第十四届全国粒子物理学术会议（2024）

UV divergences of loops, the Higgs boson’s low mass and the graviton loop in quantizing Einstein gravity

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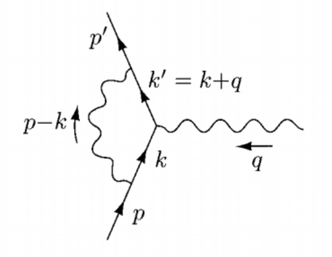
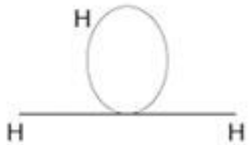
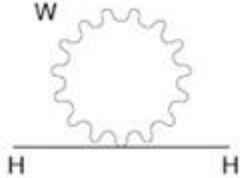
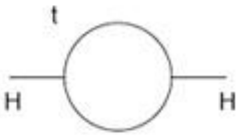
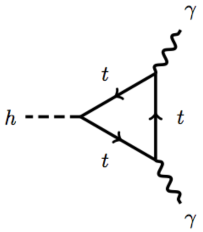
Based on arXiv:2305.18104，2403.09487

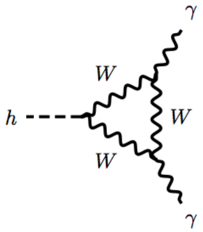








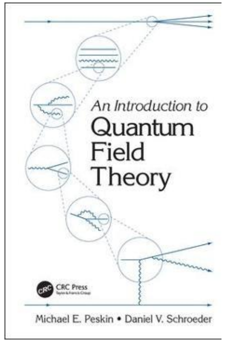


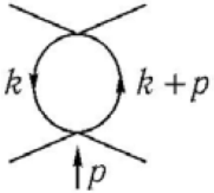


Outline:

- I. Background: UV divergences of loops
- II. Free flow of ideas --- UV-free scheme
- III. The hierarchy problem of Higgs mass
- IV. Graviton loop in Einstein gravity
- V. Summary and outlook

I. Background: UV divergences of loops





UV divergence!

$\mathcal{T}_F$

Not well-defined

Paradigm procedure

Devil or Angel?

Devil

∞

−

Devil

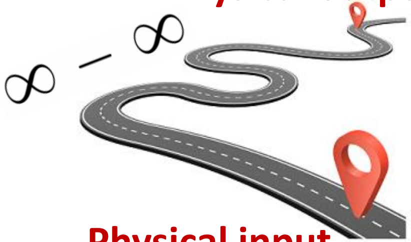
∞

=

Angel

Regularization Renormalization

Physical output



Physical input

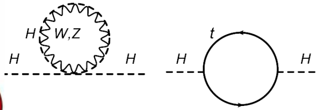
Loop road

Two devils over the **renormalization building**

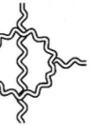
Renormalization

Log divergence is OK,
power-law divergence
is problematic

Large Devil (Higgs mass)



Huge Devil (Gravity)



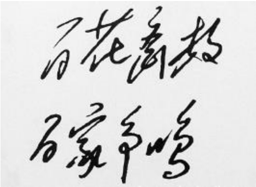
Renormalization

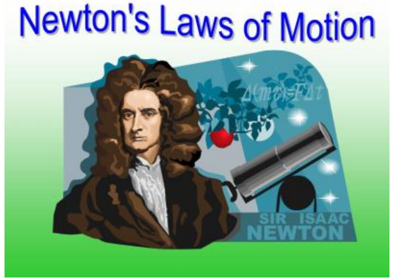
$\infty - \infty$

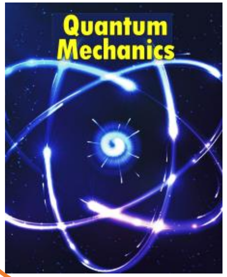
QFT (SM)

Next

II. Free flow of ideas
--- UV-free scheme







Negligible

Low-energy corrections

Loop

UV regions (Planck scale)

Loop

Negligible?!

Regularization & renormalization

本故事纯属虚构

UV-free scheme

arXiv:2305.18104

A presumption:

The physical contributions of loops are finite with contributions from UV regions being insignificant.

To obtain the physical results of loops, an equation is introduced

$$\mathcal{T}_{\rm F} \Rightarrow \mathcal{T}_{\rm P} = \left[\int d\xi_1 \cdots d\xi_i \frac{\partial \mathcal{T}_{\rm F}(\xi_1, \cdots, \xi_i)}{\partial \xi_1 \cdots \partial \xi_i}\right]_{\{\xi_1,\cdots,\xi_i\}\to 0} + C$$

(primary antiderivative + boundary constant)

or an equivalent form $\mathcal{T}_{\rm P} = \left[\int (d\xi)^n \frac{\partial^n \mathcal{T}_{\rm F}(\xi)}{\partial \xi^n}\right]_{\xi\to 0} + C,$

$\mathcal{T}_{\rm F}$

$\mathcal{T}_{\rm P}$

除非 ...

UV-free scheme:

assume that the physical transition amplitude $\mathcal{T}_{\rm P}$ with propagators can be described by an equation of

$$\mathcal{T}_{\rm P}=\Big[\int d\xi_1\cdots d\xi_i\,\frac{\partial\mathcal{T}_{\rm F}(\xi_1,\cdots,\xi_i)}{\partial\xi_1\cdots\partial\xi_i}\Big]_{\{\xi_1,\cdots,\xi_i\}\to 0}+C\,,\quad(1)$$

a. Tree-level:

the photon propagator $\frac{-ig_{\mu\nu}}{p^2+i\epsilon}$,

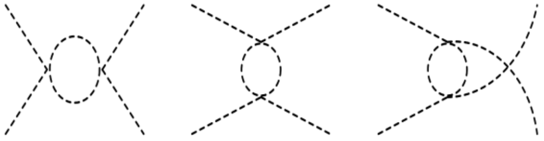
$$\mathcal{T}_{\rm F}(\xi)=\frac{-ig_{\mu\nu}}{p^2+\xi+i\epsilon},\quad \frac{\partial\mathcal{T}_{\rm F}(\xi)}{\partial\xi}=\frac{-ig_{\mu\nu}(-1)}{(p^2+\xi+i\epsilon)^2},$$

$$\Big[\int d\xi\,\frac{\partial\mathcal{T}_{\rm F}(\xi)}{\partial\xi}\Big]=\frac{-ig_{\mu\nu}}{p^2+\xi+i\epsilon},\quad \text{with}\quad C=0$$

$$\mathcal{T}_{\rm P}=\Big[\int d\xi\,\frac{\partial\mathcal{T}_{\rm F}(\xi)}{\partial\xi}\Big]_{\xi\to 0}=\frac{-ig_{\mu\nu}}{p^2+i\epsilon}$$

the gauge field propagator restored

b. Loop-level Log:



ϕ^4 theory

The physical scattering amplitude

$$\mathcal{T}_{\rm P}(s)=\Big[\int d\xi\,\frac{\partial\mathcal{T}_{\rm F}(\xi)}{\partial\xi}\Big]_{\xi\to 0}+C_1$$

$$=\Big[\frac{-\lambda^2}{2}\int d\xi\int\frac{d^4k}{(2\pi)^4}\,\frac{-i}{(k^2-m^2+\xi)^2}\,\frac{i}{(k+q)^2-m^2}\Big]_{\xi\to 0}+C_1\,,$$

$$\mathcal{T}_{\rm P}(s)=\frac{-i\lambda^2}{32\pi^2}\int_0^1 dx\,\log[m^2-x(1-x)s]+C_1\,.$$

A freedom of ξ in propagators

Considering the renormalization conditions, $s=4m^2$, $t=u=0$. ⇒ $C_1=\frac{i\lambda^2}{32\pi^2}\int_0^1 dx\,\log[m^2-4m^2x(1-x)]\,.$

No troublesome UV divergence in loop calculations!

In massless limit $\mathcal{T}_{\rm P}=\mathcal{T}_{\rm P}(s)+\mathcal{T}_{\rm P}(t)+\mathcal{T}_{\rm P}(u)$

$$s=-t=-u=\mu^2 \qquad =\frac{i\lambda^2}{32\pi^2}\Big(\log\frac{\mu^2}{s}+\log\frac{\mu^2}{-t}+\log\frac{\mu^2}{-u}\Big)$$

the n-point physical correlation function $G^{(n)}_{\rm P}$ can be set by the physical field $\phi_{\rm P}(x)$ with $\phi_{\rm P}(x)=Z^{1/2}\phi(x,\mu)$, and the rescaling factor Z is finite here. The local correlation function $G^{(n)}$ (shorthand for a full expression $G^{(n)}(\phi,\lambda,m,\cdots,\mu)$) in the perturbation expansion can be written as $G^{(n)}=Z^{-n/2}G^{(n)}_{\rm P}$. Considering $\frac{dG^{(n)}_{\rm P}}{d\mu}=0$, the variation of μ in the massless limit can be described by a relation

$$(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial\lambda}+n\gamma)G^{(n)}=0\,.$$

This is the form of the Callan-Symanzik equation [5, 6], and we have another picture about it in UV-free scheme. The μ-dependent term in UV-free scheme is from the boundary constant C. For the ϕ^4 theory in the massless limit, the one-loop result of the parameter γ is zero ($\mathcal{T}^{2p}_{\rm P}=0$). The beta function can be derived by Eq. (10), with the result

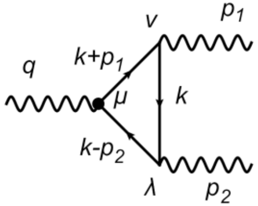
$$\beta=-i\mu\frac{\partial}{\partial\mu}\mathcal{T}_{\rm P} =\frac{3\lambda^2}{16\pi^2}+\mathcal{O}(\lambda^3)\,.$$

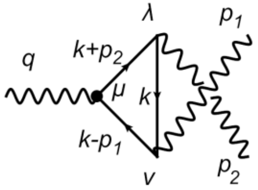
...

UV physics being free

An illustration:

electron physical charge $e=e_0+\Delta e=e_\mu+\Delta e_\mu$





γ^5 the original form

$$\partial_\mu j^{\mu 5}=iq_\mu\mathcal{T}^{\mu\nu\lambda}_{\rm P}\epsilon^*_\nu(p_1)\epsilon^*_\lambda(p_2) =-\frac{e^2}{16\pi^2}(\frac{2}{3}-2\log r)\varepsilon^{\alpha\nu\beta\lambda}F_{\alpha\nu}F_{\beta\lambda}$$

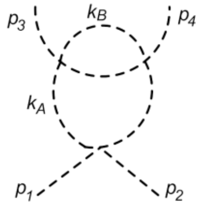
Taking $C_0=2\log r$

If $C_0=\frac{2}{3}$

SM self-consistent

charge values of quarks coincidence, or correlation?

two-loop transition



$$\mathcal{T}_{\rm P}=\Big[\int d\xi\frac{\partial\mathcal{T}_{\rm F}(\xi)}{\partial\xi}\Big]_{\xi\to0}+C$$

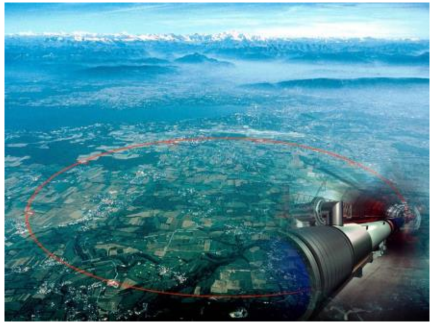
$$=\Big[\frac{(-i\lambda)^3}{2}\int d\xi\int\frac{d^4k_A}{(2\pi)^4}\frac{d^4k_B}{(2\pi)^4}\frac{i}{k_A^2-m^2}\frac{i}{(k_A+q)^2-m^2}\times\frac{-i}{(k_B^2-m^2+\xi)^2}\frac{i}{(k_B+k_A+p_3)^2-m^2}\Big]_{\xi\to0}+C$$

with $q=p_1+p_2$

Log divergences are OK

除非 ...

III. The hierarchy problem (c. Loop-level Λ^2, Λ^4)

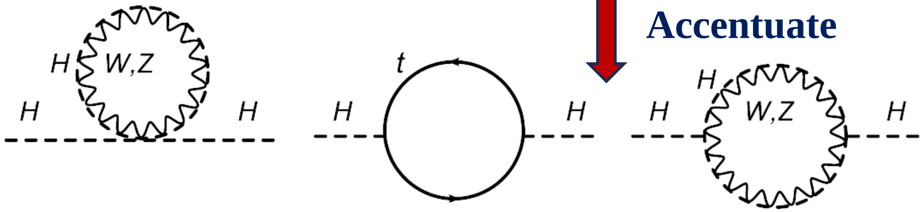


LHC

Higgs boson

125 GeV

Accentuate



The hierarchy problem

$$M_H^2 = (M_H^0)^2 + \frac{3\Lambda^2}{8\pi^2 v^2}\left[M_H^2 + 2M_W^2 + M_Z^2 - 4m_t^2\right]$$

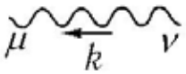
Fine-tuning!

A real problem for renormalization!

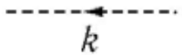
Power-law divergences (Λ^2 , Λ^4)

For W, Z

In Feynman-'t Hooft gauge (Λ^2)



$$= \frac{-ig^{\mu\nu}}{k^2 - m_A^2};$$



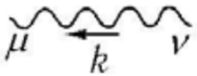
$$= \frac{i}{k^2 - m_A^2}.$$

√

Supersymmetry?

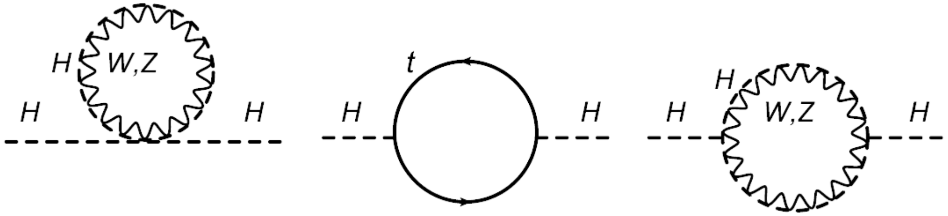
In unitarity gauge (Λ^4)

✕



$$= \frac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_A^2}\right)$$

Large Devil
(Higgs mass)



Power-law divergences (Λ^2 , Λ^4)

In UV-free scheme

Higgs in the first diagram

$$\mathcal{T}_{\rm P}^{H1} = \Big[\int d\xi_1 d\xi_2 \frac{\partial \mathcal{T}_{\rm F}^{H1}(\xi_1,\xi_2)}{\partial\xi_1\partial\xi_2}\Big]_{\{\xi_1,\xi_2\}\to 0} + C$$

$$= \Big[(-3i)\frac{m_H^2}{2v^2}\int d\xi_1 d\xi_2 \int \frac{d^4k}{(2\pi)^4} \times \frac{2i}{(k^2 - m_H^2 + \xi_1 + \xi_2)^3}\Big]_{\{\xi_1,\xi_2\}\to 0} + C\,.$$

After integral, one has

$$\mathcal{T}_{\rm P}^{H1} = i\frac{3m_H^4}{32\pi^2 v^2}(\log\frac{1}{m_H^2} + 1) + C$$

$$= i\frac{3m_H^4}{32\pi^2 v^2}(\log\frac{\mu^2}{m_H^2} + 1)\,.$$

V (V=W,Z) in **unitary gauge**

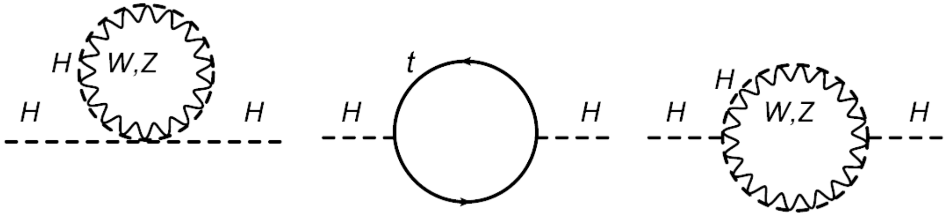
$$\mathcal{T}_{\rm P}^{V1} = \Big[\int d\xi_1 d\xi_2 d\xi_3 \frac{\partial \mathcal{T}_{\rm F}^{V1}(\xi_1,\xi_2,\xi_3)}{\partial\xi_1\partial\xi_2\partial\xi_3}\Big]_{\{\xi_1,\xi_2,\xi_3\}\to 0} + C$$

$$= \Big[i\frac{2m_V^2}{v^2 s_V}\int d\xi_1 d\xi_2 d\xi_3 \int \frac{d^4k}{(2\pi)^4} g_{\mu\nu} \times \frac{6i(g^{\mu\nu} - k^\mu k^\nu/m_V^2)}{(k^2 - m_V^2 + \xi_1 + \xi_2 + \xi_3)^4}\Big]_{\{\xi_1,\xi_2,\xi_3\}\to 0} + C\,,$$

where the symmetry factor s_V is $s_V = 1,\ 2$ for W, Z respectively. After integral, one has

$$\mathcal{T}_{\rm P}^{V1} = i\frac{2m_V^2}{v^2 s_V}\frac{m_V^2}{16\pi^2}(3\log\frac{1}{m_V^2} + \frac{5}{2}) + C$$

$$= i\frac{2m_V^2}{v^2 s_V}\frac{3m_V^2}{16\pi^2}(\log\frac{\mu^2}{m_V^2} + \frac{5}{6})\,.$$



Power-law divergences (Λ^2 , Λ^4)

top quark loop

$$\mathcal{T}_{\rm P}^{t} = -\frac{3m_t^2}{v^2}\frac{i}{4\pi^2}\int_0^1 dx[m_t^2 - p^2x(1-x)] \times (3\log\frac{1}{m_t^2 - p^2x(1-x)} + 2) + C$$
$$= -\frac{3m_t^4}{v^2}\frac{3i}{4\pi^2}\int_0^1 dx[1 - \frac{p^2}{m_t^2}x(1-x)] \times (\log\frac{\mu^2}{m_t^2 - p^2x(1-x)} + \frac{2}{3})\,.$$

Higgs in the third diagram

$$\mathcal{T}_{\rm P}^{H3} = \frac{9m_H^4}{2v^2}\frac{i}{16\pi^2}\int_0^1 dx\,\log\frac{1}{m_H^2 - x(1-x)p^2} + C$$
$$= i\frac{9m_H^4}{32\pi^2v^2}\int_0^1 dx\,\log\frac{\mu^2}{m_H^2 - x(1-x)p^2}\,.$$

V (V=W,Z) **in the third diagram**

$$\mathcal{T}_{\rm P}^{V3} = \frac{4m_V^4}{v^2 s_V}\frac{6i}{16\pi^2}\int_0^1 dx([\frac{1}{2} - \frac{p^2}{m_V^2}(x - x^2 + \frac{1}{12}) + \frac{p^4}{m_V^4}\frac{x(1-x)}{12}(20x - 20x^2 - 1)]\log\frac{1}{m_V^2 - x(1-x)p^2} + \frac{1}{12} - \frac{p^2}{12m_V^2}(22x(1-x) - 1) - \frac{p^4x(1-x)}{12m_V^4}(-21x(1-x) + 1)) + C$$
$$= \frac{m_V^4}{v^2 s_V}\frac{3i}{2\pi^2}\int_0^1 dx([\frac{1}{2} - \frac{p^2}{m_V^2}(x - x^2 + \frac{1}{12}) + \frac{p^4}{m_V^4}\frac{x(1-x)(20x-20x^2-1)}{12}]\log\frac{\mu^2}{m_V^2 - x(1-x)p^2} + \frac{1}{12} - \frac{p^2(22x(1-x)-1)}{12m_V^2} - \frac{p^4x(1-x)(-21x(1-x)+1)}{12m_V^4})$$

Considering μ in the electroweak scale,

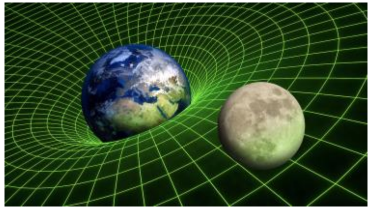
125 GeV Higgs can be obtained without fine-tuning, i.e., an alternative interpretation within SM.

Power-law divergences are OK in UV-free scheme!

除非 ...

IV. Graviton loop in Einstein gravity

$$\mathcal{S}=\int d^4X\sqrt{-g}\big[-\frac{2}{\kappa^2}R+\mathcal{L}_{\rm M}\big] \quad g_{\mu\nu}=\eta_{\mu\nu}+\kappa h_{\mu\nu}$$



Huge Devil (Gravity)

Loops: **Renormalization** → **Non-renormalizable Einstein Gravity**

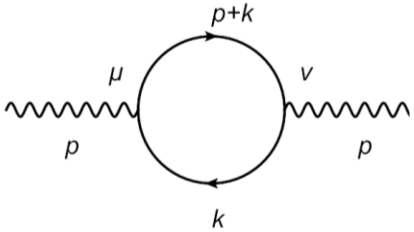
→ **Einstein Gravity cannot be quantized!?**

?

Plan B: **Another alternative method**
UV-free scheme

For the primary antiderivative $\xi-$**dependent choice**

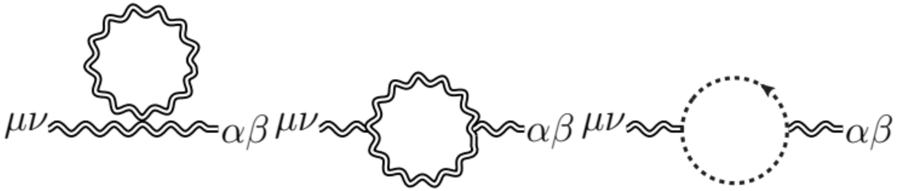
$$\mathcal{T}_{\rm P}^{t2n}=A\Big[\frac{(\xi+\Delta)^n}{n!}(\log|\xi+\Delta|-(\sum_{l=1}^{n}\frac{1}{l}))\Big]_{\xi\to 0}+C_1$$
$$=A\frac{\Delta^n}{n!}\log|\Delta|+C\,.$$



$$\mathcal{T}_{\rm P}^{\mu\nu}=-\frac{ie^2}{2\pi^2}\int_0^1 dx(p^\mu p^\nu-g^{\mu\nu}p^2)x(1-x)$$
$$\times\log(m^2-p^2x(1-x))+C^{\mu\nu}\,,$$

with the Ward identity automatically preserved by the primary antiderivative.

One-loop propagator



The $\mu\nu \leftrightarrow \alpha\beta$ asymmetry involved at one-loop level in a particle propagation means that time reversal is not invariant in quantum gravity, i.e. an arrow of time at the microscopic level.

$$\mathcal{T}_{\mathrm{P}}^{a} = \Big[i\kappa^2 \frac{i\Pi_{\mu_3\nu_3\mu_4\nu_4}}{2}\frac{i}{16\pi^2}\Big(V^{\mu_3\nu_3\mu_4\nu_4|\lambda_1\mu\nu\lambda_2\alpha\beta}p_{\lambda_1}p_{\lambda_2} \times(\xi_1-\xi_1\log\xi_1)+\frac{V^{\mu\nu\alpha\beta|\lambda_3\mu_3\nu_3\lambda_4\mu_4\nu_4}\eta_{\lambda_3\lambda_4}}{4} \times(\xi_1^2\log\xi_1-\frac{3}{2}\xi_1^2)\Big)\Big]_{\xi_1\to 0}+C_a^{\mu\nu\alpha\beta}\,. = 0.$$

$$\mathcal{T}_{\mathrm{P}}^{b} = \frac{(2i\kappa)^2}{2}\frac{i}{16\pi^2}\int_0^1 dx(-\frac{1}{4})\Big\{\frac{1}{16}[40x^2(1-x)^2p^\mu p^\nu p^\alpha p^\beta +2p^2((1-2x)^2(15x^2-15x-2)(p^\mu p^\nu\eta^{\alpha\beta}+p^\alpha p^\beta\eta^{\mu\nu}) +(10x^4-20x^3+17x^2-7x+2)(p^\nu p^\beta\eta^{\mu\alpha}+p^\mu p^\beta\eta^{\nu\alpha} +p^\nu p^\alpha\eta^{\mu\beta}+p^\mu p^\alpha\eta^{\nu\beta}))+p^4((115x^4-230x^3+103x^2 +12x+1)\eta^{\mu\nu}\eta^{\alpha\beta}+(85x^4-170x^3+139x^2-54x+3) \times(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha}))]\log\frac{1}{-p^2x(1-x)}\Big\}+C_b^{\mu\nu\alpha\beta}\,.$$

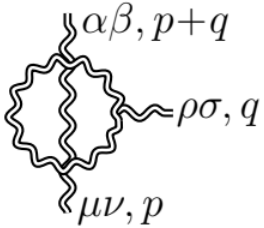
$$\mathcal{T}_{\mathrm{P}}^{c} = (-1)(i\kappa)^2\frac{4i}{16\pi^2}\int_0^1 dx(-\frac{1}{4})\Big\{\frac{1}{4}[4(4x^4-8x^3+2x^2 +2x+1)p^\mu p^\nu p^\alpha p^\beta + p^2((8x^4-16x^3+4x^2+4x-1) \times(p^\nu p^\beta\eta^{\mu\alpha}+p^\mu p^\beta\eta^{\nu\alpha}+p^\nu p^\alpha\eta^{\mu\beta}+p^\mu p^\alpha\eta^{\nu\beta}) +2x(14x^3-24x^2+13x-4)p^\mu p^\nu\eta^{\alpha\beta}+2p^\alpha p^\beta\eta^{\mu\nu} \times(14x^4-32x^3+25x^2-6x-1))+p^4(2x(11x^3-22x^2 +13x-2)(\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\nu\alpha})+(12x^4-24x^3+16x^2 -4x+1)\eta^{\mu\nu}\eta^{\alpha\beta})]\log\frac{1}{-p^2x(1-x)}\Big\}+C_c^{\mu\nu\alpha\beta}\,,$$

$n-$loop with overlapping divergences

superficial degree of divergence $2n+2$

$$\mathcal{T}_{\mathrm{P}}^{t2n} = A\frac{\Delta^n}{n!}\log|\Delta| + C$$

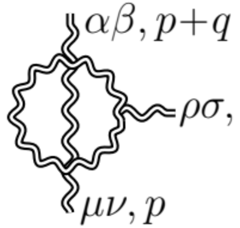
$$\mathcal{T}_{\mathrm{P}}^{\mathrm{total}} = \mathcal{T}_{\mathrm{P}}^{t2(n+1)} + \mathcal{T}_{\mathrm{P}}^{t2n} + \cdots + \mathcal{T}_{\mathrm{P}}^{t2} + \mathcal{T}_{\mathrm{P}}(\log) + \mathcal{T}_{\mathrm{P}}(\mathrm{finite})\,,$$



$$\mathcal{T}_{\mathrm{P}}^{V} = (2i)^3\kappa^5(\frac{i}{16\pi^2})^2\int_0^1 dx\int_0^1 dy\int_0^1 dz\frac{z}{2^4(1-z)^4} \times\Big\{A_3\frac{\Delta_0^3}{3!}+A_2\frac{\Delta_0^2}{2!}+A_1\Delta_0+A_0\Big\}\log\frac{1}{\Delta_0}+C^{\mu\nu\alpha\beta\rho\sigma}$$

Here Δ_0 is $\Delta_0 = b^2 - ac$, with $a = z + (1-z)x(x-1)$, $b = yzq + (1-z)x(x-1)p$, $c = yzq^2 + (1-z)x(x-1)p^2$. A_3, A_2, A_1, A_0 are coefficients related to sextic, quartic, quadratic, logarithmic divergence inputs respectively.

arXiv: 2403.09487



$$
\begin{aligned}
A_3 = \frac{z-1}{64a^8}&([440a^2 + a(1564x^2 + 1300x + 23)(z-1)\\
&+4(281x^4 - 562x^3 + 683x^2 - 402x + 273)(z-1)^2]\\
&\times\eta^{\mu\nu}(\eta^{\alpha\rho}\eta^{\beta\sigma} + \eta^{\alpha\sigma}\eta^{\beta\rho}) + [744a^2 + a(1932x^2 + 44x\\
&+1203)(z-1) + 4(297x^4 - 594x^3 + 1563x^2 - 1266x\\
&+673)(z-1)^2]\eta^{\rho\sigma}(\eta^{\alpha\nu}\eta^{\beta\mu} + \eta^{\alpha\mu}\eta^{\beta\nu}) + [440a^2\\
&+a(1564x^2 - 1100x + 2423)(z-1) + 4(281x^4\\
&-562x^3 + 683x^2 - 402x + 273)(z-1)^2]\eta^{\alpha\beta}(\eta^{\mu\rho}\eta^{\nu\sigma}\\
&+\eta^{\mu\sigma}\eta^{\nu\rho}) + [1032a^2 + a(3396x^2 - 3020x + 801)\\
&\times(z-1) + 4(591x^4 - 1182x^3 + 1101x^2 - 510x + 215)\\
&\times(z-1)^2](\eta^{\alpha\rho}\eta^{\beta\nu}\eta^{\mu\sigma} + \eta^{\alpha\nu}\eta^{\beta\rho}\eta^{\mu\sigma} + \eta^{\alpha\nu}\eta^{\beta\sigma}\eta^{\mu\rho}\\
&+\eta^{\alpha\sigma}\eta^{\beta\nu}\eta^{\mu\rho} + \eta^{\alpha\rho}\eta^{\beta\mu}\eta^{\nu\sigma} + \eta^{\alpha\mu}\eta^{\beta\rho}\eta^{\nu\sigma} + \eta^{\alpha\mu}\eta^{\beta\sigma}\eta^{\nu\rho}\\
&+\eta^{\alpha\sigma}\eta^{\beta\mu}\eta^{\nu\rho}) + [1696a^2 + a(4844x^2 + 848x + 4147)\\
&\times(z-1) + 4(787x^4 - 1574x^3 + 2521x^2 - 1734x\\
&+795)(z-1)^2]\eta^{\alpha\beta}\eta^{\mu\nu}\eta^{\rho\sigma}) .
\end{aligned}
$$

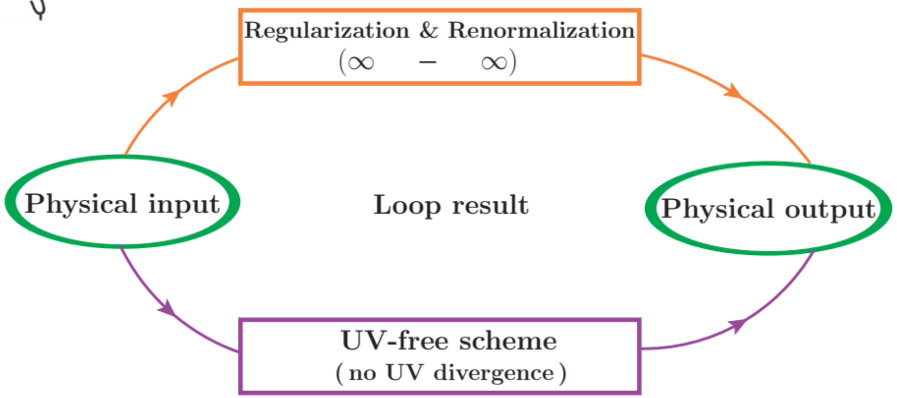
Parameter A_0 $(l = p + q)$

In the case of $p^2 = l^2 = 0$, the result is

$$
\begin{aligned}
A_0 = -\frac{(z-1)^3}{64a^8}\Big\{&16y^3z^3[a^3(8x^2-8x+7)-2a^2(4x^4-8x^3+16x^2-12x+11)yz+a(14x^4-28x^3+53x^2-39x+28)y^2z^2\\
&-14(x^2-x+1)^2y^3z^3]q^\alpha q^\beta q^\mu q^\nu q^\rho q^\sigma-8y^2z^2[a^4(6-9x+9x^2)+a^2(x-1)x(47-75x+83x^2-16x^3+8x^4)y(1-z)z\\
&-2a(x-1)x(21-25x+32x^2-14x^3+7x^4)y^2(1-z)z^2+28(x-1)x(1-x+x^2)^2y^3(1-z)z^3+a^3((x-1)x(-12+41x\\
&-41x^2)(1-z)+(-7+14x-6x^2-16x^3+8x^4)yz)](p^\rho q^\alpha q^\beta q^\mu q^\nu q^\sigma+p^\sigma q^\alpha q^\beta q^\mu q^\nu q^\rho)+8y^2z^2[a^4(7-9x+9x^2)-28\\
&\times(x-1)x(1-x+x^2)^2y^3(1-z)z^3+ay^2z^2((x-1)x(49-57x+59x^2-4x^3+2x^4)(1-z)+(-14+45x-73x^2+56x^3\\
&-28x^4)yz)+a^3((x-1)x(-3+29x-29x^2)(1-z)+(-7-9x+x^2+16x^3-8x^4)yz)+a^2yz((x-1)x(-9-21x+13x^2\\
&+16x^3-8x^4)(1-z)+(12-17x+37x^2-40x^3+20x^4)yz)](p^\beta q^\alpha q^\mu q^\nu q^\rho q^\sigma+p^\alpha q^\beta q^\mu q^\nu q^\rho q^\sigma)-4yz[2a^5+56(x-1)^2\\
&\times x^2(1-x+x^2)^2y^3(1-z)^2z^3-a^4(9(x-1)x(1-z)+2(3-7x+7x^2)yz)+2a(x-1)xy^2(1-z)z^2((x-1)x(-35+29x\\
&-17x^2-24x^3+12x^4)(1-z)+(14-45x+73x^2-56x^3+28x^4)yz)+a^2(x-1)xy(1-z)z(10(x-1)x(5-6x+6x^2)\\
&\times(1-z)+(-38+31x+9x^2-80x^3+40x^4)yz)+2a^3(-3(x-1)^2x^2(1-z)^2+(x-1)x(17-18x+18x^2)y(1-z)z\\
&+(2-3x+11x^2-16x^3+8x^4)y^2z^2)](p^\beta p^\rho q^\alpha q^\mu q^\nu q^\sigma+p^\alpha p^\rho q^\beta q^\mu q^\nu q^\sigma+p^\beta p^\sigma q^\alpha q^\mu q^\nu q^\rho+p^\alpha p^\sigma q^\beta q^\mu q^\nu q^\rho)-8yz[a^5(5-6x\\
&+6x^2)+28(x-1)^2x^2(1-x+x^2)^2y^3(1-z)^2z^3+2a^4((x-1)x(7-x+x^2)(1-z)+(-21+34x-30x^2-8x^3+4x^4)\\
&\times yz)+2a(x-1)xy^2(1-z)z^2(3(x-1)x(-7+6x-2x^2-8x^3+4x^4)(1-z)+(14-45x+73x^2-56x^3+28x^4)yz)\\
&+a^3(2(x-1)^2x^2(3+2x-2x^2)(1-z)^2+(x-1)x(-47+107x-91x^2-32x^3+16x^4)y(1-z)z+(53-50x+30x^2\\
&+40x^3-20x^4)y^2z^2)+a^2yz((x-1)^2x^2(-30+87x-79x^2-16x^3+8x^4)(1-z)^2+(x-1)x(11-30x+34x^2-8x^3\\
&+4x^4)y(1-z)z+2(-8-11x+25x^2-28x^3+14x^4)y^2z^2)]p^\alpha p^\beta q^\mu q^\nu q^\rho q^\sigma-8[a^6+28(x-1)^3x^3(1-x+x^2)^2y^3(1-z)^3\\
&\times z^3+a^5(2(x-1)x(1-x+x^2)(1-z)+(-2-5x+5x^2)yz)+2a(x-1)^2x^2y^2(1-z)^2z^2((x-1)x(-14+4x+15x^2\\
&-38x^3+19x^4)(1-z)+(14-45x+73x^2-56x^3+28x^4)yz)+a^4((1-2x)^2(x-1)^2x^2(1-z)^2+(x-1)x(7-8x+8x^2)\\
&\times y(1-z)z+(1+8x-16x^3+8x^4)y^2z^2)+a^2(x-1)xy(1-z)z((x-1)^2x^2(-1+12x-12x^2)(1-z)^2+3(x-1)x\\
&\times(-1-11x+39x^2-56x^3+28x^4)y(1-z)z+2(-8-11x+25x^2-28x^3+14x^4)y^2z^2)+a^3(x-1)x(1-z)(2(x-1)^2\\
&\times x^2(1-x+x^2)(1-z)^2+(x-1)x(10+9x-9x^2)y(1-z)z+(18-25x+79x^2-108x^3+54x^4)y^2z^2)](p^\alpha p^\beta p^\rho q^\mu q^\nu q^\sigma\\
&+p^\alpha p^\beta p^\sigma q^\mu q^\nu q^\rho)+8y^2z^2[8a^4(1-2x+2x^2)-28(x-1)x(1-x+x^2)^2y^3(1-z)z^3+ay^2z^2((x-1)x(49-88x+142x^2\\
&-108x^3+54x^4)(1-z)+(-14+45x-73x^2+56x^3-28x^4)yz)+a^3(12(x-1)x(1-x+x^2)(1-z)+(-31+88x-104x^2\\
&+32x^3-16x^4)yz)+2a^2yz((x-1)x(-16+21x-29x^2+16x^3-8x^4)(1-z)+(17-51x+69x^2-36x^3+18x^4)yz)]\\
&\times(p^\nu q^\alpha q^\beta q^\mu q^\rho q^\sigma+p^\mu q^\alpha q^\beta q^\nu q^\rho q^\sigma)-4yz[2a^5(6-11x+11x^2)+56(x-1)^2x^2(1-x+x^2)^2y^3(1-z)^2z^3+a^2(x-1)x\\
&\times y(1-z)z((x-1)x(68-113x+129x^2-32x^3+16x^4)(1-z)+(-26+117x-213x^2+192x^3-96x^4)yz)+a^4((x-1)\\
&\times x(-11+52x-52x^2)(1-z)+2(-4+7x+x^2-16x^3+8x^4)yz)+2a(x-1)xy^2(1-z)z^2(-5(-1+x)x(7-12x+20x^2\\
&-16x^3+8x^4)(1-z)+(14-45x+73x^2-56x^3+28x^4)yz)+a^3(-2(x-1)^2x^2(12-37x+37x^2)(1-z)^2+(x-1)x\\
&\times(27-31x+63x^2-64x^3+32x^4)y(1-z)z-2(2-3x+11x^2-16x^3+8x^4)y^2z^2)](p^\nu p^\rho q^\alpha q^\beta q^\mu q^\sigma+p^\nu p^\sigma q^\alpha q^\beta q^\mu q^\rho\\
&+p^\mu p^\rho q^\alpha q^\beta q^\nu q^\sigma+p^\mu p^\sigma q^\alpha q^\beta q^\nu q^\rho)-4yz[-6a^5(3-5x+5x^2)+56(x-1)^2x^2(1-x+x^2)^2y^3(1-z)^2z^3+a^4((x-1)x\\
&\times(-11-32x+32x^2)(1-z)+2(27-44x+52x^2-16x^3+8x^4)yz)+2a(x-1)xy^2(1-z)z^2((x-1)x(-42+67x-95x^2\\
&+56x^3-28x^4)(1-z)+2(14-45x+73x^2-56x^3+28x^4)yz)+a^2yz((x-1)^2x^2(7+36x-20x^2-32x^3+16x^4)(1-z)^2\\
&-8(x-1)x(13-29x+43x^2-28x^3+14x^4)y(1-z)z+4(10-21x+35x^2-28x^3+14x^4)y^2z^2)-2a^3((x-1)^2x^2(-3\\
&+31x-31x^2)(1-z)^2+(x-1)x(-21+17x-33x^2+32x^3-16x^4)y(1-z)z+(35-57x+85x^2-56x^3+28x^4)y^2z^2)]\\
&\times(p^\beta p^\nu q^\alpha q^\mu q^\rho q^\sigma+p^\alpha p^\nu q^\beta q^\mu q^\rho q^\sigma+p^\beta p^\mu q^\alpha q^\nu q^\rho q^\sigma+p^\alpha p^\mu q^\beta q^\nu q^\rho q^\sigma)-4[56(x-1)^3x^3(1-x+x^2)^2y^3(1-z)^3z^3+2a^5\\
&\times((x-1)^2x^2(1-z)+(1-x+x^2)yz)-a^4(x-1)x(1-z)((x-1)x(1-z)+4(x-1)x^2(1-z)-4(x-1)x^3(1-z)\\
&+(-10-31x+31x^2)yz)+2a(x-1)^2x^2y^2(1-z)^2z^2((x-1)x(-28+39x-53x^2+28x^3-14x^4)(1-z)+2(14-45x\\
&+73x^2-56x^3+28x^4)yz)+a^2(x-1)xy(1-z)z((x-1)^2x^2(37-41x+41x^2)(1-z)^2-2(x-1)x(38-71x+99x^2\\
&-56x^3+28x^4)y(1-z)z+4(10-21x+35x^2-28x^3+14x^4)y^2z^2)+a^3(x-1)x(1-z)((x-1)^2x^2(-5-2x+2x^2)\\
&\times(1-z)^2+8(x-1)x(6-x+x^2)y(1-z)z-2(25-36x+50x^2-28x^3+14x^4)y^2z^2)](p^\beta p^\nu p^\rho q^\alpha q^\mu q^\sigma+p^\alpha p^\nu p^\rho q^\beta q^\mu q^\sigma\\
&+p^\beta p^\nu p^\sigma q^\alpha q^\mu q^\rho+p^\alpha p^\nu p^\sigma q^\beta q^\mu q^\rho+p^\beta p^\mu p^\rho q^\alpha q^\nu q^\sigma+p^\alpha p^\mu p^\rho q^\beta q^\nu q^\sigma+p^\beta p^\mu p^\sigma q^\alpha q^\nu q^\rho+p^\alpha p^\mu p^\sigma q^\beta q^\nu q^\rho)+4[4a^6(1-x\\
&+x^2)-56(x-1)^3x^3(1-x+x^2)^2y^3(1-z)^3z^3+2a^5((x-1)x(3-8x+8x^2)(1-z)+(2-21x+13x^2+16x^3-8x^4)\\
&\times yz)+2a(x-1)^2x^2y^2(1-z)^2z^2((x-1)x(35-46x+48x^2-4x^3+2x^4)(1-z)-3(14-45x+73x^2-56x^3+28x^4)\\
&\times yz)+a^4(4(x-1)^2x^2(1-5x+5x^2)(1-z)^2+(x-1)x(61-104x+56x^2+96x^3-48x^4)y(1-z)z+2(1+9x+19x^2\\
&-56x^3+28x^4)y^2z^2)-2a^2(x-1)xy(1-z)z((x-1)^2x^2(-29+75x-67x^2-16x^3+8x^4)(1-z)^2+(x-1)x(-35+66x
\end{aligned}
$$

21 pages

Why does the UV-free scheme seem effective for power-law divergences and give another picture?



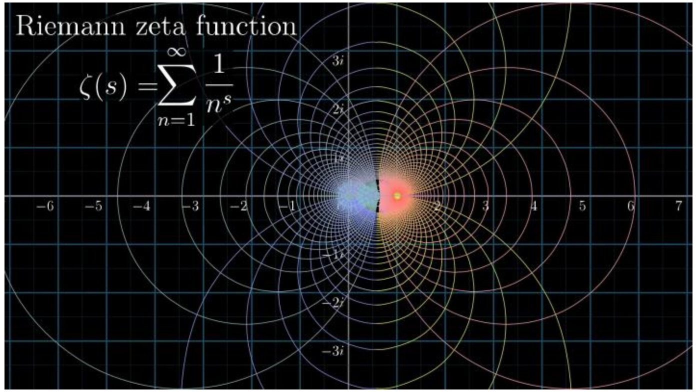
Two alternative routes of concern

The hierarchy problem

- (a) New particles (TeV) needed to cancel out UV contributions of loops to the Higgs mass
- **(b) An interpretation within SM**

SELECTED BY NATURE

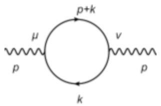
(a) *Equivalent transformation* of the loop integral from UV divergence to UV divergence mathematically expressed form (regularization), with renormalization required to remove the UV divergence.
(b) *Analytic continuation* of the transition amplitude from UV divergent $\mathcal{T}_{\rm F}$ to UV converged $\mathcal{T}_{\rm P}$ (the UV-free scheme here), without UV divergences in calculations.

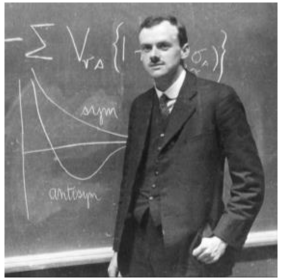


UV-free scheme — *Analytic continuation*

$$\mathcal{T}_{\rm F} \Rightarrow \mathcal{T}_{\rm P} = \left[\int (d\xi)^n \frac{\partial^n \mathcal{T}_{\rm F}(\xi)}{\partial \xi^n}\right]_{\xi\to 0} + C,$$

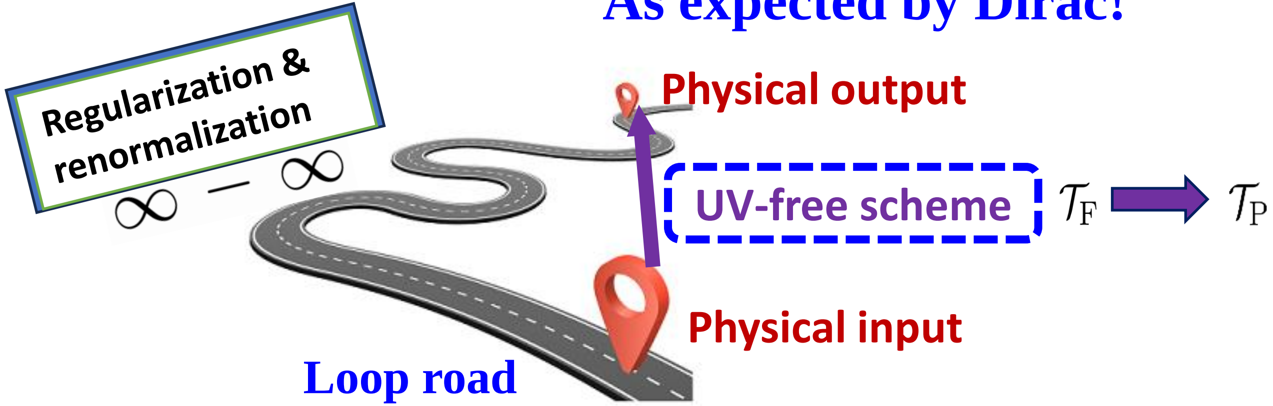
Finite input		UV divergence input (continuation)	
Tree level	Loop finite	Loop Log	Loop $\Lambda^2, \Lambda^4, \Lambda^6, \ldots$
Originally well-defined		Verified	To be verified





P. A. M. Dirac I believe the successes of the renormalization theory will be on the same footing as the successes of the Bohr orbit theory applied to one-electron problems.

As expected by Dirac!



Schemes	Tree level	Loop finite	Loop Log	Loop $\Lambda^2, \Lambda^4, \Lambda^6, \ldots$
Regularization & renormalization			OK	Problematic
UV-free scheme	OK	OK	OK	OK

Both loops of the renormalizable Standard Model and non-renormalizable Einstein gravity being OK!

V. Summary and outlook

殊途同归

Different roads
lead to the same goal

A. An alternative method --- UV-free scheme:
Finite loop results obtained without UV divergences, the original γ^5 matrix, and effective for loop Log and power−law divergence inputs.

B. To the hierarchy problem of the 125 GeV Higgs, an alternative interpretation without fine-tuning within SM.

C. It is possible to incorporate Einstein gravity into the framework of QFT.

Outlook:

It is the beginning of a new alternative method.

Thank you!