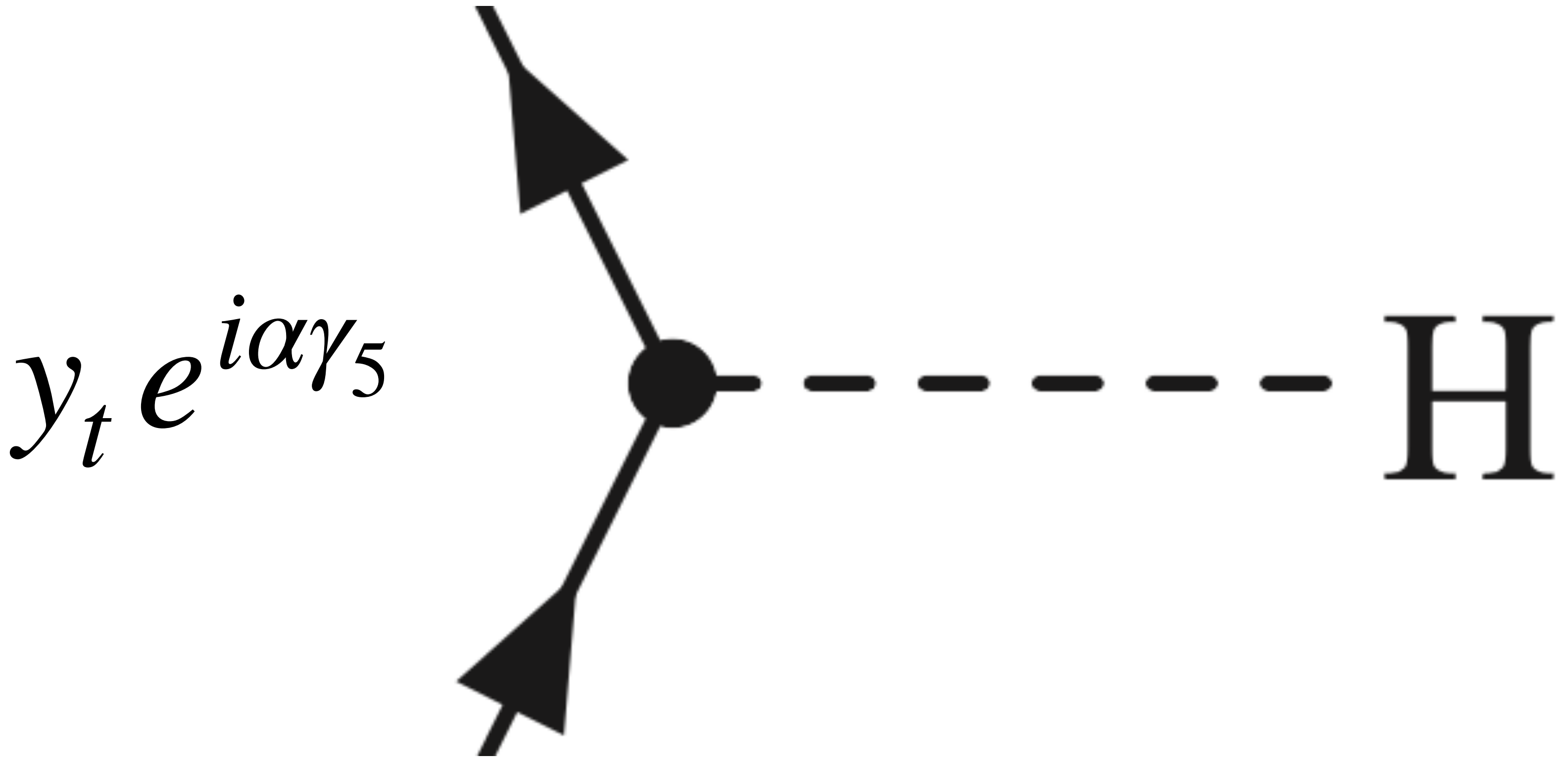
Towards NNLO calculation for high energy production of tTH

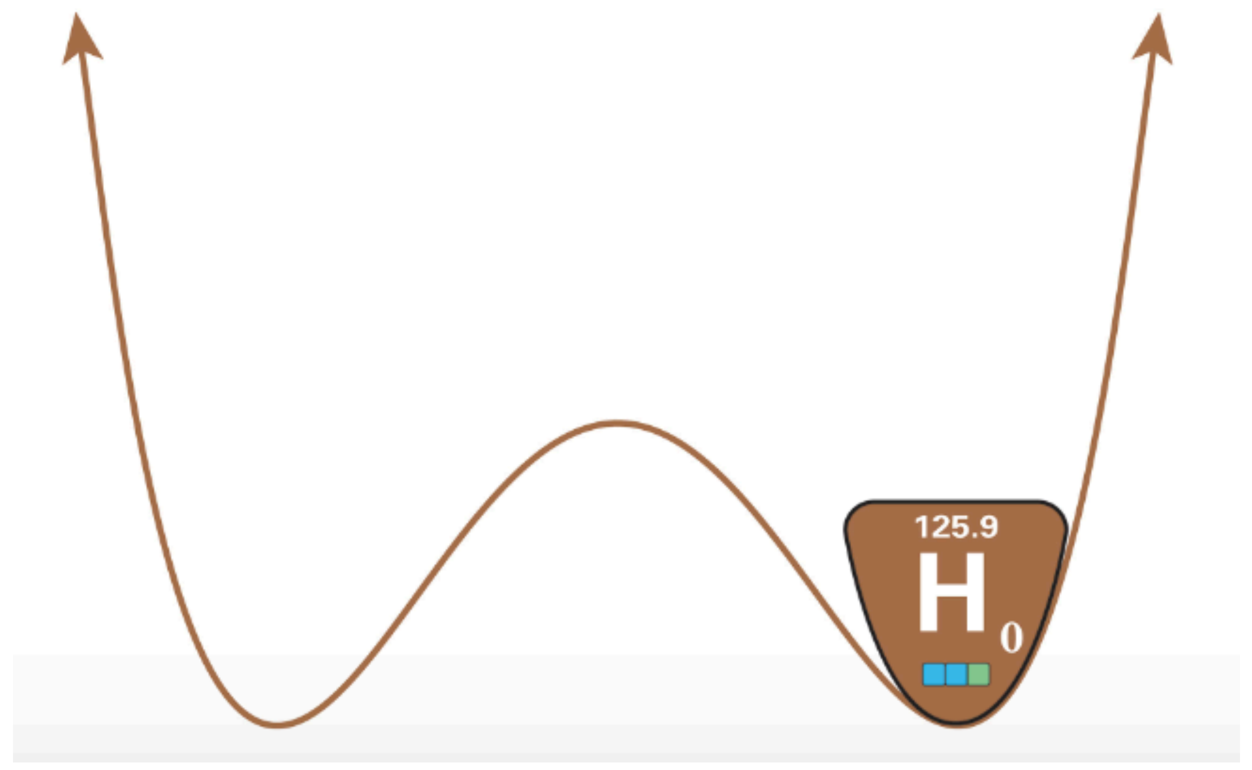
Li Lin Yang
Zhejiang University

The top quark Yukawa coupling

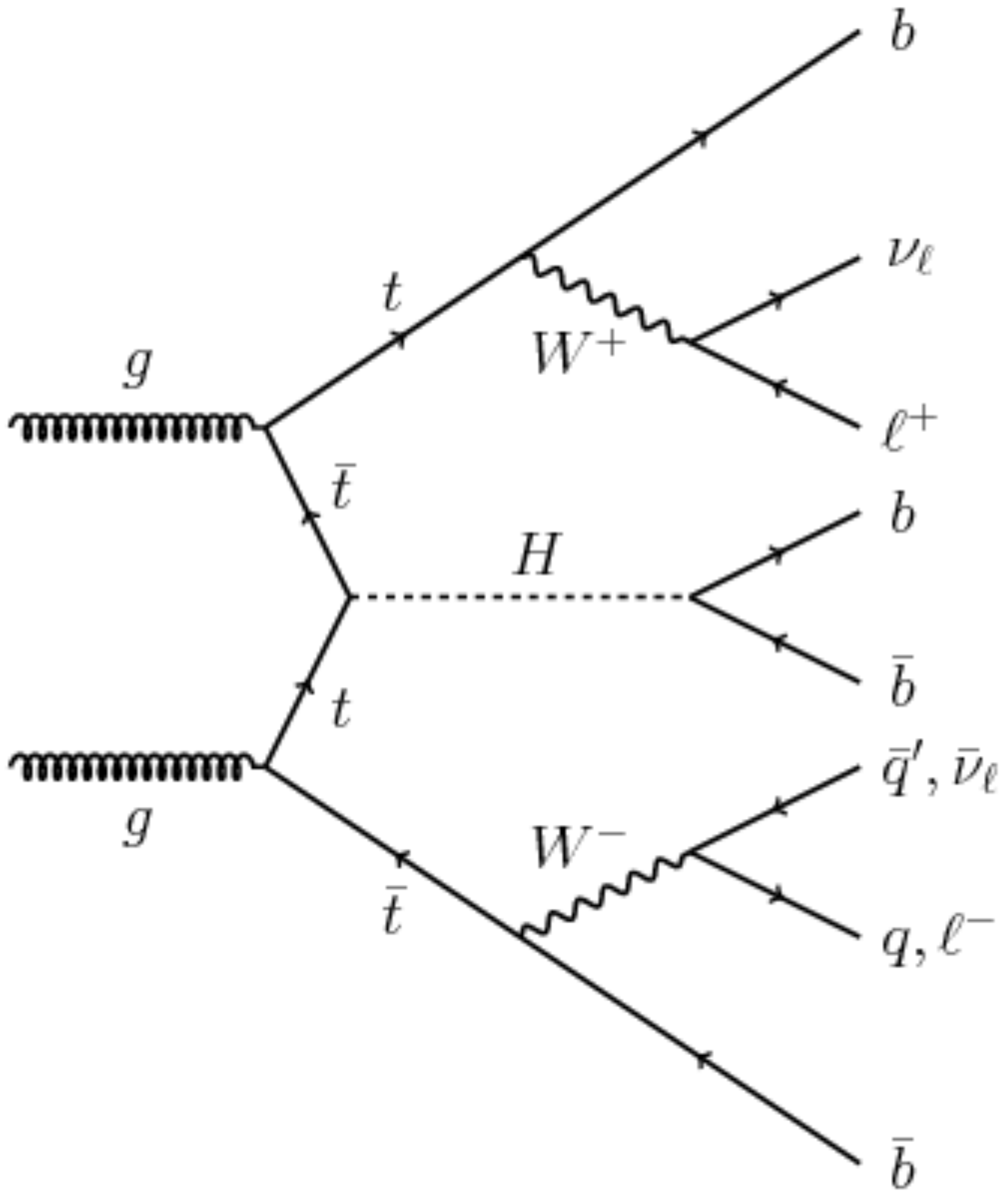


Relevant for

- Origin of masses of fundamental fermions
- Matter-anti-matter asymmetry (possible source of CP violation)
- Higgs effective potential (vacuum stability)

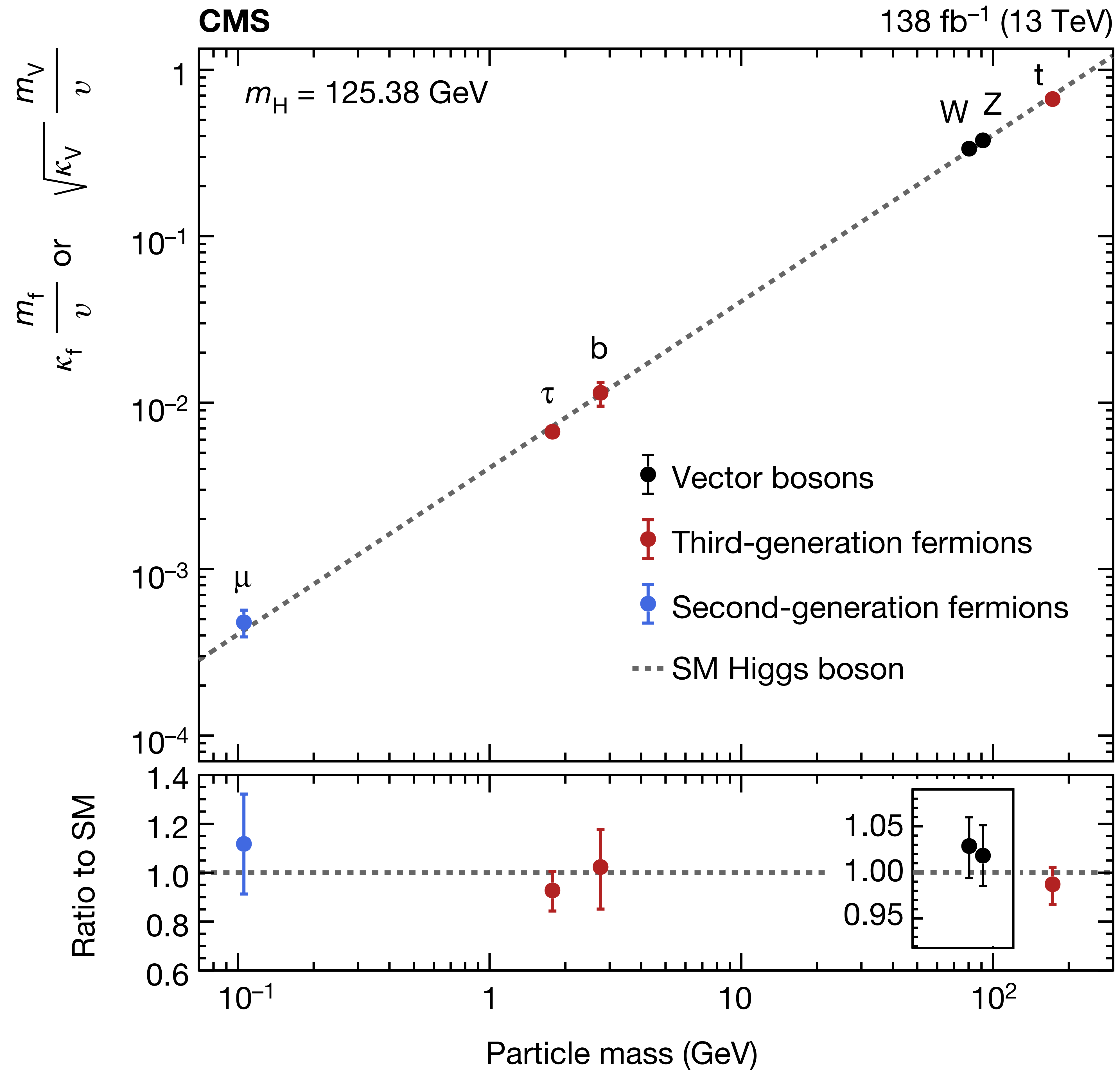


Associated tTH production



- Direct probe of top quark Yukawa coupling
- Observed in 2018 by ATLAS and CMS
- CP structure probed in 2020

The need for precision



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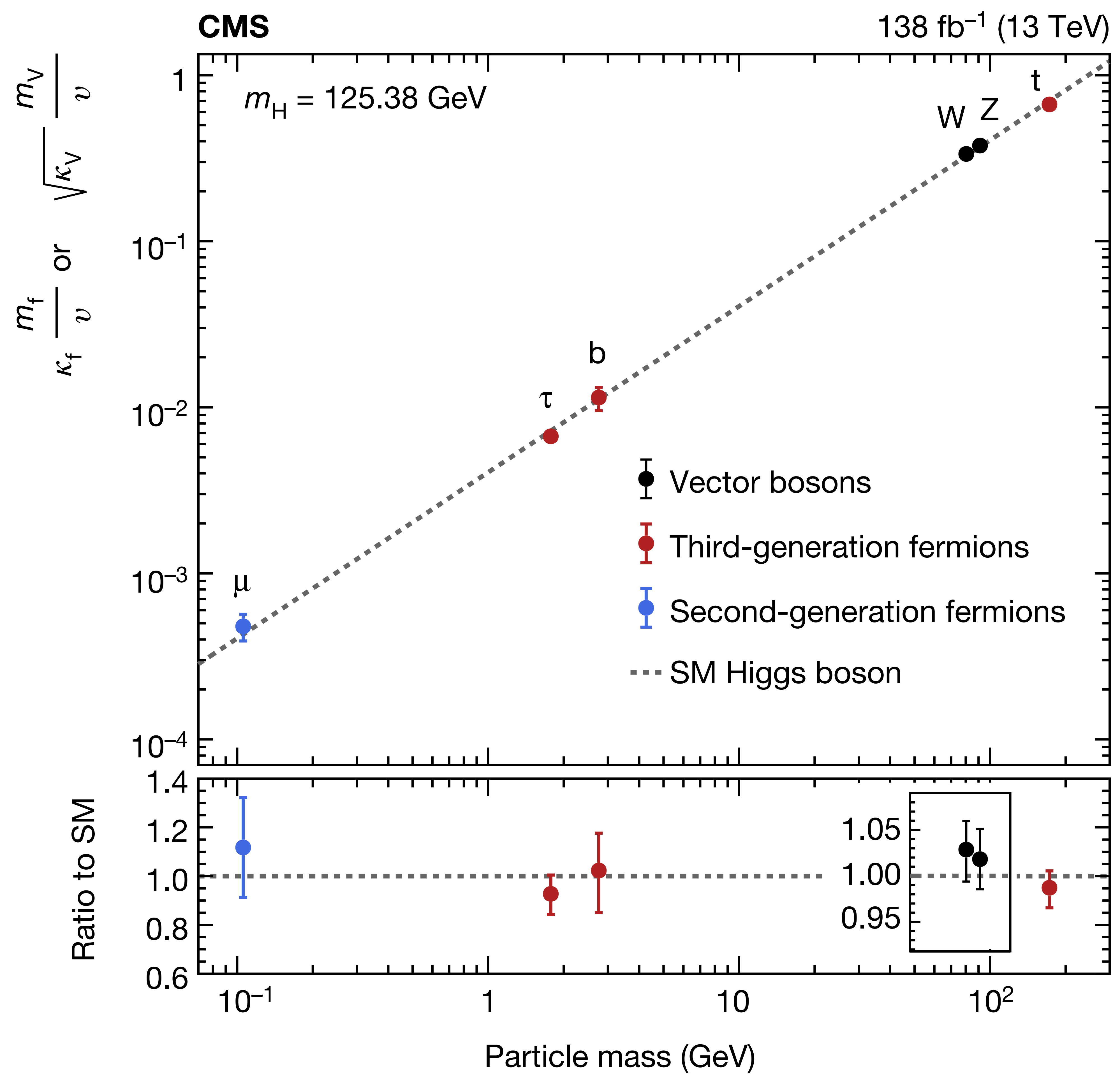
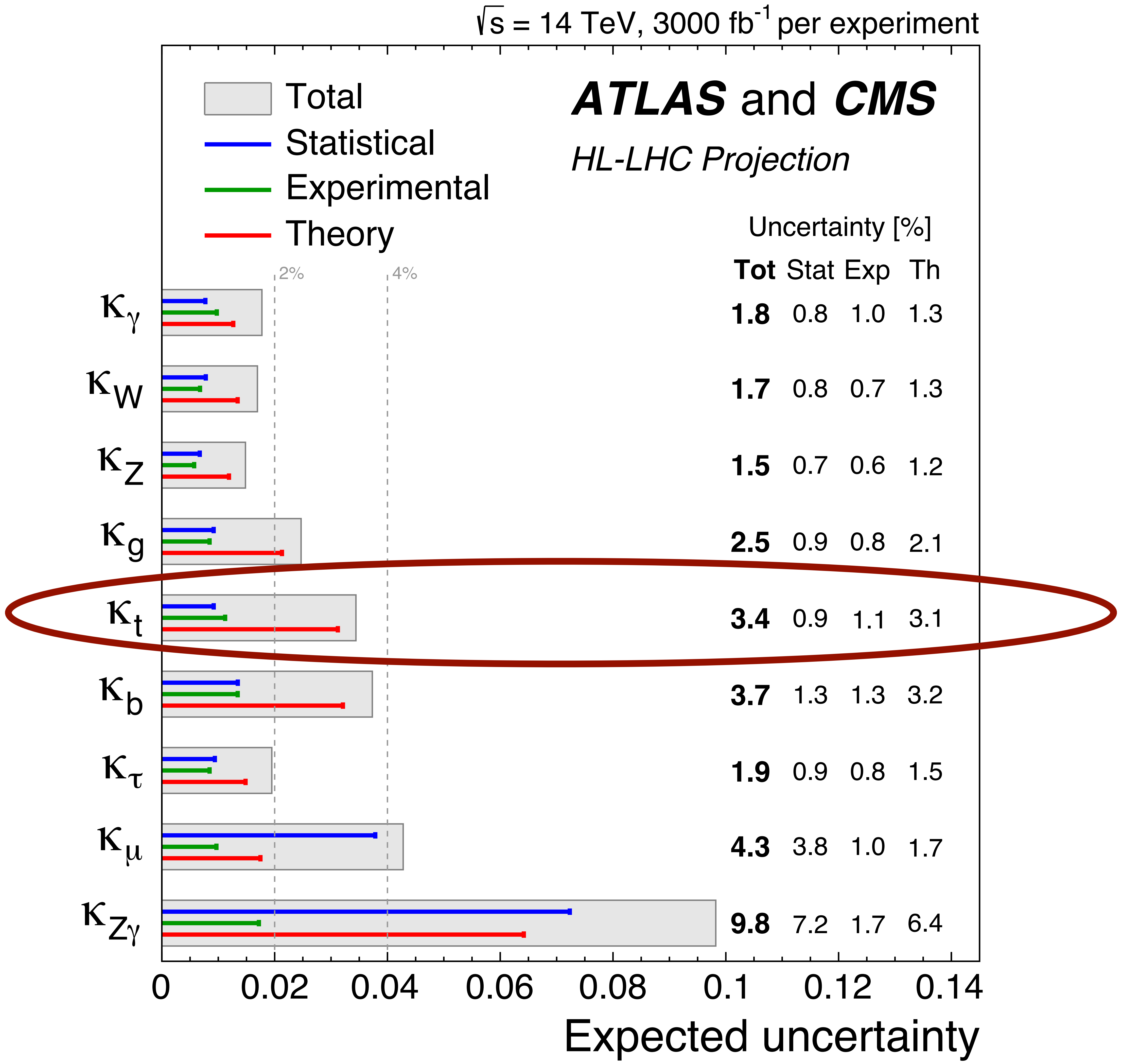
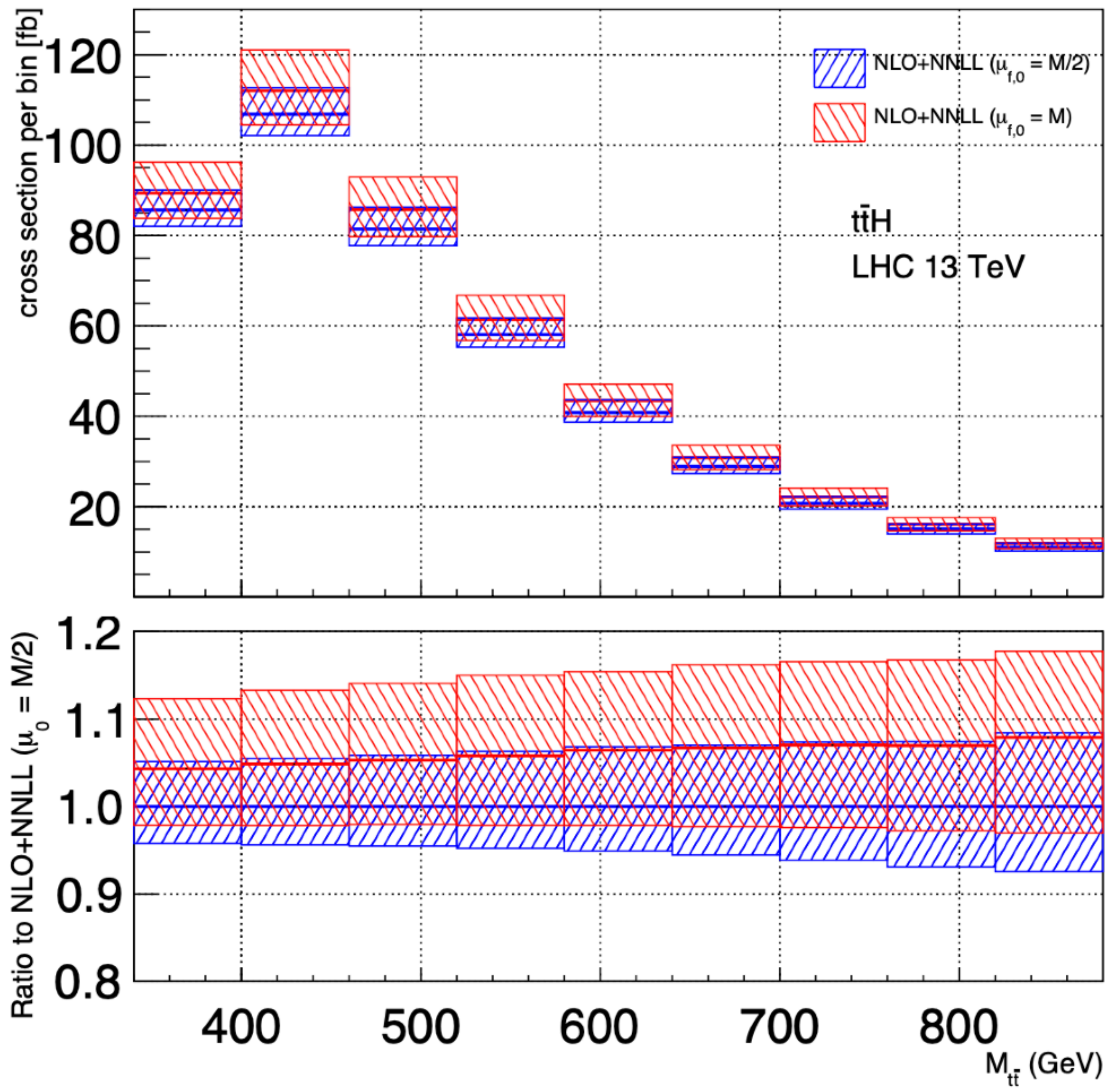




Theoretical status

- ➤ NLO + resummation — Broggio, Ferroglia, Pecjak, LLY: 1611.00049
- ➤ Coulomb corrections — Ju, LLY: 1904.08744

	13 TeV LHC (pb)	14 TeV LHC (pb)
NLO	$0.493^{+5.8\%}_{-9.2\%}$	$0.597^{+6.1\%}_{-9.2\%}$
NLL′+NLO	$0.521^{+1.9\%}_{-2.6\%}$	$0.630^{+2.3\%}_{-2.6\%}$
K-factor	1.06	1.06



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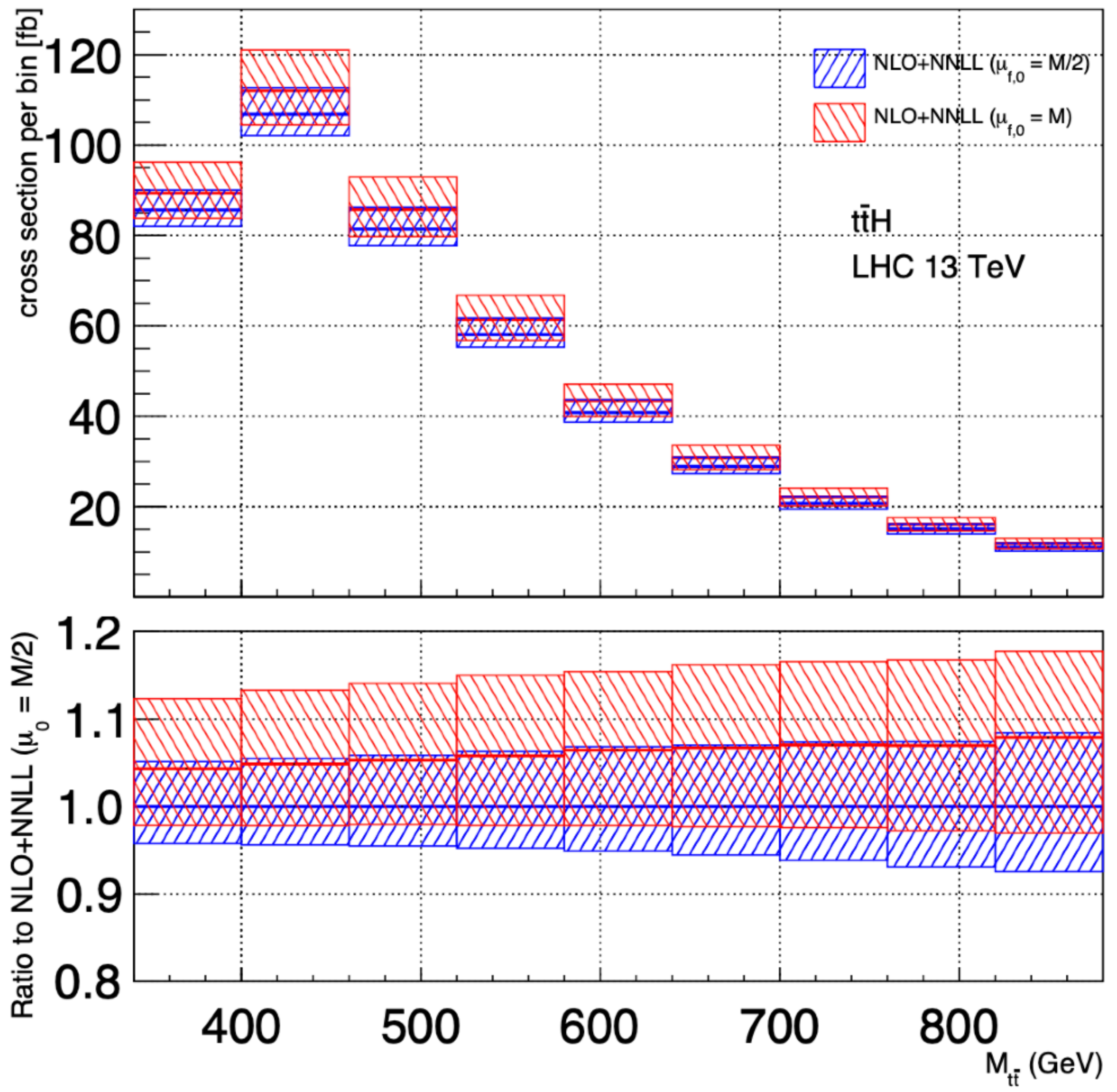

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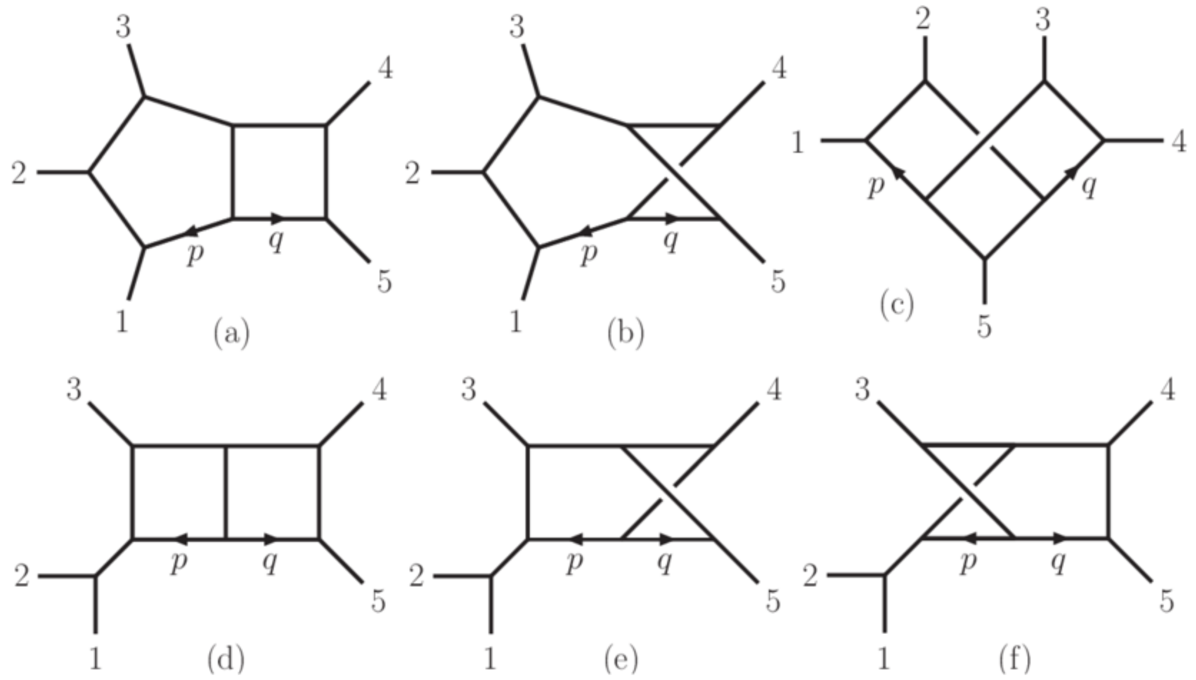
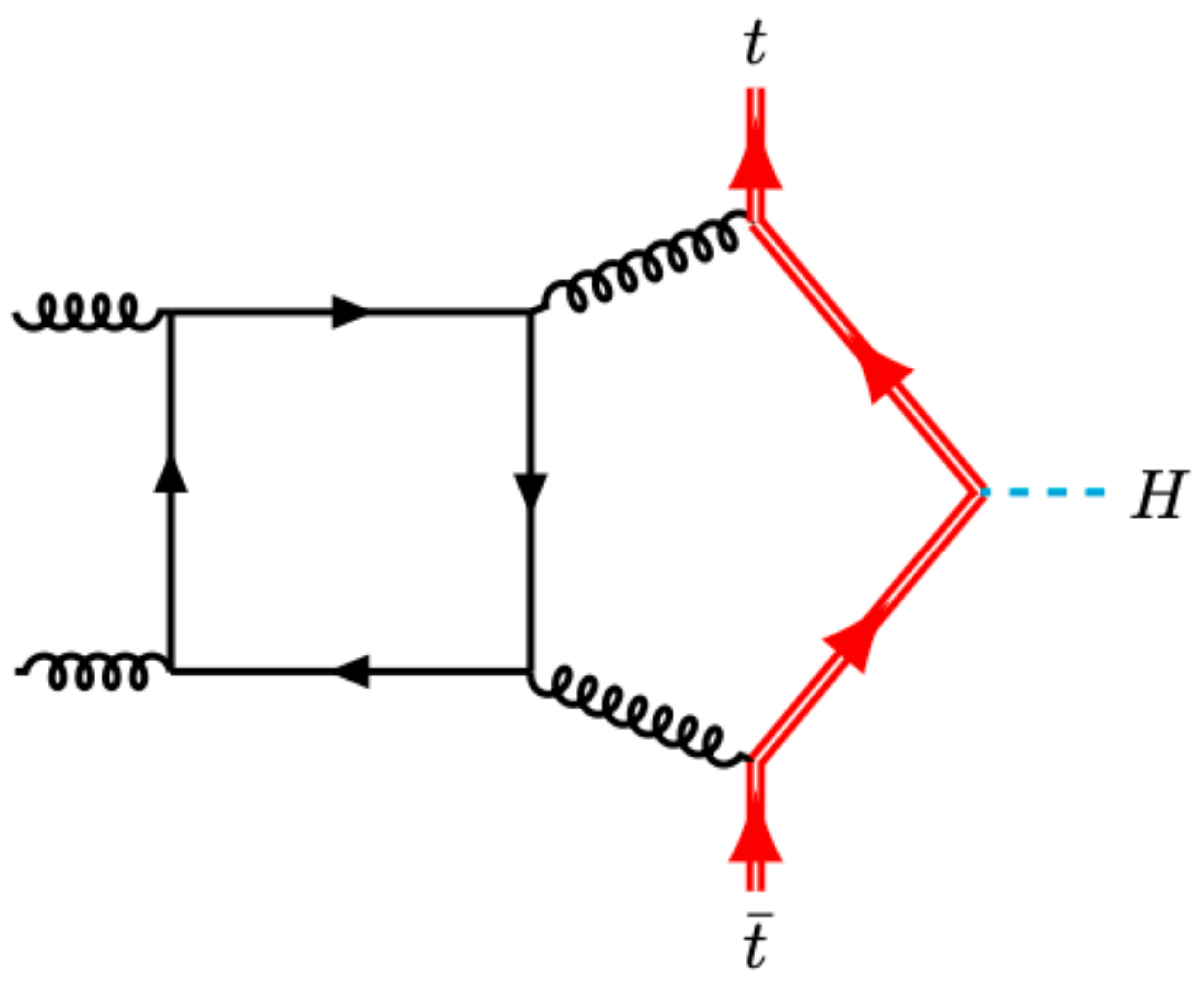
- Bottlenecks towards NNLO
  - Two-loop amplitudes
  - IR subtraction



Two-loop amplitudes for $t\bar{t}H$



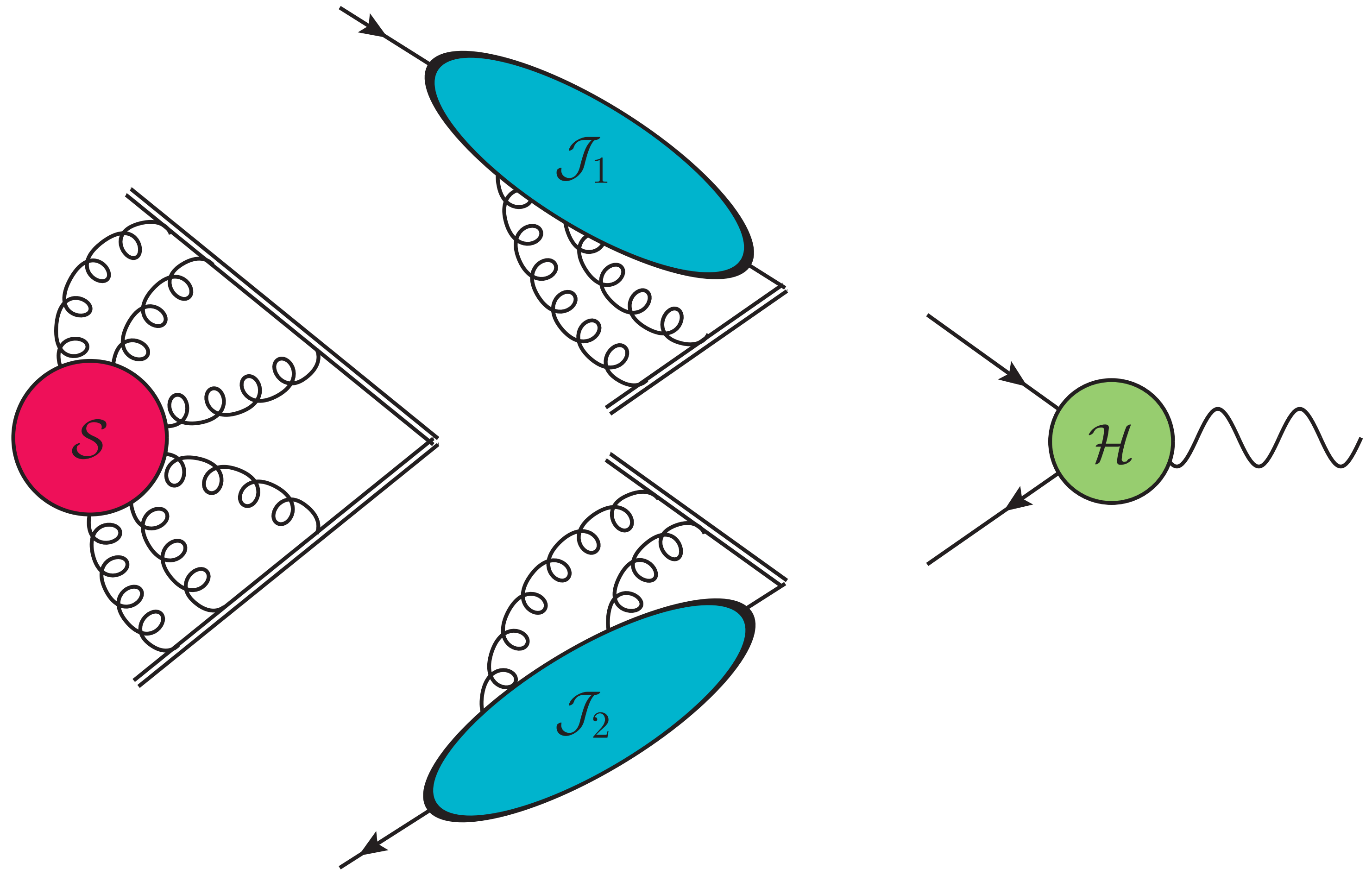
+ many more planar and non-planar families

- Two-loop five-point amplitudes with 7 scales
- Partial results for simpler families e.g.: 2312.08131, 2402.03301
- Full results require much more efforts (analytic + numeric methods)

Two-loop IR singularities

Chen, Ma, Wang, LLY, Ye: 2202.02913

IR singularities of QCD amplitudes admit a universal structure due to soft/collinear factorization



$$Z^{-1}(\epsilon)\,\mathscr{M}^{\text{UV renormalized}}(\epsilon) = \mathcal{O}(\epsilon^0)$$

Two-loop poles = Two-loop Z-factor × One-loop amplitude to ϵ^1

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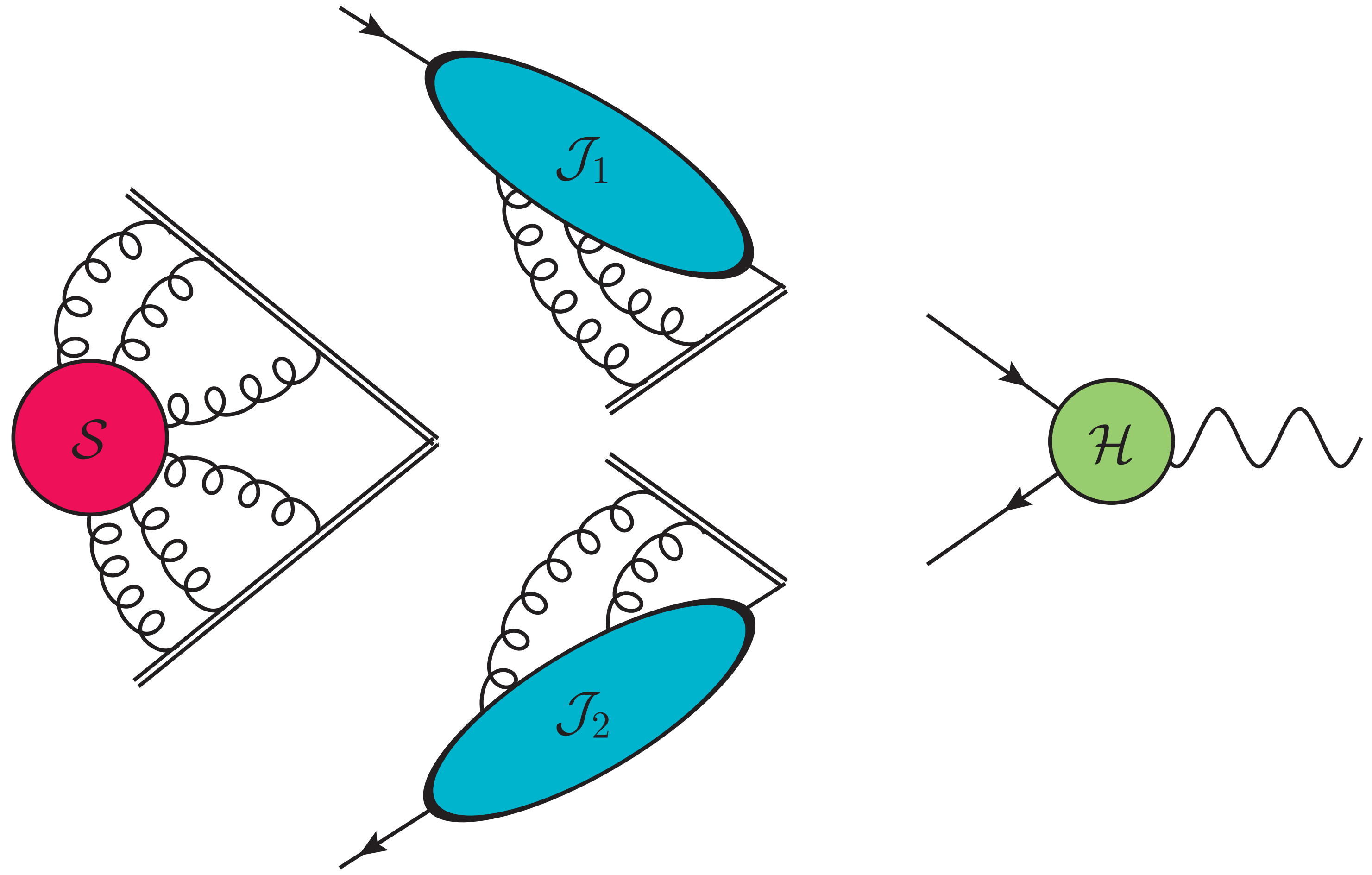

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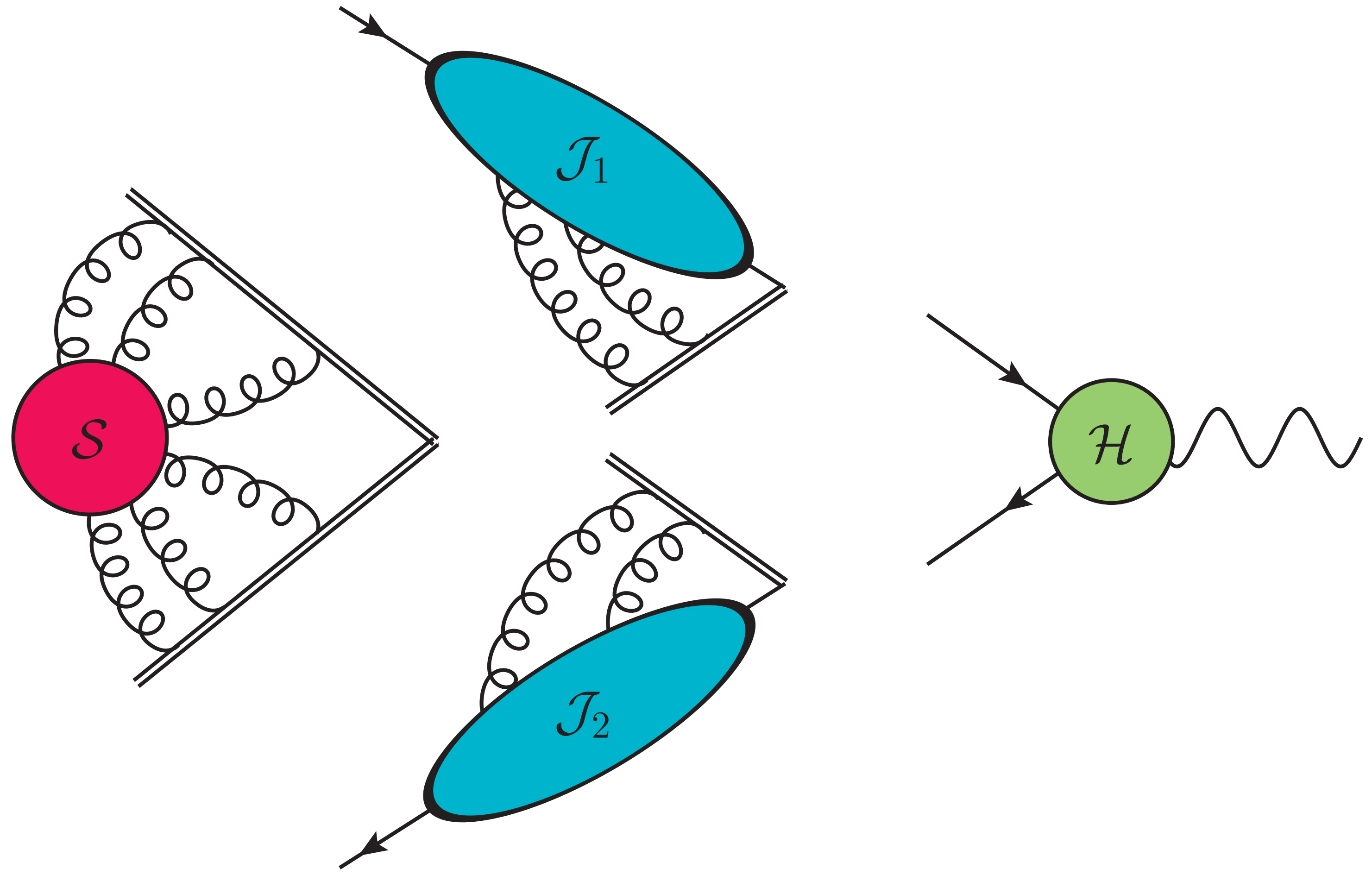

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Generically known in terms of symbols

Chen, Ma, LLY: 2201.12998
Jiang, LLY: 2303.11657

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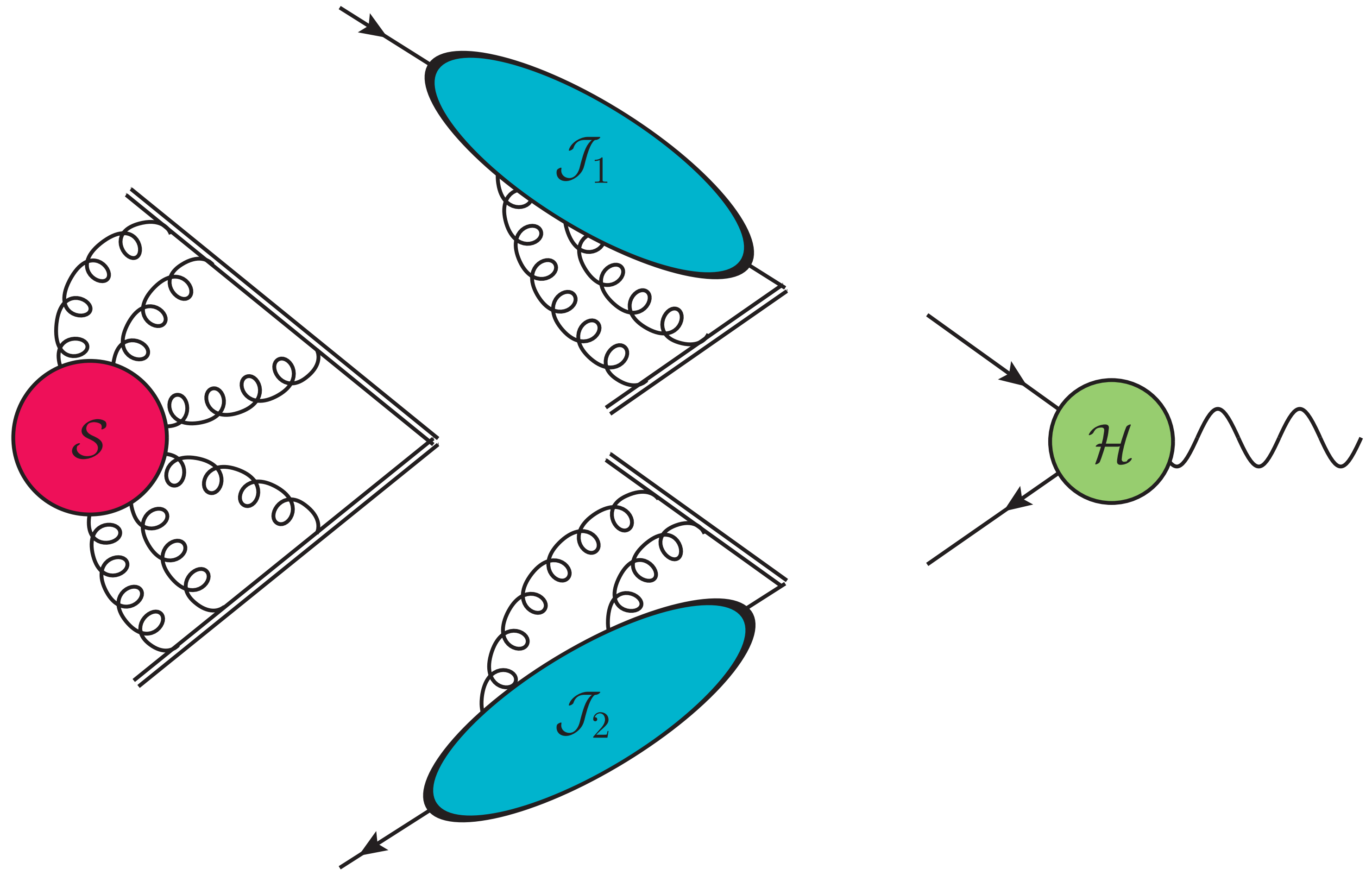

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- Predict two-loop IR poles for tTH
- Provide strong check on two-loop amplitudes
- Validate IR subtraction

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}
A^g	17.37022326	6.277797530	−162.1830217	559.8062598
B^g	−32.49510001	−34.75486260	−624.1343773	3901.332369
C^g		−9.463444735	−54.41556200	−497.5350517
D^g			143.6321997	−578.4857199
E^g_l		−20.26526047	46.54471184	−10.69967085
E^g_h			−24.23013938	79.68650479
F^g_l		37.91095001	−74.94866603	71.66904977
F^g_h			43.70151160	−132.3384924
G^g_l			4.731722368	85.25318119
G^g_h				6.363526190
H^g_l			3.860049613	−10.52987601
H^g_{lh}				8.076713126
H^g_h				
I^g_l			−7.221133335	19.49234494
I^g_{lh}				−14.56717053
I^g_h				
A^q	2.390051823	15.03938540	0.597121534	−34.95784899
B^q	−4.780103646	−22.69017086	49.54607207	106.0851578
C^q	2.390051823	7.650785464	−186.5751188	−21.39439443
D^q_l		−2.390051823	0.308675876	−6.605875838
D^q_h			6.244349191	4.860387981
E^q_l		2.390051823	1.610219156	77.52356965
E^q_h			−6.244349191	19.76269918
F^q_l				
F^q_{lh}				
F^q_h				

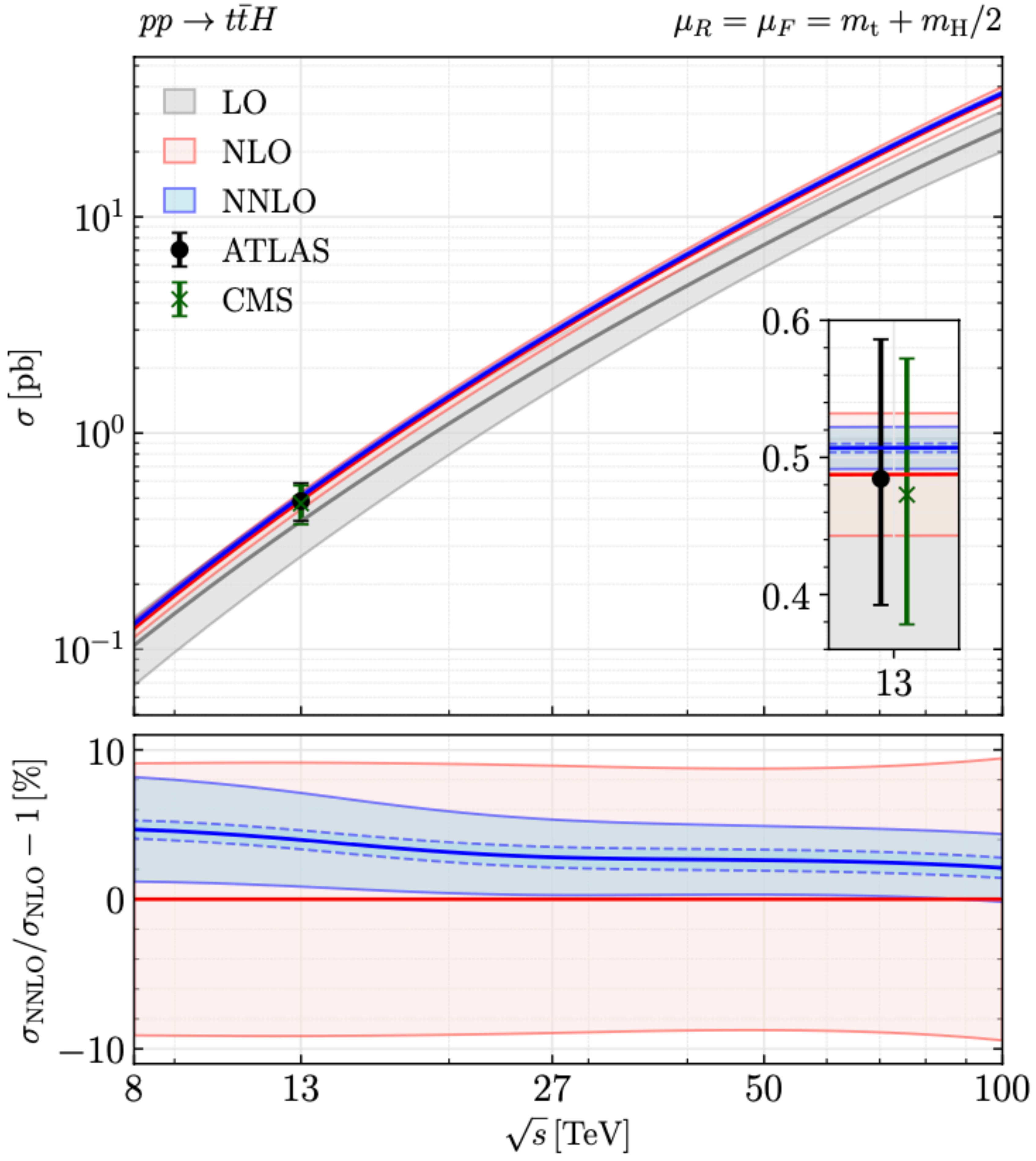
Table 1. IR poles decomposed as color coefficients for the phase-space point $x_{12} = 10$, $x_{13} = -1339/920$, $x_{14} = -2269/465$, $x_{23} = -1951/620$, $x_{24} = -1803/1810$ and $x_{34} = 5$.

Approximation with soft Higgs

Catani et al.: 2210.07846

Eikonal approximation: $2 \to 2$ kinematics

$$\mathcal{M}(\{p_i\}, k) \simeq F(\alpha_S(\mu_R); \tfrac{m_t}{\mu_R}) \, \frac{m_t}{v} \sum_{i=3,4} \frac{m_t}{p_i \cdot k} \mathcal{M}(\{p_i\})$$



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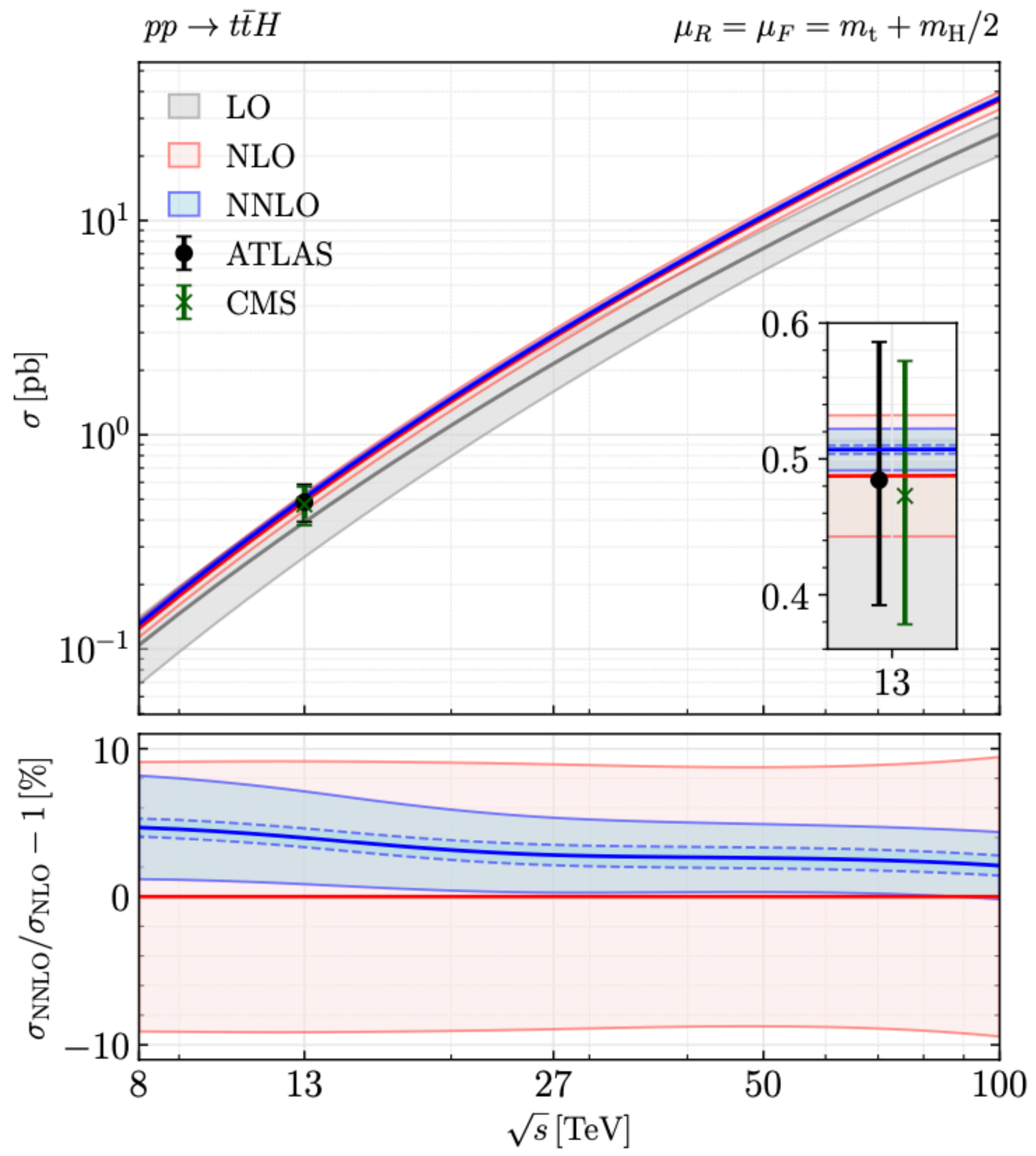
Not a good approximation for two-loop amplitudes:

- One-loop already 30% error
- Two-loop estimated 100% error



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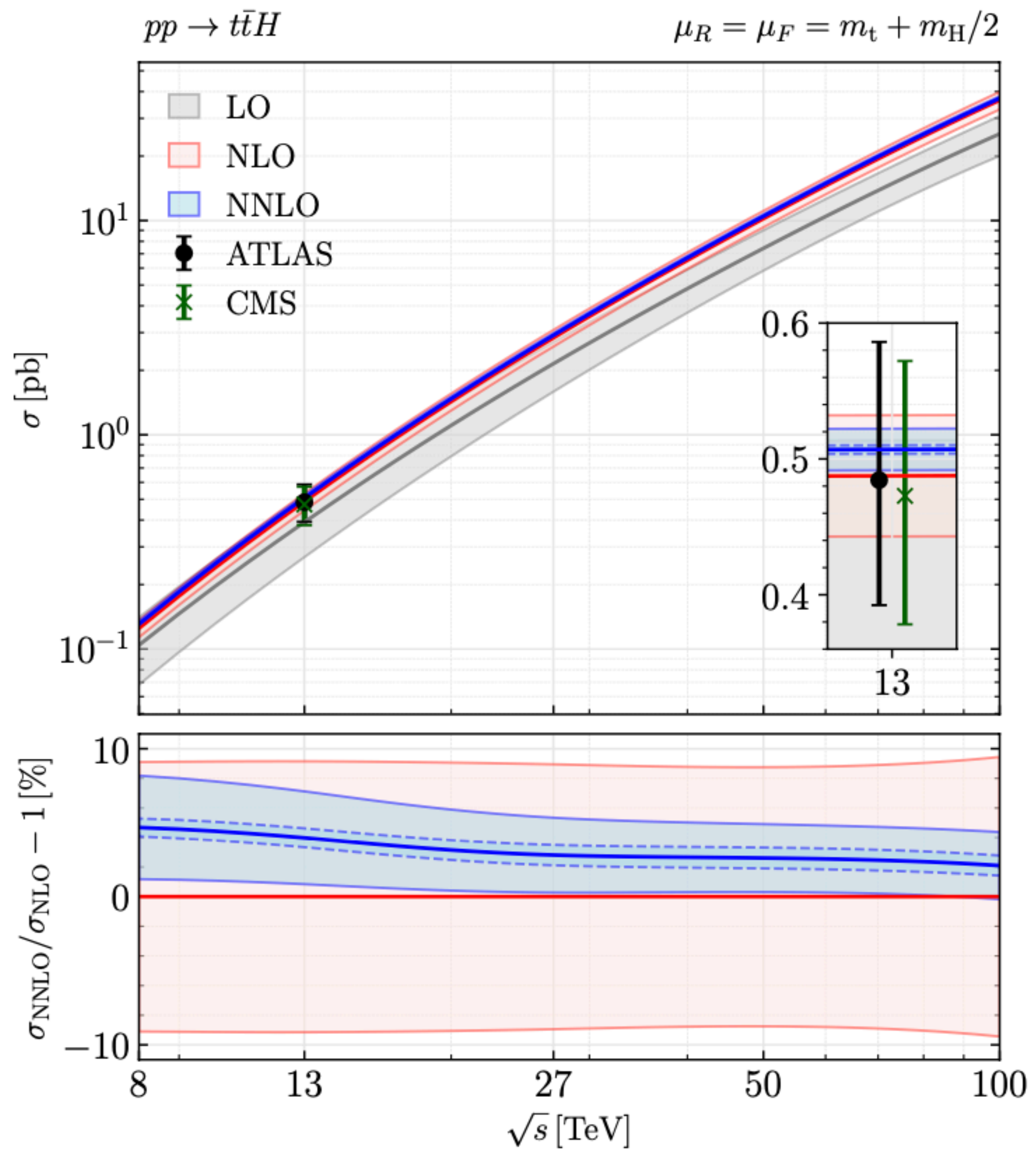
The argument was: two-loop amplitudes small for total cross section



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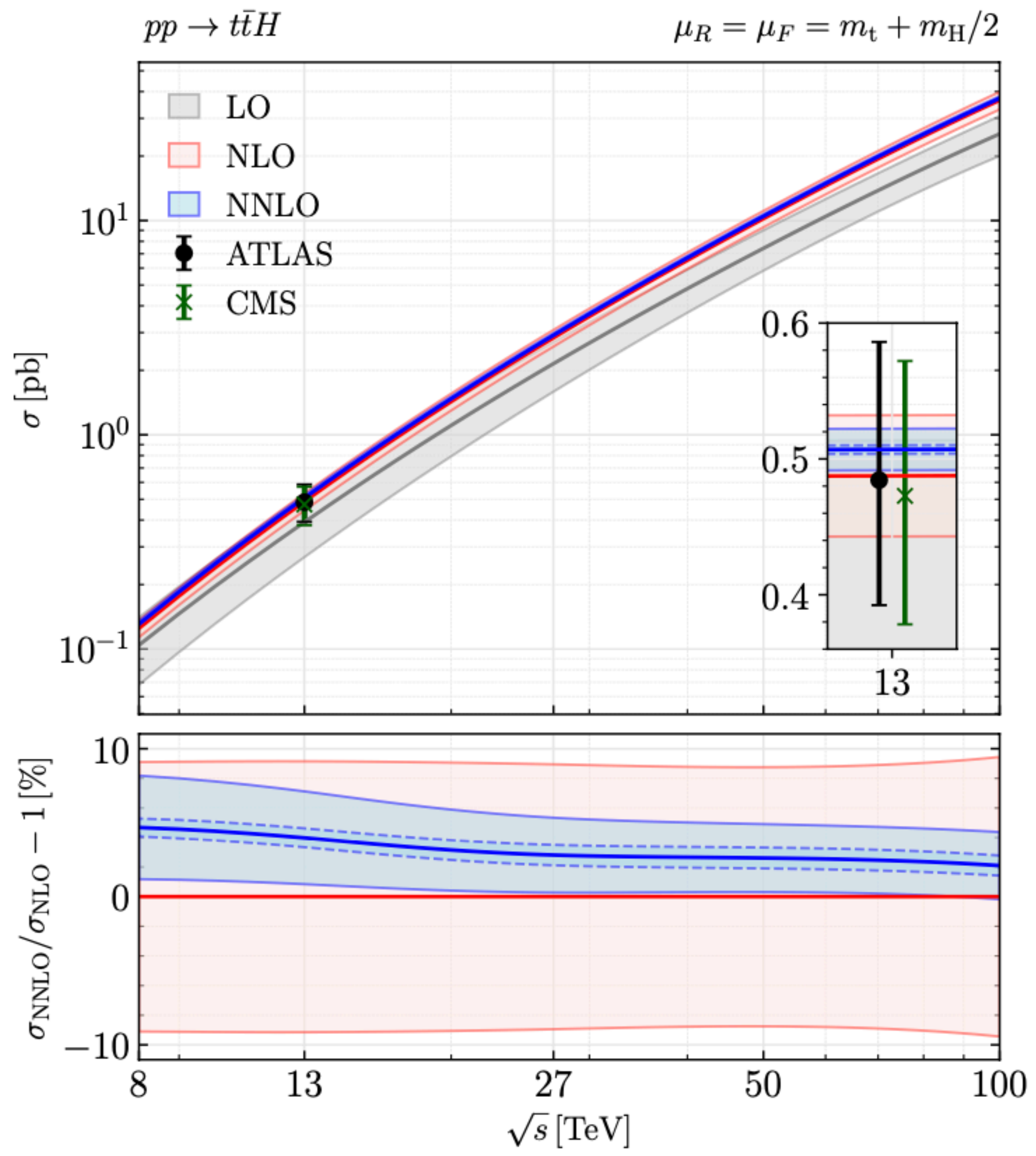

What about differential cross sections?

Approximation in the high energy limit

It is known that a massive amplitude can be factorized into a massless amplitude and a collinear factor for each leg in the high-energy limit

Mitov, Moch: hep-ph/0612149

$$\mathcal{M}^{[\mathrm{p}],(m)}\left(\{k_i\},\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right) =$$

$$\prod_{i\in\,\{\text{all legs}\}}\left(Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)\right)^{\frac{1}{2}} \times \mathcal{M}^{[\mathrm{p}],(m=0)}\left(\{k_i\},\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\varepsilon\right)$$

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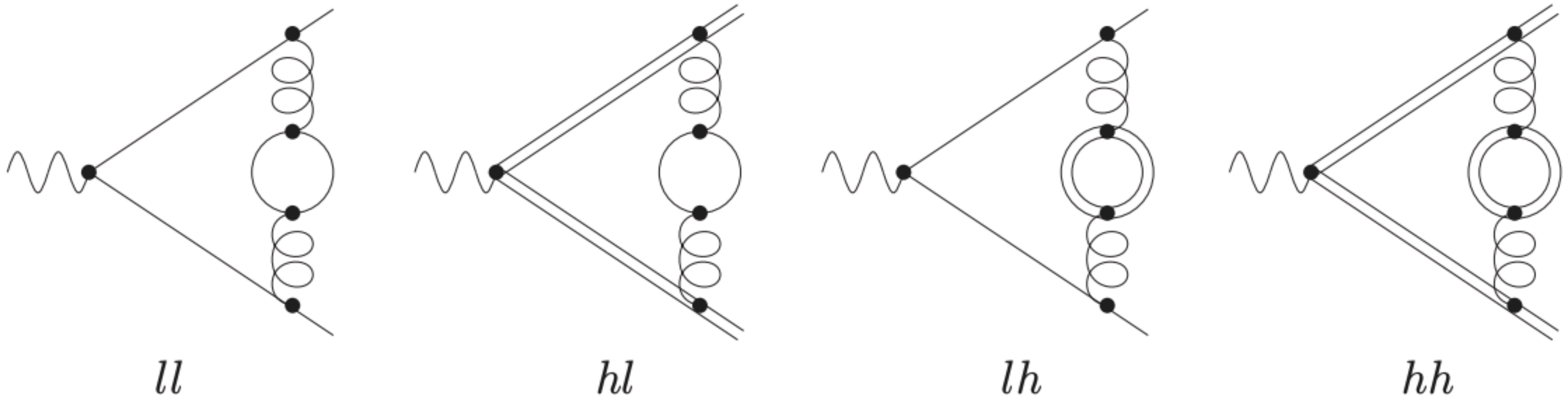
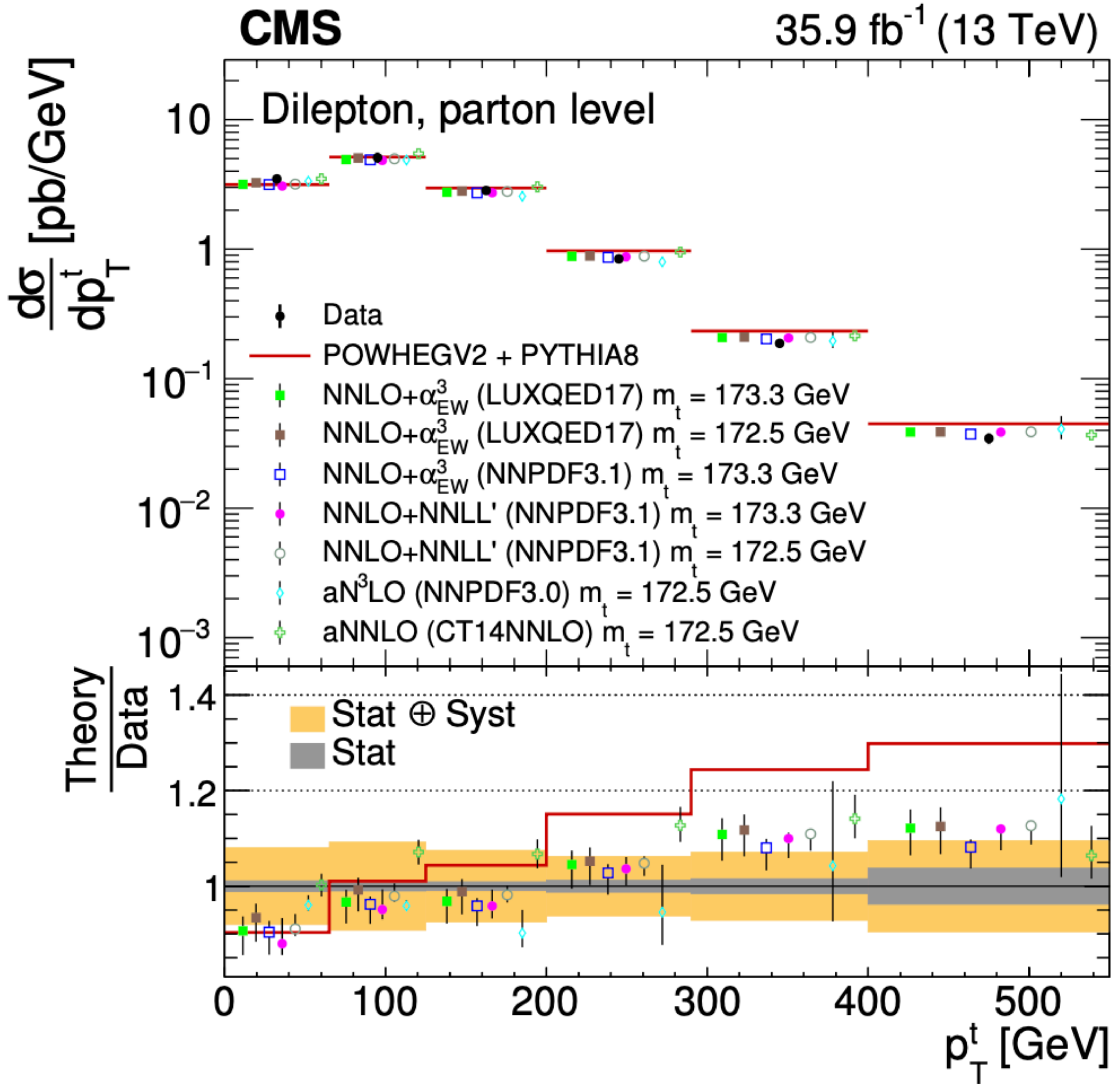


But the heavy-quark bubbles are not included!

Top quark pair production



High energy factorization has been applied in the resummation for top quark pair production

1205.3662
1306.1537
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1601.07020
1803.07623
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Best precision:
NNLO+NNLL' in QCD + NLO in EW

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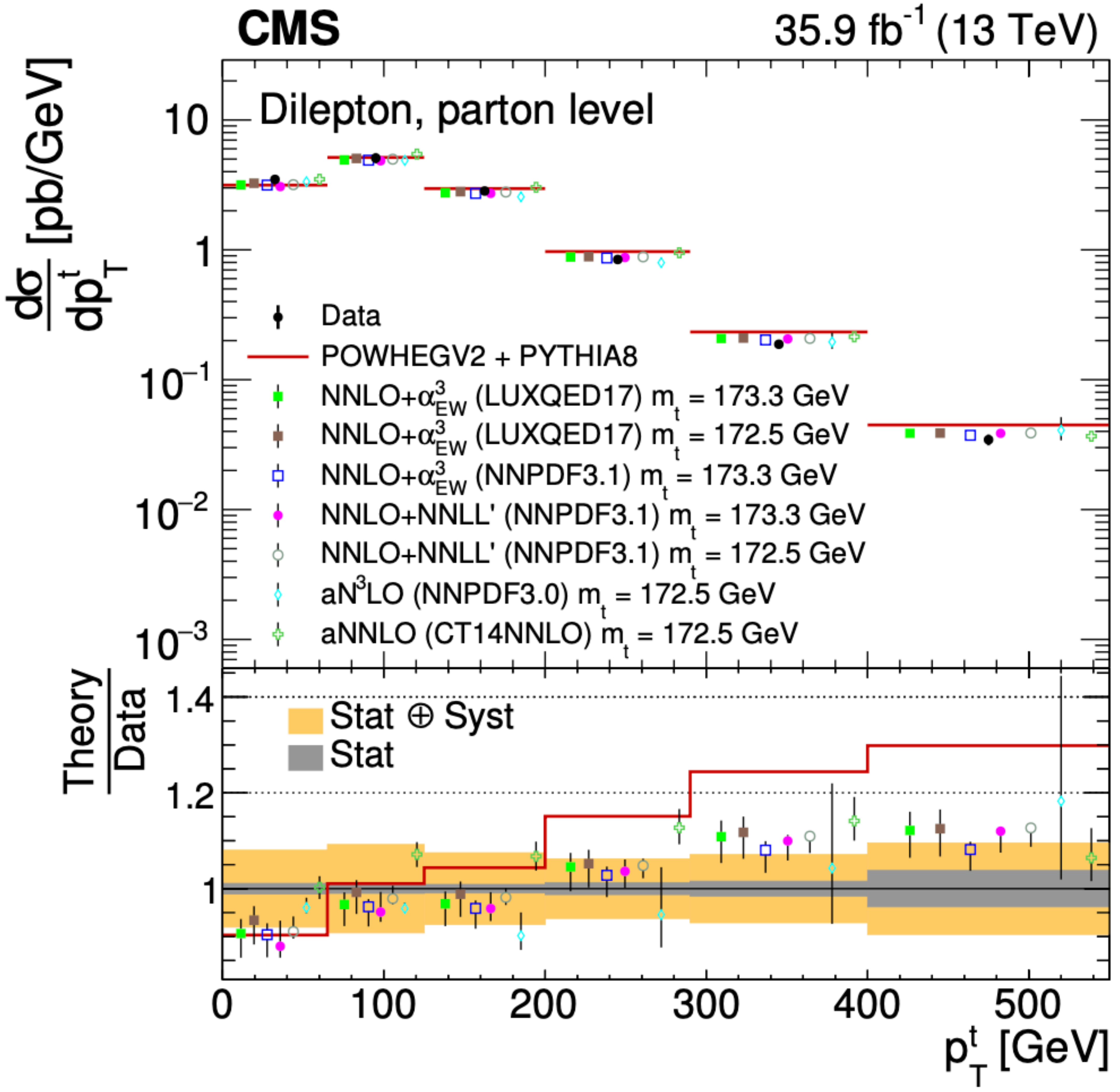

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But the factorization of heavy quark bubbles was not understood…

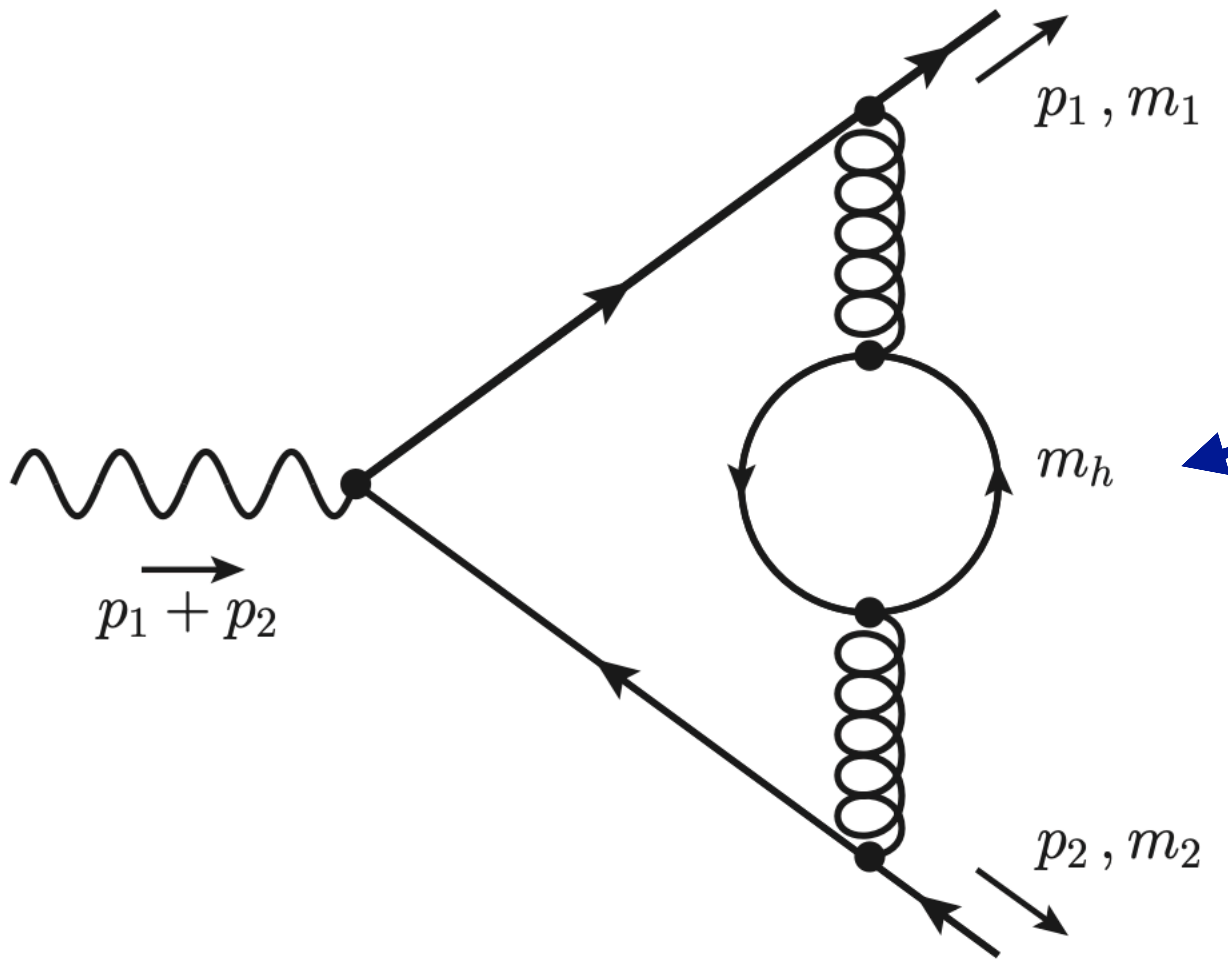
Heavy-quark bubbles

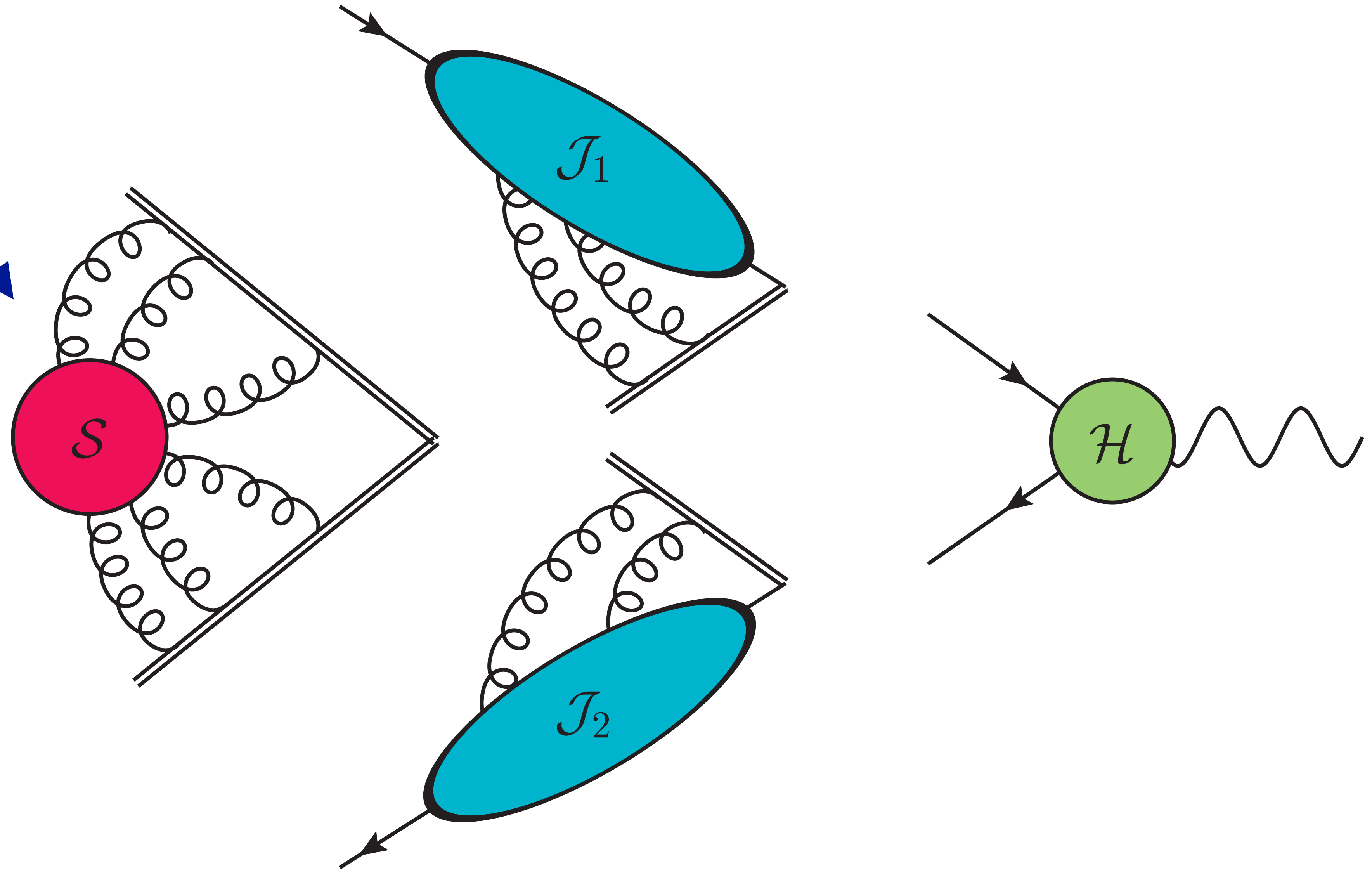
Wang, Xia, LLY, Ye: 2312.12242

A new factorization formula

$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle$$

A new soft function





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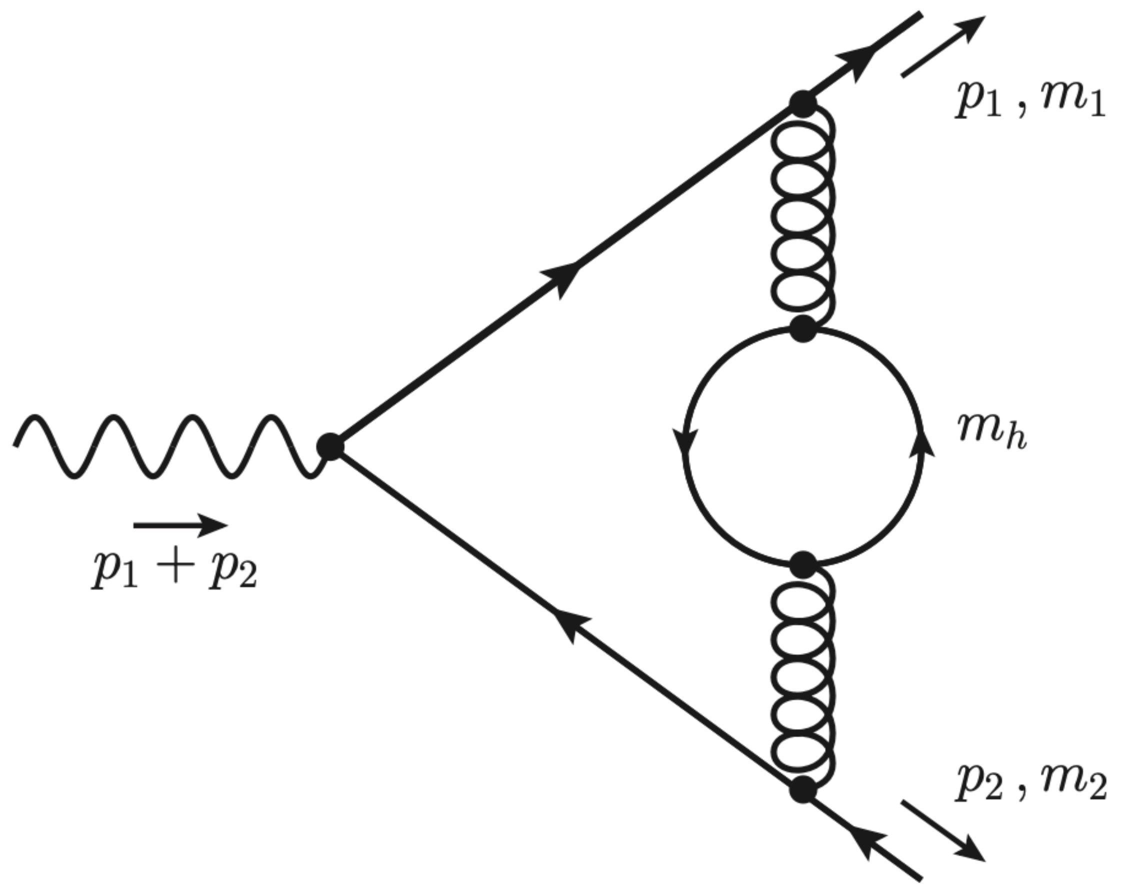


$$\text{hard}: k^\mu \sim \sqrt{|s|}\,,$$
$$n_i\text{-collinear}: (n_i\cdot k,\, \bar{n}_i\cdot k,\, k_\perp) \sim \sqrt{|s|}\,(\lambda^2,\, 1,\, \lambda)$$
$$\text{soft}: k^\mu \sim \sqrt{|s|}\,\lambda\,.$$

Rapidity divergence: analytic regulator

$$I_{\{a_i\}} \equiv \mu^{4\epsilon}\int \frac{dk_1}{(2\pi)^d}\frac{dk_2}{(2\pi)^d}\frac{1}{[k_1^2-m_h^2]^{a_1}}\frac{1}{[k_2^2-m_h^2]^{a_2}}\frac{1}{[(k_1+k_2)^2]^{a_3}}\frac{1}{[(k_1+k_2-p_1)^2-m_1^2]^{a_4}}$$
$$\times\frac{(-\tilde{\mu}^2)^\nu}{[(k_1+k_2+p_2)^2-m_2^2]^{a_5+\nu}}\frac{1}{[(k_1-p_1)^2]^{a_6}}\frac{1}{[(k_1+p_2)^2]^{a_7}}\,, \quad (3.4)$$

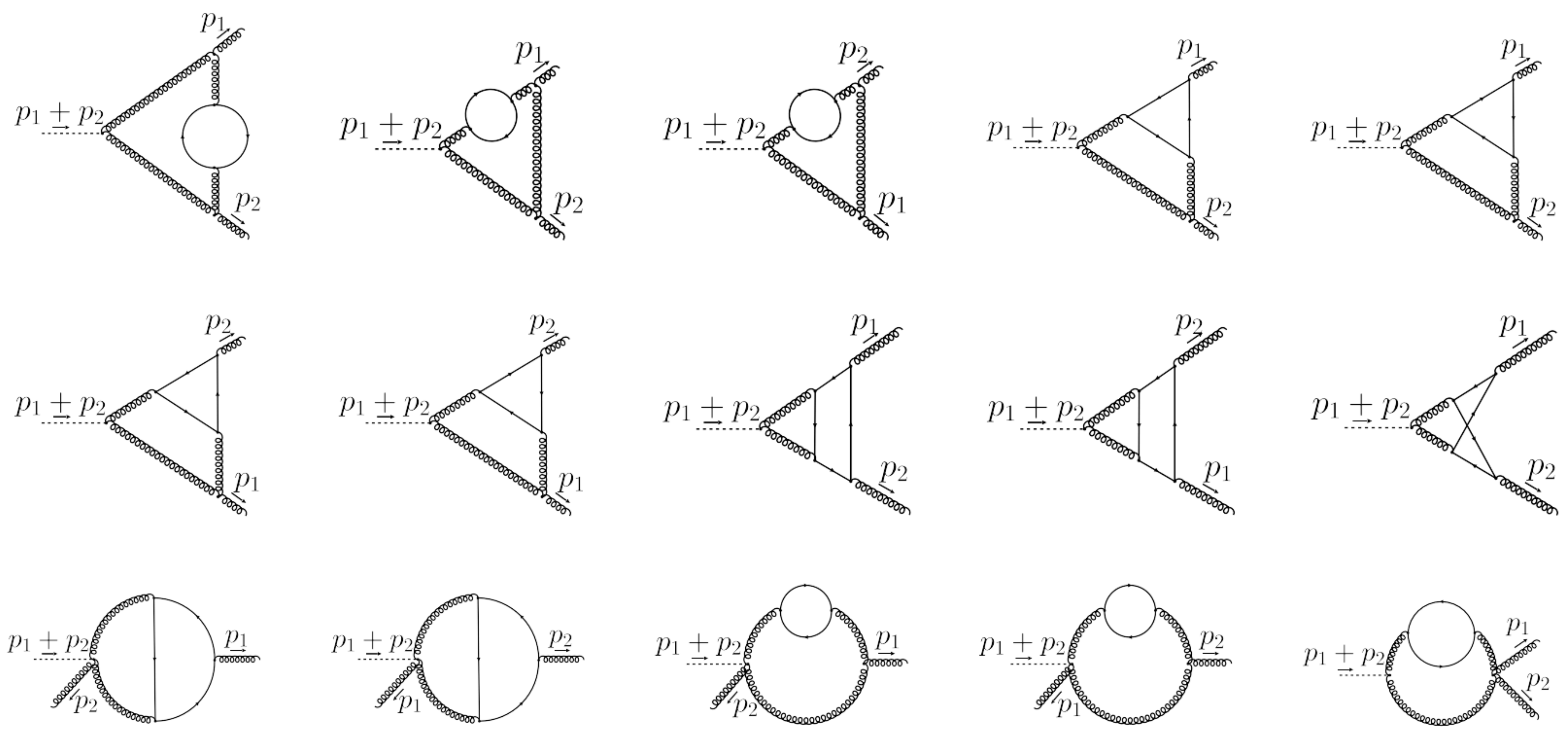
$$\boldsymbol{\mathcal{S}}(\{p\},\{m\}) = 1+\left(\frac{\alpha_s}{4\pi}\right)^2\sum_{\substack{i,j\\ i\neq j}}(-\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\sum_h \mathcal{S}^{(2)}(s_{ij},m_h^2)+\mathcal{O}(\alpha_s^3)$$

$$\mathcal{S}^{(2)}(s_{ij},m_h^2) = T_F\left(\frac{\mu^2}{m_h^2}\right)^{2\epsilon}\left(-\frac{4}{3\epsilon^2}+\frac{20}{9\epsilon}-\frac{112}{27}-\frac{4\zeta_2}{3}\right)\ln\frac{-s_{ij}}{m_h^2}$$

Validation of the new formula

Wang, Xia, LLY, Ye: 2312.12242

$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle$$



Checked in various situations:

- Quark form factors: heavy-heavy, heavy-light, light-light
- Gluon form factor
- Top quark pair amplitude

Two-loop amplitudes for tTH in the high-energy limit

Wang, Xia, LLY, Ye: 2402.00431

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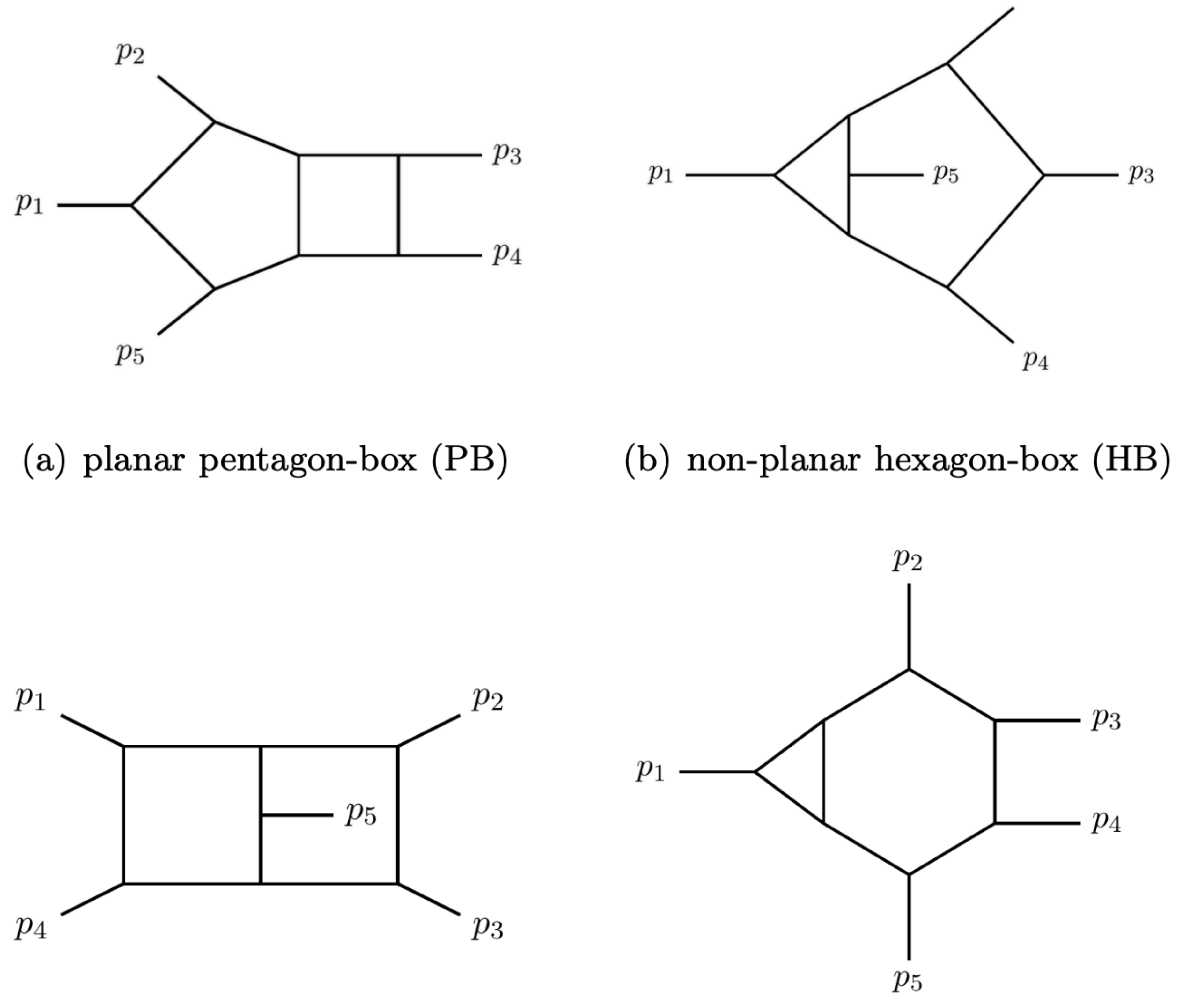


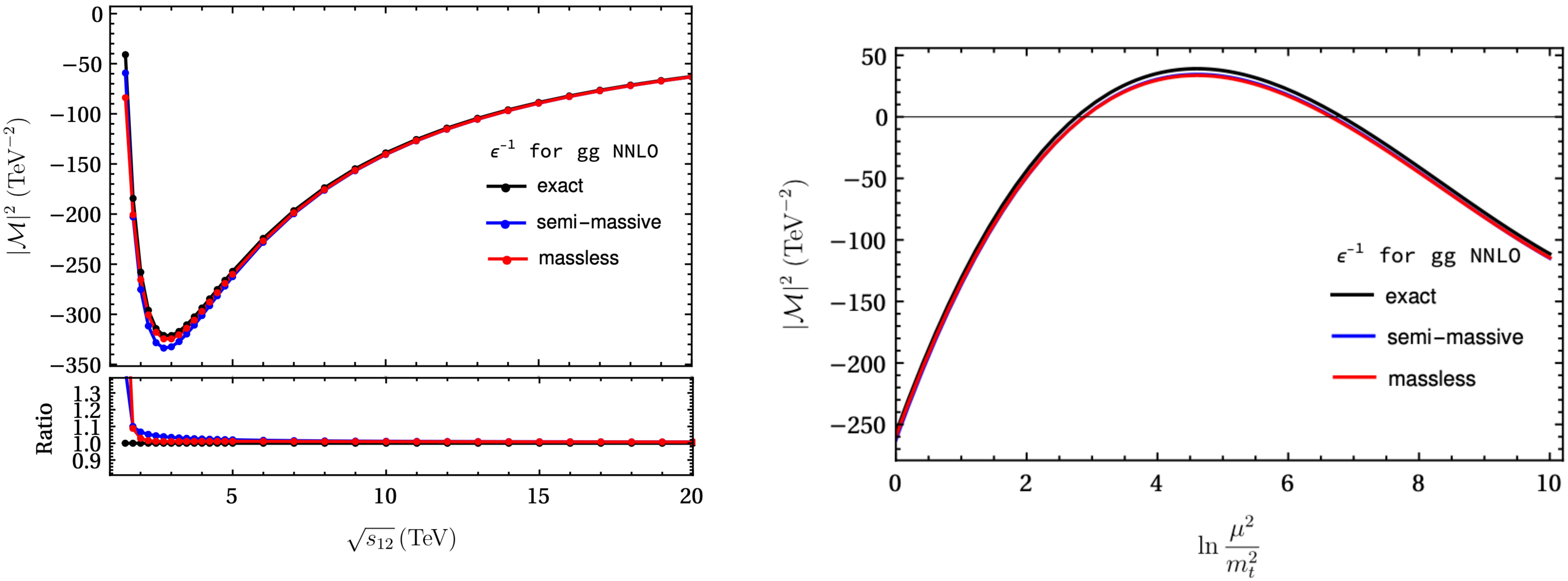
(a) planar pentagon-box (PB) (b) non-planar hexagon-box (HB)

(c) non-planar double pentagon (DP) (d) planar hexagon-triangle (HT)

- Massless amplitudes computed using standard techniques
- Very large expressions, simplified using MultivariateApart
- Fast numeric evaluation with PentagonMI

Numerical results

Wang, Xia, LLY, Ye: 2402.00431



IR poles validated against exact results in Chen, Ma, Wang, LLY, Ye: 2202.02913

Note: without the heavy quark bubble, the scale-dependence would be wrong!

Numerical results

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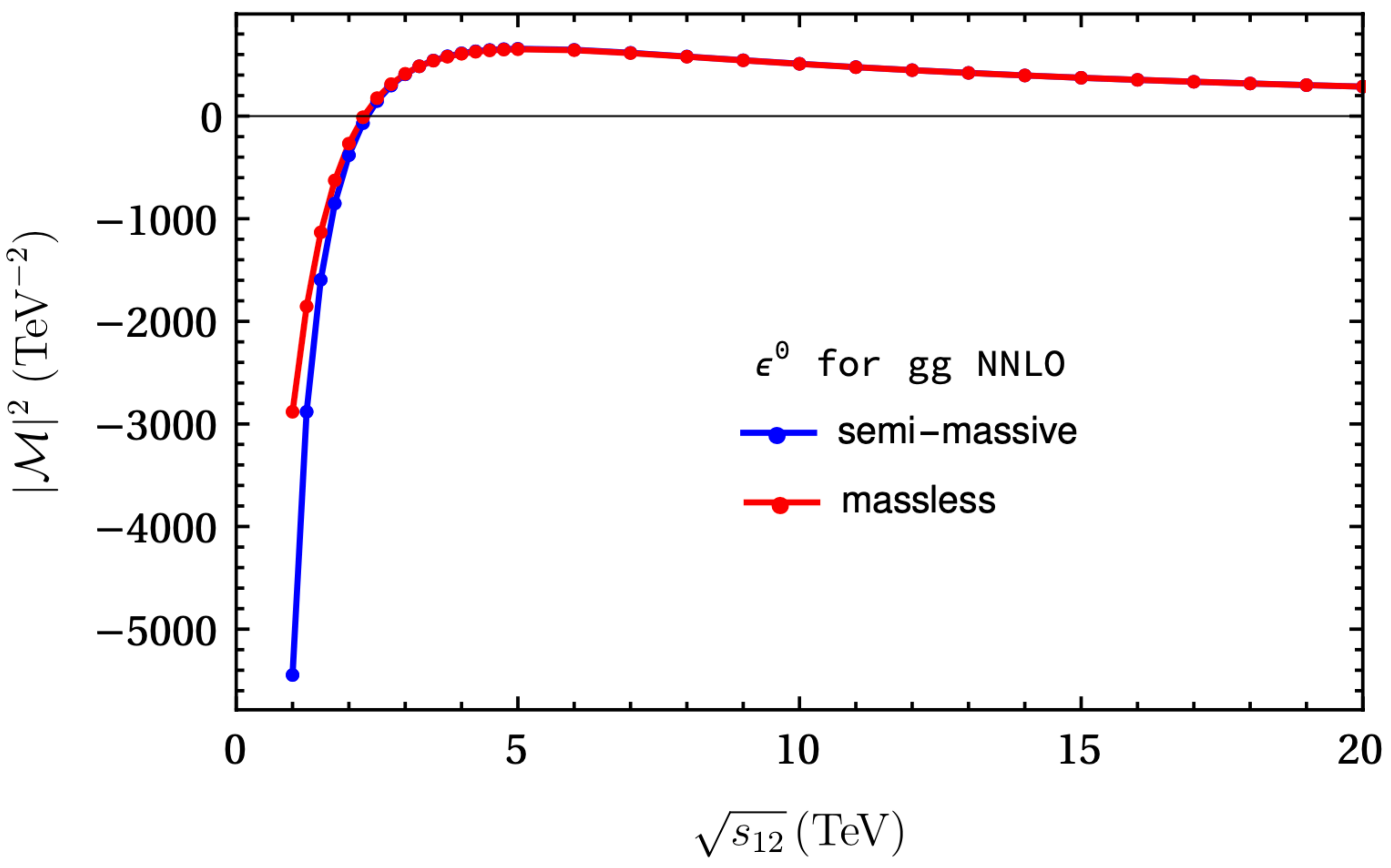


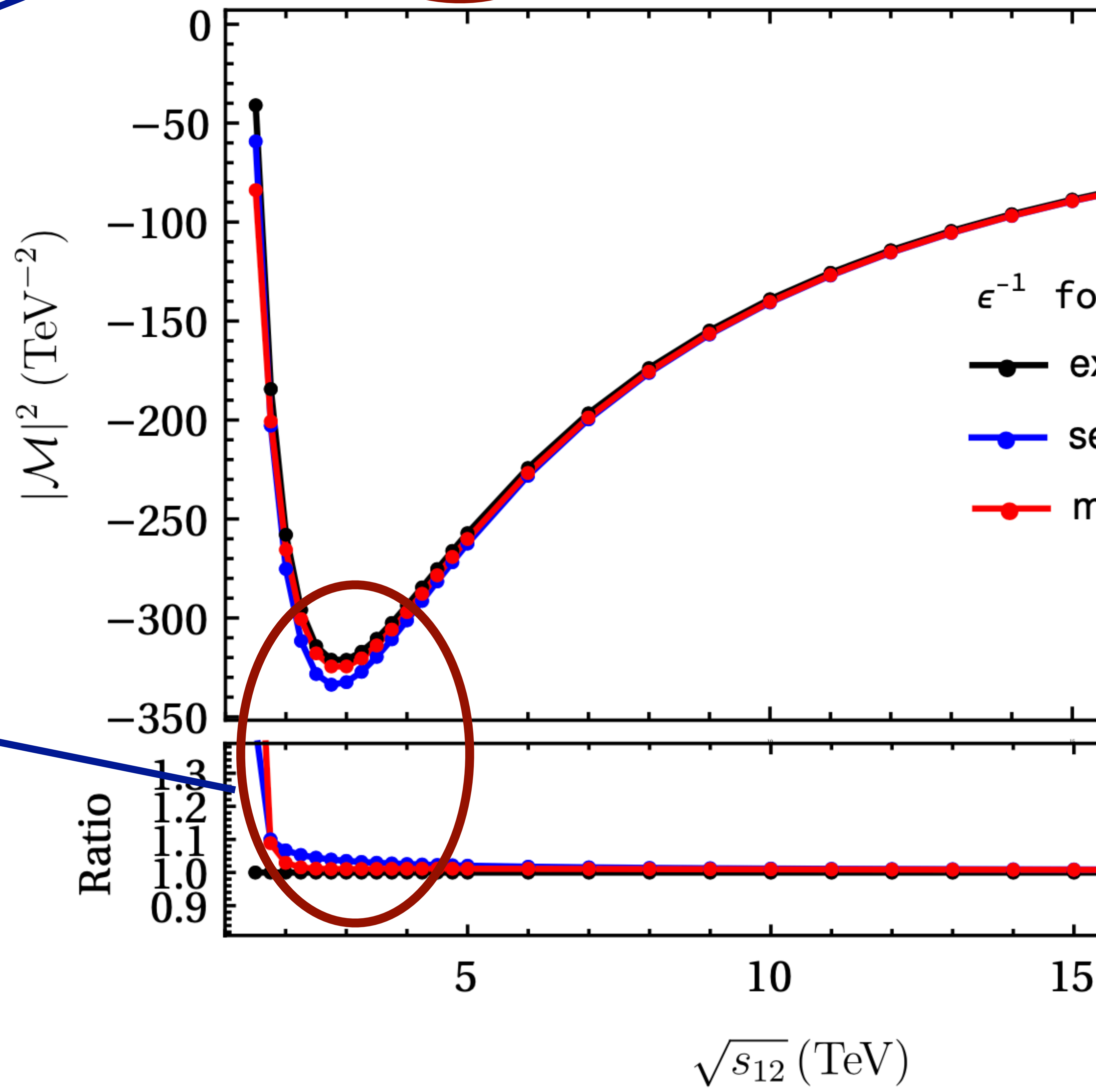
- Two-loop amplitudes at high energies are ready
- Combine with low energy approximations (threshold / soft Higgs)?
- Differential cross sections (IR subtraction)?

Towards sub-leading factorization

$$\left|\mathcal{M}^{\text{massive}}(\{p\},\{m\})\right\rangle = \prod_i \left(\mathcal{Z}_{[i]}^{(m|0)}(\{m\})\right)^{1/2} \boldsymbol{\mathcal{S}}(\{p\},\{m\}) \left|\mathcal{M}^{\text{massless}}(\{p\})\right\rangle + \mathcal{O}\left(\frac{m^2}{s_{ij}}\right)$$

Power corrections to the factorization formula

Important for intermediate energy range



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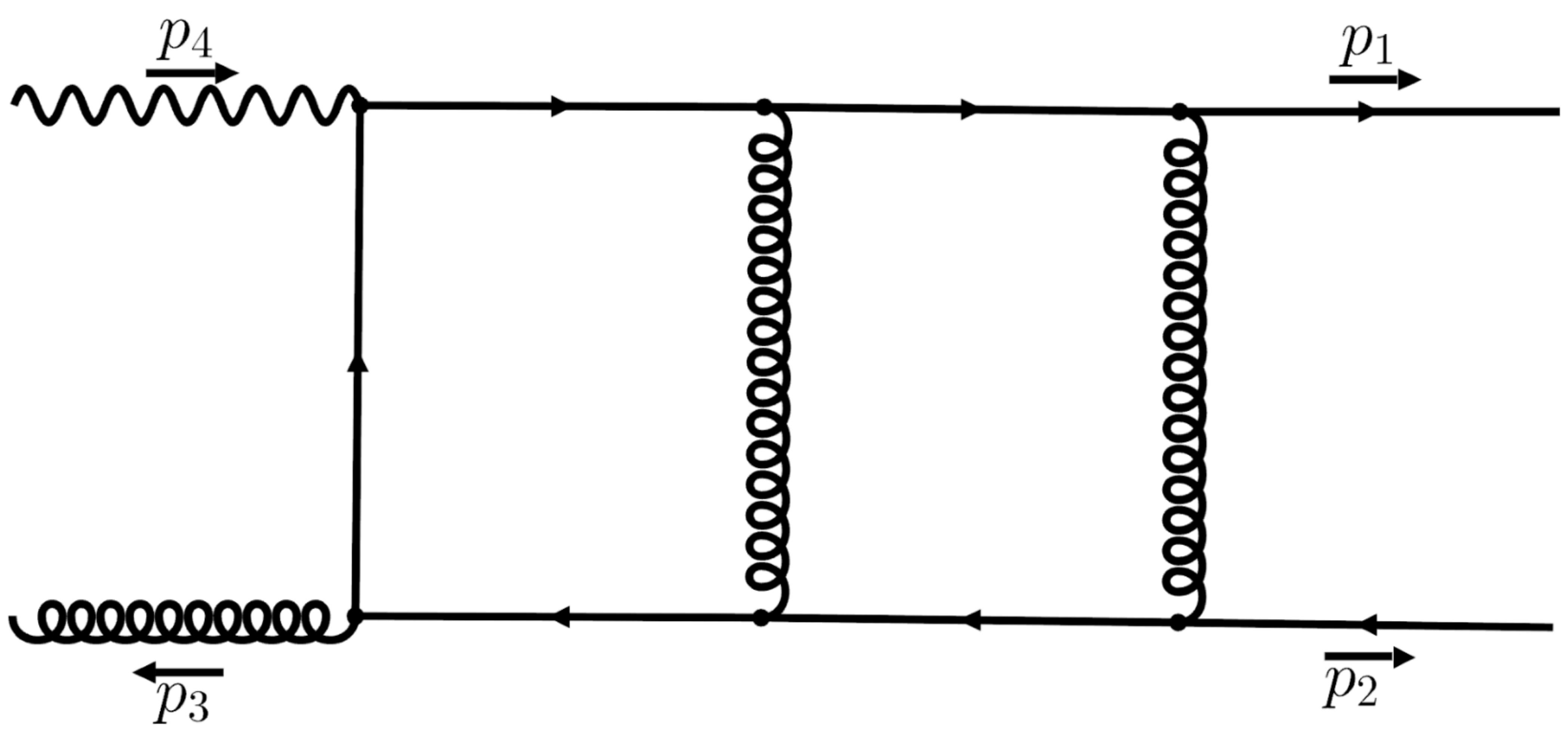
Ongoing: analyzing sub-leading corrections in $1 \to 3$ form factors using two methods



- Small-mass expansion
- Method of regions

Summary and outlook

- The tTH production is important for probing the top quark Yukawa coupling
- Theoretical status:
  - NLO+NNLL resummation for differential cross sections
  - NNLO with soft Higgs approximation for total cross section
  - Two-loop IR poles computed, but full NNLO not available
- Towards NNLO prediction at high energies
  - High energy factorization formula for QCD amplitudes
  - Applied to tTH production: approximate two-loop amplitudes now available
  - Future: sub-leading corrections to the factorization formula
  - Future: combine with real emissions (IR subtraction) for differential cross sections

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Thank you!