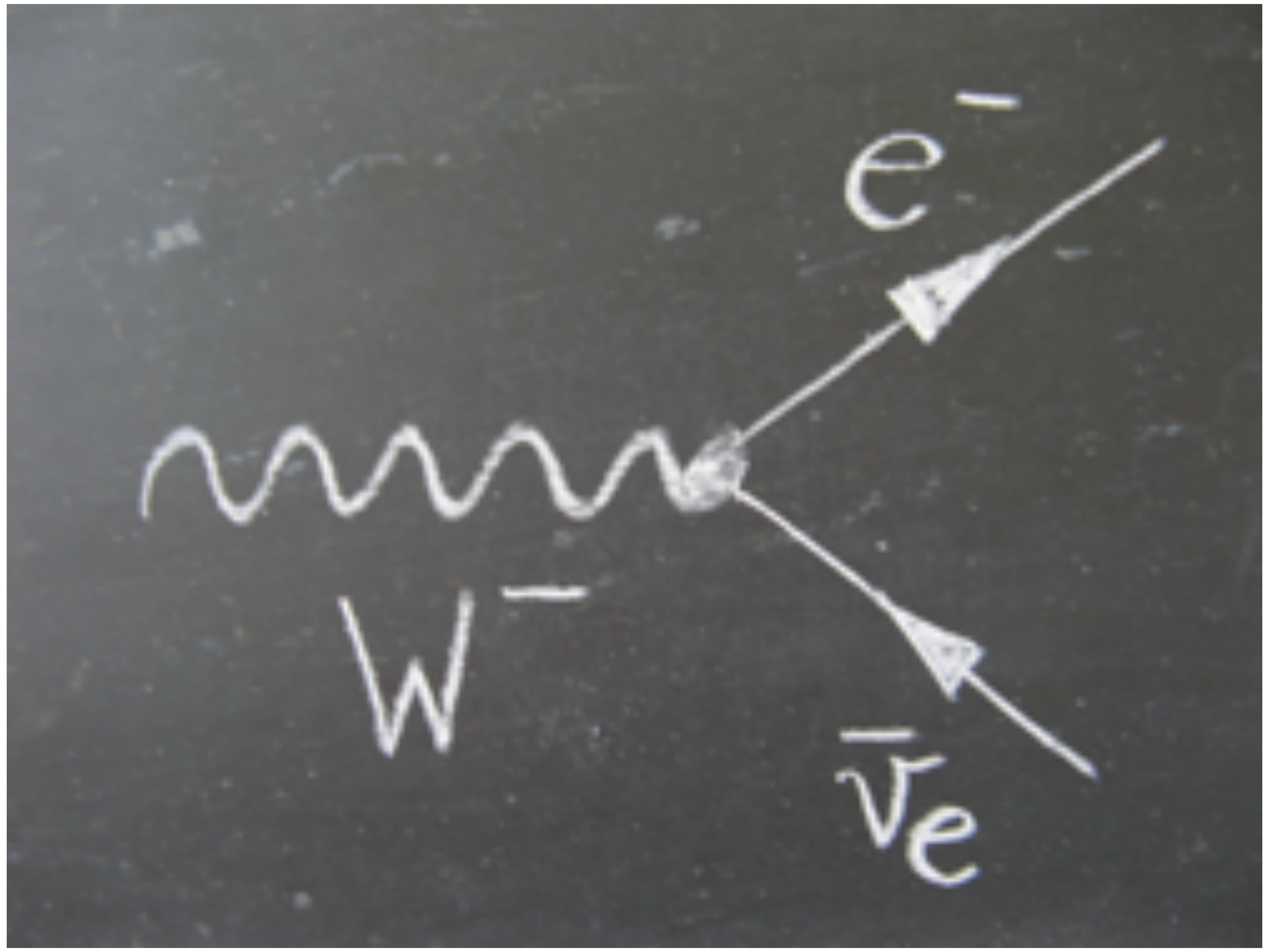
$$\hat{\rho}_W = \frac{1}{3}\hat{I}_3 + d^i\hat{S}_i + q^{ij}\hat{S}_{\{ij\}},\ \ i,j=1,2,3$$

$$\begin{aligned}\hat{\rho}_{WW} \ &= \ \frac{1}{9}\hat{I}_9 + \frac{1}{3}d_+^i\hat{S}_i^+ \otimes \hat{I}_3 + \frac{1}{3}d_-^i\hat{I}_3 \otimes \hat{S}_i^- \\ &+\frac{1}{3}q_+^{ij}\hat{S}_{\{ij\}}^+ \otimes \hat{I}_3 + \frac{1}{3}q_-^{ij}\hat{I}_3 \otimes \hat{S}_{\{ij\}}^- \\ &+C_d^{ij}\hat{S}_i^+ \otimes \hat{S}_j^- + C_{d,q}^{i,jk}\hat{S}_i^+ \otimes \hat{S}_{\{jk\}}^- \\ &+C_{q,d}^{ij,k}\hat{S}_{\{ij\}}^+ \otimes \hat{S}_k^- + C_q^{ij,k\ell}\hat{S}_{\{ij\}}^+ \otimes \hat{S}_{\{k\ell\}}^-\end{aligned}$$

WW production at Higgs factory

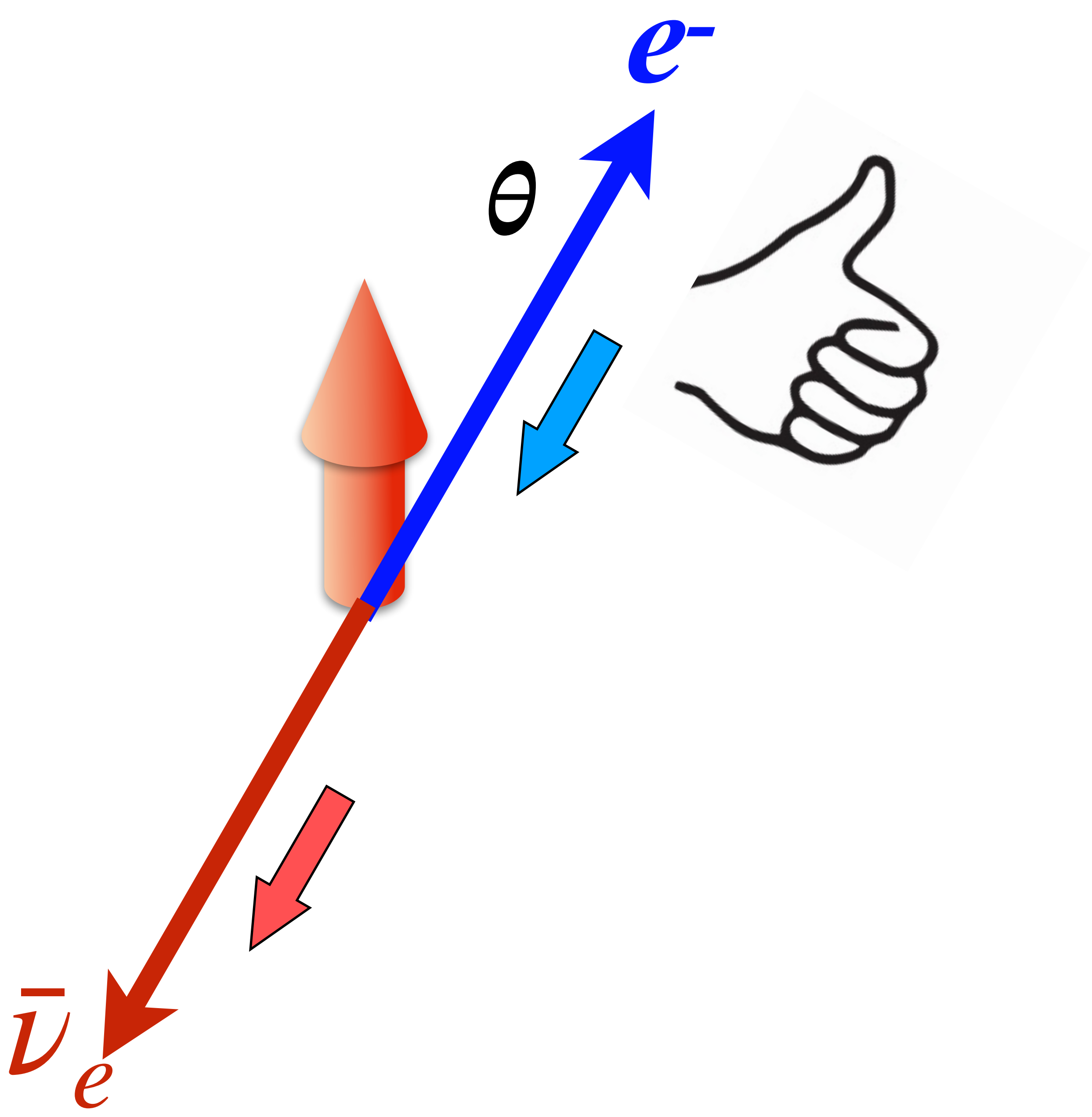
- How to measure it at Higgs factory???

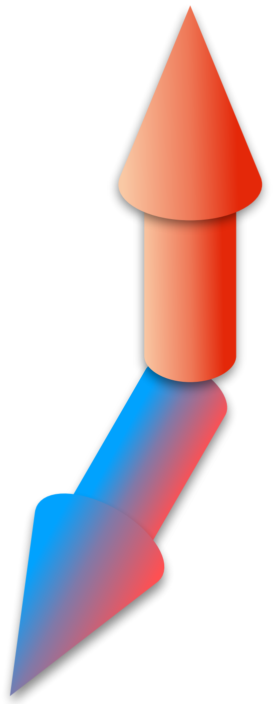
- “Measuring” the polarization direction of the W boson.



WW production at Higgs factory

- How to measure it at Higgs factory???
- “Measuring” the polarization direction of the W boson. (To be different from the condense matter physicists, we do not “measure” or “choose” the observables. We can only accept the “choice” of the nature, and reconstruct the (partial) density matrix with the results.)



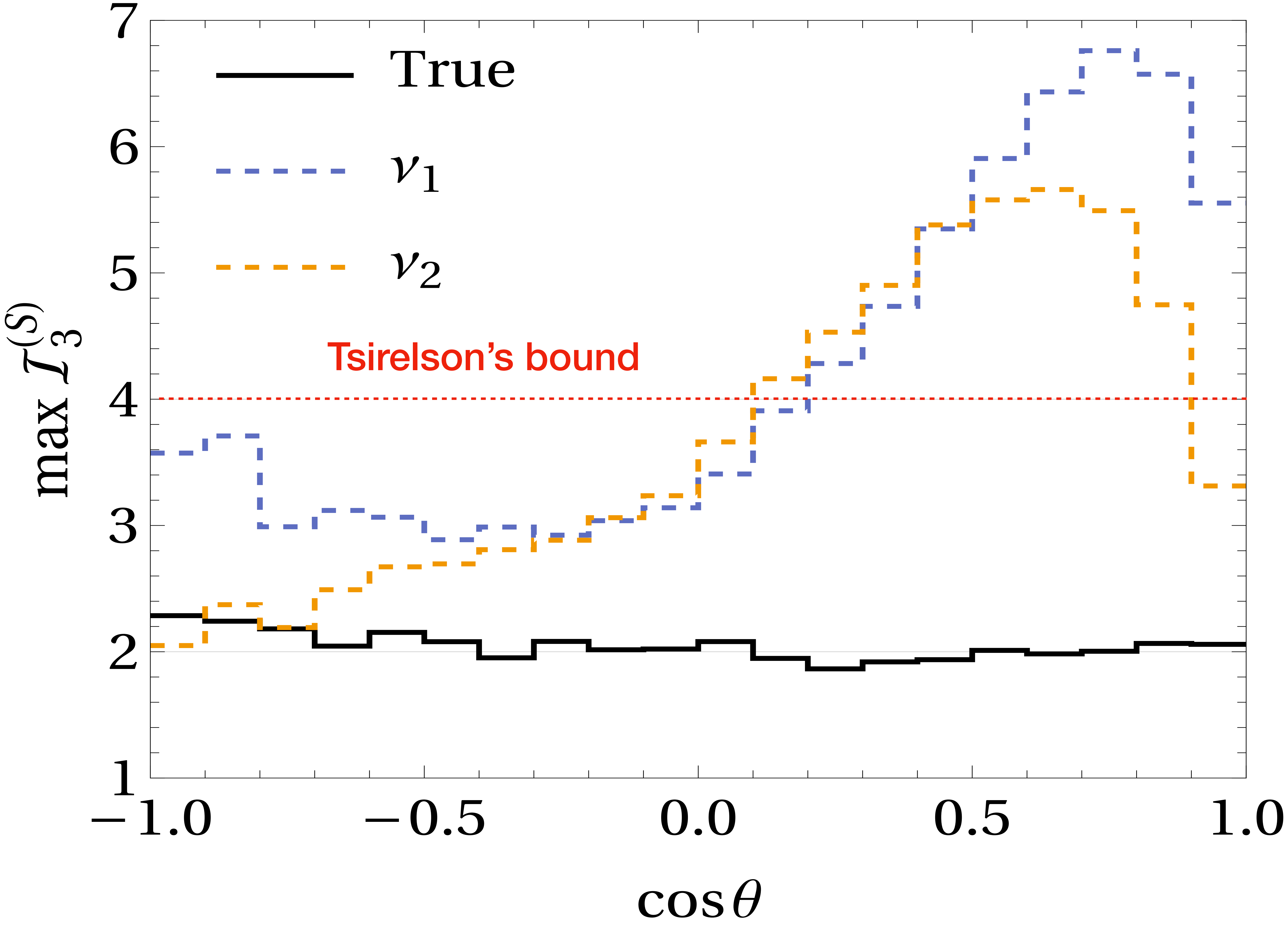


Real W polarization

Measured W polarization

WW production at Higgs factory

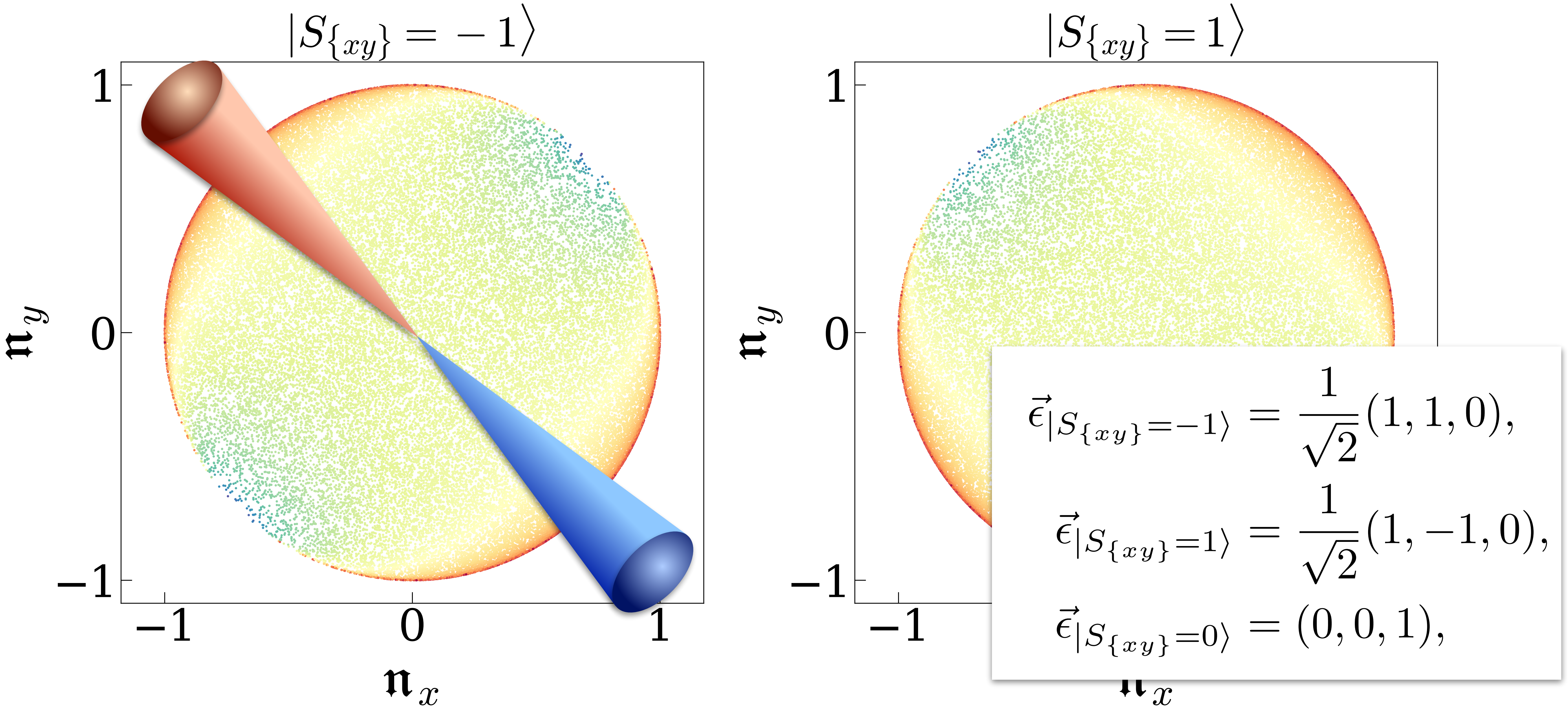
- Collider phenomenology



WW production at Higgs factory

- Collider phenomenology: from dilepton channel to semi-leptonic channel.

- Circular polarization → linear polarization.



$$\vec{\epsilon}_{|S_{\{xy\}}=-1\rangle} = \frac{1}{\sqrt{2}}(1,1,0),$$

$$\vec{\epsilon}_{|S_{\{xy\}}=1\rangle} = \frac{1}{\sqrt{2}}(1,-1,0),$$

$$\vec{\epsilon}_{|S_{\{xy\}}=0\rangle} = (0,0,1),$$

WW production at Higgs factory

- Collider phenomenology: from dilepton channel to semi-leptonic channel.
- Circular polarization → linear polarization.
- With the observables, we maximize the Bell expression by suitable choice of the directions.

$$\begin{aligned}\mathscr{I}_3(\hat{S}_{\vec{a}_1}, \hat{S}_{\vec{a}_2}; \hat{S}_{\{x_3y_3\}}, \hat{S}_{\{x_4y_4\}}) \equiv &+ \left[P(S_{\vec{a}_1} = S_{\{x_3y_3\}}) + P(S_{\{x_3y_3\}} = S_{\vec{a}_2} + 1) + P(S_{\vec{a}_2} = S_{\{x_4y_4\}}) + P(S_{\{x_4y_4\}} = S_{\vec{a}_1})\right] \\ &- \left[P(S_{\vec{a}_1} = S_{\{x_3y_3\}} - 1) + P(S_{\{x_3y_3\}} = S_{\vec{a}_2}) + P(S_{\vec{a}_2} = S_{\{x_4y_4\}} - 1) + P(S_{\{x_4y_4\}} = S_{\vec{a}_1} - 1)\right]\end{aligned}$$

WW production at Higgs factory

- Calculating the generalized Bell observable

$$
\begin{aligned}
&\mathcal{I}_3(\hat{S}_{\vec{a}_1}, \hat{S}_{\vec{a}_2}; \hat{S}_{\{x_3y_3\}}, \hat{S}_{\{x_4y_4\}}) \\
&= 2q^-_{ij}(\omega_{1i}\omega_{1j} + \omega_{2i}\omega_{2j} - 2\omega_{3i}\omega_{3j}) \\
&\quad + 2C^{dq}_{i,jk}a_{1i}(2\epsilon_{1j}\epsilon_{1k} - \epsilon_{2j}\epsilon_{2k} - \epsilon_{3j}\epsilon_{3k} + \omega_{1j}\omega_{1k} \\
&\qquad\quad - 2\omega_{2j}\omega_{2k} + \omega_{3j}\omega_{3k}) \\
&\quad + 2C^{dq}_{i,jk}a_{2i}(-2\epsilon_{1j}\epsilon_{1k} + \epsilon_{2j}\epsilon_{2k} + \epsilon_{3j}\epsilon_{3k} + 2\omega_{1j}\omega_{1k} \\
&\qquad\quad - \omega_{2j}\omega_{2k} - \omega_{3j}\omega_{3k}) \\
&\quad + 6C^{q}_{ij,kl}a_{1i}a_{1j}(-\epsilon_{2k}\epsilon_{2l} + \epsilon_{3k}\epsilon_{3l} - \omega_{1k}\omega_{1l} + \omega_{3k}\omega_{3l}) \\
&\quad + 6C^{q}_{ij,kl}a_{2i}a_{2j}(\epsilon_{2k}\epsilon_{2l} - \epsilon_{3k}\epsilon_{3l} - \omega_{2k}\omega_{2l} + \omega_{3k}\omega_{3l})
\end{aligned}
$$

$$\langle \mathfrak{n}^\pm_i \rangle = d^\pm_i,$$

$$\langle \mathfrak{q}^\pm_{ij} \rangle = \frac{2}{5} q^\pm_{ij},$$

~~$\langle \mathfrak{n}^+_i \mathfrak{n}^-_j \rangle = C^d_{ij},$~~

$$\langle \mathfrak{q}^+_{ij} \mathfrak{q}^-_{kl} \rangle = \frac{4}{25} C^q_{ij,kl},$$

$$\left\langle \mathfrak{n}^+_i \mathfrak{q}^-_{jk} \right\rangle = \frac{2}{5} C^{dq}_{i,jk},$$

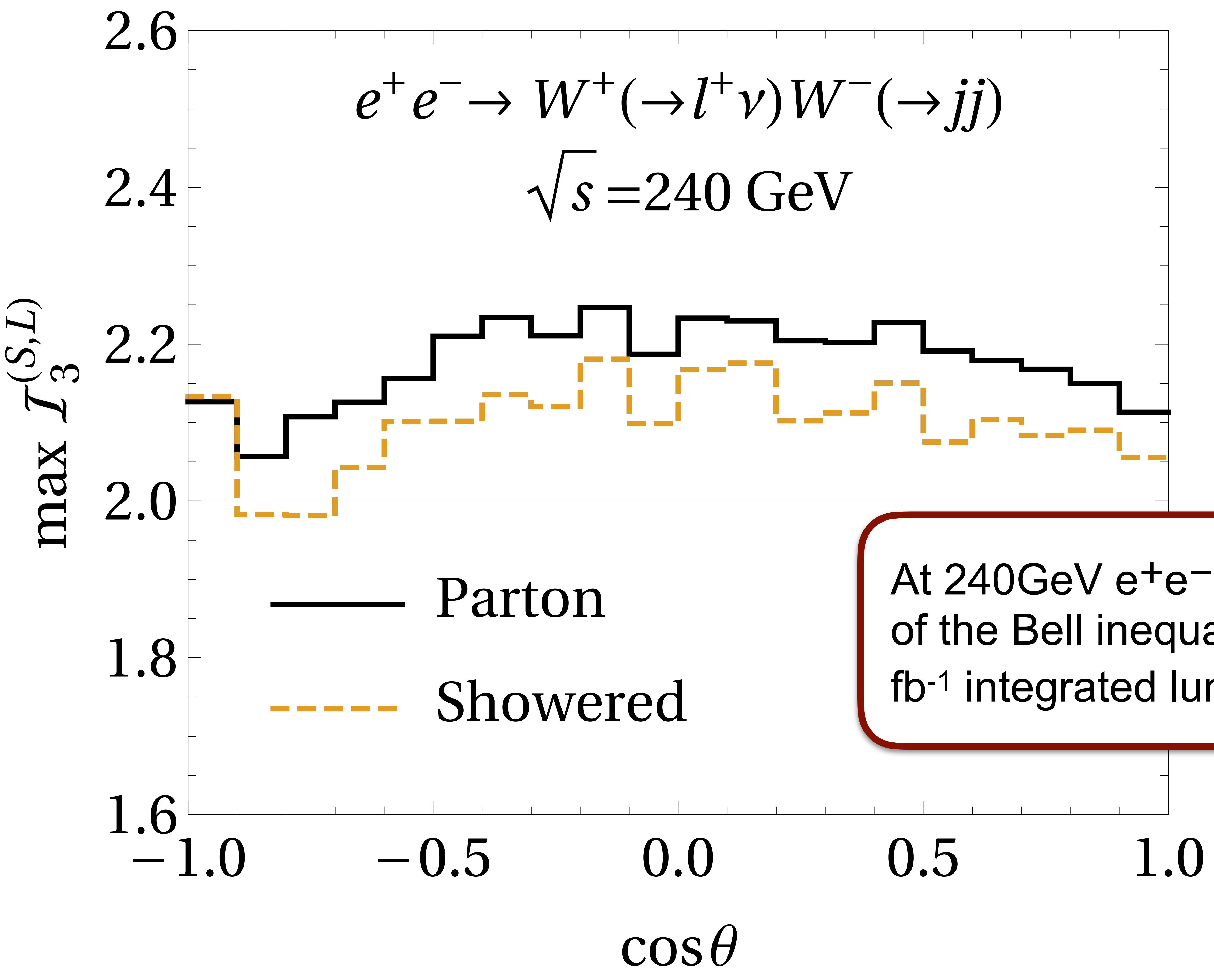
$$\langle \mathfrak{q}^+_{ij} \mathfrak{n}^-_k \rangle = \frac{2}{5} C^{qd}_{ij,k}.$$

WW production at Higgs factory

- Some details ($e^+e^- \to W^+W^- \to \ell^\pm \nu jj$)
  - 240GeV electron-positron collider
  - (LO) MADGRAPH5_AMC@NLO+PYTHIA8+FASTJET
  - 2 Exclusive jets with Durham algorithm ($E_j > 5\text{GeV}, |\eta_j| < 3.5$)
  - One isolated charged lepton ($e^\pm, \mu^\pm$) ($E_\ell > 15\text{GeV},\ |\cos\theta_\ell| < 0.98$)
  - Missing energy ($\cos\theta_{\ell\nu} < 0.2$)
  - Reconstructed *W* mass ($|m_{jj} - m_W| < 20\text{GeV}$)

WW production at Higgs factory

- The result



At 240GeV e^+e^- collider, one can verify the violation of the Bell inequality at 5.0σ significance with ~180 fb^{-1} integrated luminosity.

Conclusion and Discussion

- We provide a realistic approach to test Bell inequalities in W pair systems using a new set of Bell observables based on measuring the linear polarization of W bosons.

- Our observables depend on only part of the density matrix that can be correctly measured in the semi-leptonic decay mode of W.

- Why should we test the correlations at higher and higher scale?

Thank you!